Learning to Destroy and Repair Local Optima in Multilinear Graph QUBO Optimization

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Abstract

Graph combinatorial optimization problems such as Maximum Independent Set and Maximum Cut can be written as pseudo-Boolean quadratic objectives with multilinear polynomial forms. Keeping the same multilinear objective while relaxing the variable domain from $\{0,1\}^n$ to $[0,1]^n$ preserves all 0–1 global minimizers, and every strict local minimizer of the relaxed objective is still 0–1. This exactness makes it possible to use continuous gradient-based optimization while still targeting discrete solutions. However, the relaxed landscape remains nonconvex: direct gradient optimizers such as Adam may require many iterations to stabilize near a local optimum, while amortized neural solvers trained on the relaxation can be faster but may still settle at suboptimal binary solutions. We propose LDR, a learned destroy-and-repair framework for improving such solutions. A recurrent repair network is trained by the multilinear quadratic unconstrained binary optimization (QUBO) relaxation to construct and repair continuous states that are decoded into feasible solutions of the original discrete problem. A destroy policy then masks part of the repaired continuous state before repair is applied again, and is trained with the repair network as its environment using post-repair improvement in the decoded discrete objective as a graph-level reward. The policy learns a graph- and state-dependent destruction rate; at inference time, LDR samples a random destruction mask over the vertices eligible to be perturbed using that learned rate. The repair and destroy modules can be alternated to calibrate the perturbation scale and expose repair to damaged continuous states it will see at inference time. We give a possible explanation for why destroy-and-repair can help hard-exclusion tasks such as MIS, and evaluate the method on MIS and MaxCut. On these two tasks, destroy-and-repair improves the decoded solution produced by the base repair network, with additional inference cost that we report explicitly.

1 Introduction

Many graph combinatorial optimization problems can be expressed as quadratic objectives over binary variables. Maximum Independent Set (MIS) selects vertices with no selected edge, and Maximum Cut (MaxCut) partitions vertices to maximize the number or weight of crossing edges. These problems are NP-hard and arise in applications such as scheduling and network design, biological and social network analysis, statistical physics, and circuit layout (Garey & Johnson, 1979; Nemhauser & Wolsey, 1988; Korte & Vygen, 2018; Barahona et al., 1988). They also serve as benchmark tasks for both classical combinatorial optimization and neural optimization methods (Bengio et al., 2021). Maximum Clique has the same hard-exclusion form as MIS after taking the complement graph; we mention this equivalence for completeness but do not report separate MaxClique experiments.

Neural combinatorial optimization has developed along several lines, including supervised construction policies, reinforcement-learning solvers, unsupervised energy minimization, GFlowNet-style samplers, and diffusion-based solvers (Khalil et al., 2017; Ahn et al., 2020; Qiu et al., 2022; Zhang et al., 2023; Sun & Yang, 2023; Sanokowski et al., 2024; 2025). The unsupervised line is particularly relevant here because it does not require labeled optimal solutions: Erdős Goes Neural learns distributions over feasible sets (Karalias & Loukas, 2020), physics-inspired and principled-relaxation methods train GNNs directly from differentiable

graph energies (Schuetz et al., 2022; Wang et al., 2022), and recent diffusion-style methods optimize or sample discrete solutions without supervised labels (Sanokowski et al., 2024; 2025). We focus on pseudo-Boolean QUBO objectives whose natural relaxation is the multilinear extension of the discrete objective. This extension has a useful exactness property: the discrete and relaxed problems share the same 0–1 global minimizers, and strict local minima of the relaxed objective are still binary. In principle, one could apply direct gradient optimization such as Adam to the relaxed variables, but many iterations may be needed before the trajectory stabilizes near a local optimum. Amortized neural solvers reduce this inference cost by learning a graph-to-solution map, yet preserving optima does not imply finding them: solvers trained on relaxed objectives can repeatedly converge to stable binary or near-binary local solutions that are still suboptimal.

The obstacle addressed in this paper is the structure of improvement in the relaxed QUBO landscape. For hard-exclusion tasks, a maximal independent set cannot be improved by adding a single vertex; selected vertices must be removed before alternative unselected vertices can enter. For MaxCut, all binary assignments are feasible, but improving a local partition can still require coordinated changes to several correlated vertices. In both cases, repeatedly decoding the same neural preference or applying undirected random perturbations can waste computation on changes that the repair solver immediately reverses.

We propose LDR (**L**earned **D**estroy and **R**epair), an unsupervised framework for graph QUBO optimization. The repair network constructs a local continuous state from noise and repairs damaged states, using the original multilinear QUBO relaxation as its training objective. Since the multilinear objective has no strict fractional local minimizer, stabilized repair outputs are encouraged to polarize before decoding. The destroy network observes the graph, current continuous state, and search step, predicts one Bernoulli destruction probability per node, and is trained by post-repair score improvement. Because this reward is graph-level, default inference averages these probabilities over eligible vertices to obtain a learned destruction rate, then samples a random mask at that rate.

This paper makes the following contributions:

- We formulate iterative improvement for graph QUBO solvers under the original discrete task score, covering hard-exclusion subset selection and unconstrained partitioning.
- We introduce LDR, a learned destroy-and-repair framework that trains repair by a multilinear continuous objective and trains destroy by post-repair improvement rewards.
- We analyze why multilinear relaxations still produce difficult nonconvex landscapes and provide MIS and MaxCut experiments with explicit runtime and ablations.

2 Background and Problem Formulation

2.1 Graph QUBO Objectives

Let $G = (V, E)$ be an undirected graph with $n = |V|$. A binary vector $\boldsymbol{x} \in \{0,1\}^n$ indicates which vertices are selected.

For MIS, we use the standard penalized QUBO energy

$$H_\lambda^{\mathrm{MIS}}(\boldsymbol{x}) = \lambda \sum_{(i,j)\in E} x_i x_j - \sum_{i\in V} x_i, \qquad \lambda > 1. \tag{1}$$

The linear term rewards large selected sets, while the quadratic term penalizes selecting adjacent vertices. When $\lambda > 1$, every infeasible selected edge can be improved by deleting one endpoint, so minimizers correspond to maximum independent sets.

MaxClique is the same hard-exclusion objective on the complement graph. Let $\bar{E} = \{(i,j) : i < j, (i,j) \notin E\}$ be the set of missing edges. The corresponding objective is

$$H_\lambda^{\mathrm{MaxClique}}(\boldsymbol{x}) = \lambda \sum_{(i,j)\in \bar{E}} x_i x_j - \sum_{i\in V} x_i, \qquad \lambda > 1. \tag{2}$$

This has the same mathematical form as MIS after replacing conflicts by missing edges. We include this formulation to clarify the scope of the analysis, but the present empirical section reports MIS and MaxCut only.

For MaxCut, the binary variable has a different meaning: x_i denotes the side of the cut containing vertex i. For edge weights $w_{ij} \geq 0$, the cut value is

$$C^{\text{MaxCut}}(\boldsymbol{x}) = \sum_{(i,j)\in E} w_{ij} \left[x_i(1-x_j) + x_j(1-x_i)\right]. \tag{3}$$

We minimize the negative cut value,

$$H^{\text{MaxCut}}(\boldsymbol{x}) = -\sum_{(i,j)\in E} w_{ij} \left[x_i(1-x_j) + x_j(1-x_i)\right], \tag{4}$$

which is also a quadratic pseudo-Boolean objective. Unlike MIS, every binary assignment is feasible for MaxCut; the challenge is optimizing a partition objective rather than satisfying hard exclusion constraints.

2.2 Multilinear Continuous Relaxation

We relax the discrete objective directly by replacing binary variables with continuous variables $\boldsymbol{y} \in [0,1]^n$ in its multilinear pseudo-Boolean form. If a QUBO is written with diagonal quadratic terms, those terms are first absorbed into the linear part using $x_i^2 = x_i$ on $\{0,1\}^n$. For MIS, the resulting relaxation is

$$H_\lambda^{\text{MIS}}(\boldsymbol{y}) = \lambda \sum_{(i,j)\in E} y_i y_j - \sum_{i\in V} y_i, \qquad \boldsymbol{y} \in [0,1]^n. \tag{5}$$

The same replacement applies to MaxCut using Eq. 4. More generally, consider a pseudo-Boolean objective whose polynomial representation has been written in multilinear form on $\{0,1\}^n$. Its continuous relaxation is the multilinear extension

$$F(\boldsymbol{y}) = \mathbb{E}_{\boldsymbol{Z}\sim\prod_i \text{Bern}(y_i)} \left[H(\boldsymbol{Z})\right]. \tag{6}$$

This probabilistic view is useful because F agrees with the discrete objective H on every binary vector and is affine in each coordinate when all other coordinates are fixed. These two facts imply that the relaxation preserves the binary optimum set, but they do not make the continuous problem convex. The following theorem records both the exactness property and the binary nature of strict local minima.

Theorem 1 (0–1 optima and strict local minima of multilinear relaxations)**.** *Let $H : \{0,1\}^n \to \mathbb{R}$ be a pseudo-Boolean objective represented by its multilinear polynomial, and let $F : [0,1]^n \to \mathbb{R}$ be its multilinear extension defined by Eq. 6. Then*

$$\min_{\boldsymbol{y}\in[0,1]^n} F(\boldsymbol{y}) = \min_{\boldsymbol{x}\in\{0,1\}^n} H(\boldsymbol{x}), \tag{7}$$

and the discrete problem and the relaxed problem share all global minimizers among 0–1 vectors: a binary vector is a global minimizer of H if and only if it is a global minimizer of F over $[0,1]^n$. Moreover, every strict local minimizer of F over $[0,1]^n$ is a 0–1 vector.

The proof is in Appendix C. This result explains why continuous neural optimization can yield nearly binary solutions without supervised labels. It also clarifies the remaining difficulty. For the quadratic multilinear objectives used here, the Hessian has zero diagonal because no term contains y_i^2, but it is not the zero matrix whenever the graph has at least one active pairwise interaction. A nonzero symmetric matrix with zero trace must have both positive and negative eigenvalues, so the relaxed objective is nonconvex. Consequently, there may be many stable 0–1 local solutions, and a neural solver trained on the relaxed objective can converge to a suboptimal one.

2.3 Local Improvement Objective

Let $\mathcal{X}_G \subseteq \{0,1\}^n$ denote the task-specific valid solution set and let $s_G(\boldsymbol{x})$ be the task score: independent-set size for MIS and cut value for MaxCut. For MaxCut, $\mathcal{X}_G = \{0,1\}^n$; for MIS, $\mathcal{X}_G$ contains only feasible

selected subsets. Given a current binary solution $\hat{\boldsymbol{x}} \in \mathcal{X}_G$, the local-improvement problem is

$$\text{given } (G, \hat{\boldsymbol{x}}), \quad \text{find } \boldsymbol{x}' \in \mathcal{X}_G \text{ such that } s_G(\boldsymbol{x}') > s_G(\hat{\boldsymbol{x}}). \tag{8}$$

For the minimization forms in Eq. 1 and Eq. 4, the same goal is to find a binary solution with a smaller objective value. The current solution $\hat{\boldsymbol{x}}$ is not treated as a failed output to discard. It reveals the present solution structure, which constraints or pairwise interactions are active, and how large a perturbation the subsequent repair step may need.

3 Method

3.1 Overview

Throughout this section, $\boldsymbol{z}^t \in [0,1]^n$ denotes the continuous search state at outer destroy-repair iteration t. Given a graph G and the current state $\boldsymbol{z}^t$, LDR alternates two operations. A destroy policy D_ϕ predicts one Bernoulli destruction probability for each node from the current continuous graph state. Since the training reward is graph-level, the default inference rule uses these probabilities through their average over the vertices eligible to be perturbed, obtaining a learned destruction rate ρ^t. It then samples a random binary mask $\boldsymbol{a}^t \in \{0,1\}^n$ with rate ρ^t, where $a_i^t = 1$ means that vertex i is perturbed in the continuous state. A repair network R_θ maps the damaged continuous state back to the next repaired continuous state $\boldsymbol{z}^{t+1}$. The decoded binary solution is used for reward computation, score logging, and selecting the best returned solution, but it is not fed back as the next destroy or repair input.

Figure 1 summarizes the method: repair is pretrained from the multilinear objective, destroy learns a state-dependent perturbation rate using repair as its environment, the two modules can be alternated, and inference repeats the rate-matched destroy-repair loop. The key design choice is that destroy is not trained to make the damaged state itself good. It is trained to make the repaired state better under the original task score. Since this score is observed only at the graph level, we do not assign node-specific rewards to individual mask entries. This separates two roles: destroy chooses how much of the current state to perturb, and repair reconstructs a high-quality solution from the damaged state.

Table 4 in the appendix summarizes the main symbols used in the method.

3.2 Repair Network

The repair network is a learnable map from a graph and an input continuous state to a repaired continuous state. We use $\boldsymbol{y}^r$ for the internal repair state after r recurrent repair steps within one repair call; this index r is distinct from the outer destroy-repair iteration index t used for $\boldsymbol{z}^t$. Thus $\boldsymbol{y}^0$ is the input to repair, such as a random initial vector $\boldsymbol{u}$ or a damaged search state $\tilde{\boldsymbol{z}}^t$, and one repair step outputs one repaired value per vertex:

$$\boldsymbol{y}^{r+1} = R_\theta(G, \boldsymbol{y}^r) \in [0,1]^n. \tag{9}$$

The same map is applied recurrently, so a single set of parameters defines a multi-step repair process; $R_\theta^R(G, \boldsymbol{y}^0)$ denotes R such recurrent repair steps.

Repair is trained by minimizing the task's continuous multilinear energy. For a graph objective H_G, the per-graph relaxed loss is

$$\mathcal{L}_{\text{repair}}(\theta; G, \boldsymbol{y}) = F_G(\boldsymbol{y}) + \beta \sum_i y_i(1 - y_i), \tag{10}$$

where F_G is the multilinear extension of the discrete energy and the optional binary term encourages polarization. For MIS, F_G is Eq. 5; for MaxCut, F_G is the relaxation of Eq. 4. The MaxClique analogue is obtained from the MIS loss by replacing edges with missing edges.

The repair network optimizes continuous variables, but the objective geometry makes this a direct route to discrete repair. Since strict local minima of F_G are binary by Theorem 1, a stabilized recurrent repair trajectory is encouraged to approach a 0–1 solution. We then use a fixed task decoder only when a discrete solution must be scored or returned: for MIS, the reported evaluation constructs a maximal independent set

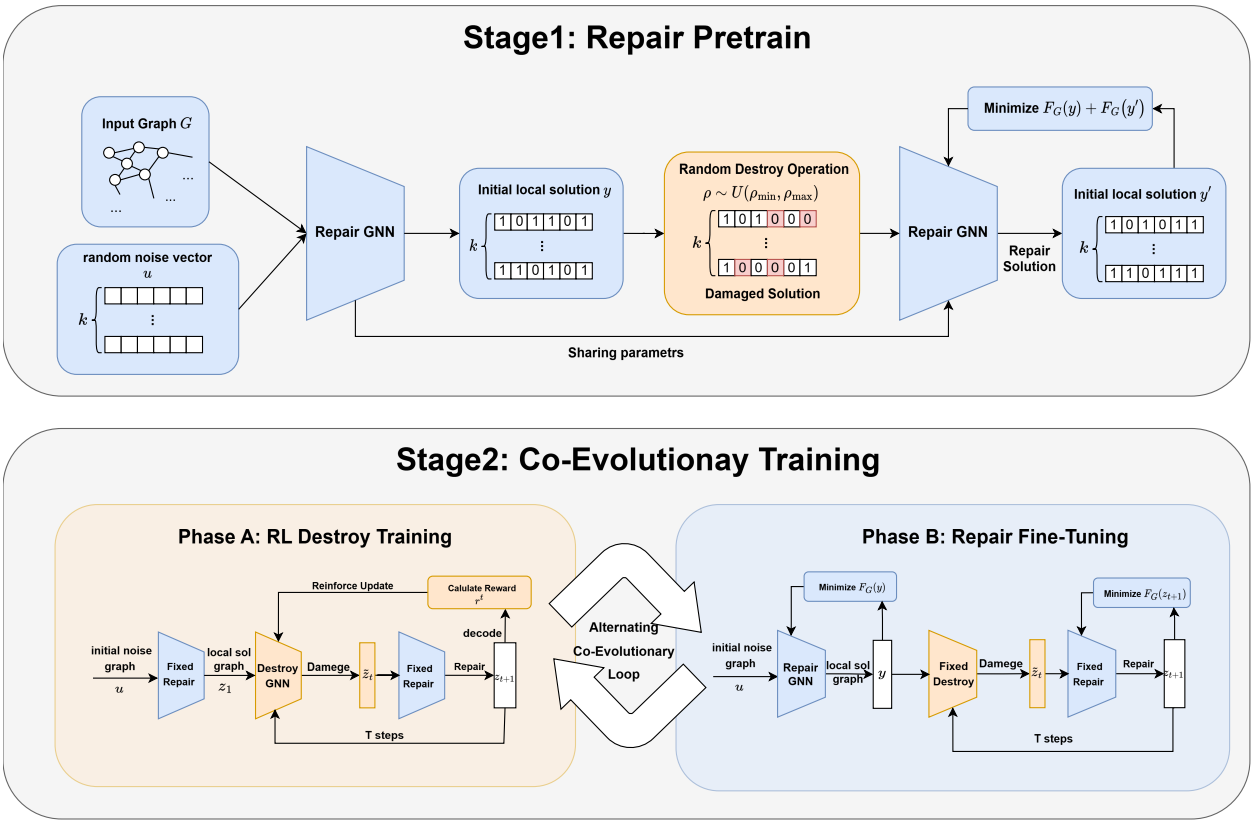

Figure 1: Overview of LDR. Stage 1 trains a recurrent repair network by the multilinear QUBO objective from both random initial states and randomly damaged continuous states. Stage 2 freezes repair and trains a graph-level destroy-rate policy whose sampled mask perturbs the current continuous state before repair. The destroy and repair stages can be alternated, and inference repeats the learned destroy-repair loop.

by repeatedly selecting the highest-scoring currently feasible vertex and masking its neighbors; for MaxCut, scores are thresholded to obtain a binary partition. This projection is shared by all MIS variants and is not an adaptive or tuned post-processing heuristic, so internal differences come from the destroy-and-repair dynamics rather than from a changing decoder.

Multi-source repair pretraining. Repair pretraining uses two sources of input states. First, the network maps a random initial continuous vector $\boldsymbol{u} \in [0,1]^n$ to a continuous state, giving it a base construction ability. Second, it receives randomly destroyed versions of its own outputs. Let $\boldsymbol{y} = R_\theta^R(G, \boldsymbol{u})$ be a current continuous state produced after R repair steps and let $\rho \in [\rho_{\min}, \rho_{\max}]$ be a sampled destruction rate. RandDestroy samples keep masks $m_i \sim \text{Bernoulli}(1-\rho)$ on active continuous coordinates, where activity means that vertex i has a large current state value, and returns $\boldsymbol{m} \odot \boldsymbol{y}$. For inactive coordinates, m_i is set to one so that random destruction focuses on variables that currently participate in the continuous repair state. The training loss sums the relaxed objective over recurrent repair outputs:

$$\mathcal{L}_{\text{pre}}(\theta) = F_G(R_\theta^R(G, \boldsymbol{u})) + F_G(R_\theta^R(G, \text{RandDestroy}(R_\theta^R(G, \boldsymbol{u})))). \tag{11}$$

This stage, shown as Stage 1 in Figure 1, is designed to make repair useful on partial states before it is paired with a learned destroy policy. It also makes the early training environment more stable than learning destroy against an untrained repair module. Algorithm 1 summarizes the repair pretraining procedure.

Algorithm 1 Repair Pretraining

Require: Training graphs $\mathcal{G}$; repair network R_θ; recurrent steps R; destruction-rate range $[\rho_{\min}, \rho_{\max}]$

Initialize repair parameters θ
for each training iteration **do**
  Sample a graph $G \in \mathcal{G}$ and an initial vector $\boldsymbol{u} \in [0,1]^n$
  Compute $\boldsymbol{y} = R_\theta^R(G, \boldsymbol{u})$
  Sample $\rho \sim \mathrm{Unif}[\rho_{\min}, \rho_{\max}]$
  Compute $\tilde{\boldsymbol{y}} = \textsc{RandDestroy}(\boldsymbol{y}; \rho)$
  Compute $\boldsymbol{y}' = R_\theta^R(G, \tilde{\boldsymbol{y}})$
  Update θ by minimizing $F_G(\boldsymbol{y}) + F_G(\boldsymbol{y}')$ with Eq. 10
end for
return pretrained repair parameters θ

3.3 Learned Destroy-Rate Policy

The destroy policy conditions on the graph, the current repaired state $\boldsymbol{z}^t$, and a scalar time or step feature τ_t. It outputs one Bernoulli destruction probability per node,

$$\boldsymbol{p}^t = D_\phi(G, \boldsymbol{z}^t, \tau_t) \in [0,1]^n. \tag{12}$$

Although Eq. 12 is written per node, the reward used to train it is graph-level: all mask entries for the same graph receive the same post-repair improvement signal. We therefore use the probabilities to choose the amount of destruction rather than to claim reliable node-specific credit. During policy training, nodewise Bernoulli masks provide the sampled actions for REINFORCE. In the default LDR inference rule, the probabilities are averaged over the eligible vertex set $\mathcal{V}^t_{\text{elig}}$ defined for the current continuous state,

$$\rho^t = \frac{1}{|\mathcal{V}^t_{\text{elig}}|} \sum_{i \in \mathcal{V}^t_{\text{elig}}} p^t_i, \tag{13}$$

and the actual inference mask is sampled uniformly at that matched rate: $a^t_i \sim \mathrm{Bernoulli}(\rho^t)$ for $i \in \mathcal{V}^t_{\text{elig}}$ and $a^t_i = 0$ otherwise. For MIS, the eligible set is defined from active coordinates of the current continuous state rather than from a decoded independent set; for MaxCut, all vertices are eligible. Direct nodewise sampling is the policy-action parameterization during training and is kept as an inference ablation, but it is not the default LDR inference rule. The damaged state for removal-style tasks is

$$\tilde{\boldsymbol{z}}^t = \boldsymbol{z}^t \odot (\boldsymbol{1} - \boldsymbol{a}^t). \tag{14}$$

For MaxCut, all vertices are eligible and the masked variables are flipped before repair. Repair then produces the next continuous state $\boldsymbol{z}^{t+1} = R_\theta^R(G, \tilde{\boldsymbol{z}}^t)$. Decoding is applied only to compute the post-repair score.

Reward. The policy reward is defined by improvement in the decoded task score:

$$r^t = s_G(\mathrm{Decode}(\boldsymbol{z}^{t+1})) - s_G(\mathrm{Decode}(\boldsymbol{z}^t)). \tag{15}$$

For MIS, s_G is the decoded independent-set size; for MaxCut, s_G is the decoded cut value. The decoded solutions in Eq. 15 are used for scoring only; the trajectory itself remains a sequence of continuous states. This reward directly matches the local-improvement objective in Eq. 8.

Policy optimization. Destroy is trained with a REINFORCE estimator (Williams, 1992; Sutton & Barto, 1998). For each graph, a training episode follows the same continuous-state rollout structure used at inference time: repair first constructs $\boldsymbol{z}^0 = R_\theta^R(G, \boldsymbol{u})$ from a random initial vector, and each subsequent destroy action is applied to the latest repair output $\boldsymbol{z}^t$. With repair frozen, this defines a finite-horizon environment of length T. The return is

$$G^t = \sum_{s=t}^{T-1} r^s. \tag{16}$$

Using a baseline b^t, the destroy loss is

$$\mathcal{L}_{\text{destroy}}(\phi) = -\sum_{t=0}^{T-1} \log \pi_\phi(\boldsymbol{a}^t \mid G, \boldsymbol{z}^t, \tau_t)(G^t - b^t). \tag{17}$$

Here π_ϕ denotes the nodewise Bernoulli mask distribution induced by Eq. 12. Gradients update destroy only; repair is frozen during this stage. Because the same return multiplies the log-probabilities of all mask decisions in a trajectory, the learned probabilities tend to encode how aggressive destruction should be for the current graph state and time step. The inference rule in Eq. 13 deliberately uses this learned rate while avoiding an unsupported claim that graph-level rewards identify which individual nodes deserve larger credit.

3.4 Alternating Co-Evolution

Training is organized into one pretraining phase followed by the alternating destroy-training and repair-fine-tuning loop in Figure 1. Algorithm 1 first trains repair on random and randomly damaged states. After that, both alternating sub-stages use the same continuous-state rollout structure as iterative inference. An episode starts with repair constructing $\boldsymbol{z}^0 = R_\theta^R(G, \boldsymbol{u})$ from noise; for $t = 0, \ldots, T-1$, destroy observes $\boldsymbol{z}^t$, samples a nodewise mask from $\boldsymbol{p}^t$, perturbs the continuous state, and repair maps the damaged state to $\boldsymbol{z}^{t+1}$. The two sub-stages differ only in which module is frozen and which loss is optimized. When repair is frozen, destroy is updated by the policy-gradient objective in Eq. 17 using decoded score improvements. When destroy is frozen, repair is fine-tuned on the same kind of destroy-repair trajectory by applying the relaxed repair loss to every repaired state in the trajectory,

$$\mathcal{L}_{\text{ft}}(\theta; G) = \sum_{t=0}^{T} \mathcal{L}_{\text{repair}}(\theta; G, \boldsymbol{z}^t). \tag{18}$$

Thus the initial constructed state $\boldsymbol{z}^0$ and all post-destroy repair outputs contribute to repair fine-tuning. In both sub-stages, the mask is computed from the continuous repair state rather than from a decoded solution; decoding is used only for the reward in destroy training. The purpose is to reduce the distribution shift between random destruction during pretraining and learned-rate destruction during search.

3.5 Iterative Inference

At test time, LDR follows the inference loop in Figure 1: it starts from an initial continuous vector $\boldsymbol{u} \in [0,1]^n$, obtains the initial search state by repair, and then repeats learned rate-matched destroy-and-repair for a fixed budget. Here t indexes outer refinement iterations:

$$\boldsymbol{z}^0 = R_\theta^R(G, \boldsymbol{u}), \tag{19}$$

$$\boldsymbol{z}^{t+1} = R_\theta^R\big(G, \text{Perturb}(\boldsymbol{z}^t, \boldsymbol{a}^t)\big), \quad a_i^t \sim \text{Bernoulli}(\rho^t), \quad \rho^t = \frac{1}{|\mathcal{V}_{\text{elig}}^t|} \sum_{i \in \mathcal{V}_{\text{elig}}^t} D_{\phi,i}(G, \boldsymbol{z}^t, \tau_t). \tag{20}$$

Here Perturb acts on the continuous state $\boldsymbol{z}^t$. Destroy never receives a decoded solution during the loop; decoding is used only to evaluate each repaired state and to maintain the returned incumbent. Algorithm 3 gives the complete inference procedure.

4 Experiments

We evaluate LDR on MIS and MaxCut. Table 1 reports external comparisons, Table 2 evaluates matched-budget destroy-policy variants, and Table 3 shows refinement trajectories. Appendix D explains the hard-exclusion rigidity that motivates destruction for MIS, while Appendix I reports hyperparameters, full statistics, a repair-depth/step-feature interaction ablation, an ER-graph check, failure-case diagnostics, selected-budget runtimes, pairwise counts, the 32-particle MIS decoder ablation, training curves, and feasibility diagnostics.

Algorithm 2 Alternating Destroy Training and Repair Fine-Tuning

Require: Training graphs $\mathcal{G}$; pretrained repair R_θ; destroy policy D_ϕ; outer rounds M; horizon T; destroy episodes N_D; repair fine-tuning episodes N_R

Initialize θ from Algorithm 1 and initialize ϕ
for $m = 1$ to M **do**
  Freeze θ
  for $i = 1$ to N_D **do**
    Sample $G \in \mathcal{G}$ and an initial vector $\boldsymbol{u} \in [0,1]^n$
    Construct $\boldsymbol{z}^0 = R_\theta^R(G, \boldsymbol{u})$
    for $t = 0$ to $T-1$ **do**
      Compute $\boldsymbol{p}^t = D_\phi(G, \boldsymbol{z}^t, \tau_t)$
      Sample a nodewise mask $\boldsymbol{a}^t$ from $\boldsymbol{p}^t$, perturb $\boldsymbol{z}^t$ to $\tilde{\boldsymbol{z}}^t$, and repair $\boldsymbol{z}^{t+1} = R_\theta^R(G, \tilde{\boldsymbol{z}}^t)$
      Compute reward $r^t = s_G(\text{Decode}(\boldsymbol{z}^{t+1})) - s_G(\text{Decode}(\boldsymbol{z}^t))$
    end for
    Update ϕ using Eq. 17 on the collected trajectory
  end for
  Freeze ϕ
  for $j = 1$ to N_R **do**
    Sample $G \in \mathcal{G}$ and an initial vector $\boldsymbol{u} \in [0,1]^n$
    Construct $\boldsymbol{z}^0 = R_\theta^R(G, \boldsymbol{u})$
    Initialize $\mathcal{L}_{\text{ft}} \leftarrow \mathcal{L}_{\text{repair}}(\theta; G, \boldsymbol{z}^0)$
    for $t = 0$ to $T-1$ **do**
      Compute $\boldsymbol{p}^t = D_\phi(G, \boldsymbol{z}^t, \tau_t)$
      Sample a nodewise mask $\boldsymbol{a}^t$ from $\boldsymbol{p}^t$, perturb $\boldsymbol{z}^t$ to $\tilde{\boldsymbol{z}}^t$, and repair $\boldsymbol{z}^{t+1} = R_\theta^R(G, \tilde{\boldsymbol{z}}^t)$
      Accumulate $\mathcal{L}_{\text{ft}} \leftarrow \mathcal{L}_{\text{ft}} + \mathcal{L}_{\text{repair}}(\theta; G, \boldsymbol{z}^{t+1})$
    end for
    Fine-tune θ by minimizing $\mathcal{L}_{\text{ft}}$
  end for
end for
return repair parameters θ and destroy parameters ϕ

4.1 Setup

Datasets. For MIS, we use RB graphs with 200–300 and 800–1200 vertices, following prior neural MIS work (Xu et al., 2005; Xu & Li, 2000). For MaxCut, we use Barabási–Albert graphs with 200–300 and 800–1200 vertices (Barabási & Albert, 1999). The fixed-budget internal comparisons use 100 validation instances and average over three evaluation seeds. Newly run external baselines in Table 1 use the full 500-instance validation split and report the average over those instances.

Baselines. The external comparison uses representative values for the same benchmark families. In Table 1, **OR**, **SL**, **UL**, **RL**, and **H** denote optimization-based, supervised-learning, unsupervised-learning, reinforcement-learning, and heuristic methods, respectively. External runtimes are not hardware-normalized; we use them for positioning and rely on fixed-budget internal tables for destroy-variant claims. The PI-GNN, CRA-PI-GNN, CPRA, and RLNN rows are newly run on the same validation splits and report average wall-clock time per graph. For MIS, PI-GNN/CRA-PI-GNN/CPRA use the fixed score-order feasibility projection used by their runners, because raw thresholding can be infeasible on dense RB graphs.

Table 2 evaluates the destroy-policy variants with the same number of repair calls: **Repair** repeats repair without destruction; **Fixed** $p = .6$ and **Uniform** $[.1, .6]$ use random graph-level destroy budgets; **Graph-rate** uses a learned graph-level probability; **Nodewise** samples directly from per-node destroy probabilities; and **LDR (Rate-matched)** averages those probabilities over eligible vertices before sampling the mask. Comparing LDR with Nodewise tests whether direct nodewise sampling adds value beyond the learned destruction amount.

Algorithm 3 Iterative Inference

Require: Test graph G; trained repair R_θ; trained destroy policy D_ϕ; repair steps R; destroy-repair iterations T

Sample or initialize $\boldsymbol{u} \in [0,1]^n$
$\boldsymbol{z}^0 = R_\theta^R(G, \boldsymbol{u})$
$\hat{\boldsymbol{x}}^0 = \text{Decode}(\boldsymbol{z}^0)$
Set incumbent $\hat{\boldsymbol{x}}_{\text{best}} \leftarrow \hat{\boldsymbol{x}}^0$
for $t = 0$ to $T-1$ **do**
  Compute per-node destruction probabilities $\boldsymbol{p}^t = D_\phi(G, \boldsymbol{z}^t, \tau_t)$
  Define the eligible vertex set $\mathcal{V}_{\text{elig}}^t$ from $\boldsymbol{z}^t$
  Compute the rate $\rho^t = |\mathcal{V}_{\text{elig}}^t|^{-1} \sum_{i \in \mathcal{V}_{\text{elig}}^t} p_i^t$
  Sample $a_i^t \sim \text{Bernoulli}(\rho^t)$ for $i \in \mathcal{V}_{\text{elig}}^t$ and set $a_i^t = 0$ otherwise
  Perturb masked coordinates in the continuous state to obtain $\tilde{\boldsymbol{z}}^t$
  Repair: $\boldsymbol{z}^{t+1} = R_\theta^R(G, \tilde{\boldsymbol{z}}^t)$
  Decode for evaluation: $\hat{\boldsymbol{x}}^{t+1} = \text{Decode}(\boldsymbol{z}^{t+1})$
  if $s_G(\hat{\boldsymbol{x}}^{t+1}) > s_G(\hat{\boldsymbol{x}}_{\text{best}})$ **then**
    $\hat{\boldsymbol{x}}_{\text{best}} \leftarrow \hat{\boldsymbol{x}}^{t+1}$
  end if
end for
return $\hat{\boldsymbol{x}}_{\text{best}}$

Metrics and budget. The primary metric is the decoded task score: independent-set size for MIS and cut value for MaxCut; higher is better. All internal methods use 256 refinement iterations, two repair steps per iteration, and 512 total repair calls. Destroy and repair operate on continuous states: MIS resets destroyed active coordinates to zero, while MaxCut flips destroyed coordinates. We log starting score, best score, best step, improvement, runtime, destroy probability, number of destroyed variables, movement diagnostics, and feasibility indicators after decoding. The binary coefficient β in Eq. 10 is set to zero, so these runs isolate the multilinear task objective and the fixed decoder.

4.2 Main Results

Table 1 gives the external reference point.

- On MIS, LDR is competitive with learning baselines but does not dominate specialized search: it is strongest on RB-[200–300] and remains below KaMIS on RB-[800–1200].
- On MaxCut, LDR gives the best listed scores on both BA splits, showing that destroy-and-repair is not limited to hard-exclusion decoding.
- Because external runtimes use different implementations and hardware assumptions, the following internal tables provide the controlled destroy-variant comparisons.

4.3 Destroy-Policy Ablations

Table 2 supports three conclusions.

- **Destroy-before-repair is the dominant source of improvement.** LDR improves over repair-only inference on all four datasets under the same repair-call budget.
- **The learned destruction rate is the useful part of the current destroy policy.** LDR is above direct nodewise sampling in all four rows, supporting the view that graph-level rewards most reliably learn a state-dependent destruction rate.
- **Simple random destruction remains a strong baseline.** Uniform random destruction improves over repair-only inference on every dataset, but LDR is higher in all four rows.

Table 1: Comparison with representative external baselines on MIS and MaxCut. Scores are independent-set sizes for MIS and cut values for MaxCut; higher is better. Runtime is wall-clock seconds per instance. Newly run rows use the 500-instance validation split; external runtimes are not hardware-normalized.

Task	Method	Type	**Small**		**Large**	
			Score ↑	Time ↓	Score ↑	Time ↓
MIS on RB graphs: Small = RB-[200–300], Large = RB-[800–1200]						
MIS	Gurobi	OR	20.21	0.41	40.90	15.624
	KaMIS	OR	20.23	10.080	**43.15**	14.760
	DGL	SL	17.36	1.534	34.50	2.868
	DIFUSCO	SL	18.52	1.926	–	–
	LTFT	UL	19.18	0.064	37.48	0.524
	PI-GNN	UL	17.21	4.596	33.77	5.552
	CRA-PI-GNN	UL	17.05	66.162	31.63	65.934
	CPRA	UL	17.53	15.988	33.49	16.029
	DiffUCO	UL	19.59	0.056	38.87	5.924
	SDDS	UL	19.62	0.040	39.32	5.762
	PPO-style	RL	18.43	0.154	32.32	0.906
	RLNN	RL	19.70	0.148	34.26	0.304
	LDR (ours)	UL/RL	**20.28**	2.43	40.58	8.84
MaxCut on BA graphs: Small = BA-[200–300], Large = BA-[800–1200]						
MaxCut	Gurobi	OR	730.87	1.020	2944.38	9.216
	ERDOES	UL	693.45	0.092	2870.34	0.338
	LTFT	UL	704.30	0.354	2864.61	2.560
	PI-GNN	UL	652.42	6.152	2796.80	6.305
	CRA-PI-GNN	UL	724.16	16.523	2930.55	16.882
	CPRA	UL	715.23	14.634	2919.91	15.023
	DiffUCO	UL	727.32	0.120	2947.53	0.454
	SDDS	UL	730.76	0.028	–	–
	RLNN	RL	728.99	0.141	2912.60	0.376
	Greedy	H	688.31	0.026	2786.00	0.374
	MFA	H	704.03	0.192	2833.86	0.872
	LDR (ours)	UL/RL	**736.42**	0.46	**2963.03**	1.38

Table 2: Destroy-policy variants with the same number of repair calls. Values are mean best decoded scores over 100 instances and three seeds. LDR (Rate-matched) averages learned nodewise probabilities over eligible vertices and samples a random mask at that learned rate.

Task	Dataset	Repair	Fixed $p = .6$	Uniform $[.1, .6]$	Graph-rate	Nodewise	LDR
MIS	RB-[200–300]	18.12	20.15	20.05	19.86	20.27	**20.28**
MIS	RB-[800–1200]	34.67	40.08	40.02	40.18	40.39	**40.58**
MaxCut	BA-[200–300]	734.36	735.40	735.34	735.18	735.37	**736.42**
MaxCut	BA-[800–1200]	2951.71	2942.80	2957.48	2944.21	2962.98	**2963.03**

The ablation pattern is consistent across tasks: destruction creates room for alternative vertices in MIS and coordinates new partitions in MaxCut. In both cases, the learned rate remains useful even when the concrete mask is randomized, matching the graph-level reward used to train destroy.

Appendix Table 7 reports an additional interaction ablation. It crosses the inner repair depth R with the step-feature design for τ_t while keeping the total number of repair calls fixed at $R \times T = 512$. This experiment is not used to claim a new mechanism; it clarifies that the main ablation evidence in Table 2 comes from destroy-before-repair and learned rate matching under a fixed repair budget.

Table 3: Search dynamics over 256 refinement steps. Each entry is the mean best decoded score over 100 instances and three seeds. Time is the cumulative wall-clock runtime at step 256. Appendix Table 11 gives the corresponding selected-budget scores and runtimes for LDR.

Task	Dataset	Method	0	8	16	32	64	128	256	Time (s)
MIS	RB-[200–300]	Repair	18.03	18.11	18.11	18.12	18.12	18.12	18.12	2.04
		Uniform [.1, .6]	18.03	19.36	19.58	19.76	19.89	19.95	20.05	2.30
		Graph-rate	18.03	19.28	19.47	19.60	19.70	19.82	19.86	2.37
		LDR	18.03	19.68	19.87	20.02	20.12	20.24	20.28	2.43
MIS	RB-[800–1200]	Repair	34.15	34.67	34.67	34.67	34.67	34.67	34.67	7.44
		Uniform [.1, .6]	34.15	37.63	38.27	38.83	39.36	39.74	40.02	8.18
		Graph-rate	34.15	37.91	38.67	39.36	39.82	40.11	40.18	8.88
		LDR	34.15	37.93	38.71	39.38	39.92	40.30	40.58	8.84
MaxCut	BA-[200–300]	Repair	729.59	734.33	734.36	734.36	734.36	734.36	734.36	0.33
		Uniform [.1, .6]	729.59	734.73	735.07	735.16	735.20	735.23	735.34	0.37
		Graph-rate	729.59	734.72	734.94	735.02	735.10	735.16	735.18	0.47
		LDR	729.59	734.87	735.13	735.28	735.36	735.37	736.42	0.46
MaxCut	BA-[800–1200]	Repair	2914.60	2949.09	2951.25	2951.71	2951.71	2951.71	2951.71	0.97
		Uniform [.1, .6]	2914.60	2945.01	2948.59	2951.47	2953.94	2955.80	2957.48	1.00
		Graph-rate	2914.60	2925.04	2927.68	2929.77	2931.92	2934.16	2944.21	1.29
		LDR	2914.60	2951.94	2956.45	2959.52	2961.61	2962.79	2963.03	1.38

4.4 Search Dynamics

Table 3 shows that repair-only inference saturates early, while destroy-and-repair continues to improve by creating new damaged states for repair. Uniform random destruction is a strong trajectory baseline, but LDR improves over it at step 256 on all four datasets, supporting the rate-matched rule under the fixed repair-call budget.

5 Related Work

LDR connects neural combinatorial optimization, continuous QUBO relaxations, and local search. Learning-based solvers include supervised, reinforcement-learning, unsupervised-energy, GFlowNet-style, and diffusion approaches (Khalil et al., 2017; Bengio et al., 2021; Zhang et al., 2023; Sun & Yang, 2023; Sanokowski et al., 2024); continuous relaxations optimize differentiable graph objectives (Karalias & Loukas, 2020; Schuetz et al., 2022; Wang et al., 2022); and classical search uses restarts, annealing, tabu moves, large-neighborhood search, and repair heuristics (Shaw, 1998; Blum & Roli, 2003; Ropke & Pisinger, 2006; Nikolaev & Jacobson, 2010). LDR differs by learning a destruction rate for a continuous repair trajectory. Appendix B gives expanded related work, and Appendix E details the relation to large-neighborhood search.

6 Discussion

Limitations. LDR targets graph QUBO settings where a current binary or near-binary solution is useful for refinement. The hard-exclusion analysis applies most directly to MIS and MaxClique, while MaxCut uses destruction as coordinated variable flipping. The experiments do not establish broad dominance across all graph-QUBO families; the theory does not prove global convergence; and the destroy-repair loop adds inference cost. Because the destroy reward is graph-level, the policy is best interpreted as a state-dependent destruction-rate estimator rather than node-specific causal credit.

Conclusion. We presented LDR, a learned destroy-and-repair framework for multilinear graph QUBO optimization. Experiments on MIS and MaxCut show that the loop improves over repair-only inference under the same repair-call budget and that rate-matched inference is strong among the tested destroy variants. The evidence is strongest for MIS-style hard-exclusion refinement, while broader graph-QUBO generality remains future work. Appendix A gives the broader-impact discussion.

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A Broader Impact Statement

This work studies generic graph optimization algorithms. Potential benefits include faster heuristic solvers for scheduling, resource allocation, and scientific computing. Potential risks arise if such solvers are used inside high-stakes allocation systems without validating objective design, constraints, and fairness implications. The method itself does not use personal data, but downstream deployments should audit whether the optimized graph objective faithfully represents the intended decision problem.

B Additional Related Work

Neural combinatorial optimization. Learning-based combinatorial optimization includes supervised construction policies, reinforcement learning, unsupervised energy minimization, GFlowNet-style methods, and diffusion-based solvers (Khalil et al., 2017; Bengio et al., 2021; Zhang et al., 2023; Sun & Yang, 2023; Sanokowski et al., 2024). LDR belongs to the unsupervised family because it does not require optimal labels, but it differs from one-shot construction by repeatedly perturbing the current continuous state and repairing it.

Continuous relaxations and QUBO. QUBO and multilinear relaxations connect binary graph objectives to continuous optimization (Punnen, 2022; Lucas, 2014; Zemel, 1981; Merz & Freisleben, 2002; Ryoo & Sahinidis, 2001). Physics-inspired GNN solvers and principled objective relaxations exploit this differentiability for graph problems (Karalias & Loukas, 2020; Schuetz et al., 2022; Wang et al., 2022). Our analysis emphasizes both sides of this relaxation: it preserves 0–1 optima and strict local minima are binary, but the relaxed landscape can still contain stable suboptimal binary solutions.

Escaping local optima. Classical methods use random restarts, simulated annealing, tabu search, large-neighborhood search, and repair-based perturbation heuristics (Shaw, 1998; Blum & Roli, 2003; Ropke & Pisinger, 2006; Nikolaev & Jacobson, 2010; Lamm et al., 2017; Pisinger & Ropke, 2010); recent sampling methods use gradient-informed or Langevin-like proposals (Grathwohl et al., 2021; Sun et al., 2021; Zhang et al., 2022; Feng & Yang, 2025). LDR is closest in spirit to large-neighborhood search, but its large-neighborhood move is implemented through a learned continuous repair operator rather than an explicit discrete subproblem solver.

C Proof of Theorem 1

For any $\boldsymbol{y} \in [0,1]^n$, let $\boldsymbol{Z} \sim \prod_{i=1}^n \mathrm{Bern}(y_i)$ be a random binary vector with independent coordinates. By construction of the multilinear extension,

$$F(\boldsymbol{y}) = \mathbb{E}\left[H(\boldsymbol{Z})\right]. \tag{21}$$

Because $\{0,1\}^n \subset [0,1]^n$ and $F(\boldsymbol{x}) = H(\boldsymbol{x})$ for every $\boldsymbol{x} \in \{0,1\}^n$,

$$\min_{\boldsymbol{y}\in[0,1]^n} F(\boldsymbol{y}) \le \min_{\boldsymbol{x}\in\{0,1\}^n} H(\boldsymbol{x}). \tag{22}$$

Conversely, for any $\boldsymbol{y} \in [0,1]^n$,

$$F(\boldsymbol{y}) = \mathbb{E}\left[H(\boldsymbol{Z})\right] \ge \min_{\boldsymbol{x}\in\{0,1\}^n} H(\boldsymbol{x}), \tag{23}$$

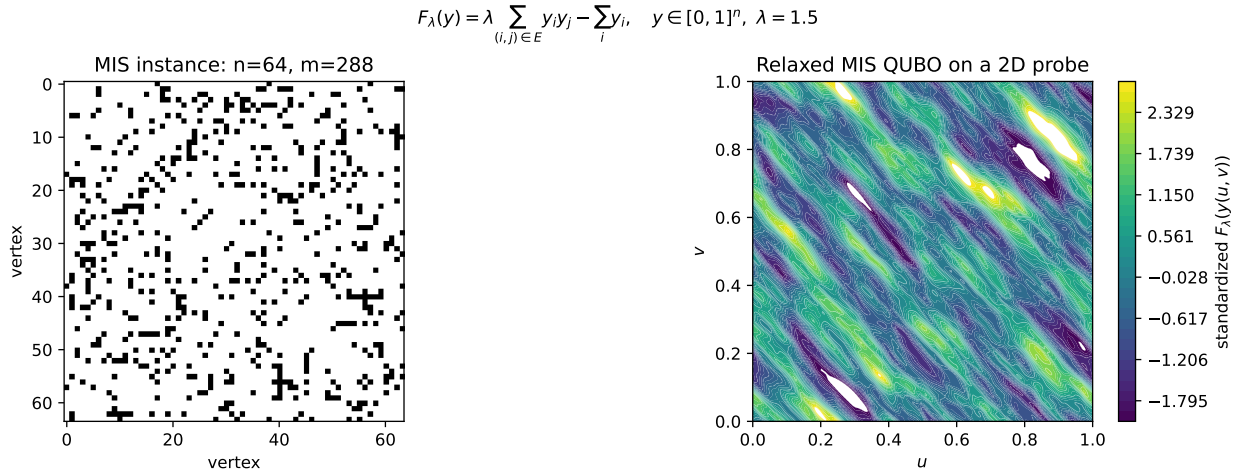

Figure 2: Relaxed-landscape visualization for a small MIS instance. The multilinear relaxation keeps binary optima but can still contain nonconvex basins and ridges, which motivates perturbing the current repaired state before applying repair again.

since an expectation is bounded below by the minimum value of the random variable's support. Taking the minimum over $\boldsymbol{y} \in [0,1]^n$ gives

$$\min_{\boldsymbol{y}\in[0,1]^n} F(\boldsymbol{y}) = \min_{\boldsymbol{x}\in\{0,1\}^n} H(\boldsymbol{x}). \tag{24}$$

Let $m = \min_{\boldsymbol{x}\in\{0,1\}^n} H(\boldsymbol{x})$ and let $\bar{\boldsymbol{y}}$ be a continuous global minimizer. Then $F(\bar{\boldsymbol{y}}) = m$. Since $H(\boldsymbol{Z}) \geq m$ almost surely and $\mathbb{E}[H(\boldsymbol{Z})] = m$, the random variable $H(\boldsymbol{Z})$ equals m on every binary assignment with positive probability under $\bar{\boldsymbol{y}}$. The distribution has nonempty support, so at least one supported 0–1 vector $\boldsymbol{x}^\star$ satisfies $H(\boldsymbol{x}^\star) = m$. Therefore $F(\boldsymbol{x}^\star) = H(\boldsymbol{x}^\star) = m$, and a 0–1 global minimizer of the continuous problem exists. Moreover, for any binary vector $\boldsymbol{x}$, $F(\boldsymbol{x}) = H(\boldsymbol{x})$. Thus $\boldsymbol{x}$ is a discrete global minimizer of H if and only if $F(\boldsymbol{x}) = m$, which is exactly the condition for the same 0–1 vector to be a global minimizer of the relaxed problem.

It remains to show that strict local minimizers are 0–1. Suppose $\boldsymbol{y}^\star$ is a strict local minimizer of F but is not binary. Then there exists a coordinate i with $0 < y_i^\star < 1$. Fixing all other coordinates, multilinearity implies that

$$\phi_i(t) = F(y_1^\star, \ldots, y_{i-1}^\star, t, y_{i+1}^\star, \ldots, y_n^\star) \tag{25}$$

is affine in t. If this affine function has positive slope, moving t slightly left decreases F; if it has negative slope, moving t slightly right decreases F; if it has zero slope, F is locally constant along that coordinate. Each case contradicts strict local minimality. Hence every strict local minimizer is a 0–1 vector.

D Structural Analysis

For MIS, local improvement has an additional structural obstruction beyond nonconvexity: once a solution is maximal, improving it requires first removing selected vertices. The same statement holds for MaxClique after replacing the graph by the conflict graph of missing edges. This argument does not apply to MaxCut, where every partition is feasible and improvement instead requires coordinated variable flips.

Proposition 1 (Hard-exclusion rigidity)**.** *Let S be a maximal independent set in G. If S' is an independent set with $|S'| > |S|$, then transforming S into S' requires removing at least one vertex from S. If $|S'| = |S| + n$ with $n \geq 1$, and the transition removes p vertices from S, then it must add $q = n + p$ vertices outside S.*

The proof is immediate: maximality means no outside vertex can be added while keeping all vertices in S. Thus any larger feasible set must first remove a selected vertex. The bookkeeping identity $q = n + p$ follows from the set-size difference.

This proposition is one motivation for learned destruction in hard-exclusion tasks. A repair network that simply follows its current selection preference may reconstruct the same maximal solution after decoding. A useful destroy policy must perturb enough of the continuous state representing the current selection pattern to make alternative additions possible, without destroying so much that repair degenerates into construction from scratch. For MaxCut, the same module is interpreted as learning how many variables to perturb so that repair can coordinate a new partition.

E Relation to Large Neighborhood Search

LDR can be viewed as a neural large-neighborhood-search-style method. Classical large neighborhood search (LNS) improves an incumbent solution by destroying part of it and then repairing the resulting partial solution through a problem-specific search or insertion procedure (Shaw, 1998; Pisinger & Ropke, 2010). Adaptive LNS further changes which destroy and repair operators are used during search (Ropke & Pisinger, 2006). Our method follows the same high-level template, but the state being perturbed and repaired is continuous. Decoding is used to score the post-repair state and to select the final incumbent solution.

The main difference is how the large-neighborhood improvement is carried out. In standard LNS, repair is often a handcrafted combinatorial routine, a constraint-programming subproblem, or an insertion heuristic. In LDR, repair is a learned recurrent GNN trained by optimizing the original multilinear continuous QUBO relaxation. The repair step therefore searches the neighborhood through a continuous objective whose strict local minima are binary by Theorem 1, followed by a fixed task decoder. This is the sense in which the large-neighborhood repair move is implemented through continuous optimization rather than an explicit discrete subproblem solver.

The learned destroy module also has a narrower role than a full adaptive LNS operator selector. Because its reward is graph-level, the current training signal does not provide reliable node-specific causal credit. We therefore interpret the destroy network primarily as learning the destruction amount: how large the neighborhood should be for the current graph, state, and search step. The default rate-matched inference rule in Eq. 13 averages the per-node probabilities over the eligible vertex set into a learned destruction rate and then samples the concrete mask randomly at that rate. This explains why the random-destroy baselines are strong: once the repair operator is effective, a nontrivial destruction budget already creates useful large neighborhoods. The benefit of LDR is to adapt that budget from data while leaving the exact destroyed vertices randomized under graph-level rewards.

F Task-Specific Objective Details

For MaxCut, there is no hard-exclusion decoder. The relaxed energy is

$$F^{\text{MaxCut}}(\boldsymbol{y}) = -\sum_{(i,j)\in E} w_{ij}\left[y_i(1-y_j) + y_j(1-y_i)\right]. \tag{26}$$

At inference time, the continuous output is thresholded to a binary partition and evaluated by Eq. 3. Because every binary partition is feasible, the destroy step is not "removing" selected vertices; it flips masked variables so that repair can coordinate a new partition.

G Implementation Details

The current prototype uses residual-aware GIN layers with layer normalization and sigmoid node heads for both repair and destroy. Repair is optimized with AdamW and gradient clipping. Destroy is trained with Bernoulli mask actions, sampled time-step features, a finite-horizon repair environment, normalized

Table 4: Main notation used in the method.

Symbol(s)	Meaning
$G = (V, E)$	Input graph with $n = \|V\|$ vertices
$\boldsymbol{x}$, $\hat{\boldsymbol{x}}$, $\boldsymbol{x}'$	Binary solution, incumbent solution, and improved candidate in the local-improvement view
$\mathcal{X}_G$	Task-specific feasible binary set; for MaxCut, $\mathcal{X}_G = \{0,1\}^n$
$H_G(\boldsymbol{x})$, $F_G(\boldsymbol{y})$	Discrete minimization energy and its multilinear continuous relaxation on $[0,1]^n$
$s_G(\boldsymbol{x})$	Maximization score used for evaluation: independent-set size or cut value
$\mathrm{Decode}(\boldsymbol{z})$	Fixed task decoder from a continuous state to a binary solution used for scoring/returning solutions
$\boldsymbol{u}$	Initial continuous state in $[0,1]^n$
R_θ, R_θ^R	One repair update and the result of applying R recurrent repair steps with shared parameters
$\boldsymbol{y}^r$, r	Internal repair state and repair-step index inside a single repair call
$\boldsymbol{z}^t$, t	Outer continuous search state and outer destroy-repair iteration index
D_ϕ	Destroy policy producing nodewise destruction probabilities from $(G, \boldsymbol{z}^t, \tau_t)$
$\boldsymbol{p}^t$	Nodewise destruction probabilities output by D_ϕ
$\mathcal{V}^t_{\mathrm{elig}}$	Vertices eligible to be perturbed at outer step t
ρ^t	Rate-matched destruction rate, averaged over $\mathcal{V}^t_{\mathrm{elig}}$
$\boldsymbol{a}^t$	Binary destroy mask; $a_i^t = 1$ means vertex i is perturbed
$\tilde{\boldsymbol{z}}^t$	Damaged continuous state after applying the destroy mask
τ_t	Normalized progress feature for the destroy policy
r^t, G^t, b^t	One-step reward, return, and baseline used in destroy policy optimization
$\mathcal{L}_{\mathrm{repair}}$, $\mathcal{L}_{\mathrm{destroy}}$, $\mathcal{L}_{\mathrm{ft}}$	Repair loss, destroy policy-gradient loss, and repair fine-tuning loss
λ, β	MIS conflict penalty and optional binary regularization coefficient
R, T	Number of recurrent repair steps per repair call and number of outer destroy-repair iterations

advantages, and gradient clipping. The reported comparisons should be read together with the runtime columns because the destroy-repair loop adds repeated network calls at inference time.

During destroy training, repair is frozen and each graph can contribute multiple sampled rollouts to the same policy update. Each rollout first constructs $\boldsymbol{z}^0$ from a random initial vector with the frozen repair network, then repeatedly applies the destroy mask and frozen repair update to produce $\boldsymbol{z}^1, \ldots, \boldsymbol{z}^T$. We use these repeated rollouts to form a leave-one-sample baseline: the baseline for one sample is the mean return of the other samples from the same graph when available, with the batch mean used as a fallback. The resulting advantages are normalized before the REINFORCE update. This is an implementation variance-reduction device; it does not change the graph-level reward definition in Eq. 15.

During repair fine-tuning, destroy is frozen and the same rollout structure is used. Repair again constructs the initial state $\boldsymbol{z}^0$ from noise, then receives each damaged continuous state produced by the frozen destroy rule and maps it to $\boldsymbol{z}^{t+1}$. The fine-tuning loss is the sum of multilinear repair losses over all repaired states in the rollout, including the initial construction output $\boldsymbol{z}^0$ and every subsequent post-destroy repair output. The decoded task score is not used in this repair loss; decoding remains only for destroy rewards and evaluation.

H Model and Evaluation Details

Node features. Both repair and destroy receive node-wise feature tensors built from the current soft state $\boldsymbol{z} \in [0,1]^{n \times K}$, where K is the number of repair samples per graph. For a node i, let d_i be its degree, $N(i)$ its neighbors, $b(i)$ its graph index in a mini-batch, and

$$n_i(\boldsymbol{z}) = \sum_{j \in N(i)} z_j, \qquad T_i(\boldsymbol{z}) = \sum_{j : b(j) = b(i)} z_j, \qquad o_i(\boldsymbol{z}) = T_i(\boldsymbol{z}) - z_i. \tag{27}$$

The universal feature builder uses a shared feature basis for MIS and MaxCut. It includes the current value z_i, degree d_i, neighbor mass n_i, graph total T_i, other-node mass o_i, non-neighbor mass $o_i - n_i$, non-neighbor support/conflict ratios, and cut-style local statistics. In particular, the expected cross-side and same-side cut masses are

$$c_i^{\mathrm{cross}} = z_i(d_i - n_i) + (1 - z_i)n_i, \qquad c_i^{\mathrm{same}} = z_i n_i + (1 - z_i)(d_i - n_i). \tag{28}$$

The feature vector also contains $c_i^{\text{cross}}/(d_i+\epsilon)$, $c_i^{\text{same}}/(d_i+\epsilon)$, $c_i^{\text{cross}}-c_i^{\text{same}}$, and a graph-level balance feature $(T_i-|V_{b(i)}|/2)/(|V_{b(i)}|+\epsilon)$. When a mask-aware repair checkpoint is used, the damage mask is appended as an additional feature channel. The small constant is $\epsilon = 10^{-6}$ in feature construction.

Residual-aware GIN backbone. Each node feature vector is first projected by a two-layer MLP with ReLU and layer normalization to obtain $h_i^{(0)} \in \mathbb{R}^{d_h}$. The backbone contains L residual-aware GIN layers. Let $\alpha = 0.5$ and let $u_i^{(\ell)}$ be the input to layer ℓ:

$$u_i^{(\ell)} = (1-\alpha)h_i^{(\ell-1)} + \alpha h_i^{(0)}. \tag{29}$$

If a repair mask m_i is supplied, the layer first gates the input as $\tilde{u}_i^{(\ell)} = m_i u_i^{(\ell)}$; otherwise $\tilde{u}_i^{(\ell)} = u_i^{(\ell)}$. When global features are enabled, the layer computes a graph mean $\bar{u}_{b(i)}^{(\ell)} = |V_{b(i)}|^{-1}\sum_{j:b(j)=b(i)} \tilde{u}_j^{(\ell)}$ and replaces $\tilde{u}_i^{(\ell)}$ by

$$q_i^{(\ell)} = \text{MLP}_{\text{global}}^{(\ell)}\left([\tilde{u}_i^{(\ell)}, \bar{u}_{b(i)}^{(\ell)}]\right). \tag{30}$$

Without global features, $q_i^{(\ell)} = \tilde{u}_i^{(\ell)}$. The GIN aggregation and residual MLP update are

$$a_i^{(\ell)} = \sum_{j\in N(i)} q_j^{(\ell)} + (1+\epsilon_\ell)q_i^{(\ell)}, \qquad h_i^{(\ell)} = \text{ReLU}\left[\text{LayerNorm}\left(\text{MLP}_\ell(a_i^{(\ell)}) + q_i^{(\ell)}\right)\right], \tag{31}$$

where ϵ_ℓ is a learned scalar initialized at zero. The final node representation uses jump-knowledge concatenation followed by a linear projection:

$$h_i^{\text{out}} = W_{\text{JK}}\left[h_i^{(1)}; \ldots; h_i^{(L)}\right]. \tag{32}$$

The repair network uses $d_h = 128$ and $L = 5$; the destroy network uses $d_h = 64$ and $L = 4$ in the reported experiments. Since the destroy reward is shared across all node-mask actions of a graph, the node features should be read mainly as a permutation-equivariant way to summarize the current state and set a graph-conditioned destroy rate.

Repair and destroy heads. The repair head is a two-layer MLP followed by a sigmoid:

$$z_i^{\text{new}} = \sigma\left(w_r^\top \text{ReLU}(W_r h_i^{\text{out}} + b_r) + c_r\right). \tag{33}$$

For recurrent repair, this update is applied repeatedly, with the output of one recurrent step becoming the input state of the next step. The destroy network uses the same backbone structure but additionally conditions on an inference time variable $\tau \in [0,1]$. In the checkpoints used here, τ is encoded by a sinusoidal embedding, passed through a two-layer MLP, and added to the projected node features before message passing. The destroy probabilities define a graph-level expected destroy budget $B_\phi = \sum_i p_i^{\text{dest}}$ and are computed as

$$p_i^{\text{dest}} = \sigma\left(w_d^\top \text{ReLU}(W_d h_i^{\text{out}} + b_d) + c_d\right). \tag{34}$$

The default LDR inference rule averages these probabilities over the eligible vertex set and samples the actual mask with the resulting rate. For MIS, destroyed active coordinates are reset to zero before repair. For MaxCut, destroyed coordinates are flipped, and a mask-aware repair checkpoint receives the flipped-variable mask as an input channel when the checkpoint architecture supports it.

Internal evaluation variants. The repair-only variant repeatedly applies the fine-tuned repair network with the same repair-call budget as LDR but never destroys variables. Fixed random destroy uses $\rho_t = 0.6$. Uniform random destroy samples $\rho_t \sim \text{Uniform}(0.1, 0.6)$ independently for each graph and refinement step. Graph-rate destroy uses a learned graph-level probability shared by all eligible vertices. Nodewise destroy samples directly from per-node Bernoulli probabilities. LDR uses rate-matched inference: it first averages the per-node probabilities over the eligible vertex set and then samples a random mask with that average probability. The pretraining ablation compares a noise-only repair checkpoint with a multi-source repair checkpoint. The fine-tuning ablation compares learned-destroy inference with the pretrained repair checkpoint and with the repair checkpoint fine-tuned over learned-destroy trajectories, where the multilinear repair loss is applied to the initial construction output and all post-destroy repair outputs.

Table 5: Evaluation hyperparameters used for the reported internal experiments.

Hyperparameter	Value
GPU	NVIDIA A100-SXM4-40GB
Datasets	RB-[200–300], RB-[800–1200], BA-[200–300], BA-[800–1200]
Split	validation
Instances per dataset	100
Evaluation seeds	0, 1, 2
Batch size	32
Repair samples per graph	8
Destroy-repair iterations	256
Repair steps per iteration	2
Total repair calls	512
Conflict coefficient λ	1.5
Binary coefficient β	0.0
Repair hidden dimension / layers	128 / 5
Destroy hidden dimension / layers	64 / 4
Global graph features in destroy	yes
Fixed random destroy probability	0.6
Uniform random destroy range	$[0.1, 0.6]$
LDR rate matching	matched per graph and step

I Additional Experimental Details

I.1 Additional Design Ablations

Table 7 checks whether repair depth and step-feature design interact. Here R is the number of recurrent repair steps inside one repair call, and T is the number of outer destroy-repair iterations. We keep the total number of repair calls fixed at $R \times T = 512$ and evaluate the full grid of $R \in \{1, 2, 4\}$ and τ_t encodings in {sinusoidal, linear, none}. The interaction does not change the main conclusion: $R = 1$ is marginally higher but slower, $R = 2$ gives a practical speed-quality trade-off, and the sinusoidal step feature gives the highest score across all tested repair depths.

I.2 Additional Distribution and Failure Diagnostics

Table 8 reports an additional MIS evaluation on ER-[700–800] graphs. The result shows that destroy-before-repair still improves over repair-only on this graph family, but the learned rate-matched rule does not outperform the strong uniform random-destruction baseline. This supports the narrower scope statement in Section 4: learned destruction is useful as a refinement mechanism, but the learned rate is not uniformly better across graph distributions.

Table 9 summarizes when LDR does or does not improve over uniform random destruction on MIS RB-[800–1200]. We bucket instances by the per-instance score difference between LDR and uniform random destruction. The worst bucket for LDR has a substantially lower learned destruction rate than the uniform baseline and is also sparser by density and average degree. The best bucket uses a more moderate learned rate and is denser. Thus the current learned-rate policy can be too conservative on some sparse instances, while it helps when the predicted perturbation scale is large enough to move away from the current maximal independent set without excessive disruption.

Table 10 compares LDR with KaMIS on the same MIS RB-[800–1200] validation instances used in our LDR evaluation. We use the larger RB split here because the small split leaves little room between LDR and the classical reference. On RB-[800–1200], the remaining gap is more informative: the largest-gap bucket is sparser and uses a lower learned destruction rate than the closest-to-KaMIS bucket. This is consistent with the limitation discussed in Appendix E: the destroy network is trained from graph-level reward and is

Table 6: Full internal results corresponding to Table 2. Scores are mean ± standard deviation over 100 instances and three seeds. $\bar{p}$ is the mean or expected graph-level destroy probability, and $\bar{d}$ is the mean number of destroyed variables per step when logged.

Task	Dataset	Method	Best score ↑	Improvement ↑	Best step	Time (s) ↓	$\bar{p}$	$\bar{d}$
MIS	RB-[200–300]	Repair	18.12 ± 1.43	0.09	0.1	2.04	–	–
		Fixed $p = .6$	20.15 ± 1.45	2.13	29.4	2.25	0.600	11.1
		Uniform $[.1, .6]$	20.05 ± 1.43	2.02	25.4	2.30	0.350	–
		Graph-rate	19.86 ± 1.46	1.83	26.7	2.37	0.243	4.4
		Nodewise	20.27 ± 1.48	2.24	16.5	2.46	0.250	4.9
		LDR	20.28 ± 1.47	2.25	26.0	2.43	0.266	5.1
		No FT	20.12 ± 1.45	1.53	17.6	2.37	–	–
		With FT	20.26 ± 1.48	2.24	16.5	2.40	–	–
		Noise only	18.26 ± 1.65	0.37	0.7	1.80	–	–
		Multi-source	19.08 ± 1.42	0.49	0.6	2.15	–	–
MIS	RB-[800–1200]	Repair	34.67 ± 2.82	0.52	0.5	7.44	–	–
		Fixed $p = .6$	40.08 ± 3.24	5.93	78.8	8.03	0.600	21.7
		Uniform $[.1, .6]$	40.02 ± 3.32	5.87	72.4	8.18	0.350	–
		Graph-rate	40.18 ± 3.36	6.03	54.3	8.88	0.123	4.8
		Nodewise	40.39 ± 3.30	6.24	87.8	8.82	0.198	7.7
		LDR	40.58 ± 3.24	6.43	92.4	8.84	0.201	7.9
		No FT	40.39 ± 3.31	6.15	97.7	8.80	–	–
		With FT	40.43 ± 3.31	6.28	95.8	8.87	–	–
		Noise only	34.44 ± 3.15	0.28	0.3	7.42	–	–
		Multi-source	34.25 ± 3.01	0.00	0.0	7.40	–	–
MaxCut	BA-[200–300]	Repair	734.36 ± 83.51	4.77	2.4	0.33	–	–
		Fixed $p = .6$	735.40 ± 83.61	5.81	35.0	0.36	0.600	150.7
		Uniform $[.1, .6]$	735.34 ± 83.59	5.75	24.4	0.37	0.350	–
		Graph-rate	735.18 ± 83.64	5.60	13.2	0.47	0.070	17.6
		Nodewise	735.37 ± 83.64	5.78	9.3	0.47	0.101	25.3
		LDR	736.42 ± 83.70	6.83	9.7	0.46	0.101	25.4
		No FT	735.37 ± 83.64	5.78	9.3	0.43	–	–
		With FT	735.37 ± 83.64	5.78	9.3	0.43	–	–
		Noise only	733.51 ± 83.55	5.38	2.4	0.33	–	–
		Multi-source	734.36 ± 83.51	4.77	2.4	0.33	–	–
MaxCut	BA-[800–1200]	Repair	2951.71 ± 350.57	37.11	9.4	0.97	–	–
		Fixed $p = .6$	2942.80 ± 346.62	28.20	125.1	1.00	0.600	597.9
		Uniform $[.1, .6]$	2957.48 ± 349.21	42.88	174.3	1.00	0.350	–
		Graph-rate	2944.21 ± 347.51	29.61	213.1	1.29	0.449	447.9
		Nodewise	2962.98 ± 350.80	48.38	69.2	1.39	0.711	708.6
		LDR	2963.03 ± 350.68	48.43	70.7	1.38	0.711	708.7
		No FT	2963.05 ± 350.84	48.45	71.3	1.34	–	–
		With FT	2963.00 ± 350.78	48.40	72.8	1.35	–	–
		Noise only	2946.50 ± 349.35	37.42	9.3	0.96	–	–
		Multi-source	2951.69 ± 350.52	37.09	9.4	0.96	–	–

Table 7: Repair-depth and step-feature interaction ablation on MIS RB-[200–300] with default rate-matched inference. Each entry reports mean best score / runtime in seconds per graph over 100 validation graphs and three seeds. Higher score is better.

Step feature	$R = 1, T = 512$	$R = 2, T = 256$	$R = 4, T = 128$
sinusoidal	20.29 / 4.54	20.28 / 2.39	20.22 / 1.35
linear	19.97 / 4.42	19.94 / 2.37	19.86 / 1.33
none	20.19 / 4.53	20.13 / 2.40	20.05 / 1.34

therefore most reliable as a destruction-rate estimator, whereas a specialized solver can use more structured and coordinated search moves.

Random destroy baseline. The fixed and uniform random baselines use explicit random masks, not untrained destroy networks. The uniform baseline samples one graph-level rate $\rho_t \sim \text{Uniform}(0.1, 0.6)$ per

Table 8: Additional MIS evaluation on ER-[700–800] graphs. Scores are mean best independent-set sizes over the evaluated validation instances; higher is better.

Method	Best score ↑
Repair only	42.18
Uniform $[.1, .6]$	**44.35**
LDR (rate-matched)	43.52

Table 9: Failure-case diagnostic for LDR versus uniform random destruction on MIS RB-[800–1200]. Buckets are formed by the per-instance difference LDR − uniform. Δ is the mean score difference in each bucket. $\bar{\rho}$ is the mean learned destruction rate used by LDR; the uniform baseline has expected rate about 0.30.

Bucket	Δ score	LDR $\bar{\rho}$	Density	Avg. degree	Best step
Best for LDR	0.33	0.240	0.111	107.8	114.4
Middle	-0.06	0.213	0.100	101.5	92.0
Worst for LDR	-0.21	0.129	0.075	75.4	71.6

graph and refinement step, and then samples the concrete mask from that rate. For MIS, random masks are sampled only over active coordinates of the current continuous state; for MaxCut, random masks are sampled over all vertices and the masked variables are flipped. LDR differs from fixed and uniform random destruction because it first computes the learned destroy policy's average per-node probability at the current graph and step, then samples a random mask with that probability. This rate-matched rule reuses the learned destruction amount while avoiding reliance on nodewise probability variation.

MIS decoder ablation. Table 14 directly checks how much the reported score-greedy maximal decoder contributes to MIS performance. Replacing the reported decoder by the weaker conflict-refinement decoder reduces the RB-[200–300] mean score from 20.28 to 20.17 while reducing runtime from 3.53s to 1.45s per graph. The 0.110-vertex gap indicates that the full greedy constructor is not the sole source of the improvement.

Training curves. Figure 3 shows training diagnostics for destroy training and repair fine-tuning on RB-[200–300]. We report the destroy stage's average episode return and the repair fine-tuning loss over epochs. The curves improve during training and do not show collapse in the selected runs.

Feasibility diagnostics. Table 13 summarizes the feasibility diagnostics. For MIS, the reported score-greedy decoder enforces feasibility and maximality. In the reported evaluation, the post-decoding conflict count is zero for every method, instance, and seed, and the feasibility and maximality indicators are both one for all MIS rows. For MaxCut, every binary partition is feasible, so the feasibility field is always one and the maximality field is unused.

Table 10: Gap diagnostic between LDR and KaMIS on MIS RB-[800–1200]. The first row reports the aggregate mean score gap and positive-gap rate. The remaining rows bucket instances by the LDR–KaMIS gap.

Bucket	Mean score gap	Positive-gap rate	Density	Avg. degree	LDR $\bar{\rho}$
All instances	2.57	88%	0.097	97.5	0.201
Closest to KaMIS	0.22	–	0.099	100.0	0.213
Middle	1.54	–	0.102	101.8	0.216
Largest gap to KaMIS	3.03	–	0.082	82.1	0.148

Table 11: LDR best-score and runtime at selected inference budgets. Each score row is followed by its cumulative runtime row in seconds.

Task	Dataset	0	8	16	32	64	128	256
MIS	RB-[200–300]	18.03	19.68	19.87	20.02	20.12	20.24	20.28
	runtime	0.01	0.08	0.15	0.30	0.60	1.20	2.43
MIS	RB-[800–1200]	34.15	37.93	38.71	39.38	39.92	40.30	40.58
	runtime	0.03	0.29	0.55	1.09	2.19	4.40	8.84
MaxCut	BA-[200–300]	729.59	734.87	735.13	735.28	735.36	735.37	736.42
	runtime	0.00	0.02	0.03	0.06	0.12	0.23	0.46
MaxCut	BA-[800–1200]	2914.60	2951.94	2956.45	2959.52	2961.61	2962.79	2963.03
	runtime	0.00	0.05	0.09	0.18	0.35	0.69	1.38

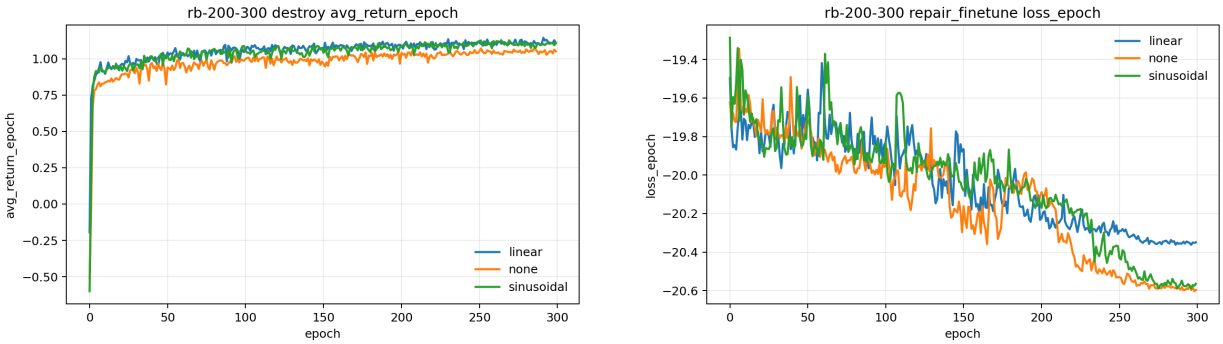

Figure 3: Training curves on RB-[200–300] for the alternating stages. Left: average episode return for destroy training, computed as the unnormalized sum of the raw post-repair score-improvement rewards. Right: multilinear repair fine-tuning loss. The destroy return improves from -0.600 to 1.110, while the repair fine-tuning loss decreases from -19.288 to -20.564.

Table 12: Pairwise comparisons over matched instance-seed pairs. The last column reports wins/ties/losses for the first method in each comparison when pairwise counts are available.

Task	Dataset	Comparison	Mean diff.	W/T/L
MIS	RB-[200–300]	LDR − Repair	2.16	283/17/0
		LDR − Fixed	0.12	43/249/8
		LDR − Uniform	0.23	–
		LDR − Graph-rate	0.42	91/200/9
		LDR − Nodewise	0.01	23/257/20
		With FT − No FT	0.14	57/229/14
		Multi-source − Noise	0.82	181/88/31
MIS	RB-[800–1200]	LDR − Repair	5.91	300/0/0
		LDR − Fixed	0.50	159/100/41
		LDR − Uniform	0.56	–
		LDR − Graph-rate	0.41	107/164/29
		LDR − Nodewise	0.19	83/178/39
		With FT − No FT	0.04	68/175/57
		Multi-source − Noise	-0.19	85/94/121
MaxCut	BA-[200–300]	LDR − Repair	2.06	–
		LDR − Fixed	1.02	–
		LDR − Uniform	1.08	–
		LDR − Graph-rate	1.24	–
		LDR − Nodewise	1.05	–
		With FT − No FT	0.00	0/300/0
		Multi-source − Noise	0.85	155/103/42
MaxCut	BA-[800–1200]	LDR − Repair	11.32	291/6/3
		LDR − Fixed	20.23	300/0/0
		LDR − Uniform	5.55	–
		LDR − Graph-rate	18.82	299/0/1
		LDR − Nodewise	0.05	80/126/94
		With FT − No FT	-0.05	47/202/51
		Multi-source − Noise	5.19	235/16/49

Table 13: Feasibility diagnostics for the main internal comparison. Feasible and maximal columns are percentages. Pre- and post-conflict columns are mean decoded conflict counts. For MIS, the reported score-greedy decoder constructs a maximal independent set. Maximality is not applicable to MaxCut because every binary partition is feasible.

Task	Dataset	Method	Feasible (%)	Maximal (%)	Pre-conflict	Post-conflict
MIS	RB-[200–300]	Repair	100.0	100.0	0.007	0.000
		Fixed $p = .6$	100.0	100.0	0.000	0.000
		Uniform $[.1, .6]$	100.0	100.0	0.003	0.000
		Graph-rate	100.0	100.0	0.003	0.000
		Nodewise	100.0	100.0	0.000	0.000
		LDR	100.0	100.0	0.007	0.000
MIS	RB-[800–1200]	Repair	100.0	100.0	0.197	0.000
		Fixed $p = .6$	100.0	100.0	0.287	0.000
		Uniform $[.1, .6]$	100.0	100.0	0.210	0.000
		Graph-rate	100.0	100.0	0.110	0.000
		Nodewise	100.0	100.0	0.043	0.000
		LDR	100.0	100.0	0.097	0.000
MaxCut	BA-[200–300]	Repair	100.0	–	0.000	0.000
		Fixed $p = .6$	100.0	–	0.000	0.000
		Uniform $[.1, .6]$	100.0	–	0.000	0.000
		Graph-rate	100.0	–	0.000	0.000
		Nodewise	100.0	–	0.000	0.000
		LDR	100.0	–	0.000	0.000
MaxCut	BA-[800–1200]	Repair	100.0	–	0.000	0.000
		Fixed $p = .6$	100.0	–	0.000	0.000
		Uniform $[.1, .6]$	100.0	–	0.000	0.000
		Graph-rate	100.0	–	0.000	0.000
		Nodewise	100.0	–	0.000	0.000
		LDR	100.0	–	0.000	0.000

Table 14: MIS decoder ablation on RB-[200–300]. All rows use the same trained LDR model, 100 validation graphs, three seeds, and a 512 repair-call budget. The reported decoder is the score-greedy maximal constructor. The weaker refinement decoder thresholds the continuous state and only removes conflicted endpoints, without greedily adding vertices.

Decoder	MIS ↑	Time ↓	Feasible
Conflict refinement	20.173	1.45s	100%
Reported greedy maximal	20.283	2.43s	100%