Efficiently Attacking Memorization Scores

Abstract

Influence estimation tools—such as memorization scores—are widely used to understand model behavior, attribute training data, and inform dataset curation. However, recent applications in data valuation and responsible machine learning raise the question: *can these scores themselves be adversarially manipulated?* In this work, we present a systematic study of the feasibility of attacking memorization-based influence estimators when valuators only have **black-box** access to scores. We design and analyze attacks for producing highly memorized samples as highly sensitive queries in the regime where a trained algorithm is accurate. Our inverse-based attacks are practical, requiring only black-box access to model outputs, incur modest computational overhead, and generalize across data modalities. We empirically validate our attack across a wide suite of image classification tasks, showing that even state-of-the-art proxies are vulnerable to targeted score manipulations. In addition, we provide a theoretical analysis of the stability of memorization scores under adversarial perturbations, revealing conditions under which influence estimates are inherently fragile. Our findings highlight critical vulnerabilities in influence-based attribution and suggest the need for robust defenses. All code can be found at `https://anonymous.4open.science/r/MemAttack-5413/`

1 Introduction

Online data market platforms, such as AWS Data Exchange (AWS), Dawex (Dawex), Xignite (XIgnite), WorldQuant (WorldQuant), are spaces where data is bought and sold. Concretely, there are three major entities in a data market: platforms, buyers, and sellers/providers (Kennedy et al., 2022; Alabi et al., 2025). The data market platform performs *data valuation* based on the acquired data from data sellers (Azcoitia & Laoutaris, 2022; Mehta et al., 2021; Agarwal et al., 2019; Fan et al., 2020; Wang & Jia, 2023). Sellers in data markets offer datasets (collections of information) that are valuable for businesses, researchers, or governments. Buyers look for data that can help them make better decisions, run more effective marketing campaigns, or improve products and services. *Data valuation* is released to buyers who use that information to buy data (Jung & Park, 2019; Ray et al., 2020).

Influence functions such as Shapley values are commonly used to price data (proportional to its valuation) in a data market (Yan & Procaccia, 2021; Song et al., 2019; Wang et al., 2020). A series of recent papers (Feldman & Zhang, 2020; Feldman, 2020; Brown et al., 2021) propose the *label memorization score* for supervised classification (in machine learning settings): in large datasets, a small subset of highly influential (memorized) training examples disproportionately affects the model's predictions and generalization capabilities, while the majority of examples have little to no impact. Clearly, this concept is relevant in data valuation, where the goal is to identify which training data points contribute most to a model's decision-making (i.e., samples with high memorization scores are more valuable). While the original proposal (Feldman & Zhang, 2020) is computationally expensive, various studies (Garg et al., 2023; Ravikumar et al., 2024; Jiang et al., 2020; Zhao & Triantafillou, 2024) propose efficient proxies.

Figure 1: An overview of how data valuation functions can be attacked.

We aim to study how malicious data sellers can alter influence valuations in data markets. In particular, we look at the robustness of *memorization-based* scoring when a valuator is only given black-box access to scores, proposing multiple attacks based on distribution shift, stability notions, and decision boundary proximity.

Recent literature showcases memorization scoring as being a prevalent objective in tasks such as data attribution (Aerni et al., 2024; Sakarvadia et al., 2024). Given the difficulty of obtaining high-quality data in data market platforms, there is a clear economic incentive for some sellers to manipulate the valuation scores (Alabi et al., 2025).

Basu et al. (2021) previously study the robustness of influence functions, but do not provide theoretical guarantees and restrict their empirical analyses to neural networks. Lai & Bayraktar (2020) provide an approach for adversarial robustness of general estimators (not influence estimators). Yadav et al. (2024) address the robustness of influence functions on adversaries that control the valuation algorithm, a strong assumption. Wang et al. (2024) attack data valuation scores in the white-box setting; they study the case where access to the precise scoring mechanism. In contrast to these works, we present the *first theoretical analysis* for vulnerability of memorized-based valuation functions. We particularly assert that when a trained algorithm exhibits high accuracy, we can characterize highly memorized samples with high sensitivity. We take advantage of the inverse operation's naturally high sensitivity to motivate a simple, computationally efficient, and effective Pseudoinverse attack to produce highly memorized samples in image classification tasks, and additionally propose other effective inverse-based attacks on text classification data. In comparison to prior work, we assume a **more realistic adversarial model**, one where the (malicious) data sellers only change the provided data, require no collusion, and are unaware of the exact (memorization-based) valuation algorithm, but the valuator only has black-box access to the scores to measure contribution. In our most effective attack, the adversary need not have knowledge of the underlying data distribution! Figure 1 illustrates the threat model. Our theoretical findings demonstrate a fundamental vulnerability in data valuation for supervised learning, and our simple and motivated attacks present an improvement over past adversarial studies on data valuation scoring.

We summarize our contributions below:

- We develop a theoretical framework for analyzing vulnerability of memorization scores to adversarial manipulation and demonstrate its implications on potential data pricing (§ 4).
- We provide a simple and efficient Pseudoinverse attack that successfully alters memorization-based scoring of image classification tasks agnostic to underlying dataset and model architecture (§ 5).
- We validate the Pseudoinverse attack with experiments on image classification tasks across various convolutional network architectures on the MNIST, SVHN, and CIFAR-10 datasets (§ 6), and present additional support over more complex transformer architecture, higher resolution image datasets, and other data modalities in textual classification in (§ C). We additionally demonstrate the improvement of manipulating memorization scores from our theoretically motivated attacks over previous naive optimization-based attacks. The Pseudoinverse attack is the strongest attack in the low-to-moderate attack-size regime.

A Note on Semantic Meaning   The image and text data produced by our inverse-based attacks, appear as noise and have low semantic meaning and consequently may not be realistically considered in a data market setting after manual inspection. We retain the belief that manual inspection alone provides limited insight into the actual contribution of a sample toward model performance and serves as a poor baseline for detecting adversarial samples. As shown by Wang et al. (2018), there exist many training samples that may appear semantically uninformative or even irrelevant to a human observer (i.e., easy to detect and discard) but nonetheless prove useful for generalization. Thus, it would be premature or misleading to discard or downweight such samples solely based on visual or semantic intuition.

Additionally, in many settings where data valuation is most critical e.g., active data curation, responsible dataset pruning, or cost-sensitive training, it is likely that the model's performance depends on the cumulative influence of a large number of individually subtle or redundant examples. This further reduces the practical viability of relying on human judgment alone. Manual inspection does not scale, especially when dealing with high-volume datasets or when the goal is to precisely quantify marginal utility of each point for fine-grained optimization. Finally, Zhang et al. (2021) demonstrates the unreliability of out-of-distribution detection based on low-likelihood models, indicating the viability of our inverse-based attacks in the data market setting. For completeness, we empirically show that our inverse-based attacks are able to avoid certain OOD detection methods in Section 6.7.

Threat Model and Out-of-Scope Attacks   Our attacks focus on (seller-side) black-box score manipulation under limited knowledge of the valuation mechanism; we do not study colluding sellers, attackers controlling the valuation platform, full white-box attackers, or joint optimization over semantic realism and score inflation (i.e., full strategic market equilibrium behavior (Azcoitia & Laoutaris, 2022; Mehta et al., 2021; Agarwal et al., 2019)).

2   Related Work

Data Markets: Azcoitia & Laoutaris (2022) give a survey on existing commercial data markets and their business models. Mehta et al. (2021) introduce pricing policies for enabling data economies. Agarwal et al. (2019) introduce a mathematical model for data marketplaces. Also, recent work surveys privacy and security vulnerabilities in data markets and offers possible design solutions (Alabi et al., 2025). Our work introduces a realistic threat model and calls for robust algorithmic solutions for future data markets.

Data Valuation: Shapley values are a classical concept in game theory which were first employed for the problem of data valuation in machine learning by Ghorbani & Zou (2019), termed Data Shapley Value, which aims to measure the influence of individual data samples on some specified performance metric. Due to computational overhead of computing Shapley Value, various influence functions have been proposed and adopted, including finding a core set of data examples (Yan & Procaccia, 2021), gradient-based approximations to the Shapley Value (Song et al., 2019), adaptations to federated learning setting (Wang et al., 2020), as well as non-Shapley based influence functions such as gradient tracing (Pruthi et al., 2020) and low-rank kernel approximation (Park et al., 2023). However, it is not always clear what target for which the influence of data samples should be measured on. Common settings involve a specific test set on which the effects of influence are measured, but a definitive target test set might not be available or might change over time, or might not even be known in certain applications. Label memorization (Feldman & Zhang, 2020), the main

metric of data valuation in this study, is a measure of self-influence that removes this particular challenge. Feldman (2020) shows that accuracy on training examples with label memorization scores is crucial to low generalization error, demonstrating that highly memorized samples strongly influence downstream tasks.

Adversarial Attacks Against Memorization: Privacy risk in the context of membership inference attack is closely related to memorization as shown previously by Choi et al. (2023). Carlini et al. (2022) find outliers to be more vulnerable to membership inference attack, whereas other prior work use poisoned images (Jagielski et al., 2020) or artificially crafted "canaries" for privacy auditing of language models (Carlini et al., 2019; Thakkar et al., 2020) to produce samples of high privacy risk, and subsequently high memorization scores. In comparison to prior adversarial studies, we present the first theoretical analysis characterizing highly memorized samples. Wang et al. (2024) present a similar setup to our study contributing the iterative optimization-based Shadow and Outlier Attack on influence scores, but is outperformed by our attack methods on self-influence (memorization) scoring which more directly attacks the core vulnerability of data valuation scoring.

3 Preliminaries and Notation

3.1 Queries: Sensitivity and Accuracy

Let $\mathcal{Z}$ be a universe or domain. e.g., $\mathcal{Z} = \mathbb{R}^3$. Also, let $\mathbf{P}$ be a distribution over $\mathcal{Z}$ from which samples $\mathbf{z} = (z_1, \cdots, z_n) \in \mathcal{Z}^n$ can be drawn. We use the notation $z_1, \ldots, z_n \leftarrow_{\mathrm{R}} \mathbf{P}$ to indicate that $z_1, \ldots, z_n$ is randomly drawn from $\mathbf{P}$. A mechanism, trained by the data evaluator or data platform, can answer *queries*, from some query family Q, about $\mathbf{P}$ or $\mathbf{z}$. For any $q \in Q$, we define the query answers on either the population level or sample-level:

$$q(\mathbf{P}) = \mathop{\mathbb{E}}_{\mathbf{z} \leftarrow_{\mathrm{R}} \mathbf{P}} [q(\mathbf{z})] \qquad \text{and} \qquad q(\mathbf{z}) = \frac{1}{n} \sum_{i \in [n]} q(z_i).$$

Query families can be separated by their sensitivities, which quantifies how much the query changes when one or more elements of the input are changed. Let $\mathbf{z} \sim \mathbf{z}'$ denote that $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}^n$ differ on at most one entry. For any query $q \in Q$ and neighboring $\mathbf{z} \sim \mathbf{z}'$, $\|q(\mathbf{z}) - q(\mathbf{z}')\|$ (the ℓ_2-norm of the difference between $q(\mathbf{z})$ and $q(\mathbf{z}')$) is the sensitivity of the query $q : \mathcal{Z}^n \to \mathcal{Z}$. We can define Δ-sensitive queries:

Definition 3.1 (Δ-Sensitive Queries). For $\Delta \geq 0$, $n \in \mathbb{N}$, these queries are specified by a function $q : \mathcal{Z}^n \to \mathcal{Z}$ where $\mathcal{Z} \subseteq \mathbb{R}^*$. Further, the queries satisfy

$$\|q(\mathbf{z}) - q(\mathbf{z}')\| \leq \Delta,$$

for every pair $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}^n$ differing in only one entry.

Q_Δ is the set of all queries with sensitivity of at most Δ.

As done in the literature on stability and adaptive data analysis (Bassily et al., 2016; Bousquet & Elisseeff, 2002; Shalev-Shwartz et al., 2010), $\mathcal{A}$ is a stateful algorithm with access to samples $z_1, \ldots, z_n \in \mathcal{Z}$. We can define an accuracy game between a stateful *adversary* $\mathcal{Q}$ and a mechanism $\mathcal{A}$, illustrated in Figure 2. The (opposing) goal of $\mathcal{Q}$ is to increase the error of the query answers provided by $\mathcal{A}$. Because $\mathcal{Q}$ and $\mathcal{A}$ are stateful, the queries and the query answers may depend on the history of past queries and past query answers.

A primary goal of machine learning is to train an algorithm/mechanism $\mathcal{A}$ that can accurately answer queries on $\mathbf{P}$. The training can use (independent) samples $z_1, \ldots, z_n \leftarrow_{\mathrm{R}} \mathbf{P}$. Here, for each $i \in [n]$, $z_i = (x_i, y_i)$ corresponds to feature-label pairs, which are used to train $\mathcal{A}$. Then a query for $\mathcal{A}$ could be: *given $\mathbf{z} \in \mathcal{Z}^n$, what is the label y_{n+1} for feature vector x_{n+1}?* Clearly, this can be encoded via the query function $q : \mathcal{Z}^n \to \mathcal{Z}$ where $q(\mathbf{z}) = q^*(\mathbf{z}, x_{n+1})$ and $q^* : \mathcal{Z}^n \times \mathcal{Z} \to \mathcal{Z}$ takes in samples and new feature vector x_{n+1}. The query answer is $z_{n+1} = (x_{n+1}, y_{n+1}) \in \mathcal{Z}$. For any $j \in [k]$, we measure query accuracy as $\|a_j - q_j(\mathbf{P})\|$ where a_j is the answer by $\mathcal{A}$—based on the samples $\mathbf{z}$ and previous query answers—and $q_j(\mathbf{P})$ is the query answer on the population level. Note that the formalism goes beyond classification: we might want to estimate the mean of a population in which case the query is the population mean and the answer could be the mean of the samples $\mathbf{z} = (z_1, \ldots, z_n)$.

Sample $z_1, \ldots, z_n \leftarrow_{\mathrm{R}} \mathbf{P}$ and let $\mathbf{z} = (z_1, \ldots, z_n)$.
For $j = 1, \ldots, k$
    $\mathcal{Q}$ outputs a query $q_j \in Q$.
    $\mathcal{A}(\mathbf{z}, \{q_t\}_{t=1}^{j})$ outputs a_j.

Figure 2: The Accuracy Game $\mathsf{Acc}_{n,k,Q}[\mathcal{A}, \mathcal{Q}]$

Since queries are meant to return answers, we measure how accurate the answers are either on the population or specific samples.

Definition 3.2 (Population Accuracy). *A mechanism $\mathcal{A}$ is (α, β)-accurate with respect to the population $\mathbf{P}$ for k, potentially adaptively,*[1] *chosen queries from Q given n samples in $\mathcal{Z}$ if for every adversary $\mathcal{Q}$,*

$$\mathop{\mathbb{P}}_{\mathsf{Acc}_{n,k,Q}[\mathcal{A}, \mathcal{Q}]} \left[\max_{j \in [k]} \|q_j(\mathbf{P}) - a_j\| \leq \alpha \right] \geq 1 - \beta.$$

In Definition 3.2, the randomness is over algorithms $\mathcal{A}, \mathcal{Q}$ and the distribution $\mathbf{P}$.

3.2 Stability Notions

A variety of stability notions (such as Max-KL stability, KL stability, and TV stability) capture how sensitive an algorithm is to changes to its input (Hellström et al., 2025). In our work, we focus on Max-KL stability but our theoretical results can be ported to other notions of stability:

Definition 3.3 (Max-KL Stability). *Let $\mathcal{A} : \mathcal{Z}^n \to \mathcal{R}$ be a randomized algorithm. We say that $\mathcal{A}$ is (ϵ, δ)-max-KL stable if for every pair of samples $\mathbf{z}, \mathbf{z}'$ that differ on exactly one element, and every $R \subseteq \mathcal{R}$,*

$$\mathbb{P}[\mathcal{A}(\mathbf{z}) \in R] \leq e^{\epsilon} \cdot \mathbb{P}[\mathcal{A}(\mathbf{z}') \in R] + \delta.$$

For any $\epsilon, \delta \geq 0$, it is easy to see that Max-KL stability is equivalent to (ϵ, δ)-*differential privacy* (Dwork et al., 2006b;a).

Post-processing refers to the notion that a property (e.g., stability) is preserved after certain modifications. All the stability notions are preserved under post-processing:

Lemma 3.1 (Post-Processing of Stability Notions (e.g., see Bun & Steinke (2016))). *Let $\mathcal{A} : \mathcal{Z}^n \to \mathcal{R}$ and $f : \mathcal{R} \to \mathcal{R}'$ be a pair of randomized algorithms. If $\mathcal{A}$ is $\{\epsilon\text{-}TV, \epsilon\text{-}KL, (\epsilon, \delta)\text{-}max\text{-}KL\}$-stable then the algorithm $f(\mathcal{A}(\mathbf{z}))$ is $\{\epsilon\text{-}TV, \epsilon\text{-}KL, (\epsilon, \delta)\text{-}max\text{-}KL\}$-stable.*

In addition, composition of disjoint databases preserves the stability notions:

Lemma 3.2 (Composition of Stability Notions (e.g., see Bun & Steinke (2016))). *Let $\mathcal{A}_1 : \mathcal{Z}^n \to \mathcal{R}$ and $\mathcal{A}_2 : \mathcal{Z}^n \to \mathcal{R}$ be a pair of randomized algorithms. If $\mathcal{A}_1, \mathcal{A}_2$ are $\{\epsilon\text{-}TV, \epsilon\text{-}KL, (\epsilon, \delta)\text{-}max\text{-}KL\}$-stable then $(\mathcal{A}_1(\mathbf{z}_1), \mathcal{A}_2(\mathbf{z}_2))$ is $\{\epsilon\text{-}TV, \epsilon\text{-}KL, (\epsilon, \delta)\text{-}max\text{-}KL\}$-stable if $\mathbf{z}_1, \mathbf{z}_2$ are disjoint.*

3.3 Memorization Scores & Proxies

Memorization reflects how much a model relies on memorizing specific training examples instead of generalizing.

Definition 3.4 (Label Memorization (Feldman & Zhang, 2020)). *For training point $z_i = (x_i, y_i)$, the memorization score is:*

$$\mathtt{mem}(\mathcal{A}, \mathbf{z}, z_i) := \Pr_{h \leftarrow \mathcal{A}(\mathbf{z})}[h(x_i) = y_i] - \Pr_{h \leftarrow \mathcal{A}(\mathbf{z}^{\setminus z_i})}[h(x_i) = y_i].$$

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[1]As in standard in the stability literature, we allow for adaptive queries. However, our experimental results in this paper are focused on non-adaptive queries.

Here, h is the (randomized) classifier obtained from training algorithm $\mathcal{A}$ on dataset $\mathbf{z}$.

However, the first formulation of memorization proposed by Feldman and Zhang requires training thousands of models and is computationally prohibitive. Zhao & Triantafillou (2024) names two computationally efficient proxies with empirically high correlation with memorization that we describe below to use in our experiments. We describe an additional proxy per risk scores computed from a membership inference attack given by Song & Mittal (2021).

1. Input Loss Curvature-based Proxies: Garg et al. (2023) observe that data samples with high loss curvature visually correspond to long-tailed, mislabeled or conflicting samples, which are more likely to be memorized.

Definition 3.5 (Loss Curvature)**.** The curvature of the loss function with respect to an input x_i is: $\texttt{curv}(\ell, i) := \text{Tr}\left(\nabla_x^2 \ell(h_\theta(x_i), y_i)\right)$. Implicitly, θ is a hypothesis trained from randomized algorithm $\mathcal{A}$, and h_θ is the model output from parameters θ. ℓ is the loss function used to train models in $\mathcal{A}$, e.g. cross entropy.

2. Learning Events-based Proxies: The learning event proxies are first introduced by Jiang et al. (2020), a class of memorization score proxies designed to measure how quickly and reliably a model learns a specific example during training. The key intuition is that a specific example that is consistent with many others should be learned quickly as the gradient steps for all consistent examples should be well aligned. Zhao & Triantafillou (2024) find a strong link between these learning events proxies and memorization scores per the Spearman correlation coefficient. As done by prior work (Jiang et al., 2020; Zhao & Triantafillou, 2024), we aggregate the following cumulative statistics over each training epoch: confidence, max confidence, entropy and binary correctness. Full details of this class of attacks are described in Appendix B.

Definition 3.6 (Learning Events Proxy)**.** Given some per-sample event function $\phi(h_\theta(x), y)$ that depends on the learning hypothesis θ (e.g., $\phi(h_\theta(x_i), y_i) = h_\theta(x_i)_{y_i}$), we define the cumulative training event proxy: $\texttt{event}(\mathcal{A}, i, \phi) = \frac{1}{T}\sum_{t=1}^{T} \phi(h_{\theta_t}(x_i), y_i)$. Here θ_t denotes the hypothesis learned at epoch t in training algorithm $\mathcal{A}$.

3. Membership Inference Attacks: Membership Inference is a topic highly related with memorization. The goal of a membership inference attack is to recover whether a particular data entry was part of an unknown training set, either by using knowledge of the model, access to the model, or in a black-box setting. Choi et al. (2023) previously shows a theoretical link between Membership Inference advantage to memorization. Using a model trained on a certain dataset as the target model in an attack, we obtain risk scores for each data entry in the dataset—the probability of inclusion in the dataset—as a proxy for memorization scoring. The intuition comes from observing that highly memorized data entries are unlike representative data entries, and have higher probability of being identified by black-box model access. Song & Mittal (2021) give formulation for privacy risk score.

Definition 3.7 (Privacy Risk Score)**.** The privacy risk score r of an input sample $z = (x, y)$ for the target machine learning model $h \leftarrow \mathcal{A}(\mathbf{z})$ is defined as the posterior probability that it is from the (random) set $\mathbf{z}$ after observing the target model's behavior. i.e., $r(z) = \Pr(z \in \mathbf{z} \mid O(h, z))$, where $O(h, z) = h(z)$ is the model's final output.

These proxy scores are independent of each other, and each proxy operates on a different scale than the other, so the raw numbers are not meaningfully comparable. We give results comparing relative scoring in Appendix C.8.

4 Manipulating Memorization Scores

We formally analyze the conditions under which data sellers can manipulate memorization scores. The contrapositive of Theorem 4.1 is that if the algorithm that is used to compute the memorization score is sufficiently accurate and does not satisfy stability guarantees, then the memorization will be high for a family of queries. Our work builds on the theory of stability notions in the literature (Hellström et al., 2025; Bassily et al., 2016; Bousquet & Elisseeff, 2002; Shalev-Shwartz et al., 2010).

For our analysis, we note that *we do not control how the algorithm $\mathcal{A}$ is trained*! Thus, our adversarial model is natural and affords the algorithm designer the ability to respond to collusion by data sellers by modifying how $\mathcal{A}$ operates on the dataset.

> We show that if the classification algorithm $\mathcal{A}$ is very accurate on the population and does not satisfy stability guarantees, then there *always* exist a new example from the data sellers that will lead to high memorization scores.

Let $q : \mathcal{Z}^n \to \mathcal{Z}$ be a query function. For any fixed dataset $\mathbf{z}$, the goal of the adversarial data seller is to consider if either $q(\mathbf{z}) \cup \mathbf{z}$ or $\mathbf{z}$ leads to a higher valuation score (via memorization).

For any algorithm $\mathcal{A}$, we study the following question: *what queries on the dataset would lead to high memorization scores?* In order to quantify the question, we consider the following memorization score on addition of a new example to the existing dataset:

$$\texttt{mem}(\mathcal{A}, \mathbf{z}, q(\mathbf{z})) := \Pr_{(x,y) \leftarrow q(\mathbf{z}), h \leftarrow \mathcal{A}(\mathbf{z} \cup q(\mathbf{z}))}[h(x) = y] - \Pr_{(x,y) \leftarrow q(\mathbf{z}), h \leftarrow \mathcal{A}(\mathbf{z})}[h(x) = y] \tag{1}$$

In Equation 1, what query functions $q : \mathcal{Z}^n \to \mathcal{Z}$ would lead to high memorization? We can measure sensitivity of a query as $\max_{\mathbf{z}, \mathbf{z}'} \|q(\mathbf{z}) - q(\mathbf{z}')\|$.

Theorem 4.1 (See Theorem A.1 in Appendix). *Let Q_Δ be a family of Δ-sensitive queries on $\mathcal{Z}^n$. Fix $\delta \in [0, 1]$ and let the dataset size $n \in \mathbb{N}$ be such that there exists $\gamma > \delta$ such that $n \geq 1/\gamma$. Then for any $\alpha, \beta \in (0, 1/10)$, there exists algorithm $\mathcal{A}$ with memorization score (i.e., $\texttt{mem}(\mathcal{A}, \mathbf{z}, q(\mathbf{z})) \leq \delta$ from Equation 1) of at most δ such that $\mathcal{A}$ is (α, β)-accurate for any query from Q_Δ but it must be the case that $\alpha \geq \gamma \Delta n$ and $\beta \geq \frac{\delta}{2\gamma}$.*

That is, there exists algorithm with memorization score (Equation 1) of at most δ such that for $\mathbf{z} \leftarrow_R \mathbf{P}^n$ and query $q \in Q_\Delta$, $\Pr[\|q(\mathbf{z}) - q(\mathbf{P})\| \geq \gamma \Delta n] \geq \frac{\delta}{2\gamma}$, where $\mathbf{P}$ is a distribution over $\mathcal{Z}$.

Remark 1. *The inverse query $q(z) = z^{-1}$ has sensitivity that approaches ∞ (i.e., let the input be non-invertible or be 0 for one-dimensional input). This motivates a subset of our attacks, namely the Pseudoinverse attack in the image classification setting, but we also consider other inverse-like attacks on textual data in Section 6.9. More generally, Theorem 4.1 implies that in order to avoid manipulation by data sellers, the algorithm $\mathcal{A}$ cannot be too accurate on the population level or the algorithm $\mathcal{A}$ must (formally) satisfy stability guarantees, such as max-KL stability. However, Theorem 4.1 does not imply that the Pseudoinverse attack is effective on arbitrary learning algorithms.*

Full Details For the full details of our proofs/analysis and high-level ideas, see Appendix A.

5 Experimental Setup

Attacks: We evaluate the robustness of data valuation methods under four distinct input-space attacks, each modifying input x while preserving the label y. The attacks are presented from low to high fidelity

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.00±0.00	0.01±0.00	0.09±0.00	0.01±0.00	0.06±0.00	0.22±0.00	0.45±0.00	0.49±0.00	0.18±0.00
OOD	0.15±0.00	0.02±0.00	0.20±0.00	0.51±0.00	0.48±0.00	0.58±0.01	0.06±0.01	-0.12±0.00	0.10±0.00
PINV	**0.20±0.00**	**0.35±0.00**	**0.25±0.00**	**0.67±0.01**	**0.79±0.00**	**0.64±0.00**	**0.30±0.02**	**0.46±0.00**	**0.58±0.01**
EMD	0.07±0.00	0.00±0.00	-0.05±0.00	0.33±0.01	0.59±0.00	0.39±0.00	-0.08±0.02	-0.05±0.00	-0.05±0.00
DF	0.00±0.00	0.01±0.00	0.00±0.00	-0.01±0.00	0.01±0.00	-0.03±0.00	-0.02±0.01	-0.03±0.00	-0.02±0.00

Table 1: **EAA on ResNet-18 architecture with attack set size of 10.** PINV outperforms all other attacks by a significant margin across dataset and proxy.

i.e., how closely the adversarially modified image represents the original. Previous work (Xing et al., 2023) demonstrates that low-fidelity images can have high utility in synthetic data, which justifies the consideration of our visually aberrated attack images. Our attacks are motivated by distributional shift, stability notions and decision boundary proximity. A detailed description of how these attacks are implemented is given in Appendix B.

1. *No Attack:* Henceforth referred to as `None`, we use this label in our results section to denote unperturbed natural data samples.
2. *Out-of-Distribution Replacement Attack:* Henceforth referred to as `OOD`, this attack replaces in-distribution samples with out-of-distribution inputs from related datasets (e.g., FashionMNIST, CIFAR-10), testing sensitivity to semantic shifts. Intuitively, a small spurt of data drawn from a separate latent distribution will be long-tailed and memorized.
3. *Pseudoinverse Attack:* Henceforth referred to as `PINV`, this attack transforms input images by computing their Moore-Penrose pseudoinverses and normalizing to image-space ranges, inducing unnatural but structured distortions. Intuitively, by treating images as information signals for a model to parse, prior work (Brown et al., 2021) suggests that there are natural tasks for which any high-accuracy algorithm would need to memorize the majority of the samples. Motivated by our theoretical results (Remark 1), we take the pseudoinverse of an image matrix in order to result in memorization of the new image.
4. *Naive EMD Attack:* Henceforth referred to as `EMD`, this attack maximizes the Wasserstein distance between original and perturbed images using a greedy per-pixel binary search heuristic over RGB intensities. Intuitively, we seek to maximize the distance between the original and perturbed image, treating image data as probability distributions.
5. *DeepFool Perturbation Attack:* Henceforth referred to as `DF`, this attack applies the DeepFool algorithm (Moosavi-Dezfooli et al., 2016; Abroshan et al., 2024) to perturb each input minimally toward the classifier's decision boundary, forcing misclassification. Intuitively, we might expect a data point close to the decision boundary to be memorized by the model.

Datasets: We conduct experiments on three canonical image classification datasets: MNIST (LeCun et al., 1998), SVHN (Netzer et al., 2011), and CIFAR-10 (Krizhevsky et al., 2009). These datasets span grayscale handwritten digits (MNIST), real-world digit photographs (SVHN), and natural scene object categories (CIFAR-10), and thus collectively test attribution methods across increasing levels of input complexity and semantic variation. We also present supplemental results over ImageNet (Russakovsky et al., 2015) and AG News (Zhang et al., 2015) in Section 6.8.

Models: We evaluate three standard deep neural net architectures: VGG-11 (Simonyan & Zisserman, 2014), ResNet-18 (He et al., 2016), and MobileNet-v2 (Sandler et al., 2018). These models represent different design paradigms: convolutional (VGG), residual (ResNet), and mobile-efficient (MobileNet). Each model is trained using standard data augmentation and optimization techniques (SGD with cross-entropy loss), with architectures matched to dataset resolution where applicable. We also present supplemental results over Vision Transformer (ViT) (Dosovitskiy et al., 2020) and BERT (Devlin et al., 2019) architecture in Section 6.8.

Experimental Setting: We evaluate on the three proxies described in § 3.3. A detailed description of scoring implementation is found in Appendix C. Each attack is evaluated over $t = 5$ independent trials. In each trial, a randomly chosen subset, which we call the *attack set*, of data points from the base dataset is

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.00±0.00	0.01±0.00	0.09±0.00	0.01±0.00	0.06±0.00	0.23±0.00	0.47±0.00	0.49±0.00	0.19±0.00
OOD	0.13±0.00	0.02±0.00	**0.14±0.00**	0.62±0.00	0.52±0.00	0.61±0.00	0.08±0.00	-0.10±0.00	0.09±0.00
PINV	**0.14±.00**	**0.14±0.00**	0.08±0.00	**0.85±0.00**	**0.81±0.00**	**0.66±0.00**	**0.29±0.01**	**0.51±0.00**	**0.79±0.00**
EMD	0.06±0.00	0.00±0.00	-0.05±0.00	0.51±0.00	0.68±0.00	0.54±0.00	-0.03±0.00	-0.03±0.00	0.01±0.00
DF	0.00±0.00	0.00±0.00	0.01±0.00	0.00±0.00	0.00±0.00	0.00±0.00	-0.02±0.00	-0.01±0.00	-0.02±0.00

Table 2: **EAA on ResNet-18 architecture with attack set of size 100.** `PINV` outperforms almost all other attacks by a significant margin across dataset and proxy.

Attack	MNIST				SVHN				CIFAR-10			
Attack set size	10	100	1000	10000	10	100	1000	10000	10	100	1000	10000
None	0.01	0.01	0.01	0.01	0.08	0.08	0.08	0.08	0.23	0.23	0.23	0.23
OOD	0.08	0.02	0.01	0.01	0.04	0.02	0.01	0.01	0.02	0.02	0.01	**0.01**
PINV	**0.62**	**0.36**	**0.20**	**0.03**	**0.58**	**0.45**	**0.13**	**0.02**	**0.43**	**0.23**	**0.09**	**0.01**

Table 3: **EAA for label memorization across dataset over VGG-11 architecture.** `PINV` raises label memorization score significantly over base scoring and `OOD` attack.

Attack	MNIST				SVHN				CIFAR-10			
Attack set size	10	100	1000	10000	10	100	1000	10000	10	100	1000	10000
None	0.01	0.01	0.01	0.01	0.07	0.07	0.07	0.07	0.10	0.10	0.11	0.11
OOD	0.06	0.02	0.01	0.01	0.06	0.01	0.01	**0.01**	0.03	0.02	0.01	0.01
PINV	**0.51**	**0.36**	**0.17**	**0.02**	**0.47**	**0.30**	**0.04**	**0.01**	**0.36**	**0.20**	**0.11**	**0.02**

Table 4: **EAA for label memorization across dataset over ResNet-18 architecture.** `PINV` raises label memorization score significantly over base scoring and `OOD` attack.

perturbed using the attack under consideration. We consider 3 sizes of attack sets: 10, 100, 1000. We then average the valuation scores of the perturbed samples and report the mean over trials as the final *attack score*.

Hardware Used: All experiments were conducted using NVIDIA A40 single GPU nodes. Training and attack procedures were implemented in PyTorch. More details can be found in Appendix C.

6 Experimental Results

We aim to answer the following questions: (1) What attack is the most effective in manipulating memorization scores?; (2) Is this attack effective across dataset and model architecture settings?

As a quick summary, we empirically validate the following claims: (1) The high sensitivity queries produced by `PINV` perturb memorization scores (and proxies) to a greater extent than other attacks we consider; (2) `PINV` outperforms other non-motivated attacks across datasets and model architectures, shown through extensive experimentation over the MNIST, SVHN and CIFAR-10 datasets using convolutional network architectures and limited results over higher resolution datasets and textual data over ImageNet and AG News dataset using transformer architectures.

We also find that as the size of the attack set reaches 10^4, `PINV` appears to lose advantage over `OOD` and `EMD`. That is, `PINV` is strongest when it inserts a small number of highly singular, atypical points but as more `PINV` points are added, they become less isolated and behave as a coherent subpopulation. On the other hand, `OOD` and `EMD` can benefit more directly from larger-scale distributional displacement. However, `PINV` does not require knowledge of the underlying data distribution (in comparison to `OOD`). While one might wonder if such adversarial modifications degrade model generalization, we find that the test accuracy does not significantly decrease; full testing analysis is presented in Appendix C.

Attack	MNIST				SVHN				CIFAR-10			
Attack set size	10	100	1000	10000	10	100	1000	10000	10	100	1000	10000
None	0.01	0.01	0.01	0.01	0.06	0.06	0.06	0.06	0.16	0.16	0.16	0.16
OOD	0.04	0.01	0.01	0.01	0.03	0.02	0.01	0.01	0.01	0.01	0.01	0.01
PINV	**0.34**	**0.34**	**0.19**	**0.02**	**0.20**	**0.12**	**0.04**	**0.02**	**0.32**	**0.18**	**0.10**	**0.02**

Table 5: **EAA for label memorization across dataset over MobileNet-v2 architecture.** `PINV` raises label memorization score significantly over base scoring and `OOD` attack.

Memorization scoring and its proxies are inherently metrics which have dependence on both individual data samples and underlying dataset. To compare performance the performance of different attacks, we present our results in accordance to *expected attack advantage* (EAA), our defined metric for comparing the memorization scoring perturbation capabilities for the attacks considered in our study.

Definition 6.1 (Expected Attack Advantage)**.** For a scoring mechanism `score` and attack set $\mathcal{S} \subseteq \mathbf{z}$, we define

$$EAA(\mathcal{S}) := \mathbb{E}_{x \in \mathcal{S}}[\texttt{score}(x)] - \mathbb{E}_{x \in \mathbf{z} \setminus \mathcal{S}}[\texttt{score}(x)]. \tag{2}$$

In particular, EAA captures the expected improvement of memorization scoring between an adversary's attack in comparison to their honest uncorrupted (no attack) data as a baseline, motivated by our threat model of an adversary without knowledge of underlying data distribution. *Concretely, EAA captures the expected difference between scoring of data samples that are attacked and data samples that are produced by `None`.* The results in the row `None` correspond to the clean-data baseline (without any attack samples) included for calibration purposes.

Note: Appendix C contains more visualizations and results over different experimental settings, including higher resolution datasets, textual data and transformer architectures.

6.1 Measuring Attack Effectiveness

Across our experimental settings, `PINV` outperforms the other attacks and produces samples that score highly relative to the base (unmodified) dataset. Tables 1 & 2 record the effectiveness of our four attacks on ResNet-18 models across difference choices of datasets and memorization scores and proxies (for an attack set sizes of 10 and 100). One can see the clear advantage of our theoretically motivated attack `PINV` in comparison to the other unmotivated attacks considered.

An Explanation: Our theoretical work suggests taking the inverse of an in-distribution sample strongly distinguishes it from the rest of the underlying dataset. We assert that it is the high sensitivity that causes the strongest scoring increase; additional experiments in Appendix C favorably compare `PINV` to random noise. On the other hand, while `DF` produces samples that are not meaningfully distinguishable from the underlying dataset, the attack is largely ineffective in perturbing memorization scoring. Based on Definition 3.4, one might believe that points that are most likely to change the decision boundary (i.e., those that lie close to it) would be more memorized. Thus, we also consider attack sets comprising of points that lie close to the decision boundary. Results of the effectiveness of `DF` across different attack sets is presented in Appendix C. We find that boundary starting points marginally improves the performance of `DF`, but not enough to beat out `PINV` which does not assume any prior knowledge of the underlying dataset. In line with our expectations, `OOD` performs well relative to baseline scoring. However, its performance compared with our `PINV` suggests that there is an underlying informational component to memorization that is not fully captured by distributional shifts.

6.2 Robustness of Attacks to Training Setting

Significance of Dataset: Figure 3 compares the effectiveness of our four attacks over confidence event scoring on the MNIST data set across all model architectures we consider. This metric best showcases the advantage of `PINV`. Similar plots that demonstrate the resilience of `PINV` across underlying dataset are presented in Appendix C. We also note that the other attacks seem to exhibit similar trends across dataset, suggesting that memorization-based scoring is resilient for evaluating data for neural networks. Across all datasets, most attacks see a monotonic increase of scoring as the size of attack set increases. We observe that most attacks display their strongest performance over CIFAR10. Our plot suggests that more complex datasets are more susceptible to attack.

Significance of Architecture: Figure 4 compares the effectiveness of our four attacks on just the ResNet-18 architecture across all benchmark datasets we consider using the over confidence event proxy metric. Similar plots that demonstrate the resilience of `PINV` across model architecture are presented in Appendix C. With the exception of `PINV`, we again observe a monotonic increase in attack performance with attack subset size across all architectures. On the VGG-11 models, `PINV` holds a weaker advantage compared to the other

Figure 3: **Confidence event scoring across datasets.** `PINV` outperforms other attacks across datasets.

models. Although the relative ordering is preserved, we still observe some variance in scoring advantage across model architectures.

Figure 4: **Confidence event scoring across model architectures.** `PINV` outperforms other attacks across model architectures.

6.3 Performance of Attacks Across Memorization Proxy

Figure 5 compares the effectiveness of our four attacks on a VGG-11 model trained on the CIFAR-10 dataset across all three memorization proxies we consider. Similar plots tracking `PINV` over different scoring metrics are found in Appendix C. Interestingly, curvature score does not exhibit the monotonic increase in performance observed in our other plots. We also find confidence event scoring to be the most susceptible to attack across all metrics considered. While we observe high variance in attack performance across metrics, the relative rankings of the attacks remain the same.

Figure 5: **Proxy scores on VGG-11 model architectures.** `PINV` outperforms other attacks across memorization proxies

6.4 Label Memorization

For completeness, we also provide a limited set of experiments for the effectiveness of the `OOD` and `PINV` attack on the more computationally intensive label memorization scoring per Feldman & Zhang (2020), in Tables 3, 4, and 5. Unlike Feldman & Zhang, we only use $n = 100$ number of models trained per scoring run due to computational constraints. Each model is trained using the same process as described in Appendix C. We demonstrate that the high sensitivity queries produced by the `PINV` attack raise label memorization scoring in accordance with our theoretical claims.

6.5 Transformer Architecture

While we focus on low resolution datasets and convolutional network architectures due to computational constraints of training models for our experiments, we assert that our main theory is agonistic of training algorithm and model architecture. We present a limited set of experiments over ImageNet data using ViT architectures in Section 6.8 showing that `PINV` efficiently manipulates memorization scores for any training algorithm. We also present a small experiment over text classification dataset AG News over BERT transformer architecture that demonstrates the effectiveness of inverse-based attacks across other data modalities such as text in Section 6.9.

6.6 Comparison to Prior Optimization-based Attacks

Using our memorization scoring functions, we demonstrate that our theoretically motivated sensitivity-based attack method `PINV` outperforms the previously existing naive optimization-based attacks on influence scores in a similar setting by Wang et al. (2024), called the Shadow and Outlier attacks. Table 6, as well as Tables 7, 8 showcase our inverse-based `PINV` better attacks memorization scores while only requiring black-box access to the data and no knowledge of the scoring mechanism. We demonstrate that our attacks exploit a previously unexplored core vulnerability in data valuation scoring methods.

Attack	Attack set size 10	Attack set size 100	Attack set size 1000
PINV	0.67	0.85	0.70
Shadow Wang et al. (2024)	0.24	0.22	0.19
Outlier Wang et al. (2024)	0.21	0.19	0.19

Table 6: **Confidence Event Advantage on MNIST**

Attack	Attack set size 10	Attack set size 100	Attack set size 1000
PINV	0.2	0.14	0.1
Shadow Wang et al. (2024)	0.01	0.01	0.01
Outlier Wang et al. (2024)	0.01	0.01	0.01

Table 7: **Loss Curvature Advantage on MNIST**

Attack	Attack set size 10	Attack set size 100	Attack set size 1000
PINV	0.30	0.30	0.20
Shadow Wang et al. (2024)	0.05	0.03	0.02
Outlier Wang et al. (2024)	0.04	0.01	0.01

Table 8: **Privacy Risk Advantage on MNIST**

We demonstrate that our sensitivity-based attack method `PINV` outperforms the naive optimization-based attacks in (Wang et al., 2024). Furthermore, the computational complexity of the `PINV` is minimal, whereas the Shadow Attack requires the training of multiple "shadow models" and many forward passes and the Outlier Attack requires many forward passes to attack an image. Finally, we argue that the `PINV` attack does not consider the threat model considered in (Wang et al., 2024), but actually a weaker threat model. In the Shadow Attack, knowledge of the training algorithm is required, and in the Outlier Attack query access to the central model for evaluating contribution is required. In our study, no knowledge of the input distribution nor training algorithm is required in our flagship `PINV` attack.

6.7 OOD Detection

Figure 6: Mahalanobis distances of MNIST and attacked `PINV` images.

The Mahalanobis distance is used in outlier detection (Ghorbani, 2019) and is a measure of the statistical distance from the center of a feature space. Figure 6 shows a sample of `PINV` attacked images scored in comparison to MNIST images. While `PINV` produces semantically strange images, the AUROC score of 0.0466 demonstrates that certain OOD detection methods fail to discern `PINV` from ground truth.

6.8 Additional Architecture: Vision Transformers

We also evaluate `PINV` on different architectures.

Attack	Attack set size 10	Attack set size 100	Attack set size 1000
None	0.01±0.00	0.01±0.00	0.01±0.00
PINV	0.03±0.00	0.03±0.00	0.04±0.00

Table 9: **Curvature score of ViT architecture on ImagetNet.** `PINV` retains its high attack scoring capability over non-convolutional networks and higher resolution datasets.

Attack	Attack set size 10	Attack set size 100	Attack set size 1000
None	0.02±0.00	0.02±0.00	0.02±0.00
PINV	0.87±0.00	0.87±0.00	0.86±0.00

Table 10: **Confidence event score of ViT architecture on ImagetNet.** `PINV` retains its high attack scoring capability over non-convolutional networks and higher resolution datasets.

Tables 9 and 10 display the curvature and confidence scoring of ImageNet (Russakovsky et al., 2015) data before and after the `PINV` attack on the Vision Transformer (ViT) architecture (Dosovitskiy et al., 2020).

Our results validate our theoretical claims that high sensitivity queries are model and dataset agnostic in attacking memorization scores.

6.9 Text Classification

While the majority of our evaluation is done on image data, we show that the `PINV` attack can also be applied to text data.

Attack	Attack set size 10	Attack set size 100	Attack set size 1000
None	0.04±0.00	0.04±0.00	0.04±0.00
INV	0.55±0.00	0.61±0.00	0.39±0.00

Table 11: **Confidence event score of BERT architecture on AG News classification dataset.** Inverse operations (`INV`) retain its high attack scoring capability over textual data.

Table 11 displays confidence scoring on AG News (Zhang et al., 2015) text classification dataset over BERT (Devlin et al., 2019) transformer architecture. The `INV` attack describes the additive inverse operation on the additive integer group modulo size vocab length described by a pretrained tokenizer. Our results show that inverse operations display high memorization scoring agnostic of data modality.

7 Conclusion

In our work we present a theoretical justification for our framework of memorization score manipulation, as well as a suite of experimental results that demonstrate the vulnerability of memorization scores and its proxies to adversarial manipulation. We empirically find that a simple and efficient attack of taking a scaled Pseudoinverse of in-distribution image data is sufficient to successfully produce images with high memorization scores, and outperforms previous naive baselines. We also give a theoretical starting point for further analysis of adversarial manipulation of memorization scores and other data attribution methods.

Impact Statement

Our study provides a practical attack on data valuation, which can potentially enable dishonest behavior in future data markets. One of our main claims in the study is a call for robust scoring mechanisms, which will mitigate potential harms. We also acknowledge the potential societal consequences presented in any machine learning study, which we do not feel must be specifically highlighted here.

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Appendix

A   Theoretical Analysis

Recall that the goal is to study the following question: *what queries on the dataset would lead to high memorization scores?* In order to quantify the question, we consider the following memorization score on addition of a new example to the existing dataset:

$$\mathtt{mem}(\mathcal{A}, \mathbf{z}, q(\mathbf{z})) := \Pr_{(x,y) \leftarrow q(\mathbf{z}), h \leftarrow \mathcal{A}(\mathbf{z} \cup q(\mathbf{z}))}[h(x) = y] - \Pr_{(x,y) \leftarrow q(\mathbf{z}), h \leftarrow \mathcal{A}(\mathbf{z})}[h(x) = y] \tag{3}$$

It is natural to consider adding $k > 1$ examples so we also consider the case where k (adaptive) queries on the existing dataset can be made:

$$\mathtt{mem}(\mathcal{A}, \mathbf{z}, \{q_i(\mathbf{z})\}_{i=1}^{k}) := \Pr_{\{(x_i, y_i)\}_{i=1}^{k} \leftarrow \{q_i(\mathbf{z})\}_{i=1}^{k}, h \leftarrow \mathcal{A}(\mathbf{z} \cup \{q_i(\mathbf{z})\}_{i=1}^{k})}[\cap_{i=1}^{k} h(x_i) = y_i]$$
$$- \Pr_{\{(x_i, y_i)\}_{i=1}^{k} \leftarrow \{q_i(\mathbf{z})\}_{i=1}^{k}, h \leftarrow \mathcal{A}(\mathbf{z})}[\cap_{i=1}^{k} h(x_i) = y_i] \tag{4}$$

We begin by restating our main theorem:

Theorem A.1. *Let Q_Δ be a family of Δ-sensitive queries on $\mathcal{Z}^n$. Fix $\delta \in [0,1]$ and let the dataset size $n \in \mathbb{N}$ be such that there exists $\gamma > \delta$ such that $n \geq 1/\gamma$.*

Then for any $\alpha, \beta \in (0, 1/10)$, there exists algorithm $\mathcal{A}$ with memorization score (i.e., $\mathtt{mem}(\mathcal{A}, \mathbf{z}, q(\mathbf{z})) \leq \delta$ from Equation 1) of at most δ such that $\mathcal{A}$ is (α, β)-accurate for any query from Q_Δ but it must be the case that $\alpha \geq \gamma \Delta n$ and $\beta \geq \frac{\delta}{2\gamma}$.

That is, there exists algorithm with memorization score (Equation 1) of at most δ such that for $\mathbf{z} \leftarrow_R \mathbf{P}^n$ and query $q \in Q_\Delta$,

$$\Pr[\|q(\mathbf{z}) - q(\mathbf{P})\| \geq \gamma \Delta n] \geq \frac{\delta}{2\gamma},$$

where $\mathbf{P}$ is a distribution over $\mathcal{Z}$.

Our theory suggests that, in order to avoid manipulation by data sellers, the algorithm $\mathcal{A}$ cannot be too accurate on the population level or the algorithm $\mathcal{A}$ must (formally) satisfy stability guarantees. e.g., max-KL stability which can be achieved via the use of differential privacy. Without such guarantees, one can effectively attack memorization scores.

High-level Idea for Proof of Theorem A.1   We will engineer a Δ-sensitive query that counts (at scale Δ) how many sample points satisfy a *sample-dependent* predicate p. The predicate p itself is produced by a *stable* $(0, \delta)$-max-KL procedure. We will use the predicate to construct Δ-sensitive queries (i.e., adding or removing a single record can change the induced query value by at most Δ). Intuitively, when p is chosen from the sample $\mathbf{z}$, the value $q(\mathbf{z})$ is large ($\geq \gamma \Delta n$) whereas the population value $q(\mathbf{P})$ remains 0. Stability keeps the *memorization score* (the advantage of re-training on an added/removed example) at most δ. And we prove that the empirical accuracy of such algorithm cannot, with high probability, be close to the population accuracy.

Proof of Theorem A.1. First, we start with the following algorithm $\mathcal{B}$ that is $(0, \delta)$-max-KL stable (Definition 3.3). $\mathcal{B}$ is defined as follows:

- For any database element e input to $\mathcal{B}$, output e with probability δ.

- Otherwise, with probability $1 - \delta$, output the empty database $\emptyset$ that is independent of the input to $\mathcal{B}$.

Input: A database $\mathbf{z} \in [0,1]^n$. We think of $\mathbf{z}$ as $\lceil \frac{1}{\gamma} \rceil$ databases of size γn each: $\mathbf{z} = (\mathbf{z}_1, \ldots, \mathbf{z}_{\lceil 1/\gamma \rceil})$.

For $1 \le i \le \lceil 1/\gamma \rceil$, let $\hat{\mathbf{z}}_i = \mathcal{B}(\mathbf{z}_i)$.

Let $p : [0,1] \to \{0,1\}$ where $p(z) = 1$ iff $\exists i$ s.t. $z \in \hat{\mathbf{z}}_i$.

Define $q_p : [0,1]^n \to \mathbb{R}$ where $q_p(\mathbf{z}) = \Delta \sum_{z \in \mathbf{z}} p(z)$ (note that q_p is a Δ-sensitive query).

Output: q_p.

Figure 7: $(0, \delta)$ max-KL stable algorithm $\mathcal{A}$.

Clearly, $\mathcal{B}$ satisfies the definition of $(0, \delta)$-max-KL stability.

Now, given a dataset $\mathbf{z} \in [0,1]^n$, we partition the dataset into $\lceil 1/\gamma \rceil$ disjoint blocks $\mathbf{z}^{(1)}, \ldots, \mathbf{z}^{(\lceil 1/\gamma \rceil)}$, each of size γn (possible since $n \ge 1/\gamma$).

Consider the stable mechanism $\mathcal{A}$ defined in Figure 7: on each block $\mathbf{z}^{(i)}$, run $\mathcal{B}$ and obtain output $\hat{\mathbf{z}}^{(i)}$. Define a predicate $p : [0,1] \to \{0,1\}$ by

$$p(u) = \mathbb{1}\left\{ u \in \bigcup_{i \in \lceil 1/\gamma \rceil} \{\hat{\mathbf{z}}^{(i)}\} \right\}.$$

Now define the Δ-sensitive query

$$q_p(\mathbf{z}) = \Delta \cdot \sum_{j=1}^{n} p(\mathbf{z}_j).$$

Since changing one coordinate alters the sum by at most Δ, q_p is indeed Δ-sensitive. By Lemma 3.2, q_p inherits the $(0, \delta)$-max-KL stability of $\mathcal{B}$. Thus, $\mathcal{A}$ satisfies $(0, \delta)$-max-KL stability since $\mathcal{B}$ is applied to disjoint databases.

Now let $\mathbf{z} = (\mathbf{z}_1, \ldots, \mathbf{z}_{\lceil 1/\gamma \rceil})$ contain i.i.d. samples from $\mathcal{Z}$, and consider the execution of $\mathcal{B}$ on $\mathbf{z}$. Note that the predicate p evaluates to 1 only on a finite number of points from $[0,1]$, an infinite universe. Therefore,

$$q_p(\mathbf{P}) = \Delta \cdot n \cdot \mathbb{P}_{u \leftarrow \mathbf{P}}[p(u) = 1] = 0.$$

Next, notice that

$$q_p(\mathbf{z}) = \gamma \Delta n \cdot |\{i : \hat{\mathbf{z}}_i = \mathbf{z}_i\}|.$$

Therefore, if there exists an i s.t. $\hat{\mathbf{z}}_i = \mathbf{z}_i$ then $q_p(\mathbf{z}) - q_p(\mathbf{P}) \ge \gamma \Delta n$. Since the probability of sampling any element in the database is δ, the probability that $q_p(\mathbf{z}) - q_p(\mathbf{P}) < \gamma \Delta n$ is at most

$$(1 - \delta)^{1/\gamma} \le e^{-\delta/\gamma} \le 1 - \frac{\delta}{2\gamma},$$

for some $\gamma > \delta$. Then, with probability at least $\frac{\delta}{2\gamma}$, algorithm $\mathcal{A}$ outputs a Δ-sensitive query q_p s.t. $q_p(\mathbf{z}) - q_p(\mathbf{P}) \ge \gamma \Delta n$.

We have already established that $\mathcal{A}$ is $(0, \delta)$-max-KL stable. Looking at Equation 1, $\mathcal{A}$ is trained with and without some element $q(\mathbf{z})$. The memorization score is computed as the difference of post-processing when $\mathcal{A}$ is trained with and without the element. By post-processing (Lemma 3.2), *any* learned hypothesis h will still differ in probabilities of classification on any input, with and without training on $q(\mathbf{z})$, by at most δ. Thus, $\mathtt{mem}(\mathcal{A}, \mathbf{z}, q(\mathbf{z})) \le \delta$.

And we have also shown that for $\mathbf{z} \leftarrow_{\mathrm{R}} \mathbf{P}^n$ and query $q \in Q_\Delta$,

$$\Pr[\|q(\mathbf{z}) - q(\mathbf{P})\| \ge \gamma \Delta n] \ge \frac{\delta}{2\gamma},$$

where $\mathbf{P}$ is a distribution over $\mathcal{Z}$. $\qquad\square$

We note that Max-KL stability is related to other notions of stability in the literature, including KL-stability and TV-stability:

Definition A.1 (TV-Stability)**.** Let $\mathcal{A} : \mathcal{Z}^n \to \mathcal{R}$ be a randomized algorithm. We say that $\mathcal{A}$ is ϵ-*TV stable* if for every pair of samples that differ on exactly one element,

$$\mathrm{d}_{\mathrm{TV}}(\mathcal{A}(\mathbf{z}), \mathcal{A}(\mathbf{z}')) = \sup_{R \subseteq \mathcal{R}} \left| \mathbb{P}\left[\mathcal{A}(\mathbf{z}) \in R\right] - \mathbb{P}\left[\mathcal{A}(\mathbf{z}') \in R\right] \right| \leq \epsilon.$$

Definition A.2 (KL-Stability)**.** Let $\mathcal{A} : \mathcal{Z}^n \to \mathcal{R}$ be a randomized algorithm. We say that $\mathcal{A}$ is ϵ-*KL-stable* if for every pair of samples $\mathbf{z}, \mathbf{z}'$ that differ on exactly one element,

$$\mathbb{E}_{r \leftarrow_{\mathrm{R}} \mathcal{A}(\mathbf{z})} \left[\log \left(\frac{\mathbb{P}\left[\mathcal{A}(\mathbf{z}) = r\right]}{\mathbb{P}\left[\mathcal{A}(\mathbf{z}') = r\right]} \right) \right] \leq 2\epsilon^2$$

It can be shown that (ϵ, δ)-max-KL stability implies $(2\epsilon + \delta)$-TV stability for $\epsilon \leq 1$ and $\delta > 0$ (Dwork et al., 2010). Because of the relations between the stability notions, we have focused on one such notion, the max-KL stability, and its properties.

B   Attack Details

For each attack, we only change the input image x, while retaining its original label y, to retain class distributions in each dataset.

B.1   OOD Replacement Attack

We first consider the robustness of data valuation to samples out-of-distribution of the original dataset. We use other known datasets to sample out-of-distribution images from. For MNIST, we attack using replacement samples from FashionMNIST. We attack SVHN with CIFAR-10 and vice-versa. The attack procedure is described in Algorithm 1.

Algorithm 1 OOD Replacement Attack

  1: **Input:** Image x in dataset S, out-of-distribution dataset S'
  2: **Output:** Image $\hat{x}$ sampled from S'
  3:
  4: Sample $x' \sim S'$
  5: **return** $\hat{x} \leftarrow x'$

B.2   Pseudoinverse Attack

We treat each image as a matrix and take its pseudoinverse as the attack sample. To prevent vanishing gradient from pixel values being too close to 0 after the pseudoinverse operation, we multiply our perturbed images by a scalar factor to return images that can be processed by our neural networks. In PyTorch, images are converted to tensors in $[0, 1]$, so our attack correspondingly scales the pseudoinverses to have unit normalization, described in Algorithm 2. For the RGB images of SVHN and CIFAR-10, we take the pseudoinverse of each channel and return the stacked pseudoinverses as our attack image. x^+ denotes the usual pseudoinverse operation in linear algebra.

Algorithm 2 Pseudoinverse Perturbation Attack

  1: **Input:** Image x
  2: **Output:** Pseudoinverse $\hat{x}$ scaled to avoid vanishing gradient
  3:
  4: **return** $\hat{x} \leftarrow \frac{x^+}{||x^+||_1}$

B.3   Naive Wasserstein Attack (Earth Mover's Distance)

We aim to produce images that have maximal Wasserstein distance from the original image as possible. To reduce computational complexity, we implement a naive greedy binary search method (over the possible range of RGB values for images) that searches per pixel for the image with maximal Wasserstein distance, described in Algorithm 3. Note that $\mu_W(x, y)$ returns the Wasserstein distance between two images x and y.

$\hat{x}_{p \leftarrow l}$ denotes the images $\hat{x}$ with pixel p having value l, and $\hat{x}_{p \leftarrow r}$ is similarly defined.

Algorithm 3 Naive EMD Attack

1: **Input:** Image x
2: **Output:** Image $\hat{x}$ with high Wasserstein distance from original image
3:
4: Initialize $\hat{x} \leftarrow \mathbf{0}$
5: **for** pixel p in x **do**
6:     $l \leftarrow 0, r \leftarrow 255$
7:     **for** $i = 1$ to 8 **do**
8:         **if** $\mu_W(\hat{x}_{p \leftarrow l}, x) < \mu_W(\hat{x}_{p \leftarrow r}, x)$ **then**
9:             $l \leftarrow \frac{l+r}{2}$
10:         **else**
11:             $r \leftarrow \frac{l+r}{2}$
12:         **end if**
13:     **end for**
14:     $\hat{x} \leftarrow \hat{x}_{p \leftarrow l}$
15: **end for**
16: **return** $\hat{x}$

B.4   DeepFool Perturbation Attack

Moosavi-Dezfooli et al. (2016) give an algorithm for DeepFool, an adversarial classification attack which perturbs an input image towards the decision boundary of a model to force misclassification. We attack data points by pushing them towards the decision boundary. The attack procedure is described in Algorithm 5, with Moosavi-Dezfooli et al. (2016)'s DeepFool subroutine described in Algorithm 4. We use overshoot parameter $\alpha = 0.02$ as in the original DeepFool paper.

Algorithm 4 DeepFool (Moosavi-Dezfooli et al.)

1: **Input:** Image x, classifier h which outputs class probabilities from an image, and $\hat{h}$ which outputs class labels from an image based on h
2: **Output:** Perturbation $\hat{r}$ that pushes x out of decision region.
3:
4: Initialize $x_0 \leftarrow x, i \leftarrow 0$.
5: **while** $\hat{h}(x_i) = \hat{h}(x_0)$ **do**
6:     **for** classes $k \neq \hat{h}(x_0)$ **do**
7:         $\mathbf{w}'_k \leftarrow \nabla h_k(x_i) - \nabla h_{\hat{h}(x_0)}(x_i)$
8:         $h'_k \leftarrow h_k(x_i) - h_{\hat{h}(x_0)}(x_i)$
9:     **end for**
10:     $\hat{l} \leftarrow \arg\min_{k \neq \hat{h}(x_0)} \frac{|h'_k|}{||\mathbf{w}'_k||_2}$
11:     $r_i \leftarrow \frac{|h'_{\hat{l}}|}{||\mathbf{w}'_{\hat{l}}||_2^2} \mathbf{w}_{\hat{l}}$
12:     $x_{i+1} \leftarrow x_i + r_i$
13:     $i \leftarrow i + 1$
14: **end while**
15: **return** $\hat{r} = \sum_i r_i$

Algorithm 5 Adversarial DeepFool Perturbation Attack

1: **Input:** Image x in dataset S, with training algorithm $\mathcal{A}$, overshoot α
2: **Output:** Image $\hat{x}$ such that model trained on S with mislabel y with minimal distance
3:
4: Train classifier $h \rightarrow \mathcal{A}(S)$
5: $\hat{r} \leftarrow \text{DeepFool}(h, x)$.
6: **return** $\hat{x} \leftarrow x + (1 + \alpha)\hat{r}$

C Additional Experimental Details

The codebase for our experiments is presented at `https://anonymous.4open.science/r/MemAttack-5413/`.

C.1 Training Setup

Over all models we use the SGD optimizer with learning rate of 10^{-2} and momentum 0.9. We use a batch size of 512 to speed up training. For models trained on MNIST we train for 5 epochs, and for the other two datasets SVHN and CIFAR10 we train for 10 epochs. All models are trained using the CrossEntropy loss function. All experiments were conducted using NVIDIA A40 single GPU nodes, and all training and attack procedures were implemented in PyTorch. Our architectures and training algorithms are not state-of-the-art since state-of-the-art training is significantly more computationally intensive.

C.2 Proxy Implementation Details

C.2.1 Loss Curvature

For the Loss Curvature calculation, we take the curvature after epoch 2 for MNIST and after epoch 5 for SVHN and CIFAR10. We follow the code from Ravikumar et al. (2024) which uses Hutchinson's trace estimator to approximate the Hessian product of randomly sampled Radamacher vectors for trace estimation, consistent with Garg et al. (2023).

C.2.2 Learning Events

We give a detailed description of the four learning events below, following Jiang et al. (2020); Zhao & Triantafillou (2024). Each is implicitly evaluated for any data sample (x, y).

1. **Confidence:** The softmax probability of $h(x)$ corresponding to ground truth label y.

2. **Max Confidence:** The highest softmax probability of $h(x)$ across all classes.

3. **Entropy:** The entropy of the output probabilities of $h(x)$.

4. **Binary Correctness:** Indicator of whether the model correctly predicts y for x (0 or 1).

For each event, we evaluate each data point after each training data point, and then take the mean of the event across all epochs to return as our final learning event score.

C.2.3 Privacy Risk Score

We closely adhere to Song & Mittal (2021) implementation for membership inference. We train our shadow models on the test split of each dataset, and then evaluate the membership of a target model trained over the training set (which is perturbed by our attacks) to calculate a score for each training data sample. The precise score comes from comparing histograms of of shadow and target models.

C.3 Additional Comparison Tables

C.3.1 VGG-11 Results

Tables 12, 13, 14 present the mean and variance of our four attacks on the VGG-11 architecture across all of our experimental settings. As discussed in § 6, PINV outperforms other attacks but loses its advantage when the attack size reaches 10^4. Although PINV is observed to lose out on its top spot in certain settings, we still observe it to consistently produce perturbed points with high advantage over non-attack points. PINV performs the best in the SVHN dataset under the VGG-11 architecture.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.00±0.00	0.01±0.00	0.05±0.00	0.01±0.00	0.07±0.00	0.24±0.00	0.50±0.00	0.49±0.00	0.32±0.00
OOD	0.11±0.00	0.01±0.00	0.25±0.00	0.57±0.01	0.52±0.01	0.61±0.00	0.20±0.00	0.05±0.01	0.26±0.00
PINV	0.11±0.00	**0.17±0.00**	**0.41±0.00**	**0.78±0.01**	**0.77±0.00**	**0.65±0.00**	**0.33±0.01**	**0.51±0.00**	**0.65±0.00**
EMD	**0.17±0.00**	0.04±0.00	0.27±0.00	0.62±0.00	0.65±0.00	0.48±0.01	0.06±0.01	0.06±0.00	0.11±0.01
DF	0.00±0.00	0.00±0.00	0.01±0.00	0.00±0.00	0.04±0.00	0.03±0.01	0.01±0.00	0.03±0.00	0.03±0.00

Table 12: **EAA on VGG-11 architecture with attack set size of 10.** PINV outperforms almost all other attacks by a significant margin across dataset and proxy.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.01±0.00	0.01±0.00	0.06±0.00	0.01±0.00	0.07±0.00	0.24±0.00	0.50±0.00	0.49±0.00	0.30±0.00
OOD	0.07±0.00	0.01±0.00	0.25±0.00	0.80±0.00	0.61±0.00	0.63±0.00	0.36±0.00	0.06±0.00	0.49±0.00
PINV	0.08±0.00	**0.23±0.00**	**0.37±0.00**	**0.85±0.00**	**0.81±0.00**	**0.66±0.00**	**0.43±0.00**	**0.51±0.00**	**0.70±0.00**
EMD	**0.12±0.00**	0.04±0.00	0.22±0.00	0.84±0.00	0.74±0.00	0.59±0.00	0.31±0.00	0.08±0.00	0.15±0.00
DF	0.00±0.00	0.00±0.00	0.00±0.00	0.00±0.00	0.00±0.00	0.00±0.00	0.01±0.00	0.01±0.00	0.01±0.00

Table 13: **EAA on VGG-11 architecture with attack set size of 100.** PINV outperforms almost all other attacks by a significant margin across dataset and proxy.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.00±0.00	0.01±0.00	0.06±0.00	0.01±0.00	0.07±0.00	0.24±0.00	0.51±0.00	9,49±0.00	0.31±0.00
OOD	0.03±0.00	0.01±0.00	0.21±0.00	0.88±0.00	0.75±0.00	**0.65±0.00**	**0.50±0.00**	0.15±0.00	0.64±0.00
PINV	**0.06±0.00**	**0.09±0.00**	0.24±0.00	0.76±0.00	**0.81±0.00**	0.64±0.00	0.42±0.00	**0.51±0.00**	**0.68±0.00**
EMD	**0.06±0.00**	0.07±0.00	**0.28±0.00**	**0.89±0.00**	0.80±0.00	**0.65±0.00**	**0.50±0.00**	0.15±0.01	0.27±0.00
DF	0.00±0.00	0.00±0.00	0.00±0.00	0.00±0.00	0.01±0.00	0.00±0.00	0.00±0.00	0.00±0.00	0.00±0.00

Table 14: **EAA on VGG-11 architecture with attack set size of 1000.** OOD, PINV, EMD all perform similarly across dataset and proxy.

C.3.2 ResNet-18 Results

Tables 1, 2, 15 present the mean and variance of our four attacks on the ResNet-18 architecture across all of our experimental settings. We again observe that PINV outperforms other attacks but slightly loses its advantage when the attack size reaches 10^4. Compared to the results over the VGG-11 architecture, it appears that PINV displays more consistent top performances over the ResNet-18 model. We again observe that PINV performs relatively the best in the SVHN dataset.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.01±0.00	0.01±0.00	0.09±0.00	0.01±0.00	0.06±0.00	0.23±0.00	0.53±0.00	0.49±0.00	0.21±0.00
OOD	0.07±0.00	0.02±0.00	**0.04±0.00**	**0.86±0.00**	0.68±0.00	0.65±0.00	0.51±0.01	**0.49±0.00**	**0.23±0.00**
PINV	**0.10±0.00**	**0.03±0.00**	0.02±0.00	0.70±0.00	**0.82±0.00**	**0.66±0.00**	**0.53±0.00**	**0.49±0.00**	0.21±0.00
EMD	0.03±0.00	0.00±0.00	-0.04±0.00	**0.86±0.00**	0.80±0.00	**0.66±0.00**	0.52±0.00	**0.49±0.00**	0.18±0.00
DF	0.00±0.00	0.00±0.00	0.00±0.00	0.00±0.00	0.01±0.00	0.00±0.00	0.52±0.00	0.48±0.00	0.21±0.00

Table 15: **EAA on ResNet-18 architecture with attack set size of 1000.** OOD, PINV, EMD all perform similarly across dataset and proxy.

C.3.3 MobileNet-v2 Results

Tables 16, 17, 18 present the mean and variance of our four attacks on the MobileNetV2 architecture across all of our experimental settings. We find that over the MobileNetV2 architecture, `PINV` displays the most resilience with the Loss Curvature proxy. Furthermore, `PINV` remains a strong performer across the other proxies.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.00±0.00	0.01±0.00	0.12±0.00	0.01±0.00	0.09±0.00	0.33±0.00	0.45±0.02	0.49±0.00	0.47±0.00
OOD	0.10±0.00	0.03±0.01	0.27±0.00	0.56±0.00	0.65±0.00	**0.54±0.00**	**0.59±0.00**	**0.50±0.00**	0.41±0.00
PINV	**0.22±0.00**	**0.19±0.00**	**0.46±0.00**	**0.74±0.00**	**0.77±0.00**	0.51±0.00	0.45±0.02	0.49±0.00	**0.47±0.00**
EMD	0.04±0.00	0.01±0.00	-0.04±0.00	0.48±0.00	0.73±0.00	0.51±0.00	0.51±0.02	0.49±0.00	0.40±0.01
DF	0.00±0.00	0.00±0.00	-0.03±0.00	0.00±0.00	0.00±0.00	-0.06±0.00	0.50±0.00	0.49±0.00	**0.47±0.02**

Table 16: **EAA on MobileNet-v2 architecture with attack set size of 10.** `PINV` outperforms almost all other attacks by a slight margin across dataset and proxy.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.00±0.00	0.01±0.00	0.11±0.00	0.01±0.00	0.09±0.00	0.33±0.00	0.46±0.01	0.48±0.00	0.40±0.01
OOD	0.10±0.00	0.02±0.00	0.22±0.00	0.66±0.00	0.62±0.00	0.53±0.00	0.53±0.02	**0.50±0.00**	**0.43±0.02**
PINV	**0.15±0.00**	**0.16±0.00**	**0.35±0.00**	**0.80±0.00**	**0.77±0.00**	**0.55±0.00**	0.46±0.01	0.48±0.00	0.40±0.00
EMD	0.03±0.00	0.01±0.00	-0.04±0.00	0.67±0.00	**0.77±0.00**	0.54±0.00	**0.61±0.00**	**0.50±0.00**	**0.43±0.01**
DF	0.00±0.00	0.00±0.00	0.01±0.00	0.00±0.00	0.00±0.00	0.02±0.00	0.60±0.01	**0.50±0.00**	0.37±0.00

Table 17: **EAA on MobileNet-v2 architecture with attack set size of 100.** `OOD`, `PINV`, `EMD` all perform similarly across dataset and proxy.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.01±0.00	0.01±0.00	0.12±0.00	0.01±0.00	0.09±0.00	0.33±0.00	0.40±0.01	0.52±0.00	0.41±0.02
OOD	0.08±0.00	0.02±0.00	0.11±0.00	0.84±0.00	0.72±0.00	**0.55±0.00**	0.40±0.01	0.50±0.00	0.34±0.01
PINV	**0.14±0.00**	**0.07±0.00**	**0.28±0.01**	0.67±0.00	**0.79±0.00**	0.54±0.00	0.40±0.01	**0.52±0.00**	0.41±0.01
EMD	0.03±0.00	0.00±0.00	-0.05±0.00	**0.87±0.00**	**0.79±0.00**	**0.55±0.00**	**0.64±0.01**	0.50±0.00	0.34±0.01
DF	0.00±0.00	0.00±0.00	0.00±0.00	0.00±0.00	0.01±0.00	-0.01±0.00	0.44±0.00	0.49±0.00	**0.44±0.01**

Table 18: **EAA on MobileNet-v2 architecture with attack set size of 1000.** `OOD`, `PINV`, `EMD` all perform similarly across dataset and proxy.

C.4 Additional Comparison Plots

C.4.1 Curvature Proxy Plots

Comparison plots of the four attacks for the Loss Curvature proxy across dataset and model architecture are presented in Figure 8. Unlike the other proxies, the absolute value of attack curvature decreases as the size of the attack set increases in `PINV`. The other attacks seem more resilient to this monotonic decrease, which follows our observations in § 6 where we assert that `PINV` loses its advantage as the attack size increases. With the exception of 7a., we see that `PINV` demonstrates superiority over the other attacks across all training settings over the Loss Curvature proxy.

Figure 8: **Attack comparison plots for Loss Curvature.** `PINV` outperforms other attacks across dataset and model architecture.

C.4.2 Confidence Proxy Plots

Comparison plots of the four attacks for the Confidence Event proxy across dataset and model architecture are presented in Figure 9. With some exceptions at attack size 10^4, we see that `PINV` strongly outperforms the other attacks in the Confidence Event proxy across all other experimental parameters. We also note that the performance of all four attacks seem consistent across attack set size, which differs from our observations in the other two proxies.

Figure 9: **Attack comparison plots for Confidence Event.** `PINV` outperforms other attacks across dataset and model architecture.

C.4.3 Privacy Score Proxy Plots

Comparison plots of the four attacks for the Risk Score proxy across dataset and model architecture are presented in Figure 10. Here we see again that `PINV` is a consistently strong performer across all experimental parameters. Outside of MNIST, `PINV` solidly trumps all other attacks. We observe that while MobileNetV2 admits a wider range of performances for the other attacks in comparison to the other architectures, but none consistently scores higher than `PINV`.

Figure 10: **Attack comparison plots for Privacy Risk.** `PINV` outperforms other attacks across dataset and model architecture.

C.5 Other Learning Events

In accordance with Zhao & Triantafillou (2024), we consider 3 learning event proxies other than the confidence event detailed in the main paper, presented in Table 19. In line with the main results, we observe that `PINV` outperforms the other attacks regardless of the learning event considered. We also find that the relative ordering of the other attacks is preserved, which affirms the results of Jiang et al. (2020); Zhao & Triantafillou (2024) which find that these learning events have high correlation with label memorization, and each other.

Attack	Max Confidence Event			Entropy Event			Binary Correctness Event		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	0.01±0.00	0.05±0.00	0.17±0.00	0.02±0.00	0.16±0.00	0.50±0.00	0.00±0.00	0.04±0.00	0.15±0.00
OOD	0.42±0.00	0.40±0.00	0.36±0.00	1.24±0.00	1.16±0.00	0.98±0.00	0.53±0.00	0.42±0.00	0.66±0.00
PINV	**0.67±0.01**	**0.72±0.00**	**0.56±0.00**	**1.91±0.04**	**1.96±0.00**	**1.56±0.00**	**0.80±0.00**	**0.79±0.00**	**0.72±0.00**
EMD	0.40±0.00	0.60±0.00	0.43±0.00	1.21±0.01	1.69±0.02	1.21±0.01	0.40±0.00	0.55±0.00	0.50±0.01
DF	0.00±0.00	0.01±0.00	0.00±0.00	0.00±0.00	0.01±0.00	0.01±0.00	0.00±0.00	0.00±0.00	0.01±0.00

Table 19: **Expected attack advantage on ResNet-18 architecture with attack set of size 100.** `PINV` outperforms almost all other attacks by a significant margin across dataset and learning event.

C.6 Random Noise

Table 20 shows `PINV` compared with random noise in curvature scoring over ResNet-18 architecture. Despite lack of semantic meaning, `PINV` supports our theory linking high sensitivity queries with high memorization.

Attack	Attack set size 10	Attack set size 100	Attack set size 1000
Random Noise	0.13±0.00	0.05±0.00	0.01±0.00
PINV	0.36±0.00	0.21±0.00	0.13±0.00

Table 20: **Curvature score of ResNet-18 architecture on CIFAR10.** `PINV` outperforms simple random noise in memorization scoring.

C.7 Boundary Points in DeepFool

One might believe that samples that are most likely to change the decision boundary would be more memorized. We consider attack sets for `DF` consisting of what we call *boundary points*. We define a sample as being closer to the boundary when the model's maximum softmax probability is smaller, indicating lower confidence for a model in its prediction on that sample. We find that this selection of attack set improves `DF`, but not enough to gain advantage over our simpler `PINV` that retains effectiveness without knowledge of the underlying dataset. Figure 11 displays the performance of `DF` on both random and boundary starting points, both of which fall short of our best performer in `PINV`.

C.8 Percentile

Table 21 displays the mean and variance of score percentile before and after our four attacks for the ResNet-18 architecture with attack size of 100. Unsurprisingly, `PINV` remains the best attack across training setting, in line with our other results. However, when looking at only percentile, the gap between `PINV` and other attacks is not as dramatic as when looking at raw scores. We find that the percentile results affirms Feldman (2020)'s observations that few samples have exhibit strong memorization in the form of a long-tail distribution. While the magnitude of curvature of our attack points is small, our attacks display effectiveness in creating highly ranked samples. `PINV` remains the most desirable attack with its computational efficiency and weaker threat model.

Figure 11: **Performance of DF on random vs. boundary starting points.** Although DF improves performance with smart selection of starting points, PINV remains superior without any knowledge of the underlying distribution.

Attack	Loss Curvature			Confidence Event			Privacy Score		
	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10	MNIST	SVHN	CIFAR-10
None	50.0±0.9	49.2±7.9	51.2±3.5	51.3±5.9	50.6±19.8	49.3±12.0	50.4±5.0	49.2±3.9	49.6±15.2
OOD	98.9±0.1	84.0±4.2	**84.1±1.8**	99.8±0.0	97.4±0.0	97.5±0.3	61.0±23.	24.5±3.9	68.0±23.5
PINV	**99.1±0.1**	**97.9±0.7**	78.2±8.6	**99.9±0.0**	**99.6±0.0**	**99.3±0.0**	**79.4±24.0**	**100.0±0.0**	**98.9±0.7**
EMD	97.7±1.0	63.3±3.5	48.7±2.4	99.8±0.0	99.0±0.0	96.7±1.1	47.9±23.7	42.1±8.3	63.8±11.6
DF	50.6±8.6	53.1±2.9	50.7±14.1	50.1±2.6	54.3±8.3	50.4±3.8	55.6±8.4	45.7±0.5	60.6±12.7

Table 21: **Average attack score percentile on ResNet-18 architecture with attack set of size 100.** PINV outperforms almost all other attacks across dataset and proxy.

C.9 Test Correctness

Table 22 displays the mean and variance of test accuracy of a model before and after each attack over $t = 5$ trials over CIFAR10. Unsurprisingly, the test accuracy does not change significantly with our attacks, considering we only perturb a small fraction (2%) of the training data set at most. Our attacks would remain undetected by any test accuracy analysis.

C.10 Larger Attack Sizes

Daniel: Tue: add some experiments for $10^4, 10^5$.

Model	VGG-11			ResNet-18			MobileNetV2		
Attack set size	10	100	1000	10	100	1000	10	100	1000
None	76.4±0.9	77.1±0.2	76.8±0.6	78.9±0.2	79.6±0.0	78.8±0.6	69.4±0.6	69.7±0.5	68.8±0.4
OOD	75.6±6.2	76.3±1.5	75.7±0.3	**79.2±0.0**	78.4±2.5	77.1±0.4	68.8±1.5	69.6±1.1	69.0±0.3
PINV	**78.0±1.0**	**76.6±0.1**	76.4±1.2	79.1±0.4	78.3±0.3	77.9±0.5	67.8±0.2	68.7±1.5	68.8±0.3
EMD	76.3±2.5	76.0±1.3	**76.5±0.43**	79.0±0.5	79.2±0.5	76.5±2.2	68.7±2.0	69.4±1.3	69.1±1.1
DF	77.1±1.7	77.2±1.3	76.6±1.0	78.9±0.6	**79.3±0.1**	**79.5±0.5**	**69.1±0.4**	**70.4±0.4**	**70.1±1.8**

Table 22: **Test accuracy before and after attack on CIFAR10.** All attacks have negligible influence on testing accuracy.