RAGs to Riches: Dual-Blind Retrieval for Distributed RAG

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Abstract

Retrieval-augmented generation (RAG) grounds large language models (LLMs) in external, updatable evidence at inference time, enabling factual and domain-specific responses without retraining. In a third-party RAG marketplace, however, this benefit is hard to realize: an LLM owner may need to retrieve from multiple proprietary providers, while providers must protect confidential corpora and users require strong query confidentiality. We introduce DUAL-BLIND RAG, a *bounded-leakage* decentralized retrieval pipeline for multi-provider RAG under a semi-honest threat model. The pipeline combines (i) query-side obfuscated SimHash screening, (ii) distributed top-candidate and block pruning, and (iii) CKKS-based encrypted reranking on a small final pool. By aggressively pruning before encrypted scoring, the system keeps the expensive stage bounded and concentrates most computation on pruned candidate buffers rather than full corpora. DUAL-BLIND RAG does not reveal raw query or document embeddings during screening and reranking, but it does expose an explicit coordination transcript, including noised query signatures, stage outcomes, and candidate-selection patterns. On MS MARCO, DUAL-BLIND RAG achieves an MRR@10 within 0.03 of a plaintext dense retriever and an nDCG@10 within 8%, while avoiding raw embedding exchange and preserving retrieval quality in decentralized third-party RAG markets.

1 Introduction

Modern machine-learning progress depends not only on algorithmic advances, but also on access to *large*, *fresh*, and *domain-specific* data. Scaling results show that more data improves generalization (Sun et al., 2017; Hoffmann et al., 2022), while data acquisition and curation have become a major production cost in deployed ML systems (Mukherjee and Tong, 2024; Appen Limited and Poll, 2022). This has motivated data marketplaces for trading *raw* datasets (Dawex, 2025; Snowflake Inc., 2025a; Amazon Web Services, 2025). Yet raw-data exchange remains difficult to monetize: buyers must often pay before they can verify utility, while providers cannot easily price which parts of a corpus create downstream value. The result is a classic lemons-market problem that raises upfront costs and depresses trust (Azcoitia and Laoutaris, 2022; Snowflake Inc., 2025b; Akerlof, 1978).

Large language models (LLMs) make this problem even more acute. Their usefulness increasingly depends on access to fresh and specialized knowledge at inference time, and retrieval-augmented generation (RAG) supplies that knowledge on demand. This suggests a different market design: instead of transferring raw corpora, independent providers expose retrieval APIs and an LLM owner purchases only the documents actually used.[1] We call this setting **Third-Party RAG**. It improves freshness and long-tail coverage while keeping proprietary corpora in place, but it introduces two privacy constraints: *(i)* providers should not expose their corpora before purchase, and *(ii)* the LLM owner should not reveal its queries. Naive embedding exchange does not satisfy these constraints, since inversion attacks such as VEC2TEXT and ALGEN can recover text from embeddings even in low-data or black-box settings (Morris et al., 2023; Chen et al., 2025) (§2.1).

We present DUAL-BLIND RAG, a decentralized retrieval pipeline for Third-Party RAG that limits practically exploitable leakage without requiring providers to share raw embeddings or corpora. The design combines *privacy-aware approximate screening*, *distributed cascade pruning*, and

[1] Early commercial instances include AWS Bedrock's BYOI "third-party-RAG" knowledge bases (Amazon Web Services (AWS), 2025), LlamaCloud's managed retrieval service (LlamaIndex Team, 2024), and Voyage AI's retrieval API (Voyage AI, 2025).

encrypted final-stage reranking: providers interact first through query-obfuscated similarity surrogates, the system progressively prunes providers, blocks, and documents through decentralized top-k coordination, and only a small surviving candidate pool is passed to homomorphic encrypted scoring. This confines the expensive encrypted stage to a bounded candidate pool while reducing leakage relative to naive dense-vector exchange. We explicitly characterize per-party leakage and target a bounded-leakage envelope under a semi-honest, non-colluding threat model. Empirically, on MS MARCO (Nguyen et al., 2016), DUAL-BLIND RAG achieves 0.304 MRR@10 (within 9% of dense cosine) and remains within 10% on five out-of-domain BEIR benchmarks (Thakur et al., 2021).

2 Background

RAG couples a *retriever* that surfaces external evidence–typically passages of text, but increasingly images, tables, code, and other modalities–with a *generator* (usually an LLM) that conditions on that evidence to produce the final answer (Lewis et al., 2020; Izacard and Grave, 2021; Zhao et al., 2023). Existing deployments typically use dense dual-encoders for recall (Karpukhin et al., 2020), late-interaction models for robustness, and lightweight rerankers (e.g. ColBERT-v2) (Santhanam et al., 2022) for precision.

2.1 Challenges

Although RAG decouples model and data, it introduces a key hurdle: **privacy of the retrieval channel**. Naive dense indexes can leak private user queries and proprietary corpus contents.

In a third-party RAG marketplace, both the consumer's query and the provider's documents are proprietary; nevertheless, publishing an index built from *raw* embeddings exposes three well-studied attack surfaces. (1) **Membership-inference** asks whether a given string was part of the corpus; shadow-training attacks achieve $> 90\%$ precision on popular sentence encoders (Carlini et al., 2021). (2) **Attribute-inference** (or property inference) estimates sensitive labels—e.g. political stance or authorship information from the embedding alone (Song and Raghunathan, 2020). While damaging, both attacks can sometimes be mitigated by clustering or k-anonymity at modest accuracy loss. (3) A far more serious threat is **embedding inversion**: reconstructing the *plain text* (or a paraphrase) that produced a vector. The seminal work "*Text Embeddings Reveal (Almost) as Much as Text*" (Morris et al., 2023) trains a conditional language model that, given only a sentence embedding, recovers on average 92% of the original content words for MiniLM and 88% for SBERT across six domains. Crucially, the attack performance scales linearly with corpus size—making it perfectly suited to batch–extract an entire data provider's documents once the index is public. This work has inspired further embedding inversion attacks, including multilingual, transferable, and few-shot cross-model variants (Huang et al., 2024; Chen et al., 2024; Tragoudaras et al., 2025; Chen et al., 2025). These results render raw vector publication fundamentally incompatible with confidential third-party RAG, motivating a private retrieval scheme.

2.2 Related Work

Several cryptographic ANN systems for private semantic search may initially appear suitable for closing these leaks, but they face important limitations in the Third-Party RAG setting. *Compass* traverses HNSW inside Path-ORAM, requiring the client to decrypt full ORAM paths at each hop, increasing extra reads beyond the final top-k and client egress (Zhu et al., 2024). *Pacmann* preserves query privacy via client-preprocessing PIR, but retrieving local HNSW subgraphs at each hop still exposes neighborhood structure beyond the purchased documents (Zhou et al., 2025a). *Wally* adds Laplace noise for differential privacy, yet under realistic budgets ($\varepsilon \approx 8$ in legal/ medical settings) document membership remains inferable (Asi et al., 2024).

Two systems, *SANNS* and *Tiptoe*, encrypt both query and corpus but do not fit our third-party RAG setting. SANNS combines FHE with garbled circuits, but its precomputed tables scale only to $\leq 10^7$ vectors and would require parallel execution across providers plus a secure cross-provider top-k merge (Chen et al., 2020). Tiptoe combines linearly homomorphic encryption with cluster pruning and scales to 360M web pages on a *single* provider-controlled cluster, but its cost grows with the number of clusters and it requires a global clustering step that would force providers to share raw embeddings (Henzinger et al., 2023). Beyond ANN protocols, several privacy-oriented RAG systems are orthogonal to our setting: they either assume a centralized trusted store (Privacy-Aware

Table 1: Can any private ANN act as a drop-in third-party RAG retriever? ✓= satisfies the criterion; ✗= fails; Δ = partially (leaks limited side-info).

Scheme (year)	Query private	Docs private	No extra vectors leaked	Global top-k across M prov?	Main technique
Compass '24 (Zhu et al., 2024)	✓	✗	Δ (10–100)	✗	ORAM-shielded HNSW
Pacmann '25 (Zhou et al., 2025a)	✓	✓	Δ (sub-graph)	✗	Piano-PIR over HNSW
Tiptoe '23 (Henzinger et al., 2023)	✓	✓	✓	✗*	LHE + cluster pruning
SANNS '20 (Chen et al., 2020)	✓	✓	✓	✗†	FHE + garbled circuits
Wally '24 (Asi et al., 2024)	✓	✗	✓	✗	DP + partially-HE
Dual-Blind RAG (ours)	✓	✓	✓	✓	LSH + CKKS FHE

* Tiptoe needs a *global* clustering step; providers would have to share raw embeddings, violating document privacy. † SANNS' garbled-circuit LUTs saturate memory beyond $\sim 10^7$ vectors, so per-provider shards cannot be merged securely.

RAG) (Zhou et al., 2025b), operate at training time (FedRAG) (Fajardo et al., 2025), rely on TEEs (C-FedRAG) (Addison et al., 2024), or defend only query privacy without protecting provider-side confidentiality (RemoteRAG) (Cheng et al., 2025).

None of these methods satisfies three core requirements of Third-Party RAG: (i) no raw query/embedding sharing, (ii) disclosure only for the retrieved top-k documents, (iii) composability across many mutually distrusting providers without a shared global graph. Our design satisfies these requirements; Table 1 summarizes the gap.

2.3 Problem Statement

We consider a *decentralized* marketplace with one **LLM owner** $\mathcal{O}$ and a set of $M > 1$ **data-providers** $\{\mathcal{P}_1, \ldots, \mathcal{P}_M\}$. Provider $\mathcal{P}_i$ holds a private document corpus $P_i = \{d_{i,1}, \ldots, d_{i,n_i}\}$ and a vector index $E_i = \{\mathbf{e}_{i,1}, \ldots, \mathbf{e}_{i,n_i}\}$, where $\mathbf{e}_{i,j} = f(d_{i,j}) \in \mathbb{R}^d$ is the embedding of document $d_{i,j}$ under a shared embedding model $f(\cdot)$. The LLM owner stores a black-box generator $\mathcal{M}$ that produces an answer a for the query $\mathbf{q}$ conditioned on a (small) set of retrieved documents $R \subseteq \bigcup_i P_i$.

Privacy constraints: We capture this practical goal in two privacy constraints:

P1 Query privacy (coordination transcript). No provider should learn the plaintext query embedding (or any equivalent function of it) beyond (i) the per-query coordination transcript required by the protocol—namely the (noised) projection signatures it receives, its own pass/-fail outcomes, and its *final encrypted reranking request workload* (i.e., how many/which local candidate identifiers it is asked to score)—and (ii) which of its own documents are ultimately purchased and returned in the final top-k.

P2 Document privacy for non-returned content. The LLM owner should learn no provider plaintext or embeddings beyond the purchased top-k results R. For candidates that are merely *scored* (e.g., in the overshoot set top-L) but not purchased, the owner learns only opaque identifiers and similarity scores (surrogate and exact), not document content or embeddings.

Threat model and explicit leakage. We assume a *semi-honest* (honest-but-curious) setting: the LLM owner and each provider follow the protocol but may try to infer extra information from the transcript. We assume secure channels and no collusion among providers. Throughout, we treat document identifiers as *opaque pseudonyms*; if identifiers or external metadata are linkable to document content, then revealing candidate IDs constitutes additional metadata leakage outside our scope.

Definition (dual-blind retrieval for non-returned content). We use the term *dual-blind* in an operational sense: (i) the owner learns no plaintext documents or document embeddings beyond the purchased top-k, and (ii) each provider learns no plaintext query or raw query embedding, but only the explicit coordination transcript required by the protocol. This notion is evaluated under the semi-honest, non-colluding threat model above.

Objective: Design a retrieval protocol under the stated constraints while minimizing: *(i)* additional leakages beyond **P1 & P2**, *(ii)* computational overhead on each provider, and *(iii)* communication cost per query. §3 details our construction.

3 Dual-Blind Decentralized Retrieval

Our goal is to retrieve the global top-k documents for a query embedding $\mathbf{q} \in \mathbb{R}^d$ held by the LLM owner while (i) avoiding exchange of raw document embeddings across providers and (ii) avoiding disclosure of the raw query embedding to providers. Rather than fully hiding the retrieval transcript, our design exposes only noised signatures, pruning outcomes, and bounded reranking requests. As illustrated in Figure 1, the four stages progressively

narrow the search space: Stage 1 prunes irrelevant providers, Stage 2 prunes irrelevant blocks within them, Stage 3 computes a distributed surrogate top-L over the remaining hot blocks, and Stage 4 applies CKKS only to that bounded pool. All query/document embeddings are d-dimensional and ℓ_2-normalized; we use $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ to denote either a query embedding $\mathbf{q}$ or a document embedding $\mathbf{e}$ depending on context.

3.1 Approximating Cosine Similarity with Random Projection Signatures

Dense retrieval ranks documents by cosine similarity between a query embedding and document embeddings. However, exchanging raw embeddings (even without plaintext) is unsafe: embedding inversion attacks such as VEC2TEXT and ALGEN can reconstruct the underlying text from vectors (Morris et al., 2023; Chen et al., 2025). To enable decentralized screening without revealing raw embeddings, we approximate cosine similarity using locality-sensitive hashing (LSH) signatures. We use Locality Sensitive Hashing (Indyk and Motwani, 1998) (SimHash (Charikar, 2002)) as an embedding-obscuring surrogate for cosine similarity between high-dimensional normalized embeddings via binary random-projection signatures. Given a normalized vector $\mathbf{x} \in \mathbb{R}^d$, we map it to a K-bit (or signed) signature by projecting onto K random hyperplanes and recording the sign of each projection. Concretely, we generate a random projection matrix $W \in \mathbb{R}^{K \times d}$ (Gaussian rows, or orthogonalized when specified in the pipeline) and define the *signed* signature $s(\mathbf{x}) = \operatorname{sign}(W\mathbf{x}) \in \{-1, +1\}^K$, where $s_i(\mathbf{x}) = +1$ iff $\langle \mathbf{x}, \mathbf{r}_i \rangle \geq 0$ and $s_i(\mathbf{x}) = -1$ otherwise, where $\mathbf{r}_i$ is the ith row of W.

The intuition is that two vectors $\mathbf{x}$ and $\mathbf{y}$ which are closely aligned (small angle θ) produce similar sign patterns, while those separated by a large angle disagree on more bits. In particular, for random hyperplanes, the collision probability is $1 - \theta/\pi$. If the Hamming distance between bit-signatures is d_H, then $\mathbb{E}[d_H/K] = \theta/\pi$. A convenient monotone similarity proxy is $\widetilde{\cos}(\mathbf{x}, \mathbf{y}) = 1 - \frac{2\, d_H}{K}$, which equals the normalized signed correlation $\frac{1}{K} \sum_{i=1}^{K} s_i(\mathbf{x})\, s_i(\mathbf{y})$ when using $s(\cdot) \in \{-1, +1\}^K$. Because retrieval only requires ranking, it suffices that $\widetilde{\cos}$ is monotone in the true cosine similarity in expectation, tightening as K increases. Importantly, the pipeline exchanges only signatures—not $\mathbf{x}$ or $\mathbf{y}$—which are many-to-one embedding surrogates. They avoid raw embedding exchange but are not secure encodings, since they still reveal a limited coordination signal and may permit coarse directional inference.

3.2 Private Semantic Similarity Reranking

To compute precise cosine similarities while preserving privacy, we use the CKKS FHE scheme (Cheon et al., 2017); with standard CKKS parameters, decrypted cosine scores closely match plaintext up to the small approximation error of rescaling. The LLM owner generates a key pair $(\mathsf{pk}, \mathsf{sk})$ and encrypts the query embedding $\mathbf{q}$ as $\mathsf{ct}_q = \mathsf{Enc}_{\mathsf{pk}}(\mathbf{q}) \in \mathsf{CKKS}(\Delta)$.

Packed (batched) encrypted dot products. A naive approach would return one CKKS ciphertext per candidate score, which makes bandwidth and ciphertext handling scale linearly with the number of reranked candidates. Rather than returning one ciphertext per candidate, providers use CKKS packing (SIMD) to amortize computation and bandwidth across multiple candidates at once. Given plaintext document embeddings $\{\mathbf{d}_i\}_{i=1}^{j}$, a provider groups them into packs of size p (up to the available CKKS slots) and homomorphically evaluates a *packed* dot-product operation that returns a ciphertext vector of p scores:

$$\begin{aligned} \mathsf{ct}^{(\ell)} &= \mathsf{EvalPackDot}(\mathsf{ct}_q, \mathbf{D}^{(\ell)}) \\ &\approx \mathsf{Enc}_{\mathsf{pk}}\big([\langle \mathbf{q}, \mathbf{d}_{(\ell,1)} \rangle, \ldots, \langle \mathbf{q}, \mathbf{d}_{(\ell,p)} \rangle]\big). \end{aligned}$$

where $\mathbf{D}^{(\ell)} \in \mathbb{R}^{d \times p}$ is the pack matrix whose columns are the p plaintext document embeddings in pack ℓ. The provider returns $\{\mathsf{ct}^{(\ell)}\}_{\ell=1}^{\lceil j/p \rceil}$ to the LLM owner, who decrypts and extracts the packed scores. Since all embeddings are pre-normalized ($\|\mathbf{q}\| = \|\mathbf{d}_i\| = 1$), the decrypted inner products are cosine similarities (up to CKKS numerical precision). Packing reduces the number of returned ciphertexts from j to $\lceil j/p \rceil$ for that provider. In Stage 4, if provider i is asked to score L_i of the global top-L candidates, the total number of returned ciphertexts across all providers is $\sum_i \lceil L_i/p \rceil$ (rather than L); this is smallest when the top-L is concentrated on a few providers and grows with the number of distinct contributing providers.

3.3 Decentralized Private Retrieval Pipeline

We combine the projection-based cosine approximation of §3.1 with the CKKS-based private cosine of §3.2. The four stages progressively narrow the search space: Stage 1 prunes irrelevant

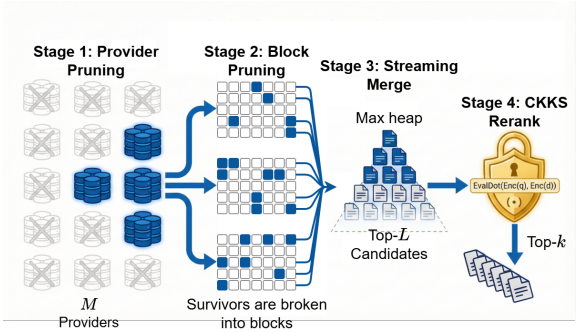

Figure 1: The Dual-Blind retrieval pipeline.

providers, Stage 2 prunes irrelevant blocks within them, Stage 3 computes a distributed surrogate top-L over the remaining hot blocks using a mid-length signature with lazy refinement for heap candidates, and Stage 4 applies CKKS only to that bounded pool. See Appendix A for bounds, derivations, and protocol details.

There are M providers $\mathcal{P}_1, \ldots, \mathcal{P}_M$, each holding a private corpus of document embeddings, and an owner holding a query vector $\mathbf{q} \in \mathbb{R}^d$. Since FHE cosine is too costly at full scale, we select an overshoot set of size $L > k$ using cheap surrogates, reranking only those L candidates by CKKS.

Envelope upper bound: Let $W \in \mathbb{R}^{K \times d}$ be an orthogonal projection and, for readability, write the query signature as the clean signed vector $s_q = \mathrm{sign}(Wq) \in \{-1, +1\}^K$ (computed as in §3.1); in the actual protocol, Stages 1–3 transmit only its obfuscated/noised version, introduced later in §4.2. Each provider precomputes, for every row w_i of W, the extremes over its local set P: $m_i^{\max} = \max_{d \in P} \langle d, w_i \rangle$ and $m_i^{\min} = \min_{d \in P} \langle d, w_i \rangle$. Define:

$$u_i = \begin{cases} m_i^{\max}, & s_q[i] = +1, \\ -m_i^{\min}, & s_q[i] = -1, \end{cases} \quad U(W, P) = \tfrac{1}{K} \textstyle\sum_{i=1}^{K} u_i.$$

Then for any document $d \in P$, the surrogate $s_W(q, d) = \frac{1}{K} \sum_{i=1}^{K} s_q[i] \cdot \langle d, w_i \rangle$ satisfies $s_W(q, d) \leq U(W, P)$; thus $U(W, P)$ is an *envelope upper bound* valid for all $d \in P$. In Stage 1 we use $W = W_c$ with P the provider's corpus; in Stage 2, $W = W_b$ we get surrogate scores per block in P. Moreover, for isotropic random rows w_i, $\mathbb{E}[\, s_q[i] \langle d, w_i \rangle \,] = c \, \langle q, d \rangle$ for some constant $c > 0$ depending only on the row distribution, so $\mathbb{E}[s_W(q, d)]$ is monotone in the true cosine.

Stage 1: Provider envelope (coarse). Using a shared orthogonal projection W_c, the LLM owner sends a K*-bit signed signature* of $\mathbf{q}$ (not $\mathbf{q}$ itself). Each provider returns a single coarse upper bound over its entire corpus. Providers with $U(W_c, P_i) \leq \tau_{\mathcal{P}}$ are discarded.

Stage 2: Block envelope (refined granularity). Surviving providers partition their corpora into blocks, repeating the envelope test with a larger projection W_b: pruning blocks with $U(W_b, b) \leq \tau_B$. The remaining "hot" providers/blocks are exactly known.

Stage 3: Streaming merge via Threshold Algorithm (TA). The LLM owner and surviving providers use two Stage 3 projections: a mid-length projection W_m to cheaply score all documents in the hot blocks, and a longer refined projection W_r used lazily during TA for documents that enter the running heap (and for provider-side next-item bounds), with $K_r > K_m > K_b > K_c$. Providers expose only (i) a streaming PEEK of their next best candidates from hot blocks and (ii) an UB call reporting the best unseen score under the current refined surrogate. The LLM owner maintains a size-L heap and runs a distributed TA loop, freezing any provider whose best unseen score cannot enter the current top-L. When all providers are frozen, the heap contains the global top-L under the refined surrogate.

Stage 4: Exact top-k via packed encrypted reranking. Given the L candidates, the LLM owner encrypts the normalized query $\mathbf{q}$ once as ct_q and sends it to the involved providers, along with the identifiers of the candidates to score. Each provider $\mathcal{P}$ computes encrypted dot products *in packs* of size p using CKKS packing (§3.2), returning only $\lceil L_{\mathcal{P}}/p \rceil$ ciphertexts for its local share of the L candidates (where $L_{\mathcal{P}}$ is the number of requested candidates held by provider $\mathcal{P}$). The LLM owner decrypts the packed scores and selects the exact top-k. Since Stage 4 computes exact cosines only on the L surviving candidates, it recovers the true top-k only if those candidates are preserved by Stages 1–3. Thus, the final exactness guarantee is conditional on the surrogate cascade, whose earlier stages are designed for high recall rather than exact preservation of the global plaintext cosine ranking.

Privacy and safety: As per the leakage envelope in §4.1, Stages 1–3 exchange only signatures/surrogates and Stage 4 uses CKKS; pruning relies on surrogate scores and envelope bounds that are sound with respect to the transmitted surrogate representation. This ensures that no candidate that could enter the final surrogate top-L is removed before

Stage 4, although true cosine top-k preservation remains empirical rather than formally guaranteed.

4 Security and Leakage Analysis

Our privacy claim is bounded leakage with explicit transcript accounting, not full end-to-end cryptographic privacy. Because the pipeline is staged (§3.3), leakage consists of the per-query coordination transcript visible to each party and the inferences it enables, such as partial recovery of query direction from signatures. In particular, the owner never sends the raw query embedding; providers see only query-side-obfuscated projection signatures produced via randomized bit flips (§4.2).

4.1 Explicit per-query leakage envelope

We analyze leakage under the semi-honest model in §2.3. Since the protocol is staged (§3.3), each stage reveals a distinct coordination transcript.

Explicit leakage to the owner (per query).

- Stage 1–2 synopsis values: per-provider envelope bounds $B_{\mathcal{P}_i}$ and per-block bounds $B_{\mathcal{P}_i,b}$ for providers/blocks it chooses to query.
- Stage 3 TA stream: identifiers of the documents surfaced by PEEK calls from hot blocks, together with their refined surrogate scores.
- Stage 4 reranking: exact cosine scores for the requested candidate identifiers (after decrypting packed CKKS outputs); no plaintext or embeddings unless a result is purchased/returned.
- Owner-side access pattern: which providers/blocks are contacted and which candidate identifiers are requested for Stage 4 scoring (visible to the owner by construction).

Explicit leakage to a provider $\mathcal{P}_i$ (per query).

- The obfuscated query signatures (randomized bit flips on the query side; see §4.2) used in Stages 1–3 and public protocol parameters (projection dimensions, L, H, CKKS public parameters, ciphertext sizes).
- Its own protocol outcomes and workload: whether it survives Stage 1, which of its blocks survive Stage 2, and the number/identifiers of its local candidates requested for Stage 4 scoring.
- Which of its own documents are ultimately purchased and returned in the final top-k (an unavoidable consequence of serving results).

Non-goal: hiding global access patterns. We do *not* hide the owner's access pattern from the owner itself: by construction, the owner sees which providers and blocks it contacts, while each provider sees only its own requests (absent collusion or traffic analysis). We also do not hide provider sizes, timing information, or distributional leakage from repeated queries (see §4.2).

4.2 Reconstruction Risk and Obfuscation

In Stages 1–3 the owner never transmits the query embedding $\mathbf{q} \in \mathbb{R}^d$ in the clear; instead it sends only *signed random-projection signatures* $\sigma_q^{(\ell)} = \mathrm{sign}(W_\ell \mathbf{q}) \in \{-1,+1\}^{K_\ell}$ (for stage-dependent projections W_ℓ). This is a strong practical improvement over sharing $\mathbf{q}$, but it is *not* one-way. Because it is a *many*-to-one mapping, a provider knowing the public projection(s) W_ℓ can treat $\sigma_q^{(\ell)}$ as 1-bit measurements and attempt partial recovery of the *direction* of $\mathbf{q}$ (e.g., coarse topic). In our threat model, the provider is the primary adversary: it observes query signatures, while the owner never sees document signatures or embeddings before purchase. To reduce this leakage without sacrificing Hamming/correlation ranking, we obfuscate each query signature with LSH∘RR ("LSHRR") (Fernandes et al., 2022). Before sending a signature $\sigma_q \in \{-1,+1\}^K$, the owner applies independent randomized response to each coordinate:

$$\tilde{\sigma}_{q,i} = \begin{cases} \sigma_{q,i}, & \text{w.p } \frac{e^\varepsilon}{e^\varepsilon+1}, \\ -\sigma_{q,i}, & \text{w.p } p_{\text{flip}} = \frac{1}{e^\varepsilon+1}, \end{cases} \quad i = 1,\dots,K$$

Each coordinate is an ε-randomized-response channel (i.e., ε-DP for one bit), and the resulting bitwise-RR construction corresponds to LSHRR (Fernandes et al., 2022). Importantly, *only the query-side signatures* are noised: providers neither perturb their document representations nor reveal document signatures or embeddings to the owner. This channel attenuates directional information by $\alpha = \tanh(\varepsilon/2) = \frac{e^\varepsilon-1}{e^\varepsilon+1}$, with $\mathbb{E}[\tilde{\sigma}_{q,i} \mid \sigma_{q,i}] = \alpha\,\sigma_{q,i}$. Thus, $\frac{1}{K}\sum_i \tilde{\sigma}_{q,i}\sigma_{d,i}$ stays monotone in expectation but grows noisier as ε decreases, capturing the privacy–utility trade-off. We study inversion risk in Appendix B and this trade-off in Appendix D.4.

5 Experimental Evaluation

We first evaluate retrieval effectiveness against plaintext retrieval, then analyze privacy, computation, latency, and scalability. The ablation study is deferred to Appendix D, where we show the main quality–cost trade-offs and the mechanisms that keep computation and communication sublinear.

5.1 Experimental Setup

Datasets, embeddings, and baseline. We evaluate retrieval on MS MARCO-dev (6,980 queries) and five BEIR datasets (*nfcorpus*, *trec-covid*, *quora*, *arguana*, *scidocs*). All dense methods use `all-mpnet-base-v2` embeddings (768-d, L2-normalized for cosine). We compare against: (i) Dense-exact cosine similarity (brute-force), (ii) HNSW approximate dense search, and (iii) BM25 sparse retrieval. Implementation details, index parameters, and tuning ranges are in Appendix C.2. We report MRR@10, nDCG@10, Recall@100, and Recall@1 for: (i) Dual-Blind RAG light: Stages 1–3 (no encrypted reranking), and (ii) Dual-Blind RAG: Stages 1–4, our full pipeline. Approximation fidelity results that motivate signature lengths are in Appendix C.1.

5.2 Retrieval Effectiveness

Unless stated otherwise, we evaluate under a decentralized setup with $M = 10$ providers using a *non-IID* (size-skewed) corpus split (Dirichlet $\alpha = 1$). We use $B = 512$ blocks per provider in Stage 2, reranking depth $L = 50$, signature lengths $(K_c, K_b, K_m, K_r) = (124, 256, 512, 1024)$ for Stages 1–3, PRIVHASH/LSHRR noise $\varepsilon = 1.5$, and τ_B tuned to prune 80% of blocks (keep 20%). In Appendix D.1-D.4, we ablate each knob (provider split, B, τ_B and ε) to expose the accuracy–cost trade-offs.

MS MARCO (main). Table 2 reports end-to-end retrieval quality in the default decentralized setting. Despite dual privacy and decentralization, DUAL-BLIND RAG retains most of Dense-exact performance, reaching MRR@10= 0.304 and nDCG@10= 0.366 (91% and 93% of Dense-exact), while nearly matching Recall@100 (0.855 vs. 0.864). Recall@1 also matches HNSW (both 0.191) and is only 0.004 below Dense-exact, suggesting the cascade rarely filters out truly relevant items. Comparing DUAL-BLIND RAG LIGHT (Stages 1–3) with the full pipeline isolates the remaining gap. The light variant is close to BM25 on MS MARCO (MRR@10 0.245 vs. 0.241), indicating that Stage 3 mid-signature scoring is the main source of ranking distortion. Stage 4 reranking then substantially improves head quality while preserving near-dense recall, consistent with correcting the final order within a bounded candidate pool. See Appendix D.1 for details.

Table 2: Retrieval effectiveness on MS MARCO. Best non-private baseline in **bold**, our results in gray.

Method	MRR@10	nDCG@10	Recall@100	Recall@1
Dense exact	**0.334**	**0.395**	**0.864**	**0.195**
BM25	0.241	0.277	0.655	0.103
HNSW	0.333	0.394	0.864	0.191
Dual-Blind RAG light	0.245	0.284	0.657	0.113
Dual-Blind RAG	0.304	0.366	0.855	0.191

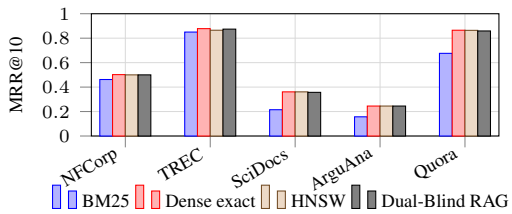

Figure 2: BEIR head-ranking effectiveness (MRR@10) across five tasks. Full breakdown in Appendix C.3.

BEIR (generalization). Figure 2 summarizes BEIR effectiveness across five diverse datasets. We first verified that our baseline runs reproduce standard BEIR reference numbers closely (typically within ≈ 0.01 absolute on key metrics), so differences are attributable to the privacy-preserving cascade rather than evaluation artifacts.

Overall, DUAL-BLIND RAG preserves most of dense retrieval's advantage over BM25 under domain shift. Recall remains high across tasks, consistent with conservative Stage 1/2 screening, while most of the remaining gap to Dense-exact is in head ranking, which Stage 4 reranking targets. Dataset-specific results and anomalies are in Appendix C.3.

5.3 Privacy-Preserving Analysis

Our protocol avoids raw embedding exchange, but the Stage 1–3 transcript still reveals query signatures and is thus not fully cryptographically private. To quantify inversion risk, we evaluate VEC2TEXT (Morris et al., 2023) and ALGEN (Chen et al., 2025) under our exact threat channel, using three attacker inputs: the raw query embedding, the clean Stage 3 refined signature, and the default noised signature (PRIVHASH/LSHRR, K_r=1024, ε=1.5). Figure 3 reports mean semantic cosine with 95% bootstrap confidence intervals on MS MARCO-dev; details are in Appendix B. For both attacks and encoders, reconstruction drops monotonically from raw embeddings to clean signatures to noised signatures, showing that sign compression and randomized-response noise both reduce invertibility. Overall, the Stage 1–3 transcript is substantially harder to invert than raw embed-

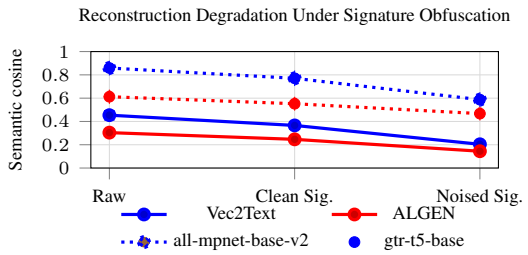

Figure 3: SOTA query-reconstruction from raw embeddings to clean/noised signatures, measured as semantic cosine between reconstructed and original embeddings

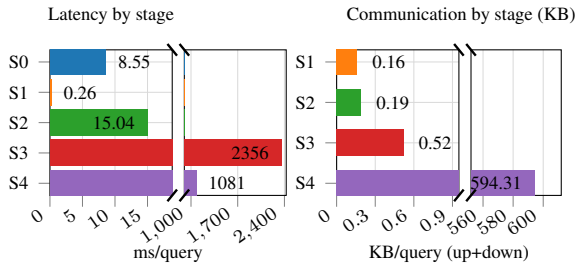

Figure 4: Stage-wise online (per-query) latency and communication under the main setting, with a broken linear x-axis to avoid compressing smaller stages. S0 is query encoding.

dings, though residual bounded leakage remains, consistent with §4.2.

5.4 Latency, Communication, and Offline Preprocessing

We report costs under a *simulated distributed deployment*, separating one-time offline preprocessing from online per-query costs. Offline costs are amortized and excluded from per-query results.

Offline one-time preprocessing. Offline preprocessing is amortized across queries; details are in Appendix C.4, and the dominant recurring costs are online (primarily Stages 3–4)

Online latency and bandwidth (per query). Figure 4 shows that our full pipeline (Stages 0–4) takes 3.46 s per query online and exchanges 0.58 MB total communication (0.291 MB uplink + 0.291 MB downlink). Stage 3 (mid-signature scoring) dominates compute: 2.36 s (68.2%) is spent in streaming merge/scoring, while Stage 4 (reranking) adds 1.08 s (31.3%) for CKKS reranking. Stages 0–2 (encoding, bound evaluation) together are negligible (< 30 ms, $< 1\%$), confirming that synopsis tests are cheap and that the online bottleneck lies in candidate evaluation and cryptographic refinement.

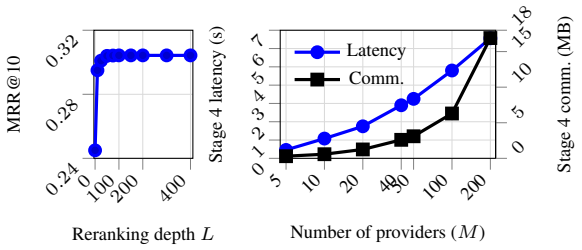

Figure 5: Sensitivity of the final reranking stage. Left: effect of reranking depth L on head quality (MRR@10). Right: provider scaling (M): Stage 4 latency (left axis) and Stage 4 communication (right axis) per query.

Stage 4 dominates communication and scales with the active top-L pool. Figure 4 shows that nearly all per-query traffic comes from Stage 4 (≈ 0.58 MB), while Stages 1–3 together require < 1 KB. This cost remains bounded in practice because Stage 4 reranks only the top-L candidates produced by Stage 3, and Figure 5 (left) shows a clear elbow: most head-quality gains are achieved by $L \approx 50$, after which improvements saturate. Stage 4 is further restricted to the *candidate-owning* providers in that top-L pool, so its cost depends on the number of active contributing providers rather than on the full corpus size. Accordingly, Figure 5 (right) shows that as M increases, Stage 4 latency and communication both rise: $\mathbb{E}[\#\text{prov S2}]$ grows from 2.0 at $M{=}5$ to 65.4 at $M{=}200$, Stage 4 latency from 0.46 to 6.56 s/query, and Stage 4 communication from 0.97 to 16.9 MB/query. Over this tested range, both curves increase markedly but slower than linearly in M, consistent with the top-L pool being spread across more providers as the marketplace becomes more fragmented.

6 Conclusion and Future Work

Dual-Blind RAG enables decentralized, document-protecting retrieval with sublinear per-query cost, recovering over 90% of plaintext centralized retriever performance on MS MARCO and BEIR through a four-stage pipeline with homomorphic reranking. A rent-only model—where LLM owners pay per grounded answer without acquiring the document—could complement the current buy-to-keep framework by enabling on-demand access without data transfer. Hybrid FHE/secure-enclave designs or faster homomorphic methods could further reduce reranking overhead. Overall, Dual-Blind RAG helps lay the foundations for practical privacy-first knowledge markets.

Limitations

Our privacy claims are bounded-leakage and operational rather than full end-to-end cryptographic guarantees for the entire retrieval transcript. However, we explicitly characterize the coordination transcript exposed to each party and show empirically that strong privacy protection against raw embedding exchange can coexist with competitive retrieval quality. In Stages 1–3, providers observe noised query signatures, local pruning outcomes, and reranking workload, which may still support coarse semantic or directional inference, especially under repeated queries. We expect some of this leakage to overlap with the semantic inference a provider could already make from observing which of its own documents are highly scored or ultimately retrieved, although we do not claim formal equivalence. Our results suggest that practical multi-provider RAG can be supported without centralizing corpora or exposing raw embeddings, while leaving stronger malicious-model and repeated-query protections to future work.

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A Decentralized Private Retrieval Pipeline

Query-side PRIVHASH/LSHRR is applied in all signature stages. In our implementation, the owner never transmits a raw query signature in any stage. Whenever the protocol requires a signed query signature $\sigma_q = \mathrm{sign}(Wq) \in \{-1,+1\}^K$ for some stage-specific projection W, the owner first applies independent randomized response (PRIVHASH/LSHRR) to obtain a *noised* signature

$$\tilde{\sigma}_{q,i} = \begin{cases} \sigma_{q,i}, & \text{w.p. } \frac{e^{\varepsilon}}{e^{\varepsilon}+1}, \\ -\sigma_{q,i}, & \text{w.p. } p_{\text{flip}} = \frac{1}{e^{\varepsilon}+1}, \end{cases} \quad i = 1, \ldots, K$$

and *only* $\tilde{\sigma}_q$ is sent to providers (Stages 1–3). Document signatures are never noised and are never revealed to the owner.

Signed envelope pruning soundness (with noised query signature). Fix a projection matrix $W \in \mathbb{R}^{K\times d}$ with rows $w_1, \ldots, w_K$ and a set of documents P (either a provider corpus or a block), where all embeddings are ℓ_2-normalized. Let the owner compute the signed query signature $\sigma_q = \mathrm{sign}(Wq) \in \{-1,+1\}^K$ and transmit the *noised* signature

$$\tilde{\sigma}_q \in \{-1,+1\}^K$$

obtained by applying PRIVHASH/LSHRR as defined above. Each provider precomputes projection extremes on P:

$$m_i^{\max} = \max_{d\in P}\langle d, w_i\rangle, \qquad m_i^{\min} = \min_{d\in P}\langle d, w_i\rangle.$$

Define the per-row envelope contribution with respect to the received (noised) sign:

$$u_i = \begin{cases} m_i^{\max}, & \tilde{\sigma}_q[i] = +1, \\ -m_i^{\min}, & \tilde{\sigma}_q[i] = -1, \end{cases}$$

$$U(W, P; \tilde{\sigma}_q) = \frac{1}{K}\sum_{i=1}^{K} u_i.$$

Consider the signed surrogate score that is *actually evaluated against the received signature*:

$$\tilde{s}_W(q, d; \tilde{\sigma}_q) = \frac{1}{K}\sum_{i=1}^{K} \tilde{\sigma}_q[i]\cdot\langle d, w_i\rangle.$$

For each i and any $d \in P$: if $\tilde{\sigma}_q[i] = +1$, then $\tilde{\sigma}_q[i]\langle d, w_i\rangle = \langle d, w_i\rangle \le m_i^{\max} = u_i$; if $\tilde{\sigma}_q[i] = -1$, then $\tilde{\sigma}_q[i]\langle d, w_i\rangle = -\langle d, w_i\rangle \le -m_i^{\min} = u_i$. Summing over i and dividing by K yields $\tilde{s}_W(q, d; \tilde{\sigma}_q) \le U(W, P; \tilde{\sigma}_q)$ for all $d \in P$. Therefore, pruning any set P whenever $U(W, P; \tilde{\sigma}_q) \le \tau$ is *sound* with respect to the surrogate induced by the transmitted (noised) signature.

A.1 Stage 1: Provider Envelope Test

1) **Pre-computation (by each provider $\mathcal{P}$).** For the current provider $\mathcal{P}$, let P denote its local corpus. Before any queries arrive, each provider computes and stores $(m_i^{\min}, m_i^{\max})$ for every row w_i of $W_c \in \mathbb{R}^{K_{\text{coarse}}\times d}$:

$$m_i^{\max} = \max_{d\in P}\langle d, w_i\rangle,$$
$$m_i^{\min} = \min_{d\in P}\langle d, w_i\rangle, \qquad i = 1, \ldots, K_{\text{coarse}}.$$

This is $2K_{\text{coarse}}$ float32 values, i.e., $2K_{\text{coarse}} \cdot 4$B (about 1 kB when K_{coarse}=124).

2) **Query arrives (owner → providers).** The owner computes the signed coarse signature $\sigma_q^c = \mathrm{sign}(W_c q) \in \{-1,+1\}^{K_{\text{coarse}}}$ and transmits the *noised* version

$$\tilde{\sigma}_q^c \in \{-1,+1\}^{K_{\text{coarse}}},$$

obtained by applying PRIVHASH/LSHRR (defined at the start of this section). Only $\tilde{\sigma}_q^c$ is sent.

3) **Envelope computation (at provider $\mathcal{P}$).** Given $\tilde{\sigma}_q^c$, the provider forms

$$u_i = \begin{cases} m_i^{\max}, & \tilde{\sigma}_q^c[i] = +1, \\ -m_i^{\min}, & \tilde{\sigma}_q^c[i] = -1, \end{cases}$$

$$B_{\mathcal{P}} = U(W_c, P; \tilde{\sigma}_q^c) = \frac{1}{K_{\text{coarse}}}\sum_{i=1}^{K_{\text{coarse}}} u_i.$$

By the soundness argument above, $B_{\mathcal{P}}$ upper-bounds the induced surrogate $\tilde{s}_{W_c}(q, d; \tilde{\sigma}_q^c)$ for all $d \in P$.

4) **Filtering (owner side).** The provider returns the single float $B_{\mathcal{P}}$. The owner keeps provider $\mathcal{P}$ iff $B_{\mathcal{P}} > \tau_{\mathcal{P}}$ (where $\tau_{\mathcal{P}}$ is the Stage 1 threshold).

A.2 Stage 2: Block–Level Envelope Test

Each surviving provider partitions its corpus into B blocks. We re-apply the signed envelope test within each block using $W_b \in \mathbb{R}^{K_{\text{block}} \times d}$ with $K_{\text{block}} > K_{\text{coarse}}$.

1) **Block extremes (pre-computation).** For each block b and each row $w_{b,i}$ of W_b, store

$$m_{b,i}^{\max} = \max_{d \in b} \langle d, w_{b,i} \rangle,$$
$$m_{b,i}^{\min} = \min_{d \in b} \langle d, w_{b,i} \rangle, \qquad i = 1, \ldots, K_{\text{block}}.$$

Storage per provider is $2BK_{\text{block}}$ float32 values.

2) **Block signature (owner → provider).** Owner computes $\sigma_q^b = \text{sign}(W_b q) \in \{-1, +1\}^{K_{\text{block}}}$ and transmits the *noised* version

$$\tilde{\sigma}_q^b \in \{-1, +1\}^{K_{\text{block}}},$$

via PRIVHASH/LSHRR (defined at the start of this section).

3) **Envelope per block (at provider).** For each block b, define

$$u_{b,i} = \begin{cases} m_{b,i}^{\max}, & \tilde{\sigma}_q^b[i] = +1, \\ -m_{b,i}^{\min}, & \tilde{\sigma}_q^b[i] = -1, \end{cases}$$

$$B_{\mathcal{P},b} = \frac{1}{K_{\text{block}}} \sum_{i=1}^{K_{\text{block}}} u_{b,i}.$$

Then for all $d \in b$, $\tilde{s}_{W_b}(q, d; \tilde{\sigma}_q^b) \leq B_{\mathcal{P},b}$.

4) **Block filtering (owner side).** Each provider returns $\{B_{\mathcal{P},b}\}$ (one float per block). The owner discards any block b with $B_{\mathcal{P},b} \leq \tau_B$; the remaining "hot" blocks advance to Stage 3.

A.3 Stage 3: Distributed Streaming with K_m Ordering and K_r Heap Scoring

After Stage 2, the owner knows the surviving providers and their hot blocks. Stage 3 builds a global overshoot set of size L using a two-signature procedure: a shorter *mid* signature (K_m) to define provider-local stream order, and a refined signature (K_r) to score documents inserted into the global heap.

Two Stage-3 signature lengths. Let

$$W_m \in \mathbb{R}^{K_m \times d}, \qquad W_r \in \mathbb{R}^{K_r \times d},$$

with $K_r > K_m$ in our implementation. Each provider precomputes and stores both document signatures:

$$\sigma_m(d) = \text{sign}(W_m d) \in \{-1, +1\}^{K_m}, \quad (1)$$
$$\sigma_r(d) = \text{sign}(W_r d) \in \{-1, +1\}^{K_r}, \quad (2)$$

packed as bits. For each query, the owner computes $\sigma_m(q), \sigma_r(q)$ and transmits only their noised versions

$$\tilde{\sigma}_m(q), \tilde{\sigma}_r(q)$$

after applying PRIVHASH/LSHRR (as defined at the start of this section).

1) **Provider-local stream order (uses K_m).** For each surviving provider $\mathcal{P}$ and its hot documents $\mathcal{D}_{\mathcal{P}}^{\text{hot}}$, define

$$\tilde{s}_m(q, d) = 1 - \frac{2}{K_m} d_H\big(\tilde{\sigma}_m(q), \sigma_m(d)\big).$$

Provider $\mathcal{P}$ sorts $\mathcal{D}_{\mathcal{P}}^{\text{hot}}$ by $\tilde{s}_m$ (descending), yielding a stream

$$\pi_{\mathcal{P}}(1), \pi_{\mathcal{P}}(2), \ldots$$

(implemented as a merged ordering over hot blocks).

2) **PEEK/UB scores for heap updates (uses K_r).** At stream pointer $t_{\mathcal{P}}$, provider $\mathcal{P}$ returns the next H docs from the K_m-ordered stream and scores them with the refined signature:

$$\tilde{s}_r(q, d) = 1 - \frac{2}{K_r} d_H\big(\tilde{\sigma}_r(q), \sigma_r(d)\big).$$

Interface:

1. $\text{PEEK}_{\mathcal{P}}(H) \rightarrow \{(d, \tilde{s}_r(q, d))\}$ for $d \in \{\pi_{\mathcal{P}}(t_{\mathcal{P}}+1), \ldots, \pi_{\mathcal{P}}(t_{\mathcal{P}}+H)\}$.

2. $\text{UB}_{\mathcal{P}}() \rightarrow \tilde{s}_r\big(q, \pi_{\mathcal{P}}(t_{\mathcal{P}}+1)\big)$ (or $-\infty$ if exhausted).

So, documents pushed into the global heap are scored with K_r signatures (not K_m).

3) **Owner-side heap loop.** The owner maintains a size-L min-heap keyed by refined scores $\tilde{s}_r$ and threshold

$$\tau_L = \text{current } L\text{-th best refined score}$$

($-\infty$ until heap is full). Repeatedly: (i) call $\textsc{peek}_{\mathcal{P}}(H)$ on active providers and update heap; (ii) query $\textsc{ub}_{\mathcal{P}}()$ and freeze provider $\mathcal{P}$ when $\textsc{ub}_{\mathcal{P}}() \le \tau_L$. When all providers are frozen, the heap contents form the Stage 3 overshoot candidate set, which is sent to Stage 4 for exact reranking.

A.4 Stage 4: Final Top-k reranking with Packed CKKS

By the end of Stage 3, the owner has L candidate document identifiers. Let provider $\mathcal{P}$ hold $L_{\mathcal{P}}$ of these candidates.

1. The owner encrypts the normalized query $\mathbf{q}$ once as $\mathsf{ct}_q = \mathsf{Enc}_{\mathsf{pk}}(\mathbf{q})$ and sends ct_q to each involved provider, along with the identifiers of the candidates to score.

2. Each provider groups its $L_{\mathcal{P}}$ candidates into packs of size p (up to CKKS slot capacity) and evaluates a packed dot-product:

$$\mathsf{ct}^{(\ell)} \approx \mathsf{Enc}_{\mathsf{pk}}\Big([\langle \mathbf{q}, \mathbf{d}_{(\ell,1)}\rangle, \ldots, \langle \mathbf{q}, \mathbf{d}_{(\ell,p)}\rangle]\Big), \quad \ell = 1, \ldots, \left\lceil \frac{L_{\mathcal{P}}}{p} \right\rceil.$$

   The provider returns only $\lceil L_{\mathcal{P}}/p \rceil$ ciphertexts instead of $L_{\mathcal{P}}$.

3. The owner decrypts the packed ciphertexts, extracts the (approximate) cosine scores, and selects the exact top-k.

Packing reduces Stage 4 bandwidth and ciphertext handling from $O(L_{\mathcal{P}})$ to $O(\lceil L_{\mathcal{P}}/p \rceil)$ ciphertexts per provider, while preserving the same final top-k outcome (up to CKKS numerical precision).

B Risk of Reconstruction and Obfuscation Measures Discussion

What an attacker sees. For each stage $\ell \in \{1, 2, 3\}$ the provider observes only a (potentially noised) query signature $\tilde{\sigma}_q^{(\ell)} \in \{-1, +1\}^{K_\ell}$ together with its *local* coordination transcript (e.g., whether it survives, and how many candidate ids it is asked to score). It does *not* observe the plaintext query embedding $\mathbf{q}$, any document embeddings from other providers, or any decrypted Stage 4 scores. The reconstruction risk we consider here is: given public W_ℓ and observed $\tilde{\sigma}_q^{(\ell)}$, how well can a provider estimate the *direction* of $\mathbf{q}$?

Baseline intuition: one-bit observations can recover coarse direction. The one-bit compressed-sensing analysis of Plan–Vershynin (Plan and Vershynin, 2013) implies that, for random hyperplanes and suitable estimators, directional recovery improves with the number of bits: the mean-square error scales as $\Theta(d/k)$ and thus the expected angular error decreases like $\sqrt{d/k}$. For $d \in \{768, 1024\}$ and $k \approx 1000$ this corresponds to a fairly wide but still semantically informative cone, often sufficient for coarse topical inference. This theoretical intuition motivates our use of query-side obfuscation, but by itself it does not tell us how vulnerable the actual Stage 1–3 transcript is to modern text reconstruction attacks.

Query-side obfuscation via bitwise randomized response (PrivHash/LSHRR). To mitigate inversion, we apply independent ε-randomized response (RR) to *query signatures only* before transmission:

$$\tilde{\sigma}_{q,i} = \begin{cases} \sigma_{q,i}, & \text{w.p. } \frac{e^{\varepsilon}}{e^{\varepsilon}+1}, \\ -\sigma_{q,i}, & \text{w.p. } p_{\text{flip}} = \frac{1}{e^{\varepsilon}+1}, \end{cases}$$

which is exactly Warner's RR channel per bit and is the "bitwise RR" component in LSHRR (Fernandes et al., 2022).[2] Define $\alpha = \tanh(\varepsilon/2) = \frac{e^{\varepsilon}-1}{e^{\varepsilon}+1}$ so that $\mathbb{E}[\tilde{\sigma}_{q,i} \mid \sigma_{q,i}] = \alpha\, \sigma_{q,i}$. For similarity estimation one can debias by dividing by α; however our pipeline only requires ranking, so we treat ε as a direct privacy–utility knob and evaluate both the induced inversion resistance (Figure 3) and the retrieval trade-off separately in Appendix D.4.

Empirical protocol with SOTA inversion attacks. The Stage 1–3 threat channel is not access to raw embeddings but access to observed query signatures. We therefore evaluate inversion under three attacker inputs: (i) the raw query embedding, which serves as an upper-bound baseline, (ii) the clean signature σ_q, and (iii) the noised signature $\tilde{\sigma}_q$ used in our default protocol (K=1024, ε=1.5). We

[2] We report ε as the *per-bit* RR parameter. Under naive composition, releasing k independently randomized bits yields a worst-case $k\varepsilon$ bound in standard local DP; Fernandes *et al.* analyze the combined LSH∘RR construction under an extended-DP view tailored to angular similarity and show that this worst-case bound is pessimistic for the similarity task.

use the MS MARCO-dev queries and two victim encoders: `all-mpnet-base-v2` (our default) and `gtr-t5-base`. As downstream textual inversion attacks we use VEC2TEXT (Morris et al., 2023) and ALGEN (Chen et al., 2025). Because both attacks natively operate on dense embeddings, for clean and noised signatures we first recover a dense proxy embedding via optimization-based one-bit recovery and then apply the same inversion pipeline. We score each reconstruction by re-encoding the generated text with the corresponding victim encoder and measuring cosine similarity to the original query embedding. Figure 3 reports means with 95% bootstrap confidence intervals.

Observed degradation from signatures and RR noise. Across all four attack/encoder combinations we observe the same monotone ordering: raw embedding > clean signature > noised signature. For `all-mpnet-base-v2`, VEC2TEXT falls from 0.454 on raw embeddings to 0.366 on clean signatures and 0.204 on noised signatures, while ALGEN falls from 0.304 to 0.246 to 0.144. This corresponds to raw-to-noised reductions of approximately 55% and 53%, respectively. For `gtr-t5-base`, absolute values are higher—VEC2TEXT $0.859 \rightarrow 0.770 \rightarrow 0.587$, ALGEN $0.612 \rightarrow 0.552 \rightarrow 0.467$—but the same degradation persists, with raw-to-noised drops of about 32% and 24%. We therefore interpret the result primarily within each encoder: `gtr-t5-base` is simply easier to invert in absolute terms, whereas the security question for our protocol is whether the representation released in Stages 1–3 is less recoverable than the raw embedding it replaces.

Implication for the threat model. This experiment directly matches the provider-side inversion channel in our protocol: providers see signatures, not raw embeddings, and query-side RR noise further perturbs those signatures before transmission. The results therefore provide empirical support for the design claim that signature compression already reduces direct inversion risk, and PRIVHASH/LSHRR noise further suppresses semantic recoverability while preserving enough ranking signal for coordination. At the same time, noised-signature recovery is *not* zero, which is consistent with our stated non-goal of eliminating all semantic leakage from the coordination transcript.

Natural semantic leakage remains (by design). Even without any explicit inversion attempt, a provider can sometimes infer coarse query intent from *which of its own documents* appear most similar under the (possibly noised) signature—e.g., if the provider's top candidates are medical abstracts, the query is likely medical. This is an instance of the *coordination transcript leakage* we declare in the main threat model: our goal is to prevent disclosure of plaintext embeddings or non-purchased document content, not to eliminate inevitable high-level semantic inference from the (few) candidates that become highly scored.

CKKS isolation and the effect of ciphertext packing. Stage 4 uses CKKS so that providers never see $\mathbf{q}$ nor any decrypted dot products; only the owner decrypts scores. Moreover, providers return *packed* ciphertexts (SIMD): instead of sending one ciphertext per candidate, a provider batches up to p candidates per ciphertext and returns only $\lceil j/p \rceil$ ciphertexts for its j requested candidates. Packing therefore reduces Stage 4 bandwidth and per-candidate overhead by approximately a factor of p, while preserving the same cryptographic isolation assumptions. As usual, if an adversary were to obtain sufficiently many *plaintext* inner products $\langle \mathbf{q}, \mathbf{y}_i \rangle$ against known linearly independent $\mathbf{y}_i$, then $\mathbf{q}$ could be solved by linear algebra; in our pipeline, such plaintext values are only ever available to the owner after decryption.

C Evaluation Details

C.1 Approximation Fidelity Details

As shown in Fig. 6, global correlations increase steeply from $K = 128 \rightarrow 512$ and then plateau at $K = 1024$. The binned MSE concentrates in the high-similarity tail ($[0.6, 1.0]$), which motivates using short signatures for aggressive early pruning and a longer signature right before exact reranking. Errors are essentially flat below 0.3 cosine and rise again in the negative tail (approx. $[-0.4, -0.2]$), consistent with LSH behavior.

C.2 Baseline and Index Hyper-Parameters

Table 3 collects baseline/index settings and the default cascade configuration used throughout §5.2 unless explicitly swept. Dense exact uses FAISS in exhaustive mode; BM25 follows standard Anserini defaults with RM3; HNSW settings follow common practice and were lightly tuned on MS MARCO `train`.

Unless noted, hyperparameters are tuned once on MS MARCO-`train` and then frozen for

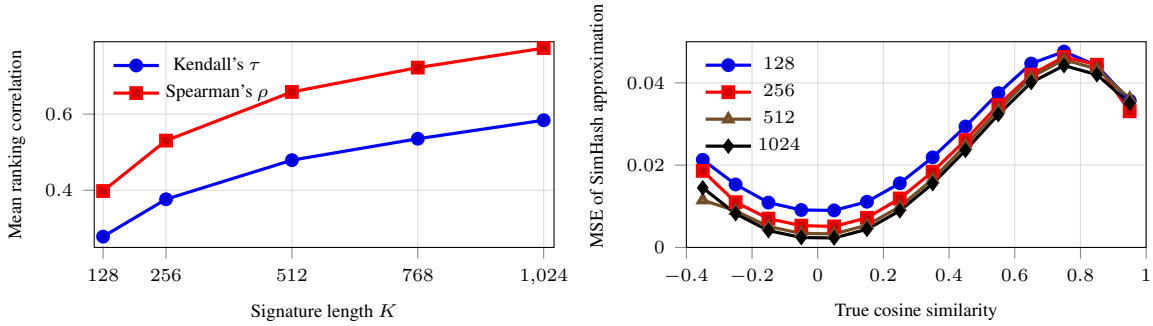

Figure 6: **Approximation fidelity of SimHash.** Left: global ranking fidelity (Kendall's τ and Spearman's ρ) improves with signature length and begins to plateau at larger K. Right: binned MSE of cosine approximation versus true cosine for $K \in \{128, 256, 512, 1024\}$, with largest errors in the high-similarity region.

Table 3: Baseline and index hyper-parameters (default unless otherwise stated).

Component	Setting
Dense exact (FAISS)	IVF-Flat with n_{list}=1 (exhaustive), inner-product
HNSW	M=32, `ef_construction`=256, `ef_search`=64
BM25 (Anserini)	k_1=0.9, b=0.4, RM3 enabled (default params)
Embeddings	`all-mpnet-base-v2`, d=768, L2-normed for cosine
Our cascade	$(K_c, K_b, K_m, K_r) = (124, 256, 512, 1024)$, B=512, τ_B=80% of blocks pruned, L=50, M=10, ε=1.5
CKKS rerank	Depth 1, modulus 218-bit, scale 2^{40} (TenSEAL)

MS MARCO-dev and BEIR.

C.3 BEIR Results

Across general-domain (NFCorpus) and biomedical (TREC-COVID) retrieval, dense embeddings substantially outperform BM25, and our private cascade preserves most of this gain. For instance, on NFCorpus, Recall@100 rises from 0.209 (BM25) to 0.333 (Dense-exact), while our pipeline incurs only a modest drop relative to dense retrieval (about 0.03 in Recall@100 and 0.005 in nDCG@10). Similarly on TREC-COVID, embeddings improve Recall@100 from 0.040 to 0.051, and the private cascade remains close, suggesting that successive provider- and block-level screening filters aggressively without systematically eliminating relevant passages even under domain shift. SciDocs is a challenging scientific-paper retrieval task where absolute scores are low for all methods (e.g., nDCG@10 $\approx$ 0.077 under dense baselines). Here, we observe that our pipeline is competitive with (and in our runs slightly exceeds) HNSW on Recall@100 (e.g., 0.089 vs. 0.081), indicating that the cascade can maintain broad candidate coverage in difficult regimes where approximate index traversal may miss relevant regions. ArguAna exhibits very high Recall@100 but near-zero Recall@1 across baselines, reflecting that many candidates are topically similar and the single truly relevant sentence is hard to place at rank 1. Our method increases Recall@100 (e.g., to 0.903) but Recall@1 remains low (e.g., 0.022), suggesting that this dataset's bottleneck is fine-grained answer selection rather than neighborhood search. Finally, on the large Quora corpus ($\approx$ 552K passages), embedding-based methods saturate at high recall, and our approach trails Dense-exact only slightly (e.g., $\approx$ 0.02 in Recall@100 and $\approx$ 0.05 in MRR@10), consistent with decentralization and privacy imposing only a modest penalty at scale.

C.4 Offline Preprocessing Details

Table 4 reports the one-time preprocessing cost of materializing stage-specific signatures and synopsis structures used by the cascade. As in the main paper, these costs are paid once and amortized across queries. Storage is dominated by signature materialization and grows roughly linearly with signature length K, while the Stage 1–2 synopsis structures remain comparatively compact.

Peak preprocessing memory is 28.1 GB RSS. Overall, this supports the claim in §5.4 that the main recurring overheads are online—especially Stages 3–4 of streaming and encrypted reranking—while the offline footprint is amortized.

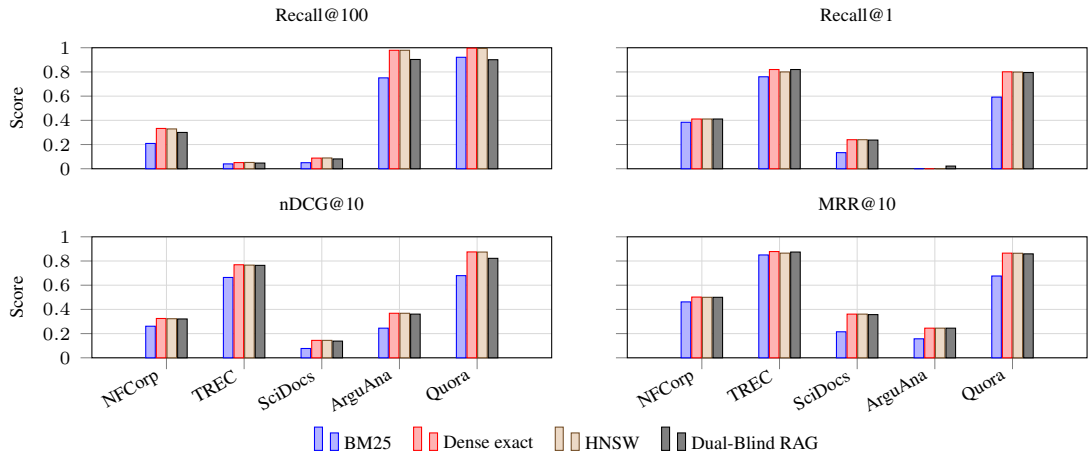

Figure 7: **BEIR effectiveness across five tasks.** Metrics shown: Recall@100, Recall@1, nDCG@10, and MRR@10.

Table 4: Offline preprocessing and storage footprint (one-time, amortized). Peak RSS: 28.1 GB.

Artifact	Variant	Build time	Size on disk
Signatures Stage 1	coarse (K_c=124)	4.22 s	135.2 MiB
Signatures Stage 2	block (K_b=256)	6.69 s	270.3 MiB
Signatures Stage 3	mid (K_m=512)	10.76 s	539.6 MiB
	refined (K_r=1024)	20.49 s	1079.3 MiB
Provider envelopes	S1 synopsis	18.86 s	0.06 MiB
Block synopses	S2 synopsis	27.50 s	44.0 MiB

D Additional Ablations and Sensitivity

We report sensitivity studies and detailed ablations under the same overall evaluation protocol. Our ablations show that (i) the final encrypted Stage 4 reranking provides the largest gain in head ranking quality but drives most bandwidth, (ii) Stage 2 block pruning and Stage 3 TA streaming are the key mechanisms that keep compute and communication sublinear, and (iii) the main quality–cost knobs are the block pruning aggressiveness and reranking depth L. We also find that retrieval quality is stable as the number of providers grows, but cryptographic cost increases when the top-L pool spans many providers; non-IID provider splits typically help by enabling earlier pruning. Two key messages we want readers to remember: (1) effectiveness close to dense/HNSW with dual privacy, and (2) a controllable accuracy–cost frontier driven mainly by Stage 2 pruning and Stage 4 depth L, with Stage 3 streaming dominating online compute. Unless stated otherwise, results use the default decentralized configuration: M=10 providers, a non-IID Dirichlet split with α=1.0, B=512 blocks/provider, Stage 2 keep=20% (prune 80%), L=50, $(K_c, K_b, K_m, K_r) = (124, 256, 512, 1024)$, and PRIVHASH/LSHRR noise ε=1.5. All latency/communication numbers are measured on a fixed profiled query subset and are intended to support *relative* comparisons.

D.1 Do the stages matter? (Component Ablations)

Table 5 isolates the contribution of Stage 2 pruning, Stage 3 TA pruning, and Stage 4 reranking (FHE) under the default setting.

Stage 4 (FHE) improves head quality at a clear cost. Comparing *Light* to *Full*, Stage 4 increases MRR@10 from 0.245 to 0.304 and nDCG@10 from 0.284 to 0.366, indicating that exact reranking corrects hash-induced mis-orderings in the head. This gain comes with additional compute and bandwidth (2.38 s → 3.46 s and 0.001 MB → 0.58 MB), consistent with Stage 4 dominating per-query communication.

Stage 2 is primarily a cost reducer (and is conservative). Removing block pruning increases end-to-end latency by ∼21% (3.46 s → 4.18 s) with essentially unchanged quality, showing that Stage 2 filters substantial downstream work while rarely over-pruning at the tuned keep rate.

TA pruning in Stage 3 is essential for scalability. Disabling TA termination causes a ∼3× latency blow-up (3.46 s → 10.45 s) and a large communication increase (0.58 MB → 6.97 MB), because scanning the full hot region expands both the amount of streaming coordination in Stage 3 and the number of candidate-owning providers that must participate in Stage 4. This confirms that TA pruning is not merely an optimization but a key mechanism keeping compute and coordination sublinear in the

Table 5: Component ablations on MS MARCO-dev.

Variant	MRR@10	nDCG@10	R@100	R@1	Total ms	Comm MB
Full (Stg. 1–4, FHE)	0.304	0.366	0.855	0.191	3460	0.58
Light (Stg. 1–3)	0.245	0.284	0.657	0.113	2380	0.001
w/o Stage 2 (no block prune)	0.305	0.367	0.856	0.191	4180	0.61
w/o TA pruning (scan-all hot blocks)	0.298	0.358	0.860	0.186	10450	6.97

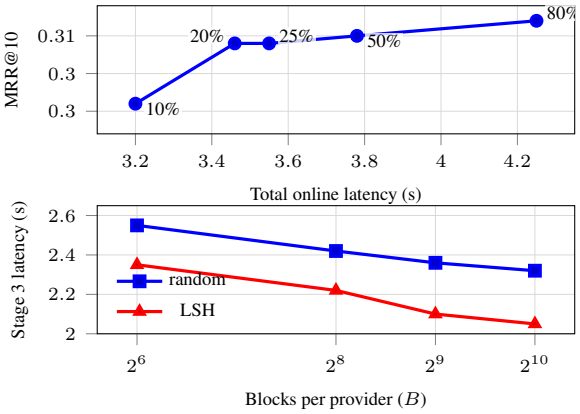

Figure 8: Stage 2 block pruning ablation. (a) Quality–latency trade-off induced by pruning aggressiveness at B=512 (labels denote keep%). (b) Effect of block granularity B on Stage 3 latency at keep= 25%, comparing random vs. LSH partitions.

hot region size.

D.2 Stage 2 Block Pruning: Trade-off and Block Design

We ablate Stage 2 along two axes: (i) *pruning aggressiveness*, implemented as a target keep-fraction (keep $\in \{10\%, 20\%, 25\%, 50\%, 80\%\}$ at B=512), and (ii) *block construction*, varying the number of blocks per provider $B \in \{64, 256, 512, 1024\}$ and using either random or LSH-based partitions. Unless noted, we keep ε=1.5, L=50, M=10, and the default Dirichlet α=1.0 split.

Pruning aggressiveness yields a clean accuracy–latency knob. Figure 8(a) shows a Pareto frontier: moderate pruning (keep $\geq$ 25%) preserves head quality, while aggressive pruning (keep= 10%) further reduces latency but begins to degrade head ranking, reflecting over-pruning once the hot region becomes too small.

Block granularity and partitioning shape downstream Stage 3 work. Figure 8(b) shows Stage 3 latency at keep= 25%. Finer partitions (larger B) can reduce downstream exposure by allowing more precise block pruning. LSH partitions typically tighten the hot region relative to random at the same keep rate.

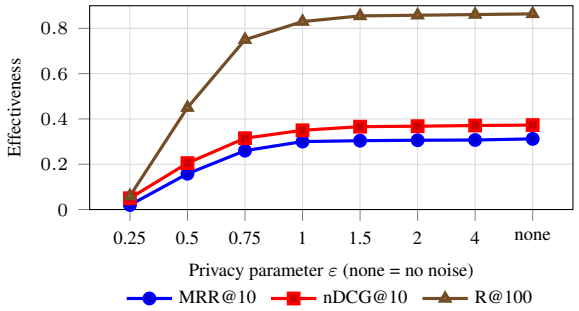

Figure 9: Effect of PrivHash signature noise on effectiveness. We sweep ε and place "none" (no noise) after the largest ε for reference.

D.3 Robustness to Provider Splits

We evaluate robustness to how the corpus is partitioned across providers by sweeping: (i) IID (random-even), (ii) size-skewed Dirichlet splits with varying concentration α (smaller α = more skew), and (iii) topic-based splits with different topic resolutions ("bits"). We keep the total corpus fixed and use the same protocol settings (M=10, ε=1.5, L=50, B=512). Table 6 reports effectiveness, pruning statistics, and the resulting Stage 4 (FHE) cost.

Interpretation (high level). Near-IID splits are the hardest case for pruning (many providers survive Stage 1/2), so Stage 4 cost is highest. As non-IID structure increases, the cascade concentrates computation on fewer providers, reducing Stage 4 coordination while maintaining effectiveness.

D.4 Privacy/Noise Sensitivity (PrivHash/LSHRR)

We sweep the PrivHash privacy parameter ε (equivalently, the signature bit-flip probability p_{flip}) to quantify the privacy–utility trade-off, keeping the rest of the pipeline fixed (M=10, L=50, B=512, same split). Figure 9 shows a clear elbow around $\varepsilon \approx 1$: utility collapses at very strong noise, recovers rapidly between $\varepsilon \in [0.75, 1.5]$, and then saturates. We use ε=1.5 as the default point beyond the elbow.

Table 6: Provider split robustness. The **Dirichlet** α=1.0 row corresponds to the default setting used in the main paper.

Split	**MRR@10**	**nDCG@10**	**R@100**	**R@1**	$\mathbb{E}[\#\mathrm{prov}_{\mathrm{S1}}]$	$\mathbb{E}[\#\mathrm{prov}_{\mathrm{S2}}]$	**S4 ms**	**S4 MB**
IID (random-even)	0.292	0.354	0.848	0.187	10.00	9.60	2190	1.74
Dirichlet (α=10)	0.296	0.358	0.850	0.188	10.00	9.10	2050	1.60
Dirichlet (α=1.0)	0.304	0.366	0.855	0.191	5.10	2.90	1080	0.58
Dirichlet (α=0.1)	0.306	0.369	0.858	0.192	4.30	2.30	950	0.52
Topic-even (bits=10)	0.289	0.349	0.846	0.184	10.00	10.00	1980	1.52
Topic-even (bits=14)	0.297	0.357	0.850	0.187	10.00	10.00	1880	1.45
Topic-even (bits=16)	0.302	0.363	0.852	0.189	10.00	10.00	1760	1.36
Topic+Size (single-bucket, bits=16)	0.305	0.367	0.856	0.191	8.60	3.60	1120	0.62