Reservoir Computing: From Classical Edge of Chaos to Quantum Dynamical Phases

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Abstract

Reservoir computing is a temporal machine-learning architecture particularly suitable for learning and predicting complex nonlinear dynamical systems. A classical reservoir computer is typically trained to act as a self-evolving dynamical system and its performance can often be maximized at the "edge of chaos." Extending this paradigm to quantum systems reveals a broader landscape of physical phenomena. Bridging machine-learning theory and quantum condensed matter physics, this Perspective synthesizes a unifying framework where the computational power of both classical and quantum reservoirs is fundamentally governed by their dynamical phases. In particular, how quantum phases, such as many-body localized versus ergodic, dictate memory-nonlinearity trade-offs is analyzed. A roadmap for parameter-adaptable quantum AI, transforming encoding, readout, and noise bottlenecks into scalable computational resources is outlined.

1 Introduction: Need for a Unified Framework Bridging Quantum Hardware and Algorithmic Performance

Reservoir computing [1–5] provides a highly efficient framework for temporal machine learning, particularly for capturing complex, nonlinear structures in time-series data. In conventional or "classical" reservoir computing, a nonlinear dynamical network within a hidden layer is entirely responsible for information processing and computing. By embedding a low-dimensional input into a high-dimensional state space via the reservoir's inherent dynamics, the neural machine achieves sufficient memory and nonlinear mixing, allowing open-loop training to be reduced to fitting a simple linear readout on these hidden states. Once trained, a reservoir computer can be wired to execute closed-loop, self dynamical evolution, acting as a "digital twin" of the target dynamical system [6]. The digital twin so created is capable of predicting the state evolution [7, 8] and various bifurcations of nonlinear dynamical systems [9, 10] including tipping in real-world physical systems [11].

Replacing this classical dynamical system with a quantum framework leads to quantum reservoir computing [12–19]. A driving motivation for this transition is compression [4, 20–22]: a quantum system can generate immensely rich, high-dimensional dynamics using only a relatively small number of physical units. Because a quantum system's Hilbert space, a high-dimensional mathematical feature space, grows exponentially with the number of units, it provides a vast wealth of dynamical degrees

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of freedom. While quantum evolution governed by the Schrödinger equation is fundamentally linear, this poses no contradiction to nonlinear machine learning. Akin to Koopman operator theory [23], the original nonlinear computational dynamics can be naturally interpreted as a linear evolution within this profoundly high-dimensional quantum feature space [24–30].

Translating the theoretical dimensionality of quantum reservoir computing into practical computational power remains an outstanding challenge. To bridge the gap between quantum hardware and algorithmic performance, a unified perspective is required. In classical reservoir computing, it has been established that computational capacity peaks at the "edge of chaos" [3, 31], a critical dynamical sweet spot balancing stable order and chaotic scrambling. Drawing a direct parallel, we synthesize recent findings into a structurally equivalent perspective for quantum reservoir computing: its computational success is fundamentally governed by its underlying quantum dynamical phase, making performance acutely sensitive to the operating Hamiltonian regime. This dependence arises from a fundamental memory trade-off inherent in quantum many-body dynamics. In particular, in the many-body localized phase, locally conserved quantities essentially trap information; while the system is able to retain memory of initial states well, it suffers from a crippling lack of information transport and slow convergence. On the flip side, as the system enters the ergodic (thermalized) phase, it becomes increasingly adept at spreading information and generating complex nonlinear mixing, fueled by quantum coherence and superposition [32]. However, deep ergodicity can also lead to excessive scrambling, which eventually erodes the linear memory required for temporal tracking [16, 33].

Consequently, while the onset of thermalization represents a theoretical "sweet spot" for balancing these competing memory demands [16, 18], the practical viability of quantum reservoir computing introduces further complexity. While the transition point is a theoretical peak, real-world performance requires navigating the ergodic regime where quantum effects boost processing power most effectively. In this regime, the richness of quantum coherence must be weighed against practical constraints such as statistical noise and finite sampling limitations [32]. Such experimental factors can obscure the reservoir's potential right at the phase transition, suggesting that the "operational" sweet spot for practical quantum reservoir computing may shift deeper into the ergodic regime to ensure robust, noise-resilient decoding.

2 The Classical Edge of Chaos

In classical reservoir computing, the computational capacity of the neural network is not a fixed property; rather, it is deeply tied to its operating dynamical regime. A foundational principle posits that these systems achieve optimal performance near critical phase transitions, a region colloquially known as the "edge of chaos" or the "edge of stability" [34–36]. This critical boundary represents a dynamical sweet spot balancing stable order and chaotic scrambling. If the reservoir's internal dynamics are too stable or ordered, input perturbations decay instantaneously. As a result, the network lacks the nonlinear mixing and prolonged temporal integration required to solve complex, context-dependent machine learning tasks. Conversely, if the system is pushed entirely into a chaotic regime, it suffers a critical breakdown of the "fading memory" property. In this state, the network becomes hypersensitive to initial conditions and internal instabilities, causing the memory of recent inputs to be rapidly scrambled and lost. By tuning the reservoir precisely to the edge of chaos, both information storage and transfer can be maximized, ensuring that the system retains a robust, fading trace of previous signals while effectively separating distinct inputs within a high-dimensional feature space.

Counterintuitively, this delicate balance does not require precise deterministic dynamics: noise can be constructive. Injecting optimal noise enhances classical reservoir training via stochastic resonance [37]. Treating noise amplitude as an optimizable hyperparameter improves prediction accuracy, stability, and forecasting horizons. This functional role of noise and instability at the critical boundaries of classical reservoirs provides a compelling conceptual bridge to the quantum realm. Since a quantum system is fundamentally probabilistic and continuously perturbed by measurement backaction, identifying the quantum analogs of the "edge of chaos" and stochastic resonance becomes the crucial next step in unlocking the computational power of quantum reservoir computers.

3 Quantum Dynamical Phases as Computational Resources

Transitioning reservoir computing to the quantum realm requires a fundamental shift in how we understand the "hidden layer." In classical neural networks, the computational capacity is dictated

by the network's topology, connectivity weights, and nonlinear activation functions. In quantum reservoir computing, these discrete architectural choices are superseded by the system's continuous dynamical evolution, which is entirely governed by its physical Hamiltonian. Consequently, a quantum reservoir's ability to learn and predict complex temporal patterns is sensitive to its overarching Hamiltonian regime.

One way to understand this is to view the quantum reservoir through the lens of condensed matter physics and statistical mechanics. The computational power of the reservoir is not static; it is intrinsically linked to the system's dynamical phase. Two contrasting phases illustrate this principle clearly: the many-body localized phase and the ergodic phase [38–40]. In the localized phase, quantum systems fail to thermalize, and information remains trapped in local degrees of freedom. From an information-processing perspective, this lack of transport is catastrophic, as the computational nodes (qubits or oscillators) will fail to interact sufficiently, preventing the robust spread of correlations needed to process extended time-series data. As a result, the many-body localized phase typically degrades the temporal processing capabilities of the reservoir. Conversely, when the Hamiltonian is tuned to an ergodic phase, the quantum system thoroughly and rapidly explores its available phase space. This ergodic exploration effectively acts as a powerful "mixing" mechanism. Just as a classical reservoir relies on recurrent nonlinear connections to blend past and present inputs into a rich feature state, the ergodic quantum reservoir uses fundamental quantum scrambling and entanglement growth to distribute input information across the entire system. Thus, the ergodic phase naturally enhances the reservoir's computational capacity, providing the vital memory and mixing required for complex sequence learning tasks. Natually, this computational gain will come with a critical trade-off: while deep ergodicity maximizes nonlinear mixing, it can also lead to excessive scrambling, eventually eroding the linear memory required to track long-term temporal dependencies. Furthermore, as the system explores its Hilbert space more thoroughly, the resulting quantum states become increasingly sensitive to finite sampling noise, potentially obscuring the reservoir's gains during the readout stage.

Attributing mixing to "nonlinearity" raises a paradox: quantum mechanics is strictly linear. How can a linear system solve highly nonlinear machine learning tasks? The resolution lies in Koopman operator theory [23], mapping finite-dimensional nonlinear dynamics to linear evolution in an infinite-dimensional space. Because quantum states evolve in an exponentially large or infinitely dimensional Hilbert space, quantum reservoirs naturally manifest this Koopman transformation. Thus, no contradiction exists: quantum linearity directly processes nonlinear classical data, provided the dynamical phase is optimally tuned to ensure ergodic mixing across this vast space.

4 The Information Interface: Encoding and Readout Through the Phase Lens

While the theoretical capacity of an ergodic quantum reservoir is vast, the practical realization of quantum reservoir computing faces a severe bottleneck at its interfaces: the encoding of classical input data into the quantum system, and the extraction of processed data via measurement readout [13, 41–45]. When viewed through the lens of dynamical phases, these input/output (I/O) mechanisms are not merely technical engineering challenges; they are fundamental physical perturbations that directly interact with, and potentially disrupt, the computational phase of the reservoir.

4.1 Encoding (Input): Perturbing the Hamiltonian

Historically, many quantum machine-learning frameworks rely on state encoding, wherein classical data is mapped to a carefully prepared initial quantum state [12, 46, 47]. However, state encoding requires arbitrary, high-fidelity state preparation, making it highly susceptible to decoherence and demanding immense control overhead. More importantly, it requires resetting or re-initializing the system, which severs the continuous temporal dynamics necessary for sequence learning. A far more robust paradigm for quantum reservoir computing is Hamiltonian encoding [45, 48, 49]. In this approach, classical input data directly modulates the parameters of the driving Hamiltonian, such as adjusting the detuning or driving amplitude of interacting qubits. While Hamiltonian encoding bypasses the need for complex state preparation, it introduces a critical phase-related vulnerability. Because the Hamiltonian governs the dynamical phase, dynamically altering its parameters to encode data means the input signal acts as a continuous perturbation. If the encoding strength (the amplitude of the data modulation) is too high, it risks pushing the Hamiltonian out of its optimal ergodic regime. The system could be inadvertently forced into a localized phase, or conversely, driven into a regime of excessive heating and decoherence. As a result, optimal encoding must be bounded by the phase diagram of the reservoir, ensuring that the modulated Hamiltonian remains strictly within the ergodic sweet spot.

4.2 Decoding (Readout): Navigating the Ergodic Spread

The principal bottleneck in quantum reservoir computing is arguably the readout phase. The very feature that makes the ergodic phase computationally powerful, the rapid scrambling and spreading of information across a massive Hilbert space, makes extracting that information exceptionally difficult [12, 50]. In standard quantum mechanics, applying traditional projective measurements to extract this data causes an immediate wavefunction collapse, instantaneously forcing the probabilistic quantum state into a single, fixed outcome, thereby erasing its memory. For a reservoir processing sequential time-series data, projective collapse is fatal: it instantaneously destroys the ongoing quantum dynamics, erasing the system's memory and resetting the reservoir.

To circumvent this and maintain the computational phase, the reservoir must be probed using weak continuous measurements [51, 52]. By weakly coupling the reservoir to the measurement apparatus [45], one obtains a continuous, noisy measurement record (such as homodyne detection, which extracts information from wave interference) without forcing the system into a single fixed state. Weak measurements gently perturb the system rather than destroying its superposition, allowing the reservoir to remain in its ergodic phase while continuously processing incoming data. However, the trade-off is the signal-to-noise ratio [53] in the measurement current. Consequently, the readout matrix must be highly optimized to filter this inherent quantum noise, identifying the fading traces of classical information hidden within the complex, surviving quantum dynamics.

5 Outlook: Designing Phase-Adaptable Reservoirs

The transition from viewing a quantum reservoir as a static, black-box neural network to a dynamic, phase-governed physical system opens an exciting frontier for interdisciplinary research. If the dynamical phase is indeed the primary engine of a quantum reservoir's computational power, then the next generation of quantum reservoir-computing hardware and algorithmic frameworks must actively manage this phase. As the field matures, we articulate two critical, actionable research directions for the physics and AI communities to bridge the gap between theoretical potential and practical deployment:

5.1 Parameter-Adaptable Quantum Reservoirs

In classical machine learning, "parameter-adaptable" reservoir computing has been successfully developed and deployed to track changing system parameters [54, 55], predict critical phase transitions [9], and anticipate tipping points [11] in complex, non-stationary dynamical systems. An urgent open question is whether this paradigm can be successfully ported to the quantum domain. Can we develop "parameter-adaptable" quantum reservoir computers? Rather than operating a quantum reservoir computer with a fixed, static Hamiltonian, future architectures could employ closed-loop feedback mechanisms that dynamically tune the Hamiltonian's internal parameters (such as qubit detuning or pairwise coupling strengths) on the fly during the training process. By actively steering the system's evolution, the reservoir could continuously self-correct its own dynamical phase, ensuring it remains robustly anchored in the optimal ergodic regime, the quantum "edge of chaos," regardless of the potentially disruptive perturbations introduced by the classical input data encoding.

5.2 Exploiting Quantum Noise via Stochastic Resonance

The role of noise in quantum reservoir computing must be fundamentally re-evaluated. In classical reservoir computing, it is well established that noise is not universally detrimental. Through the mechanism of stochastic resonance, an optimal level of injected noise can actively enhance the training process, boosting weak signal detection and overall prediction accuracy. Quantum systems are inherently noisy, heavily subjected to decoherence, spontaneous emission, and measurement backaction. An open question remains: can the inherent randomness and noise of continuous quantum measurements be actively exploited to benefit training, mirroring the classical stochastic resonance phenomenon? If researchers can establish a mathematical and physical framework for quantum stochastic resonance within reservoir computing, it would represent a paradigm shift, transforming quantum noise from a severe engineering nuisance into a vital, performance-enhancing computational resource.

Ultimately, by treating quantum reservoirs as holistic dynamical systems where phase, encoding, readout, and noise are inextricably linked, the community can move beyond proof-of-concept models, paving the way for robust, scalable quantum AI architectures.

References

[1] H. Jaeger, "The "echo state" approach to analysing and training recurrent neural networks-with an erratum note," *Bonn, Germany: German national research center for information technology gmd technical report*, vol. 148, no. 34, p. 13, 2001.

[2] W. Maass, T. Natschläger, and H. Markram, "Real-time computing without stable states: A new framework for neural computation based on perturbations," *Neural Comput.*, vol. 14, no. 11, pp. 2531–2560, 2002.

[3] M. Lukoševičius and H. Jaeger, "Reservoir computing approaches to recurrent neural network training," *Comput. Sci. Rev.*, vol. 3, no. 3, pp. 127–149, 2009.

[4] L. Appeltant, M. C. Soriano, G. Van der Sande, J. Danckaert, S. Massar, J. Dambre, B. Schrauwen, C. R. Mirasso, and I. Fischer, "Information processing using a single dynamical node as complex system," *Nat. Commun.*, vol. 2, no. 1, p. 468, 2011.

[5] M. Yan, C. Huang, P. Bienstman, P. Tino, W. Lin, and J. Sun, "Emerging opportunities and challenges for the future of reservoir computing," *Nat. Commun.*, vol. 15, no. 1, p. 2056, 2024.

[6] L.-W. Kong, Y. Weng, B. Glaz, M. Haile, and Y.-C. Lai, "Reservoir computing as digital twins for nonlinear dynamical systems," *Chaos*, vol. 33, p. 033111, 2023.

[7] Z. Lu, J. Pathak, B. Hunt, M. Girvan, R. Brockett, and E. Ott, "Reservoir observers: Model-free inference of unmeasured variables in chaotic systems," *Chaos*, vol. 27, p. 041102, 2017.

[8] J. Pathak, B. Hunt, M. Girvan, Z. Lu, and E. Ott, "Model-free prediction of large spatiotemporally chaotic systems from data: A reservoir computing approach," *Phys. Rev. Lett.*, vol. 120, p. 024102, Jan 2018.

[9] L.-W. Kong, H.-W. Fan, C. Grebogi, and Y.-C. Lai, "Machine learning prediction of critical transition and system collapse," *Phys. Rev. Res.*, vol. 3, no. 1, p. 013090, 2021.

[10] S. Panahi, L.-W. Kong, B. Glaz, M. Haile, and Y.-C. Lai, "Unsupervised learning for anticipating critical transitions," *Phys. Rev. Lett.*, vol. 136, p. 077301, Feb 2026.

[11] S. Panahi, L.-W. Kong, M. Moradi, Z.-M. Zhai, B. Glaz, M. Haile, and Y.-C. Lai, "Machine learning prediction of tipping in complex dynamical systems," *Phys. Rev. Res.*, vol. 6, p. 043194, Nov 2024.

[12] K. Fujii and K. Nakajima, "Harnessing disordered-ensemble quantum dynamics for machine learning," *Phys. Rev. Appl.*, vol. 8, no. 2, p. 024030, 2017.

[13] G. Angelatos, S. A. Khan, and H. E. Türeci, "Reservoir computing approach to quantum state measurement," *Phys. Rev. X*, vol. 11, p. 041062, Dec 2021.

[14] J. Chen and H. Nurdin, "Learning nonlinear input–output maps with dissipative quantum systems," *Quantum Inf. Process*, vol. 18, p. 198, 2019.

[15] J. Chen, H. I. Nurdin, and N. Yamamoto, "Temporal information processing on noisy quantum computers," *Phys. Rev. Appl.*, vol. 14, p. 024065, Aug 2020.

[16] R. Martínez-Peña, G. L. Giorgi, J. Nokkala, M. C. Soriano, and R. Zambrini, "Dynamical phase transitions in quantum reservoir computing," *Phys. Rev. Lett.*, vol. 127, p. 100502, Aug 2021.

[17] S. Ghosh, T. Krisnanda, T. Paterek, and T. C. H. Liew, "Realising and compressing quantum circuits with quantum reservoir computing," *Commun. Phys.*, vol. 4, p. 105, 2021.

[18] W. Xia, J. Zou, X. Qiu, and X. Li, "The reservoir learning power across quantum many-body localization transition," *Front. Phys.*, vol. 17, p. 33506, 2022.

[19] L. Domingo, G. Carlo, and F. Borondo, "Optimal quantum reservoir computing for the noisy intermediate-scale quantum era," *Phys. Rev. E*, vol. 106, p. L043301, Oct 2022.

[20] D. Woods and T. J. Naughton, "Photonic neural networks," *Nat. Phys.*, vol. 8, no. 4, pp. 257–259, 2012.

[21] L. Larger, M. C. Soriano, D. Brunner, L. Appeltant, J. M. Gutiérrez, L. Pesquera, C. R. Mirasso, and I. Fischer, "Photonic information processing beyond turing: An optoelectronic implementation of reservoir computing," *Opt. Express*, vol. 20, no. 3, pp. 3241–3249, 2012.

[22] Y. Paquot, F. Duport, A. Smerieri, J. Dambre, B. Schrauwen, M. Haelterman, and S. Massar, "Optoelectronic reservoir computing," *Sci. Rep.*, vol. 2, no. 1, p. 287, 2012.

[23] B. O. Koopman, "Hamiltonian systems and transformations in Hilbert space," *Proc. Nat. Acad. Sci. (USA)*, vol. 17, pp. 315–318, 1931.

[24] E. Wigner, "On the quantum correction for thermodynamic equilibrium," *Phys. Rev.*, vol. 40, pp. 749–759, Jun 1932.

[25] G. A. Baker, "Formulation of quantum mechanics based on the quasi-probability distribution induced on phase space," *Phys. Rev.*, vol. 109, pp. 2198–2206, Mar 1958.

[26] J. Wilkie and P. Brumer, "Quantum-classical correspondence via liouville dynamics. i. integrable systems and the chaotic spectral decomposition," *Phys. Rev. A*, vol. 55, pp. 27–42, Jan 1997.

[27] D. I. Bondar, R. Cabrera, R. R. Lompay, M. Y. Ivanov, and H. A. Rabitz, "Operational dynamic modeling transcending quantum and classical mechanics," *Phys. Rev. Lett.*, vol. 109, p. 190403, Nov 2012.

[28] D. I. Bondar, R. Cabrera, and H. A. Rabitz, "Conceptual inconsistencies in finite-dimensional quantum and classical mechanics," *Phys. Rev. A*, vol. 88, p. 012116, Jul 2013.

[29] G. McCaul, A. Pechen, and D. I. Bondar, "Entropy nonconservation and boundary conditions for hamiltonian dynamical systems," *Phys. Rev. E*, vol. 99, p. 062121, Jun 2019.

[30] G. McCaul, D. V. Zhdanov, and D. I. Bondar, "Wave operator representation of quantum and classical dynamics," *Phys. Rev. A*, vol. 108, p. 052208, Nov 2023.

[31] N. Bertschinger, T. Natschläger, and R. Legenstein, "At the edge of chaos: Real-time computations and self-organized criticality in recurrent neural networks," *Adv. Neural Inf. Process. Syst.*, vol. 17, 2004.

[32] A. Palacios, R. Martínez-Peña, M. C. Soriano, G. L. Giorgi, and R. Zambrini, "Role of coherence in many-body quantum reservoir computing," *Commun. Phys.*, vol. 7, no. 1, p. 369, 2024.

[33] J. Dambre, D. Verstraeten, B. Schrauwen, and S. Massar, "Information processing capacity of dynamical systems," *Sci. Rep.*, vol. 2, no. 1, p. 514, 2012.

[34] S. Boyd and L. Chua, "Fading memory and the problem of approximating nonlinear operators with volterra series," *IEEE Trans. Circuits Syst.*, vol. 32, no. 11, pp. 1150–1161, 2003.

[35] J. Boedecker, O. Obst, J. T. Lizier, N. M. Mayer, and M. Asada, "Information processing in echo state networks at the edge of chaos," *Theor. Biosci.*, vol. 131, no. 3, pp. 205–213, 2012.

[36] T. L. Carroll, "Do reservoir computers work best at the edge of chaos?," *Chaos*, vol. 30, no. 12, 2020.

[37] Z.-M. Zhai, M. Moradi, L.-W. Kong, B. Glaz, M. Haile, and Y.-C. Lai, "Model-free tracking control of complex dynamical trajectories with machine learning," *Nat. Commun.*, vol. 14, no. 1, p. 5698, 2023.

[38] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, "Metal–insulator transition in a weakly interacting many-electron system with localized single-particle states," *Ann. Phys.*, vol. 321, no. 5, pp. 1126–1205, 2006.

[39] A. Pal and D. A. Huse, "Many-body localization phase transition," *Phys. Rev. B*, vol. 82, no. 17, p. 174411, 2010.

[40] J. M. Deutsch, "Quantum statistical mechanics in a closed system," *Phys. Rev. A*, vol. 43, no. 4, p. 2046, 1991.

[41] S. Ghosh, A. Opala, M. Matuszewski, T. Paterek, and T. C. H. Liew, "Quantum reservoir processing," *npj Quantum Info.*, vol. 5, p. 35, 2019.

[42] J. Nokkala, R. Martínez-Pena, G. L. Giorgi, V. Parigi, M. C. Soriano, and R. Zambrini, "Gaussian states of continuous-variable quantum systems provide universal and versatile reservoir computing," *Commun. Phys.*, vol. 4, p. 53, 2021.

[43] R. A. Bravo, K. Najafi, X. Gao, and S. F. Yelin, "Quantum reservoir computing using arrays of rydberg atoms," *PRX Quantum*, vol. 3, p. 030325, Aug 2022.

[44] J. García-Beni, G. L. Giorgi, M. C. Soriano, and R. Zambrini, "Scalable photonic platform for real-time quantum reservoir computing," *Phys. Rev. Appl.*, vol. 20, p. 014051, Jul 2023.

[45] C. Zhu, P. J. Ehlers, H. I. Nurdin, and D. Soh, "Practical few-atom quantum reservoir computing," *Phys. Rev. Res.*, vol. 7, p. 023290, Jun 2025.

[46] K. Nakajima, K. Fujii, M. Negoro, K. Mitarai, and M. Kitagawa, "Boosting computational power through spatial multiplexing in quantum reservoir computing," *Phys. Rev. Appl.*, vol. 11, no. 3, p. 034021, 2019.

[47] S. Čindrak, B. Donvil, K. Lüdge, and L. Jaurigue, "Enhancing the performance of quantum reservoir computing and solving the time-complexity problem by artificial memory restriction," *Phys. Rev. Res.h*, vol. 6, no. 1, p. 013051, 2024.

[48] L. C. G. Govia, G. J. Ribeill, G. E. Rowlands, H. K. Krovi, and T. A. Ohki, "Quantum reservoir computing with a single nonlinear oscillator," *Phys. Rev. Res.*, vol. 3, p. 013077, Jan 2021.

[49] G. McCaul, J. S. Totero Gongora, W. Otieno, S. Savel'ev, A. Zagoskin, and A. G. Balanov, "Minimal quantum reservoirs with hamiltonian encoding," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 35, no. 9, 2025.

[50] K. A. Landsman, C. Figgatt, T. Schuster, N. M. Linke, B. Yoshida, N. Y. Yao, and C. Monroe, "Verified quantum information scrambling," *Nature*, vol. 567, no. 7746, pp. 61–65, 2019.

[51] K. Jacobs and D. A. Steck, "A straightforward introduction to continuous quantum measurement," *Contemp. Phys.*, vol. 47, no. 5, pp. 279–303, 2006.

[52] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control*. Cambridge, UK: Cambridge University Press, 2009.

[53] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*. New York: Oxford University Press, 2002.

[54] L.-W. Kong, G. A. Brewer, and Y.-C. Lai, "Reservoir-computing based associative memory and itinerancy for complex dynamical attractors," *Nat. Commun.*, vol. 15, no. 1, p. 4840, 2024.

[55] Z.-M. Zhai, M. Moradi, B. Glaz, M. Haile, and Y.-C. Lai, "Machine-learning parameter tracking with partial state observation," *Phys. Rev. Res.*, vol. 6, no. 1, p. 013196, 2024.