LLM-Guided Search for Deletion-Correcting Codes

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Abstract

Finding deletion-correcting codes of maximum size has been an open problem for over 70 years, even for a single deletion. In this paper, we adapt FunSearch, a large language model (LLM)-guided evolutionary search, to discover functions that construct large deletion-correcting codes. For a single deletion, our search discovers a construction provably equivalent to the conjectured-optimal Varshamov-Tenengolts code. We study design choices for LLM-guided evolutionary search and find that, perhaps surprisingly, compute is better allocated to sampling more functions than extending reasoning per function or using larger models. We also find that evolving descriptions along with code is not beneficial, and propose deduplicating logically identical functions during evolution, which we find critical for search diversity. These results demonstrate the potential of LLM-guided evolutionary search for information theory and code design and represent the first application of such methods for constructing error-correcting codes.

1 Introduction

Large language models (LLMs) are increasingly used for scientific discovery and problem solving, in particular for mathematical and algorithmic problems (Fawzi et al., 2022; Mankowitz et al., 2023; Guo et al., 2025b). An important class of methods for discovering new algorithms and mathematics is LLM-guided evolutionary search (Romera-Paredes et al., 2024; Novikov et al., 2025; Lange et al., 2025; Yu et al., 2025; Assumpção et al., 2025). Romera-Paredes et al. (2024) demonstrated that LLM-guided evolutionary search can discover new algorithmic solutions for problems that are difficult to solve but easy to evaluate. Their method, FunSearch (Function Space Search), represents combinatorial problems as Python code and searches for algorithmic solutions, improving on best-known results for the cap set and online bin-packing problems.

In this paper, we adapt FunSearch to find new error-correcting codes and study FunSearch variants to see which design choices enable solving algorithmic problems efficiently.

Error-correcting codes enable reliable communication and data recovery from storage media (such as HDDs and SSDs), even in the presence of errors and defects. In a typical coding scheme, an encoder maps information to a codeword, which is corrupted by errors during transmission, and a decoder attempts to recover the original message. While substitutions and erasures are well understood with optimal encoding and decoding algorithms approaching known theoretical limits, deletions are significantly more challenging. Deletions shift subsequent symbols, disrupting the memoryless property typically assumed in coding theory.

For a fixed number of correctable errors, better codes have larger code sizes. Despite significant effort, determining the maximum code size for a fixed number of adversarial deletions has proven difficult using traditional human-driven approaches to information theory. A class of codes known as Varshamov-Tenengolts (VT) codes (Varshamov & Tenengolts, 1965) achieves the maximum code size for correcting a single deletion as the code length approaches infinity (Levenshtein, 1966). However, for finite code lengths, the gap to the upper bound on code size is large even at moderate lengths (Kulkarni & Kiyavash, 2013). Although VT codes are conjectured to be maximum-size for all lengths and a single deletion, their optimality has only been proven for lengths up to 11 (Butenko et al., 2002; Nakasho et al., 2023; Sloane, 2002).

LLMs and evolutionary search offer a promising new approach to constructing deletion-correcting codes, and we adapt FunSearch for this problem as follows. We set up the LLM to generate priority functions

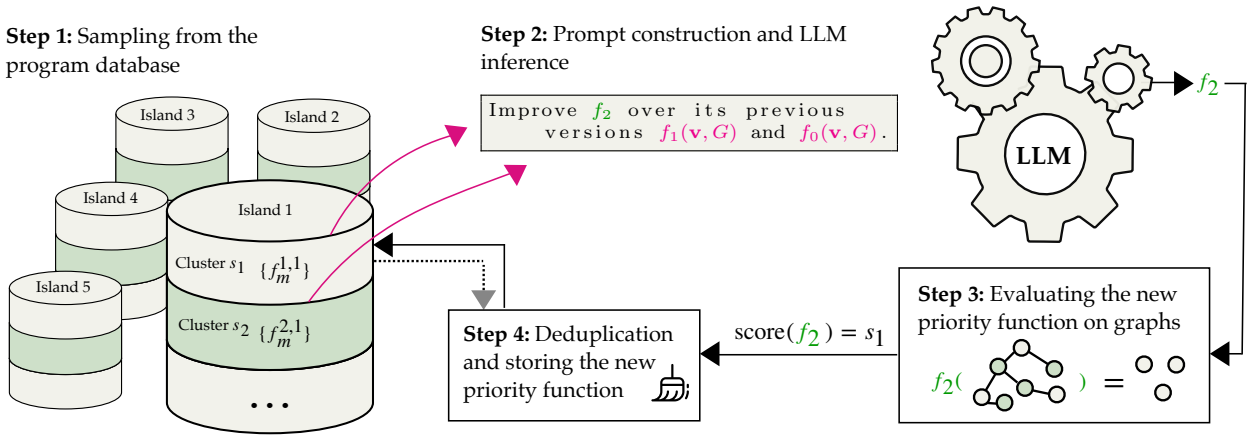

Figure 1: FunSearch for finding deletion-correcting codes iteratively refines a priority function through evolutionary search guided by a pretrained LLM. In each iteration, a few-shot prompt is constructed by sampling from the database. The LLM generates a new priority function, which is evaluated by greedily constructing deletion-correcting codes for different code lengths and numbers of deletions. If executable and not a duplicate, the function is added to the database.

that greedily construct codes by assigning priorities to sequences and iteratively adding the highest-priority sequence while ensuring error-correcting constraints are satisfied. We focus on binary deletion-correcting codes, where many fundamental questions remain open (Sloane, 2002), and additionally consider quaternary insertion-deletion-substitution (edit) codes motivated by biological sequencing and DNA data storage (Bar-Lev et al., 2024). There, we outperform a recent neural approach (Guo et al., 2025a).

While FunSearch relies on sampling many functions, viewing the LLM as a source of diversity with occasionally useful ideas, subsequent work has focused on investing more compute per function, reporting better sample efficiency but without controlling for total inference budget (Liu et al., 2024; Ye et al., 2024). Under a fixed inference budget, investing more compute per function reduces the number of functions sampled, raising the question of whether discovery comes from diversity through sampling or reasoning about the problem. We study this trade-off and find that for our problem, sampling more priority functions is more effective than investing more reasoning per function.

Our main contributions are:

- For a single deletion, our search matches conjectured-optimal VT code sizes for all tested lengths, including one function we prove optimal for any length. For multiple deletions and quaternary edit codes, our functions outperform explicit constructions (Helberg & Ferreira, 2002; Abdel-Ghaffar et al., 2011), search-based methods (Landjev & Haralambiev, 2007; Swart & Ferreira, 2003), and a recently proposed neural method (Guo et al., 2025a), though they do not match the best-known sizes obtained by general-purpose graph solvers.

- We study design choices for LLM-guided evolutionary search and find that, perhaps surprisingly, for our problem, sampling more functions is more effective than investing more reasoning per function, and co-evolving natural language descriptions with code hurts overall search quality. Moreover, we propose to deduplicate logically identical functions during evolution, which we find to be critical, and without it we could not find any function matching VT code sizes.

- Since sampling many priority functions is more effective for our problem, scaling the search is important. Unlike existing implementations (Sharma, 2025; Liu et al., 2024; Ye et al., 2024; Assumpção et al., 2025) that parallelize on a single node with API-based inference, our implementation scales across multiple nodes via asynchronous message passing and uses batched LLM inference with locally hosted models. We release our code to facilitate future discovery by LLM-guided evolutionary search.

Our results demonstrate the potential of LLM-guided evolutionary search for information and coding theory, and provide initial evidence that investing more compute per function does not necessarily improve search quality. However, our current approach does not scale well to long codes, a limitation we discuss in more detail later.

2 Related work

We review related work on LLM-guided evolutionary search and deletion-correcting codes.

2.1 Related work on LLM-guided evolutionary search

As mentioned, our work builds on existing approaches to LLM-guided evolutionary search, in particular FunSearch (Romera-Paredes et al., 2024). Lehman et al. (2023) first demonstrate the effectiveness of using LLMs to aid evolutionary algorithms, using the LLM as an intelligent mutator for automatic data generation. AlphaEvolve (Novikov et al., 2025) extends FunSearch by evolving entire codebases rather than individual functions. Other applications of LLM-guided evolutionary search include neural architecture search (Zheng et al., 2023; Liu et al., 2025; Chen et al., 2023; Nasir et al., 2024), prompt optimization (Yang et al., 2023; Fernando et al., 2024), reward design for reinforcement learning (Ma et al., 2023; Hazra et al., 2024), scientific discovery in mathematics (Georgiev et al., 2025), and systems research (Cheng et al., 2025).

The most relevant application to finding error-correcting codes is heuristic design for combinatorial optimization. Liu et al. (2024) propose EoH, which improves performance and sample efficiency over FunSearch by evolving both algorithmic summaries and code jointly. Ye et al. (2024) introduce ReEvo, which incorporates reflection into the search by prompting the LLM to compare previously generated solutions. Dat et al. (2024) propose two diversity metrics and find that FunSearch and ReEvo stagnate in local optima due to low diversity, while EoH trades off diversity for performance.

2.2 Related work on deletion-correcting codes

The best-known single-deletion-correcting codes are VT codes (Varshamov & Tenengolts, 1965). For code length n and parameter $a \in \mathbb{Z}$, the $\mathrm{VT}_a(n)$ code is defined as the set of binary sequences $\mathbf{v} \in \{0,1\}^n$ satisfying $\sum_{i=1}^{n} iv_i \equiv a \pmod{n+1}$, where each bit v_i is weighted by its position i in the sequence. Levenshtein (1966) proved that the $\mathrm{VT}_0(n)$ code is asymptotically optimal and proposed a linear-time decoding algorithm. The $\mathrm{VT}_0(n)$ code is also conjectured to be largest for all finite lengths, but this has only been proven for lengths up to $n = 11$ (for $n \leq 8$, Sloane (2002); for $n \leq 10$, Butenko et al. (2002); for $n \leq 11$, Nakasho et al. (2023)).

For multiple deletions $s > 1$, the optimal redundancy is known to be between $s \log n$ and $2s \log n$ (Levenshtein, 2002). Several constructions exist that are asymptotically optimal. Brakensiek et al. (2018) achieve $O(s^2 \log s \log n)$ redundancy for any number of deletions s, Gabrys & Sala (2019) and Sima et al. (2020b) achieve $8 \log n + o(\log n)$ and $7 \log n + o(\log n)$ for $s = 2$, Sima & Bruck (2021) achieve $8s \log n + o(\log n)$ for any s (non-systematic), Sima et al. (2020a) achieve $4s \log n + o(\log n)$ for any s with a systematic construction, and Guruswami & Håstad (2021) achieve $4 \log n + O(\log \log n)$ for $s = 2$.

These asymptotically optimal constructions all follow a common approach, where the codeword is partitioned into short blocks and block boundaries and contents are protected separately. Block boundaries are typically recovered either by majority voting over multiple synchronization patterns (Brakensiek et al., 2018; Gabrys & Sala, 2019) or by using generalized VT codes for constrained indicator vectors (Sima & Bruck, 2021; Sima et al., 2020a; Guruswami & Håstad, 2021). For block contents, the constructions rely on short deletion-correcting codes, which are nontrivial to construct and require block lengths to be short enough for exhaustive search. Therefore, progress on finite-length deletion-correcting codes directly improves these asymptotic constructions, which motivates the finite-length regime we consider in this paper.

For finite-length multiple deletion-correcting codes, Helberg & Ferreira (2002) extend VT codes with an explicit construction, later proven to correct multiple deletions by Abdel-Ghaffar et al. (2011), though the resulting code sizes are small for longer lengths. Swart & Ferreira (2003) find larger codes for two deletions and lengths $n \leq 12$ by greedy search over random permutations in a run-length representation. Similarly,

```
"""
Finds large independent set in graph G where nodes are binary strings of length n.
Nodes in G are connected if they share a subsequence of length at least n-s.
Improve f1 over its previous versions below.
"""
import numpy as np
import networkx as nx

def f0(v, G):
    """Returns the priority with which we want to add node v. Higher priority nodes are
    added first."""
    return 0.0

def f1(v, G):
    """Improved version of f0"""
```

Figure 2: Prompt with function f_0 that initializes all islands.

Landjev & Haralambiev (2007) search in the space of all binary sequences, constructing codes for $s = 2, 3, 4, 5$ deletions and lengths $n \leq 30$.

3 Problem statement

We consider the problem of constructing large n-bit, s-deletion-correcting codes.

A deletion-correcting code is a set of sequences such that, even if an adversary deletes s bits from a sequence, the original sequence can still be uniquely recovered. A subsequence of length $n - s$ is obtained by deleting s bits while preserving the order of the remaining bits. Unique recovery is not possible if two sequences share a common subsequence. Thus, an n-bit, s-deletion-correcting code is a set $\mathcal{C} \subseteq \{0, 1\}^n$ such that no two distinct codewords $\mathbf{c}, \mathbf{c}' \in \mathcal{C}$ share a common subsequence.

The problem of constructing n-bit, s-deletion-correcting codes can be reduced to finding an independent set in a graph G defined as follows (Shannon, 2003). Let G be an undirected graph where each node is a binary sequence of length n, and two nodes are connected by an edge if and only if they share a common subsequence of length at least $n - s$. Each choice of code length n and deletion count s defines a problem instance with a corresponding graph G. An independent set $\mathcal{I}$ in graph G is a subset of nodes such that no two are connected by an edge. An n-bit, s-deletion-correcting code is an independent set in graph G.

To construct deletion-correcting codes, we greedily build independent sets $\mathcal{I}$ in graph G. Let $f(\mathbf{v}, G)$ be a priority function that assigns a real-valued priority to each node $\mathbf{v}$. At each step, we select the node with the highest priority, add it to $\mathcal{I}$, and remove the node and its neighbors from G. We break ties by selecting the lexicographically smallest node (with 0 smaller than 1). The size of the resulting independent set depends on the choice of the priority function f.

In this formulation, constructing large n-bit, s-deletion-correcting codes reduces to designing a priority function f that maximizes the size of the independent set $\mathcal{I}$. Finding a maximum independent set is NP-complete in general (Lovász & Plummer, 2009) (see Appendix C for a complexity analysis of our specific problem instances).

4 Method

We adapt FunSearch (Romera-Paredes et al., 2024) with a deduplication step to optimize the priority function f to construct large deletion-correcting codes. While conceptually simple, without our deduplication step, we could not find any functions matching conjectured-optimal sizes. We deduplicate at the output level, which requires less compute and is more interpretable than approaches that use embedding models or LLM-as-a-judge (Liu et al., 2025; Lange et al., 2025).

FunSearch works iteratively: in each iteration, we sample existing priority functions as few-shot examples, prompt a pretrained LLM to generate a new one, and evaluate it on a set of search instances (i.e., specific code lengths n and deletion counts s). See Figure 1 for an illustration.

4.1 Sampling from the database

We organize the priority functions in a database divided into independent sub-databases called islands. We sample few-shot examples from a single island j and store new functions back on the same island, allowing the islands to explore independently.

Within each island, we group priority functions into clusters. Two functions belong to the same cluster i if they achieve the same independent set sizes across all search instances. All functions in a cluster share the same score, which becomes the cluster's score_i.

To sample a priority function, we first sample an island j uniformly at random. We then sample a cluster i from island j with probability

$$p_i = \frac{e^{\text{score}_i\,/T_j}}{\sum_{i'} e^{\text{score}_{i'}\,/T_j}}, \quad \text{where } T_j = T\left(1 - \frac{n_j \bmod P}{P}\right).$$

The temperature T_j balances exploration and exploitation on island j. It depends on an initial temperature T, the number of stored functions n_j, and a sampling period P. As more functions are stored, the temperature decreases, shifting from uniform sampling (exploration) to favoring high-scoring clusters (exploitation). The temperature resets every P functions to periodically reintroduce exploration. We then sample a function from the cluster, favoring shorter ones.

4.2 Prompt construction and LLM inference

We construct few-shot prompts by sampling twice from the database to obtain two priority functions. Sampling is done without replacement for diverse examples. The functions are sorted by their cluster scores, with the lower-scoring one first and the higher-scoring one second as target for improvement. The prompt is framed as a code completion task, ending with the header of a new function for the LLM to complete. The initial prompt is shown in Figure 2.

We use StarCoder2-15B (Lozhkov et al., 2024) to generate new priority functions, an open-access model with 15 billion parameters trained on The Stack v2 dataset (775B tokens from 600+ programming languages) and additional tokens from sources like pull requests, issues, Jupyter notebooks, and Stack Overflow, totaling 913B tokens.

4.3 Evaluating the new priority function on graphs

We evaluate each new priority function on a set of search instances, each defined by a code length n and deletion count s. For each instance, we greedily construct an independent set $\mathcal{I}$ by iteratively adding the highest-priority node to $\mathcal{I}$ and removing it and its neighbors from G. We pass G as an immutable object to the priority function as otherwise the LLM generates functions that remove edges from G to obtain a larger independent set. This is a form of reward hacking that has also been observed in other works (Georgiev et al., 2025; Zhang et al., 2025).

We discard functions that are not executable (e.g., due to syntax errors). We assign each executable priority function a score based on the independent set size achieved on the search instance with the largest code length n, which we found to outperform alternatives such as averaging sizes. After the search, we evaluate the functions achieving the largest independent sets across all search instances on held-out instances with unseen code lengths or deletion counts to assess generalization.

4.4 Deduplication and storing the new priority function

We store a new priority function on the same island j from which the few-shot examples were sampled and assign it to a cluster based on the independent set sizes it achieves across all search instances, creating a new

```
def f(v, G, n, s):
    # The condition ord(a) > 125 has no effect, as the ASCII values of '0' and '1' are
    always below 125.
    v = ''.join(['-' * (ord(a) > 125) + a for a in list(v)])
    onepositions = [c for c, d in reversed(list(enumerate(v, start=-len(v)))) if d == '1']
    negonesum = sum([-c for c in onepositions])
    # Maximum of negonesum is (n-1)/2 for n odd and n/2 for n even, which is always < n, so
    taking mod n does not change the priority
    finalans = (⌊negonesum/((n + s) · 1)⌋ % n)
    return finalans
```

Figure 3: Discovered priority function provably equivalent to the conjectured optimal $\mathrm{VT}_0(n)$ code. We added comments for clarity.

cluster if no match exists. Before adding a function to a cluster, we check for duplicates, i.e. functions that implement the same logic with different syntax. Determining if two functions are duplicates is non-trivial. We find that comparing at the output level is sufficient and consider two functions duplicates if the hash of all priority scores assigned to all sequences of the shortest code length matches an existing function in the cluster, and discard the new function if a matching hash exists.

4.5 Island initialization and resets

Each island is initialized with the same trivial priority function f_0 shown in Figure 2, which assigns equal priority to all sequences. To allow information exchange between islands, we periodically reset them. During a reset, we discard the stored priority functions in the worst-performing half of islands, where performance is measured by the highest score_i on each island. We then re-initialize each discarded island with the best-performing function from a random surviving island. We reset islands after every R stored functions rather than after a fixed time interval as in Romera-Paredes et al. (2024), to separate the reset logic from the rate at which functions are generated and evaluated.

5 Experiments

We first test whether LLM-guided evolutionary search can match conjectured-optimal code sizes for a single deletion and rediscover the $\mathrm{VT}_0(n)$ code, as well as find new optimal constructions distinct from any $\mathrm{VT}_a(n)$ code within the search range ($n \leq 11$), though these do not generalize to longer lengths.

We then use reasoning models to discover new two- and three-deletion-correcting codes, studying the trade-off between sampling more priority functions versus spending more compute per function. We find that sampling more functions is more effective than reasoning more per function.

Motivated by applications in DNA data storage and biological sequencing, we additionally search for quaternary edit codes, finding codes that outperform a recent neural approach (Guo et al., 2025a).

We report results over three runs with fixed random seeds for LLM generation and database sampling, though runs are not fully deterministic due to distributed execution. Our primary metric is the normalized gap to best-known code sizes, averaged over search instances. We report the top-50 mean gap, averaged over the 50 best unique clusters and runs, as a measure of overall search quality. To assess generalization and compare against baselines, we select the function with minimum normalized gap on search instances and evaluate it on a set of held-out instances. We summarize computational costs for each run in Appendix A.

Baselines. We compare against explicit constructions from coding theory (Varshamov & Tenengolts, 1965; Helberg & Ferreira, 2002; Abdel-Ghaffar et al., 2011), search-based methods over all binary sequences (Landjev & Haralambiev, 2007), and a greedy construction over 10^5 random orderings. We additionally compare against general-purpose graph solvers: OnlineMIS and ReduMIS from KaMIS (Dahlum et al., 2016; Lamm et al., 2016) (single-threaded, 3 seeds, 24-hour timeout, best result reported), and Gurobi (Gurobi Optimization, LLC, 2025) as an integer linear programming (ILP) solver (16 threads, 24-hour timeout). We

Table 1: Code sizes for binary single-deletion-correcting codes. Bold indicates best known, T/O no solution found within 48 h (24 h, extended to 48 h if no solution returned).

	Code length n														
Method	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
LP upper bound	**10**	17	**30**	53	96	175	321	593	1104	2251	4202	7882	14845	28059	53202
ILP upper bound	**10**	**16**	**30**	**52**	**94**	173	320	592	1533	2883	5441	T/O	T/O	T/O	T/O
ILP lower bound	**10**	**16**	**30**	**52**	**94**	**172**	242	436	737	1345	2468	4521	8345	15495	28832
MIS solver: OnlineMIS	**10**	**16**	**30**	**52**	**94**	143	254	456	816	1469	2657	4843	8908	16439	30440
MIS solver: ReduMIS	**10**	**16**	**30**	**52**	**94**	**172**	**316**	457	812	1453	2634	4803	8793	16180	30111
Random greedy	**10**	**16**	26	42	69	116	201	351	620	1111	2003	3636	6644	12215	22544
$\mathrm{VT}_0(n)$ codes	**10**	**16**	**30**	**52**	**94**	**172**	**316**	**586**	**1096**	**2048**	**3856**	**7286**	**13798**	**26216**	**49940**
Ours (search over $s = 1$)	**10**	**16**	**30**	**52**	**94**	**172**	**316**	**586**	**1096**	**2048**	**3856**	**7286**	**13798**	**26216**	**49940**
Ours (search over $s \in \{1, 2\}$)	**10**	**16**	**30**	**52**	**94**	**172**	**316**	**586**	**1096**	2042	2474	4539	8373	15451	28697

do not directly compare against neural maximum independent set solvers, as classical solvers currently still outperform them (Böther et al., 2022; Wu et al., 2025). For quaternary edit codes, we additionally compare against DoDo-Code (Guo et al., 2025a), a recent neural approach.

Upper Bounds. As upper bounds, we solve the linear programming (LP) relaxation of the fractional transversal problem, which gives the tightest bounds for the regimes we consider (Kulkarni & Kiyavash, 2013; Fazeli et al., 2015). Graph statistics for all problem instances are summarized in Appendix G.

5.1 Single-Deletion Correction

We search over code lengths $n \in [6, 11]$, where the maximum code sizes are known and equal to those achieved by the $\mathrm{VT}_0(n)$ code. Smaller code lengths make the problem trivial, while larger lengths result in prohibitive computational and memory costs. Each search runs for 500K iterations, using LLM and evolutionary search hyperparameters from Table 5 in Appendix B, which we found to perform best in smaller-scale experiments.

Table 1 shows that our search matches $\mathrm{VT}_0(n)$ code sizes across all code lengths we consider, while general-purpose graph solvers match optimal sizes only for small lengths $n < 13$, with the gap increasing with n. We find three types of priority functions: (i) functions that construct the same code as $\mathrm{VT}_0(n)$ for all lengths we could verify (including the function in Figure 3, which we prove in Appendix D constructs $\mathrm{VT}_0(n)$ for any length n), (ii) functions that construct the $\mathrm{VT}_0(n)$ code at even lengths and its bitwise complement $\mathrm{VT}_{\frac{n+1}{2}}(n)$ at odd lengths (e.g., Figure 9), and (iii) novel constructions distinct from any $\mathrm{VT}_a(n)$ code within the search range (e.g., Figure 10), though these do not generalize to longer lengths.

While VT codes likely appear in the training data, we argue this is a genuine discovery rather than memorization, as the discovered functions use different construction logic than VT codes typically use.

5.1.1 Ablations

Our default configuration deduplicates priority functions, includes graph G as input to the function, and scores functions on the code size achieved at the largest search length $n = 11$. We ablate each design choice below.

Deduplication. Without deduplication, none of the three runs find a priority function that achieves optimal code sizes on all search lengths, compared to 2 out of 3 runs with deduplication. Figure 4 (right) shows that without deduplication, the search discovers fewer unique signatures, consistent with 65.8% ($\pm$5.4%) of stored functions per island being duplicates.

Graph input. Figure 4 (left) shows that the top-50 mean gap is larger for runs without graph G as input to the priority functions, consistent with only 1 out of 3 runs finding functions that achieves optimal code sizes on all search lengths, compared to 2 out of 3 with graph input. Moreover, with graph input, we additionally discover novel constructions distinct from any $\mathrm{VT}_a(n)$ code, such as the function in Figure 10, though these do not generalize to held-out lengths.

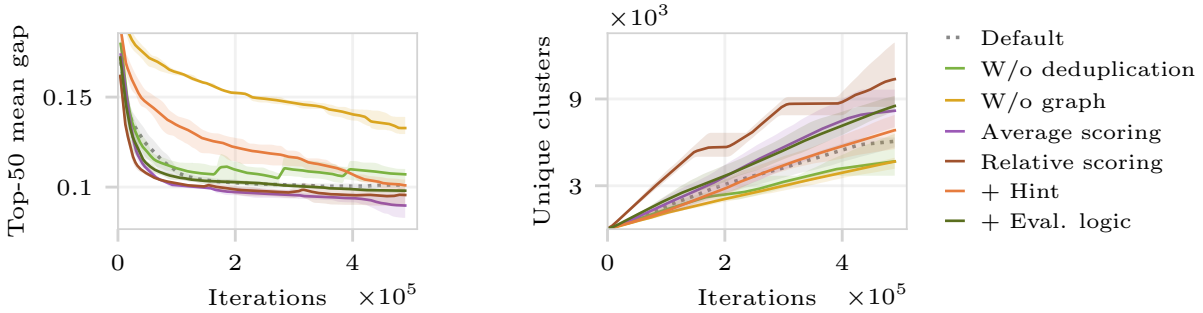

Figure 4: Ablations on single-deletion-correcting codes. Left: gap of the top-50 stored functions (lower is better). Right: unique clusters discovered across all islands (higher is better). Lines are means and shaded bands are min-max ranges across 3 runs.

Scoring function. We consider two alternative scoring functions: (i) averaging code sizes over all lengths $n \in [6, 11]$, and (ii) the average normalized gap to best-known sizes, weighting each length equally. Figure 4 (left) shows that both alternatives achieve a lower top-50 mean gap than the default, which is expected since they directly optimize for average performance. However, our default scoring finds priority functions that achieve optimal code sizes on all search lengths in 2 out of 3 runs, compared to 1 out of 3 for average and 0 out of 3 for normalized gap scoring. This suggests that functions performing well on larger code lengths generalize well to shorter lengths.

Prompt augmentation. Motivated by position-dependent weighting in VT codes, we add a short instruction to the prompt: *"Consider properties of the binary string 'node', such as specific patterns and the number of ones and zeros."* Under this augmentation, the search discovers the priority function in Figure 3, which is provably equivalent to the $\mathrm{VT}_0(n)$ code (see Appendix D for the proof).

Including the evaluation logic as prompt augmentation, i.e., the greedy construction loop that sorts nodes by priority and iteratively builds the independent set, does not find a function achieving optimal code sizes on all search lengths.

5.2 Two-Deletion Correction

We search over code lengths $n \in [7, 12]$ with the same setup as for single-deletion correction and augment the docstring of each few-shot example with the per-instance normalized gap to the best-known code size found by general-purpose graph solvers. We evaluate StarCoder2-15B as a baseline and the Qwen3 series (8B, 14B, 32B), which supports toggling reasoning on and off. Each search runs for a fixed inference budget of \$120, where cost is measured as $C = T_{\text{in}}\, p_{\text{in}} + T_{\text{out}}\, p_{\text{out}}$, with T_{in} and T_{out} the input and output token counts and p_{in}, p_{out} their respective prices per token using Together AI pricing.

Table 2 shows the code sizes achieved by our priority function in Figure 12 and baselines for two-deletion correction. Our priority function outperforms the explicit construction of Helberg & Ferreira (2002) across all lengths we consider. Compared to search-based methods in sequence space (Landjev & Haralambiev, 2007), our function achieves competitive or larger code sizes at longer held-out lengths, despite not being directly optimized for them. This suggests that searching in function space generalizes better than searching in sequence space, which becomes exponentially harder with code length. Across all discovered functions, we match the code sizes achieved by general-purpose graph solvers at search lengths $n \in [7, 12]$, though our single best function constructs smaller codes at $n = 9, 11$ within the search range and at all held-out lengths. We analyze the priority function in Appendix F.

5.2.1 Ablations

We consider two axes of investing more compute per function: model size and reasoning per function. For model size, we compare the Qwen3 series (8B, 14B, 32B) and StarCoder2-15B as a code-only baseline. For reasoning per function, we consider prompting strategies with varying levels of reasoning before generating

Table 2: Code sizes for binary two- and three-deletion-correcting codes. Bold indicates best known, T/O no solution found within 48 h, OOM exceeded 1 TB memory limit.

	Code length n												
Method	6	7	8	9	10	11	12	13	14	15	16	17	18
2-deletion correction													
LP upper bound	**4**	**5**	**8**	12	20	33	55	93	160	277	487	862	1539
ILP upper bound	**4**	**5**	**8**	**11**	**16**	**24**	53	T/O	T/O	T/O	T/O	T/O	T/O
ILP lower bound	**4**	**5**	**8**	**11**	**16**	**24**	34	**46**	72	114	189	316	525
MIS solver: OnlineMIS	**4**	**5**	**7**	**11**	**16**	**24**	36	**46**	**87**	**136**	**215**	346	563
MIS solver: ReduMIS	**4**	**5**	**7**	**11**	**16**	**24**	**37**	**56**	**87**	135	**215**	**349**	**567**
Random greedy	**4**	**5**	**7**	10	14	20	30	44	66	102	162	258	416
Search heuristics (Landjev & Haralambiev, 2007)	**4**	**5**	**7**	**11**	**16**	21	31	49	75	109	176	286	485
Explicit construction (Helberg & Ferreira, 2002)	3	4	5	6	8	9	11	15	18	22	30	35	43
Ours (search over $s = 2$)	**4**	**5**	**7**	10	**16**	23	34	48	76	120	194	311	520
Ours (search over $s \in \{1, 2\}$)	**4**	4	7	10	15	21	33	49	78	123	200	332	539
3-deletion correction													
LP upper bound	**2**	**2**	**4**	**5**	7	10	**16**	24	38	61	99	163	273
ILP upper bound	**2**	**2**	**4**	**5**	**6**	**8**	**16**	T/O	T/O	T/O	T/O	T/O	OOM
ILP lower bound	**2**	**2**	**4**	**5**	**6**	**8**	11	13	17	24	36	OOM	OOM
MIS solver: OnlineMIS	**2**	**2**	**4**	**5**	**6**	**8**	**12**	**16**	**22**	**30**	**42**	**60**	T/O
MIS solver: ReduMIS	**2**	**2**	**4**	**5**	**6**	**8**	**12**	**16**	T/O	T/O	T/O	T/O	T/O
Random greedy	**2**	**2**	**4**	**5**	**6**	**8**	10	13	17	24	32	45	65
Search heuristics (Landjev & Haralambiev, 2007)	**2**	**2**	**4**	**5**	**6**	7	10	12	15	24	31	48	71
Explicit construction (Helberg & Ferreira, 2002)	**2**	**2**	3	4	4	5	6	8	8	9	11	15	16
Ours (search over $s = 3$)	**2**	**2**	**4**	**5**	**6**	**8**	**12**	**16**	20	25	37	52	79
Ours (search over $s \in \{1, 2\}$)	**2**	**2**	**4**	4	5	7	**12**	14	19	28	39	57	**81**

the priority function: (a) code only, i.e., no reasoning, (b) generating a natural language description before code, either an algorithmic summary or a comparative reflection (Ye et al., 2024), (c) co-evolving code and algorithmic summary jointly (Liu et al., 2024), and (d) all of the above with reasoning toggled on. For Qwen3 models without reasoning, we increase maximum new tokens from 246 (StarCoder2-15B) to 2048, and to 16000 with reasoning enabled.

Model choice and size. Figure 5 (left) shows that within the Qwen3 series, the 14B model performs best overall, while the 32B model performs surprisingly poorly, with an execution failure rate above 95% for all prompting strategies. Larger models tend to generate more complex priority functions: the 8B model writes short, flat functions with type errors as the main failure mode, while the 32B model generates overly complex code (e.g., generator expressions inside list comprehensions inside loops), with syntax errors and timeouts as the main failure mode (e.g., due to missing brackets in nested expressions).

Regarding model choice, we find that reasoning models improve the search. Comparing models of similar size, Figure 5 (left) shows that Qwen3-14B achieves a lower top-50 mean gap than StarCoder2-15B, likely due to fewer failed executions and more unique clusters. The primary failure modes of StarCoder2-15B are type confusions early in the search, and placeholder implementations, syntax errors, and truncated functions exceeding the token limit later in the search. [1]

Effect of natural language descriptions. Figure 5 (center) shows that for Qwen3-14B, the type of description matters: generating a comparative reflection before code improves performance over code only, while an algorithmic summary does not. Figure 7 in Appendix E shows that for the 8B model, differences are smaller and all prompting strategies perform similarly to code only, while for the 32B model, both description types outperform code only, though the model performs worse overall regardless of prompting strategy.

Co-evolving natural language with code. Figure 5 (center) shows that for Qwen3-14B, co-evolving an algorithmic summary jointly with code results in a worse top-50 mean gap, and we find this to be consistent across all Qwen model sizes (see Figure 7, Appendix E). We observe that co-evolved descriptions become increasingly abstract over time, for example *"Fuse multi-scale structural analysis with reinforcement learning-*

[1] We also ran StarCoder2-15B with maximum new tokens matched to Qwen3-14B (2048) to reduce execution failures from truncated functions, but found worse performance on both minimum and top-50 mean gap.

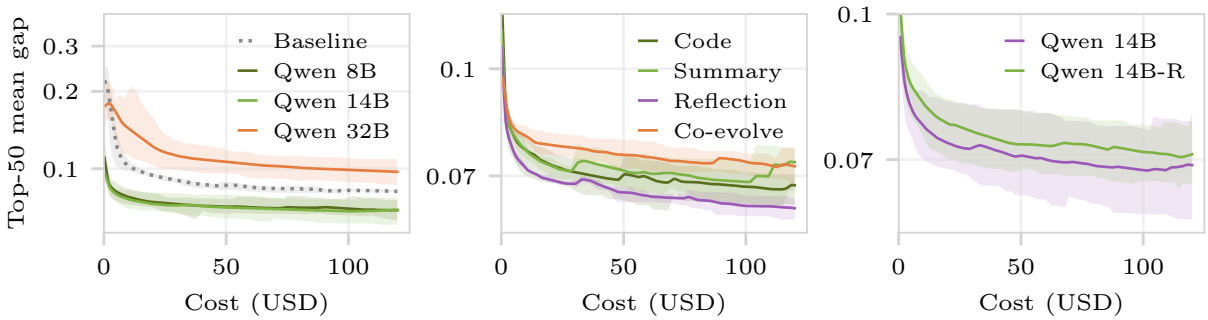

Figure 5: Trade-off between generating more functions and investing more compute per function. Left: top-50 mean gap averaged over prompting strategies for Qwen3 (8B, 14B, 32B), with StarCoder2-15B as baseline. Center: prompting strategies for Qwen3-14B: *Code* (direct solution), *Summary* (algorithmic summary first), *Co-evolve* (summary in few-shot examples), *Reflection* (comparative analysis). Right: enabling reasoning (-R) for Qwen3-14B, averaged over prompting strategies. Lines are means, shaded bands are min-max ranges.

inspired adaptivity to dynamically balance immediate gains against future expansion potential during greedy selection." This prompts the model to attempt ideas it cannot correctly implement, resulting in higher failure rates of 71%, 76%, and 98% for the 8B, 14B, and 32B models respectively, compared to 14%, 50%, and 99% for code only.

Extended reasoning. Figure 5 (right) shows that for Qwen3-14B, toggling reasoning on does not improve solution quality and, under a fixed inference budget, has a worse top-50 mean gap by reducing the number of priority functions evaluated. Figure 8 in Appendix E shows that this result is consistent across model sizes. Runs with reasoning enabled evaluate on average 2.1×, 1.4×, and 8.7× fewer priority functions for Qwen3-8B, 14B, and 32B, respectively. Interestingly, even when controlling for the number of priority functions evaluated, reasoning performs similarly or worse than non-reasoning. By inspection, we find that while reasoning traces explore complex and novel ideas, the model frequently concludes that more complex approaches are too computationally expensive and that novel ideas lack sufficient justification, converging instead to generic low-degree heuristics. As a proxy for algorithm simplicity, we compute average lines of code, percentage of generated functions with helper functions, and cyclomatic complexity (number of conditionals such as if and while), finding that reasoning runs consistently generate priority functions with fewer lines of code (25.8 vs. 30.4), fewer helper functions (25.9% vs. 41.4%), and lower cyclomatic complexity (7.1 ± 3.4 vs. 7.3 ± 3.6) across all models and prompting strategies. This suggests that, for the problem we consider, investing compute in generating more priority functions is more effective than investing more compute per function.

5.3 Generalization Across Deletion Counts

We now study whether a function optimized jointly for single- and two-deletion correction generalizes to three-deletion correction. We search over code lengths $n \in [9, 11]$ for $s = 1$ and $n \in [10, 12]$ for $s = 2$, and evaluate the best function on three-deletion-correcting codes of lengths $n \in [6, 18]$, comparing against a function directly optimized for $s = 3$ on lengths $n \in [8, 13]$.

Table 2 shows that on search lengths $n \in [8, 13]$, the function in Figure 14 jointly optimized for $s = 1, 2$ has a gap of 10.28% to best-known code sizes, versus zero gap for the function in Figure 13 directly optimized for $s = 3$. However, on held-out lengths, the joint function achieves a lower gap of 5.41% compared to 8.50% for the function optimized for $s = 3$. We observe a similar trend for two deletions, where the joint function constructs strictly larger codes for all held-out lengths $n > 12$. This suggests that jointly optimizing across deletion counts can act as a regularizer, helping discover functions that generalize better to unseen code lengths, and it is consistent with Georgiev et al. (2025), who find that searching jointly over related problem instances improves generalization.

Table 3: Code sizes for quaternary single-edit-correcting codes. T/O indicates no solution found within 48 h, OOM exceeded 1 TB memory limit, "–" results not reported in the original paper.

	Code length n					
Method	6	7	8	9	10	11
LP upper bound	215	744	2621	9362	33825	123361
ILP upper bound	212	853	4527	131072	T/O	OOM
ILP lower bound	106	311	1027	3457	11743	OOM
MIS solver: OnlineMIS	112	345	1119	3678	12399	42080
MIS solver: ReduMIS	113	352	1130	3717	12489	T/O
Random greedy	91	269	844	2747	9182	31554
DoDo-Code (Guo et al., 2025a)	–	275	900	3011	10414	36368
Ours (search over $s = 1$)	101	321	1038	3443	11743	40604

For $s = 1$, Table 1 shows that the joint function matches optimal code sizes for $n \leq 14$, outperforming general-purpose graph solvers, but constructs smaller codes than the function optimized for $s = 1$ at longer lengths. Interestingly, the joint search, as well as the search for two- and three-deletion-correcting codes, converges to graph-based priority functions, while the search optimized for $s = 1$ converges to functions that use only algebraic properties of the binary string.

5.4 Extension to Insertion-Deletion-Substitution Codes and Non-Binary Alphabets

While our approach currently does not scale well to moderate or long code lengths, short codes are practically relevant in biological sequencing and DNA data storage. In biological sequencing, short barcodes are used for sample identification and need to be distinguishable under insertions, deletions, and substitutions (edit errors) (Craig et al., 2008). In DNA data storage, current synthesis technologies do not support long DNA strands and information must be broken into short pieces that are stored in unordered pools. These sequences require indices that are similarly distinguishable under edit errors to allow correct reordering (Antkowiak et al., 2020) and random access (Organick et al., 2018). Therefore, we additionally search for quaternary single-edit-correcting codes using the best-performing configuration from our two-deletion experiments.

Table 3 shows the code sizes achieved by our priority function in Figure 11 and baselines for quaternary single-edit-correcting codes. Our function outperforms DoDo-Code (Guo et al., 2025a) across all lengths considered. DoDo-Code uses the same greedy construction as our approach, but with a fixed, hand-designed heuristic that adds sequences with the smallest edit ball first, where edit balls are approximated via learned edit distance embeddings. Compared to general-purpose graph solvers, our function constructs codes of similar size to those obtained by the ILP solver, but smaller than those obtained by the KaMIS solvers.

6 Conclusion and limitations

In this work, we showed that LLM-guided evolutionary search is well-suited for discovering error-correcting codes, rediscovering conjectured-optimal codes for single binary deletions and finding new codes for multiple binary deletions and quaternary edit correction. We studied key design choices and found that, for our problem, sampling more functions is more effective than investing more compute per function, and that deduplication is critical for search diversity.

A key limitation of our approach is the poor scalability of the evaluator, which makes evolutionary search infeasible for moderate to large code lengths. The evaluator must compute priorities for exponentially many sequences as code length increases, and for graph-based priority functions, it must also construct the full graph storing all sequences and pairwise edges, which quickly becomes memory-prohibitive.

Nonetheless, searching in function space generalizes better than previous approaches (Landjev & Haralambiev, 2007; Swart & Ferreira, 2003) that search over all sequences. Moreover, our priority functions can be mathematically analyzed to determine code sizes without explicit construction, as demonstrated by the priority function we prove constructs the same codes as the largest VT code for any length.

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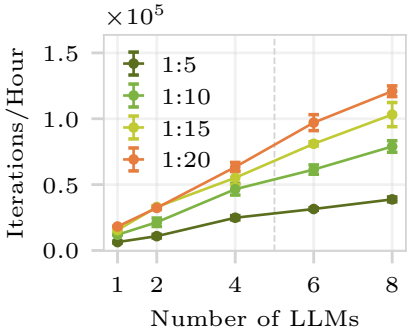

Figure 6: Iterations per hour for different LLM-to-evaluator ratios in our distributed implementation of FunSearch. Ratios to the right of the dashed line use two nodes.

Table 4: Average computational cost per run (number of runs in parentheses) where -R denotes reasoning enabled.

Model	GPUs	Cores	Wall time (h)	GPU hours	CPU hours	Throughput (tok/s)
StarCoder2-15B (10)	4×A100-80GB	96	7.84 ± 0.28	31.16 ± 1.15	39.68 ± 13.77	3957.3 ± 117.2
Qwen3-8B (2)	4×H100-94GB	96	19.91 ± 7.04	44.18 ± 2.10	1221.33 ± 620.77	3795.8 ± 1373.7
Qwen3-14B (10)	4×H100-94GB	96	22.99 ± 5.59	57.42 ± 5.35	798.83 ± 224.08	3514.6 ± 1089.6
Qwen3-32B (12)	4×H100-94GB	96	12.84 ± 1.08	30.47 ± 2.21	$6.71 \pm 4.01^*$	1809.2 ± 220.8
Qwen3-8B-R (2)	4×A100-40GB	128	25.41 ± 1.82	101.26 ± 7.09	632.22 ± 146.56	4534.5 ± 292.9
Qwen3-14B-R (10)	4×H100-94GB	96	25.65 ± 3.80	95.77 ± 6.42	613.15 ± 200.88	4330.9 ± 944.2
Qwen3-32B-R (5)	4× H100-94GB	96	46.73 ± 1.10	182.28 ± 3.68	$1.64 \pm 0.44^*$	767.6 ± 18.1

*Low CPU hours for Qwen3-32B and Qwen3-32B-R are due to the high execution failure rate (>95%) observed with this model, which causes evaluators to return almost immediately without constructing a valid code.

A Implementation details

We implement FunSearch using RabbitMQ for parallelization via asynchronous message passing. The implementation consists of multiple LLMs and evaluators, and a single program database, each running as independent workers communicating through message queues. The database constructs prompts and sends them to the LLM queue, the LLMs generate new priority functions and forward them to the evaluator queue, and the evaluators compute scores and return results to the database. Table 4 summarizes the computational cost per run for each model used in our experiments.

Our implementation differs from existing open-source implementations such as OpenEvolve (Sharma, 2025), EoH (Liu et al., 2024), ReEvo (Ye et al., 2024), and CodeEvolve (Assumpção et al., 2025) in two ways. First, existing implementations parallelize by iteration, running multiple independent copies of the full loop (generate, evaluate, and store) as separate processes on a single node, where each copy blocks until the current step completes. In contrast, we parallelize by task using asynchronous message passing, so no worker has to wait for another (e.g., evaluators process previously generated functions while the LLM generates new ones) and workers can be distributed across multiple nodes. Second, existing implementations rely on API-based inference, where requests are processed individually, while we use batched LLM inference with locally hosted models.

We measure throughput in iterations per hour, which depends on the number of LLMs and evaluators. We allocate one GPU (NVIDIA A100, 80GB) per LLM, one CPU core per evaluator, and one additional core for the database, with a 30-second evaluator timeout. Throughput is measured on the single-deletion-correcting code search ($s = 1$, $n \in [6, 11]$) using StarCoder2-15B. Figure 6 shows throughput at different LLM-to-evaluator ratios, measured over four 20-minute windows (mean ± SD). For each ratio, we report results at the optimal configuration found via grid search over LLM batch size (25, 50, 100, 150), evaluator prefetch

Table 5: Hyperparameters used in our experiments, selected based on smaller-scale experiments.

Hyperparameter	Value
Maximum new tokens	246
Repetition penalty	1.22
Temperature	0.9444
Top-p	0.7778
Initial temperature T	0.1
Sampling period P	30K
Functions before reset R	1.2K

count (5, 15, 30), and parallel workers per evaluator (1, 2, 3). The LLM prefetch count is fixed at twice the batch size, allowing the next batch to buffer during GPU inference, which dominates processing time. We scale from 1 to 8 LLMs across one or two nodes, scaling evaluators proportionally.

Throughput scales near-linearly with the number of LLMs, showing that message passing overhead is negligible compared to the cost of LLM inference and evaluation (i.e., the workload is embarrassingly parallel). At a ratio of 1:20, for example, throughput increases from approximately 18K iterations per hour with 1 GPU to 120K with 8 GPUs across two nodes, with no significant overhead from scaling to multiple nodes. We achieve the highest throughput at the largest tested ratio of 20 evaluators per LLM and expect further gains from additional evaluators up to a point. However, we note that the optimal ratio is specific to the exact setup, as it depends on the hardware, the LLM used, the evaluator timeout, and the problem instances evaluated (e.g., harder instances take longer to evaluate relative to LLM inference time). Our implementation also supports dynamically scaling the number of LLMs and evaluators during execution based on message load and available resources.

B Hyperparameter optimization

We optimize hyperparameters for both the LLM and the evolutionary search on the single-deletion-correcting code search ($s = 1$, $n \in [6, 11]$) using StarCoder2-15B. For the LLM, we optimize maximum new tokens, repetition penalty, temperature, and top-p. For the evolutionary search, we optimize initial temperature T, sampling period P, and the number of functions R stored before an island reset. Table 5 summarizes the best values, which we use for all experiments in the main paper.

B.1 LLM hyperparameters

We run two independent grid searches, varying maximum new tokens and repetition penalty while keeping temperature and top-p fixed, and vice versa. We measure the average improvement in independent set sizes constructed by the best priority functions across all islands, relative to the trivial initialization. Each run is evaluated after one hour using one GPU and 40 CPUs.

For maximum new tokens, we consider values in $[60, 300]$, and for repetition penalty, values in $[1.0, 2.0]$, both divided into 10 equally spaced grid points. Temperature and top-p are fixed at 0.2 and 0.95, respectively, as in Section 7.1.3 of Lozhkov et al. (2024). Low repetition penalties combined with high maximum new tokens often result in the LLM repeating the code completion task, generating multiple function headers with minor variations or trivial return statements instead of a single, improved function. Repetition penalties above 1.22 fail to generate executable functions. While we achieve competitive results with maximum new tokens between 60 and 140 and repetition penalties between 1.05 and 1.11, we observe the largest improvement with 246 maximum new tokens and a repetition penalty of 1.22. Since we only need to find one good priority function to discover a new maximum code size, we use a maximum of 246 new tokens and a repetition penalty of 1.22 for all subsequent experiments.

For temperature and top-p, we consider values in $[0.5, 1.5]$ and $[0.6, 1.0]$, respectively, with 10 equally spaced grid points, while keeping maximum new tokens at 246 and repetition penalty at 1.22. Higher variability in token sampling (larger temperature and top-p values) leads to more variable code sizes across runs, but also

to larger best-found sizes. More deterministic sampling generates more syntactically correct functions but does not lead to larger code sizes. These findings align with the hypothesis of Romera-Paredes et al. (2024) that the LLM contributes by exploring diverse function solutions, occasionally generating good executable functions but often producing unusable outputs. We achieve the best results at a temperature of 0.9444 and a top-p of 0.7778.

B.2 Evolutionary search hyperparameters

We perform independent grid searches over initial temperature T, sampling period P, and the number of functions R stored before an island reset, using the best-performing LLM hyperparameters. We measure performance as a binary outcome: whether or not we find a priority function that constructs optimal code sizes on all search instances. Each run is evaluated after 400K iterations or stops early if such a function is found.

For initial temperatures $T \in \{0.05, 0.1, 0.3, 0.5, 1\}$ with fixed sampling period $P = 30K$ and $R = 1.2K$ functions stored before a reset, we find a priority function that constructs optimal code sizes on all search instances only when $T = 0.1$. With $T = 0.1$, we find such a function after ∼116K iterations, with 20.7% of generated functions stored at the end of the search. When $T = 0.05$, 0.3, 0.5, or 1, the percentages of stored functions are 18.6%, 19.3%, 12.0%, and 10.0%, respectively.

For sampling periods $P \in \{5K, 10K, 30K, 50K, 100K\}$, with $T = 0.1$ and $R = 1.2K$, adjusting the sampling period does not improve performance beyond $P = 30K$. With $P = 5K$, we find a priority function that constructs optimal code sizes on all search instances after ∼194K iterations, with 18.1% stored at the end of the search. With $P = 50K$, we find such a function after ∼132K iterations, with 23.0% stored. When $P = 10K$ or $P = 100K$, we do not find such a function after 400K iterations, with 13.0% and 19.8% stored, respectively.

For $R \in \{300, 600, 1.2K, 2.4K, 5K\}$, with $T = 0.1$ and $P = 30K$, varying R does not improve performance beyond $R = 1.2K$. With $R = 300$, we find a priority function that constructs optimal code sizes on all search instances after ∼251K iterations, with 18.2% stored at the end of the search. With $R = 600$, we find such a function after ∼197K iterations, with 19.9% stored. When $R = 2.4K$ or $R = 5K$, we do not find such a function within 400K iterations, with 19.2% and 19.6% stored, respectively.

We also experiment with dynamically decreasing the LLM sampling temperature to balance exploration and exploitation during the search. The temperature starts at 0.94 and decreases as more functions are stored on the island from which the prompt is sampled, reaching zero at $D \in \{5K, 10K, 20K, 50K\}$ stored functions. While this approach slightly increases the number of executable functions, it does not improve search efficiency compared to a fixed temperature. With $D = 5K$, we find a priority function that constructs optimal code sizes on all search instances after ∼247K iterations, with 22.6% stored at termination. When $D = 10K$, $20K$, or $50K$, we do not find such a function within 400K iterations, with 21.1%, 17.2%, and 21.4% stored, respectively.

C Problem complexity

Finding a maximum independent set in a general graph is NP-complete (Lovász & Plummer, 2009). If we know the maximum code size C^*, we must evaluate all $\binom{2^n}{C^*}$ subsets of that size. For code length $n = 6$ (the smallest length we consider), deletion count $s = 1$, and $C^* = 10$, this already exceeds 151 billion subsets. If we do not know the maximum code size, we must consider all possible subset sizes, giving 2^{2^n} subsets in total. Verifying whether a single subset of size k forms a valid code requires checking that no two sequences share a common subsequence of length $n - s$, which takes $O(n^2)$ time per pair using dynamic programming (Cormen et al., 2022), and $O(k^2n^2)$ time in total. This gives a worst-case complexity of $O(2^{2^n} \cdot k^2n^2)$ for exhaustive search over all subsets.

D Equivalence to VT codes

In this section, we prove that our discovered priority function f in Figure 3 constructs the largest $\mathrm{VT}_0(n)$ code for any code length n under greedy construction with lexicographic tie-breaking.

For a single deletion, the function assigns priority to a binary sequence $\mathbf{x} \in \{0,1\}^n$ as

$$f(\mathbf{x}) = \left\lfloor \frac{W(\mathbf{x})}{n+1} \right\rfloor \quad \text{where} \quad W(\mathbf{x}) = \sum_{i=1}^{n} (n-i+1)x_i. \tag{1}$$

Since $(n-i+1) \equiv -i \pmod{n+1}$, we have $\mathbf{x} \in \mathrm{VT}_a(n)$ with $a \equiv -W(\mathbf{x}) \pmod{n+1}$ and it suffices to show that any sequence $\mathbf{w} \notin \mathrm{VT}_0(n)$ conflicts with some sequence $\mathbf{v} \in \mathrm{VT}_0(n)$ that the greedy algorithm selects first.

By the maximality of VT codes (Cullina et al., 2012), there exists a sequence $\mathbf{v} \in \mathrm{VT}_0(n)$ that shares a length-$(n-1)$ subsequence $\mathbf{z}$ with $\mathbf{w}$. Write $\mathbf{v} = I_{b_v}^{j_v}(\mathbf{z})$ and $\mathbf{w} = I_{b_w}^{j_w}(\mathbf{z})$, where $b_v, b_w \in \{0,1\}$ and j_v, j_w are the bits and positions inserted into $\mathbf{z}$ to obtain sequences $\mathbf{v}$ and $\mathbf{w}$, respectively.

Claim 1. $f(\mathbf{v}) \in \{f(\mathbf{w}),\ f(\mathbf{w})+1\}$.

Proof of Claim 1. Let $q = f(\mathbf{w})$ and $r = W(\mathbf{w}) \bmod (n+1)$, so that $W(\mathbf{v}) = (n+1)f(\mathbf{v})$ and $W(\mathbf{w}) = (n+1)q + r$ with $r \in \{1, \ldots, n\}$. Then

$$W(\mathbf{v}) - W(\mathbf{w}) = (n+1)(f(\mathbf{v}) - q) - r.$$

We bound $|W(\mathbf{v}) - W(\mathbf{w})|$ by case analysis on (b_v, b_w). Let $m \leq n-1$ denote the number of 1-bits in $\mathbf{z}$.

Case A: $b_v = b_w = 0$. Inserting a 0-bit at different positions shifts at most m ones by one position, giving $|W(\mathbf{v}) - W(\mathbf{w})| \leq m \leq n-1$.

Case B: $b_v = b_w = 1$. Inserting a 1-bit at position j adds $n-j+1$ to $W(\mathbf{z})$, ranging from $m+1$ to n, giving $|W(\mathbf{v}) - W(\mathbf{w})| \leq n-(m+1) \leq n-1$.

Case C: $b_v \neq b_w$. Without loss of generality, assume $b_v = 1$ and $b_w = 0$. Then $W(\mathbf{v}) = W(\mathbf{z}) + (n-j_v+1)$ and $W(\mathbf{z}) - m \leq W(\mathbf{w}) \leq W(\mathbf{z})$, giving $1 \leq W(\mathbf{v}) - W(\mathbf{w}) \leq (n-j_v+1) + m \leq n$.

In all cases, $|W(\mathbf{v}) - W(\mathbf{w})| \leq n$, so $|(n+1)(f(\mathbf{v}) - q) - r| \leq n$. Since $r \in \{1, \ldots, n\}$, this gives $f(\mathbf{v}) - q \in \{0,1\}$. □

Claim 2. If $f(\mathbf{v}) = f(\mathbf{w})$, then $\mathbf{v} <_{\text{lex}} \mathbf{w}$.

Proof of Claim 2. If $f(\mathbf{v}) = f(\mathbf{w})$, then $W(\mathbf{w}) = W(\mathbf{v}) + r > W(\mathbf{v})$. We show that $\mathbf{w} <_{\text{lex}} \mathbf{v}$ implies $W(\mathbf{w}) \leq W(\mathbf{v})$, a contradiction, so we must have $\mathbf{v} <_{\text{lex}} \mathbf{w}$.

Case A: $b_v = b_w = 0$ with $j_w < j_v$ (so $\mathbf{w} <_{\text{lex}} \mathbf{v}$). For positions $i \in [j_w, j_v)$, bits contribute weight $(n-i)$ in $\mathbf{w}$ versus $(n-i+1)$ in $\mathbf{v}$, giving $W(\mathbf{v}) - W(\mathbf{w}) = \sum_{i=j_w}^{j_v-1} z_i \geq 0$.

Case B: $b_v = b_w = 1$ with $j_w > j_v$ (so $\mathbf{w} <_{\text{lex}} \mathbf{v}$). The inserted 1-bit contributes $(n-j_v+1)$ to $W(\mathbf{v})$ and $(n-j_w+1)$ to $W(\mathbf{w})$. Since $j_w > j_v$, we have $W(\mathbf{v}) > W(\mathbf{w})$.

Case C: If $b_v = 1$ and $b_w = 0$, then $W(\mathbf{v}) \geq W(\mathbf{w}) + 1$ from Case C of Claim 1. If $b_v = 0$ and $b_w = 1$, then $\mathbf{w}$ cannot be lexicographically smaller than $\mathbf{v}$.

In all cases, $\mathbf{w} <_{\text{lex}} \mathbf{v}$ implies $W(\mathbf{w}) \leq W(\mathbf{v})$, contradicting $W(\mathbf{w}) > W(\mathbf{v})$. □

By Claims 1 and 2, $\mathbf{v}$ either has strictly higher priority or is lexicographically smaller at the same priority, so the greedy algorithm selects sequence $\mathbf{v}$ before $\mathbf{w}$ for all $\mathbf{w} \notin \mathrm{VT}_0(n)$, completing the proof.

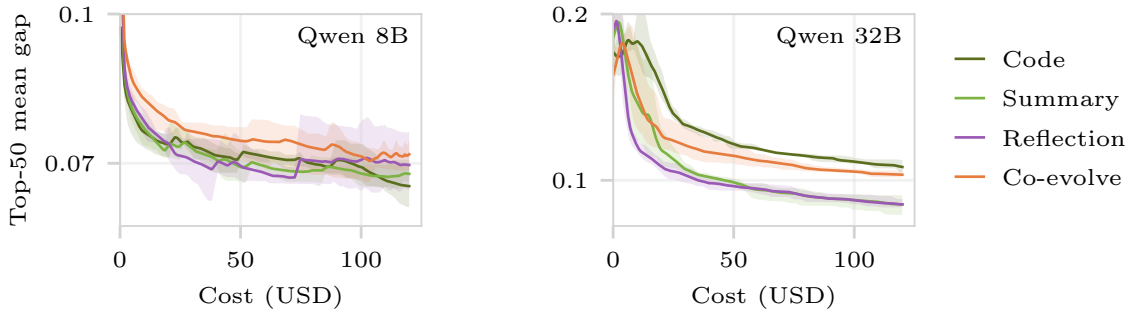

Figure 7: Effect of natural language descriptions for Qwen3-8B and Qwen3-32B. Lines are means, shaded bands are min-max ranges across 3 runs.

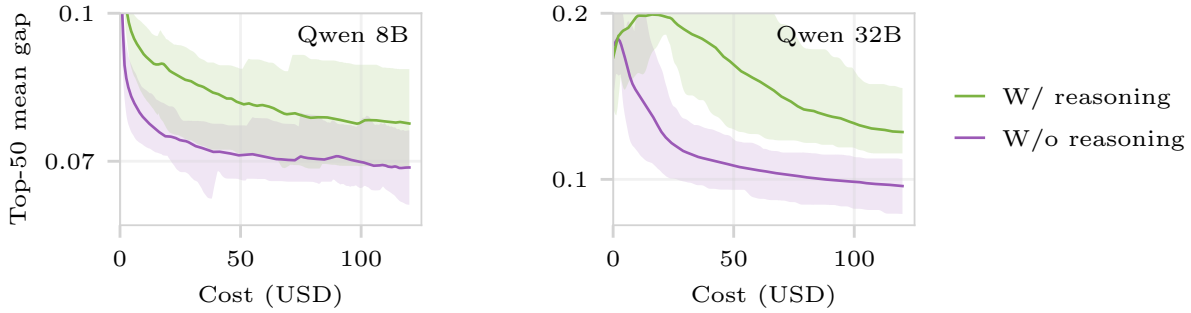

Figure 8: Effect of enabling reasoning for Qwen3-8B and Qwen3-32B, averaged over prompting strategies. Lines are means, shaded bands are min-max ranges across 12 runs.

E Additional results

In this section, we provide additional results on the effect of natural language descriptions and extended reasoning for the Qwen3-8B and Qwen3-32B models.

Figure 7 shows that, consistent with Qwen3-14B, co-evolving an algorithmic summary with code hurts overall search quality for both model sizes. For the 8B model, differences across prompting strategies are smaller and all strategies perform similarly to code only. Figure 8 shows that for both the smaller 8B and the larger 32B model, investing compute in sampling more functions is more effective than extended reasoning per function under a fixed inference budget.

F Discovered priority functions

In this section, we analyze the logic of the discovered priority functions, focusing on the best-performing functions found in the two-deletion correction searches as a representative example.

The best-performing function for two-deletion correction is shown in Figure 12 and computes the priority as a product of six heuristics with different exponents. To analyze the contribution of each heuristic, we ablate each individually by setting its exponent to zero and measuring the mean relative decrease in code size over lengths $n \in [6, 12]$ and $s = 2$. We observe the largest average decrease in code sizes of 27% when removing the heuristic that assigns higher priority to low-degree nodes, a common greedy heuristic for maximum independent set problems. The second largest average decrease of 15% occurs when removing the heuristic that assigns higher priority to sequences with a smaller deletion ball, i.e., fewer distinct subsequences obtained by deleting up to s bits. This is consistent with prior work (Guo et al., 2025a), which also uses the edit ball size as a heuristic for constructing large edit codes. Removing the heuristic that assigns higher priority to

```
def priority(node, n, s):
    count = 0
    for i in range(len(node)):
        for j in range(i + 1, len(node) + 1):
            substr = ''.join(node[i:j])
            count += substr.count("0") ** 3
    return -(6 * count / 2 ** (n + 1) - n ** 2)
```

Figure 9: Priority function equal to the $\mathrm{VT}_0(n)$ code at even lengths and its complement $\mathrm{VT}_{\frac{n+1}{2}}(n)$ at odd lengths.

```
def priority(node, G, n, s):
    ones = sum(int(c) for c in node)
    max_run = 0
    run = 0
    for c in node:
        if c == '1':
            run += 1
            max_run = max(max_run, run)
        else:
            run = 0
    is_sorted = (max_run == ones) if ones > 0 else True

    scores = []
    for nb in G.neighbors(node):
        score = sum(-ord(nb[-i]) * pow(i, (n - (i + 1)))
                    * (1 + 0.2 * min(i, n + 1 - i))
                    for i in range(1, len(nb) + 1))
        scores.append(score)
    nb_max = max(scores) if scores else 0

    eff_ones = n // 2 if ones == n else ones
    base = nb_max * eff_ones * eff_ones
    return base + 1e-12 * (1 if is_sorted else 0)
```

Figure 10: Priority function that constructs single-deletion-correcting codes matching best-known sizes for $n \leq 12$, with codes distinct from all $\mathrm{VT}_a(n)$ codes at lengths $n = 6, 7, 9, 10, 11$.

sequences with low binary entropy, i.e., an unbalanced number of zeros and ones, decreases code sizes on average by 6.3%. This is consistent with prior work showing that uniform input distributions do not achieve the capacity of the deletion channel (Drmota et al., 2012).

The remaining heuristics contribute less to code sizes. Removing the heuristic that assigns higher priority to sequences whose average Hamming distance to their neighbors is close to $n/2$ decreases code sizes on average by 3.7%, removing the heuristic that assigns higher priority to sequences with lower variance in Hamming distance to their neighbors decreases code sizes on average by 1.9%, and removing the heuristic that assigns higher priority to sequences whose neighbors have low average degree decreases code sizes on average by 0.6%.

The other functions found in the search for two-deletion-correcting codes that achieve equal minimum normalized gap on search instances $n \in [7, 12]$ follow similar logic, using different weightings of the heuristics, a subset of them, or combining them as a weighted sum rather than a product.

```python
def priority(node, G, n, s):
    chars = [ord(c) for c in node]

    degree_penalty = -len(list(G.neighbors(node)))

    unique_chars, counts = np.unique(chars, return_counts=True)
    probabilities = counts / n
    shannon_entropy = -np.sum(probabilities * np.log2(probabilities + 1e-10))

    char_positions = {}
    for index, char in enumerate(chars):
        if char not in char_positions:
            char_positions[char] = []
        char_positions[char].append(index)

    distribution_spread = 0
    for positions in char_positions.values():
        mean_pos = np.mean(positions)
        dist_from_mean = [(pos - mean_pos)**2 for pos in positions]
        distribution_spread += np.sqrt(np.mean(dist_from_mean))

    lex_order_weight = -float(int(''.join(map(str, chars))))

    final_priority = (
        0.4 * degree_penalty +
        0.35 * shannon_entropy +
        0.15 * distribution_spread +
        0.1 * lex_order_weight
    )

    return final_priority
```

Figure 11: Priority function that constructs quaternary single-edit-correcting codes with minimum normalized gap to best-known sizes across all runs and search instances $n \in [6, 9]$.

```python
def priority(node, G, n, s):

    degree = G.degree[node]
    total_nodes = len(G.nodes())

    ham_distances = []
    for neighbor in G.neighbors(node):
        dist = sum(bit1 != bit2 for bit1, bit2 in zip(node, neighbor))
        ham_distances.append(dist)

    avg_ham_dist = sum(ham_distances) / len(ham_distances) if ham_distances else 0
    var_ham_dist = sum((d - avg_ham_dist)**2 for d in ham_distances) / len(ham_distances) if
     ham_distances else 0

    bits = list(map(int, node))
    ones_count = sum(bits)
    zeros_count = n - ones_count
    eps = 1e-6
    prob_one = max(min(ones_count / n, 1 - eps), eps)
    prob_zero = max(min(zeros_count / n, 1 - eps), eps)
    info_content = - (prob_one * np.log2(prob_one) + prob_zero * np.log2(prob_zero))

    def count_deletion_variants():
        variants = set()
        from itertools import combinations

        for k in range(1, min(s, 3)+1):
            for indices_to_remove in combinations(range(n), k):
                remaining_seq = ''.join([node[i] for i in range(n) if i not in
    indices_to_remove])
                variants.add(remaining_seq)

        return len(variants) / (2 ** min(s, 3))

    del_pattern_diversity = count_deletion_variants()

    coverages = [G.degree[nb] for nb in G.neighbors(node)] if G.has_node(node) else []
    mean_neighbor_degree = sum(coverages) / len(coverages) if coverages else 0
    influence_metric = 1 / (mean_neighbor_degree + 1)

    final_score = (
        (-degree / total_nodes) *
        (del_pattern_diversity ** 2.7) *
        (influence_metric ** 1.4) *
        (info_content + 0.5) *
        ((var_ham_dist + 1) ** 0.4) *
        (abs(avg_ham_dist - n/2) ** 0.7)
    )

    return final_score
```

Figure 12: Priority function that constructs two-deletion-correcting codes with minimum normalized gap to best-known sizes across all runs and search instances $n \in [7, 12]$.

```python
def priority(node, G, n, s):
    degree = G.degree.get(node, 0)
    value = int(node, 2)
    num_ones = bin(value).count('1')
    density = num_ones / n
    connectivity_score = -math.log(degree + 1) if degree > 0 else 0
    if 0 < density < 1:
        info_content = -(density * math.log2(density) + (1 - density) * math.log2(1 - density))
    else:
        info_content = 0

    def evaluate_subseq_uniqueness():
        total_observed = 0
        unique_count = 0
        for length in range(max(1, s-3), min(n, s+4)+1):
            samples_per_segment = 1 if length <= 4 else 2
            seen_segments = set()
            for pos in range(samples_per_segment, n-length+1, samples_per_segment):
                segment = node[pos:pos+length]
                if segment not in seen_segments:
                    seen_segments.add(segment)
                    unique_count += 1
                total_observed += 1
        return unique_count / total_observed if total_observed > 0 else 0

    seq_uniq = evaluate_subseq_uniqueness()

    def compute_structural_balance():
        scores = []
        for scale in [1, 2, 4]:
            chunks = n // scale
            matched_chunks = 0
            for chunk_idx in range(chunks):
                front_chunk = node[chunk_idx*scale : (chunk_idx+1)*scale]
                back_chunk = node[-((chunk_idx+1)*scale) : -chunk_idx*scale]
                if front_chunk == back_chunk:
                    matched_chunks += 1
            scores.append(matched_chunks / chunks)
        border_matches = sum(1 for i in range(min(5, n//2)) if node[i] == node[n-1-i])
        edge_ratio = border_matches / min(5, n//2)
        return np.mean(scores + [edge_ratio])

    struct_balance = compute_structural_balance()

    def measure_neighborhood_diversity():
        reference_value = value
        hamming_dists = []
        for neighbor in G.adj[node]:
            comp_val = int(neighbor, 2)
            hdistance = bin(reference_value ^ comp_val).count('1')
            hamming_dists.append(hdistance)
        if not hamming_dists:
            return 0
        avg_dist = np.mean(hamming_dists)
        std_dist = np.std(hamming_dists)
        normalized_std = std_dist / (np.sqrt(n) + 1e-6)
        return 1/(normalized_std + 1e-6)
    div_measures = measure_neighborhood_diversity()

    def detect_pattern_replication():
        window_counts = {}
        for win_start in range(n):
            for win_end in range(win_start+2, n+1):
                current_window = node[win_start:win_end]
                count = window_counts.get(current_window, 0)
                window_counts[current_window] = count + 1
        frequencies = list(window_counts.values())
        if not frequencies:
            return 0
        return max(frequencies)/len(frequencies)

    repetitiveness = detect_pattern_replication()
    baseline_noise = 0.0015
    shape_factor = (1 + s/(n**0.8)) ** 0.7
    center_offset = abs(density - 0.5)**0.6 * (1 + 0.07*s)
    adapt_randomness = np.random.normal(loc=0, scale=baseline_noise * shape_factor * center_offset)

    final_priority = ( connectivity_score * 0.24 + info_content * 0.26 + seq_uniq * 0.31 + struct_balance * 0.13 + div_measures * 0.04 + (1 -
      repetitiveness) * 0.01 + adapt_randomness * 0.01 )
    return final_priority
```

Figure 13: Priority function that constructs three-deletion-correcting codes matching best-known sizes obtained with general-purpose graph solvers on search instances $n \in [7, 12]$.

```
def priority(node, G, n, s):
    degree = G.degree[node]
    neighbor_degrees = [G.degree[nbr] for nbr in G.neighbors(node)]
    avg_neighbor_degree = np.mean(neighbor_degrees) if neighbor_degrees else 0

    connective_strength = -np.log1p(degree + avg_neighbor_degree)
    neighborhood_size_cost = -np.log1p(degree + 1)
    num_nodes = len(G.nodes())
    density_scale = np.sqrt(num_nodes) if num_nodes > 10 else 1

    resiliency = 0
    for pos, bit in enumerate(node):
        if bit == '1':
            geo_influence = ((pos + 1) * (n - pos))
            resiliency += np.log2(geo_influence + 1e-6)

    def calc_robustness(string, window=5):
        substrings = set()
        for i in range(len(string)):
            for j in range(i + 1, min(i + window, len(string))):
                substrings.add(string[i:j])
        return len(substrings) / max(window * len(string) // 2, 1)

    subseq_robustness = calc_robustness(node[::-1])

    unique_subs = set()
    base_window = min(s + 2, 5)
    for wsize in [base_window, base_window // 2]:
        for start in range(n - wsize + 1):
            for end in range(start + 1, min(start + wsize, n) + 1):
                unique_subs.add(node[start:end])
    subs_diversity = len(unique_subs) / max(len(unique_subs), 100)

    rng = np.random.RandomState(42)
    unique_states = set()
    if s < len(node):
        sample_rate = min(s * 2, 50)
        for _ in range(sample_rate):
            deletes = rng.choice(len(node), size=s, replace=False)
            remaining = ''.join(c for idx, c in enumerate(node) if idx not in deletes)
            unique_states.add(remaining)
        for i in range(min(5, len(node))):
            remaining = node[:i] + node[i + s:]
            unique_states.add(remaining)
    recoverability_score = len(unique_states) / max(max(1, len(node) - s), 100)

    return -(connective_strength * 0.25 +
             subs_diversity * 0.20 +
             resiliency * 0.17 +
             (avg_neighbor_degree / density_scale) * 0.12 +
             neighborhood_size_cost * 0.09 +
             recoverability_score * 0.07 +
             subseq_robustness * 0.05)
```

Figure 14: Priority function jointly optimized for single- and two-deletion-correcting codes and evaluated for single-, two-, and three-deletion correction.

Table 6: Statistics for the graphs used in our experiments. Each graph has q^n nodes corresponding to all sequences of length n over an alphabet of size q, with edges between pairs of sequences at Levenshtein distance $\leq 2s$ (deletion codes) or edit distance ≤ 2 (insertion-deletion-substitution codes). Density is the ratio of edges to the maximum possible number of edges.

Binary single-deletion ($s = 1$, $q = 2$)					Quaternary edit ($s = 1$, $q = 4$)				
n	Nodes	Edges	Avg. deg.	Den. (%)	n	Nodes	Edges	Avg. deg.	Den. (%)
6	64	543	17	26.9	6	4 096	392 358	192	4.7
7	128	1 471	23	18.1	7	16 384	2 206 374	269	1.6
8	256	3 839	30	11.8	8	65 536	11 815 590	361	0.55
9	512	9 727	38	7.4	9	262 144	60 992 166	465	0.18
10	1 024	24 063	47	4.6	10	1 048 576	305 965 734	584	0.056
12	4 096	139 263	68	1.7	11	4 194 304	1 500 162 726	715	0.017
14	16 384	761 855	93	0.57					
16	65 536	3 997 695	122	0.19					
18	262 144	20 316 159	155	0.059					
20	1 048 576	100 663 295	192	0.018					
Binary 2-deletion ($s = 2$, $q = 2$)					Binary 3-deletion ($s = 3$, $q = 2$)				
n	Nodes	Edges	Avg. deg.	Den. (%)	n	Nodes	Edges	Avg. deg.	Den. (%)
6	64	1 494	47	74.1	6	64	1 906	60	94.5
7	128	5 173	81	63.6	7	128	7 361	115	90.6
8	256	17 183	134	52.6	8	256	27 868	218	85.4
9	512	54 895	214	42.0	9	512	103 272	403	78.9
10	1 024	169 162	330	32.3	10	1 024	373 784	730	71.4
12	4 096	1 460 525	713	17.4	12	4 096	4 531 232	2 213	54.0
14	16 384	11 342 596	1 385	8.5	14	16 384	49 323 673	6 021	36.8
16	65 536	80 964 349	2 471	3.8	16	65 536	484 191 170	14 776	22.5
18	262 144	540 270 870	4 122	1.6	18	262 144	4 331 830 631	33 049	12.6

G Graph Statistics

Table 6 summarizes graph statistics for all problem instances we consider. For one and two deletions, the graphs become sparse at longer code lengths and kernelization-based MIS solvers from KaMIS find competitive solutions. For three deletions, the graphs are still dense even at longer code lengths (e.g., 12.6% at code length 18 with over 4.3 billion edges) and KaMIS times out without finding a solution.