CITE: Anytime-Valid Statistical Inference in LLM Self-Consistency

Hirofumi Ota [1 2] **Naoto Iwase** [3] **Yuki Ichihara** [4 5] **Junpei Komiyama** [5 2] **Masaaki Imaizumi** [1 2 6]

Abstract

Large language models often improve reasoning by sampling multiple outputs and aggregating their final answers, but precise and efficient control of error levels remains a challenging task. In particular, deciding when to stop sampling remains difficult when the stopping rule is data-dependent and the set of possible response labels is not known in advance. We study anytime-valid certification of a prespecified target answer as the unique mode of the model's response distribution, a guarantee distinct from answer correctness. We propose the Certification by Intersection-union Testing with E-processes (CITE) algorithm, which provably controls false certification at any prescribed level under arbitrary data-driven stopping, without requiring prior knowledge of the answer category set. We also prove a category-set-size-free stopping-time rate, establish matching minimax lower bounds up to constants in the main regime, and extend the construction to confidence-weighted voting. Simulations and LLM self-consistency experiments show empirical error control and improved certification in diffuse-tail settings.

1. Introduction

1.1. Background and Motivation

Large language models (LLMs) have demonstrated increasingly strong performance on a wide range of tasks (Brown et al., 2020; Chowdhery et al., 2023; Achiam et al., 2023; Touvron et al., 2023), especially from the use of *test-time compute*. Prompting methods such as chain-of-thought and least-to-most decomposition encourage models to expose intermediate reasoning steps, while inference-time strategies such as self-consistency and tree-of-thought style search improve robustness by sampling, exploring, and aggregating multiple candidate reasoning trajectories (Wei et al., 2022; Zhou et al., 2023; Wang et al., 2023; Yao et al., 2023; Brown et al., 2024; Snell et al., 2025).

A notion of *adaptive self-consistency* has gained attention to reduce computational cost of sampling growing approximately linearly with the number of sampled trajectories. A seminal approach is *majority voting* (Wang et al., 2023), which is simple, model-agnostic, and does not require a separate verifier, reward model, or additional training. In particular, the method adapts to the case that easy questions often stabilize after only a few samples, whereas hard questions remain ambiguous and may benefit from continued sampling. As extensions, (Aggarwal et al., 2023) studies a stopping method based on posterior agreement among sampled answers; (Li et al., 2024) uses low-entropy windows as a practical stopping signal; (Wang et al., 2025b) and (Wang et al., 2025a) allocate budgets using difficulty estimates or bandit-style uncertainty across sets of queries; and confidence-aware variants use response-level confidence to improve aggregation and stopping (Taubenfeld et al., 2025; Aghazadeh et al., 2025).

One challenge of the literature is to handle the situation where we must precisely control error level ε of certifying a modal response, those that arise when the risk of failure is high.

A typical real example is the emergency-room triage on data such as ER-REASON (Mehandru et al., 2025), where each LLM-output acuity score determines the time-to-physician for an arriving patient and each disposition determines admission-versus-discharge. In this case, if a modal-response certificate is used operationally, the false-certification probability must be bounded at a prescribed level ε. At present, it is difficult to control the certification errors with precision; therefore, keeping errors below a certain threshold requires performing an excessive number of inferences, which directly leads to a massive increase in computational costs.

Based on the situation, we have the following question: *can we derive a data-driven stopping rule for adaptive self-*

[1]The University of Tokyo, Tokyo, Japan [2]RIKEN Advanced Intelligence Project, Tokyo, Japan [3]Nagoya University, Nagoya, Japan [4]NARA Institute of Science and Technology (NAIST), Nara, Japan [5]Mohamed bin Zayed University of Artificial Intelligence (MBZUAI), Abu Dhabi, UAE [6]Kyoto University, Kyoto, Japan. Correspondence to: Hirofumi Ota <hirofumi-ota@g.ecc.u-tokyo.ac.jp>, Masaaki Imaizumi <imaizumi@g.ecc.u-tokyo.ac.jp>.

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consistency that controls the error level of certifying a modal response by LLMs?

1.2. Our Study

In this study, we propose the *Certification by Intersection-union Testing with E-processes* (CITE), which is an *anytime-valid* and *category-agnostic* sequential test for controlling the Type-I error of falsely certifying a prespecified response label as the unique mode. We note the important features: (i) *Anytime validity*: CITE works with data-driven stopping time and the error guarantee must hold uniformly over the time, not at a single prespecified sample size. (ii) *Category-agnostic*, CITE works even when the response category is unknown a priori and grows as sampling proceeds, which can adapt to new responses from LLMs. CITE gives a stopping time based on the three conditions on empirical probabilities: (i) the target is larger than the runner-up, (ii) the target itself is large, and (iii) the unobserved category is small, using anytime-valid e-process machinery (Ville, 1939; Grünwald et al., 2024; Ramdas and Wang, 2025). Figure 1 gives a graphical abstract.

Our contributions are as follows.

- **Anytime-valid category-agnostic error guarantee**: CITE controls Type-I error of certifying a label as the unique mode uniformly over all stopping times at any prescribed level ε (Theorem 4.1). The error guarantee is valid without the knowledge of answer categories.
- **Category-size-free stopping-time**: We show that the stopping time by CITE has an order which is free from category size (Theorem 4.3), which is

  contrast to several baseline methods. We also derive a matching minimax lower bound (Theorem 4.4).
- **Extension to weighted case**: We extend CITE to handle a case with weighted observations and gaps. We also derive an anytime-valid Type-I control and yield the same rate of the stopping time (Section 5).
- **Empirical performance**: Simulations and LLM experiments (Section 6) confirm that CITE controls Type-I error empirically, matches or improves on baselines on three LLM benchmarks.

2. Problem Formulation

2.1. Setup and Observation

Let $\mathcal{A}$ be a countable (possibly infinite) set of response categories. We consider an unknown categorical distribution P on $\mathcal{A}$, and write $p_a := P(X = a)$ for its probability mass function for $a \in \mathcal{A}$. Let $p_{(1)} \geq p_{(2)} \geq \cdots$ denote the decreasing rearrangement of $(p_a)_{a\in\mathcal{A}}$, and let $\mathcal{F}_t := \sigma(X_1, \ldots, X_t)$ be the natural filtration.

Suppose that we observe an i.i.d. sequence $X_1, X_2, \ldots$ from P:

Assumption 2.1 (i.i.d. categorical data)**.** *$(X_t)_{t\geq 1}$ are i.i.d. draws from a categorical distribution P on a countable set $\mathcal{A}$.*

We define some notations for statistics from the observations. For each $a \in \mathcal{A}$ and $t \geq 1$, let

$$N_t(a) := \sum_{i=1}^{t} \mathbf{1}\{X_i = a\}, \qquad \widehat{p}_t(a) := \frac{N_t(a)}{t},$$

denote the count and empirical frequency, and also

$$\mathcal{A}_t := \{a \in \mathcal{A} : N_t(a) \geq 1\}$$

denotes a set of observed categories by time t.

Our definition of P has a distinctive feature: A may contain a countably infinite number of categories, and since $\mathcal{A}$ is unknown, information about the set of categories cannot be used in decision-making. This setup is effective for representing the complex response patterns of LLMs.

2.2. Statistical Testing

We study the fixed-target unique-mode testing problem. For a target category $r \in \mathcal{A}$ fixed *a priori*, i.e. independent of the data, we design a sequential test of the unique-mode hypothesis that r strictly dominates every other category.

Definition 2.2 (Null and alternative hypothesis)**.** *For a fixed target category $r \in \mathcal{A}$, set*

$$H_{0,r} := \big\{P : p_r \leq \sup_{a\neq r} p_a\big\}, \;\; H_{1,r} := \big\{P : p_r > \sup_{a\neq r} p_a\big\}.$$

$H_{1,r}$ is the unique-mode alternative, i.e. r strictly dominates every other category. Under any $P \in H_{0,r}$ there exists a deterministic witness $a^\star \neq r$ such that $p_{a^\star} \geq p_r$ holds. A procedure certifies *r whenever it rejects $H_{0,r}$ in favor of $H_{1,r}$.*

To handle this testing, we consider certifying the mode through a sequential procedure with the observations using the following criteria.

Definition 2.3 (Sequential certification procedure)**.** *A sequential certification procedure for target r is a sequence $(D_t)_{t\geq 1}$ of $\mathcal{F}_t$-measurable decision rules, where $D_t = 1$ means that, after observing $X_1, \ldots, X_t$, the procedure certifies r as the unique mode. Its stopping time is*

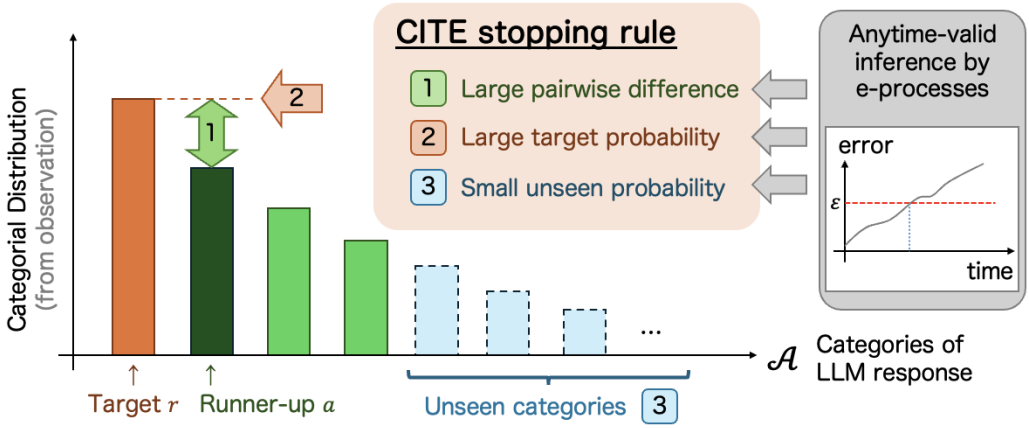
$$\tau := \inf\{t \geq 1 : D_t = 1\},$$

with $\inf \emptyset = \infty$. The procedure has anytime-valid Type-I error control at level ε if it holds that

$$\sup_{P\in H_{0,r}} \mathbb{P}_P(\tau < \infty) \leq \varepsilon. \tag{1}$$

If $\tau < \infty$, the procedure certifies r as the unique mode.

Figure 1. Overview of the stopping rule by CITE. With the countably infinite set of categories $\mathcal{A}$, CITE tests the hypothesis of whether the target category $r \in \mathcal{A}$ is a mode or not.



Importantly, the sampling time t is not fixed in advance, hence The bound (1) is time-uniform. At each time t, the procedure either continues sampling or stops and certifies, using only the data observed so far. The Type-I guarantee controls the probability that such a data-dependent rule ever falsely certifies r.

Remark 2.4. *We note the originality of our setting. While similar settings have been considered in statistics (Good, 1953; Painsky, 2025) and LLM test-time compute (Aggarwal et al., 2023; Li et al., 2024; Komiyama et al., 2026), to the best of our knowledge, an anytime valid sequential testing for the infinite and unknown category set has not been investigated. This setting describes with the nature of LLMs and represents a novel approach in terms of hypothesis testing.*

3. CITE: Certification by Intersection-union Testing with E-processes

We present our proposed method, CITE. Here, the target $r \in \mathcal{A}$ and the error level of certification $\varepsilon > 0$ is fixed before observing data as formalized in (Kim and Ramdas, 2025). Our strategy involves using three data-driven stochastic processes to estimate the components of the distribution uniformly over time. In particular, we consider $\mathcal{F}_t$-measurable stochastic processes E_t, L_t and U_t with their forms will be derived in the following sections, which satisfies the following conditions under the null hypothesis $H_{0,r}$:

1. **Pairwise evaluation**: For $a \in \mathcal{A}$, E_t evaluates an relative size of p_r against p_a, that is, for any $\alpha \in (0,1)$, $\sup_{t\geq 1} E_t \leq 1/\alpha$ holds with probability at least $1-\alpha$.
2. **Lower bound of the target probability** p_r: the second process L_t bounds p_r below, that is, $\sup_{t\geq 1} L_t \leq p_r$ holds.
3. **Upper bound of unseen categories**: the third process U_t is an upper bound of probability of unseen categories, that is, $\inf_{t\geq 1} U_t \geq \sup_{a\in\mathcal{A}:N_t(a)=0} p_a$ holds.

Note that these stochastic processes are calculated from observed values $X_1, ..., X_t$ without the knowledge of P.

Using these processes, we define our stopping rule: if any of the conditions above satisfied by these stochastic processes is violated at time t, we reject the null hypothesis $H_{0,r}$ and stop as $\tau = t$.

Rigorously, the stopping rule, with $P \in H_{0,r}$ and $a^\star \neq r$ satisfying $p_{a^\star} \geq p_r$, provides the following inclusion:

$$\{\tau < \infty\} \subseteq \underbrace{\{\exists t : E_t \geq 3/\varepsilon\}}_{\text{(i) pairwise}} \cup \underbrace{\{\exists t : L_t(r) > p_r\}}_{\text{(ii) target probability}} \cup \underbrace{\{\exists a, t : N_t(a) = 0, p_a > U_t\}}_{\text{(iii) unseen categories}}. \quad (2)$$

In the following sections, we will define the stochastic processes E_t, L_t, and U_t in detail and evaluate these three events.

3.1. Component (i): Pairwise E-Values

We give a specific form of the stochastic process E_t by the notion of e-process in the testing-by-betting style of Ramdas et al. (2023). For the fixed target $r \in \mathcal{A}$ and a competing category $a \neq r$, we consider a centered indicator $Z_i^{(r,a)} := \mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\} \in \{-1, 0, +1\}$ with a betting parameter $\lambda \in (0,1)$, and define the *pairwise e-process* as

$$E_t^{(r,a)}(\lambda) := \prod_{i=1}^{t} \left(1 + \lambda Z_i^{(r,a)}\right) = (1+\lambda)^{N_t(r)}(1-\lambda)^{N_t(a)}. \quad (3)$$

The closed form depends on the data only through $(N_t(r), N_t(a))$. Under the null hypothesis, i.e. $p_r \le p_a$, it holds that $\sup_{t\ge1} E_t^{(r,a)}(\lambda) \le 1/\alpha$ with probability at least $1-\alpha$; namely, it is a Nonnegative Super-Martingale (NSM) process.

We define the stochastic process E_t as the pairwise mixture e-process against the empirical runner-up. We extend $E_t^{(r,a)}(\lambda)$ by integrating λ out. Since the optimal λ is $(p_r - p_a)/(p_r + p_a)$ (Proposition C.6) depends on unknown values,

we therefore mix over a finite grid $\Lambda_{\mathrm{pw}} \subset (0,1)$ with positive weights (w_λ) summing to at most 1. Namely, we define the pairwise mixture e-process as follows:

Definition 3.1 (pairwise mixture e-process)**.** *For each competitor $a \ne r$, define*

$$E_t = E_t^{(r,\widehat{a}_t)} := \sum_{\lambda\in\Lambda_{\mathrm{pw}}} w_\lambda E_t^{(r,\widehat{a}_t)}(\lambda), \tag{4}$$

$$\textit{where} \quad \widehat{a}_t := \arg\max_{a\in\mathcal{A}_t\setminus\{r\}} N_t(a). \tag{5}$$

While $E_t^{(r,\widehat{a}_t)}$ is not a NSM process, $E_t^{(r,a)}$ is NSM since it is a convex combination of NSM processes:

Corollary 3.2. *Under $p_r \le p_a$, for any $\alpha \in (0,1)$, we have $\sup_{t\ge1} E_t^{(r,a)} \le 1/\alpha$ with probability at least $1-\alpha$.*

3.2. Component (ii): Lower Confidence Bound on the Target Probability p_r

We develop a time-uniform lower confidence bound (LCB) on the target probability p_r to handle *hidden competitors*, i.e., categories $h \in \mathcal{A}$ with $p_h > p_r$ that have never appeared in the sample.

For each candidate value $q \in (0,1]$, we test a hypothesis "$p_r \le q$" by betting on the centered indicator $\mathbf{1}\{X_i = r\} - q$, which is an analogous of $\mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\}$ in Component (i). Fix a finite grid $\Lambda_r \subset (0,\infty)$ with positive weights (v_λ) summing to at most 1, and for $q \in (0,1]$, we consider $\Lambda_r(q) := \{\lambda \in \Lambda_r : \lambda < 1/q\}$, thus each factor $1 + \lambda(\mathbf{1}\{X_i = r\} - q)$ is positive. Then, we define the mixture e-process

$$M_t(q) := \sum_{\lambda\in\Lambda_r(q)} v_\lambda \prod_{i=1}^{t} \big(1 + \lambda(\mathbf{1}\{X_i = r\} - q)\big), \tag{6}$$

and it follows $\mathbb{P}(\sup_t M_t(q) \ge 1/\alpha) \le \alpha$ for any $\alpha \in (0,1)$ under $p_r \le q$ (Corollary 3.2).

Moreover, $q \mapsto M_t(q)$ is nonincreasing (Lemma C.3), so the set of q at which the wealth exceeds $1/\alpha$ is a downset, making the following definition well-posed.

Definition 3.3 (Lower confidence bound on p_r)**.** *Given $\varepsilon > 0$, at time t, we define the lower confidence bound on p_r as*

$$L_t = L_t(r) := \sup\Big(\{0\} \cup \big\{q \in (0, \widehat{p}_t(r)] : M_t(q) \ge 3/\varepsilon\big\}\Big).$$

Proposition 3.4 (LCB validity)**.** *Under Assumption 2.1, it holds that $\mathbb{P}\big(\exists t \ge 1 : L_t(r) > p_r\big) \le \varepsilon/3$.*

Algorithm 1 Stopping rule by CITE

Input: target label $r \in \mathcal{A}$; parameter grid $\Lambda_{\mathrm{pw}} \subset (0,1)$ with weights (w_λ); LCB grid $\Lambda_r \subset (0,\infty)$ with weights (v_λ); error level $\varepsilon > 0$.

Output: stopping time τ and certification decision.

1. Initialise $N_0(a) \leftarrow 0$ for all $a \in \mathcal{A}$; $E_0^{(r,a)}(\lambda) \leftarrow 1$ for all $a \ne r$, $\lambda \in \Lambda_{\mathrm{pw}}$.
2. **For** $t = 1, 2, \ldots$ **do**
   - 2a. Draw X_t and set $N_t(a) \leftarrow N_{t-1}(a) + \mathbf{1}\{X_t = a\}$ for all $a \in \mathcal{A}$.
   - 2b. STEP(I): PAIRWISE E-PROCESSES.
     If $\mathcal{A}_t \setminus \{r\} = \emptyset$, set $P_t^{\mathrm{pw}} \leftarrow$ `true` (the unseen condition U_t then governs whether $\tau = t$) and skip to Step (II);
     otherwise, set $\widehat{a}_t \leftarrow \arg\max_{a\in\mathcal{A}_t\setminus\{r\}} N_t(a)$ and, for each $\lambda \in \Lambda_{\mathrm{pw}}$, compute
     $$E_t = E_t^{(r,\widehat{a}_t)} \leftarrow \textstyle\sum_\lambda w_\lambda E_t^{(r,\widehat{a}_t)}(\lambda),$$
     $$E_t^{(r,\widehat{a}_t)}(\lambda) \leftarrow (1+\lambda)^{N_t(r)}(1-\lambda)^{N_t(\widehat{a}_t)},$$
     and set $P_t^{\mathrm{pw}} \leftarrow \big\{E_t \ge 3/\varepsilon\big\}$.
   - 2c. STEP(II): LOWER CONFIDENCE BOUND ON p_r. Set $\widehat{p}_t(r) \leftarrow N_t(r)/t$ and
     $$L_t = L_t(r) \leftarrow \sup\big(\{0\} \cup \{q \in (0, \widehat{p}_t(r)] : M_t(q) \ge 3/\varepsilon\}\big).$$
   - 2d. STEP(III): UNSEEN UPPER BOUND. Set $U_t \leftarrow \min\{u \in (0,1] : u^{-1}(1-u)^t \le \varepsilon/3\}$.
   - 2e. **If** P_t^{pw} **and** $L_t > U_t$ **then** return ($\tau \leftarrow t$, "certify r as unique mode").

3.3. Component (iii): Unseen Upper Bound

We develop an upper bound for the probability of any currently unseen category, simultaneously over all categories and times. A category of mass $u \in (0,1)$ remains unseen after t i.i.d. draws with probability $(1-u)^t$, and a *probability-weighted* union bound turns this fact into a time-

uniform *per-category* upper bound, bypassing the need to estimate the aggregate missing mass.

Definition 3.5 (Unseen upper bound)**.** *An upper bound of the unseen probabilities is defined as*

$$U_t := \min\{u \in (0,1] : u^{-1}(1-u)^t \le \varepsilon/3\}.$$

The threshold balances two forces: $(1-u)^t$ decays exponentially in u, while the u^{-1} factor pays the union-bound cost over rare categories. As a result, we obtain the following guarantee:

Proposition 3.6 (Simultaneous validity of the unseen bound)**.** *Under Assumption 2.1, we obtain*

$$\mathbb{P}\big(\exists a \in \mathcal{A},\ \exists t \ge 1 : N_t(a) = 0 \text{ and } p_a > U_t\big) \le \varepsilon/3.$$

3.4. Stopping Rule

We define the CITE stopping rule by combining these three stochastic processes above, which are uniform in time. In particular, we design the stopping rule to follow the inclusion of the events as (2). Algorithm 1 shows the pseudocode of the CITE stopping rule.

Definition 3.7 (CITE stopping rule)**.** *With given $\varepsilon > 0$, CITE stops at the first time both **(a) Pairwise rejection:** $E_t \ge 3/\varepsilon$ at the empirical runner-up, and **(b) LCB exceeds unseen bound:** $L_t > U_t$, hold simultaneously, i.e. we define the stopping time as*

$$\tau := \inf\{t \ge 1 : E_t \ge 3/\varepsilon \text{ and } L_t > U_t\}. \tag{7}$$

If $\tau < \infty$, the procedure certifies r as the unique mode.

4. Theoretical Guarantees

We establish three theoretical guarantees for CITE. First, the false certification probability is controlled at the prescribed anytime-valid level for the fixed target r (Theorem 4.1). Second, under the unique-mode alternative, the expected stopping time is bounded by a category-set-size-free rate depending on (δ, p_r) but not on $|\mathcal{A}|$ (Theorem 4.3). Third, a combined information-theoretic lower bound shows that this rate is optimal up to constants and logarithmic terms (Theorem 4.4).

4.1. Main Theorems

We first provide the sequential version of Type-I error control on the stopping time τ given in Definition 3.7.

Theorem 4.1 (Type-I error control)**.** *Let $\varepsilon \in (0,1)$ be a prespecified significance level. Under Assumption 2.1, the stopping time τ of Definition 3.7 satisfies*

$$\sup_{P \in H_{0,r}} \mathbb{P}_P(\tau < \infty) \le \varepsilon.$$

The proof, deferred to Appendix B.6, fixes a null witness $a^\star \ne r$ satisfying $p_{a^\star} \ge p_r$. If this witness has been observed by the stopping time, runner-up monotonicity implies that the fixed-competitor e-process $E_t^{(r,a^\star)}$ must have crossed $3/\varepsilon$. If the witness is still unseen, then the inequality $L_t > U_t$ can hold only if either the LCB or unseen bound has failed. This yields (2) and a union bound over the three $\varepsilon/3$ components.

The above result on the Type-I error control does not provide any guarantee regarding the time required to certify the true mode. Under the alternative, write the modal gap as $\delta := p_r - p_{(2)} > 0$. We now show that $\mathbb{E}[\tau]$ is bounded by a function of (δ, p_r) alone, independent of $|\mathcal{A}|$.

Condition 4.2. *Λ_{pw} contains some $\lambda_{\mathrm{pw}} \in [\delta/8, \delta/4]$, and Λ_r contains some $\lambda_r \in [1/32, 1/16]$.*

Theorem 4.3 (Stopping-time bound)**.** *Suppose Assumption 2.1 holds and Condition 4.2 with $\delta > 0$, and the selected grid weights $w_{\lambda_{\mathrm{pw}}}, v_{\lambda_r}$ are bounded below by a constant independent of δ, p_r, ε. Then, we have $\mathbb{P}(\tau < \infty) = 1$ and*

$$\mathbb{E}[\tau] = O\left(\frac{\log(1/\varepsilon) + \log(1/\delta)}{\delta^2} + \frac{\log(1/\varepsilon) + \log(1/p_r)}{p_r}\right), \tag{8}$$

with universal constants independent of δ, p_r, $|\mathcal{A}|$.

We next state the matching lower bound used to interpret Theorem 4.3. For $p \in (0, 1/2]$ and $\delta \in (0, p)$, define

$$\mathcal{P}(p,\delta) := \{P : p_r = p, \quad p_{(2)} \le p - \delta\}.$$

Theorem 4.4 (Combined minimax lower bound)**.** *Fix $p \in (0, 1/2]$ and $\delta \in (0, p)$. Assume either that $\mathcal{A}$ is countably infinite, or that it contains at least $\lceil (1-p)/(p-\delta) \rceil + 2$ distinct non-target categories. Let τ' be any level-ε sequential certification procedure for the fixed target r, i.e., $\sup_{Q \in H_{0,r}} \mathbb{P}_Q(\tau' < \infty) \le \varepsilon$. Assume that, for every $P \in \mathcal{P}(p,\delta)$, $\mathbb{P}_P(\tau' < \infty) = 1$ and $\mathbb{E}_P[\tau'] < \infty$ hold. Then, there is a universal constant $c > 0$ such that*

$$\sup_{P \in \mathcal{P}(p,\delta)} \mathbb{E}_P[\tau'] \ge c \log(1/\varepsilon) \left(\frac{p}{\delta^2} + \frac{1}{p}\right).$$

Comparing the lower bound of Theorem 4.4 with the upper bound (8), we show that the CITE stopping time achieves the minimax optimal order up to logarithmic factors whenever p_r is bounded away from zero.

In the very small-p_r regime, the current upper bound can be worse in the pairwise term; closing this gap is left open.

In Appendix F, we give extensions of the analysis: sharper bounds with refined constant (Theorem F.2), a growth rate of the e-process (Theorem F.8); a stopping time bound with an geometric grid (Theorem F.5); and a top-k extension for the Type-I error and the stopping-time bound (Theorem F.9).

5. Extension to Confidence-Weighted Voting

5.1. Setup and Weighted CITE

We extend CITE to the confidence-weighted case: suppose we observe a pair (X_i, W_i) with a confidence score $W_i \in [0,1]$, e.g. the score from logits or a verifier; see (Taubenfeld et al., 2025; Fu et al., 2025). In this case, our purpose is to certify the *weighted unique mode* $\arg\max_a \mu_a$ with $\mu_a := \mathbb{E}[W_i \mathbf{1}\{X_i = a\}]$ and weighted modal gap $\Delta_w := \mu_r - \mu_\star$, where $\mu_\star := \sup_{a \neq r} \mu_a$.

Assumption 5.1 (i.i.d. weighted categories)**.** *$(X_i, W_i)_{i\geq 1}$ are i.i.d. with $X_i \in \mathcal{A}$ and $W_i \in [0,1]$.*

We develop *Weighted CITE* (W-CITE) for this setting. The weighted null and alternative are

$$H_{0,r}^{(w)} := \{P : \mu_r \leq \mu_\star\}, \quad H_{1,r}^{(w)} := \{P : \mu_r > \mu_\star\}. \quad (9)$$

W-CITE extends the three components of Section 3 to the weighted masses $(\mu_a)_{a\in\mathcal{A}}$.

Component (i): Pairwise E-Values. We consider the weighted indicator $\widetilde{Z}_i^{(r,a)} := W_i(\mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\})$ and a corresponding weighted pairwise e-process and its mixture as

$$\widetilde{E}_t^{(r,a)} := \sum_{\lambda\in\Lambda_{\mathrm{pw}}} w_\lambda \widetilde{E}_t^{(r,a)}(\lambda), \quad (10)$$

$$\widetilde{E}_t^{(r,a)}(\lambda) := \prod_{i=1}^{t} \big(1 + \lambda \widetilde{Z}_i^{(r,a)}\big).$$

Because runner-up monotonicity need not hold with heterogeneous weights, W-CITE uses the all-competitors check $\min_{a\in\mathcal{A}_t\setminus\{r\}} \widetilde{E}_t^{(r,a)} \geq 3/\varepsilon$.

Component (ii): LCB on Target Weighted Mass. The LCB targets μ_r via the increment $W_i\mathbf{1}\{X_i = r\} - q$. Let $\widehat{\mu}_t(r) := \frac{1}{t}\sum_{i=1}^t W_i \mathbf{1}\{X_i = r\}$. For $q \in (0,1]$, define

$$\widetilde{M}_t(q) := \sum_{\lambda\in\Lambda_r(q)} v_\lambda \widetilde{M}_t(q,\lambda),$$

$$\widetilde{M}_t(q,\lambda) := \prod_{i=1}^{t} \big(1 + \lambda(W_i\mathbf{1}\{X_i = r\} - q)\big),$$

and set $\widetilde{L}_t(r) := \sup(\{0\} \cup \{q \in (0, \widehat{\mu}_t(r)] : \widetilde{M}_t(q) \geq 3/\varepsilon\})$.

Component (iii): Unseen Upper Bound. The unseen bound U_t is unchanged since $\mu_a \leq p_a$.

Stopping Time. W-CITE stops when the weighted pairwise check passes for every observed competitor and the weighted LCB exceeds the unseen bound, $\widetilde{L}_t(r) > U_t$; if no non-target competitor has been observed, the pairwise part is treated as vacuous. This all-competitors check raises the per-round cost from $O(1)$ to $O(|\mathcal{A}_t|)$, since runner-up monotonicity (Lemma B.3) fails under heterogeneous weights. We write $\tau^{(w)}$ for the resulting stopping time, and give the validity analysis in Appendix D.

5.2. Theoretical Guarantee for Weighted CITE

We develop theoretical analysis for W-CITE, including the Type-I error and the stopping time.

Theorem 5.2 (Type-I error, weighted case)**.** *Under Assumption 5.1,* $\sup_{P\in H_{0,r}^{(w)}} \mathbb{P}_P(\tau^{(w)} < \infty) \leq \varepsilon$.

Condition 5.3 (Weighted grid coverage)**.** *Λ_{pw} contains some $\lambda_{\mathrm{pw}} \in [\Delta_w/8, \Delta_w/4]$ and Λ_r contains some $\lambda_r \in [1/32, 1/16]$.*

Theorem 5.4 (Weighted stopping-time bound)**.** *Suppose Assumption 5.1 holds with $\Delta_w > 0$ and Condition 5.3, and the grid weights $w_{\lambda_{\mathrm{pw}}}, v_{\lambda_r}$ are bounded below by a constant independent of $\Delta_w, \mu_r, \varepsilon$. Then we have $\mathbb{P}(\tau^{(w)} < \infty) = 1$ and*

$$\mathbb{E}[\tau^{(w)}] \quad (11)$$
$$= O\left(\frac{\log(\varepsilon^{-1}) + \log(\Delta_w^{-1})}{\Delta_w^2} + \frac{\log(\log(\varepsilon^{-1}) + \log(\mu_r^{-1})}{\mu_r}\right).$$

The weighted guarantee mirrors the unweighted theory (8) with (p_r, δ) replaced by (μ_r, Δ_w). The bound suggests that weighting can improve stopping time when it increases the effective gap Δ_w without reducing the target weighted mass μ_r too much.

6. Empirical Validation

We evaluate CITE on simulated multinomial distributions and on LLM self-consistency benchmarks. The experiments are designed to evaluate the following quantities: empirical false certification (Type-I error) under null targets, empirical power measured by certification probability under alternatives, and stopping time or sample efficiency, both on synthetic multinomial models and on LLM answer distributions. Baselines are **Bonferroni** (fixed-sample union bound), **KR** (Kim and Ramdas, 2025) (fixed-target sample-split test), and **MMC** (Cordero-Encinar and Duncan, 2025) (sequential leader-tracking), all at level $\varepsilon = 0.05$. We do not include adaptive stopping methods without theoretical guarantees of Type-I error control. Detailed protocols, per-setting tables, and ablations on component scaling and the confidence-weighted W-CITE are deferred to Appendices G and H.

Simulations. We evaluate CITE on five multinomial settings (Appendix G.1). First, we check false certification

under the fixed-target null. In Case B, across all five settings and all sample sizes, CITE has empirical false-certification rate zero, consistent with Theorem 4.1; the same is true for W-CITE and fixed-sample Bonferroni. By contrast, KR shows small nonzero false-certification rates on Settings 3–4, up to 0.002, and MMC shows nonzero false certifications in the same regimes, reaching 0.006 on Setting 3 and 0.014 on Setting 4 (Table 3).

Second, we evaluate certification power and stopping time under the alternative. In the easy concentrated Setting 2, all methods saturate quickly: at $N = 64$, CITE certifies with rate 0.890 and mean stopping time $\bar{\tau} \approx 36$, while Bonferroni and KR achieve rates 0.928 and 0.946, respectively. In the diffuse Setting 1, CITE is more efficient than Bonferroni before saturation, with certification rates 0.188 vs. 0.100 at $N = 64$ and 0.892 vs. 0.804 at $N = 128$, reaching full certification by $N = 256$ with $\bar{\tau} \approx 90$. In the moderate LLM-like Setting 5, CITE reaches rate 0.544 at $N = 256$ and 0.932 at $N = 512$ with $\bar{\tau} \approx 234$, whereas MMC remains at rate at most 0.002 through $N = 2048$. KR often has higher finite-sample power, but it is a fixed-sample baseline rather than an anytime-valid stopping rule. In the near-tie Setting 3 and very diffuse Setting 4, all methods remain low-power within the evaluated budget range.

Finally, the component-level simulations are consistent with the two terms in Theorem 4.3. When p_r is varied at fixed gap, the LCB–unseen component follows the predicted p_r^{-1} dependence; in the reported sweep, $\bar{\tau}_{\mathrm{lu}} p_r$ stays between roughly 14 and 20. When δ is varied at fixed $p_r = 0.24$, the LCB–unseen component stays approximately constant, around $\bar{\tau}_{\mathrm{lu}} \approx 70$, while the pairwise component is the part most affected by the modal gap (Appendix G.3).

LLM self-consistency. We evaluate the performance of Qwen3-30B (`Qwen3-30B-A3B-Instruct-2507`) (Qwen Team, 2025) and gpt-oss-20b (`openai/gpt-oss-20b`) (OpenAI, 2025) on AIME 2026 (Dekoninck et al., 2026), FrontierScience-Olympiad (Wang et al., 2026), and the clinical benchmark ER-REASON (Mehandru et al., 2025). Figure 2 plots mean accuracy against the sample budget; the horizontal axis is the mean stopping time for the sequential methods (CITE, W-CITE, MMC) and the prefix sample size N for the fixed-sample baselines (Bonferroni, KR).

When the target is set to the empirical runner-up rather than the mode, the maximum false-certification rate over all evaluated methods, problems, and budgets is 0.052 on gpt-oss-20b/ER-REASON and at most 0.012 on every other tested (model, dataset) pair; both lie within Monte Carlo error of the nominal level $\varepsilon = 0.05$ given the multiplicity of comparisons. On Qwen3-30B/ER-REASON, fixed-sample Bonferroni saturates at certification rate 0.111 for $N \geq 256$, whereas CITE attains certification rate ≥ 0.98 at mean stopping time $\bar{\tau} \approx 12$. This gives a direct stopping-time diagnostic: in this concentrated-answer regime, CITE certifies after roughly a dozen samples on average, while the fixed-sample baseline remains weak even at much larger budgets. At the largest evaluated budget $N = 1024$, the CITE certification rate exceeds that of MMC by a factor of approximately 2 on FrontierScience-Olympiad (where the empirical answer set contains about 250 distinct values per problem on gpt-oss-20b), while on ER-REASON the two methods differ by at most 0.054 across all budgets, attained on gpt-oss-20b. Relative to the KR baseline, the CITE certification rate is lower by at most 0.087, attained on Qwen3-30B/FrontierScience at $N \in \{256, 512\}$, while retaining anytime validity, whereas KR, by construction, is not valid under adaptive stopping. Per-problem and per-budget tables and the corresponding results for the W-CITE are deferred to Appendix H.

7. Conclusion

We introduced CITE for sequentially certifying the unique mode of a discrete distribution on a countable, possibly infinite category set. Our theory guarantees that anytime-valid Type-I error control, the stopping time has the category-set-size free order, and the order matches the information-theoretic lower bound up to logarithmic factors. We also extend CITE to confidence-weighted voting and develop its weighted version W-CITE. In our experiments, both numerical simulations and real LLM environments demonstrated that CITE achieves high accuracy while controlling Type-I errors.

Limitations and future work. CITE provides statistical guarantee that a prespecified label is the unique mode of the model's response distribution. This is logically distinct from, and complementary to, *answer correctness*: the two compose, and neither subsumes the other, so a downstream system can combine our anytime-valid mode certificate with any separate correctness signal. Within this scope, a natural extension is to certify the empirically observed top category $\widehat{r}_t \in \arg\max_a N_t(a)$, i.e., a data-adaptive target; this requires a Bonferroni-type correction over the random sequence of empirical leaders, paralleling the construction of Cordero-Encinar and Duncan (2025), and whether the category-size-free rate of Theorem 4.3 survives this correction is left open.

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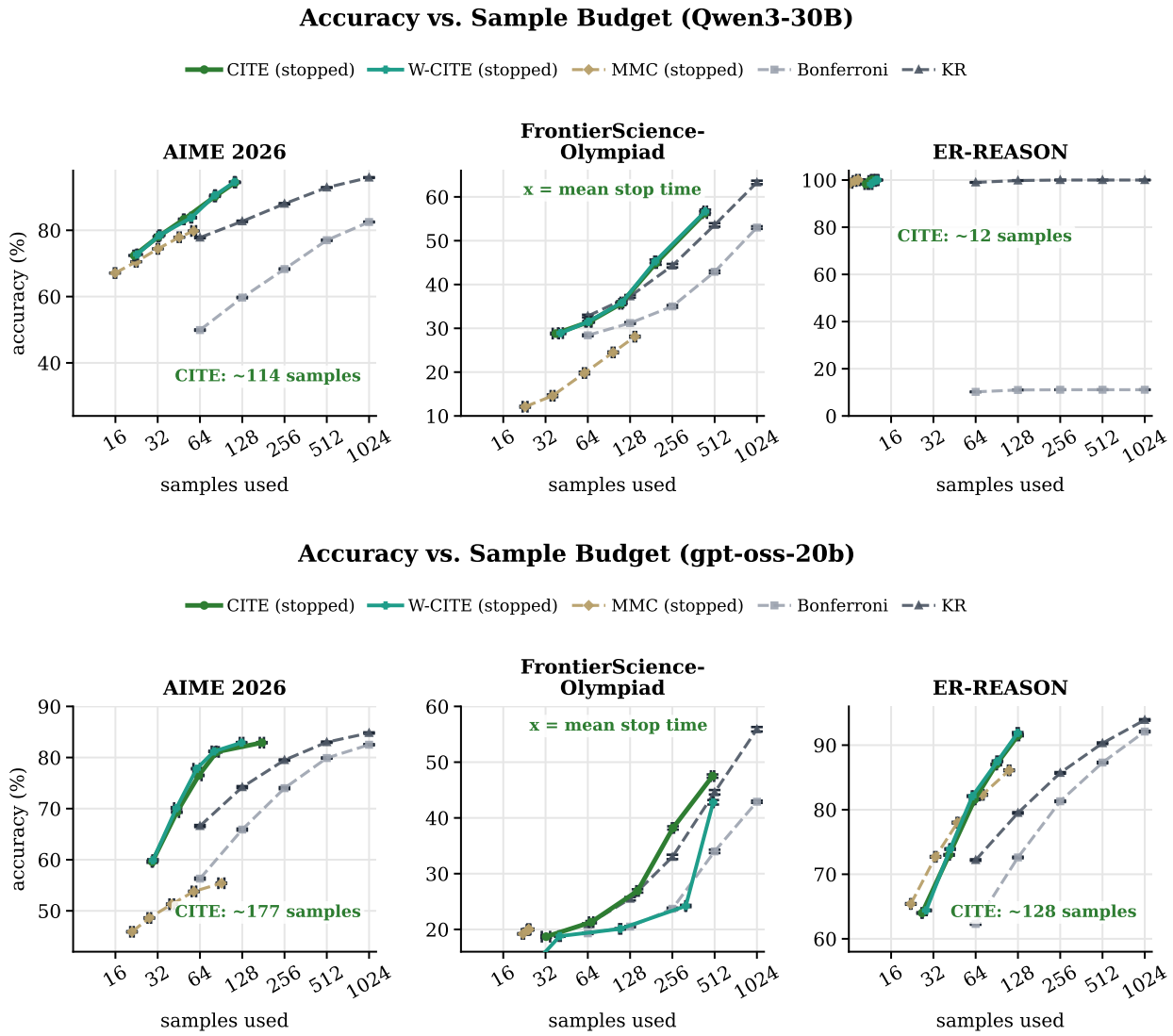

Figure 2. Mean accuracy vs. sample budget on Qwen3-30B (top) and gpt-oss-20b (bottom), across AIME 2026, FrontierScience-Olympiad, and ER-REASON. For the sequential methods (CITE, W-CITE, MMC) the horizontal axis is the mean stopping time; for the fixed-sample baselines (Bonferroni, KR) it is the prefix sample size N.

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A. Related Works

Self-consistency and test-time compute for LLM reasoning. Self-consistency and related inference-time scaling methods improve LLM reasoning by sampling multiple reasoning trajectories and aggregating their terminal answers, typically by majority voting (Wei et al., 2022; Zhou et al., 2023; Wang et al., 2023; Yao et al., 2023; Brown et al., 2024; Snell et al., 2025). These methods motivate our statistical formulation: after normalization, repeated LLM outputs are viewed as i.i.d. draws from an unknown categorical distribution over terminal response labels. Our work is not a new prompting, search, or reranking method. Instead, it studies when one may stop sampling and certify, with a prescribed error level, that a prespecified response label is the unique mode of this distribution. This is a guarantee about the model-induced response distribution, and is distinct from semantic correctness of the answer.

Adaptive self-consistency and early stopping. A growing line of work reduces the cost of self-consistency by adapting the number of sampled trajectories to the apparent difficulty of each query. Adaptive-Consistency uses posterior agreement among sampled answers as a lightweight stopping signal (Aggarwal et al., 2023); Early-Stopping Self-Consistency stops when a low-entropy window indicates sufficient agreement (Li et al., 2024); difficulty-adaptive and dynamic variants allocate inference budgets using difficulty estimates, answer-distribution uncertainty, or reasoning-path quality (Wang et al., 2025b;a); and confidence-aware methods use response-level confidence or verifier-like scores to improve aggregation and stopping (Taubenfeld et al., 2025; Aghazadeh et al., 2025; Fu et al., 2025). These methods are primarily designed to improve the accuracy–cost tradeoff. By contrast, CITE is designed for statistical certification: it controls the probability of falsely certifying the target as the unique mode under arbitrary data-driven stopping, at a user-specified level ε.

Certified self-consistency. The closest LLM-specific statistical work is the Martingale Majority Certificate (MMC) of Cordero-Encinar and Duncan (2025), which develops finite-sample and anytime-valid certificates for self-consistency and uses martingale arguments to determine when enough samples have been drawn. MMC provides an important first step toward certified self-consistency, but it certifies a stronger majority/residual-tail condition than the fixed-target unique-mode property considered here. Our target differs in two respects. First, CITE tests a fixed-target unique-mode alternative, $p_r > \sup_{a \neq r} p_a$, whereas MMC-style certificates are naturally tied to a stronger majority or residual-tail domination condition. These two notions coincide only in non-diffuse regimes. In LLM self-consistency, however, the modal answer may have substantially larger mass than every individual competitor while still being smaller than the aggregate mass of the long tail. Proposition E.1 shows that on a provable least-favourable family in this regime, the MMC certifies with probability at most the nominal level ε. Second, CITE explicitly handles the possibility that a high-mass competitor has not yet been observed. This unseen-category component is essential when the set of normalized free-form answers is unknown and grows during sampling.

Anytime-valid inference, e-values, and confidence sequences. Our construction builds on the classical martingale view of sequential testing (Ville, 1939) and the modern theory of e-values, e-processes, testing by betting, and confidence sequences (Howard et al., 2020; 2021; Shafer, 2021; Vovk and Wang, 2021; Lindon and Malek, 2022; Grünwald et al., 2024; Ramdas and Wang, 2025). These tools provide tests and confidence statements that remain valid under continuous monitoring and data-dependent stopping. CITE instantiates this machinery for a composite categorical unique-mode problem: it combines pairwise e-processes against observed competitors with a time-uniform lower confidence bound for the target mass and an upper bound for unseen competitors. The resulting intersection-union test yields a certification rule whose Type-I error is valid uniformly over all stopping times.

Best-arm identification and discrete argmin inference. The fixed-target unique-mode problem is related to fixed-confidence best-arm identification and pure-exploration bandits (Garivier and Kaufmann, 2016; Kaufmann et al., 2016; Kaufmann and Koolen, 2021). Those works characterize the sample complexity of identifying the best arm under sequential sampling and often rely on a known finite set of arms. Our setting is different: each LLM query returns a single categorical draw from the entire response distribution, the category set is not known in advance, and a previously unseen response can become a relevant competitor. The closest fixed-target finite-dimensional inference problem is the dimension-agnostic discrete argmin test of Kim and Ramdas (2025), which we include as a baseline. CITE differs by providing an anytime-valid stopping rule for a countable and unknown category set, rather than a fixed-sample or sample-split test over a given finite vector.

Unseen species, missing mass, and support-agnostic inference. The need to reason about unobserved response labels connects our work to the classical unseen-species and missing-mass literature, including Good–Turing and Good–Toulmin estimation (Good, 1953; Good and Toulmin, 1956), concentration bounds for the missing mass (McAllester and Ortiz, 2003; Berend and Kontorovich, 2013), and minimax prediction of unseen species (Orlitsky et al., 2016). These works estimate or bound quantities such as the aggregate probability of unseen outcomes or the number of new species. CITE uses a different object: for certification of a unique mode, it is enough to rule out the existence of a single unseen competitor whose mass could exceed the target. The unseen component of CITE is therefore tailored to sequential modal certification rather than to estimating the entire missing mass or support size.

Confidence-weighted aggregation. Recent work shows that response-level confidence can improve self-consistency by weighting or prioritizing high-confidence reasoning paths (Taubenfeld et al., 2025; Aghazadeh et al., 2025; Fu et al., 2025). Our weighted extension formalizes a complementary question: when the observations are pairs (X_i, W_i) with bounded confidence weights, can one certify that a target label is the unique weighted mode? W-CITE extends the same anytime-valid and category-agnostic guarantee to this weighted setting, with the weighted gap replacing the unweighted modal gap in the stopping-time analysis.

B. Proofs for Section 3

Let $\alpha_{\mathrm{pw}}, \alpha_r, \alpha_u > 0$ be error levels of each component satisfying $\alpha_{\mathrm{pw}} + \alpha_r + \alpha_u \le \varepsilon$. The main text uses the equal split

$$\alpha_{\mathrm{pw}} = \alpha_r = \alpha_u = \varepsilon/3.$$

For readability, the Appendix proofs keep the symbols $\alpha_{\mathrm{pw}}, \alpha_r, \alpha_u$ and substitute the equal split only at the end.

We prepare some supportive results as follow.

For an $(\mathcal{F}_t)$-adapted nonnegative process $(M_t)_{t\ge 0}$ with $\mathbb{E}_P[M_t \mid \mathcal{F}_{t-1}] \le M_{t-1}$ and $M_0 \le 1$ (an NSM), Ville's inequality (Ville, 1939) gives the time-uniform tail bound

$$P\Big(\sup_{t\ge 0} M_t \ge 1/\alpha\Big) \le \alpha, \qquad \alpha \in (0,1], \tag{12}$$

which we invoke repeatedly in what follows.

Lemma B.1 (Attainment of a positive supremum)**.** *Let $(x_a)_{a\in\mathcal{B}}$ be a nonnegative summable sequence indexed by a countable set $\mathcal{B}$. If $\sup_{a\in\mathcal{B}} x_a > 0$, then the supremum is attained.*

Proof. Let $s := \sup_{a\in\mathcal{B}} x_a > 0$. If no index attained s, then for each n one could choose a distinct $a_n \in \mathcal{B}$ with $x_{a_n} > s/2$. This would contradict $\sum_{a\in\mathcal{B}} x_a < \infty$. □

Lemma B.2 (Pairwise NSM)**.** *Fix $a \ne r$ and $\lambda \in (0,1)$. Under any P with $p_r \le p_a$, the process $(E_t^{(r,a)}(\lambda))_{t\ge 0}$ is an NSM with $E_0^{(r,a)}(\lambda) = 1$.*

Lemma B.3 (Runner-up monotonicity)**.** *For any $a, b \in \mathcal{A}_t \setminus \{r\}$ with $N_t(a) \ge N_t(b)$ and any $\lambda \in (0,1)$, it holds that*

$$E_t^{(r,a)}(\lambda) \le E_t^{(r,b)}(\lambda).$$

Consequently, we have

$$E_t^{(r,a)} \le E_t^{(r,b)}.$$

B.1. Proof of Lemma B.2

Proof. We have $E_0 = 1$ (empty product) and $E_t \ge 0$ since each factor $1 + \lambda Z_i \ge 1 - \lambda > 0$. For the supermartingale property:

$$\begin{aligned}\mathbb{E}[E_t^{(r,a)}(\lambda) \mid \mathcal{F}_{t-1}] &= E_{t-1}^{(r,a)}(\lambda) \cdot (1 + \lambda \mathbb{E}[Z_t^{(r,a)}]) \\ &= E_{t-1}^{(r,a)}(\lambda) \cdot (1 + \lambda(p_r - p_a)),\end{aligned}$$

using the i.i.d. property. Under $p_r \le p_a$, the factor $1 + \lambda(p_r - p_a) \le 1$. □

B.2. Proof of Corollary 3.2

Proof. For each $\lambda \in \Lambda_{\mathrm{pw}}$, $(E_t^{(r,a)}(\lambda))_{t\geq 0}$ is an NSM under $p_r \leq p_a$ by Lemma B.2. Since

$$E_t^{(r,a)} = \sum_{\lambda\in\Lambda_{\mathrm{pw}}} w_\lambda E_t^{(r,a)}(\lambda)$$

is a nonnegative weighted sum with total weight at most one, it is also an NSM and satisfies $E_0^{(r,a)} \leq 1$. Ville's inequality (12) then gives

$$\mathbb{P}_P\left(\sup_{t\geq 0} E_t^{(r,a)} \geq \alpha^{-1}\right) \leq \alpha.$$

□

B.3. Proof of Lemma B.3

Proof. By (3), $E_t^{(r,a)}(\lambda) = (1+\lambda)^{N_t(r)}(1-\lambda)^{N_t(a)}$. The factor $(1+\lambda)^{N_t(r)}$ depends only on r and is common to both $E_t^{(r,a)}(\lambda)$ and $E_t^{(r,b)}(\lambda)$. Since $0 < 1-\lambda < 1$ and $N_t(a) \geq N_t(b)$,

$$(1-\lambda)^{N_t(a)} \leq (1-\lambda)^{N_t(b)},$$

whence $E_t^{(r,a)}(\lambda) \leq E_t^{(r,b)}(\lambda)$. Multiplying by the nonnegative weights w_λ and summing over $\lambda \in \Lambda_{\mathrm{pw}}$ gives

$$E_t^{(r,a)} \leq E_t^{(r,b)}.$$

□

B.4. Proof of Proposition 3.4

Proof. If $p_r = 0$, then $N_t(r) = 0$ a.s. for all t, so $\widehat{p}_t(r) = 0$ and $L_t(r) = 0 = p_r$ a.s.; the event $\{\exists t : L_t(r) > p_r\}$ has probability 0. Assume $p_r > 0$ in the remainder.

Step 1: Monotonicity in q. For any $0 < q_1 < q_2 \leq 1$, $\Lambda_r(q_2) \subseteq \Lambda_r(q_1)$ and each factor $1 + \lambda(\mathbf{1}\{X_i = r\} - q)$ is nonincreasing in q; hence $q \mapsto M_t(q)$ is nonincreasing.

Step 2: NSM at $q = p_r$. Set $M_t(q,\lambda) := \prod_{i=1}^t (1 + \lambda(\mathbf{1}\{X_i = r\} - q))$. For $\lambda \in \Lambda_r(p_r)$, every factor $1 + \lambda(\mathbf{1}\{X_i = r\} - p_r) \geq 1 - \lambda p_r > 0$, and $\mathbb{E}[1 + \lambda(\mathbf{1}\{X_i = r\} - p_r)] = 1$, so $(M_t(p_r,\lambda))_{t\geq 0}$ is a nonnegative martingale with initial value 1. The mixture

$$M_t(p_r) := \sum_{\lambda\in\Lambda_r(p_r)} v_\lambda M_t(p_r,\lambda)$$

is therefore an NSM with $M_0(p_r) = \sum_{\lambda\in\Lambda_r(p_r)} v_\lambda \leq 1$ (Corollary-style argument analogous to Corollary 3.2). Ville's inequality (12) gives

$$\mathbb{P}\Big(\sup_{t\geq 0} M_t(p_r) \geq 1/\alpha_r\Big) \leq \alpha_r. \tag{13}$$

Step 3: Event inclusion. Suppose $L_t(r) > p_r$ for some $t \geq 1$. By the definition of $L_t(r)$, there exists $q \in (p_r, \widehat{p}_t(r)]$ with $M_t(q) \geq 1/\alpha_r$. Step 1 then implies $M_t(p_r) \geq M_t(q) \geq 1/\alpha_r$. Hence

$$\{\exists t \geq 1 : L_t(r) > p_r\} \subseteq \{\sup_{t\geq 0} M_t(p_r) \geq 1/\alpha_r\},$$

and the desired bound follows from (13). □

B.5. Proof of Proposition 3.6

Proof. For a fixed integer $t \geq 1$, define $f_t(u) := u^{-1}(1-u)^t$ for $u \in (0,1]$. Since $(\log f_t)'(u) = -1/u - t/(1-u) < 0$, f_t is strictly decreasing on $(0,1]$.

Now fix $a \in \mathcal{A}$ with $p_a > 0$. Since U_t is nonincreasing in t, define $t_*(a) := \inf\{t \geq 1 : p_a > U_t\}$ (possibly $+\infty$). If $t_*(a) = \infty$, then $p_a \leq U_t$ for all t, so the event $\{\exists t : N_t(a) = 0, p_a > U_t\}$ is empty. Otherwise, $t_*(a) < \infty$ and $p_a > U_{t_*(a)}$. For any $t \geq t_*(a)$ with $p_a > U_t$, we have $N_t(a) = 0$ only if $N_{t_*(a)}(a) = 0$ (since counts are nondecreasing), so the event $\{\exists t : N_t(a) = 0, p_a > U_t\}$ is contained in $\{N_{t_*(a)}(a) = 0\}$. By definition of $U_{t_*(a)}$, we have $f_{t_*(a)}(U_{t_*(a)}) \leq \alpha_u$. Since $p_a > U_{t_*(a)}$ and $f_{t_*(a)}$ is strictly decreasing, $f_{t_*(a)}(p_a) = p_a^{-1}(1-p_a)^{t_*(a)} < f_{t_*(a)}(U_{t_*(a)}) \leq \alpha_u$, hence

$$\mathbb{P}\big(N_{t_*(a)}(a) = 0\big) = (1-p_a)^{t_*(a)} < \alpha_u p_a.$$

A probability-weighted union bound over the countable category set gives

$$\mathbb{P}\Big(\exists a \in \mathcal{A}, \exists t \geq 1 : N_t(a) = 0 \text{ and } p_a > U_t\Big) \leq \sum_{a \in \mathcal{A}} \alpha_u p_a = \alpha_u. \qquad \square$$

B.6. Proof of Theorem 4.1

Proof. Fix any $P \in H_{0,r}$. We choose a deterministic $a^\star \neq r$ such that

$$p_{a^\star} \geq p_r.$$

We decompose the Type-I event according to whether $a^\star$ has been observed by time τ:

$$\{\tau < \infty\} \subseteq \underbrace{\{\tau < \infty,\ a^\star \in \mathcal{A}_\tau\}}_{E_1} \cup \underbrace{\{\tau < \infty,\ a^\star \notin \mathcal{A}_\tau\}}_{E_2}. \tag{14}$$

Step 1 (bounding $\mathbb{P}(E_1)$). On E_1, the witness $a^\star$ is observed at time τ, so $\mathcal{A}_\tau \setminus \{r\} \neq \emptyset$. Since CITE stops at τ, the pairwise part of the stopping rule gives

$$E_\tau^{(r,\widehat{a}_\tau)} = E_\tau \geq \alpha_{\text{pw}}^{-1}.$$

By the definition of the empirical runner-up, $N_\tau(\widehat{a}_\tau) \geq N_\tau(a^\star)$. Lemma B.3 then gives

$$E_\tau^{(r,\widehat{a}_\tau)} \leq E_\tau^{(r,a^\star)}.$$

Thus

$$E_\tau^{(r,a^\star)} \geq \alpha_{\text{pw}}^{-1}.$$

Since $a^\star$ is deterministic under P and $p_r \leq p_{a^\star}$, Corollary 3.2 and Ville's inequality yield

$$\mathbb{P}_P(E_1) \leq \mathbb{P}_P\left(\sup_{t \geq 0} E_t^{(r,a^\star)} \geq \alpha_{\text{pw}}^{-1}\right) \leq \alpha_{\text{pw}}. \tag{15}$$

Step 2 (bounding $\mathbb{P}(E_2)$). On E_2, CITE stops at τ, so $L_\tau(r) > U_\tau$, and $a^\star$ is unseen at τ. Define the two failure events

$$F_r := \{\exists t \geq 1 : L_t(r) > p_r\},$$
$$F_u := \{\exists a \in \mathcal{A},\ \exists t \geq 1 : N_t(a) = 0 \text{ and } p_a > U_t\}.$$

Propositions 3.4 and 3.6 give $\mathbb{P}_P(F_r) \leq \alpha_r$ and $\mathbb{P}_P(F_u) \leq \alpha_u$. On $E_2 \setminus (F_r \cup F_u)$, we have $L_\tau(r) \leq p_r$ and, because $a^\star$ is unseen at τ, $p_{a^\star} \leq U_\tau$. Combining these inequalities with $L_\tau(r) > U_\tau$ yields

$$p_{a^\star} \leq U_\tau < L_\tau(r) \leq p_r,$$

which contradicts $p_{a^\star} \geq p_r$. Hence, $E_2 \subseteq F_r \cup F_u$, and

$$\mathbb{P}_P(E_2) \leq \mathbb{P}_P(F_r) + \mathbb{P}_P(F_u) \leq \alpha_r + \alpha_u. \tag{16}$$

Step 3 (union bound). Combining (15)–(16),

$$\mathbb{P}_P(\tau < \infty) \leq \mathbb{P}_P(E_1) + \mathbb{P}_P(E_2) \leq \alpha_{\text{pw}} + \alpha_r + \alpha_u \leq \varepsilon. \qquad \square$$

C. Power Analysis for CITE

Throughout this section, $X_1, X_2, \ldots$ are i.i.d. on the countable category set $\mathcal{A}$, and the fixed target r is the unique mode. Write

$$p_{(2)} := \sup_{a \neq r} p_a, \qquad \delta := p_r - p_{(2)} > 0.$$

For the analysis, define the conservative competitor count

$$R_t := \max_{a \neq r} N_t(a),$$

where zero-count competitors are included in the maximum, and define

$$E_t^\circ := \sum_{\lambda \in \Lambda_{\mathrm{pw}}} w_\lambda (1+\lambda)^{N_t(r)} (1-\lambda)^{R_t}.$$

The corresponding conservative stopping time is

$$\tau^\circ := \inf \left\{ t \geq 1 : E_t^\circ \geq \alpha_{\mathrm{pw}}^{-1} \text{ and } L_t(r) > U_t \right\}.$$

This stopping time is no earlier than the implemented CITE stopping time: replacing the observed runner-up count by $R_t = \max_{a \neq r} N_t(a)$ can only make the pairwise condition harder, while the LCB–unseen condition is unchanged. Hence

$$\tau \leq \tau^\circ \quad \text{pathwise},$$

and it suffices to upper-bound $\mathbb{E}[\tau^\circ]$.

C.1. Auxiliary lemmas for the clean stopping-time bound

Lemma C.1 (A deterministic logarithmic inequality)**.** *Let $A > 0$, $C > 0$, and $t > 0$. If*

$$t \geq 2A \log\big((AC) \vee e\big),$$

then

$$t \geq A \log(Ct).$$

Proof. Set $u = t/A$. The assumption gives $u \geq 2\log((AC) \vee e) \geq 2$, so $\log(CA) \leq u/2$ and $\log u \leq u/2$. Therefore

$$\log(Ct) = \log(CA) + \log u \leq u = t/A.$$

□

Lemma C.2 (Expected maximum competitor count)**.** *For every integer $t \geq 1$,*

$$\mathbb{E}[R_t] \leq t p_{(2)} + 2\sqrt{2t \log(t+1)}.$$

Proof. For $a \neq r$, set $f_a(x) := \mathbf{1}\{x = a\}$. Since $p_a \leq p_{(2)}$,

$$\mathbb{E}[R_t] - t p_{(2)} \leq \mathbb{E} \sup_{a \neq r} \sum_{i=1}^{t} \{ f_a(X_i) - \mathbb{E} f_a(X_i) \}.$$

By symmetrization, the right-hand side is at most

$$2\mathbb{E}\mathbb{E}_\sigma \sup_{a \neq r} \sum_{i=1}^{t} \sigma_i f_a(X_i),$$

where σ_i are i.i.d. Rademacher variables independent of the sample. Conditional on the sample, the vectors

$$(f_a(X_1), \ldots, f_a(X_t)), \qquad a \neq r,$$

have disjoint supports and hence at most $t+1$ distinct values, each with Euclidean norm at most $\sqrt{t}$. The standard exponential maximal inequality gives

$$\mathbb{E}_\sigma \sup_{a \neq r} \sum_{i=1}^{t} \sigma_i f_a(X_i) \leq \sqrt{2t \log(t+1)}.$$

Combining the displays proves the claim. □

Lemma C.3 (LCB monotonicity and inversion)**.** *Fix $t \geq 1$. For*

$$M_t(q, \lambda) := \prod_{i=1}^{t} \big\{1 + \lambda(\mathbf{1}\{X_i = r\} - q)\big\},$$

the following hold.

(a) *The map $q \mapsto M_t(q)$ is nonincreasing on $(0, 1]$.*

(b) *If $M_t(q) > 1$, then $\widehat{p}_t(r) > q$.*

(c) *If $M_t(q) \geq \alpha_r^{-1}$, then $L_t(r) \geq q$.*

Proof. For (a), if $q_1 < q_2$, then $\Lambda_r(q_2) \subseteq \Lambda_r(q_1)$, and each remaining factor $1 + \lambda(\mathbf{1}\{X_i = r\} - q)$ is nonincreasing in q. Hence $M_t(q_1) \geq M_t(q_2)$.

For (b), suppose $\widehat{p}_t(r) \leq q$. For fixed $\lambda \in \Lambda_r(q)$, write

$$\phi(\lambda) := \widehat{p}_t(r) \log(1 + \lambda(1 - q)) + (1 - \widehat{p}_t(r)) \log(1 - \lambda q).$$

Then $\log M_t(q, \lambda) = t\phi(\lambda)$, $\phi(0) = 0$, $\phi'(0) = \widehat{p}_t(r) - q \leq 0$, and ϕ is concave on $[0, 1/q)$. Therefore $M_t(q, \lambda) \leq 1$ for every admissible λ, so

$$M_t(q) \leq \sum_{\lambda \in \Lambda_r(q)} v_\lambda \leq 1.$$

This proves the contrapositive.

For (c), if $M_t(q) \geq \alpha_r^{-1} > 1$, then (b) gives $\widehat{p}_t(r) > q$, so q belongs to the set defining $L_t(r)$. Hence $L_t(r) \geq q$. □

Lemma C.4 (Quadratic lower bound for $\log(1 + x)$)**.** *For every $x \in [-1/2, 1/2]$,*

$$\log(1 + x) \geq x - x^2.$$

Proof. Let $h(x) := \log(1+x) - x + x^2$. Then $h(0) = 0$ and $h'(x) = x(1+2x)/(1+x)$, which is nonpositive on $[-1/2, 0]$ and nonnegative on $[0, 1/2]$. Thus $h(x) \geq 0$ on $[-1/2, 1/2]$. □

Proposition C.5 (Small-bet pairwise drift)**.** *Assume $\delta > 0$ and*

$$0 < \lambda \leq \frac{\delta}{2(p_r + p_{(2)})}.$$

Then, for every $a \neq r$ with $p_a \leq p_{(2)}$,

$$\mathbb{E}\left[\log\{1 + \lambda(\mathbf{1}\{X = r\} - \mathbf{1}\{X = a\})\}\right] \geq \frac{\lambda\delta}{2}.$$

Proof. Since $\delta \leq p_r + p_{(2)}$, the assumed bound gives $\lambda \leq 1/2$. Let

$$\xi := \lambda(\mathbf{1}\{X = r\} - \mathbf{1}\{X = a\}) \in [-1/2, 1/2].$$

By Lemma C.4,

$$\mathbb{E}[\log(1 + \xi)] \geq \lambda(p_r - p_a) - \lambda^2(p_r + p_a).$$

Using $p_a \leq p_{(2)}$ gives

$$\mathbb{E}[\log(1 + \xi)] \geq \lambda\delta - \lambda^2(p_r + p_{(2)}) \geq \lambda\delta/2.$$

□

C.2. Proof of Theorem 4.3

Proof. We prove the bound for τ°; since $\tau \le \tau^\circ$ pathwise, the same expectation bound holds for the implemented CITE stopping time.

Step 1: pairwise condition. Let $\lambda := \lambda_{\mathrm{pw}} \in [\delta/8, \delta/4]$ be the grid point from Condition 4.2, with weight w_λ, and set

$$B_{\mathrm{pw}} := \log\{1/(\alpha_{\mathrm{pw}} w_\lambda)\}.$$

For this fixed λ, define

$$S_t := N_t(r)\log(1+\lambda) + R_t \log(1-\lambda).$$

Since $E_t^\circ \ge w_\lambda \exp(S_t)$, the pairwise condition $E_t^\circ \ge \alpha_{\mathrm{pw}}^{-1}$ holds whenever $S_t \ge B_{\mathrm{pw}}$.

Using Lemma C.2 and $\log(1-\lambda) < 0$,

$$\mathbb{E}[S_t] \ge t\{p_r \log(1+\lambda) + p_{(2)} \log(1-\lambda)\} - C\lambda\sqrt{t\log(t+1)}.$$

Because $p_r + p_{(2)} \le 1$, $\lambda \le \delta/4$, and $\log(1+x) \ge x - x^2$ on $[-1/2, 1/2]$,

$$p_r \log(1+\lambda) + p_{(2)} \log(1-\lambda) \ge \lambda\delta - \lambda^2(p_r + p_{(2)}) \ge c\delta^2$$

for a universal constant $c > 0$. Hence

$$\mathbb{E}[S_t] \ge c_0\delta^2 t - C_0\delta\sqrt{t\log(t+1)}.$$

Changing one observation changes both $N_t(r)$ and R_t by at most one, so S_t is $C_1\delta$-Lipschitz in each sample. McDiarmid's inequality gives

$$\mathbb{P}\{S_t < \mathbb{E}[S_t] - x\} \le \exp\left\{-\frac{c_1 x^2}{t\delta^2}\right\}.$$

Taking $x = C_2\delta\sqrt{t\log(et)}$ and using Lemma C.1, we obtain, for a universal constant C,

$$\mathbb{P}\left(E_t^\circ < \alpha_{\mathrm{pw}}^{-1}\right) \le (et)^{-2}, \qquad t \ge T_{\mathrm{pw}} := C\frac{B_{\mathrm{pw}} + \log(1/\delta)}{\delta^2}. \tag{17}$$

Step 2: LCB–unseen condition. Set $q_0 := p_r/2$. Condition 4.2 gives $\lambda_r \in [1/32, 1/16] \subset \Lambda_r(q_0)$ with weight v_{λ_r}. The unseen bound satisfies $U_t \le p_r/4$ once

$$t \ge Cp_r^{-1}\log\{1/(\alpha_u p_r)\}.$$

Let

$$A_t := \left\{\frac{3}{4}p_r t \le N_t(r) \le \frac{5}{4}p_r t\right\}.$$

Bernstein's inequality gives

$$\mathbb{P}(A_t^c) \le 2\exp(-c_2 p_r t).$$

On A_t, the single LCB factor

$$M_t(q_0, \lambda_r) = \prod_{i=1}^{t}\{1 + \lambda_r(\mathbf{1}\{X_i = r\} - q_0)\}$$

satisfies

$$\log M_t(q_0, \lambda_r) \ge c_3 p_r t$$

for a universal constant $c_3 > 0$. Therefore

$$\widetilde{M}_t(q_0) \ge v_{\lambda_r} M_t(q_0, \lambda_r) \ge \alpha_r^{-1}$$

whenever

$$t \ge Cp_r^{-1}\log\{1/(\alpha_r v_{\lambda_r})\}.$$

By Lemma C.3, this implies $L_t(r) \geq q_0 > U_t$. Combining this with the Bernstein tail and Lemma C.1, we obtain

$$\mathbb{P}\{L_t(r) \leq U_t\} \leq (et)^{-2} \tag{18}$$

for all

$$t \geq T_{\text{rare}} := C\frac{\log(1/\alpha_r) + \log(1/\alpha_u) + \log(1/v_{\lambda_r}) + \log(1/p_r)}{p_r}.$$

Step 3: combine. Let

$$T_{\text{cl}} := \max\{T_{\text{pw}}, T_{\text{rare}}\}.$$

For all $t \geq T_{\text{cl}}$,

$$\mathbb{P}(\tau^\circ > t) \leq 2(et)^{-2}.$$

The tail-sum identity gives

$$\mathbb{E}[\tau^\circ] \leq T_{\text{cl}} + 1.$$

Finally, substituting $\alpha_{\text{pw}} = \alpha_r = \alpha_u = \varepsilon/3$ and using the assumption that w_λ and v_{λ_r} are bounded below by constants,

$$\mathbb{E}[\tau] \leq \mathbb{E}[\tau^\circ] = O\left(\frac{\log(1/\varepsilon) + \log(1/\delta)}{\delta^2} + \frac{\log(1/\varepsilon) + \log(1/p_r)}{p_r}\right),$$

with constants independent of δ, p_r, and $|\mathcal{A}|$. □

C.3. Oracle pairwise growth rate

Proposition C.6 (Closed form for the oracle pairwise growth rate)**.** *For $\lambda \in (0,1)$, define*

$$\rho(\lambda) := p_r \log(1+\lambda) + p_{(2)} \log(1-\lambda).$$

Then ρ is strictly concave on $(0,1)$. If $p_{(2)} > 0$, its unique maximizer is

$$\lambda^\star = \frac{p_r - p_{(2)}}{p_r + p_{(2)}} = \frac{\delta}{p_r + p_{(2)}},$$

and the maximal value is

$$\rho^\star := \rho(\lambda^\star) = p_r \log\frac{2p_r}{p_r + p_{(2)}} + p_{(2)} \log\frac{2p_{(2)}}{p_r + p_{(2)}}.$$

If $p_{(2)} = 0$, then $\rho(\lambda) = p_r \log(1+\lambda)$ is increasing on $(0,1)$ and

$$\sup_{0<\lambda<1} \rho(\lambda) = p_r \log 2.$$

Proof. We have

$$\rho'(\lambda) = \frac{p_r}{1+\lambda} - \frac{p_{(2)}}{1-\lambda}, \qquad \rho''(\lambda) = -\frac{p_r}{(1+\lambda)^2} - \frac{p_{(2)}}{(1-\lambda)^2} < 0.$$

Thus ρ is strictly concave. When $p_{(2)} > 0$, solving $\rho'(\lambda) = 0$ gives $\lambda^\star = (p_r - p_{(2)})/(p_r + p_{(2)})$, and substituting this value gives the stated formula for $\rho^\star$. The case $p_{(2)} = 0$ is immediate. □

Lemma C.7 (Stopped KL identity for the swapped alternative)**.** *Assume $p_{(2)} > 0$. Fix*

$$a^\star \in \arg\max_{a \neq r} p_a,$$

and let Q be obtained from P by swapping the masses of r and $a^\star$:

$$q_r = p_{(2)}, \qquad q_{a^\star} = p_r, \qquad q_a = p_a \quad \text{for } a \notin \{r, a^\star\}.$$

Let P^τ and Q^τ denote the laws of the stopped experiment $(\tau, X_1, \ldots, X_\tau)$ under P and Q. If $\mathbb{E}_P[\tau] < \infty$, then

$$D(P^\tau \| Q^\tau) = \mathbb{E}_P[\tau]\delta \log\frac{p_r}{p_{(2)}}.$$

Proof. Let $\ell(x) := \log(P(x)/Q(x))$. Since ℓ is bounded (both P and Q have positive masses on the same support), the stopping factor cancels in the likelihood ratio $dP^\tau/dQ^\tau = \prod_{i\le\tau}(P(X_i)/Q(X_i))$, and Wald's identity for i.i.d. bounded summands yields

$$D(P^\tau\|Q^\tau) = \mathbb{E}_P\left[\sum_{i=1}^{\tau}\ell(X_i)\right] = \mathbb{E}_P[\tau]\mathbb{E}_P[\ell(X_1)].$$

A direct calculation gives $\mathbb{E}_P[\ell(X_1)] = p_r\log(p_r/p_{(2)}) + p_{(2)}\log(p_{(2)}/p_r) = \delta\log(p_r/p_{(2)})$, completing the proof. □

Corollary C.8 (Information-theoretic lower bound)**.** *Assume $p_{(2)} > 0$. Let ε be the Type-I error level of a sequential test for $H_{0,r}$, and assume that under the alternative P one has*

$$\mathbb{P}_P(\tau < \infty) = 1 \qquad \textit{and} \qquad \mathbb{E}_P[\tau] < \infty.$$

Then

$$\mathbb{E}_P[\tau] \ge \frac{\log(1/\varepsilon)}{\delta\log(p_r/p_{(2)})}.$$

In particular, if $\delta/p_r \to 0$, then

$$\mathbb{E}_P[\tau] \ge \frac{p_r\log(1/\varepsilon)}{\delta^2}\big(1+o(1)\big).$$

Proof. Let Q be the swap from Lemma C.7, so $Q \in H_{0,r}$ and Type-I validity gives $Q(\tau < \infty) \le \varepsilon$. Set $A := \{\tau < \infty\}$, so $P(A) = 1$. By data processing followed by Lemma C.7,

$$\log\frac{1}{\varepsilon} \le \mathrm{kl}\big(P(A), Q(A)\big) \le D(P^\tau\|Q^\tau) = \mathbb{E}_P[\tau]\delta\log(p_r/p_{(2)}),$$

which is the first claim. For the small-gap expansion, $\delta\log(p_r/p_{(2)}) = \delta\log(1+\delta/p_{(2)}) = \delta^2/p_r + o(\delta^2/p_r)$ as $\delta/p_r \to 0$. □

Lemma C.9 (Forward I-projection of P onto $H_{0,r}$)**.** *Assume $p_{(2)} > 0$ and pick any $a^\star \in \arg\max_{a\ne r} p_a$ (this set may have ties). Every forward I-projection*

$$P^\star \in \arg\min_{Q\in H_{0,r}} D(P\|Q)$$

arises by midpointing r with one of the strongest competitors; for the choice $a^\star$ above, the corresponding projection is the midpoint distribution

$$p_r^\star = p_{a^\star}^\star = \frac{p_r + p_{(2)}}{2}, \qquad p_a^\star = p_a \quad \textit{for } a \notin \{r, a^\star\},$$

and the per-sample KL divergence is

$$D(P\|P^\star) = p_r\log\frac{2p_r}{p_r+p_{(2)}} + p_{(2)}\log\frac{2p_{(2)}}{p_r+p_{(2)}}.$$

In particular, as $\delta/p_r \to 0$,

$$D(P\|P^\star) = \frac{\delta^2}{4p_r}\big(1+o(1)\big).$$

Proof. $H_{0,r} = \bigcup_{a\ne r}\{q_r \le q_a\}$ is a (possibly countable) union of half-spaces, hence not itself convex. We minimise $D(P\|\cdot)$ over each face $\{q_r \le q_a\}$ separately (on each face $D(P\|\cdot)$ is strictly convex and strictly positive since $P \notin H_{0,r}$), and then take the smallest of the per-face minima. The optimum on each face lies on its boundary $\{q_r = q_a\}$. We organise the proof in three steps.

Step 1 (per-pair midpoint projection). For each $a \ne r$ with $p_a < p_r$, the I-projection of P onto the affine subspace $\{q_r = q_a\}$ is given in closed form by

$$q_r = q_a = \frac{p_r + p_a}{2}, \qquad q_b = p_b \quad \text{for } b \notin \{r, a\}. \tag{19}$$

The Lagrangian on the simplex with multiplier μ has first-order conditions $q_b = p_b/\mu$ and $q_r = q_a = (p_r + p_a)/(2\mu)$; the normalisation $\sum_b q_b = 1/\mu = 1$ forces $\mu = 1$, yielding (19), and the positive-definite Hessian $\mathrm{diag}(p/q^2)$ ensures uniqueness. The KL value of this projection is

$$\Phi(p_a) := p_r \log \frac{2p_r}{p_r + p_a} + p_a \log \frac{2p_a}{p_r + p_a}. \tag{20}$$

Step 2 (the optimal competitor is $a^\star$). Differentiating (20) gives $\Phi'(p_a) = \log \frac{2p_a}{p_r+p_a} \le 0$ on $(0, p_r)$, so Φ is non-increasing in p_a and is minimised over $\{p_a : a \neq r\}$ at $p_a = p_{(2)} = p_{a^\star}$. After midpointing on $a^\star$, the remaining masses satisfy $q_b = p_b \le p_{(2)} < (p_r + p_{(2)})/2 = q_r = q_{a^\star}$ for $b \notin \{r, a^\star\}$, confirming that the candidate lies in $H_{0,r}$. Hence midpointing r with the chosen $a^\star$ yields a global forward I-projection $P^\star$ with $D(P\|P^\star) = \Phi(p_{(2)})$ (in case of ties at $p_{(2)}$ each choice of strongest competitor gives a distinct minimiser with the same KL value), proving the first two displays.

Step 3 (small-gap expansion). Write $q^\star := (p_r + p_{(2)})/2 = p_r - \delta/2$ and $x := \delta/(2q^\star) \in (0, 1)$, so $p_r/q^\star = 1 + x$ and $p_{(2)}/q^\star = 1 - x$. The closed form factorises as $\Phi(p_{(2)}) = q^\star f(x)$, where

$$f(x) := (1 + x)\log(1 + x) + (1 - x)\log(1 - x).$$

f is even with $f''(x) = 2/(1 - x^2) = 2 + 2x^2 + O(x^4)$, hence $f(x) = x^2 + x^4/6 + O(x^6)$ about 0. Substituting,

$$D(P\|P^\star) = q^\star x^2 + O(q^\star x^4) = \frac{\delta^2}{4q^\star} + O\left(\frac{\delta^4}{q^{\star 3}}\right),$$

and $q^\star = p_r(1 - \delta/(2p_r)) = p_r(1 + o(1))$ as $\delta/p_r \to 0$ yields the third display. □

Lemma C.10 (Stopped KL identity for the I-projection)**.** *Under the assumptions of Lemma C.9, if $\mathbb{E}_P[\tau] < \infty$, then*

$$D(P^\tau \| (P^\star)^\tau) = \mathbb{E}_P[\tau] D(P\|P^\star).$$

Proof. P and $P^\star$ are mutually absolutely continuous on $\mathcal{A}$ (since $q^\star \in (0, 1)$ when $p_r \in (0, 1)$, and other masses agree), and the log-likelihood $\ell(x) := \log(P(x)/P^\star(x))$ is bounded. The stopping-event factor cancels in the likelihood ratio, so Fubini and independence give $D(P^\tau\|(P^\star)^\tau) = \mathbb{E}_P[\tau]\mathbb{E}_P[\ell(X_1)] = \mathbb{E}_P[\tau]D(P\|P^\star)$, exactly as in Lemma C.7. □

Corollary C.11 (Sharp information-theoretic lower bound)**.** *Under the hypotheses of Corollary C.8,*

$$\mathbb{E}_P[\tau] \ge \frac{\log(1/\varepsilon)}{D(P\|P^\star)} = \frac{4p_r \log(1/\varepsilon)}{\delta^2}\big(1 + o(1)\big) \qquad (\delta/p_r \to 0).$$

Proof. Apply the same data-processing argument as in Corollary C.8 with $Q = P^\star$ in place of the swap distribution. Since $P^\star \in H_{0,r}$ (Lemma C.9), Type-I validity gives $P^\star(\tau < \infty) \le \varepsilon$, and the data-processing inequality combined with Lemma C.10 yields $\log(1/\varepsilon) \le \mathbb{E}_P[\tau]D(P\|P^\star)$. The asymptotic constant follows from Lemma C.9. □

C.4. Information-Theoretic Lower Bound for the Unseen-Category Component

Lemma C.12 (Stopped KL identity for a hidden-competitor alternative)**.** *Fix a distribution P on a countable category set $\mathcal{A}$ with $p_r \in (0, 1)$, and let $h \in \mathcal{A}$ satisfy $h \neq r$ and $p_h = 0$. Define a second distribution Q by*

$$q_r = \frac{p_r}{2}, \qquad q_h = \frac{p_r}{2}, \qquad q_a = p_a \quad \textit{for } a \notin \{r, h\}.$$

Let P^τ and Q^τ denote the laws of the stopped experiment $(\tau, X_1, \ldots, X_\tau)$ under P and Q. If $\mathbb{E}_P[\tau] < \infty$, then

$$D(P^\tau\|Q^\tau) = \mathbb{E}_P[\tau] p_r \log 2.$$

Proof. By the same Wald argument as Lemma C.7, $D(P^\tau\|Q^\tau) = \mathbb{E}_P[\tau] \cdot \mathbb{E}_P[\ell(X_1)]$ with $\ell(x) := \log(P(x)/Q(x))$, where we set $\ell(h) := 0$ (an arbitrary finite value on the P-null event $\{X = h\}$ that does not affect any P-expectation). By construction $\ell(r) = \log 2$ and $\ell(x) = 0$ for $x \neq r$, so $\mathbb{E}_P[\ell(X_1)] = p_r \log 2$. □

Corollary C.13 (Information-theoretic lower bound for the unseen-category component)**.** *Fix $p \in (0,1)$ and $\delta \in (0,p)$, and assume the category set $\mathcal{A}$ contains at least $\lceil (1-p)/(p-\delta) \rceil + 2$ distinct non-target categories (so that an unused "hidden" label is available; this holds automatically on countably infinite category sets). Define*

$$\mathcal{P}(p,\delta) := \left\{ P : p_r = p, p_{(2)} \le p - \delta \right\}.$$

Let τ be any stopping time corresponding to a sequential test such that

$$\sup_{Q \in H_{0,r}} \mathbb{P}_Q(\tau < \infty) \le \varepsilon,$$

and assume that, for every $P \in \mathcal{P}(p,\delta)$,

$$\mathbb{P}_P(\tau < \infty) = 1 \qquad \textit{and} \qquad \mathbb{E}_P[\tau] < \infty.$$

Then

$$\sup_{P \in \mathcal{P}(p,\delta)} \mathbb{E}_P[\tau] \ge \frac{\log(1/\varepsilon)}{p \log 2}.$$

Proof. Pick $m \ge (1-p)/(p-\delta)$ distinct categories $b_1, \dots, b_m, h \in \mathcal{A} \setminus \{r\}$ and set $P(r) = p$, $P(h) = 0$, $P(b_j) = (1-p)/m$, so $P \in \mathcal{P}(p,\delta)$. The hidden-competitor distribution $Q(r) = Q(h) = p/2$, $Q(b_j) = (1-p)/m$ lies in $H_{0,r}$, so $Q(\tau < \infty) \le \varepsilon$. Data processing combined with Lemma C.12 then gives $\log(1/\varepsilon) \le D(P^\tau \| Q^\tau) = \mathbb{E}_P[\tau] p \log 2$. □

Proof of Theorem 4.4. Fix $p \in (0, 1/2]$, $\delta \in (0,p)$, and a level-ε sequential certifier τ' satisfying the assumptions of the corollary. For notational simplicity, write $\tau := \tau'$ within this proof, and let

$$\mathcal{P}(p,\delta) := \left\{ P : p_r = p, \quad p_{(2)} \le p - \delta \right\}.$$

We combine two distinct least favorable subfamilies of $\mathcal{P}(p,\delta)$.

Step 1: Discrimination-hard instance. Because $p \le 1/2$, the quantity $1 - 2p + \delta$ is nonnegative. Choose an integer

$$m \ge \frac{1 - 2p + \delta}{p - \delta},$$

and distinct categories

$$a^\star, b_1, \dots, b_m \in \mathcal{A} \setminus \{r\}.$$

Define P_{pw} by

$$P_{\mathrm{pw}}(r) = p, \qquad P_{\mathrm{pw}}(a^\star) = p - \delta, \qquad P_{\mathrm{pw}}(b_j) = \frac{1 - 2p + \delta}{m} \quad (j = 1, \dots, m),$$

and $P_{\mathrm{pw}}(a) = 0$ otherwise. Then $P_{\mathrm{pw}} \in \mathcal{P}(p,\delta)$ and its runner-up satisfies $p_{(2)} = p - \delta > 0$. Corollary C.8 therefore gives

$$\mathbb{E}_{P_{\mathrm{pw}}}[\tau] \ge \frac{\log(1/\varepsilon)}{\delta \log\big(p/(p-\delta)\big)}.$$

If $\delta \le p/2$, then

$$\log \frac{p}{p-\delta} = -\log\left(1 - \frac{\delta}{p}\right) \le \frac{\delta/p}{1 - \delta/p} \le \frac{2\delta}{p},$$

whence

$$\mathbb{E}_{P_{\mathrm{pw}}}[\tau] \ge \frac{p \log(1/\varepsilon)}{2\delta^2}.$$

Step 2: Exploration-hard instance. Corollary C.13 provides another distribution $P_{\mathrm{un}} \in \mathcal{P}(p,\delta)$ such that

$$\mathbb{E}_{P_{\mathrm{un}}}[\tau] \ge \frac{\log(1/\varepsilon)}{p \log 2}.$$

Step 3: Combine the two bounds. If $\delta \le p/2$, then $\max\{x,y\} \ge (x+y)/2$ applied to the two lower bounds gives the result directly. If $\delta > p/2$, then $p/\delta^2 \le 4/p$, so $p/\delta^2 + 1/p \le 5/p$, and Corollary C.13 alone gives $\sup_P \mathbb{E}_P[\tau] \ge \log(1/\varepsilon)/(p \log 2) \ge (5 \log 2)^{-1} \log(1/\varepsilon)(p/\delta^2 + 1/p)$. Since $(5 \log 2)^{-1} > 1/4$, the same constant works in both cases. □

D. Detailed Power Analysis for the W-CITE

This appendix contains the proofs for the confidence-weighted extension in Section 5.2. We keep the notation parallel to the unweighted analysis, replacing $(p_r, p_{(2)}, \delta)$ by $(\mu_r, \mu_\star, \Delta_w)$ where appropriate.

Lemma D.1 (Expected maximum weighted competitor penalty)**.** *Fix $\lambda \in (0,1)$ and define $g_\lambda(w) := -\log(1-\lambda w)$ for $w \in [0,1]$. For each competitor $a \neq r$, let $H_t^{(\lambda)}(a) := \sum_{i=1}^t g_\lambda(W_i)\mathbf{1}\{X_i = a\}$, $v_\lambda(a) := \mathbb{E}[g_\lambda(W)\mathbf{1}\{X = a\}]$, and set $R_t^{(\lambda)} := \max_{a\neq r} H_t^{(\lambda)}(a)$, $v_\lambda^\star := \sup_{a\neq r} v_\lambda(a)$. Then, for every integer $t \geq 1$,*

$$\mathbb{E}[R_t^{(\lambda)}] \leq t v_\lambda^\star + 2\sqrt{2}\big(-\log(1-\lambda)\big)\sqrt{t\log(t+1)}.$$

Lemma D.2 (Weighted LCB inversion at a fixed time)**.** *Fix an integer $t \geq 1$ and a number $q \in (0,1]$.* (a) *If $\widetilde{M}_t(q) > 1$, then $\widehat{\mu}_t(r) > q$.* (b) *If $\widetilde{M}_t(q) \geq 1/\alpha_r$, then $\widetilde{L}_t(r) \geq q$.*

D.1. Weighted pairwise NSM property (Section 5, item (i))

Proof. Fix $a \neq r$ and $\lambda \in (0,1)$. Since $W_i \in [0,1]$, each factor

$$1 + \lambda W_i\big(\mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\}\big)$$

is nonnegative. Under $\mu_r \leq \mu_a$,

$$\mathbb{E}\left[1 + \lambda W_i\big(\mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\}\big) \mid \mathcal{F}_{i-1}\right] = 1 + \lambda(\mu_r - \mu_a) \leq 1,$$

using independence of (X_i, W_i) from $\mathcal{F}_{i-1}$. Therefore $(\widetilde{E}_t^{(r,a)}(\lambda))_{t\geq 0}$ is an NSM. □

D.2. Weighted mixture NSM (Section 5, item (i))

Proof. For each fixed $a \neq r$ and each $\lambda \in \Lambda_{\mathrm{pw}}$, $(\widetilde{E}_t^{(r,a)}(\lambda))_{t\geq 0}$ is an NSM under $\mu_r \leq \mu_a$ by the preceding proof. Since the weights (w_λ) are nonnegative and sum to at most one,

$$\widetilde{E}_t^{(r,a)} = \sum_{\lambda\in\Lambda_{\mathrm{pw}}} w_\lambda \widetilde{E}_t^{(r,a)}(\lambda)$$

is also an NSM with initial value at most one. Ville's inequality gives

$$\mathbb{P}_P\left(\sup_{t\geq 0} \widetilde{E}_t^{(r,a)} \geq \alpha^{-1}\right) \leq \alpha.$$

□

D.3. Weighted LCB monotonicity (Section 5, item (ii))

Proof. The proof is identical to the monotonicity part of Lemma C.3: $\Lambda_r(q_2) \subseteq \Lambda_r(q_1)$ and each factor $1 + \lambda(W_i\mathbf{1}\{X_i = r\} - q)$ is pointwise larger when q is smaller, so $\widetilde{M}_t(q_1) \geq \widetilde{M}_t(q_2)$. □

D.4. Weighted LCB validity (Section 5, item (ii))

Proof. If $\mu_r = 0$, then $W_i\mathbf{1}\{X_i = r\} = 0$ almost surely for all i, so $\widehat{\mu}_t(r) = 0$ and $\widetilde{L}_t(r) = 0$ almost surely. Hence $\{\exists t : \widetilde{L}_t(r) > \mu_r\}$ is empty. Assume $\mu_r > 0$ in the remainder.

The proof parallels that of Proposition 3.4 (Appendix B.4), with the following substitutions: $\mathbf{1}\{X_i = r\}$ becomes $W_i\mathbf{1}\{X_i = r\}$, p_r becomes μ_r, $\widehat{p}_t(r)$ becomes $\widehat{\mu}_t(r)$, and the mixture weights (w_λ) become (v_λ). At the true value $q = \mu_r$, one has $\mathbb{E}[W_i\mathbf{1}\{X_i = r\} - \mu_r] = 0$, so each $\widetilde{M}_t(\mu_r, \lambda)$ is a nonnegative martingale with unit initial value, and Ville's inequality gives $\mathbb{P}(\sup_t \widetilde{M}_t(\mu_r) \geq 1/\alpha_r) \leq \alpha_r$. The monotonicity from the weighted LCB monotonicity property then implies $\{\exists t : \widetilde{L}_t(r) > \mu_r\} \subseteq \{\sup_t \widetilde{M}_t(\mu_r) \geq 1/\alpha_r\}$. □

D.5. Weighted unseen bound validity (Section 5, item (iii))

Proof. Since $\mu_a \le p_a$ for every $a \in \mathcal{A}$,

$$\Big\{\exists a, \exists t : N_t(a) = 0 \text{ and } \mu_a > U_t\Big\} \subseteq \Big\{\exists a, \exists t : N_t(a) = 0 \text{ and } p_a > U_t\Big\}.$$

The probability of the latter event is at most α_u by Proposition 3.6. □

D.6. Proof of Theorem 5.2

Proof. The proof parallels that of Theorem 4.1 (Appendix B.6), with p_a replaced by μ_a throughout and Lemma B.3 replaced by the all-competitors check (runner-up monotonicity fails for weighted observations).

Fix $P \in H_{0,r}^{(w)}$. If $\mathcal{A} \setminus \{r\} = \emptyset$, then the weighted null is vacuous. Otherwise, since

$$\sum_{a \in \mathcal{A}} \mu_a = \mathbb{E}[W] \le 1,$$

Lemma B.1 implies that any positive value of $\mu_\star = \sup_{a \ne r} \mu_a$ is attained. If $\mu_\star > 0$, choose $a^\star \ne r$ with $\mu_{a^\star} = \mu_\star \ge \mu_r$. If $\mu_\star = 0$, then $P \in H_{0,r}^{(w)}$ implies $\mu_r = 0$, and any $a^\star \ne r$ is a valid witness with $\mu_{a^\star} \ge \mu_r$. Decompose

$$\{\tau^{(w)} < \infty\} \subseteq \underbrace{\{\tau^{(w)} < \infty,\ a^\star \in \mathcal{A}_{\tau^{(w)}}\}}_{E_1} \cup \underbrace{\{\tau^{(w)} < \infty,\ a^\star \notin \mathcal{A}_{\tau^{(w)}}\}}_{E_2}.$$

Step 1 (bounding $\mathbb{P}(E_1)$). On E_1, the weighted pairwise stopping condition at $\tau^{(w)}$ applied to *every* observed competitor gives $\widetilde{E}_{\tau^{(w)}}^{(r,a^\star)} \ge 1/\alpha_{\mathrm{pw}}$. The process $(\widetilde{E}_t^{(r,a^\star)})_{t \ge 0}$ is an NSM under $\mu_r \le \mu_{a^\star}$ by the weighted mixture NSM property (Section 5, item (i)), with initial value ≤ 1. Ville's inequality (12) yields $\mathbb{P}(E_1) \le \alpha_{\mathrm{pw}}$.

Step 2 (bounding $\mathbb{P}(E_2)$). Define

$$F_r := \{\exists t \ge 1 : \widetilde{L}_t(r) > \mu_r\}, \quad F_u := \{\exists a \in \mathcal{A},\ \exists t \ge 1 : N_t(a) = 0,\ \mu_a > U_t\}.$$

Weighted LCB validity gives $\mathbb{P}(F_r) \le \alpha_r$, and weighted unseen-bound validity gives $\mathbb{P}(F_u) \le \alpha_u$. On $E_2 \setminus (F_r \cup F_u)$, the LCB–unseen part of the weighted stopping rule gives $\widetilde{L}_{\tau^{(w)}}(r) > U_{\tau^{(w)}}$; combined with F_r^c and F_u^c (recall $a^\star$ is unseen at $\tau^{(w)}$),

$$\mu_{a^\star} \le U_{\tau^{(w)}} < \widetilde{L}_{\tau^{(w)}}(r) \le \mu_r,$$

contradicting $\mu_{a^\star} \ge \mu_r$. Hence $E_2 \subseteq F_r \cup F_u$ and $\mathbb{P}(E_2) \le \alpha_r + \alpha_u$.

Step 3 (union bound). $\mathbb{P}(\tau^{(w)} < \infty) \le \mathbb{P}(E_1) + \mathbb{P}(E_2) \le \alpha_{\mathrm{pw}} + \alpha_r + \alpha_u \le \varepsilon$. □

D.7. Proof of Theorem 5.4: almost-sure stopping part

Proof. Lemma D.3 (a) gives $\gamma_{\mathrm{pw}} := \inf_{a \ne r} \mathbb{E}[\log(1 + \lambda_{\mathrm{pw}} W(\mathbf{1}\{X = r\} - \mathbf{1}\{X = a\}))] \ge \lambda_{\mathrm{pw}} \Delta_w / 2 > 0$, and Lemma D.3 (c) with $q_0 = \mu_r/2$ gives $\nu_r := \mathbb{E}[\log(1 + \lambda_r(W\mathbf{1}\{X = r\} - q_0))] \ge \lambda_r \mu_r / 4 > 0$. We now show that these positive drifts force $\mathbb{P}(\tau^{(w)} < \infty) = 1$.

Fix $\lambda := \lambda_{\mathrm{pw}}$, and let w_{pw} denote its mixture weight in Section 5, item (i).

Step 1: Pairwise condition. For each i, set $A_i := \log(1 + \lambda W_i)\mathbf{1}\{X_i = r\}$ and $B_i(a) := -\log(1 - \lambda W_i)\mathbf{1}\{X_i = a\}$, so that

$$\log \widetilde{E}_t^{(r,a)}(\lambda) = \textstyle\sum_{i=1}^t A_i - \sum_{i=1}^t B_i(a). \tag{21}$$

Write $u := \mathbb{E}[A_1]$, $v_a := \mathbb{E}[B_1(a)]$, and $v^\star := \sup_{a \ne r} v_a$. The definition of γ_{pw} gives $u - v^\star \ge \gamma_{\mathrm{pw}} > 0$.

We control $\max_{a \ne r} t^{-1} \sum_i B_i(a)$ on the countable category set by a tail-truncation argument. Since $\sum_{a \ne r} v_a \le -\log(1 - \lambda) < \infty$, for each $m \ge 1$ there is a finite $F_m \subset \mathcal{A} \setminus \{r\}$ with $\sum_{a \notin F_m} v_a < 1/m$. Define the tail process $R_t^{(m)} :=$

$\sum_i -\log(1-\lambda W_i)\mathbf{1}\{X_i \notin F_m \cup \{r\}\}$. By the Strong Law of Large Numbers, $t^{-1}\sum_i A_i \to u$, $t^{-1}\sum_i B_i(a) \to v_a$ for each fixed a, and $t^{-1}R_t^{(m)} \to \sum_{a\notin F_m} v_a < 1/m$, all almost surely. On the probability-one intersection of these countably many events, $\limsup_t \max_{a\neq r} t^{-1}\sum_i B_i(a) \le \max\{v^\star, 1/m\}$ for every m; letting $m\to\infty$ gives the limsup $\le v^\star$. Combining with (21),

$$\liminf_t \min_{a\neq r} t^{-1}\log \widetilde{E}_t^{(r,a)}(\lambda) \ge u - v^\star \ge \gamma_{\mathrm{pw}} > 0.$$

Since $\mathcal{A}_t \setminus \{r\} \subseteq \mathcal{A}\setminus\{r\}$ and $\widetilde{E}_t^{(r,a)} \ge w_{\mathrm{pw}}\widetilde{E}_t^{(r,a)}(\lambda)$, the positive liminf of the normalized log e-process implies there exists a.s. a finite T'_{pw} after which $\min_{a\in\mathcal{A}_t\setminus\{r\}} \widetilde{E}_t^{(r,a)} \ge 1/\alpha_{\mathrm{pw}}$.

Step 2: Weighted LCB condition. Fix $\lambda := \lambda_r$ with mixture weight $v_{\lambda_r} > 0$ and set $C_i := \log(1+\lambda(W_i\mathbf{1}\{X_i = r\} - q_0))$; each C_i is bounded since $\lambda < 1/q_0$ (Lemma D.3(c)) and $W_i\mathbf{1}\{X_i = r\} \in [0,1]$. Part (c) of Lemma D.3 gives $\mathbb{E}[C_i] = \nu_r \ge \lambda\mu_r/4 > 0$, so by the SLLN $t^{-1}\log \widetilde{M}_t(q_0,\lambda) = t^{-1}\sum_{i=1}^t C_i \to \nu_r$ a.s., hence $\widetilde{M}_t(q_0,\lambda)\to\infty$ and $\widetilde{M}_t(q_0) \ge v_{\lambda_r}\widetilde{M}_t(q_0,\lambda) \ge 1/\alpha_r$ eventually a.s. An independent SLLN on $W_i\mathbf{1}\{X_i = r\}$ gives $\widehat{\mu}_t(r)\to\mu_r > q_0$, so $\widehat{\mu}_t(r) \ge q_0$ eventually, and the weighted LCB construction then yields $\widetilde{L}_t(r) \ge q_0$ for all large t almost surely. Let T'_r denote the resulting (random, a.s. finite) eventual time after which both $\widetilde{M}_t(q_0) \ge 1/\alpha_r$ and $\widetilde{L}_t(r) \ge q_0$ hold.

Step 3: Unseen upper bound. The sequence $(U_t)_{t\ge1}$ is deterministic and $U_t \downarrow 0$ as $t\to\infty$. Since $q_0 > 0$, there exists a deterministic index T_u such that

$$U_t < q_0 \qquad \forall t \ge T_u.$$

Step 4: Combine. Choose t large enough that $t \ge T'_{\mathrm{pw}} \vee T'_r \vee T_u$. Then the pairwise threshold is met, $\widetilde{L}_t(r) \ge q_0 > U_t$, and both stopping conditions hold, giving $\mathbb{P}(\tau^{(w)} < \infty) = 1$. □

Lemma D.3 (Weighted small-bet drift bounds)**.** *Assume* $\Delta_w > 0$.

(a) ***Pairwise:*** *if* $\lambda \in \Lambda_{\mathrm{pw}}$ *satisfies* $\lambda \le \Delta_w/\big(2(\mu_r+\mu_\star)\big)$*, then*

$$\inf_{a\neq r} \mathbb{E}\big[\log\big(1+\lambda W(\mathbf{1}\{X=r\} - \mathbf{1}\{X=a\})\big)\big] \ge \lambda\Delta_w/2 > 0.$$

(b) ***LCB:*** *for any* $q_0 \in (0,\mu_r)$ *and any* $\lambda \in \Lambda_r$ *with* $\lambda \le (\mu_r - q_0)/(2\mu_r)$*, one has* $\lambda < 1/q_0$ *and*

$$\mathbb{E}\big[\log\big(1+\lambda(W\mathbf{1}\{X=r\} - q_0)\big)\big] \ge \lambda(\mu_r - q_0)/2 > 0.$$

(c) ***Universal LCB grid point:*** *with* $q_0 := \mu_r/2$ *in part (b), the condition reduces to* $\lambda_r \in (0, 1/4]$*, and*

$$\mathbb{E}\big[\log\big(1+\lambda_r(W\mathbf{1}\{X=r\} - \mu_r/2)\big)\big] \ge \lambda_r\mu_r/4 > 0.$$

Proof. All three parts follow from the quadratic lower bound $\log(1+\xi) \ge \xi - \xi^2$ on $[-1/2, 1/2]$ (Lemma C.4), exactly as in Proposition C.5, with two additional ingredients: $W \in [0,1]$ implies $W^2 \le W$, and the squared indicators satisfy $(\mathbf{1}\{X=r\} - \mathbf{1}\{X=a\})^2 = \mathbf{1}\{X=r\} + \mathbf{1}\{X=a\}$.

(a) Set $\xi_a := \lambda W(\mathbf{1}\{X=r\} - \mathbf{1}\{X=a\})$, so $|\xi_a| \le \lambda \le 1/2$. Taking expectations of $\log(1+\xi_a) \ge \xi_a - \xi_a^2$ and using $W^2 \le W$,

$$\mathbb{E}[\log(1+\xi_a)] \ge \lambda(\mu_r - \mu_a) - \lambda^2(\mu_r+\mu_a) \ge \lambda\Delta_w - \lambda^2(\mu_r+\mu_\star).$$

The hypothesis gives $\lambda^2(\mu_r+\mu_\star) \le \lambda\Delta_w/2$, hence $\mathbb{E}[\log(1+\xi_a)] \ge \lambda\Delta_w/2$ uniformly in a.

(b) Since $\mu_r \le 1$, $(\mu_r - q_0)/(2\mu_r) < 1/2 < 1/q_0$, so the hypothesis implies $\lambda < 1/2$ and $\lambda < 1/q_0$. Setting $Y := W\mathbf{1}\{X=r\}$ and $\xi := \lambda(Y - q_0)$, the same quadratic bound and $Y^2 \le Y$ yield

$$\mathbb{E}[\log(1+\xi)] \ge \lambda(\mu_r - q_0) - \lambda^2\mu_r \ge \lambda(\mu_r - q_0)/2.$$

(c) Specializing (b) to $q_0 = \mu_r/2$ gives $(\mu_r - q_0)/(2\mu_r) = 1/4$, so $\lambda_r \le 1/4$ verifies the hypothesis and the conclusion follows. □

D.8. Proof of Lemma D.1

Proof. The proof follows the same symmetrization argument as Lemma C.2, applied to the bounded function class $\{f_a(x, w) := g_\lambda(w)\mathbf{1}\{x = a\} : a \neq r\}$ in place of $\{\mathbf{1}\{x = a\}\}$. The key modifications are:

- The centering term $tp_{(2)}$ becomes $tv^\star_\lambda$, where $v^\star_\lambda := \sup_{a\neq r} \mathbb{E}[g_\lambda(W)\mathbf{1}\{X = a\}]$.
- The per-coordinate bound $\|v_a\|_\infty \leq 1$ becomes $\|v_a\|_\infty \leq g_\lambda(1) = -\log(1-\lambda)$, so the sub-Gaussian parameter scales by $-\log(1-\lambda)$.
- The Rademacher complexity bound remains $O(\sqrt{t\log(t+1)})$ because there are still at most $t+1$ distinct vectors in the function class evaluated on t samples.

Optimizing the Donsker–Varadhan parameter s as in Lemma C.2 yields the stated bound. □

D.9. Proof of Lemma D.2

Proof. **(a)** Assume $\widehat{\mu}_t(r) \leq q$. Set $Y_i := W_i\mathbf{1}\{X_i = r\}$. By Jensen's inequality (concavity of log), $t^{-1}\log \widetilde{M}_t(q, \lambda) \leq \log(1 + \lambda(\widehat{\mu}_t(r) - q)) \leq 0$ for every $\lambda \in \Lambda_r(q)$. Summing gives $\widetilde{M}_t(q) \leq \sum v_\lambda \leq 1$, contradicting $\widetilde{M}_t(q) > 1$.

(b) If $\widetilde{M}_t(q) \geq 1/\alpha_r > 1$, part (a) gives $\widehat{\mu}_t(r) > q$, so $q \in (0, \widehat{\mu}_t(r)]$ and q belongs to the set defining $\widetilde{L}_t(r)$; hence $\widetilde{L}_t(r) \geq q$. □

D.10. Proof of Theorem 5.4: expected stopping-time bound

Proof. We use the same structure as the proof of Theorem 4.3, with $(p_r, p_{(2)}, \delta)$ replaced by $(\mu_r, \mu_\star, \Delta_w)$.

Step 1: pairwise condition. Let $\lambda := \lambda_{\mathrm{pw}} \in [\Delta_w/8, \Delta_w/4]$ and let w_λ be its mixture weight. Set

$$B^{(w)}_{\mathrm{pw}} := \log\{1/(\alpha_{\mathrm{pw}} w_\lambda)\}.$$

For each competitor $a \neq r$, write

$$A_i := \log(1 + \lambda W_i)\mathbf{1}\{X_i = r\}, \qquad B_i(a) := -\log(1 - \lambda W_i)\mathbf{1}\{X_i = a\}.$$

The relevant fixed-λ log pairwise statistic is

$$S^{(w)}_t := \sum_{i=1}^t A_i - \max_{a\neq r} \sum_{i=1}^t B_i(a).$$

If $S^{(w)}_t \geq B^{(w)}_{\mathrm{pw}}$, then the weighted all-competitors pairwise condition holds at time t.

Lemma D.3 gives the drift lower bound

$$\inf_{a\neq r} \mathbb{E}\log\{1 + \lambda W(\mathbf{1}\{X = r\} - \mathbf{1}\{X = a\})\} \geq \lambda\Delta_w/2 \geq c\Delta_w^2.$$

Combining this with Lemma D.1 yields

$$\mathbb{E}[S^{(w)}_t] \geq c_0\Delta_w^2 t - C_0\Delta_w\sqrt{t\log(t+1)}.$$

Changing one observation changes $S^{(w)}_t$ by at most $C_1\Delta_w$, so McDiarmid's inequality and Lemma C.1 imply that, for a universal constant C,

$$\mathbb{P}\{\text{weighted pairwise condition fails at time } t\} \leq (et)^{-2}$$

whenever

$$t \geq T^{(w)}_{\mathrm{pw}} := C\frac{\log\{1/(\alpha_{\mathrm{pw}} w_\lambda)\} + \log(1/\Delta_w)}{\Delta_w^2}.$$

Step 2: LCB–unseen condition. Set $q_0 := \mu_r/2$. The grid condition gives $\lambda_r \in [1/32, 1/16] \subset \Lambda_r(q_0)$ with weight v_{λ_r}. Let

$$Y_i := W_i \mathbf{1}\{X_i = r\}, \qquad A_t := \left\{ \frac{3}{4}\mu_r t \le \sum_{i=1}^{t} Y_i \le \frac{5}{4}\mu_r t \right\}.$$

Since $Y_i \in [0,1]$ and $\mathbb{E}Y_i = \mu_r$, Bernstein's inequality gives

$$\mathbb{P}(A_t^c) \le 2\exp(-c\mu_r t).$$

On A_t, the single weighted LCB factor satisfies

$$\log \widetilde{M}_t(q_0, \lambda_r) = \sum_{i=1}^{t} \log\{1 + \lambda_r(Y_i - q_0)\} \ge c'\mu_r t$$

for a universal constant $c' > 0$. Thus

$$\widetilde{M}_t(q_0) \ge v_{\lambda_r} \widetilde{M}_t(q_0, \lambda_r) \ge \alpha_r^{-1}$$

once

$$t \ge C\mu_r^{-1} \log\{1/(\alpha_r v_{\lambda_r})\}.$$

Moreover, the unchanged unseen bound satisfies $U_t \le \mu_r/4 < q_0$ once

$$t \ge C\mu_r^{-1} \log\{1/(\alpha_u \mu_r)\}.$$

Combining these bounds with Lemma C.1, we obtain

$$\mathbb{P}\{\widetilde{L}_t(r) \le U_t\} \le (et)^{-2}$$

for all

$$t \ge T_{\mathrm{rare}}^{(w)} := C \frac{\log(1/\alpha_r) + \log(1/\alpha_u) + \log(1/v_{\lambda_r}) + \log(1/\mu_r)}{\mu_r}.$$

Step 3: combine. Let

$$T_{\mathrm{cl}}^{(w)} := \max\{T_{\mathrm{pw}}^{(w)}, T_{\mathrm{rare}}^{(w)}\}.$$

For all $t \ge T_{\mathrm{cl}}^{(w)}$,

$$\mathbb{P}(\tau^{(w)} > t) \le 2(et)^{-2}.$$

The tail-sum identity gives

$$\mathbb{E}[\tau^{(w)}] \le T_{\mathrm{cl}}^{(w)} + 1.$$

Substituting $\alpha_{\mathrm{pw}} = \alpha_r = \alpha_u = \varepsilon/3$ and using the selected-weight lower bounds yields

$$\mathbb{E}[\tau^{(w)}] = O\left(\frac{\log(1/\varepsilon) + \log(1/\Delta_w)}{\Delta_w^2} + \frac{\log(1/\varepsilon) + \log(1/\mu_r)}{\mu_r} \right).$$

□

Corollary D.4 (Independent confidence scores)**.** *Suppose W is independent of X and let $\bar{w} := \mathbb{E}[W] > 0$. If r is the unweighted unique mode with gap $\delta := p_r - p_{(2)} > 0$, then*

$$\mu_r = \bar{w} p_r, \qquad \mu_\star = \bar{w} p_{(2)}, \qquad \Delta_w = \bar{w}\delta.$$

Thus Theorem 5.4 applies whenever the weighted grid condition holds, equivalently when

$$\lambda_{\mathrm{pw}} \in \left[\frac{\bar{w}\delta}{8}, \frac{\bar{w}\delta}{4} \right] \quad \textit{and} \quad \lambda_r \in [1/32, 1/16]$$

are available on the grids.

D.11. Proof of Corollary D.4

Proof. Independence gives

$$\mu_a = \mathbb{E}[W\mathbf{1}\{X = a\}] = \bar{w}p_a \qquad \forall a \in \mathcal{A}.$$

Therefore

$$\mu_r = \bar{w}p_r, \qquad \mu_\star = \bar{w}p_{(2)}, \qquad \Delta_w = \bar{w}\delta.$$

Substituting these identities into the grid condition and the rate in Theorem 5.4 proves the claim. □

Proposition D.5 (Recovery of the unweighted CITE)**.** *Suppose $W_i \equiv 1$ almost surely. Then $\mu_a = p_a$ for every $a \in \mathcal{A}$; $\widetilde{E}_t^{(r,a)}(\lambda) = (1+\lambda)^{N_t(r)}(1-\lambda)^{N_t(a)}$; $\widehat{\mu}_t(r) = \widehat{p}_t(r)$; and the weighted stopping rule agrees with the all-competitors version of the unweighted CITE, with runner-up monotonicity reducing the pairwise check to the empirical runner-up.*

D.12. Proof of Proposition D.5

Proof. **Claim (i): Mass equality.** If $W_i \equiv 1$ almost surely, then

$$\mu_a = \mathbb{E}[W\mathbf{1}\{X = a\}] = \mathbb{E}[\mathbf{1}\{X = a\}] = p_a \quad \forall a \in \mathcal{A}.$$

Claim (ii): Pairwise e-process recovery. Under $W_i \equiv 1$, the weighted e-process simplifies:

$$\widetilde{Z}_i^{(r,a)} = W_i(\mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\}) = \mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\}.$$

Thus

$$\widetilde{E}_t^{(r,a)}(\lambda) = \prod_{i=1}^{t}\big(1 + \lambda\widetilde{Z}_i^{(r,a)}\big) = \prod_{i=1}^{t}\big(1 + \lambda(\mathbf{1}\{X_i = r\} - \mathbf{1}\{X_i = a\})\big) = (1+\lambda)^{N_t(r)}(1-\lambda)^{N_t(a)}.$$

Claim (iii): Empirical mean equality.

$$\widehat{\mu}_t(r) = \frac{1}{t}\sum_{i=1}^{t} W_i\mathbf{1}\{X_i = r\} = \frac{1}{t}\sum_{i=1}^{t}\mathbf{1}\{X_i = r\} = \widehat{p}_t(r).$$

Substituting $W_i \equiv 1$ into the weighted LCB e-process yields

$$\widetilde{M}_t(q,\lambda) = \prod_{i=1}^{t}(1 + \lambda(\mathbf{1}\{X_i = r\} - q)),$$

which is identical to the unweighted LCB e-process after dropping the tilde.

Claim (iv): Stopping rule recovery. From (ii), in the unweighted specialization the weighted pairwise condition

$$\widetilde{E}_t^{(r,a)} = \sum_{\lambda} w_\lambda \widetilde{E}_t^{(r,a)}(\lambda) \geq 1/\alpha_{\mathrm{pw}} \quad \text{for all } a \in \mathcal{A}_t \setminus \{r\}$$

checks the same pairwise e-values as the unweighted all-competitors CITE. By Lemma B.3, those e-values are monotone in the competitor count, so among observed competitors the minimum is attained at the empirical runner-up. Thus the all-competitors pairwise check reduces to a single empirical runner-up, recovering the original unweighted fixed-target CITE. □

E. Comparison with Martingale Majority Certificates

E.1. The MMC Hypothesis

The top-m MMC (Cordero-Encinar and Duncan, 2025) simultaneously tests the leader against $m-1$ pairwise competitors and the aggregated residual mass. The alternative certified upon stopping is

$$\mathcal{H}_{\mathrm{MMC},m} := \{P : p_{(1)} > p_{(i)} \forall i = 2,\ldots,m\} \cap \{P : p_{(1)} > s_m\}, \qquad s_m := \sum_{j>m} p_{(j)}. \tag{22}$$

Proposition E.1 (Strict inclusion)**.** *Suppose the target r coincides with the (unique) modal label $p_r = p_{(1)}$, equivalently r is the unique maximiser of $a \mapsto p_a$. Then for every $m \geq 2$, $\mathcal{H}_{\mathrm{MMC},m} \subsetneq H_{1,r}$; without the unique-mode-target identification, this strict inclusion must be read with $H_{1,r}$ replaced by $\bigcup_{r' \in \mathcal{A}} H_{1,r'}$.*

Remark E.2 (The diffuse-tail regime)**.** *Distributions in $H_{1,r} \setminus \mathcal{H}_{\mathrm{MMC},m}$ have a unique mode that does not dominate the aggregate tail: $p_{(1)} > p_{(2)}$ but $p_{(1)} \leq s_m$. Several of our LLM self-consistency settings (Section 6) fall into this regime, with the mode-answer carrying 20–30% of the mass and thousands of alternative answers sharing the remainder. For each fixed m, the MMC controls its certification rate at level α on any such P: its residual-mass channel at that m tests the null $\{p_{(1)} \leq s_m\}$, which is true under P, so the corresponding e-process cannot grow past the rejection threshold with probability $> \alpha$. An MMC variant that tries multiple m in parallel pays a Bonferroni-style cost over the active tuples (Appendix E.3).*

E.2. The Switching-Null Challenge

At each round n, the MMC selects the empirical leader A_{n-1} and runner-up B_{n-1} (both $\mathcal{F}_{n-1}$-measurable), maintaining e-processes: e_n^{run} (leader vs. runner-up) and e_n^{oth} (leader vs. residual mass). The per-round nulls are:

$$R_n := \{p_{A_{n-1}} \leq p_{B_{n-1}}\}, \quad O_n := \Big\{p_{A_{n-1}} \leq \sum\nolimits_{j \notin \{A_{n-1}, B_{n-1}\}} p_j\Big\}. \tag{23}$$

Theorem 3.1 of Cordero-Encinar and Duncan (2025) establishes that (e_n^{run}) is an NSM under the all-rounds null $H_0^{\mathrm{run}} := \bigcap_{n \geq 1} R_n$, and similarly for the residual-mass channel. Corollary 3.2 proves

$$\sup_{P \in H_0} \mathbb{P}_P(N < \infty) \leq \varepsilon, \qquad H_0 := H_0^{\mathrm{run}} \cup H_0^{\mathrm{oth}}. \tag{24}$$

The intersection-union structure (stop requires *both* channels, null is a *union*) gives ε without Bonferroni.

The challenge lies in bridging from (24) to the adaptive misclassification guarantee $\mathbb{P}(A_{\tau-1} \neq a^\star) \leq \varepsilon$ for the certified pre-round leader (cf. Proposition E.3 below). At any fixed time n, misclassification of the pre-round leader satisfies

$$\{A_{n-1} \neq a^\star\} \subseteq R_n \cup O_n. \tag{25}$$

However, Corollary 3.2 controls the all-rounds event

$$H_0 = \Big(\bigcap_{t \geq 1} R_t\Big) \cup \Big(\bigcap_{t \geq 1} O_t\Big),$$

which is logically distinct from the existential per-round inclusion:

$$(\forall t R_t) \vee (\forall t O_t) \quad \not\Longleftarrow \quad \exists n (R_n \vee O_n). \tag{26}$$

Bridging this gap requires a *switching-null reduction* showing that misclassification at τ is contained in the all-rounds null. Since the labels (A_{n-1}, B_{n-1}) are data-dependent and may switch while e-processes accumulate evidence without restart, this reduction is nontrivial but feasible. It introduces Bonferroni costs absent from CITE.

E.3. Tuple-Indexed Construction

To handle the switching-null challenge, *index e-processes by tuple*: let

$$\mathcal{T} := \big\{(m, a, b_1, \ldots, b_{m-1}) : m \geq 2,\ a \in \mathcal{A},\ b_1, \ldots, b_{m-1} \in \mathcal{A} \setminus \{a\} \text{ pairwise distinct}\big\}$$

denote the (countable) set of admissible tuples and assign each $\ell = (m, a, b_1, \ldots, b_{m-1}) \in \mathcal{T}$ its own e-process, with multiplicative factor 1 on rounds where the current tuple differs from ℓ.

Proposition E.3 (Tuple-indexed repair)**.** *Assume the data-generating distribution has a unique mode $a^\star \in \arg\max_{a \in \mathcal{A}} p_a$. Let $\{\alpha_\ell\}_{\ell \in \mathcal{T}}$ satisfy $\sum_\ell \alpha_\ell \leq \varepsilon$. For each ℓ, maintain separate tuple-indexed e-processes $E_t^{\ell,i}$ (pairwise, slot i) and $E_t^{\ell,\mathrm{oth}}$ (residual mass), active only when $L_{t-1} = \ell$. Define*

$$\tau := \inf\Big\{n : \exists \ell \in \mathcal{T} \text{ s.t. } L_{n-1} = \ell, E_n^{\ell,i} \geq 1/\alpha_\ell \forall i, E_n^{\ell,\mathrm{oth}} \geq 1/\alpha_\ell\Big\}.$$

Let $A_{\tau-1}$ denote the leader component of the active tuple $L_{\tau-1}$ (the pre-round empirical leader at the certifying time). Then $\mathbb{P}(\tau < \infty, A_{\tau-1} \neq a^\star) \leq \varepsilon$.

The Bonferroni cost. Each tuple receives error allocation α_ℓ, yielding threshold $1/\alpha_\ell$. With $\pi_\ell := \alpha_\ell/\varepsilon$:

$$\log \frac{1}{\alpha_\ell} = \underbrace{\log \frac{1}{\varepsilon}}_{\text{base}} + \underbrace{\log \frac{1}{\pi_\ell}}_{\text{multiplicity penalty}} . \tag{27}$$

Under uniform allocation over K tuples, the multiplicity penalty is $\log K$. With L_t observed categories: $K_{m,t} = L_t\binom{L_t-1}{m-1} = O(L_t^m)$, yielding additional sample complexity of order $m \log L_t/I_\ell$, where I_ℓ denotes the per-observation KL information.

Contrast with CITE. CITE avoids this multiplicity cost: with r fixed a priori, Ville's inequality applies to a single e-process with threshold $1/\alpha_{\text{pw}}$ and no multiplicity correction.

E.4. Structural Comparison Summary

	Tuple-Indexed MMC	**CITE (ours)**
Hypothesis	$p_{(1)} > p_{(i)}$ for all $i \le m$ and $p_{(1)} > s_m$	$p_{(1)} > p_{(2)}$ (exact unique mode)
Target selection	Adaptive (tuple may switch)	Fixed target r a priori
Multiplicity cost	$\log K_{m,t}$ threshold inflation	None
Diffuse-tail power	$\le \alpha$ ($p_{(1)} \le s_m \Rightarrow$ null true; Ville bound)	Full (pairwise margin only)
Unseen control	Aggregated into residual mass	Explicit: $L_t(r) > U_t$

Table 1. Structural comparison of the tuple-indexed MMC and CITE.

E.5. Proof of Proposition E.1

Proof. Inclusion $\mathcal{H}_{\text{MMC},m} \subseteq H_{1,r}$. Any $P \in \mathcal{H}_{\text{MMC},m}$ satisfies $p_{(1)} > p_{(i)}$ for every $i \in \{2,\dots,m\}$, and $p_{(1)} > s_m \ge p_{(j)}$ for every $j > m$; combining, $p_r = p_{(1)}$ strictly dominates every other category, i.e. $P \in H_{1,r}$.

Strict inclusion. Fix $m \ge 2$ and $K \ge 2m$, and consider the perturbed-uniform distribution

$$p_{(1)} := \frac{1}{K} + \eta, \qquad p_{(i)} := \frac{1}{K} - \frac{\eta}{K-1} \quad (i \ge 2), \qquad \eta \in (0, \tfrac{1}{K}),$$

so that $\sum_i p_{(i)} = 1$ and $p_{(1)} > p_{(2)}$. Then $P \in H_{1,r}$ (since r is strictly the unique mode), while

$$s_m = \sum_{j>m} p_{(j)} = (K-m)\left(\tfrac{1}{K} - \tfrac{\eta}{K-1}\right) \ge \tfrac{K-m}{K} - \eta \ge \tfrac{1}{2} - \eta > \tfrac{1}{K} + \eta = p_{(1)}$$

for sufficiently small η (using $K \ge 2m$ and $K \ge 2$). Hence $P \notin \mathcal{H}_{\text{MMC},m}$, proving strict inclusion. □

E.6. Proof of Proposition E.3

Proof. Fix a tuple $\ell = (a, b_1, \dots, b_{m-1}) \in \mathcal{T}$ with leader a and slots $b_1, \dots, b_{m-1}$. Define its stopping time and misclassification event

$$\tau_\ell := \inf\{n \ge 1 : L_{n-1} = \ell,\ E_n^{\ell,i} \ge 1/\alpha_\ell\ \forall i,\ E_n^{\ell,\text{oth}} \ge 1/\alpha_\ell\}, \quad G_\ell := \{\tau = \tau_\ell < \infty,\ a \ne a^\star\}.$$

Suppose G_ℓ occurs. Since $a \ne a^\star$ and $a^\star$ lies in $\mathcal{A}$, exactly one of the following holds:

Case (i): $a^\star = b_i$ for some $i \in \{1,\dots,m-1\}$. Then the pairwise null for slot i is $R^{(\ell,i)} := \{p_a \le p_{b_i}\}$, which is *true* because $p_a < p_{a^\star} = p_{b_i}$ (as $a^\star$ is the unique mode). Under $R^{(\ell,i)}$ the pairwise e-process $(E_n^{\ell,i})_{n\ge0}$ is an NSM with initial value ≤ 1, so Ville's inequality gives $\mathbb{P}(E_{\tau_\ell}^{\ell,i} \ge 1/\alpha_\ell) \le \alpha_\ell$. Hence $\mathbb{P}(G_\ell \cap \text{Case (i)}) \le \alpha_\ell$.

Case (ii): $a^\star \notin \{a, b_1, \dots, b_{m-1}\}$. Then the residual-mass null $O^{(\ell)} := \{p_a \le \sum_{j \notin \{a,b_1,\dots,b_{m-1}\}} p_j\}$ is true, since the sum on the right side includes $p_{a^\star} > p_a$. Under $O^{(\ell)}$, $(E_n^{\ell,\text{oth}})_{n\ge0}$ is an NSM with initial value ≤ 1, and Ville gives $\mathbb{P}(E_{\tau_\ell}^{\ell,\text{oth}} \ge 1/\alpha_\ell) \le \alpha_\ell$. Hence $\mathbb{P}(G_\ell \cap \text{Case (ii)}) \le \alpha_\ell$.

Combining both cases, $\mathbb{P}(G_\ell) \le \alpha_\ell$.

Mutual exclusion across tuples. At the stopping time τ, only the tuple $L_{\tau-1}$ has $L_{\tau-1} = \ell$, so the events $\{\tau = \tau_\ell, L_{\tau_\ell - 1} = \ell\}$ are disjoint across ℓ. Therefore

$$\mathbb{P}(\tau < \infty,\ A_{\tau-1} \neq a^\star) = \sum_{\ell \in \mathcal{T}} \mathbb{P}(G_\ell) \le \sum_{\ell \in \mathcal{T}} \alpha_\ell \le \varepsilon. \qquad \square$$

F. Additional Theoretical Results

This appendix collects additional results used to interpret and extend the main theory: a sharp oracle pairwise benchmark (F.1), an adaptive geometric grid (F.2), a growth-rate optimality result for the oracle pairwise bet and its finite mixture (F.3), and a top-k extension (F.4).

F.1. Sharp oracle pairwise benchmark

The main text compares the stopping-time upper bound with the minimax lower bound at the level of rates. Here we isolate a sharper constant-level statement for an oracle pairwise comparison between the target and its true runner-up. This result is not a sharp-constant theorem for the full CITE stopping time; rather, it shows that the pairwise e-process has the correct leading constant when both the true runner-up and the oracle betting parameter are available.

Proposition F.1 (Identity and small-gap expansion of $\rho^\star$)**.** *The oracle pairwise growth rate $\rho^\star := \max_{\lambda \in (0,1)} \big[p_r \log(1+\lambda) + p_{(2)} \log(1-\lambda)\big]$ (Proposition C.6) coincides exactly with the forward I-projection KL of P onto $H_{0,r}$:*

$$\rho^\star = D(P \| P^\star) = p_r \log \frac{2p_r}{p_r + p_{(2)}} + p_{(2)} \log \frac{2p_{(2)}}{p_r + p_{(2)}}. \tag{28}$$

In particular, by Lemma C.9,

$$\rho^\star = \frac{\delta^2}{2(p_r + p_{(2)})} + O\left(\frac{\delta^4}{(p_r + p_{(2)})^3}\right) = \frac{\delta^2}{4p_r}\big(1 + o(1)\big).$$

Proof. Substituting $\lambda^\star = (p_r - p_{(2)})/(p_r + p_{(2)})$ from Proposition C.6 into the definition of $\rho^\star$ yields the closed form of $D(P\|P^\star)$ in Lemma C.9, since $1 \pm \lambda^\star = 2p_r/(p_r + p_{(2)})$ and $2p_{(2)}/(p_r + p_{(2)})$ respectively. Both displays now follow directly from Lemma C.9 after substituting $q^\star = (p_r + p_{(2)})/2$. $\square$

Theorem F.2 (Sharp constant for the oracle pairwise benchmark)**.** *Assume $p_{(2)} > 0$, $p_r \in (0, 1/2]$, and $\delta/p_r \to 0$. Suppose*

$$\lambda^\star := \frac{\delta}{p_r + p_{(2)}} \in \Lambda_{\mathrm{pw}}, \qquad w_{\lambda^\star} > 0.$$

Let $a^\star \in \arg\max_{a \neq r} p_a$, and let $P^\star \in H_{0,r}$ be the forward I-projection of P from Lemma C.9. Define

$$\rho^\star := \max_{\lambda \in (0,1)} \{p_r \log(1+\lambda) + p_{(2)} \log(1-\lambda)\}.$$

Then

$$\rho^\star = D(P\|P^\star) = \frac{\delta^2}{4p_r}\{1 + o(1)\}.$$

For the oracle pairwise hitting time

$$\sigma^\star_{\mathrm{pw}} := \inf\left\{t \ge 1 : w_{\lambda^\star} E_t^{(r,a^\star)}(\lambda^\star) \ge \alpha_{\mathrm{pw}}^{-1}\right\},$$

we have

$$\mathbb{E}_P[\sigma^\star_{\mathrm{pw}}] \le \frac{\log\{1/(\alpha_{\mathrm{pw}} w_{\lambda^\star})\}}{\rho^\star}\{1 + o(1)\}.$$

Conversely, any level-ε sequential certifier τ' for $H_{0,r}$ with $\mathbb{P}_P(\tau' < \infty) = 1$ and $\mathbb{E}_P[\tau'] < \infty$ obeys

$$\mathbb{E}_P[\tau'] \geq \frac{\log(1/\varepsilon)}{\rho^\star}.$$

In particular, if $\alpha_{\mathrm{pw}} = c\varepsilon$ for a constant $c \in (0, 1]$ and $\log(1/w_{\lambda^\star}) = o\{\log(1/\varepsilon)\}$, the oracle pairwise upper bound and the information lower bound share the leading constant $4p_r/\delta^2$.

Proof. Proposition F.1 gives

$$\rho^\star = D(P\|P^\star) = \frac{\delta^2}{4p_r}\{1 + o(1)\}.$$

(a) Upper bound. Define

$$Y_i := \log\{1 + \lambda^\star Z_i^{(r,a^\star)}\}.$$

Then the Y_i are i.i.d., bounded, and $\mathbb{E}_P[Y_i] = \rho^\star > 0$. Let

$$B := \log\{1/(\alpha_{\mathrm{pw}} w_{\lambda^\star})\}.$$

At the hitting time $\sigma^\star_{\mathrm{pw}}$,

$$\sum_{i=1}^{\sigma^\star_{\mathrm{pw}}} Y_i \geq B.$$

Moreover, the crossing overshoot is at most the largest positive increment, so

$$\sum_{i=1}^{\sigma^\star_{\mathrm{pw}}} Y_i \leq B + \log(1 + \lambda^\star).$$

Since the increments are bounded and the drift is positive, Wald's identity gives

$$\rho^\star \mathbb{E}_P[\sigma^\star_{\mathrm{pw}}] = \mathbb{E}_P\left[\sum_{i=1}^{\sigma^\star_{\mathrm{pw}}} Y_i\right] \leq B + \log(1 + \lambda^\star).$$

As $\lambda^\star \to 0$ in the small-gap regime,

$$\mathbb{E}_P[\sigma^\star_{\mathrm{pw}}] \leq \frac{\log\{1/(\alpha_{\mathrm{pw}} w_{\lambda^\star})\} + o(1)}{\rho^\star}.$$

Substituting the expansion of $\rho^\star$ proves the oracle upper bound.

(b) Lower bound. Apply the sequential change-of-measure argument to the I-projection $P^\star \in H_{0,r}$. Type-I validity gives $P^\star(\tau' < \infty) \leq \varepsilon$, while $\mathbb{P}_P(\tau' < \infty) = 1$. By data processing and Lemma C.10,

$$\log(1/\varepsilon) \leq \mathbb{E}_P[\tau'] D(P\|P^\star) = \mathbb{E}_P[\tau']\rho^\star.$$

Hence

$$\mathbb{E}_P[\tau'] \geq \frac{\log(1/\varepsilon)}{\rho^\star}.$$

(c) Ratio. With $\alpha_{\mathrm{pw}} = c\varepsilon$,

$$\frac{\log(1/(\alpha_{\mathrm{pw}} w_{\lambda^\star}))}{\log(1/\varepsilon)} = 1 + \frac{\log(1/(c w_{\lambda^\star}))}{\log(1/\varepsilon)} \to 1$$

using $\alpha_{\mathrm{pw}} = c\varepsilon$ and $\log(1/w_{\lambda^\star}) = o\{\log(1/\varepsilon)\}$. □

Remark F.3 (Scope of the oracle result)**.** *Theorem F.2 is an oracle benchmark for comparing r with its true runner-up. It is not a sharp-constant theorem for the full CITE stopping time, which also uses the empirical runner-up, the LCB–unseen check, and a finite grid.*

F.2. Adaptive geometric grid

Condition 4.2 requires a grid point in $[\delta/8, \delta/4]$, which depends on the unknown gap δ. A geometric grid gives a uniform version of the same rate over all $\delta \geq \delta_0$.

Condition F.4 (Geometric grid)**.** *For a fixed minimum gap $\delta_0 \in (0, 1]$, set*

$$\Lambda_{\mathrm{pw}}^{\mathrm{geo}} := \{2^{-k} : 1 \leq k \leq K\}, \qquad K := \lceil \log_2(8/\delta_0) \rceil ,$$

with uniform weights $w_\lambda = 1/K$.

Theorem F.5 (Adaptive grid rate)**.** *Suppose Assumption 2.1 holds and CITE uses $\Lambda_{\mathrm{pw}}^{\mathrm{geo}}$ in the pairwise component. Keep the LCB-grid and selected-weight assumptions of Theorem 4.3. Then, uniformly over all P with modal gap $\delta \geq \delta_0$,*

$$\mathbb{E}[\tau] = O\left(\frac{\log(1/\varepsilon) + \log(\log(8/\delta_0) \vee e) + \log(1/\delta)}{\delta^2} + \frac{\log(1/\varepsilon) + \log(1/p_r)}{p_r} \right),$$

with constants independent of δ, p_r, and $|\mathcal{A}|$.

Proof. Fix P with $\delta \geq \delta_0$ and set

$$k^\star := \lfloor \log_2(8/\delta) \rfloor , \qquad \lambda := 2^{-k^\star}.$$

Then $\lambda \in [\delta/8, \delta/4]$ and, since $\delta \geq \delta_0$, we have $k^\star \leq K$. Hence $\lambda \in \Lambda_{\mathrm{pw}}^{\mathrm{geo}}$.

The proof of Theorem 4.3 applies with this grid point. The only change is the pairwise mixture weight:

$$\log \frac{1}{\alpha_{\mathrm{pw}} w_\lambda} = \log \frac{1}{\alpha_{\mathrm{pw}}} + \log K = \log \frac{1}{\alpha_{\mathrm{pw}}} + O\{\log(\log(8/\delta_0) \vee e)\}.$$

Substituting this into the pairwise part of the stopping-time bound, and keeping the LCB–unseen part unchanged, proves the display. □

Remark F.6. *The geometric grid removes the need to know δ exactly, but it still requires a lower design scale δ_0. Removing this design parameter would require a separate adaptive-betting analysis, which we do not pursue here.*

F.3. Growth-rate optimality of the oracle pairwise bet

We record a class-restricted growth-rate optimality property for the oracle pairwise bet, in the GRO/GROW sense of Grünwald et al. (2024). The comparison is restricted to predictable linear bets in the pairwise statistic.

Definition F.7 (Class-restricted GRO/GROW)**.** *Fix a pair (r, a) with $r \neq a$, a class $\mathcal{Q}$ of alternatives, and a class $\mathcal{E}$ of e-processes for the pairwise null $H_0^{(a)} = \{P : p_r \leq p_a\}$. An element $E^\star \in \mathcal{E}$ is* GRO/GROW within $\mathcal{E}$ with respect to $\mathcal{Q}$ *if, for every fixed horizon T,*

$$E^\star \in \arg\max_{E \in \mathcal{E}} \inf_{Q \in \mathcal{Q}} \mathbb{E}_Q[\log E_T].$$

Theorem F.8 (GRO bet and mixture near-optimality)**.** *Let $\mathcal{E}_Z$ denote the class of processes $E_t = \prod_{i=1}^t (1 + \lambda_i Z_i^{(r,a)})$ with predictable bets $\lambda_i \in [0, 1)$ measurable with respect to $\mathcal{F}_{i-1} := \sigma(X_1, \ldots, X_{i-1})$. Every $E \in \mathcal{E}_Z$ is an e-process for the composite null $H_0^{(a)} = \{P : p_r \leq p_a\}$ in the sense of Definition F.7.*

Fix a simple alternative Q with $\mathbb{P}_Q(X = r) = p$, $\mathbb{P}_Q(X = a) = q$, and $p > q > 0$ (the boundary case $q = 0$ is degenerate: $\lambda^\star = 1$ lies on the boundary of the open betting interval $[0, 1)$, so the supremum is approached only in the limit $\lambda \uparrow 1$ and is not attained in $\mathcal{E}_Z$), and define

$$\lambda^\star := \frac{p - q}{p + q}, \qquad \rho^\star := p \log(1 + \lambda^\star) + q \log(1 - \lambda^\star).$$

(a) ***GRO bet.*** *For every $T \geq 1$ and every $E \in \mathcal{E}_Z$, $\mathbb{E}_Q[\log E_T] \leq T\rho^\star$, with equality if and only if $\lambda_i = \lambda^\star$ holds Q-almost surely for every $i \leq T$. In particular, $E_t^{(r,a)}(\lambda^\star)$ is the unique growth-rate maximizer within $\mathcal{E}_Z$ under the simple alternative Q.*

(b) ***Mixture near-optimality.*** *For any finite grid $\Lambda_{\mathrm{pw}} \subset (0,1)$ with $\lambda^\star \in \Lambda_{\mathrm{pw}}$ and positive weights (w_λ) summing to at most 1, the mixture $E_t^{(r,a)} := \sum_{\lambda \in \Lambda_{\mathrm{pw}}} w_\lambda E_t^{(r,a)}(\lambda)$ satisfies, for every $T \geq 1$,*

$$T\rho^\star - \log \frac{1}{w_{\lambda^\star}} \leq \mathbb{E}_Q[\log E_T^{(r,a)}] \leq T\rho^\star.$$

The mixture therefore has the same asymptotic expected log-growth rate as the oracle fixed-$\lambda^\star$ bet, with the additive log-weight term $\log(1/w_{\lambda^\star})$.

Proof. Preliminary: e-process property. For any $P \in H_0^{(a)}$, X_i is independent of $\mathcal{F}_{i-1}$ with $\mathbb{E}_P[Z_i^{(r,a)}] = p_r - p_a \leq 0$, so

$$\mathbb{E}_P[1 + \lambda_i Z_i^{(r,a)} \mid \mathcal{F}_{i-1}] = 1 + \lambda_i(p_r - p_a) \leq 1$$

($\lambda_i \geq 0$). Thus (E_t) is a nonneg. supermartingale with $E_0 = 1$, and optional stopping gives $\mathbb{E}_P[E_\tau] \leq 1$ for any stopping time τ.

Part (a): GRO bet. Fix $E \in \mathcal{E}_Z$. For each i, λ_i is $\mathcal{F}_{i-1}$-measurable and X_i is independent of $\mathcal{F}_{i-1}$ under Q, taking values r, a, and "other" with probabilities p, q, and $1-p-q$ (in which last case $Z_i = 0$ and the log term vanishes). Conditioning on $\mathcal{F}_{i-1}$,

$$\mathbb{E}_Q\big[\log(1 + \lambda_i Z_i^{(r,a)}) \mid \mathcal{F}_{i-1}\big] = p\log(1+\lambda_i) + q\log(1-\lambda_i) =: \rho(\lambda_i).$$

Summing over $i = 1, \ldots, T$ via the tower property gives

$$\mathbb{E}_Q[\log E_T] = \sum_{i=1}^{T} \mathbb{E}_Q[\rho(\lambda_i)]. \tag{29}$$

By Proposition C.6, ρ is strictly concave on $(0,1)$ with unique maximiser $\lambda^\star$, so $\rho(\lambda_i) \leq \rho^\star$ pointwise. Substituting into (29) yields $\mathbb{E}_Q[\log E_T] \leq T\rho^\star$, with equality iff $\lambda_i = \lambda^\star$ Q-a.s. for each i.

Lower bound for part (b).

For every $\lambda \in \Lambda_{\mathrm{pw}}$, $E_t^{(r,a)}(\lambda) > 0$, hence $E_T^{(r,a)} \geq w_{\lambda^\star} E_T^{(r,a)}(\lambda^\star)$ Q-a.s. Taking logs and Q-expectations and applying (a) to the constant bet $\lambda_i \equiv \lambda^\star$:

$$\mathbb{E}_Q[\log E_T^{(r,a)}] \geq \log w_{\lambda^\star} + T\rho^\star = T\rho^\star - \log(1/w_{\lambda^\star}).$$

Upper bound for part (b), via variational duality. Let $P^\star$ be the boundary distribution

$$P^\star(X = r) = P^\star(X = a) := \tfrac{p+q}{2}, \qquad P^\star(X = b) := Q(X = b) \text{ for } b \notin \{r, a\}.$$

The Lagrangian argument of Lemma C.9, applied to $H_0^{(a)} = \{P : p_r \leq p_a\}$ in place of $H_{0,r}$, identifies $P^\star$ as the forward I-projection of Q onto $H_0^{(a)}$, with the closed form

$$D(Q\|P^\star) = p\log\frac{2p}{p+q} + q\log\frac{2q}{p+q} = p\log(1+\lambda^\star) + q\log(1-\lambda^\star) = \rho^\star, \tag{30}$$

the second equality from $1 \pm \lambda^\star = 2p/(p+q)$ and $2q/(p+q)$.

Writing $W := \sum_{\lambda \in \Lambda_{\mathrm{pw}}} w_\lambda \leq 1$, $E_T^{(r,a)}$ is an e-process for $H_0^{(a)}$ (mixture of e-processes), so $\mathbb{E}_{P^\star}[E_T^{(r,a)}] \leq W \leq 1$. Decomposing the Q-expectation via change of measure and applying Jensen to the concave log,

$$\begin{aligned}\mathbb{E}_Q[\log E_T^{(r,a)}] &= \mathbb{E}_Q\left[\log\left(E_T^{(r,a)} \cdot \frac{d(P^\star)^T}{dQ^T}\right)\right] + \mathbb{E}_Q\left[\log\frac{dQ^T}{d(P^\star)^T}\right] \\ &\leq \log \mathbb{E}_{P^\star}[E_T^{(r,a)}] + D(Q^T\|(P^\star)^T) \leq 0 + TD(Q\|P^\star) = T\rho^\star,\end{aligned}$$

where the last line uses $\mathbb{E}_{P^\star}[E_T^{(r,a)}] \leq 1$, tensorisation of KL, and (30). Combining with the lower bound finishes the proof. □

F.4. Top-k mode certification

Fix a target set $S \subseteq \mathcal{A}$ of size $k \geq 1$ with $\mathcal{A} \setminus S \neq \emptyset$ (so that the outsider competitor set is meaningful). We extend CITE to certify that S contains the top-k categories in the sense

$$H_{1,S}^{(k)} := \{P : \min_{s \in S} p_s > \sup_{a \notin S} p_a\}, \qquad H_{0,S}^{(k)} := \neg H_{1,S}^{(k)}.$$

Top-k CITE. Let

$$\delta_S := \min_{s \in S} p_s - \sup_{a \notin S} p_a, \qquad p_S := \min_{s \in S} p_s.$$

For each $s \in S$ and $a \notin S$, define $E_t^{(s,a)}$ as in Definition 3.1, and let $L_t(s)$ be the LCB for p_s.

At time t, if $\mathcal{A}_t \setminus S \neq \emptyset$, define

$$\widehat{a}_t^{\text{out}} \in \arg\max_{a \in \mathcal{A}_t \setminus S} N_t(a)$$

with deterministic tie-breaking. Define

$$\mathsf{PW}_t^{(k)} := \begin{cases} \text{true}, & \mathcal{A}_t \setminus S = \emptyset, \\ \left\{ \min_{s \in S} \widetilde{E}_t^{(s, \widehat{a}_t^{\text{out}})} \geq \alpha_{\text{pw}}^{-1} \right\}, & \mathcal{A}_t \setminus S \neq \emptyset. \end{cases}$$

The top-k CITE stopping time is

$$\tau^{(k)} := \inf \left\{ t \geq 1 : \mathsf{PW}_t^{(k)} \text{ and } \min_{s \in S} L_t(s) > U_t \right\}. \tag{31}$$

Theorem F.9 (Top-k Type-I and power)**.** *Run the top-k CITE* (31) *under the same component-budget and selected-grid-weight bounded-below assumptions as Theorem 4.3, with the same per-component budget as the single-target CITE,* $\alpha_{\text{pw}} + \alpha_r + \alpha_u \leq \varepsilon$.

(a) ***Type-I:*** *Under Assumption 2.1,* $\sup_{P \in H_{0,S}^{(k)}} \mathbb{P}_P(\tau^{(k)} < \infty) \leq \varepsilon$.

(b) ***Power:*** *Suppose $\delta_S > 0$ and the grids satisfy the analogue of Condition 4.2 with δ_S in place of δ. Define the threshold*

$$T_{\text{cl}}^{(k)} := C_0 \left(\frac{\log(k/\varepsilon) + \log(1/\delta_S)}{\delta_S^2} + \frac{\log(k/\varepsilon) + \log(1/p_S)}{p_S} \right)$$

for a universal constant C_0. For fixed k*, as $\varepsilon \to 0$,*

$$\mathbb{E}[\tau^{(k)}] \leq T_{\text{cl}}^{(k)} \big(1 + o(1)\big) = O(T_{\text{cl}}^{(k)}).$$

The upper bound extends to growing k provided $k = O(T_{\text{cl}}^{(k)})$, e.g. when $\delta_S^{-2} \log(k/\varepsilon)$ dominates k.

Proof. (a) Type-I. Fix any $P \in H_{0,S}^{(k)}$. Choose $s^\star \in S$ such that

$$p_{s^\star} = \min_{s \in S} p_s.$$

Since $P \in H_{0,S}^{(k)}$,

$$p_{s^\star} \leq \sup_{a \notin S} p_a.$$

If this supremum is positive, Lemma B.1 gives $a^\star \notin S$ with $p_{a^\star} \geq p_{s^\star}$. If $p_{s^\star} = 0$, any outsider $a^\star \notin S$ can be used.

Decompose

$$\{\tau^{(k)} < \infty\} \subseteq \underbrace{\{\tau^{(k)} < \infty, a^\star \in \mathcal{A}_{\tau^{(k)}}\}}_{E_1} \cup \underbrace{\{\tau^{(k)} < \infty, a^\star \notin \mathcal{A}_{\tau^{(k)}}\}}_{E_2}.$$

On E_1, the procedure's pairwise condition holds at $\tau^{(k)}$ for *every* $s \in S$, including the witness $s^\star$: $E_{\tau^{(k)}}^{(s^\star, \widehat{a}^{\mathrm{out}}_{\tau^{(k)}})} \geq 1/\alpha_{\mathrm{pw}}$. Runner-up monotonicity (Lemma B.3) restricted to outsiders (since $a^\star \in \mathcal{A}_{\tau^{(k)}} \setminus S$ implies $N_{\tau^{(k)}}(a^\star) \leq N_{\tau^{(k)}}(\widehat{a}^{\mathrm{out}}_{\tau^{(k)}})$) gives $E_{\tau^{(k)}}^{(s^\star, a^\star)} \geq E_{\tau^{(k)}}^{(s^\star, \widehat{a}^{\mathrm{out}}_{\tau^{(k)}})} \geq 1/\alpha_{\mathrm{pw}}$. Since $s^\star, a^\star$ are deterministic and $(E_t^{(s^\star, a^\star)})_{t \geq 0}$ is an NSM with initial value ≤ 1 under $p_{s^\star} \leq p_{a^\star}$ (Corollary 3.2), Ville's inequality (12) yields $\mathbb{P}(E_1) \leq \alpha_{\mathrm{pw}}$.

On E_2, condition (b) at $\tau^{(k)}$ holds for the witness $s^\star$: $L_{\tau^{(k)}}(s^\star) > U_{\tau^{(k)}}$. Define the two failure events

$$F_r := \{\exists t : L_t(s^\star) > p_{s^\star}\}, \qquad F_u := \{\exists a, t : N_t(a) = 0,\ p_a > U_t\}.$$

Proposition 3.4 (applied to the single index $s^\star$) gives $\mathbb{P}(F_r) \leq \alpha_r$, and Proposition 3.6 gives $\mathbb{P}(F_u) \leq \alpha_u$. On $E_2 \setminus (F_r \cup F_u)$, $L_{\tau^{(k)}}(s^\star) \leq p_{s^\star}$ and $p_{a^\star} \leq U_{\tau^{(k)}}$ (since $a^\star \notin \mathcal{A}_{\tau^{(k)}}$), yielding the contradiction $p_{a^\star} \leq U_{\tau^{(k)}} < L_{\tau^{(k)}}(s^\star) \leq p_{s^\star}$, contrary to $p_{a^\star} \geq p_{s^\star}$. Hence $\mathbb{P}(E_2) \leq \alpha_r + \alpha_u$.

A union bound gives $\mathbb{P}(\tau^{(k)} < \infty) \leq \alpha_{\mathrm{pw}} + \alpha_r + \alpha_u \leq \varepsilon$.

(b) Power. The single-target CITE of Definition 3.7 applied to a target $s \in S$ is *not* appropriate here, because its pairwise check ranges over all of $\mathcal{A} \setminus \{s\}$ and may include competitors $s' \in S$ with $p_{s'} \geq p_s$, on which the pairwise null is true; the resulting stopping time would be $+\infty$ a.s. Instead, fix $s \in S$ and consider the *outsider-restricted single-target CITE for target* s, defined exactly as Definition 3.7 except that the empirical runner-up $\widehat{a}_t$ is replaced by $\widehat{a}_t^{\mathrm{out}} := \arg\max_{a \in \mathcal{A}_t \setminus S} N_t(a)$ and the pairwise threshold is k/α_{pw}, the LCB threshold is k/α_r, the unseen budget α_u is unchanged. We adopt the same convention as for the actual top-k procedure: if $\mathcal{A}_t \setminus S = \emptyset$, the per-$s$ pairwise condition is treated as vacuously satisfied at round t. The tightened thresholds k/α_{pw} and k/α_r are *strictly stronger sufficient* requirements than the actual top-k stopping criteria (which use the original thresholds $1/\alpha_{\mathrm{pw}}$ and the LCB inequality $L_t(s) > U_t$); they are introduced solely so that a Bonferroni union bound over $s \in S$ on the per-target failure events yields a uniform ε-level conclusion. Write τ_s^{out} for its stopping time.

We use a fixed-time tail bound rather than a max-of-first-hitting-times argument (the latter fails because pairwise e-processes and LCBs are not monotone in t). For each fixed $t \geq 1$,

$$\{\tau^{(k)} > t\} \subseteq \bigcup_{s \in S} \left(\left\{ \mathcal{A}_t \setminus S \neq \emptyset,\ E_t^{(s, \widehat{a}_t^{\mathrm{out}})} < \alpha_{\mathrm{pw}}^{-1} \right\} \cup \{L_t(s) \leq U_t\} \right). \tag{32}$$

Apply the proof of Theorem 4.3 to the *outsider-restricted single-target CITE for target* s, with $\mathcal{A} \setminus \{r\}$ replaced by $\mathcal{A} \setminus S$ in the pairwise/LCB analysis (the unseen bound U_t is unchanged since it is already over all of $\mathcal{A}$): the runner-up monotonicity (Lemma B.3) holds within any subset, the LCB construction is local to target s, and the effective modal gap $p_s - \max_{a \notin S} p_a \geq \delta_S$ and target mass $p_s \geq p_S$. With per-s thresholds tightened to k/α_{pw} for the pairwise check and k/α_r for the LCB (a Bonferroni split over $|S| = k$), the same McDiarmid + Bernstein arguments give, for each fixed s and each $t \geq T_{\mathrm{cl}}^{(k)}$,

$$\mathbb{P}\big(E_t^{(s, \widehat{a}_t^{\mathrm{out}})} < 1/\alpha_{\mathrm{pw}} \mathrm{or} L_t(s) \leq U_t\big) \leq 2(et)^{-2},$$

where $T_{\mathrm{cl}}^{(k)} = C_0\big((\log(k/\varepsilon) + \log(1/\delta_S))/\delta_S^2 + (\log(k/\varepsilon) + \log(1/p_S))/p_S\big)$ for a universal constant C_0. Substituting into (32) and applying a union bound over $s \in S$,

$$\mathbb{P}(\tau^{(k)} > t) \leq 2k(et)^{-2} \qquad \text{for all } t \geq T_{\mathrm{cl}}^{(k)}.$$

The tail-sum identity ($\sum_{t \geq T} 1/t^2 \leq 2/T$ for $T \geq 2$) yields

$$\mathbb{E}[\tau^{(k)}] \leq T_{\mathrm{cl}}^{(k)} + \sum_{t > T_{\mathrm{cl}}^{(k)}} 2k(et)^{-2} \leq T_{\mathrm{cl}}^{(k)}\big(1 + o(1)\big) \quad (\varepsilon \to 0,\ k \text{ fixed}),$$

which gives the claimed bound. □

Remark F.10. *The top-k extension preserves the runner-up reduction within the "outsider" set, so the per-round cost is k pairwise checks and k LCB updates, plus one shared unseen bound. The case $k = 1$ recovers CITE of Definition 3.7, with Type-I and rate guarantees given by Theorems 4.1 and 4.3 respectively.*

G. Simulation Details

G.1. Distribution Settings

Table 2 summarizes the five distribution settings used in Section 6. In each setting, the tail probabilities follow a shifted-Zipf distribution capped so that the stated modal gap $\delta = p_{(1)} - p_{(2)}$ is preserved exactly.

Table 2. Distribution settings for the simulation study.

Setting	Name	K	p_r	δ	Tail
1	LLM-like Zipf (diffuse)	5000	0.24	0.215	$\text{Zipf}_{1.1}$ on 4987 labels
2	Concentrated (strong mode)	100	0.60	0.450	Uniform on 94 labels
3	Near-tie (small gap)	500	0.12	0.010	$\text{Zipf}_{1.3}$ on 497 labels
4	Very diffuse (huge category set)	10000	0.06	0.010	$\text{Zipf}_{1.0}$ on 9998 labels
5	Moderate LLM-like	1000	0.35	0.150	$\text{Zipf}_{1.2}$ on 997 labels

G.2. Results for All Reported Settings

Type-I error control. Across all five settings and all budgets $N \in \{64, 128, 256, 512, 1024, 2048\}$, CITE, W-CITE, and Bonferroni have empirical false-certification rate exactly zero in every Case B experiment. KR also has zero false certifications in Settings 1, 2, and 5, with only small nonzero rates on Settings 3–4. MMC shows nonzero false certifications in the same two regimes, illustrating that leader-tracking certificates are not designed for the fixed-target null considered here. Table 3 reports the full per-budget Case B false-certification rates.

Table 3. Case B false-certification rates by budget. Entries are empirical false-certification rates at level $\varepsilon = 0.05$ under the fixed-target null.

Setting	N	Bonferroni	CITE	W-CITE	KR	MMC
1: LLM-like Zipf	64	.000	.000	.000	.000	.000
	128	.000	.000	.000	.000	.000
	256	.000	.000	.000	.000	.000
	512	.000	.000	.000	.000	.000
	1024	.000	.000	.000	.000	.000
	2048	.000	.000	.000	.000	.000
2: Concentrated	64	.000	.000	.000	.000	.000
	128	.000	.000	.000	.000	.000
	256	.000	.000	.000	.000	.000
	512	.000	.000	.000	.000	.000
	1024	.000	.000	.000	.000	.000
	2048	.000	.000	.000	.000	.000
3: Near-tie	64	.000	.000	.000	.002	.000
	128	.000	.000	.000	.000	.000
	256	.000	.000	.000	.000	.002
	512	.000	.000	.000	.002	.004
	1024	.000	.000	.000	.002	.006
	2048	.000	.000	.000	.002	.006
4: Very diffuse	64	.000	.000	.000	.000	.000
	128	.000	.000	.000	.000	.012
	256	.000	.000	.000	.000	.012
	512	.000	.000	.000	.000	.012
	1024	.000	.000	.000	.002	.014
	2048	.000	.000	.000	.000	.014
5: Moderate LLM-like	64	.000	.000	.000	.000	.000
	128	.000	.000	.000	.000	.000
	256	.000	.000	.000	.000	.000
	512	.000	.000	.000	.000	.000
	1024	.000	.000	.000	.000	.000
	2048	.000	.000	.000	.000	.000

Power (Case A). Tables 4–8 report certification rates for all five settings. Setting 2 saturates quickly for all methods, whereas Settings 3 and 4 are the most difficult regimes. Setting 5 highlights a diffuse-tail regime in which CITE substantially outperforms MMC at moderate budgets. Bonferroni can be slightly more powerful in the easiest concentrated regime at the smallest budget, but CITE matches or improves on it in the remaining reported regimes and budgets.

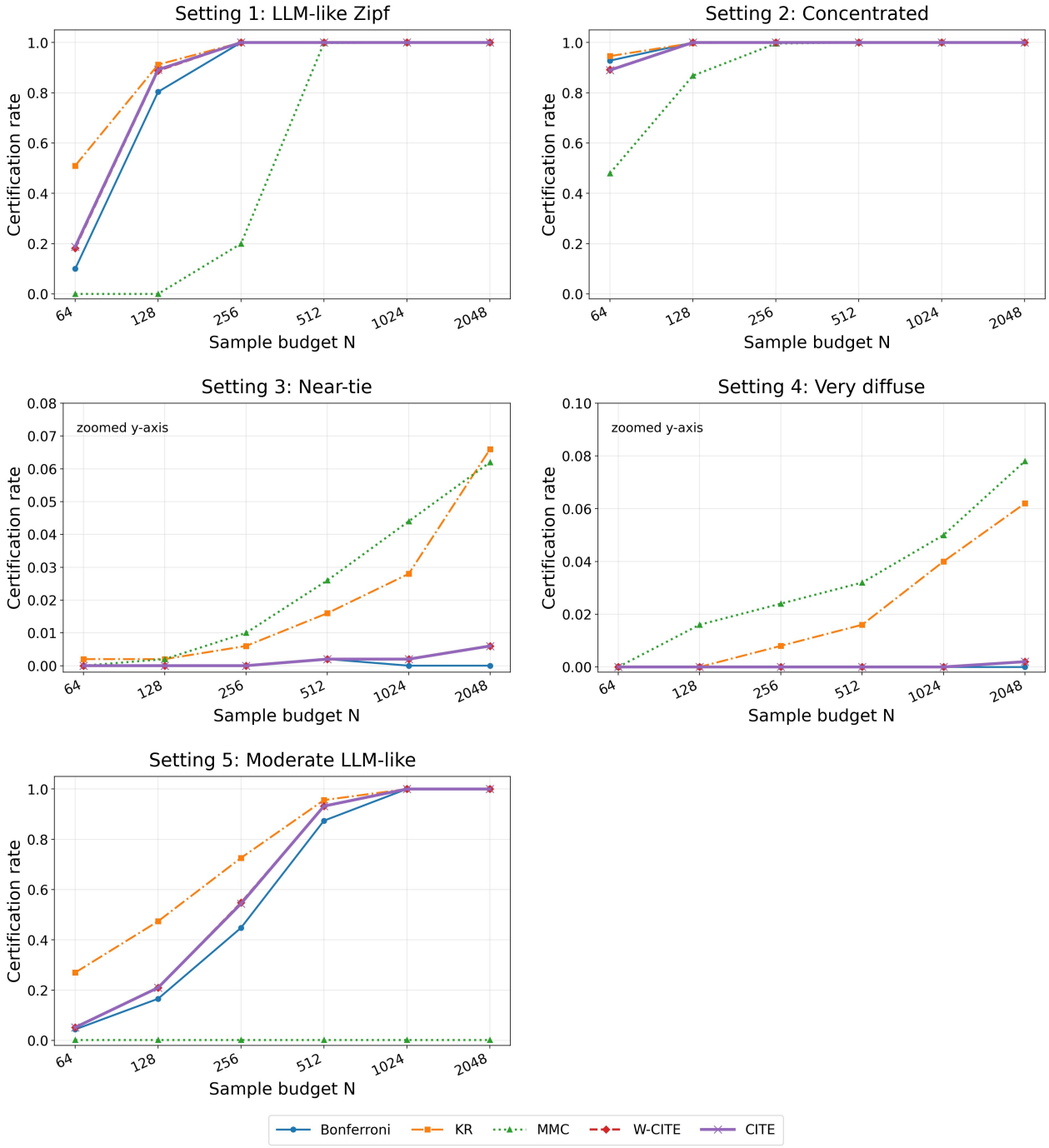

Figure 3. Certification rate versus sample budget N for CITE, W-CITE, fixed-sample Bonferroni, the sample-split fixed-target test KR (Kim and Ramdas, 2025), and the leader-tracking MMC (Cordero-Encinar and Duncan, 2025), across all five reported simulation settings. The panels for Settings 3 and 4 use zoomed vertical axes because all methods remain low-power over the evaluated budget range. CITE matches or exceeds Bonferroni in most regimes, with the easy concentrated Setting 2 saturating quickly for both methods. Settings 3–4 are also the regimes where KR and MMC show nonzero false certifications under Case B (Table 3).

Table 4. Setting 1 (LLM-like Zipf): certification rate, Case A.

N	Bonferroni	CITE	W-CITE	KR	MMC	$\bar{\tau}_{\text{CITE}}$
64	.100	.188	.182	.510	.000	53
128	.804	.892	.888	.912	.000	84
256	1.00	1.00	1.00	1.00	.200	90
512	1.00	1.00	1.00	1.00	.998	90
1024	1.00	1.00	1.00	1.00	1.00	90
2048	1.00	1.00	1.00	1.00	1.00	90

Table 5. Setting 2 (Concentrated, strong mode): certification rate, Case A.

N	Bonferroni	CITE	W-CITE	KR	MMC	$\bar{\tau}_{\text{CITE}}$
64	.928	.890	.892	.946	.480	36
128	1.00	1.00	1.00	.998	.868	41
256	1.00	1.00	1.00	1.00	.996	41
512	1.00	1.00	1.00	1.00	1.00	41
1024	1.00	1.00	1.00	1.00	1.00	41
2048	1.00	1.00	1.00	1.00	1.00	41

Table 6. Setting 3 (Near-tie, $\delta = 0.01$): certification rate, Case A.

N	Bonferroni	CITE	W-CITE	KR	MMC
64	.000	.000	.000	.002	.000
128	.000	.000	.000	.002	.002
256	.000	.000	.000	.006	.010
512	.002	.002	.002	.016	.026
1024	.000	.002	.002	.028	.044
2048	.000	.006	.006	.066	.062

Table 7. Setting 4 (Very diffuse, huge category set): certification rate, Case A.

N	Bonferroni	CITE	W-CITE	KR	MMC	$\bar{\tau}_{\text{CITE}}$
64	.000	.000	.000	.000	.000	—
128	.000	.000	.000	.000	.016	—
256	.000	.000	.000	.008	.024	—
512	.000	.000	.000	.016	.032	—
1024	.000	.000	.000	.040	.050	—
2048	.000	.002	.002	.062	.078	1149

Table 8. Setting 5 (Moderate LLM-like): certification rate, Case A.

N	Bonferroni	CITE	W-CITE	KR	MMC	$\bar{\tau}_{\text{CITE}}$
64	.044	.052	.052	.270	.002	49
128	.166	.210	.210	.474	.002	84
256	.448	.544	.548	.726	.002	151
512	.874	.932	.932	.956	.002	234
1024	1.00	1.00	1.00	1.00	.002	259
2048	1.00	1.00	1.00	1.00	.002	259

G.3. Bottleneck Analysis

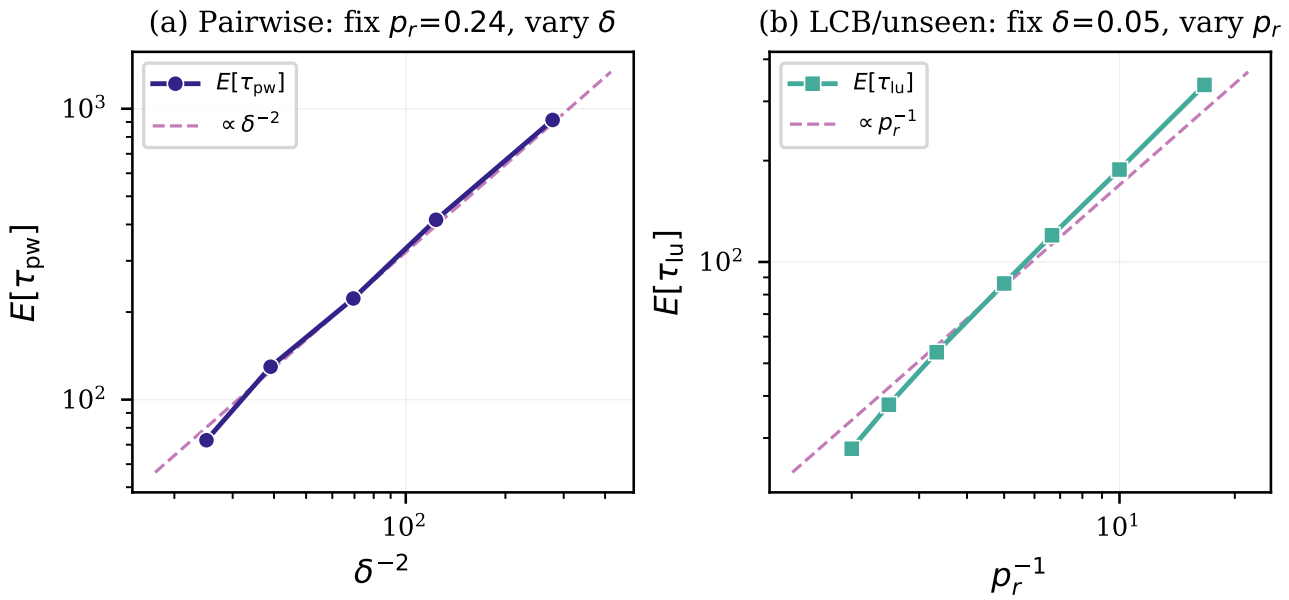

Figure 4. Component-level diagnostics for the two terms in the stopping-time upper bound of Theorem 4.3, on log-log axes. Dashed reference lines show the theoretical slopes. (a) Fixing $p_r = 0.24$ and varying δ, the pairwise component $\mathbb{E}[\tau_{\text{pw}}]$ (first time the pairwise e-value crosses $1/\alpha_{\text{pw}}$) is consistent with the δ^{-2} term. (b) Fixing $\delta = 0.05$ and varying p_r, the LCB/unseen component $\mathbb{E}[\tau_{\text{lu}}]$ (first time $L_t(r) > U_t$) is consistent with the p_r^{-1} term.

G.4. Logistic Weight Model Saturation

As discussed in Section 6, the logistic weight model can saturate when K is large. For Settings 1 and 5, sweeping $\beta \in \{0.5, 1, 2, 4, 8, 16\}$ produces certification rates and stopping times that are nearly unchanged relative to unweighted CITE. The effective gap ratio Δ_w/μ_r is essentially the same as δ/p_r for $\beta \geq 2$. This occurs because the top categories are already assigned weights close to one, so the logistic transformation does not substantially separate the mode from its runner-up.

G.5. Rank-Based Weight Model

The rank-based model assigns $E[W \mid X = a] = 0.95 \cdot e^{-\gamma \cdot \text{rank}(a)}$ for the top $K_0 = 10$ categories and $w_{\text{low}} = 0.1$ otherwise, with additive $\text{Uniform}(-0.05, 0.05)$ noise clipped to $[0.01, 1]$.

Table 9 reports detailed results for Setting 5 at $N = 512$. We omit a separate figure here to avoid duplicating Figure 3; the table directly shows how the effective weighted gap and stopping time change with the rank-decay parameter.

Table 9. Setting 5: rank-based W-CITE at $N = 512$ in the supplementary weight sweep.

γ	W-CITE rate	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{CITE}}$	Δ_w/μ_r	δ/p_r
0.0	.920	237	235	0.429	0.429
0.1	.990	185	235	0.483	0.429
0.2	1.00	142	235	0.532	0.429
0.5	1.00	71	235	0.653	0.429
1.0	1.00	48	235	0.790	0.429
2.0	1.00	45	235	0.923	0.429
3.0	1.00	45	235	0.972	0.429

In this synthetic rank-based model, performance improves as the effective gap ratio Δ_w/μ_r increases. A modest $\gamma = 0.2$ already reaches full certification in this sweep and reduces $\bar{\tau}$ by about 40%; at $\gamma = 1.0$, the reduction is about 80%. For

Table 10. Certification rate on LLM self-consistency outputs (Case A, target = mode), averaged across problems. $\bar{K}_N$: Monte Carlo mean number of observed answer categories at budget N. $\bar{\tau}$ columns report mean stopping time among certified replicates.

Model	Dataset	N	Bonferroni	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
Qwen3-30B	AIME 2026	64	0.499±0.002	0.724±0.001	0.727±0.001	0.778±0.002	0.671±0.001	7.1±0.0	21.8±0.3	22.8±0.2	15.9±0.3
		128	0.597±0.002	0.781±0.001	0.785±0.001	0.827±0.002	0.705±0.001	10.2±0.0	32.4±0.5	33.0±0.5	22.5±0.5
		256	0.683±0.002	0.833±0.002	0.838±0.002	0.880±0.002	0.744±0.001	14.5±0.0	49.1±0.6	55.5±0.8	32.1±0.6
		512	0.770±0.002	0.902±0.001	0.906±0.001	0.929±0.001	0.779±0.001	19.9±0.0	81.8±1.3	82.0±1.1	45.6±1.1
		1024	0.825±0.001	0.944±0.001	0.946±0.001	0.959±0.001	0.798±0.000	25.7±0.0	113.7±1.8	113.2±1.4	57.8±2.2
	FrontierScience-Olympiad	64	0.284±0.002	0.288±0.002	0.289±0.002	0.328±0.003	0.121±0.002	28.0±0.0	37.9±1.9	40.9±1.6	22.9±0.9
		128	0.312±0.002	0.315±0.002	0.315±0.002	0.373±0.003	0.146±0.003	46.0±0.0	65.5±2.5	64.3±2.9	35.9±1.1
		256	0.350±0.003	0.356±0.003	0.359±0.003	0.443±0.004	0.198±0.003	73.7±0.1	110.0±2.7	112.0±2.5	60.6±1.5
		512	0.429±0.003	0.449±0.004	0.453±0.004	0.536±0.004	0.245±0.002	112.9±0.1	195.5±3.3	193.5±3.2	97.0±2.2
		1024	0.530±0.003	0.563±0.004	0.567±0.004	0.633±0.004	0.281±0.002	160.0±0.1	436.1±15.0	439.2±12.3	138.5±2.9
	ER-REASON	64	0.102±0.000	0.983±0.000	0.983±0.000	0.990±0.000	0.988±0.000	1.1±0.0	10.6±0.0	11.5±0.0	8.2±0.0
		128	0.110±0.000	0.996±0.000	0.996±0.000	0.998±0.000	0.996±0.000	1.1±0.0	11.3±0.0	12.2±0.0	8.8±0.0
		256	0.111±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.1±0.0	11.8±0.0	12.6±0.0	9.3±0.0
		512	0.111±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.1±0.0	11.8±0.0	12.7±0.0	9.3±0.1
		1024	0.111±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.1±0.0	11.8±0.0	12.7±0.0	9.3±0.1
gpt-oss-20b	AIME 2026	64	0.563±0.002	0.596±0.002	0.599±0.002	0.666±0.002	0.459±0.001	12.6±0.0	29.3±0.2	29.6±0.2	21.0±0.6
		128	0.659±0.002	0.693±0.001	0.701±0.002	0.742±0.002	0.486±0.001	18.3±0.0	43.8±1.6	43.5±1.6	27.9±0.8
		256	0.740±0.002	0.765±0.001	0.778±0.001	0.795±0.001	0.513±0.001	25.7±0.0	62.8±2.9	60.7±3.2	40.3±1.5
		512	0.799±0.001	0.811±0.001	0.812±0.001	0.830±0.001	0.538±0.001	34.8±0.0	84.2±2.9	80.7±3.1	58.0±2.1
		1024	0.825±0.001	0.829±0.001	0.829±0.001	0.848±0.001	0.554±0.001	44.0±0.0	176.7±6.0	126.6±3.7	91.0±4.5
	FrontierScience-Olympiad	64	0.193±0.001	0.187±0.002	0.147±0.002	0.209±0.002	0.192±0.001	37.8±0.0	32.3±2.2	28.7±0.5	22.1±0.4
		128	0.205±0.001	0.213±0.002	0.188±0.001	0.255±0.003	0.199±0.000	65.1±0.1	67.6±2.4	40.4±0.7	24.0±0.5
		256	0.237±0.002	0.269±0.003	0.201±0.001	0.330±0.004	0.200±0.000	108.5±0.1	144.6±2.9	108.9±1.8	24.3±0.5
		512	0.340±0.003	0.382±0.003	0.242±0.002	0.446±0.004	0.200±0.000	172.0±0.1	258.7±4.5	319.0±2.3	24.3±0.5
		1024	0.429±0.002	0.475±0.003	0.428±0.004	0.559±0.004	0.200±0.000	250.3±0.1	494.7±13.2	499.9±3.7	24.3±0.5
	ER-REASON	64	0.623±0.001	0.640±0.001	0.644±0.001	0.722±0.001	0.654±0.001	3.2±0.0	26.6±0.2	28.4±0.2	22.1±0.2
		128	0.726±0.001	0.730±0.001	0.739±0.001	0.795±0.001	0.727±0.001	3.6±0.0	41.4±0.5	42.1±0.5	33.1±0.6
		256	0.813±0.001	0.816±0.001	0.821±0.001	0.857±0.001	0.780±0.001	3.8±0.0	63.0±0.7	60.7±0.6	47.5±0.7
		512	0.873±0.001	0.870±0.001	0.875±0.001	0.903±0.001	0.823±0.001	4.0±0.0	89.7±1.0	91.5±1.6	71.4±1.4
		1024	0.921±0.001	0.915±0.001	0.919±0.001	0.939±0.001	0.861±0.001	4.2±0.0	128.3±2.3	127.5±2.2	110.8±2.1

Setting 1, the rank-based model with $\gamma \geq 0.1$ reaches 99.6% certification at $N = 128$ (versus 88.6% unweighted) with $\bar{\tau} \approx 72$ (versus 92).

H. LLM Self-Consistency Details

Table 10 reports the certification rate averaged across problems for each (model, dataset, budget) under Case A (target = mode); Table 11 reports the corresponding Case B (target = runner-up) certification rates as a sanity check on Type-I error. For each (model, problem) pair we draw 1,000 i.i.d. samples, then bootstrap 500 replicates of each budget $N \in \{64, 128, 256, 512, 1024\}$ from the empirical answer pool to compute certification rates.

H.1. Model settings

Sampling parameters. Qwen3-30B-A3B-Instruct-2507: `temperature=0.7`, `top_p=0.8`, `top_k=20`. gpt-oss-20b: `temperature=0.4`, `top_p=0.9`, `top_k=20`, `reasoning_effort=low`.

Response probability weight for W-CITE. For each sample i, let $(y_1, \ldots, y_{T_i})$ denote the tokens of the extracted answer span (the final `\boxed{...}` contents for AIME and FrontierScience, the `Triage:` ⟨`acuity`⟩ | `Disposition:` ⟨`disposition`⟩ string for ER-REASON). The W-CITE weight is the response probability used by the weighted variant of self-consistency (Wang et al., 2023),

$$w_i = \prod_{t=1}^{T_i} p_\theta \left(y_t \mid y_{<t}, \text{prompt} \right),$$

i.e., the model's own likelihood of the extracted answer under token-by-token factorization. Weights are not renormalized across samples.

H.2. Dataset protocols

AIME 2026. All 30 problems of AIME 2026 (Dekoninck et al., 2026) (AIME I and AIME II combined), each with an integer gold answer in $[0, 999]$. Prompt is the problem statement with a per-model suffix: "Solve in under 100 tokens. Put

Table 11. Certification rate on LLM self-consistency outputs (Case B, target = runner-up), averaged across problems. $\bar{K}_N$: Monte Carlo mean number of observed answer categories at budget N. $\bar{\tau}$ columns report mean stopping time among certified replicates.

Model	Dataset	N	Bonferroni	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
Qwen3-30B	AIME 2026	64	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.000±0.000	7.1±0.0	—	—	17.2±0.7
		128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	10.2±0.0	—	—	17.2±0.7
		256	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	14.5±0.0	—	—	17.2±0.7
		512	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	19.9±0.0	—	—	17.2±0.7
		1024	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	25.7±0.0	—	—	17.2±0.7
	FrontierScience-Olympiad	64	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.000±0.000	28.0±0.0	—	—	26.0±0.0
		128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	46.0±0.0	—	—	26.0±0.0
		256	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	73.7±0.1	—	—	26.0±0.0
		512	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	112.9±0.1	—	—	26.0±0.0
		1024	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.000±0.000	160.0±0.1	—	—	26.0±0.0
	ER-REASON	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	1.1±0.0	—	—	7.0±0.0
		128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	1.1±0.0	—	—	7.0±0.0
		256	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	1.1±0.0	—	—	7.0±0.0
		512	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	1.1±0.0	—	—	7.0±0.0
		1024	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	1.1±0.0	—	—	7.0±0.0
gpt-oss-20b	AIME 2026	64	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.000	0.000±0.000	12.6±0.0	9.0±0.0	9.0±0.0	26.0±7.8
		128	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.000	0.000±0.000	18.3±0.0	40.0±31.0	40.0±31.0	26.0±7.8
		256	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.000	0.000±0.000	25.7±0.0	40.0±31.0	40.0±31.0	26.0±7.8
		512	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.000±0.000	34.8±0.0	40.0±31.0	40.0±31.0	26.0±7.8
		1024	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.000±0.000	44.0±0.0	354.5±15.5	485.5±13.4	26.0±7.8
	FrontierScience-Olympiad	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	37.8±0.0	—	—	—
		128	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.000±0.000	65.1±0.1	—	—	—
		256	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	108.5±0.1	—	—	—
		512	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.000±0.000	172.0±0.1	—	—	—
		1024	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	250.3±0.1	—	—	—
	ER-REASON	64	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.000	0.001±0.000	3.2±0.0	24.6±5.3	30.0±9.3	16.0±1.6
		128	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.000	0.001±0.000	3.6±0.0	28.1±6.3	36.8±11.5	16.0±1.6
		256	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.001±0.000	3.8±0.0	28.1±6.3	36.8±11.5	21.1±3.2
		512	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.001±0.000	4.0±0.0	28.1±6.3	36.8±11.5	21.1±3.2
		1024	0.000±0.000	0.000±0.000	0.000±0.000	0.001±0.000	0.001±0.000	4.2±0.0	123.4±44.4	36.8±11.5	27.1±6.6

your final answer within `\boxed{}`." (Qwen3-30B-A3B-Instruct-2507) or "Do not show your reasoning. Put only the final answer within `\boxed{}`." (gpt-oss-20b). Answers are extracted from the last `\boxed{...}` and matched by exact integer equality.

FrontierScience-Olympiad. FrontierScience-Olympiad (Wang et al., 2026) contains 100 olympiad-level physics, chemistry, and biology problems with numeric or symbolic answers. We use a 10-problem subset obtained by taking every 10th problem (problems 10, 20, …, 100). Prompt suffix is the same as for AIME 2026. Each extracted answer is compared against the gold by an LLM self-judge (the generating model decides whether its own output is equivalent to the gold).

ER-REASON. ER-REASON (Mehandru et al., 2025) contains 3,984 emergency-room encounters. We use the 72-encounter subset with expert-authored clinical rationales, which the paper designates as the primary evaluation set. Both models use a two-turn chat prompt in place of the `\boxed{}` suffix. Turn 1 asks the model to assign an ESI acuity level (1–5) from the chief complaint, demographics, and triage vital signs. Turn 2, which echoes the Turn 1 acuity, asks for the disposition (one of 12 labels) given the full ED provider note and all historical notes. The final answer is parsed as the fixed string "Triage: ⟨acuity⟩ | Disposition: ⟨disposition⟩".

H.3. Per-problem certification rates: Qwen3-30B-A3B-Instruct-2507

AIME 2026.

Table 12. Per-problem certification rate on AIME 2026 (30 problems) (Case A, target = mode). **Bold**: CITE > Bonferroni. $\bar{K}_N$ is the Monte Carlo mean number of observed answer categories at budget N.

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P6 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
gold=441, mode=441 ✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P20 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
gold=190, mode=190 ✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P30 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.0±0.0	7.0±0.0
gold=393, mode=243 ✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.0±0.0	7.0±0.0

Table 12 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.0±0.0	7.0±0.0
P1 (δ=0.998)	64	0.066±0.011	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.1±0.0	9.1±0.0	10.0±0.0	7.0±0.0
gold=277, mode=277 ✓	128	0.130±0.015	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.1±0.0	9.1±0.0	10.0±0.0	7.0±0.0
	256	0.224±0.019	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.2±0.0	9.1±0.0	10.0±0.0	7.0±0.0
	512	0.406±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.4±0.0	9.1±0.0	10.0±0.0	7.0±0.0
	1024	0.638±0.021	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.6±0.0	9.1±0.0	10.0±0.0	7.0±0.0
P7 (δ=0.995)	64	0.186±0.017	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.2±0.0	9.1±0.0	10.1±0.0	7.1±0.0
gold=396, mode=396 ✓	128	0.326±0.021	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.4±0.0	9.1±0.0	10.1±0.0	7.1±0.0
	256	0.530±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.6±0.0	9.1±0.0	10.1±0.0	7.1±0.0
	512	0.794±0.018	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.1±0.0	10.1±0.0	7.1±0.0
	1024	0.960±0.009	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.5±0.0	9.1±0.0	10.1±0.0	7.1±0.0
P24 (δ=0.994)	64	0.234±0.019	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.3±0.0	9.2±0.0	10.2±0.0	7.1±0.0
gold=669, mode=669 ✓	128	0.406±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.5±0.0	9.2±0.0	10.2±0.0	7.1±0.0
	256	0.652±0.021	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	9.2±0.0	10.2±0.0	7.1±0.0
	512	0.886±0.014	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.5±0.0	9.2±0.0	10.2±0.0	7.1±0.0
	1024	0.982±0.006	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	3.2±0.0	9.2±0.0	10.2±0.0	7.1±0.0
P21 (δ=0.987)	64	0.430±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.5±0.0	9.4±0.1	11.0±0.1	7.3±0.1
gold=50, mode=50 ✓	128	0.706±0.020	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.4±0.1	11.0±0.1	7.3±0.1
	256	0.912±0.013	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	9.4±0.1	11.0±0.1	7.3±0.1
	512	0.986±0.005	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	3.6±0.0	9.4±0.1	11.0±0.1	7.3±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.4±0.0	9.4±0.1	11.0±0.1	7.3±0.1
P5 (δ=0.976)	64	0.508±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.5±0.0	9.6±0.1	10.5±0.1	7.5±0.1
gold=65, mode=65 ✓	128	0.792±0.018	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.8±0.0	9.6±0.1	10.5±0.1	7.5±0.1
	256	0.950±0.010	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	9.6±0.1	10.5±0.1	7.5±0.1
	512	0.994±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.6±0.1	10.5±0.1	7.5±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.6±0.1	10.5±0.1	7.5±0.1
P25 (δ=0.975)	64	0.714±0.020	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	9.9±0.1	10.8±0.1	7.8±0.1
gold=850, mode=850 ✓	128	0.924±0.012	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	3.3±0.1	9.9±0.1	10.8±0.1	7.8±0.1
	256	0.992±0.004	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	4.9±0.1	9.9±0.1	10.8±0.1	7.8±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.0±0.1	9.9±0.1	10.8±0.1	7.8±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	8.9±0.1	9.9±0.1	10.8±0.1	7.8±0.1
P15 (δ=0.970)	64	0.752±0.019	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	10.1±0.1	12.8±0.1	7.9±0.1
gold=83, mode=1 ✗	128	0.920±0.012	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	10.1±0.1	12.8±0.1	7.9±0.1
	256	0.996±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	4.1±0.0	10.1±0.1	12.8±0.1	7.9±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.2±0.0	10.1±0.1	12.8±0.1	7.9±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	10.1±0.1	12.8±0.1	7.9±0.1
P12 (δ=0.956)	64	0.878±0.015	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.9±0.1	10.5±0.1	11.3±0.1	8.3±0.1
gold=161, mode=161 ✓	128	0.988±0.005	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	4.5±0.1	10.5±0.1	11.3±0.1	8.3±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.9±0.1	10.5±0.1	11.3±0.1	8.3±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	10.0±0.1	10.5±0.1	11.3±0.1	8.3±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	12.8±0.1	10.5±0.1	11.3±0.1	8.3±0.1
P8 (δ=0.910)	64	0.968±0.008	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	11.1±0.1	12.0±0.1	8.6±0.1
gold=244, mode=244 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.6±0.0	11.1±0.1	12.0±0.1	8.6±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	11.1±0.1	12.0±0.1	8.6±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.5±0.0	11.1±0.1	12.0±0.1	8.6±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.8±0.0	11.1±0.1	12.0±0.1	8.6±0.1
P22 (δ=0.891)	64	0.992±0.004	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	4.6±0.1	12.2±0.2	12.9±0.1	10.4±0.2
gold=754, mode=754 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.9±0.1	12.2±0.2	12.9±0.1	10.4±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	10.3±0.1	12.2±0.2	12.9±0.1	10.4±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	14.8±0.1	12.2±0.2	12.9±0.1	10.4±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	20.3±0.1	12.2±0.2	12.9±0.1	10.4±0.2
P3 (δ=0.859)	64	0.998±0.002	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	12.8±0.2	13.7±0.2	10.1±0.2
gold=79, mode=79 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.1±0.0	12.8±0.2	13.7±0.2	10.1±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.6±0.0	12.8±0.2	13.7±0.2	10.1±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	12.8±0.2	13.7±0.2	10.1±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	12.8±0.2	13.7±0.2	10.1±0.2
P2 (δ=0.850)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.7±0.1	13.7±0.2	14.3±0.2	13.6±0.3
gold=62, mode=62 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	9.9±0.1	13.7±0.2	14.3±0.2	13.6±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	13.6±0.1	13.7±0.2	14.3±0.2	13.6±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	17.4±0.1	13.7±0.2	14.3±0.2	13.6±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	21.0±0.1	13.7±0.2	14.3±0.2	13.6±0.3
P26 (δ=0.776)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.994±0.003	12.0±0.1	16.5±0.2	17.7±0.2	23.5±0.6
gold=132, mode=132 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	19.6±0.1	16.5±0.2	17.7±0.2	23.8±0.6
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	30.2±0.2	16.5±0.2	17.7±0.2	23.8±0.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	43.1±0.2	16.5±0.2	17.7±0.2	23.8±0.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	57.4±0.1	16.5±0.2	17.7±0.2	23.8±0.6
P27 (δ=0.728)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.994±0.003	9.8±0.1	17.4±0.2	17.9±0.2	21.4±0.6
gold=223, mode=223 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	15.7±0.1	17.4±0.2	17.9±0.2	21.8±0.6
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	24.1±0.1	17.4±0.2	17.9±0.2	21.8±0.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	34.7±0.1	17.4±0.2	17.9±0.2	21.8±0.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	46.4±0.1	17.4±0.2	17.9±0.2	21.8±0.6
P28 (δ=0.695)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.944±0.010	10.5±0.1	19.1±0.2	20.9±0.2	28.1±0.7
gold=107, mode=12 ✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	15.2±0.1	19.1±0.2	20.9±0.2	30.9±0.8
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	20.5±0.1	19.1±0.2	20.9±0.2	30.9±0.8
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	26.7±0.1	19.1±0.2	20.9±0.2	30.9±0.8
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	32.6±0.1	19.1±0.2	20.9±0.2	30.9±0.8
P11 (δ=0.690)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.990±0.004	8.6±0.1	19.1±0.3	20.5±0.3	23.3±0.6
gold=896, mode=896 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	12.8±0.1	19.1±0.3	20.5±0.3	23.8±0.6
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	19.1±0.1	19.1±0.3	20.5±0.3	23.8±0.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	27.2±0.1	19.1±0.3	20.5±0.3	23.8±0.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	35.7±0.1	19.1±0.3	20.5±0.3	23.8±0.6
P14 (δ=0.429)	64	0.976±0.007	**0.990±0.004**	0.990±0.004	0.996±0.003	0.014±0.005	24.5±0.1	33.8±0.5	35.2±0.4	17.9±5.4
gold=681, mode=681 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.018±0.006	39.4±0.2	34.2±0.5	35.7±0.5	33.1±11.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.018±0.006	60.8±0.2	34.2±0.5	35.7±0.5	33.1±11.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.020±0.006	87.7±0.2	34.2±0.5	35.7±0.5	56.5±25.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.022±0.007	116.8±0.2	34.2±0.5	35.7±0.5	105.1±53.8
P16 (δ=0.392)	64	0.808±0.018	0.550±0.022	0.564±0.022	0.792±0.018	0.682±0.021	2.0±0.0	35.3±1.0	36.2±0.9	29.0±1.0
gold=178, mode=32 ✗	128	0.988±0.005	0.922±0.012	0.928±0.012	0.948±0.010	0.946±0.010	2.0±0.0	57.2±1.5	57.2±1.4	46.0±1.5
	256	1.000±0.000	0.998±0.002	1.000±0.000	0.996±0.003	0.998±0.002	2.0±0.0	65.0±1.9	64.8±1.9	52.6±1.9
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	65.4±1.9	64.8±1.9	53.1±2.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	65.4±1.9	64.8±1.9	53.1±2.0
P13 (δ=0.319)	64	0.574±0.022	0.394±0.022	0.466±0.022	0.648±0.021	0.176±0.017	5.9±0.0	45.1±0.9	44.8±0.8	38.6±1.8
gold=39, mode=39 ✓	128	0.930±0.011	0.872±0.015	0.898±0.014	0.932±0.011	0.518±0.022	6.7±0.0	70.2±1.3	66.2±1.3	77.9±2.1
	256	1.000±0.000	0.996±0.003	0.998±0.002	0.998±0.002	0.962±0.009	7.3±0.0	81.4±1.8	75.2±1.7	123.5±2.7
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.7±0.0	82.1±1.9	75.6±1.7	129.7±3.0

Table 12 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	8.0±0.0	82.1±1.9	75.6±1.7	129.7±3.0
P23 (δ=0.312)	64	0.570±0.022	0.544±0.022	0.548±0.022	0.768±0.019	0.006±0.003	19.3±0.1	43.7±0.8	44.3±0.7	8.7±1.7
gold=245, mode=245 ✓	128	0.942±0.010	**0.946±0.010**	0.954±0.009	0.964±0.008	0.006±0.003	29.7±0.2	62.5±1.2	62.8±1.2	8.7±1.7
	256	1.000±0.000	1.000±0.000	1.000±0.000	0.998±0.002	0.006±0.003	44.8±0.2	67.6±1.5	67.2±1.5	8.7±1.7
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	65.0±0.2	67.6±1.5	67.2±1.5	8.7±1.7
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	89.1±0.2	67.6±1.5	67.2±1.5	8.7±1.7
P4 (δ=0.199)	64	0.152±0.016	0.096±0.013	0.100±0.013	0.318±0.021	0.182±0.017	4.8±0.0	38.8±2.5	41.8±2.2	35.8±1.8
gold=70, mode=70 ✓	128	0.352±0.021	0.268±0.020	0.274±0.020	0.526±0.022	0.338±0.021	6.1±0.1	76.4±2.9	76.8±2.7	64.3±2.7
	256	0.706±0.020	0.594±0.022	0.618±0.022	0.788±0.018	0.606±0.022	7.9±0.1	141.2±4.0	142.0±3.8	121.5±4.2
	512	0.978±0.007	0.944±0.010	0.950±0.010	0.962±0.009	0.924±0.012	10.2±0.1	219.8±5.7	213.5±5.4	201.4±6.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	12.7±0.1	241.6±6.8	232.6±6.4	232.9±7.6
P19 (δ=0.158)	64	0.098±0.013	0.050±0.010	0.048±0.010	0.262±0.020	0.106±0.014	2.9±0.0	28.3±2.9	29.9±2.7	21.3±2.1
gold=279, mode=279 ✓	128	0.210±0.018	0.106±0.014	0.096±0.013	0.374±0.022	0.166±0.017	3.6±0.0	68.5±5.7	66.5±5.8	52.0±4.8
	256	0.482±0.022	0.300±0.020	0.288±0.020	0.590±0.022	0.326±0.021	4.8±0.0	147.8±5.7	149.9±5.8	122.4±6.5
	512	0.814±0.017	0.688±0.021	0.664±0.021	0.814±0.017	0.660±0.021	6.0±0.0	276.0±7.2	278.7±7.4	253.0±8.3
	1024	0.988±0.005	0.960±0.009	0.950±0.010	0.974±0.007	0.940±0.011	7.1±0.0	399.0±10.9	409.8±11.1	393.1±12.0
P10 (δ=0.154)	64	0.050±0.010	**0.058±0.010**	0.078±0.012	0.270±0.020	0.054±0.010	10.7±0.1	39.2±2.4	39.4±2.1	42.7±3.3
gold=156, mode=156 ✓	128	0.130±0.015	**0.152±0.016**	0.222±0.019	0.412±0.022	0.154±0.016	16.1±0.1	76.9±3.9	76.0±3.0	78.2±3.6
	256	0.358±0.021	**0.374±0.022**	0.516±0.022	0.624±0.022	0.400±0.022	24.4±0.1	145.6±5.0	141.0±4.2	148.4±4.6
	512	0.760±0.019	**0.776±0.019**	0.902±0.013	0.860±0.016	0.770±0.019	35.4±0.1	263.8±6.8	236.6±6.1	258.5±6.9
	1024	0.996±0.003	**0.998±0.002**	1.000±0.000	0.984±0.006	0.980±0.006	47.9±0.1	360.1±10.0	274.6±7.7	355.0±10.3
P29 (δ=0.131)	64	0.008±0.004	0.008±0.004	0.008±0.004	0.112±0.014	0.000±0.000	14.9±0.1	59.2±3.1	59.2±3.1	—
gold=157, mode=12 ✗	128	0.102±0.014	0.086±0.013	0.090±0.013	0.302±0.021	0.000±0.000	17.7±0.1	100.0±3.5	100.3±3.4	—
	256	0.500±0.022	0.450±0.022	0.452±0.022	0.684±0.021	0.000±0.000	20.4±0.1	176.6±3.4	173.9±3.4	—
	512	0.954±0.009	0.954±0.009	0.952±0.010	0.962±0.009	0.000±0.000	23.5±0.1	268.6±4.8	266.9±4.9	—
	1024	1.000±0.000	1.000±0.000	1.000±0.000	0.998±0.002	0.000±0.000	26.4±0.1	283.8±5.6	282.9±5.7	—
P17 (δ=0.095)	64	0.008±0.004	**0.016±0.006**	0.016±0.006	0.134±0.015	0.000±0.000	19.4±0.1	54.2±4.8	55.4±3.8	—
gold=243, mode=243 ✓	128	0.046±0.009	**0.058±0.010**	0.058±0.010	0.246±0.019	0.000±0.000	29.0±0.2	87.7±4.7	88.5±4.6	—
	256	0.140±0.016	**0.214±0.018**	0.196±0.018	0.464±0.022	0.000±0.000	43.1±0.2	169.3±5.8	166.1±6.2	—
	512	0.404±0.022	**0.538±0.022**	0.504±0.022	0.744±0.020	0.000±0.000	62.1±0.2	300.4±7.7	305.0±8.2	—
	1024	0.812±0.017	**0.878±0.015**	0.846±0.016	0.930±0.011	0.000±0.000	82.6±0.2	467.4±11.9	478.5±12.2	—
P18 (δ=0.065)	64	0.004±0.003	0.002±0.002	0.002±0.002	0.032±0.008	0.000±0.000	13.2±0.1	28.0±0.0	27.0±0.0	—
gold=503, mode=505 ✗	128	0.014±0.005	0.014±0.005	0.016±0.006	0.080±0.012	0.000±0.000	18.1±0.1	79.0±10.8	80.4±10.3	—
	256	0.044±0.009	**0.056±0.010**	0.068±0.011	0.216±0.018	0.000±0.000	24.8±0.1	169.3±12.3	174.1±11.1	—
	512	0.116±0.014	**0.154±0.016**	0.182±0.017	0.408±0.022	0.000±0.000	33.0±0.1	306.3±14.2	311.9±13.5	—
	1024	0.364±0.022	**0.430±0.022**	0.490±0.022	0.694±0.021	0.000±0.000	40.9±0.1	603.9±18.0	599.2±16.6	—
P9 (δ=0.024)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.018±0.006	0.000±0.000	20.4±0.1	—	—	—
gold=29, mode=29 ✓	128	0.000±0.000	0.000±0.000	0.000±0.000	0.022±0.007	0.000±0.000	29.1±0.1	—	—	—
	256	0.000±0.000	0.000±0.000	0.008±0.004	0.050±0.010	0.000±0.000	41.3±0.2	—	227.5±16.8	—
	512	0.004±0.003	**0.014±0.005**	0.032±0.008	0.120±0.015	0.000±0.000	56.3±0.2	344.6±32.0	357.6±26.8	—
	1024	0.016±0.006	**0.040±0.009**	0.098±0.013	0.196±0.018	0.000±0.000	72.1±0.1	580.6±46.9	628.5±33.7	—

FrontierScience-Olympiad.

Table 13. Per-problem certification rate on FrontierScience-Olympiad (10 problems, every-10 subset) (Case A, target = mode). **Bold**: CITE $>$ Bonferroni. $\bar{K}_N$ is the Monte Carlo mean number of observed answer categories at budget N.

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P60 (δ=0.630)	64	1.000±0.000	0.996±0.003	0.998±0.002	0.994±0.003	1.000±0.000	2.7±0.0	23.5±0.5	23.4±0.5	18.2±0.5
gold=6.98, mode=6.49 ✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.1±0.0	23.7±0.5	23.5±0.5	18.2±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.4±0.0	23.7±0.5	23.5±0.5	18.2±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.8±0.0	23.7±0.5	23.5±0.5	18.2±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.3±0.0	23.7±0.5	23.5±0.5	18.2±0.5
P50 (δ=0.524)	64	0.994±0.003	0.994±0.003	0.994±0.003	0.992±0.004	0.156±0.016	14.1±0.1	29.4±0.4	30.1±0.4	34.7±2.1
gold=0.2, mode=0.2 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.354±0.021	19.2±0.1	29.7±0.4	30.4±0.4	70.3±2.7
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.756±0.019	24.6±0.1	29.7±0.4	30.4±0.4	133.9±3.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.992±0.004	30.4±0.1	29.7±0.4	30.4±0.4	181.8±4.9
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	36.2±0.1	29.7±0.4	30.4±0.4	184.8±5.1
P30 (δ=0.347)	64	0.800±0.018	**0.840±0.016**	0.858±0.016	0.930±0.011	0.006±0.003	26.0±0.1	42.1±0.6	43.5±0.5	10.7±1.9
✗	128	0.998±0.002	**1.000±0.000**	1.000±0.000	1.000±0.000	0.006±0.003	41.1±0.2	47.6±0.8	47.9±0.7	10.7±1.9
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	61.6±0.2	47.6±0.8	47.9±0.7	10.7±1.9
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	86.9±0.2	47.6±0.8	47.9±0.7	10.7±1.9
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	113.2±0.2	47.6±0.8	47.9±0.7	10.7±1.9
P70 (δ=0.116)	64	0.038±0.009	0.028±0.007	0.030±0.008	0.162±0.016	0.040±0.009	8.3±0.1	42.2±4.7	47.3±3.7	43.7±3.4
✗	128	0.080±0.012	0.076±0.012	0.080±0.012	0.306±0.021	0.094±0.013	11.3±0.1	76.6±5.3	77.5±4.6	73.3±4.5
	256	0.200±0.018	0.178±0.017	0.196±0.018	0.430±0.022	0.210±0.018	15.3±0.1	137.6±6.7	138.8±6.3	133.2±6.3
	512	0.494±0.022	0.476±0.022	0.518±0.022	0.696±0.021	0.446±0.022	20.6±0.1	290.7±8.9	287.4±8.3	267.1±9.7
	1024	0.870±0.015	0.850±0.016	0.882±0.014	0.920±0.012	0.804±0.018	26.6±0.1	484.1±12.6	464.5±11.9	472.0±13.5
P100 (δ=0.112)	64	0.006±0.003	**0.014±0.005**	0.010±0.004	0.128±0.015	0.004±0.003	17.5±0.1	36.1±8.0	47.0±5.7	7.0±0.0
✗	128	0.042±0.009	**0.060±0.011**	0.060±0.011	0.244±0.019	0.004±0.003	25.2±0.1	89.7±6.4	95.3±5.0	7.0±0.0
	256	0.264±0.020	**0.306±0.021**	0.310±0.021	0.562±0.022	0.004±0.003	35.4±0.1	182.2±4.6	181.6±4.3	7.0±0.0
	512	0.662±0.021	**0.726±0.020**	0.726±0.020	0.860±0.016	0.004±0.003	47.1±0.1	298.5±6.3	296.3±6.3	7.0±0.0
	1024	0.972±0.007	**0.978±0.007**	0.978±0.007	0.980±0.006	0.004±0.003	58.2±0.1	394.5±9.2	391.4±9.2	7.0±0.0
P90 (δ=0.057)	64	0.000±0.000	**0.004±0.003**	0.004±0.003	0.050±0.010	0.000±0.000	21.7±0.1	54.0±7.0	54.0±7.0	—
✗	128	0.000±0.000	**0.010±0.004**	0.006±0.003	0.080±0.012	0.000±0.000	33.2±0.2	87.8±14.5	72.0±18.4	—
	256	0.024±0.007	**0.040±0.009**	0.042±0.009	0.206±0.018	0.000±0.000	51.8±0.2	164.6±12.8	175.8±12.2	—
	512	0.072±0.012	**0.148±0.016**	0.138±0.015	0.380±0.022	0.000±0.000	79.8±0.3	331.7±14.3	329.7±14.7	—
	1024	0.244±0.019	**0.388±0.022**	0.394±0.022	0.638±0.021	0.000±0.000	115.5±0.2	591.1±17.7	606.5±17.6	—
P80 (δ=0.044)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.028±0.007	0.000±0.000	30.8±0.1	—	—	—
✗	128	0.000±0.000	**0.004±0.003**	0.004±0.003	0.098±0.013	0.000±0.000	45.0±0.2	103.5±4.5	103.5±4.5	—
	256	0.014±0.005	**0.034±0.008**	0.040±0.009	0.216±0.018	0.000±0.000	63.0±0.2	184.5±10.8	186.1±9.9	—
	512	0.058±0.010	**0.136±0.015**	0.144±0.016	0.378±0.022	0.000±0.000	85.6±0.2	346.4±14.0	339.3±13.8	—
	1024	0.216±0.018	**0.398±0.022**	0.396±0.022	0.616±0.022	0.000±0.000	110.4±0.2	608.2±16.5	606.3±17.4	—
P10 (δ=0.009)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	52.6±0.1	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	93.0±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.002	0.000±0.000	156.6±0.3	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.006±0.003	0.000±0.000	248.5±0.4	—	—	—
	1024	0.000±0.000	**0.004±0.003**	0.004±0.003	0.054±0.010	0.000±0.000	360.0±0.4	613.0±38.0	613.0±38.0	—
P20 (δ=0.007)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	58.6±0.1	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	109.8±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.006±0.003	0.000±0.000	198.6±0.3	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.028±0.007	0.000±0.000	338.1±0.4	—	—	—

Table 13 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
	1024	0.000±0.000	**0.006±0.003**	0.010±0.004	0.078±0.012	0.000±0.000	520.0±0.5	704.0±127.6	733.4±87.0	—
P40 (δ=0.007)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	47.1±0.1	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.002	0.000±0.000	79.1±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.004±0.003	0.000±0.000	126.6±0.3	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.016±0.006	0.000±0.000	188.3±0.3	—	—	—
	1024	0.000±0.000	**0.006±0.003**	0.006±0.003	0.048±0.010	0.000±0.000	255.4±0.3	865.3±62.6	875.3±72.5	—

ER-REASON.

Table 14. Per-problem certification rate on ER-REASON (Case A, target = mode). **Bold**: CITE > Bonferroni. $\bar{K}_N$ is the Monte Carlo mean number of observed answer categories at budget N.

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P1 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P2 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P3 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P4 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P5 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P6 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P7 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P8 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P9 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P10 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P11 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P12 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P13 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P14 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P15 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P16 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0

Table 14 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P17 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P18 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P19 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P20 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P21 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P22 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P23 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P24 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P25 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P26 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P27 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
P29 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P30 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P32 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P34 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P36 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P37 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P38 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P39 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0

Table 14 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\mathrm{CITE}}$	$\bar{\tau}_{\mathrm{W\text{-}CITE}}$	$\bar{\tau}_{\mathrm{MMC}}$
P41 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P43 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P44 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P45 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P46 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P47 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P48 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P49 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P50 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P51 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P52 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P53 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P54 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P55 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P56 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P57 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	9.0±0.0	7.0±0.0
P59 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P60 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P61 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P62 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0

Table 14 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P63 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P64 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✓	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P65 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P66 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P67 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P69 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P70 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P71 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P72 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	10.0±0.0	7.0±0.0
P35 (δ=0.918)	64	0.934±0.011	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	11.1±0.2	11.9±0.1	8.6±0.1
✗	128	0.994±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.1±0.2	11.9±0.1	8.6±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.1±0.2	11.9±0.1	8.6±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.1±0.2	11.9±0.1	8.6±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.1±0.2	11.9±0.1	8.6±0.1
P58 (δ=0.908)	64	0.956±0.009	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.4±0.2	12.2±0.1	8.7±0.1
✗	128	0.994±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.4±0.2	12.2±0.1	8.7±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.4±0.2	12.2±0.1	8.7±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.4±0.2	12.2±0.1	8.7±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.4±0.2	12.2±0.1	8.7±0.1
P33 (δ=0.754)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	17.1±0.4	16.9±0.3	12.7±0.3
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	17.1±0.4	16.9±0.3	12.7±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	17.1±0.4	16.9±0.3	12.7±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	17.1±0.4	16.9±0.3	12.7±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	17.1±0.4	16.9±0.3	12.7±0.3
P31 (δ=0.718)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	18.2±0.4	18.1±0.3	13.8±0.3
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	18.2±0.4	18.1±0.3	13.8±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	18.2±0.4	18.1±0.3	13.8±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	18.2±0.4	18.1±0.3	13.8±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	18.2±0.4	18.1±0.3	13.8±0.3
P42 (δ=0.704)	64	1.000±0.000	0.998±0.002	1.000±0.000	0.998±0.002	0.996±0.003	2.0±0.0	20.4±0.5	19.8±0.4	15.5±0.4
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	20.5±0.5	19.8±0.4	15.8±0.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	20.5±0.5	19.8±0.4	15.8±0.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	20.5±0.5	19.8±0.4	15.8±0.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	20.5±0.5	19.8±0.4	15.8±0.4
P28 (δ=0.470)	64	0.950±0.010	0.758±0.019	0.820±0.017	0.870±0.015	0.844±0.016	2.0±0.0	33.9±0.8	33.5±0.7	26.6±0.8
✗	128	0.998±0.002	0.988±0.005	0.996±0.003	0.990±0.004	0.990±0.004	2.0±0.0	45.2±1.1	41.9±1.1	35.3±1.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	46.2±1.2	42.3±1.1	36.4±1.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	46.2±1.2	42.3±1.1	36.4±1.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	46.2±1.2	42.3±1.1	36.4±1.3
P68 (δ=0.438)	64	0.890±0.014	0.686±0.021	0.626±0.022	0.840±0.016	0.788±0.018	2.0±0.0	36.0±0.9	37.6±0.9	28.2±0.9
✓	128	0.994±0.003	0.976±0.007	0.964±0.008	0.978±0.007	0.970±0.008	2.0±0.0	51.0±1.3	54.8±1.3	39.8±1.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	53.7±1.5	58.6±1.5	43.4±1.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	53.7±1.5	58.6±1.5	43.4±1.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	53.7±1.5	58.6±1.5	43.4±1.6
P40 (δ=0.326)	64	0.600±0.022	0.356±0.021	0.354±0.021	0.590±0.022	0.494±0.022	2.0±0.0	37.0±1.2	37.5±1.2	30.5±1.1
✗	128	0.908±0.013	0.736±0.020	0.732±0.020	0.864±0.015	0.768±0.019	2.0±0.0	65.9±1.7	65.8±1.7	53.1±1.8
	256	1.000±0.000	0.980±0.006	0.980±0.006	0.968±0.008	0.976±0.007	2.0±0.0	91.7±2.5	92.0±2.5	80.0±2.8
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	95.5±2.7	95.6±2.7	85.0±3.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	95.5±2.7	95.6±2.7	85.0±3.1

H.4. Per-problem certification rates: gpt-oss-20b

AIME 2026.

Table 15. Per-problem certification rate on AIME 2026 (30 problems) (Case A, target = mode). **Bold**: CITE > Bonferroni. $\bar{K}_N$ is the Monte Carlo mean number of observed answer categories at budget N.

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\mathrm{CITE}}$	$\bar{\tau}_{\mathrm{W\text{-}CITE}}$	$\bar{\tau}_{\mathrm{MMC}}$
P6 (δ=0.995)	64	0.222±0.019	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.2±0.0	9.2±0.0	9.2±0.0	7.2±0.0
gold=441, mode=441 ✓	128	0.348±0.021	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.4±0.0	9.2±0.0	9.2±0.0	7.2±0.0
	256	0.548±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.6±0.0	9.2±0.0	9.2±0.0	7.2±0.0
	512	0.792±0.018	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.1±0.0	9.2±0.0	9.2±0.0	7.2±0.0
	1024	0.968±0.008	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.6±0.0	9.2±0.0	9.2±0.0	7.2±0.0
P1 (δ=0.970)	64	0.636±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.7±0.0	9.6±0.1	9.6±0.1	7.5±0.1
gold=277, mode=277 ✓	128	0.852±0.016	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.1±0.0	9.6±0.1	9.6±0.1	7.5±0.1
	256	0.976±0.007	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.4±0.0	9.6±0.1	9.6±0.1	7.5±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.8±0.0	9.6±0.1	9.6±0.1	7.5±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.3±0.0	9.6±0.1	9.6±0.1	7.5±0.1
P24 (δ=0.904)	64	0.992±0.004	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	5.4±0.1	12.1±0.1	12.4±0.1	10.7±0.2
gold=669, mode=669 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	8.8±0.1	12.1±0.1	12.4±0.1	10.7±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	13.6±0.1	12.1±0.1	12.4±0.1	10.7±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	19.4±0.1	12.1±0.1	12.4±0.1	10.7±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	24.9±0.1	12.1±0.1	12.4±0.1	10.7±0.2
P16 (δ=0.893)	64	0.976±0.007	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	11.8±0.2	11.8±0.2	9.2±0.1
gold=178, mode=178 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.4±0.0	11.8±0.2	11.8±0.2	9.2±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	11.8±0.2	11.8±0.2	9.2±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.2±0.0	11.8±0.2	11.8±0.2	9.2±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	11.8±0.2	11.8±0.2	9.2±0.1
P15 (δ=0.878)	64	0.998±0.002	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	5.3±0.1	12.9±0.2	12.9±0.2	11.3±0.3
gold=83, mode=1 ✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.8±0.1	12.9±0.2	12.9±0.2	11.3±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	11.3±0.1	12.9±0.2	12.9±0.2	11.3±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	15.8±0.1	12.9±0.2	12.9±0.2	11.3±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	20.7±0.1	12.9±0.2	12.9±0.2	11.3±0.3
P22 (δ=0.868)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.8±0.1	13.5±0.1	13.5±0.1	13.5±0.3
gold=754, mode=754 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	13.1±0.1	13.5±0.1	13.5±0.1	13.5±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	20.8±0.1	13.5±0.1	13.5±0.1	13.5±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	30.6±0.1	13.5±0.1	13.5±0.1	13.5±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	41.0±0.1	13.5±0.1	13.5±0.1	13.5±0.3
P2 (δ=0.777)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.4±0.1	15.9±0.2	15.9±0.2	16.6±0.4
gold=62, mode=62 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	11.0±0.1	15.9±0.2	15.9±0.2	16.6±0.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	16.0±0.1	15.9±0.2	15.9±0.2	16.6±0.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	22.1±0.1	15.9±0.2	15.9±0.2	16.6±0.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	28.6±0.1	15.9±0.2	15.9±0.2	16.6±0.4
P18 (δ=0.741)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.996±0.003	6.7±0.1	17.0±0.2	17.0±0.2	17.4±0.4
gold=503, mode=1 ✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	9.0±0.1	17.0±0.2	17.0±0.2	17.6±0.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	11.3±0.1	17.0±0.2	17.0±0.2	17.6±0.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	13.5±0.1	17.0±0.2	17.0±0.2	17.6±0.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	15.6±0.1	17.0±0.2	17.0±0.2	17.6±0.4
P7 (δ=0.733)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.958±0.009	12.4±0.1	17.6±0.2	17.6±0.2	27.0±0.7
gold=396, mode=396 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	19.9±0.1	17.6±0.2	17.6±0.2	29.2±0.8
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	30.2±0.2	17.6±0.2	17.6±0.2	29.2±0.8
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	43.1±0.2	17.6±0.2	17.6±0.2	29.2±0.8
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	56.5±0.1	17.6±0.2	17.6±0.2	29.2±0.8
P8 (δ=0.700)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	20.9±0.4	20.9±0.4	18.1±0.4
gold=244, mode=244 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	20.9±0.4	20.9±0.4	18.1±0.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	20.9±0.4	20.9±0.4	18.1±0.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	20.9±0.4	20.9±0.4	18.1±0.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	20.9±0.4	20.9±0.4	18.1±0.4
P3 (δ=0.699)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.998±0.002	4.7±0.0	20.5±0.4	20.4±0.3	20.4±0.5
gold=79, mode=79 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.7±0.0	20.5±0.4	20.4±0.3	20.5±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.6±0.0	20.5±0.4	20.4±0.3	20.5±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.4±0.0	20.5±0.4	20.4±0.3	20.5±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	8.2±0.0	20.5±0.4	20.4±0.3	20.5±0.5
P30 (δ=0.675)	64	1.000±0.000	0.998±0.002	0.998±0.002	0.998±0.002	1.000±0.000	2.4±0.0	21.9±0.5	21.9±0.5	16.0±0.5
gold=393, mode=243 ✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	22.0±0.5	22.0±0.5	16.0±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.2±0.0	22.0±0.5	22.0±0.5	16.0±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.8±0.0	22.0±0.5	22.0±0.5	16.0±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.4±0.0	22.0±0.5	22.0±0.5	16.0±0.5
P5 (δ=0.488)	64	0.940±0.011	0.804±0.018	0.804±0.018	0.928±0.012	0.876±0.015	3.8±0.1	32.1±0.8	32.1±0.8	26.1±0.8
gold=65, mode=65 ✓	128	0.996±0.003	0.988±0.005	0.988±0.005	0.996±0.003	0.994±0.003	5.2±0.1	41.3±1.1	41.3±1.1	32.9±1.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.5±0.1	42.6±1.2	42.6±1.2	33.7±1.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	10.6±0.1	42.6±1.2	42.6±1.2	33.7±1.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	14.5±0.1	42.6±1.2	42.6±1.2	33.7±1.2
P19 (δ=0.444)	64	0.972±0.007	0.964±0.008	0.964±0.008	0.960±0.009	0.032±0.008	16.6±0.1	36.3±0.5	36.3±0.5	24.4±4.8
gold=279, mode=279 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.050±0.010	23.4±0.1	37.6±0.6	37.6±0.6	51.1±8.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.086±0.013	31.6±0.1	37.6±0.6	37.6±0.6	107.0±11.9
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.220±0.019	39.8±0.1	37.6±0.6	37.6±0.6	288.7±15.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.530±0.022	47.3±0.1	37.6±0.6	37.6±0.6	548.1±16.4
P20 (δ=0.415)	64	0.840±0.016	0.748±0.019	0.752±0.019	0.870±0.015	0.684±0.021	7.5±0.1	37.1±0.7	37.1±0.7	39.0±0.9
gold=190, mode=190 ✓	128	1.000±0.000	0.996±0.003	0.998±0.002	0.988±0.005	0.992±0.004	10.6±0.1	48.9±1.1	48.8±1.1	53.4±1.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	14.5±0.1	49.2±1.1	49.0±1.1	54.1±1.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	18.7±0.1	49.2±1.1	49.0±1.1	54.1±1.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	22.3±0.1	49.2±1.1	49.0±1.1	54.1±1.2
P28 (δ=0.402)	64	0.878±0.015	0.850±0.016	0.898±0.014	0.922±0.012	0.070±0.011	13.9±0.1	38.0±0.6	37.9±0.6	32.9±3.2
gold=107, mode=12 ✗	128	1.000±0.000	1.000±0.000	1.000±0.000	0.996±0.003	0.166±0.017	18.3±0.1	44.0±0.8	42.0±0.8	75.1±4.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.484±0.022	22.5±0.1	44.0±0.8	42.0±0.8	155.0±4.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.880±0.015	27.0±0.1	44.0±0.8	42.0±0.8	245.5±5.8
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	31.3±0.1	44.0±0.8	42.0±0.8	290.5±7.6
P12 (δ=0.345)	64	0.630±0.022	0.624±0.022	0.624±0.022	0.794±0.018	0.116±0.014	16.0±0.1	38.5±0.8	38.2±0.8	38.4±2.4
gold=161, mode=161 ✓	128	0.964±0.008	**0.972±0.007**	0.970±0.008	0.978±0.007	0.326±0.021	25.1±0.1	56.8±1.3	56.4±1.3	75.9±2.6
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.756±0.019	37.9±0.2	59.4±1.4	59.2±1.5	140.5±3.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.984±0.006	55.1±0.2	59.4±1.4	59.2±1.5	183.6±4.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	74.0±0.2	59.4±1.4	59.2±1.5	191.0±5.2
P10 (δ=0.315)	64	0.592±0.022	**0.654±0.021**	0.656±0.021	0.838±0.016	0.006±0.003	24.2±0.1	44.3±0.7	44.3±0.7	7.0±0.0
gold=156, mode=168 ✗	128	0.972±0.007	**0.982±0.006**	0.982±0.006	0.990±0.004	0.006±0.003	39.5±0.2	57.9±1.1	57.8±1.1	7.0±0.0
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	61.9±0.2	59.3±1.1	59.3±1.1	7.0±0.0
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	88.9±0.2	59.3±1.1	59.3±1.1	7.0±0.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.006±0.003	114.6±0.2	59.3±1.1	59.3±1.1	7.0±0.0
P29 (δ=0.314)	64	0.652±0.021	0.604±0.022	0.602±0.022	0.788±0.018	0.004±0.003	14.7±0.1	44.6±0.7	45.0±0.7	27.0±11.0
gold=157, mode=0 ✗	128	0.974±0.007	0.974±0.007	0.974±0.007	0.982±0.006	0.004±0.003	18.7±0.1	59.4±1.0	59.9±1.0	27.0±11.0
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.004±0.003	23.1±0.1	61.6±1.2	62.1±1.2	27.0±11.0
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.004±0.003	27.6±0.1	61.6±1.2	62.1±1.2	27.0±11.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.004±0.003	31.9±0.1	61.6±1.2	62.1±1.2	27.0±11.0

Table 15 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P26 (δ=0.264)	64	0.368±0.022	**0.422±0.022**	0.420±0.022	0.650±0.021	0.000±0.000	20.5±0.1	44.8±0.9	45.3±0.8	—
gold=132, mode=66 ✗	128	0.824±0.017	**0.860±0.016**	0.860±0.016	0.926±0.012	0.000±0.000	30.1±0.1	68.6±1.3	69.2±1.3	—
	256	0.994±0.003	**0.998±0.002**	0.998±0.002	0.998±0.002	0.000±0.000	41.6±0.1	81.3±1.9	81.9±1.9	—
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.000±0.000	54.9±0.2	81.7±1.9	82.3±1.9	—
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.000±0.000	68.3±0.1	81.7±1.9	82.3±1.9	—
P25 (δ=0.182)	64	0.110±0.014	0.106±0.014	0.126±0.015	0.370±0.022	0.002±0.002	15.1±0.1	48.6±1.5	48.6±1.4	7.0±0.0
gold=850, mode=850 ✓	128	0.330±0.021	**0.344±0.021**	0.366±0.022	0.632±0.022	0.002±0.002	20.7±0.1	80.4±2.1	80.5±2.1	7.0±0.0
	256	0.774±0.019	**0.782±0.018**	0.804±0.018	0.874±0.015	0.002±0.002	27.3±0.1	139.8±3.1	137.3±3.1	7.0±0.0
	512	0.998±0.002	**1.000±0.000**	1.000±0.000	0.988±0.005	0.002±0.002	34.2±0.1	180.7±4.4	173.8±4.3	7.0±0.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.002±0.002	41.1±0.1	180.7±4.4	173.8±4.3	7.0±0.0
P11 (δ=0.142)	64	0.018±0.006	**0.036±0.008**	0.036±0.008	0.278±0.020	0.000±0.000	30.6±0.2	48.6±2.1	48.5±2.1	—
gold=896, mode=896 ✓	128	0.190±0.018	**0.278±0.020**	0.280±0.020	0.568±0.022	0.000±0.000	49.2±0.2	90.8±2.0	90.3±2.0	—
	256	0.634±0.022	**0.760±0.019**	0.758±0.019	0.854±0.016	0.000±0.000	75.7±0.2	150.7±2.9	149.6±2.9	—
	512	0.960±0.009	**0.984±0.006**	0.984±0.006	0.988±0.005	0.000±0.000	109.3±0.3	195.1±4.5	194.0±4.5	—
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.000±0.000	145.2±0.2	201.2±4.9	200.1±4.9	—
P27 (δ=0.141)	64	0.014±0.005	**0.018±0.006**	0.022±0.007	0.176±0.017	0.000±0.000	22.5±0.1	47.9±3.9	53.4±2.9	—
gold=223, mode=13 ✗	128	0.134±0.015	**0.182±0.017**	0.234±0.019	0.438±0.022	0.000±0.000	33.3±0.2	97.9±2.4	95.6±2.0	—
	256	0.594±0.022	**0.668±0.021**	0.780±0.019	0.808±0.018	0.000±0.000	48.7±0.2	168.6±3.0	160.6±2.7	—
	512	0.974±0.007	**0.986±0.005**	0.996±0.003	0.984±0.006	0.000±0.000	68.2±0.2	223.5±4.5	195.2±3.8	—
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.000±0.000	89.5±0.2	228.3±4.8	196.7±3.9	—
P17 (δ=0.141)	64	0.040±0.009	0.038±0.009	0.066±0.011	0.242±0.019	0.000±0.000	19.4±0.1	52.2±1.9	52.9±1.3	—
gold=243, mode=32 ✗	128	0.180±0.017	**0.204±0.018**	0.364±0.022	0.474±0.022	0.000±0.000	26.4±0.1	93.8±2.6	90.2±1.8	—
	256	0.634±0.022	**0.668±0.021**	0.900±0.013	0.802±0.018	0.000±0.000	34.7±0.1	160.1±3.0	139.9±2.4	—
	512	0.962±0.009	**0.974±0.007**	1.000±0.000	0.968±0.008	0.000±0.000	43.7±0.1	215.4±4.6	155.8±3.1	—
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.000±0.000	53.0±0.1	226.6±5.5	155.8±3.1	—
P23 (δ=0.068)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.020±0.006	0.000±0.000	21.3±0.1	—	—	—
gold=245, mode=20 ✗	128	0.006±0.003	**0.008±0.004**	0.008±0.004	0.104±0.014	0.000±0.000	28.6±0.1	103.8±12.6	103.8±12.6	—
	256	0.044±0.009	**0.068±0.011**	0.074±0.012	0.276±0.020	0.000±0.000	37.5±0.1	196.2±8.0	196.3±7.6	—
	512	0.262±0.020	**0.360±0.021**	0.368±0.022	0.616±0.022	0.000±0.000	47.7±0.1	359.2±7.7	356.4±7.8	—
	1024	0.744±0.020	**0.814±0.017**	0.808±0.018	0.882±0.014	0.000±0.000	57.1±0.1	566.0±11.1	555.4±11.0	—
P21 (δ=0.039)	64	0.004±0.003	0.002±0.002	0.002±0.002	0.080±0.012	0.024±0.007	8.7±0.1	48.0±0.0	48.0±0.0	24.8±5.2
gold=50, mode=9 ✗	128	0.002±0.002	**0.004±0.003**	0.004±0.003	0.098±0.013	0.032±0.008	13.7±0.1	60.5±12.5	60.5±12.5	39.5±7.9
	256	0.000±0.000	**0.006±0.003**	0.006±0.003	0.120±0.015	0.034±0.008	21.0±0.1	115.7±55.6	115.7±55.6	47.6±11.0
	512	0.004±0.003	**0.012±0.005**	0.012±0.005	0.176±0.017	0.038±0.009	30.5±0.1	236.7±62.7	236.7±62.7	76.2±22.1
	1024	0.020±0.006	**0.034±0.008**	0.036±0.008	0.264±0.020	0.054±0.010	40.8±0.1	586.9±75.8	574.6±69.5	274.6±64.2
P4 (δ=0.026)	64	0.000±0.000	**0.002±0.002**	0.002±0.002	0.066±0.011	0.002±0.002	9.9±0.1	19.0±0.0	19.0±0.0	60.0±0.0
gold=70, mode=65 ✗	128	0.002±0.002	**0.004±0.003**	0.004±0.003	0.088±0.013	0.004±0.003	12.7±0.1	58.5±39.5	58.5±39.5	67.0±7.0
	256	0.006±0.003	**0.008±0.004**	0.006±0.003	0.092±0.013	0.010±0.004	16.0±0.1	146.0±53.3	121.3±66.8	129.8±26.4
	512	0.016±0.006	0.016±0.006	0.014±0.005	0.114±0.014	0.014±0.005	19.8±0.1	245.5±47.1	249.1±54.2	176.4±35.4
	1024	0.014±0.005	**0.022±0.007**	0.022±0.007	0.178±0.017	0.026±0.007	23.6±0.1	373.4±75.7	410.4±77.3	392.5±72.4
P13 (δ=0.022)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.002	0.000±0.000	8.5±0.1	—	—	—
gold=39, mode=502 ✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.004±0.003	0.000±0.000	10.2±0.1	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.004±0.003	0.000±0.000	12.2±0.1	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.018±0.006	0.000±0.000	13.9±0.0	—	—	—
	1024	0.002±0.002	0.002±0.002	0.002±0.002	0.034±0.008	0.000±0.000	14.8±0.0	661.0±0.0	661.0±0.0	—
P14 (δ=0.012)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	29.8±0.1	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	42.5±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.018±0.006	0.000±0.000	58.5±0.2	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.034±0.008	0.000±0.000	77.7±0.2	—	—	—
	1024	0.000±0.000	**0.004±0.003**	0.000±0.000	0.060±0.011	0.000±0.000	97.3±0.2	861.5±134.5	—	—
P9 (δ=0.002)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	35.0±0.1	—	—	—
gold=29, mode=13 ✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	53.1±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.002	0.000±0.000	77.5±0.2	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.016±0.006	0.000±0.000	108.4±0.3	—	—	—
	1024	0.000±0.000	**0.006±0.003**	0.000±0.000	0.028±0.007	0.000±0.000	142.0±0.2	798.0±49.8	—	—

FrontierScience-Olympiad.

Table 16. Per-problem certification rate on FrontierScience-Olympiad (10 problems) (Case A, target = mode). **Bold**: CITE > Bonferroni. $\bar{K}_N$ is the Monte Carlo mean number of observed answer categories at budget N.

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P60 (δ=0.763)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.2±0.1	16.1±0.3	17.0±0.3	14.3±0.3
gold=6.98, mode=6.98 ✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.3±0.1	16.1±0.3	17.0±0.3	14.3±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	9.8±0.1	16.1±0.3	17.0±0.3	14.3±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	12.3±0.1	16.1±0.3	17.0±0.3	14.3±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	14.4±0.0	16.1±0.3	17.0±0.3	14.3±0.3
P50 (δ=0.474)	64	0.926±0.012	0.862±0.015	0.474±0.022	0.934±0.011	0.916±0.012	6.8±0.1	33.4±0.7	40.3±0.9	30.0±0.7
gold=0.2, mode=0.2 ✓	128	0.998±0.002	0.994±0.003	0.882±0.014	0.996±0.003	0.994±0.003	9.9±0.1	39.6±0.9	63.7±1.4	33.7±0.9
	256	1.000±0.000	1.000±0.000	0.998±0.002	1.000±0.000	1.000±0.000	14.1±0.1	40.2±1.0	74.9±1.9	34.3±0.9
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	18.7±0.1	40.2±1.0	75.3±1.9	34.3±0.9
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	23.0±0.1	40.2±1.0	75.3±1.9	34.3±0.9
P80 (δ=0.107)	64	0.002±0.002	**0.004±0.003**	0.000±0.000	0.122±0.015	0.000±0.000	38.1±0.2	47.5±6.5	—	—
✓	128	0.052±0.010	**0.124±0.015**	0.000±0.000	0.354±0.021	0.000±0.000	64.6±0.2	103.3±2.4	—	—
	256	0.280±0.020	**0.468±0.022**	0.000±0.000	0.700±0.020	0.000±0.000	104.7±0.3	168.9±3.3	—	—
	512	0.860±0.016	**0.954±0.009**	0.050±0.010	0.952±0.010	0.000±0.000	160.3±0.3	261.3±5.0	448.2±9.4	—
	1024	1.000±0.000	1.000±0.000	0.514±0.022	1.000±0.000	0.000±0.000	224.4±0.3	277.4±5.8	737.7±9.9	—
P30 (δ=0.077)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.022±0.007	0.000±0.000	36.4±0.1	—	—	—
✗	128	0.002±0.002	**0.012±0.005**	0.000±0.000	0.116±0.014	0.000±0.000	56.4±0.2	111.3±9.3	—	—
	256	0.086±0.013	**0.192±0.018**	0.014±0.005	0.370±0.022	0.000±0.000	83.3±0.2	199.0±4.2	234.7±5.1	—
	512	0.500±0.022	**0.700±0.020**	0.338±0.021	0.816±0.017	0.000±0.000	115.9±0.2	332.7±5.7	398.9±5.9	—
	1024	0.984±0.006	**0.990±0.004**	0.878±0.015	0.986±0.005	0.000±0.000	149.9±0.2	422.3±7.9	586.3±9.0	—
P40 (δ=0.036)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.002	0.000±0.000	40.8±0.2	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.038±0.009	0.000±0.000	66.1±0.2	—	—	—
	256	0.000±0.000	**0.022±0.007**	0.000±0.000	0.108±0.014	0.000±0.000	102.0±0.3	221.8±11.4	—	—
	512	0.028±0.007	**0.108±0.014**	0.028±0.007	0.286±0.020	0.000±0.000	149.2±0.3	351.3±13.1	476.7±8.0	—
	1024	0.204±0.018	**0.398±0.022**	0.756±0.019	0.624±0.022	0.000±0.000	203.1±0.3	649.5±16.3	759.0±7.6	—
P90 (δ=0.029)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.002±0.002	0.000±0.000	53.5±0.1	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.016±0.006	0.000±0.000	97.3±0.2	—	—	—
	256	0.000±0.000	**0.006±0.003**	0.000±0.000	0.042±0.009	0.000±0.000	169.0±0.3	221.7±11.6	—	—
	512	0.008±0.004	**0.038±0.009**	0.002±0.002	0.220±0.019	0.000±0.000	276.8±0.4	391.5±20.7	498.0±0.0	—
	1024	0.092±0.013	**0.292±0.020**	0.136±0.015	0.540±0.022	0.000±0.000	412.3±0.4	732.0±15.8	823.7±16.0	—

Table 16 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P10 (δ=0.020)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.004±0.003	0.000±0.000	45.3±0.2	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.026±0.007	0.000±0.000	77.7±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.052±0.010	0.000±0.000	129.6±0.3	—	—	—
	512	0.004±0.003	**0.020±0.006**	0.000±0.000	0.114±0.014	0.000±0.000	206.2±0.4	376.7±25.1	—	—
	1024	0.004±0.003	**0.056±0.010**	0.000±0.000	0.258±0.020	0.000±0.000	300.6±0.4	649.1±44.6	—	—
P70 (δ=0.017)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	41.4±0.2	—	—	—
✓	128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	68.1±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.016±0.006	0.000±0.000	108.3±0.3	—	—	—
	512	0.000±0.000	**0.002±0.002**	0.000±0.000	0.052±0.010	0.000±0.000	163.0±0.3	300.0±0.0	—	—
	1024	0.002±0.002	**0.012±0.005**	0.000±0.000	0.150±0.016	0.000±0.000	227.0±0.3	696.8±105.3	—	—
P20 (δ=0.008)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	48.2±0.2	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.004±0.003	0.000±0.000	84.0±0.2	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.010±0.004	0.000±0.000	141.2±0.3	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.016±0.006	0.000±0.000	225.8±0.4	—	—	—
	1024	0.000±0.000	**0.004±0.003**	0.000±0.000	0.032±0.008	0.000±0.000	329.8±0.4	968.5±22.5	—	—
P100 (δ=0.002)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	61.8±0.1	—	—	—
✗	128	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	119.3±0.1	—	—	—
	256	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	222.9±0.2	—	—	—
	512	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	392.2±0.4	—	—	—
	1024	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	0.000±0.000	618.9±0.4	—	—	—

ER-REASON.

Table 17. Per-problem certification rate on ER-REASON (72 problems) (Case A, target = mode). **Bold**: CITE > Bonferroni. $\bar{K}_N$ is the Monte Carlo mean number of observed answer categories at budget N.

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P8 (δ=1.000)	64	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.1±0.0	7.0±0.0
✗	128	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.1±0.0	7.0±0.0
	256	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.1±0.0	7.0±0.0
	512	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.1±0.0	7.0±0.0
	1024	0.000±0.000	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.0±0.0	9.0±0.0	11.1±0.0	7.0±0.0
P47 (δ=0.996)	64	0.104±0.014	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.1±0.0	9.1±0.0	11.1±0.0	7.1±0.0
✗	128	0.196±0.018	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.2±0.0	9.1±0.0	11.1±0.0	7.1±0.0
	256	0.372±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.4±0.0	9.1±0.0	11.1±0.0	7.1±0.0
	512	0.616±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.6±0.0	9.1±0.0	11.1±0.0	7.1±0.0
	1024	0.870±0.015	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	9.1±0.0	11.1±0.0	7.1±0.0
P54 (δ=0.996)	64	0.106±0.014	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.1±0.0	9.1±0.0	11.1±0.0	7.1±0.0
✓	128	0.208±0.018	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.2±0.0	9.1±0.0	11.1±0.0	7.1±0.0
	256	0.384±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.4±0.0	9.1±0.0	11.1±0.0	7.1±0.0
	512	0.632±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.6±0.0	9.1±0.0	11.1±0.0	7.1±0.0
	1024	0.864±0.015	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	9.1±0.0	11.1±0.0	7.1±0.0
P41 (δ=0.988)	64	0.304±0.021	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.3±0.0	9.3±0.1	11.2±0.0	7.2±0.0
✓	128	0.520±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.5±0.0	9.3±0.1	11.2±0.0	7.2±0.0
	256	0.812±0.017	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.8±0.0	9.3±0.1	11.2±0.0	7.2±0.0
	512	0.962±0.009	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.3±0.1	11.2±0.0	7.2±0.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.3±0.1	11.2±0.0	7.2±0.0
P2 (δ=0.982)	64	0.418±0.022	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.4±0.0	9.3±0.1	11.2±0.0	7.2±0.0
✓	128	0.662±0.021	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.7±0.0	9.3±0.1	11.2±0.0	7.2±0.0
	256	0.878±0.015	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	9.3±0.1	11.2±0.0	7.2±0.0
	512	0.992±0.004	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.3±0.1	11.2±0.0	7.2±0.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	9.3±0.1	11.2±0.0	7.2±0.0
P60 (δ=0.951)	64	0.822±0.017	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	10.1±0.1	12.2±0.1	7.9±0.1
✓	128	0.970±0.008	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.1±0.0	10.1±0.1	12.2±0.1	7.9±0.1
	256	0.998±0.002	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	10.1±0.1	12.2±0.1	7.9±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.4±0.0	10.1±0.1	12.2±0.1	7.9±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.6±0.0	10.1±0.1	12.2±0.1	7.9±0.1
P38 (δ=0.950)	64	0.832±0.017	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.1±0.0	10.4±0.1	12.3±0.1	8.2±0.1
✓	128	0.968±0.008	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.5±0.0	10.4±0.1	12.3±0.1	8.2±0.1
	256	0.998±0.002	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.8±0.0	10.4±0.1	12.3±0.1	8.2±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	10.4±0.1	12.3±0.1	8.2±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	10.4±0.1	12.3±0.1	8.2±0.1
P52 (δ=0.944)	64	0.852±0.016	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.5±0.1	12.7±0.1	8.1±0.1
✗	128	0.974±0.007	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	10.5±0.1	12.7±0.1	8.1±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.4±0.0	10.5±0.1	12.7±0.1	8.1±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.6±0.0	10.5±0.1	12.7±0.1	8.1±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.9±0.0	10.5±0.1	12.7±0.1	8.1±0.1
P5 (δ=0.940)	64	0.874±0.015	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	10.5±0.1	12.4±0.1	8.1±0.1
✓	128	0.984±0.006	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.5±0.1	12.4±0.1	8.1±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.5±0.1	12.4±0.1	8.1±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.5±0.1	12.4±0.1	8.1±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.5±0.1	12.4±0.1	8.1±0.1
P45 (δ=0.940)	64	0.874±0.015	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	1.9±0.0	10.4±0.1	12.5±0.1	8.1±0.1
✗	128	0.992±0.004	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.4±0.1	12.5±0.1	8.1±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.4±0.1	12.5±0.1	8.1±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.4±0.1	12.5±0.1	8.1±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	10.4±0.1	12.5±0.1	8.1±0.1
P43 (δ=0.934)	64	0.936±0.011	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	10.7±0.1	11.6±0.1	8.5±0.1
✗	128	0.996±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	3.3±0.0	10.7±0.1	11.6±0.1	8.5±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.7±0.0	10.7±0.1	11.6±0.1	8.5±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	10.7±0.1	11.6±0.1	8.5±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	10.7±0.1	11.6±0.1	8.5±0.1
P34 (δ=0.887)	64	0.970±0.008	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	11.9±0.2	14.0±0.1	9.2±0.1
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.1±0.0	11.9±0.2	14.0±0.1	9.2±0.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	11.9±0.2	14.0±0.1	9.2±0.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.4±0.0	11.9±0.2	14.0±0.1	9.2±0.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	11.9±0.2	14.0±0.1	9.2±0.1
P6 (δ=0.880)	64	0.992±0.004	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.9±0.0	12.5±0.2	14.7±0.2	9.9±0.2
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.1±0.0	12.5±0.2	14.7±0.2	9.9±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.2±0.0	12.5±0.2	14.7±0.2	9.9±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.4±0.0	12.5±0.2	14.7±0.2	9.9±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.6±0.0	12.5±0.2	14.7±0.2	9.9±0.2

Table 17 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\mathrm{CITE}}$	$\bar{\tau}_{\mathrm{W\text{-}CITE}}$	$\bar{\tau}_{\mathrm{MMC}}$
P22 (δ=0.875)	64	0.990±0.004	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	12.5±0.2	15.1±0.1	10.1±0.2
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.3±0.0	12.5±0.2	15.1±0.1	10.1±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.6±0.0	12.5±0.2	15.1±0.1	10.1±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	12.5±0.2	15.1±0.1	10.1±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	12.5±0.2	15.1±0.1	10.1±0.2
P10 (δ=0.872)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.5±0.0	12.7±0.2	14.4±0.1	10.3±0.2
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	12.7±0.2	14.4±0.1	10.3±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.2±0.0	12.7±0.2	14.4±0.1	10.3±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.4±0.0	12.7±0.2	14.4±0.1	10.3±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.6±0.0	12.7±0.2	14.4±0.1	10.3±0.2
P28 (δ=0.861)	64	0.996±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	4.4±0.0	13.5±0.2	15.8±0.1	11.5±0.2
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.2±0.0	13.5±0.2	15.8±0.1	11.5±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.6±0.0	13.5±0.2	15.8±0.1	11.5±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.9±0.0	13.5±0.2	15.8±0.1	11.5±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	13.5±0.2	15.8±0.1	11.5±0.2
P26 (δ=0.858)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.5±0.0	13.4±0.2	15.8±0.2	11.2±0.2
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	13.4±0.2	15.8±0.2	11.2±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.2±0.0	13.4±0.2	15.8±0.2	11.2±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.4±0.0	13.4±0.2	15.8±0.2	11.2±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.6±0.0	13.4±0.2	15.8±0.2	11.2±0.2
P25 (δ=0.856)	64	0.996±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	3.1±0.0	13.3±0.2	15.2±0.2	10.9±0.2
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.4±0.0	13.3±0.2	15.2±0.2	10.9±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.8±0.0	13.3±0.2	15.2±0.2	10.9±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.2±0.0	13.3±0.2	15.2±0.2	10.9±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.6±0.0	13.3±0.2	15.2±0.2	10.9±0.2
P31 (δ=0.822)	64	0.996±0.003	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.5±0.0	14.0±0.2	14.6±0.2	10.7±0.2
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	14.0±0.2	14.6±0.2	10.7±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.5±0.0	14.0±0.2	14.6±0.2	10.7±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	14.0±0.2	14.6±0.2	10.7±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.5±0.0	14.0±0.2	14.6±0.2	10.7±0.2
P21 (δ=0.817)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.2±0.0	14.6±0.2	16.3±0.2	12.1±0.2
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.6±0.0	14.6±0.2	16.3±0.2	12.1±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.1±0.0	14.6±0.2	16.3±0.2	12.1±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.7±0.0	14.6±0.2	16.3±0.2	12.1±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.2±0.0	14.6±0.2	16.3±0.2	12.1±0.2
P1 (δ=0.800)	64	0.998±0.002	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	15.0±0.3	16.6±0.2	11.3±0.3
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	15.0±0.3	16.6±0.2	11.3±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	15.0±0.3	16.6±0.2	11.3±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	15.0±0.3	16.6±0.2	11.3±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	15.0±0.3	16.6±0.2	11.3±0.3
P24 (δ=0.790)	64	0.998±0.002	**1.000±0.000**	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	15.3±0.3	17.3±0.2	11.8±0.2
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.1±0.0	15.3±0.3	17.3±0.2	11.8±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.7±0.0	15.3±0.3	17.3±0.2	11.8±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.2±0.0	15.3±0.3	17.3±0.2	11.8±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.6±0.0	15.3±0.3	17.3±0.2	11.8±0.2
P71 (δ=0.781)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.6±0.0	15.5±0.3	16.0±0.3	11.9±0.2
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.8±0.0	15.5±0.3	16.0±0.3	11.9±0.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	15.5±0.3	16.0±0.3	11.9±0.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	15.5±0.3	16.0±0.3	11.9±0.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	15.5±0.3	16.0±0.3	11.9±0.2
P53 (δ=0.771)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	16.6±0.3	18.2±0.2	14.7±0.3
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.5±0.0	16.6±0.3	18.2±0.2	14.7±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.9±0.0	16.6±0.3	18.2±0.2	14.7±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.0±0.0	16.6±0.3	18.2±0.2	14.7±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.0±0.0	16.6±0.3	18.2±0.2	14.7±0.3
P56 (δ=0.757)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.1±0.0	17.4±0.4	18.0±0.3	13.0±0.3
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.1±0.0	17.4±0.4	18.0±0.3	13.0±0.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.2±0.0	17.4±0.4	18.0±0.3	13.0±0.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.4±0.0	17.4±0.4	18.0±0.3	13.0±0.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	17.4±0.4	18.0±0.3	13.0±0.3
P11 (δ=0.691)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.998±0.002	5.3±0.0	20.3±0.3	21.1±0.3	22.9±0.6
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.7±0.0	20.3±0.3	21.1±0.3	23.0±0.6
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.9±0.0	20.3±0.3	21.1±0.3	23.0±0.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	20.3±0.3	21.1±0.3	23.0±0.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	20.3±0.3	21.1±0.3	23.0±0.6
P46 (δ=0.679)	64	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	21.1±0.4	21.2±0.3	18.9±0.4
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	21.1±0.4	21.2±0.3	18.9±0.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	21.1±0.4	21.2±0.3	18.9±0.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	21.1±0.4	21.2±0.3	18.9±0.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	21.1±0.4	21.2±0.3	18.9±0.4
P15 (δ=0.666)	64	1.000±0.000	0.996±0.003	0.996±0.003	0.998±0.002	0.998±0.002	4.8±0.0	21.3±0.4	22.9±0.4	19.8±0.4
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.3±0.0	21.5±0.4	23.1±0.4	19.9±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.7±0.0	21.5±0.4	23.1±0.4	19.9±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.9±0.0	21.5±0.4	23.1±0.4	19.9±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	21.5±0.4	23.1±0.4	19.9±0.5
P42 (δ=0.655)	64	1.000±0.000	0.998±0.002	0.996±0.003	0.998±0.002	0.994±0.003	2.7±0.0	22.0±0.5	23.3±0.4	16.8±0.4
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.9±0.0	22.1±0.5	23.5±0.4	17.1±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	22.1±0.5	23.5±0.4	17.1±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	22.1±0.5	23.5±0.4	17.1±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	22.1±0.5	23.5±0.4	17.1±0.5
P68 (δ=0.645)	64	1.000±0.000	0.998±0.002	0.998±0.002	0.998±0.002	0.998±0.002	3.3±0.0	23.0±0.5	24.2±0.4	18.1±0.4
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.7±0.0	23.1±0.5	24.3±0.4	18.2±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	23.1±0.5	24.3±0.4	18.2±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	23.1±0.5	24.3±0.4	18.2±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	23.1±0.5	24.3±0.4	18.2±0.5
P51 (δ=0.642)	64	1.000±0.000	0.994±0.003	0.998±0.002	0.994±0.003	0.998±0.002	2.3±0.0	22.8±0.5	24.3±0.4	17.0±0.5
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.4±0.0	23.1±0.6	24.4±0.4	17.1±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.7±0.0	23.1±0.6	24.4±0.4	17.1±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.2±0.0	23.1±0.6	24.4±0.4	17.1±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.6±0.0	23.1±0.6	24.4±0.4	17.1±0.5
P16 (δ=0.602)	64	1.000±0.000	0.992±0.004	0.994±0.003	0.992±0.004	0.998±0.002	3.3±0.0	26.5±0.6	26.7±0.5	20.7±0.5
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.7±0.0	26.8±0.6	26.9±0.5	20.8±0.5
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.1±0.0	26.8±0.6	26.9±0.5	20.8±0.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.4±0.0	26.8±0.6	26.9±0.5	20.8±0.5
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.6±0.0	26.8±0.6	26.9±0.5	20.8±0.5
P62 (δ=0.573)	64	0.998±0.002	0.972±0.007	0.984±0.006	0.976±0.007	0.980±0.006	4.1±0.0	27.4±0.5	28.2±0.5	26.1±0.6

Table 17 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\mathrm{CITE}}$	$\bar{\tau}_{\mathrm{W\text{-}CITE}}$	$\bar{\tau}_{\mathrm{MMC}}$
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.2±0.0	28.6±0.6	28.9±0.5	26.9±0.6
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.4±0.0	28.6±0.6	28.9±0.5	26.9±0.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.7±0.0	28.6±0.6	28.9±0.5	26.9±0.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.9±0.0	28.6±0.6	28.9±0.5	26.9±0.6
P32 (δ=0.566)	64	0.992±0.004	0.958±0.009	0.952±0.010	0.980±0.006	0.974±0.007	2.0±0.0	29.0±0.7	30.6±0.6	21.8±0.6
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	31.0±0.8	33.0±0.8	23.2±0.7
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	31.0±0.8	33.0±0.8	23.2±0.7
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	31.0±0.8	33.0±0.8	23.2±0.7
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	31.0±0.8	33.0±0.8	23.2±0.7
P50 (δ=0.545)	64	0.994±0.003	0.984±0.006	0.982±0.006	0.976±0.007	0.974±0.007	3.3±0.0	30.4±0.6	31.8±0.5	27.2±0.6
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.5±0.0	31.1±0.6	32.5±0.6	28.3±0.7
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	31.1±0.6	32.5±0.6	28.3±0.7
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.5±0.0	31.1±0.6	32.5±0.6	28.3±0.7
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.2±0.0	31.1±0.6	32.5±0.6	28.3±0.7
P17 (δ=0.538)	64	0.982±0.006	0.938±0.011	0.928±0.012	0.968±0.008	0.952±0.010	3.2±0.0	29.7±0.7	31.5±0.6	22.9±0.6
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.7±0.0	32.7±0.8	34.8±0.8	25.5±0.8
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	32.7±0.8	34.8±0.8	25.5±0.8
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	32.7±0.8	34.8±0.8	25.5±0.8
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	32.7±0.8	34.8±0.8	25.5±0.8
P14 (δ=0.504)	64	0.976±0.007	0.920±0.012	0.890±0.014	0.962±0.009	0.862±0.015	5.1±0.0	34.2±0.6	36.9±0.6	35.0±0.8
✗	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.998±0.002	6.1±0.0	37.7±0.8	41.4±0.8	41.4±1.0
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.9±0.0	37.7±0.8	41.4±0.8	41.6±1.0
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.7±0.0	37.7±0.8	41.4±0.8	41.6±1.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	8.4±0.0	37.7±0.8	41.4±0.8	41.6±1.0
P20 (δ=0.502)	64	0.960±0.009	0.880±0.015	0.898±0.014	0.934±0.011	0.910±0.013	3.7±0.0	32.7±0.7	34.1±0.6	26.3±0.7
✗	128	1.000±0.000	0.998±0.002	1.000±0.000	0.998±0.002	1.000±0.000	4.7±0.0	38.1±0.9	38.7±0.9	31.2±1.0
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.1	38.3±1.0	38.7±0.9	31.2±1.0
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	7.2±0.0	38.3±1.0	38.7±0.9	31.2±1.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	8.1±0.0	38.3±1.0	38.7±0.9	31.2±1.0
P3 (δ=0.493)	64	0.972±0.007	0.936±0.011	0.932±0.011	0.960±0.009	0.728±0.020	5.8±0.0	31.9±0.6	33.7±0.5	36.0±0.8
✓	128	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.982±0.006	6.0±0.0	34.9±0.8	36.8±0.7	48.6±1.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	34.9±0.8	36.8±0.7	50.3±1.3
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	34.9±0.8	36.8±0.7	50.3±1.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	34.9±0.8	36.8±0.7	50.3±1.3
P40 (δ=0.458)	64	0.926±0.012	0.760±0.019	0.788±0.018	0.888±0.014	0.834±0.017	2.0±0.0	33.9±0.8	34.5±0.7	26.8±0.8
✗	128	0.998±0.002	0.986±0.005	0.990±0.004	0.986±0.005	0.984±0.006	2.0±0.0	44.9±1.1	43.8±1.0	35.3±1.2
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	46.5±1.3	45.0±1.2	37.3±1.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	46.5±1.3	45.0±1.2	37.3±1.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	46.5±1.3	45.0±1.2	37.3±1.4
P49 (δ=0.458)	64	0.910±0.013	0.716±0.020	0.690±0.021	0.868±0.015	0.814±0.017	2.0±0.0	34.5±0.8	35.8±0.8	27.4±0.8
✗	128	1.000±0.000	0.976±0.007	0.972±0.007	0.978±0.007	0.978±0.007	2.0±0.0	47.7±1.2	50.3±1.2	37.4±1.3
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	49.9±1.4	53.0±1.4	39.7±1.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	49.9±1.4	53.0±1.4	39.7±1.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	49.9±1.4	53.0±1.4	39.7±1.4
P59 (δ=0.430)	64	0.888±0.014	0.772±0.019	0.782±0.018	0.868±0.015	0.780±0.019	4.5±0.0	37.4±0.7	37.7±0.7	37.3±0.8
✗	128	1.000±0.000	0.986±0.005	0.986±0.005	0.994±0.003	0.986±0.005	5.0±0.0	47.4±1.1	46.8±1.0	46.8±1.1
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.4±0.0	48.7±1.2	48.0±1.1	48.2±1.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.6±0.0	48.7±1.2	48.0±1.1	48.2±1.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.9±0.0	48.7±1.2	48.0±1.1	48.2±1.2
P33 (δ=0.428)	64	0.910±0.013	0.808±0.018	0.786±0.018	0.874±0.015	0.538±0.022	5.3±0.0	37.9±0.7	38.1±0.6	39.8±1.0
✗	128	0.998±0.002	0.996±0.003	0.992±0.004	0.992±0.004	0.942±0.010	5.6±0.0	46.0±1.0	47.0±1.0	60.9±1.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.8±0.0	46.4±1.0	47.8±1.0	65.8±1.6
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	46.4±1.0	47.8±1.0	65.8±1.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	6.0±0.0	46.4±1.0	47.8±1.0	65.8±1.6
P29 (δ=0.392)	64	0.760±0.019	0.520±0.022	0.538±0.022	0.776±0.019	0.634±0.022	2.9±0.0	36.7±1.0	37.9±0.9	30.3±1.0
✓	128	0.986±0.005	0.926±0.012	0.936±0.011	0.948±0.010	0.940±0.011	3.4±0.0	60.6±1.5	59.7±1.4	50.2±1.6
	256	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.8±0.0	67.4±1.8	65.7±1.7	56.7±1.9
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	67.4±1.8	65.7±1.7	56.7±1.9
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	67.4±1.8	65.7±1.7	56.7±1.9
P37 (δ=0.380)	64	0.746±0.019	0.482±0.022	0.486±0.022	0.746±0.019	0.604±0.022	2.0±0.0	36.5±1.0	38.1±0.9	29.3±1.0
✗	128	0.972±0.007	0.894±0.014	0.902±0.013	0.928±0.012	0.914±0.013	2.0±0.0	62.0±1.5	62.4±1.4	51.0±1.6
	256	1.000±0.000	0.998±0.002	0.998±0.002	1.000±0.000	0.998±0.002	2.0±0.0	72.0±1.9	71.7±1.9	60.8±2.1
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	72.4±2.0	72.1±1.9	61.2±2.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	2.0±0.0	72.4±2.0	72.1±1.9	61.2±2.2
P12 (δ=0.364)	64	0.748±0.019	0.508±0.022	0.554±0.022	0.742±0.020	0.586±0.022	3.0±0.0	40.7±0.9	40.6±0.8	38.3±0.9
✗	128	0.984±0.006	0.914±0.013	0.950±0.010	0.940±0.011	0.938±0.011	3.1±0.0	62.9±1.4	60.5±1.3	57.9±1.4
	256	1.000±0.000	1.000±0.000	1.000±0.000	0.994±0.003	1.000±0.000	3.2±0.0	70.8±1.7	65.5±1.6	64.5±1.8
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.4±0.0	70.8±1.7	65.5±1.6	64.5±1.8
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.6±0.0	70.8±1.7	65.5±1.6	64.5±1.8
P30 (δ=0.362)	64	0.702±0.020	0.464±0.022	0.492±0.022	0.680±0.021	0.580±0.022	2.3±0.0	39.9±1.1	39.6±1.0	31.4±1.0
✗	128	0.960±0.009	0.854±0.016	0.884±0.014	0.920±0.012	0.886±0.014	2.5±0.0	64.1±1.5	63.0±1.5	53.2±1.7
	256	1.000±0.000	0.998±0.002	0.998±0.002	0.994±0.003	0.998±0.002	2.8±0.0	78.6±2.1	74.1±1.9	65.9±2.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	78.9±2.1	74.5±2.0	66.3±2.3
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	78.9±2.1	74.5±2.0	66.3±2.3
P9 (δ=0.331)	64	0.588±0.022	0.418±0.022	0.462±0.022	0.680±0.021	0.542±0.022	3.2±0.0	40.9±1.1	41.3±0.9	32.7±1.0
✓	128	0.932±0.011	0.780±0.019	0.832±0.017	0.876±0.015	0.824±0.017	3.5±0.0	65.9±1.6	65.0±1.5	53.8±1.7
	256	1.000±0.000	0.988±0.005	0.994±0.003	0.992±0.004	0.988±0.005	3.7±0.0	85.8±2.2	80.2±2.0	72.1±2.4
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	88.2±2.4	81.5±2.2	74.7±2.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	4.0±0.0	88.2±2.4	81.5±2.2	74.7±2.6
P65 (δ=0.330)	64	0.576±0.022	0.424±0.022	0.458±0.022	0.688±0.021	0.536±0.022	3.9±0.0	38.9±1.1	39.3±0.9	32.2±1.0
✗	128	0.904±0.013	0.782±0.018	0.822±0.017	0.876±0.015	0.802±0.018	4.5±0.0	64.2±1.6	62.5±1.5	52.9±1.7
	256	0.998±0.002	0.984±0.006	0.998±0.002	0.990±0.004	0.986±0.005	4.9±0.0	84.8±2.3	80.4±2.2	74.6±2.5
	512	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.3±0.0	88.1±2.6	80.9±2.3	77.7±2.8
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.7±0.0	88.1±2.6	80.9±2.3	77.7±2.8
P72 (δ=0.288)	64	0.462±0.022	0.238±0.019	0.230±0.019	0.530±0.022	0.350±0.021	2.3±0.0	36.1±1.5	37.0±1.4	28.1±1.3
✗	128	0.796±0.018	0.570±0.022	0.568±0.022	0.756±0.019	0.614±0.022	2.7±0.0	71.8±2.1	72.1±2.0	57.1±2.2
	256	0.984±0.006	0.942±0.010	0.944±0.010	0.960±0.009	0.926±0.012	3.1±0.0	113.1±2.8	113.5±2.8	97.8±3.2
	512	1.000±0.000	1.000±0.000	1.000±0.000	0.994±0.003	1.000±0.000	3.6±0.0	125.3±3.5	125.3±3.5	115.0±4.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.9±0.0	125.3±3.5	125.3±3.5	115.0±4.1
P63 (δ=0.280)	64	0.442±0.022	0.250±0.019	0.280±0.020	0.516±0.022	0.346±0.021	3.0±0.0	41.8±1.3	42.6±1.1	36.6±1.2
✗	128	0.832±0.017	0.620±0.022	0.668±0.021	0.796±0.018	0.668±0.021	3.0±0.0	74.4±1.8	73.9±1.7	64.7±1.8
	256	0.994±0.003	0.958±0.009	0.966±0.008	0.954±0.009	0.956±0.009	3.0±0.0	109.2±2.6	104.2±2.5	98.0±2.8
	512	1.000±0.000	0.998±0.002	0.998±0.002	0.998±0.002	0.998±0.002	3.0±0.0	117.2±3.1	110.8±3.0	107.2±3.4
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	118.2±3.2	111.8±3.1	108.2±3.5
P55 (δ=0.233)	64	0.276±0.020	0.158±0.016	0.158±0.016	0.432±0.022	0.274±0.020	3.6±0.0	39.8±1.6	39.9±1.5	37.6±1.4
✗	128	0.600±0.022	0.414±0.022	0.412±0.022	0.664±0.021	0.514±0.022	4.1±0.0	77.4±2.3	76.8±2.3	64.4±2.1
	256	0.938±0.011	0.828±0.017	0.826±0.017	0.874±0.015	0.846±0.016	4.5±0.0	132.3±3.2	132.9±3.2	114.3±3.5

Table 17 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
	512	0.998±0.002	0.992±0.004	0.992±0.004	0.994±0.003	0.988±0.005	4.8±0.0	163.6±4.2	164.8±4.3	146.4±4.7
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.0±0.0	167.2±4.6	168.4±4.6	151.5±5.1
P19 (δ=0.228)	64	0.250±0.019	0.124±0.015	0.130±0.015	0.408±0.022	0.226±0.019	2.9±0.0	35.9±2.1	38.8±1.7	28.1±1.6
✗	128	0.506±0.022	0.302±0.021	0.322±0.021	0.616±0.022	0.386±0.022	3.6±0.0	72.9±2.9	73.0±2.6	56.8±2.8
	256	0.854±0.016	0.702±0.020	0.728±0.020	0.828±0.017	0.710±0.020	4.6±0.0	138.9±3.6	139.0±3.6	117.6±4.1
	512	0.988±0.005	0.968±0.008	0.980±0.006	0.982±0.006	0.960±0.009	5.5±0.0	192.7±5.0	189.2±4.9	175.2±5.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	5.9±0.0	205.6±5.8	197.6±5.5	192.3±6.5
P23 (δ=0.224)	64	0.290±0.020	0.156±0.016	0.152±0.016	0.382±0.022	0.234±0.019	3.0±0.0	44.9±1.8	45.3±1.6	42.1±1.5
✗	128	0.620±0.022	0.370±0.022	0.370±0.022	0.634±0.022	0.428±0.022	3.0±0.0	75.0±2.3	75.3±2.2	65.3±2.1
	256	0.944±0.010	0.802±0.018	0.798±0.018	0.886±0.014	0.770±0.019	3.0±0.0	135.6±3.3	135.6±3.3	119.2±3.5
	512	0.996±0.003	0.988±0.005	0.988±0.005	0.996±0.003	0.984±0.006	3.0±0.0	172.0±4.5	171.9±4.4	164.4±4.9
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	3.0±0.0	177.4±4.9	177.3±4.9	172.3±5.6
P36 (δ=0.224)	64	0.236±0.019	0.142±0.016	0.136±0.015	0.428±0.022	0.202±0.018	4.3±0.0	38.1±1.8	39.2±1.6	32.5±1.7
✗	128	0.460±0.022	0.334±0.021	0.334±0.021	0.612±0.022	0.382±0.022	5.4±0.0	72.0±2.6	73.1±2.5	62.5±2.6
	256	0.828±0.017	0.720±0.020	0.720±0.020	0.812±0.017	0.702±0.020	6.4±0.0	136.9±3.7	137.8±3.6	121.1±3.9
	512	0.984±0.006	0.970±0.008	0.970±0.008	0.980±0.006	0.960±0.009	7.1±0.0	189.4±5.1	189.3±5.0	181.5±5.6
	1024	1.000±0.000	1.000±0.000	1.000±0.000	0.998±0.002	1.000±0.000	7.6±0.0	202.9±6.1	203.2±6.1	200.3±6.9
P64 (δ=0.215)	64	0.242±0.019	0.098±0.013	0.102±0.014	0.358±0.021	0.198±0.018	2.1±0.0	35.4±2.4	38.5±2.1	29.3±1.8
✓	128	0.508±0.022	0.276±0.020	0.292±0.020	0.552±0.022	0.360±0.021	2.1±0.0	77.6±3.0	77.2±2.7	61.1±2.9
	256	0.878±0.015	0.638±0.021	0.676±0.021	0.782±0.018	0.624±0.022	2.2±0.0	141.7±3.8	140.7±3.6	116.6±4.3
	512	0.992±0.004	0.946±0.010	0.954±0.009	0.968±0.008	0.932±0.011	2.4±0.0	206.8±5.3	198.2±5.1	193.1±6.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	0.998±0.002	0.998±0.002	2.6±0.0	230.4±6.8	217.7±6.3	223.0±7.7
P39 (δ=0.207)	64	0.186±0.017	0.098±0.013	0.096±0.013	0.340±0.021	0.024±0.007	6.2±0.0	46.1±1.8	47.8±1.7	30.6±4.9
✗	128	0.538±0.022	0.366±0.022	0.332±0.021	0.638±0.021	0.056±0.010	6.5±0.0	84.8±2.2	83.1±2.2	72.4±7.6
	256	0.890±0.014	0.818±0.017	0.780±0.019	0.876±0.015	0.130±0.015	6.9±0.0	138.6±2.9	141.2±3.0	146.6±9.3
	512	0.996±0.003	0.996±0.003	0.986±0.005	0.992±0.004	0.384±0.022	7.3±0.0	172.7±4.2	181.8±4.4	313.9±10.2
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.790±0.018	7.7±0.0	174.8±4.5	187.6±4.9	523.6±12.5
P58 (δ=0.205)	64	0.208±0.018	0.110±0.014	0.116±0.014	0.334±0.021	0.024±0.007	5.0±0.0	47.0±1.9	47.9±1.5	32.7±6.1
✗	128	0.506±0.022	0.340±0.021	0.346±0.021	0.628±0.022	0.088±0.013	5.4±0.0	81.7±2.3	81.4±2.2	77.5±5.1
	256	0.910±0.013	0.776±0.019	0.758±0.019	0.878±0.015	0.342±0.021	5.7±0.0	142.0±3.2	140.0±3.2	168.6±5.0
	512	1.000±0.000	0.996±0.003	0.994±0.003	0.996±0.003	0.814±0.017	6.0±0.0	182.5±4.4	184.3±4.5	288.0±6.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.996±0.003	6.0±0.0	184.1±4.5	186.6±4.7	354.0±8.2
P69 (δ=0.188)	64	0.132±0.015	0.060±0.011	0.152±0.016	0.280±0.020	0.010±0.004	5.4±0.0	43.3±2.5	45.9±1.4	23.4±9.0
✗	128	0.414±0.022	0.240±0.019	0.494±0.022	0.502±0.022	0.014±0.005	5.7±0.0	85.0±2.7	81.1±1.8	41.7±14.1
	256	0.862±0.015	0.712±0.020	0.912±0.013	0.830±0.017	0.018±0.006	5.9±0.0	153.1±3.2	127.6±2.8	71.2±22.6
	512	0.992±0.004	0.984±0.006	0.998±0.002	0.982±0.006	0.024±0.007	6.0±0.0	200.2±4.3	143.3±3.5	142.7±42.1
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.062±0.011	6.0±0.0	207.0±4.9	144.2±3.6	516.9±60.6
P27 (δ=0.179)	64	0.074±0.012	0.030±0.008	0.052±0.010	0.190±0.018	0.004±0.003	6.7±0.0	44.4±4.3	50.9±2.1	7.0±0.0
✗	128	0.366±0.022	0.216±0.018	0.304±0.021	0.476±0.022	0.004±0.003	6.9±0.0	91.7±2.5	90.2±2.0	7.0±0.0
	256	0.836±0.017	0.720±0.020	0.834±0.017	0.814±0.017	0.004±0.003	7.0±0.0	158.8±2.9	149.0±2.7	7.0±0.0
	512	1.000±0.000	0.996±0.003	1.000±0.000	0.990±0.004	0.004±0.003	7.0±0.0	207.7±4.3	177.4±3.7	7.0±0.0
	1024	1.000±0.000	1.000±0.000	1.000±0.000	1.000±0.000	0.004±0.003	7.0±0.0	209.0±4.4	177.4±3.7	7.0±0.0
P7 (δ=0.142)	64	0.088±0.013	0.040±0.009	0.026±0.007	0.160±0.016	0.042±0.009	3.1±0.0	37.6±3.3	38.0±3.7	29.7±3.0
✗	128	0.238±0.019	0.100±0.013	0.072±0.012	0.340±0.021	0.136±0.015	3.1±0.0	76.7±5.1	77.8±5.8	75.9±4.2
	256	0.534±0.022	0.324±0.021	0.212±0.018	0.578±0.022	0.336±0.021	3.2±0.0	162.4±5.3	159.2±6.6	145.0±5.2
	512	0.864±0.015	0.684±0.021	0.560±0.022	0.814±0.017	0.684±0.021	3.4±0.0	272.6±6.8	291.5±7.4	262.2±7.3
	1024	0.994±0.003	0.958±0.009	0.888±0.014	0.964±0.008	0.946±0.010	3.6±0.0	397.8±10.8	451.0±11.7	390.5±11.4
P61 (δ=0.128)	64	0.074±0.012	0.040±0.009	0.032±0.008	0.168±0.017	0.098±0.013	2.0±0.0	28.9±4.0	38.5±3.8	21.4±2.1
✓	128	0.160±0.016	0.082±0.012	0.074±0.012	0.286±0.020	0.140±0.016	2.0±0.0	65.9±6.4	75.3±6.1	44.8±4.7
	256	0.384±0.022	0.174±0.017	0.166±0.017	0.390±0.022	0.204±0.018	2.0±0.0	137.6±8.4	146.2±8.1	91.3±7.8
	512	0.758±0.019	0.388±0.022	0.388±0.022	0.610±0.022	0.372±0.022	2.0±0.0	277.7±10.6	285.6±10.1	226.4±12.2
	1024	0.974±0.007	0.808±0.018	0.812±0.017	0.890±0.014	0.766±0.019	2.0±0.0	522.4±13.8	524.0±13.4	502.3±15.9
P48 (δ=0.119)	64	0.052±0.010	0.016±0.006	0.034±0.008	0.176±0.017	0.070±0.011	2.4±0.0	38.0±6.0	42.8±2.9	26.7±2.8
✗	128	0.118±0.014	0.042±0.009	0.078±0.012	0.242±0.019	0.100±0.013	2.6±0.0	76.0±7.8	76.2±5.4	50.4±5.7
	256	0.256±0.020	0.106±0.014	0.198±0.018	0.382±0.022	0.148±0.016	3.0±0.0	145.9±9.4	150.5±6.9	99.6±9.5
	512	0.574±0.022	0.304±0.021	0.474±0.022	0.580±0.022	0.312±0.021	3.4±0.0	296.3±10.7	285.0±8.8	250.4±13.1
	1024	0.898±0.014	0.716±0.020	0.876±0.015	0.840±0.016	0.658±0.021	3.6±0.0	557.8±14.0	484.7±12.3	512.4±16.1
P18 (δ=0.114)	64	0.042±0.009	0.036±0.008	0.038±0.009	0.148±0.016	0.074±0.012	4.2±0.0	31.1±5.1	33.7±4.0	25.3±2.9
✗	128	0.104±0.014	0.066±0.011	0.104±0.014	0.236±0.019	0.106±0.014	4.8±0.0	61.2±6.8	75.0±5.2	45.3±5.0
	256	0.252±0.019	0.162±0.016	0.230±0.019	0.390±0.022	0.206±0.018	5.0±0.0	138.0±8.1	140.0±6.5	119.4±8.3
	512	0.504±0.022	0.338±0.021	0.500±0.022	0.588±0.022	0.322±0.021	5.0±0.0	263.5±10.9	273.4±9.0	212.7±11.6
	1024	0.890±0.014	0.706±0.020	0.890±0.014	0.866±0.015	0.648±0.021	5.0±0.0	512.8±15.0	473.9±12.6	480.5±16.9
P13 (δ=0.114)	64	0.026±0.007	0.002±0.002	0.002±0.002	0.064±0.011	0.002±0.002	3.0±0.0	49.0±0.0	48.0±0.0	47.0±0.0
✗	128	0.060±0.011	0.014±0.005	0.020±0.006	0.104±0.014	0.016±0.006	3.0±0.0	95.7±10.5	95.1±7.9	89.8±7.8
	256	0.232±0.019	0.082±0.012	0.106±0.014	0.280±0.020	0.084±0.012	3.0±0.0	189.1±8.3	182.0±7.1	176.7±7.9
	512	0.644±0.021	0.298±0.020	0.410±0.022	0.560±0.022	0.262±0.020	3.0±0.0	344.3±9.5	343.9±8.2	331.8±11.0
	1024	0.952±0.010	0.782±0.018	0.868±0.015	0.866±0.015	0.716±0.020	3.0±0.0	584.8±11.7	541.6±11.0	588.6±12.7
P57 (δ=0.109)	64	0.022±0.007	0.012±0.005	0.018±0.006	0.122±0.015	0.002±0.002	7.5±0.0	39.5±4.6	38.9±3.5	64.0±0.0
✗	128	0.106±0.014	0.064±0.011	0.062±0.011	0.240±0.019	0.002±0.002	7.9±0.0	87.4±4.9	83.0±5.8	64.0±0.0
	256	0.270±0.020	0.194±0.018	0.150±0.016	0.448±0.022	0.002±0.002	8.0±0.0	161.5±6.4	148.7±7.7	64.0±0.0
	512	0.602±0.022	0.480±0.022	0.388±0.022	0.702±0.020	0.002±0.002	8.0±0.0	290.1±8.1	295.4±9.8	64.0±0.0
	1024	0.950±0.010	0.884±0.014	0.758±0.019	0.922±0.012	0.002±0.002	8.0±0.0	496.1±12.5	517.0±13.8	64.0±0.0
P44 (δ=0.073)	64	0.002±0.002	0.000±0.000	0.000±0.000	0.028±0.007	0.002±0.002	4.4±0.0	—	—	7.0±0.0
✗	128	0.014±0.005	0.002±0.002	0.000±0.000	0.044±0.009	0.006±0.003	4.8±0.0	127.0±0.0	—	75.3±35.5
	256	0.036±0.008	0.010±0.004	0.000±0.000	0.106±0.014	0.006±0.003	5.3±0.0	178.6±19.8	—	75.3±35.5
	512	0.146±0.016	0.056±0.010	0.008±0.004	0.234±0.019	0.012±0.005	5.9±0.0	364.4±20.0	385.2±54.8	225.7±71.7
	1024	0.454±0.022	0.246±0.019	0.034±0.008	0.512±0.022	0.028±0.007	6.5±0.0	701.6±20.3	694.4±56.4	529.3±85.1
P4 (δ=0.071)	64	0.000±0.000	0.000±0.000	0.000±0.000	0.040±0.009	0.000±0.000	7.1±0.0	—	—	—
✗	128	0.004±0.003	0.000±0.000	0.002±0.002	0.054±0.010	0.000±0.000	7.5±0.0	—	89.0±0.0	—
	256	0.044±0.009	0.016±0.006	0.026±0.007	0.146±0.016	0.000±0.000	7.8±0.0	197.2±13.8	186.0±13.9	—
	512	0.210±0.018	0.110±0.014	0.210±0.018	0.358±0.021	0.000±0.000	8.0±0.0	381.2±13.6	367.8±9.2	—
	1024	0.608±0.022	0.416±0.022	0.678±0.021	0.680±0.021	0.000±0.000	8.0±0.0	678.7±15.5	634.8±12.1	—
P66 (δ=0.070)	64	0.034±0.008	0.016±0.006	0.014±0.005	0.120±0.015	0.052±0.010	2.0±0.0	23.4±6.2	31.1±8.6	25.4±3.8
✗	128	0.072±0.012	0.026±0.007	0.028±0.007	0.120±0.015	0.066±0.011	2.0±0.0	52.0±11.4	62.9±10.0	41.1±6.3
	256	0.088±0.013	0.042±0.009	0.044±0.009	0.186±0.017	0.078±0.012	2.0±0.0	104.8±17.6	103.8±14.3	66.0±11.1
	512	0.222±0.019	0.084±0.012	0.118±0.014	0.262±0.020	0.114±0.014	2.0±0.0	254.7±26.0	288.1±20.8	171.1±22.7
	1024	0.454±0.022	0.174±0.017	0.232±0.019	0.442±0.022	0.172±0.017	2.0±0.0	546.2±34.7	549.0±28.6	383.8±36.8
P35 (δ=0.068)	64	0.028±0.007	0.018±0.006	0.014±0.005	0.114±0.014	0.042±0.009	3.1±0.0	27.2±4.5	30.9±5.3	21.8±2.7
✗	128	0.054±0.010	0.028±0.007	0.022±0.007	0.146±0.016	0.052±0.010	3.6±0.0	51.9±9.8	57.3±11.9	35.5±6.2
	256	0.068±0.011	0.048±0.010	0.028±0.007	0.188±0.017	0.064±0.011	4.1±0.0	108.8±15.5	86.5±18.1	63.1±11.8
	512	0.162±0.016	0.082±0.012	0.042±0.009	0.286±0.020	0.100±0.013	4.4±0.0	227.3±25.3	178.4±33.2	176.8±24.0
	1024	0.334±0.021	0.180±0.017	0.072±0.012	0.444±0.022	0.176±0.017	4.7±0.0	538.3±34.7	439.2±57.1	434.2±36.2
P67 (δ=0.026)	64	0.010±0.004	0.004±0.003	0.004±0.003	0.050±0.010	0.026±0.007	3.4±0.0	34.0±1.0	34.0±1.0	31.2±5.1
✗	128	0.008±0.004	0.006±0.003	0.006±0.003	0.072±0.012	0.028±0.007	3.7±0.0	54.3±20.3	54.0±20.0	35.4±6.3
	256	0.020±0.006	0.010±0.004	0.010±0.004	0.086±0.013	0.030±0.008	3.9±0.0	93.8±26.6	92.2±25.8	43.9±10.3
	512	0.022±0.007	0.012±0.005	0.016±0.006	0.108±0.014	0.032±0.008	4.0±0.0	132.2±44.1	172.9±42.8	62.6±21.1
	1024	0.046±0.009	0.012±0.005	0.018±0.006	0.154±0.016	0.034±0.008	4.0±0.0	132.2±44.1	219.3±59.9	91.1±34.7

Table 17 – continued

Problem	N	Bonf.	CITE	W-CITE	KR	MMC	$\bar{K}_N$	$\bar{\tau}_{\text{CITE}}$	$\bar{\tau}_{\text{W-CITE}}$	$\bar{\tau}_{\text{MMC}}$
P70 (δ=0.004)	64	0.008±0.004	0.004±0.003	0.006±0.003	0.032±0.008	0.008±0.004	3.6±0.0	47.0±7.0	51.0±6.4	41.5±4.4
✗	128	0.006±0.003	0.006±0.003	0.008±0.004	0.044±0.009	0.014±0.005	3.9±0.0	65.0±18.4	63.0±12.8	56.6±7.7
	256	0.002±0.002	**0.006±0.003**	0.008±0.004	0.050±0.010	0.014±0.005	4.4±0.0	65.0±18.4	63.0±12.8	56.6±7.7
	512	0.006±0.003	0.006±0.003	0.012±0.005	0.054±0.010	0.014±0.005	4.8±0.0	65.0±18.4	188.3±80.0	56.6±7.7
	1024	0.012±0.005	0.008±0.004	0.018±0.006	0.058±0.010	0.016±0.006	5.0±0.0	210.8±146.3	361.2±110.3	130.5±74.2

I. Reproducibility Statement

Hardware and inference server. Real-LLM experiments are conducted using NVIDIA A100 80 GB GPUs with vLLM as the inference server.

Data and code access. All code will be open-sourced upon publication. Datasets and models are publicly available (Table 18). ER-REASON requires PhysioNet credentialed access.

Name	Reference
Models	
Qwen3-30B-A3B-Instruct-2507 (Qwen Team, 2025)	`https://huggingface.co/Qwen/Qwen3-30B-A3B-Instruct-2507`
gpt-oss-20b (OpenAI, 2025)	`https://huggingface.co/openai/gpt-oss-20b`
Datasets	
AIME 2026 (Dekoninck et al., 2026)	`https://huggingface.co/datasets/MathArena/aime_2026`
FrontierScience-Olympiad (Wang et al., 2026)	`https://huggingface.co/datasets/openai/frontierscience`
ER-REASON (Mehandru et al., 2025)	`https://physionet.org/content/er-reason/1.0.0/`

Table 18. List of models and datasets used in the LLM self-consistency experiments.