BTBS-LNS: BINARIZED-TIGHTENING, BRANCH AND SEARCH ON LEARNING LNS POLICIES FOR MIP

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ABSTRACT

Learning to solve large-scale Mixed Integer Program (MIP) problems is an emerging research topic, and policy learning-based Large Neighborhood Search (LNS) has been a popular paradigm. However, the explored space of LNS policy is often limited even in the training phase, making the learned policy sometimes wrongly fix some potentially important variables early in the search, leading to local optimum in some cases. Moreover, many methods only assume binary variables to deal with. We present a practical approach, termed Binarized-Tightening Branch-and-Search for Large Neighborhood Search (BTBS-LNS). It comprises three key techniques: 1) the "Binarized Tightening" technique for integer variables to handle their wide range by binary encoding and bound tightening; 2) an attention-based tripartite graph to capture global correlations among variables and constraints for an MIP instance; 3) an extra branching network as a global view, to identify and optimize wrongly-fixed backdoor variables at each search step. Experiments show its superior performance over the open-source solver SCIP and LNS baselines. Moreover, it performs competitively with, and sometimes better than the commercial solver Gurobi (v9.5.0), especially on the MIPLIB2017 benchmark chosen by Hans Mittelmann, where our method can deliver 10% better primal gaps compared with Gurobi in a 300s cut-off time.

1 INTRODUCTION AND RELATED WORK

Mixed-integer programming (MIP) is a well-established optimization problem. In many cases, feasible or even optimal solutions are required under strong time limits, and thus efficiently finding high-quality solutions is of great importance. Recently, machine learning for combinatorial optimization has been an emerging topic (Bengio et al., 2021) with prominent success in different tasks, e.g. graph matching (Yan et al., 2020), and ML4MIP is also an emerging field (Zhang et al., 2023).

A variety of deep learning-based solving methods were proposed to deal with specific MIP problems, including construction methods (Ma et al., 2019; Xing & Tu, 2020; Fu et al., 2021; Zhang et al., 2020; Khalil et al., 2017; Xin et al., 2021) and iterative-based refinements (Wu et al., 2021b; Chen & Tian, 2019; Lu et al., 2019; Li et al., 2020). While they cannot be directly applied to a wider scope of MIP problems, and thus learning the solving policies for general MIP problems has also been intensively studied, in which the primal heuristics catch more attention, including Large Neighborhood Search (LNS) (Wu et al., 2021a; Song et al., 2020; Nair et al., 2020a) and Local Branching (LB) (Liu et al., 2022). This paper focuses on LNS for solving general MIP problems – the powerful yet expensive iteration-based heuristics (Hendel, 2022).

Traditional LNS methods usually explore a complex neighborhood by predefined heuristics (Gendreau et al., 2010), in which the heuristic selection is a long-standing challenging task, especially for general MIP problems, which may require heavy efforts to design valid heuristics. Learning-based methods provide a possible direction. Both Imitation Learning (IL) (Song et al., 2020) and Reinforcement Learning (RL) (Wu et al., 2021a; Nair et al., 2020a) showed effectiveness in learning decomposition-based LNS policies. While there still exist some challenges. The performance of the learned policies may significantly degrade when applied to general integers due to the vast scale of candidate values (compared to binary variables), leading to a large complexity in optimization. Moreover, the learned policies may be trapped in local optimum for complicated cases.

Table 1: Comparison of our method to existing works, and it achieves the SOTA performance on public benchmarks.

References	Applicability	Approach	Addressing Local Optima	Training
Huang et al. (2023b)	Binary	LNS	Adaptive Neighborhood Size	Contrastive Learning
Liu et al. (2022)	Binary	Local Branching	RL-based Branching Size	Regression + RL
Wu et al. (2021a), Nair et al. (2020a)	Binary	LNS	/	RL
Song et al. (2020)	Binary	LNS	/	Imitation & RL
Hendel (2022)	General MIP	ALNS (Heuristic in B&B)	Adaptive Control for Multiple Heuristics	Multi-armed Bandit
Sonnerat et al. (2021)	General MIP	LNS	Adaptive Neighborhood Size	Imitation
BTBS-LNS (Ours)	General MIP	Branching on top of LNS	**Step-wise Global Information**	**RL (LNS) + Imitation (Branching)**

In this paper, we propose a Binarized-Tightening, Branch and Search-based LNS approach (**BTBS-LNS**) for general MIP problems. Specifically, we design the "Binarized Tightening" algorithm to deal with the optimization for general integer variables, where we first binarize the general integer variables and express them with the resulting bit sequence, and then tighten the bound of original variables w.r.t. the LNS decision along with the current solution. This way, the variable bounds can be tightened and effectively explored at a controlled complexity. Based on our binarization formulation, we further employed an attention-based tripartite graph (Ding et al., 2020) to encode the MIP instances and improved the attention architecture by removing the softmax normalization, which allows us to fully preserve the raw weights between neighboring nodes. Meanwhile, to enhance exploration and optimize some wrongly-fixed backdoor variables (Williams et al., 2003; Khalil et al., 2022) by the learned LNS policy, we leverage an extra branching network at each step, providing branching decisions at the global view to help escape local optimum. In a nutshell, this paper can be characterized by the following bullets, which we believe are common building blocks:

1) Bound Tightening for MIP. We propose a new "Binarized Tightening" scheme for general MIP problems with an efficient embodiment of variable encoding and bound tightening techniques.

2) Combining global information step by step with LNS. To assist the learned policy in escaping the local optimum efficiently, we devise an extra variable branching mechanism to select and optimize the LNS wrongly fixed backdoor variables, by contrast with the global optimum. The hybrid branch and search policy greatly enhances exploration and show efficiency.

3) Problem encoding with improved attention architecture. We employ an attention-based tripartite graph to encode MIP problems and capture correlations using an improved attention approach, which demonstrates empirical effectiveness.

4) Strong empirical results. Experiments on seven MIP problems show that our method consistently outperforms the LNS baselines and open-source SCIP (Gamrath et al., 2020). On MIPLIB2017 benchmark[1] chosen by Hans Mittelmann, it even achieves superior performance over Gurobi, purely taking SCIP as the baseline solver. It can further boost Gurobi when taking Gurobi as the baseline solver (see Appendix A.4).

We summarize the key related works in Table 1, and elaborate on more details in Appendix A.1. **Source code will be publicly available.**

2 PRELIMINARIES

Mixed Integer Program (MIP) is in general defined as:

$$\begin{aligned} \min \quad & \mathbf{c}^\top \mathbf{x} \\ s.t. \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & x_i \in \{0,1\}, \forall i \in \mathcal{B}; x_j \in Z^+, \forall j \in \mathcal{G}; x_k \geq 0, \forall k \in \mathcal{C} \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is a vector of n decision variables; $\mathbf{c} \in \mathbb{R}^n$ denotes the vector of objective coefficients. $\mathbf{A}\mathbf{x} \leq \mathbf{b}$ denotes the overall m linear constraints, where $\mathbf{A} \in \mathbb{R}^{m \times n}$ represents the incidence matrix, with $\mathbf{b} \in \mathbb{R}^m$. For general MIP instances, the index set of n variables $\mathcal{N} := \{1, ..., n\}$ can be partitioned into three sets, binary variable set $\mathcal{B}$, general integer variable set $\mathcal{G}$ and continuous variable set $\mathcal{C}$. MIP presents greater challenges compared to integer programming (Wu et al., 2021a) as the continuous variables may require distinct optimization policies with integer variables.

[1] https://plato.asu.edu/bench.html

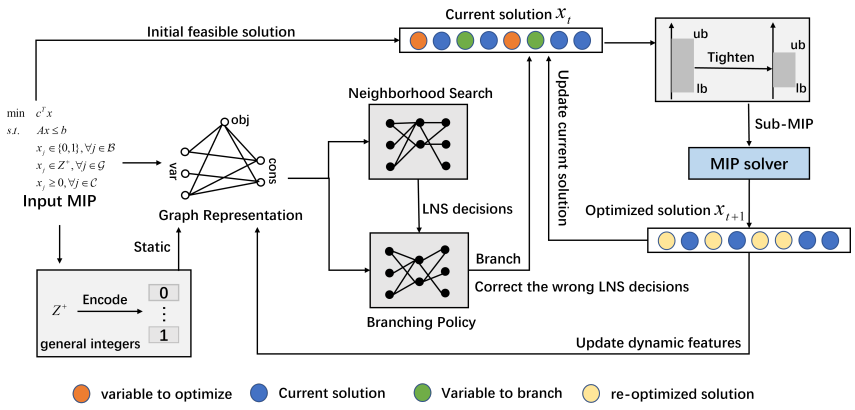

Figure 1: Overview of **BTBS-LNS**. First, we propose "Binarize Tightening" to handle general integer variables. The *Binarize* mechanism can binary-encode the variables and split them into sub-optimization bits. With the bit-wise decision by LNS, the variable bounds can be refined by bound tightening. Second, we devise a branching network on top of LNS to select wrongly fixed backdoor variables at a global view, which may help efficiently escape local optimum in some cases.

Large Neighborhood Search (LNS) is a powerful yet expensive heuristic for MIP (Gendreau et al., 2010). It takes the best feasible solution so far $\mathbf{x}^*$ as input and searches for the local optimum in its neighborhood:

$$\mathbf{x}' = \arg \min_{\mathbf{x} \in N(\mathbf{x}^*)} \{\mathbf{c}^\top \mathbf{x}\} \tag{2}$$

where $N(\cdot)$ is a predefined neighborhood - the search scope at each step, and $\mathbf{x}'$ denotes the optimized solution within $N(\mathbf{x}^*)$, obtained by destroying and re-optimization from the current solution.

Compared to local search heuristics, LNS can be more effective by using a broader neighborhood. However, the selection of neighborhood function $N(\cdot)$ is nontrivial. Heuristic methods mainly rely on problem-specific operators, e.g., 2-opt (Flood, 1956) in TSP, which call for considerable trial-and-error and domain knowledge (Papadimitriou & Steiglitz, 1998). The currently popular learning-based approaches mainly focus on binary variables and may be trapped in local optimum due to the learning complexity. In this paper, we propose a binarized-tightening branch-and-search LNS approach, designed to address general MIP problems. It may efficiently escape local optimum when the LNS decisions are unreliable in some scenarios.

3 METHODOLOGY

3.1 OVERVIEW

Fig. 1 presents the overview of our approach. The input is a MIP instance, with its initial feasible solution $\mathbf{x}_0$ generated by a baseline solver. General integer variables are first encoded into binary substitute variables, and the instance is subsequently represented as a tripartite graph (Ding et al., 2020), which is then fed into the large neighborhood search network, selecting the variable subsets that may need to be optimized at each step, with the remaining variables fixed or bound-tightened (**see Sec. 3.2 and 3.3**). Additionally, we devise an extra branching network to select some wrongly-fixed backdoor variables by the learned LNS policy, to help escape local optimum. With the sequential decisions of the branch and search policy and the resulting tightened variable bounds, an off-the-shelf solver, e.g. SCIP, is applied to obtain the optimized feasible solution $\mathbf{x}_{t+1}$. Iterations continue until the time limit is reached, and the optimized solutions can be obtained.

In general, the neighborhood search policy and branching policy are trained sequentially, where the training details are described in Sec. 3.3 and Sec. 3.4, respectively. They optimize the current solution from different views and may remedy the local search drawbacks in some cases.

3.2 THE BINARIZED TIGHTENING SCHEME

Variables in general MIP instances can be divided into three categories: binary, general integer (with arbitrary large value), and continuous variables. Previous studies mainly focused on the binary vari-

ables $(0/1)$. Limited values greatly simplify the optimization, making it easier to deal with compared to the general integer variables, and some learning frameworks have proved their effectiveness (Wu et al., 2021a; Song et al., 2020). In this paper, we concentrated on more general MIP problems, especially for general integer variables.

An intuitive method is to directly migrate some efficient binary LNS approaches, e.g., Wu et al. (2021a), to general integers. In this way, different types of variables are equally treated. At each step, we fix some of the variables (no matter what type the variable belongs to), and solve the sub-MIP with a baseline solver e.g. SCIP (Gamrath et al., 2020) or Gurobi (G., 2020). However, empirical results revealed that the simplified generalized LNS approach (e.g., RL-LNS (Wu et al., 2021a)) is much slower and significantly underperforms the MIP solvers, e.g., Gurobi. (see Table 4 and Fig. 3 for detail comparison.)

To address these challenges, we propose the so-called "Binarized Tightening" scheme for MIP. The idea is to confine the variables within a narrow range around the current solution, rather than directly fixing them, to balance exploration and exploitation. It shares similar insights with local search, which relies on the current best solution to guide the search, thus avoiding blind search throughout the entire solution space. Specifically, we represent each general integer variable with $d = \lceil \log_2 (ub - lb) \rceil$ binary variables at a decreasing magnitude, where ub and lb are original variable upper and lower bounds, respectively. The subsequent optimization is applied to the substitute binary variables, indicating whether the current solution is reliable or not. In this way, we transform the LNS for the original variable to multiple decisions on substitution variables. Note that the unbounded variables where ub or lb does not exist, will not be encoded and remain a single variable.

Algorithm 1 Bound tightening for binarized integer variable x_i

Require: Initial lower and upper bound of x_i: lb, ub;
Current solution value: $x_i = p$;
Binary LNS decision for x_i: a_i^t for unbounded variables, and $\{a_{i,j}^t | j = 1, 2, ..., d\}$ for others.
Ensure: Tightened lb, ub

1: **if** x_i unbounded **then**
2: **if** lb existed **and** $a_i^t = 0$ **then**
3: $ub = 2p - lb$
4: **else if** ub existed **and** $a_i^t = 0$ **then**
5: $lb = 2p - ub$
6: **end if**
7: **else**
8: $d = \lceil \log_2 (ub - lb) \rceil$
9: **for** $j = 0 : d$ **do**
10: **if** $a_{i,j}^t = 0$ **then**
11: $lb = \max(lb, p - 1/2(ub - lb))$;
12: $ub = \min(ub, p + 1/2(ub - lb))$;
13: **else**
14: break;
15: **end if**
16: **end for**
17: **end if**

The decision for each substitute variable can be obtained from the LNS policy (see Sec. 3.3), where 0 means the variable indicates reliability at the current encoded bit, and 1 means it still needs exploration. We design a bound-tightening scheme to fully use the bit-wise decisions in Alg. 1 (see Appendix. A.8 for a small numerical example). Specifically, let $a_{i,j}^t$ represent the decision for the jth substitute variable of variable i at step t. Decisions $a_{i,j}^t$ for all j are checked, and the upper and lower bounds will be tightened around the current solution whenever $a_{i,j}^t = 0$, as in Line 11-12. Therefore, more fixed substitute variables can contribute to tighter bounds. In our embodiment, variables that sit far from both bounds can have a significantly wider exploration scope than close-to-bound variables, as they showed no explicit "preference" on either bound direction, which is significantly different from Nair et al. (2020b) (see Appendix A.1 for detailed discussion). Tightening on either bound when the current solution sits precisely at the midpoint of variable bounds, may contribute to performance degradation, which conceptually drives us to design the bound tightening scheme, tightening the bounds on the far side iteratively.

In addition, as for unbounded variables, meticulous analysis of MIPLIB2017 benchmark (Gleixner et al., 2021) revealed that all unbounded variables within the instances are characterized by unbounded in only one direction, which means that either lb or ub will exist for all general integer variables (otherwise it will be free to optimize in our implementation). In this respect, we define a virtual upper (lower) bound when $a_i^t = 0$ as in Line 3 and 5, which share similar insights with regular variables to put the current solution at precisely the midpoint of the updated bounds.

3.3 GRAPH-BASED LNS POLICY PARAMETERIZATION

A bipartite graph is recently popularly utilized in Gasse et al. (2019), Nair et al. (2020b), and Wu et al. (2021a) to represent the MIP instance states. However, the objective is not explicitly considered, which may contribute to performance degradation in some cases, e.g., when all discrete variables do not exist in the objectives (Yoon, 2022). To capture the correlations between objectives with variables and constraints reasonably, we propose to describe the input instance as a tripartite graph $\mathcal{G} = (\mathcal{V}, \mathcal{C}, \mathcal{O}, \mathcal{E})$, where $\mathcal{V}$, $\mathcal{C}$, and $\mathcal{O}$ denote the variable, constraint, and objective nodes, and $\mathcal{E}$ denotes the edges. The features of nodes and edges can refer to Appendix A.2, where the new objective node representations are defined as the average states of corresponding variables.

We parameterize the policy $\pi_\theta(a_t|s_t)$ by an attention-based Graph Convolution Network (GCN). Different from Graph Attention Networks (GATs) utilized in (Veličković et al., 2018; Ding et al., 2020), we remove the *softmax* normalization to fully reserve the raw weights between neighboring nodes and edges, capturing the contributions for each node to the final objectives (see Table 2, 4 for comparison with traditional message passing mechanism: **LNS-ATT**). The $\mathcal{C} \to \mathcal{V}$ passing is as follows (likewise for others):

$$\mathbf{h}_i^{t+1} = f_{\mathcal{CV}}\left(\text{CONCAT}\left(\mathbf{h}_i^t, \frac{\sum_{j \in \mathcal{C} \cap N_i} w_{ij}^t (\mathbf{h}_j^t + \mathbf{h}_{e_{ij}}^t)}{|\mathcal{C} \cap N_i|}\right)\right) \tag{3}$$

where $\mathbf{h}_i^t$ and $\mathbf{h}_{e_{ij}}^t$ are the features of node i and edge (i, j) at step t; $f_{\mathcal{CV}}$ is a 2-layer perceptron with relu activation that maps the current states to the next iteration $\mathbf{h}_i^{t+1}$; N_i denotes the neighborhood nodes of i and $|\mathcal{C} \cap N_i|$ is the counts of neighborhood constraint nodes for node i, used to normalize the weighted sum neighboring features; w_{ij}^t denotes the weighted coefficient between node i and node j at step t, measuring their correlations as follows, where $\mathbf{W}_{\mathcal{CV}}$ is the weight matrix between constraint and variable.

$$w_{ij}^t = \sigma_s(\mathbf{W}_{\mathcal{CV}} \cdot \text{CONCAT}(\mathbf{h}_i^t, \mathbf{h}_{e_{ij}}^t, \mathbf{h}_j^t)) \tag{4}$$

At each graph attention layer, the message passing between different types of nodes are $\mathcal{V} \to \mathcal{O}$, $\mathcal{O} \to \mathcal{C}$, $\mathcal{V} \to \mathcal{C}$, $\mathcal{C} \to \mathcal{O}$, $\mathcal{O} \to \mathcal{V}$, $\mathcal{C} \to \mathcal{V}$, which are calculated as Eq. 3 sequentially. In this way, after K iterations, the features for both the nodes and edges are updated. We finally process the variable nodes by a multi-layer perceptron and the output value can be regarded as the *destroy* probability for each variable at this step, serving as the neighborhood search policy in Fig. 1. It is trained with Q-actor-critic by RL, following the same protocol with Wu et al. (2021a), while with the following differences:

States: We adopt an attentional tripartite graph to capture correlations among variables, constraints, and objectives. The features are gathered in Table 7 in the Appendix.

Actions: For the general variable x_i with d substitutes, the LNS decision at step t will contain d binary actions $a_{i,j}^t$, indicating the current solution reliable or not at each encoded bit j (see Alg. 1).

Transition and rewards: We follow the same protocol as in (Wu et al., 2021a), where the next state s_{t+1} is obtained by the baseline solver, and the reward is defined as objective improvements.

3.4 STEP-WISE GLOBAL INFORMATION BY BRANCHING

As discussed above, previous single-policy approaches were easily trapped in local optimum at an early stage in some complicated tasks, due to the learning complexity and limited exploration even in the training phase. To remedy this issue, an intuition is to select and optimize those wrongly fixed backdoor variables by LNS policy at each step. With this insight, we proposed to learn an extra branching network with imitation learning on top of LNS to filter out those variables at each step. Note that it was only applied to binary variables which are more likely to be backdoors that were fixed earlier, leading to local optima.

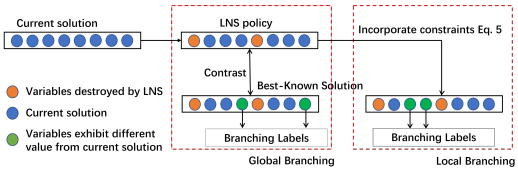

Figure 2: Global branching **vs** Local branching on different label collection schemes

The most critical issue for the branching policy learning is the collection of branching variable labels. In other words, we need to figure out how to identify the potentially wrongly-fixed variables at each step. We proposed two different variants, which deal with the issue in global and local view respectively as in Fig. 2:

Algorithm 2 Offline training of branching policy for LNS

Require: graph-based states $S = \{s_t | t = 1, 2, ..., n\}$
LNS decisions at each step $N = \{n_t | t = 1, 2, ..., n\}$
branching variable labels $B = \{b_t | t = 1, 2, ..., n\}$ collected from the global or local branching;
Ensure: trained policy $\pi_\theta(B|S, N)$
// Samples are collected by resolving the training instances, along with the learned LNS;
Let $D = \{((s_t, n_t), b_t) | t = 1, 2, ..., n\}$.
// train the model;
Initialize all learnable parameters θ;
while stopping criteria not meet **do**
  Randomly select a batch of instances D_C from D;
  Optimize θ by minimizing cross-entropy loss;
end while

Global branching (BTBS-LNS-G): It gathers labels from the fixed variables by LNS at each step and contrasts them with the global optimal solution. Variables that exhibit differing values between these solutions are indicative of potentially misclassified variables within the current LNS decisions from a global perspective. Since the global optimal solution may be too difficult to acquire in a reasonable time, it was replaced by the best-known solution obtained across various approaches within the same time budget.

Local branching (BTBS-LNS-L): Different from the global view contrast, it gathers labels by incorporating the following local branching constraints (Liu et al., 2022) at each step:

$$\sum_{i \in \mathcal{B} \cap \mathcal{F}} |x_i^{t+1} - x_i^t| \leq k \tag{5}$$

where $\mathcal{F}$ is the currently fixed variables set by LNS. With this extra constraint, the re-defined sub-MIP can be solved by the baseline solver, and up to K changed fixed variables will be selected at a local view as the branching variable labels at the current step. The selected variables are regarded as locally wrongly fixed variables by LNS.

Algorithm 3 Branch and search at the tth step

Require: Number of variables n;
LNS decisions $N^t = \{n_i^t | i = 1, 2, ..., n\}$;
branching decisions $B^t = \{b_i^t | i = 1, 2, ..., n\}$;
variable set $\mathbf{x} = \{x_i | i = 1, 2, ..., n\}$;
best solution at the tth step $\mathbf{x}^t = \{x_i^t | i = 1, 2, ..., n\}$;
The ratio for branching variables r;
Ensure: $\mathbf{x}^{t+1}$;
Let $D = \emptyset$;
for $j = 0 : n$ **do**
  if x_i is general integer variable **then**
    Tighten the bound as in Alg. 1 using $n_{i,j}^t$ (with d separate decisions for each substitute variable);
  else
    if $b_i^t = 1$ and x_i is binary variable **then**
      $D = D \cup \{i\}$;
    else
      if $n_i^t = 0$ **then**
        Fix the value $x_i^{t+1} = x_i^t$;
      end if
    end if
  end if
end for
add constraint $\sum_{i \in D} |x_i^{t+1} - x_i^t| \leq rn$ to sub-MIP;
Optimize $\mathbf{x}^{t+1}$ with the solver;

With the collected labels for each variable, branch (1) or not (0), the branching network can be trained offline. The inputs are tripartite graph-based features (see Table 7 in the Appendix), where we additionally append the LNS decisions made by the learned LNS policy as variable features, as we only focused on the fixed variables for extra branching. Note that the input states are collected by resolving the training instances, along with the learned LNS policy. The labels are also gathered within the resolving at each step. Then the graph-based features are fed into a similar graph attention network as described in Sec. 3.3 to update the node/edge representations. We finally process the variable nodes by a multi-layer perceptron (MLP) and the output value can be regarded as the branching probability for each variable at this step. Cross-entropy loss was utilized to train the branching network to bring the outputs closer to the collected labels, with the pipeline as in Alg. 2.

Except for the label collection scheme, **BTBS-LNS-L** and **BTBS-LNS-G** remain all the same. The branching policy takes effect on top of LNS, enhancing exploration and optimizing its wrongly fixed backdoor variables at each step. The pipeline for the hybrid framework is given in Alg. 3, where

Table 2: Comparison on binary Integer Programming (IP) problems: SC, MIS, CA, MC. We also let SCIP run for a longer time (500s with SCIP (500s) and 1000s with SCIP (1000s), respectively). So for Gurobi and our **BTBS-LNS** in other tables.

Methods	Set Covering (SC)		Maximal Independent Set (MIS)		Combinatorial Auction (CA)		Maximum Cut (MC)	
	Gap%	PI	Gap%	PI	Gap%	PI ($\times 10^3$)	Gap%	PI
SCIP	3.23	20225	0.25	312.25	4.71	3312.4	8.01	15193
SCIP (500s)	1.40	/	0.18	/	3.36	/	7.11	/
SCIP (1000s)	1.06	/	0.09	/	2.40	/	6.87	/
U-LNS	3.84	22459	1.50	1145.4	9.42	4003.0	6.72	11565
R-LNS	4.17	23015	1.29	693.45	6.92	3631.2	6.33	10923
FT-LNS	3.48	20988	1.42	1103.7	9.83	4123.6	6.30	10554
DINS	3.97	22735	1.24	657.5	4.48	3337.4	5.75	10006
GINS	3.81	22197	0.75	683.6	6.90	3599.8	5.41	9765.0
RINS	3.63	21835	1.32	816.5	7.33	3843.4	6.04	10277
RENS	2.35	19112	0.79	792.36	4.40	3125.2	5.29	9116
RL-LNS	1.29	17623	0.07	182.63	2.36	2271.6	4.25	6538
Branching	1.72	18007	0.07	183.44	3.09	2492.7	3.99	6104
GNN-GBDT	1.78	18169	0.22	295.43	2.24	2206.9	4.85	7492
CL-LNS	0.92	17025	0.07	182.99	2.05	2198.5	3.03	3883.5
LNS-TG	0.66	16828	0.08	182.24	2.32	2247.8	3.05	4782.6
LNS-Branch	1.11	17234	0.09	182.19	2.36	2275.3	3.73	5840.0
LNS-ATT	0.65	16714	0.07	182.10	2.23	2231.5	2.99	3975.1
BTBS-LNS-L	0.47	16234	0.05	181.47	2.18	2196.8	1.99	2518
BTBS-LNS-G	**0.35**	**16205**	0.05	178.35	**1.43**	**1998.9**	**0.59**	**785**
Gurobi	0.75	16796	**0**	**173.15**	1.44	2075.4	0.62	842

Table 3: Generalization to large-scale binary IP instances using the trained policies from small problems in Sec. 4.2

Methods	Set Covering (SC2)		Maximal Independent Set (MIS2)		Combinatorial Auction (CA2)		Maximum Cut (MC2)	
	Gap%	PI	Gap%	PI	Gap%	PI($\times 10^3$)	Gap%	PI
SCIP	4.51	14953	3.45	9542.1	17.87	12312	8.38	30039
SCIP (500s)	2.74	/	0.86	/	8.18	/	8.26	/
SCIP (1000s)	1.37	/	0.52	/	5.13	/	8.13	/
U-LNS	3.96	14268	0.97	2778.5	8.53	8032.5	7.03	24862
R-LNS	3.94	14392	0.71	2079.3	6.34	7050.0	6.52	22450
FT-LNS	4.49	14885	0.96	2765.6	9.08	8324.2	6.44	22347
DINS	2.99	13916	0.65	1935.4	6.11	6848.5	7.02	24815
GINS	3.14	14008	0.69	2011.5	6.74	7433.7	6.52	22477
RINS	2.95	13793	0.58	1844.7	6.55	7129.3	6.75	23619
RENS	2.78	13465	0.55	1782.6	6.02	6735.2	6.23	20959
RL-LNS	1.66	13007	0.51	1524.7	4.13	5933.4	3.20	8449.6
Branching	1.53	12916	0.55	1769.4	4.52	6142.7	3.19	7857.3
GNN-GBDT	1.78	13069	0.55	1549.3	3.44	5508.9	2.79	6533.7
CL-LNS	1.41	12914	0.41	1298.5	3.51	5621.7	2.83	7184.1
BTBS-LNS-L	**0.51**	**12431**	0.04	543.69	**1.67**	**4800.3**	1.45	3385.9
BTBS-LNS-G	0.68	12498	0.02	515.28	1.89	5012.6	1.44	3397.5
Gurobi	0.71	12528	**0.01**	**495.88**	3.60	5723.5	**1.01**	**2195.6**
Methods	Set Covering (SC4)		Maximal Independent Set (MIS4)		Combinatorial Auction (CA4)		Maximum Cut (MC4)	
	Gap%	PI	Gap%	PI	Gap%	PI($\times 10^3$)	Gap%	PI
SCIP	5.41	15524	3.45	22745	16.61	25275	8.71	78510
SCIP (500s)	4.21	/	3.44	/	16.61	/	8.69	/
SCIP (1000s)	3.05	/	3.03	/	16.61	/	8.46	/
U-LNS	3.42	14814	1.41	9759.0	7.42	16470	7.39	68245
R-LNS	3.26	14747	0.98	7745.5	6.19	15875	6.98	64712
FT-LNS	3.75	14882	1.30	9150.3	8.30	17328	7.02	65329
DINS	3.23	14725	1.03	7982.4	5.02	14789	6.97	64593
GINS	3.28	14782	0.85	7244.7	5.99	15538	7.04	65778
RINS	2.96	14599	1.09	8218.0	5.78	15309	6.89	63575
RENS	2.95	14573	0.82	6972.1	5.17	14916	6.85	62998
RL-LNS	3.73	14866	0.57	5365.1	3.52	13572	3.76	39645
Branching	3.39	14689	0.64	5744.8	3.37	13349	4.21	42718
GNN-GBDT	3.45	14169	0.59	5233.5	2.86	12853	4.52	45423
CL-LNS	3.39	14325	0.45	4533.4	2.99	13025	3.29	37384
BTBS-LNS-L	**0.84**	**13716**	0.07	**2140.4**	**1.39**	**11128**	1.52	21195
BTBS-LNS-G	1.20	13789	0.11	2636.9	1.46	11705	**1.51**	**20984**
Gurobi	1.22	13795	**0.04**	2215.7	12.61	21959	5.38	51298

we fix or tighten the bounds for some variables by the LNS policy (see Line 4, 10), and select some variables that were labeled 1 by the branching policy (see Line 6, 7) for extra branching. The hybrid branch and search policy work together to formulate the sub-MIP at each step.

4 Experiments

4.1 Settings and Protocols

Peer methods. We compare with the following baselines in a 200s time limit by default.

1) SCIP (v7.0.3), Gurobi (v9.5.0): state-of-the-art open source and commercial solvers, and were fine-tuned with the aggressive mode to focus on improving the objectives.

2) U-LNS(Wu et al., 2021a), R-LNS(Song et al., 2020): randomized LNS following its implementation as in Wu et al. (2021a) and Song et al. (2020).

3) DINS (Ghosh, 2007), GINS (Maher et al., 2017), RINS (Danna et al., 2005) and RENS (Berthold, 2014): heuristic-based LNS policies that were common utilized.

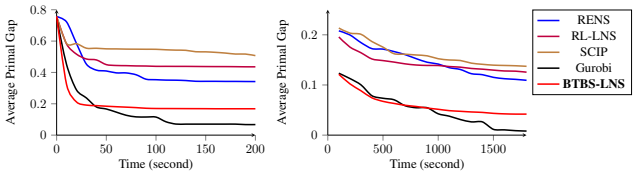

Figure 3: Performance on Item (Left) & AMIPLIB (Right).

4) FT-LNS (Song et al., 2020), RL-LNS (Wu et al., 2021a), Branching (Sonnerat et al., 2021), CL-LNS (Huang et al., 2023b) and GNN-GBDT (Ye et al., 2023): some learning-based LNS policies with imitation learning or RL, following the same protocol as its original implementation.

5) LNS-TG, LNS-Branch, LNS-IBT, LNS-IT, LNS-ATT: Degraded versions of **BTBS-LNS**, where we i) replace the tripartite graph with bipartite graph (**LNS-TG**); ii) remove the extra branching (**LNS-Branch**); iii) remove the binarized encoding (**LNS-IBT**) and bound tightening (**LNS-IT**); iv) replace the attention-based graph network with widely used GAT (Veličković et al., 2018) (**LNS-ATT**). Refer to Appendix A.2 for details.

6) BTBS-LNS-F: A variant of **BTBS-LNS**, where we replace our bound tightening mechanism with (Nair et al., 2020b).

Instances. It covers both binary and MIP problems. We follow (Wu et al., 2021a) to test four NP-hard binary Integer Programming Problems: Set Covering (SC), Maximal Independent Set (MIS), Combinatorial Auction (CA), and Maximum Cut (MC). We generate 200, 20, and 100 instances as training, validation, and testing sets, respectively. To evaluate the generalization ability, we also generate scale-transfer test instances, such as SC2 and MIS4 in Table 3. The suffix number refers to instance scales, for which the details are gathered in Table 8 in Appendix A.2.

We also test on two MIP datasets in Machine Learning for Combinatorial Optimization (ML4CO) competition[2]: Balanced Item Placement (**Item**) and Anonymous MIPLIB (**AMIPLIB**), on their official testing instances. Balanced Item Placement contained 1050 binary variables, 33 continuous variables, and 195 constraints per instance. The anonymous MIPLIB consists of a curated set of instances from MIPLIB2017, a long-standing benchmark for MIP solvers with diverse distributions, in which general integer variables are included. We also show empirical results on the whole MIPLIB2017 benchmark set in Sec. 4.5, where our **BTBS-LNS** even surpasses Gurobi on average.

Hyperparameters. We run experiments on an Intel 2.50GHz CPU. Performance comparison on CPU **vs** GPU version of our approach is given in Appendix A.5. All the approaches were evaluated with three different seeds, and the average performance was reported (see detailed stability analysis in Appendix A.6). We use the open-source SCIP (v7.0.3) as the baseline solver by default (recall the blue box in Fig. 1). Gurobi version experiments are gathered in Appendix A.4. We train 20 epochs for each instance, with 50 iterations per epoch and a 2s re-optimization time limit per iteration. LNS and branching are trained sequentially, with RL (see Sec. 3.3) and imitation learning (see Sec. 3.4), respectively. The graph convolutional layers were set as $K = 2$ for both policies, with 64-dimensional latent representations for the nodes and edges. Specifically for branching, we set the max branching variables $k = 50$ in Eq. 5 for the local branching variant. In the inference phase, the branching variable ratio r in Alg. 3 is empirically set to 10% for both branching variants. **BTBS-LNS by default denotes the local branching variant BTBS-LNS-L throughout this paper**.

Evaluation metric. We calculate the average primal gap (Nair et al., 2020b) to measure the gap between the current solution $\mathbf{x}$ and the best-known solution $\mathbf{x}^*$ found by all methods among the N testing instances, within a fixed time limit T_0:

$$gap = \frac{1}{N}\sum_{i=1}^{N}\frac{|\mathbf{c}_i^\top \mathbf{x}_i - \mathbf{c}_i^\top \mathbf{x}_i{}^*|}{\max\{|\mathbf{c}_i^\top \mathbf{x}_i|, |\mathbf{c}_i^\top \mathbf{x}_i{}^*|\}} \tag{6}$$

We also calculate the average Primal Integral (PI, (Huang et al., 2023b; Achterberg et al., 2012)) to evaluate the anytime performance within the time limit:

$$PI = \frac{1}{N}\sum_{i=1}^{N}\left(\int_{t=0}^{T_0}\mathbf{c}_i^\top \mathbf{x}_i^t dt - T_0\mathbf{c}_i^\top \mathbf{x}_i^*\right) \tag{7}$$

[2]https://www.ecole.ai/2021/ml4co-competition/

where $\mathbf{x}_i^t$ denotes the best solution within t for instance i.

4.2 Overall Performance Evaluation

Table 2 compares the results on integer programming. As can be seen, compared with SCIP and all competing LNS baselines, both **BTBS-LNS-G** and **BTBS-LNS-L** achieve consistently superior performance across all problems. LNS-TG, LNS-Branch, and LNS-ATT are degraded versions of **BTBS-LNS**, and they all perform slightly worse, revealing the effectiveness of the attention-based tripartite graph and the extra branching policy. Comparing the two variants, **BTBS-LNS-G** delivers consistently superior performance over **BTBS-LNS-L**, and it even surpasses the leading commercial solver on SC, CA, and MC. **Note that detailed anytime performance on these instances are shown in Fig. 6 to Fig. 9 in Appendix A.7**, further revealing the effectiveness of **BTBS-LNS**.

We also test our method on two NP-hard MIP problems, and the results are gathered in Table 4. Note that the anytime primal gap comparison is also shown in Fig. 3. Our method consistently outperforms SCIP and the competing LNS baselines and is slightly worse than Gurobi, capable of finding even better solutions for around 27% test instances on both Item and AMIPLIB.

Table 4: Performance on MIP instances.

Methods	Item			AMIPLIB
	Obj	Gap%	PI	Gap%
SCIP	23.33	50.73	4152.4	13.72
SCIP (500s)	19.83	39.41	/	/
SCIP (1000s)	17.02	31.05	/	/
U-LNS	20.39	44.29	3685.6	15.73
R-LNS	20.04	43.64	3485.0	14.96
RL-LNS	20.04	43.58	3498.5	12.55
DINS	18.08	37.23	3075.9	13.10
GINS	19.78	42.11	3514.7	13.64
RINS	20.53	44.88	3662.5	13.89
RENS	17.51	34.18	2925.0	11.75
Branching	18.84	40.12	3237.6	12.95
LNS-TG	18.05	37.85	3090.5	6.45
LNS-Branch	20.12	43.90	3537.0	9.32
LNS-ATT	15.54	26.91	2512.8	5.45
LNS-IBT	/	/	/	7.63
LNS-IT	/	/	/	7.65
BTBS-LNS-L	13.82	16.82	2030.3	4.19
BTBS-LNS-G	13.45	15.78	1912.5	4.35
BTBS-LNS-F	/	/	/	7.01
Gurobi	**12.67**	**6.73**	**1895.6**	**0.81**

For the AMIPLIB problem, which contains a curated set of instances from MIPLIB, we split the instances into train, validation, and test sets by 70%, 15%, and 15% with cross-validation. Policies learned from diverse training instances are directly applied to the test set. We increase the solving and re-optimization time limit at each step to 1800s and 60s for the instances, as they are too large to be solved. Different from Wu et al. (2021a), we consistently utilize open-source SCIP as the baseline solver. As seen from Table 4 and Fig. 3, our method significantly outperforms SCIP and LNS baselines and even delivers slightly better performance than Gurobi at an early stage. LNS-IBT, LNS-IT and **BTBS-LNS-F** achieve significantly inferior performance than our **BTBS-LNS**, showing the effect of the "Binarized Tightening" technique and its superiority over Nair et al. (2020b).

4.3 Problem-scale Generalization Ability Study

We test the generalization ability in line with (Wu et al., 2021a) with a 200s time limit. We directly use the trained policies on small-scale problems in Sec. 4.2, with results shown in Table 3.

As can be seen, the two variants show similar performance on the generalized instances. Compared with SCIP and all the competing LNS baselines, our approach still delivers significantly superior performance, showing a better generalization ability. As the problem sizes become larger, it can produce even better results than Gurobi on SC2, SC4, CA2, CA4, and MC4, and only slightly inferior on the remaining 3 groups. It suggests that our policies can sometimes be more efficient for larger instances than the leading commercial solver. Notably, there is a significant gap between **BTBS-LNS** and Gurobi for Combinatorial Auction (CA), particularly on CA4.

Table 5: Evaluation on CA against Gurobi.

Methods	CA2		CA4	
	Obj	Gap%	Obj	Gap%
Gurobi	-218245	3.60	-389396	12.61
Gurobi(500s)	-224245	0.95	-431626	3.14
Gurobi(1000s)	**-225629**	**0.33**	-436188	2.11
BTBS-LNS	-222590	1.67	-439431	1.39
BTBS-LNS(500s)	-225108	0.56	**-445563**	**0**

We further increase the time limit to 500s and 1000s respectively on CA, with results shown in Table 5. Our method consistently outperforms Gurobi with the same time limit. For CA4, it can even produce better solutions with a much shorter time limit. It empirically requires over 3 hours for Gurobi to deliver the same primal gap on CA4, being $58\times$ slower than our method.

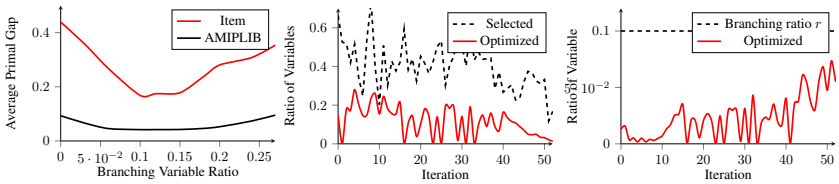

Figure 4: Impact of different branching ratios (Left). Selected & Optimized variables by the LNS (Middle) & Branching (Right) on Balanced Item Placement instances. The upper dotted line denotes the selected variable ratio that can be re-optimized by the learned Search & Branch policy, while the Solid red line denotes the variable ratio with solution value changes.

Table 6: Performance comparison on the whole MIPLIB2017 benchmark set.

	SCIP	SCIP(600s)	SCIP(900s)	U-LNS	R-LNS	FT-LNS	**BTBS-LNS**	**BTBS-LNS**-F	Gurobi
Gap%	15.15	11.08	8.79	16.26	15.94	13.07	**1.75**	3.11	1.98

4.4 Branching Policy Study by Variable Ratios

To enhance exploration, an extra branching policy was developed to incorporate global information and help the learned LNS policy escape local optimum. Fig. 4 (**Left**) depicts the impact of branching variables ratios r (see Alg. 3).

When the ratio $r < 0.1$, a larger size leads to better performance, optimizing some wrong decisions made by the learned LNS. Fig. 4 (**Right**) depicts the filtered and updated variable ratios. As can be seen, with the increasing iterations, more and more LNS fixed variables were re-optimized by the extra branching policy, indicating the LNS decisions were somewhat unreliable in some cases. In other words, branching on top of LNS was quite necessary to make up for some potential wrong decisions. However, when the branching size becomes extremely large, the performance significantly degrades limited by the solving ability.

4.5 Experiments on MIPLIB2017 Benchmark

To further evaluate our proposed approach on some heterogeneous and hard instances, we also evaluated the whole MIPLIB2017 benchmark set from Hans Mittelmann. It contains 240 instances with diverse distributions and difficulties. We compared different methods in a 300s time limit, which is the geometric mean of solving time of the solved instances with SCIP, and the re-optimization time for each iteration was set as 5s. Other hyperparameters remain the same as **AMIPLIB** in Sec. 4.2. We perform **cross-validation** to make a full comparison on all the instances and split them into training, validation, and testing sets by 70%, 15%, and 15% at each round. Policies learned from diverse training instances are directly applied to the test set.

The overall comparison results were gathered in Table 6. As can be seen, our proposed **BTBS-LNS** can deliver 10% better primal gaps compared with Gurobi and achieve significantly better results compared with all the competing baselines. Compared with Gurobi, it can deliver better solutions on 12.4% instances, and obtained equally better solutions on 77% instances, indicating its effectiveness and generalization ability. Furthermore, we notice that **BTBS-LNS-F** performs slightly inferior to Gurobi and our approach, further revealing the superior performance of our Binarized Tightening technique over Nair et al. (2020b). **Detailed per-instance comparisons are gathered in Appendix A.8**. We also conducted specific experiments on unbounded variables from MIPLIB2017, which are given in Appendix A.3.

5 Conclusion and Outlook

We have proposed a binarized tightening branch and search approach to learn LNS policies. It was designed to efficiently deal with general MIP problems and delivers superior performance over numerous competing baselines, including MIP solvers, learning and heuristic-based LNS approaches, on ILP, MIP datasets, and even heterogeneous instances from MIPLIB2017. Sufficient ablation studies demonstrate the effectiveness of each component. Considering the potential of our proposed **BTBS-LNS** on large-scale and cross-distribution instances, the applications in real-world scenarios may be our future direction.

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A APPENDIX

A.1 FURTHER DISCUSSION ON RELATED WORK

The main contributions of our **BTBS-LNS** are the general applicability and the addressing for local optima. Most LNS-based approaches (Liu et al., 2022; Nair et al., 2020a; Ding et al., 2020; Song et al., 2020; Wu et al., 2021a) solely deal with the binary programming problems due to their simplicity. Recently, some studies have tried to address the general MIP problems (Hendel, 2022; Sonnerat et al., 2021; Paulus et al., 2022a), in which Nair et al. (2020b) proposed a similar "bound tightening" technique. They differ from our approach in the following aspects. On one hand, the binary decision for each encoded variable was only applied for bound tightening in our approach, rather than directly fixed similar to Nair et al. (2020b). And on the other hand, the current solution value was also considered in bound tightening decisions in our approach. Variables that sit far from both bounds may have a significantly wider exploration scope than close-to-bound variables, as they showed no explicit "preference" in either direction. In addition, our approach can easily transfer to unbounded variables. We made a detailed comparison between the two approaches in Table 4 and Table 6. As can be seen, our **BTBS-LNS** consistently outperforms **BTBS-LNS-F**, demonstrating the effectiveness of our novel "Binarized Tightening" technique.

As for the local optima challenge, a few studies have tried Adaptive Neighborhood Size (ANS) (Huang et al., 2023a;b; Sonnerat et al., 2021) or hybrid heuristics control (Hendel, 2022), while it still requires hand-crafted hyperparameters, which are essential but difficult to determine. To address it more adaptively, we proposed to combine global-view information on top of LNS. When trapped in local optima, extra branching has the potential to select those wrongly fixed backdoor variables by the learned LNS policy for re-optimization. It is important to note that the concept of branching extends beyond the confines of the local branching (Sonnerat et al., 2021) and we also devised a novel variant termed "global branching", which can deliver even better performance in some cases. In addition, the major difference between our hybrid framework and the pure local branching approach (Sonnerat et al., 2021) lies in that we concentrate solely on variables fixed by LNS to correct its decisions, rather than the whole variable set. This specificity arises from the observation that LNS frequently converges to local optima when a limited number of backdoor variables are inaccurately fixed. Empirical results in Table 2, 3, and 4 demonstrated that our **BTBS-LNS** consistently outperforms the Branching baseline by Sonnerat et al. (2021).

We further review other studies related to ours, which can be divided into two categories: One is learning-based methods for specific MIP problems and the other is for general MIP problems.

Policy learning for specific MIP Problems: MIP problems cover numerous real-world tasks in many fields (Paschos, 2014) and quite a few studies attempt to solve certain types of problems, such as Traveling Salesman Problem (TSP) and Vehicle Routing Problems (VRP) (Li et al., 2021; Lu et al., 2019), etc. The algorithms can be divided into construction methods and learned improvement heuristics.

Construction methods usually attempt to directly learn approximate optimal solutions, like Graph Pointer Networks (GPNs) (Ma et al., 2019) and Monte Carlo tree search (Xing & Tu, 2020; Fu et al., 2021) for TSP instances. Compared to construction models, methods that learn improvement heuristics can often deliver better performance, by learning to iteratively improve the solution (Wu et al., 2021a). The improvement heuristics can be a guide for the next solution selection (Wu et al., 2021b), or policy to pick heuristics (Chen & Tian, 2019), or refinement from the current solution (Lu et al., 2019; Li et al., 2020). In general, both the learned improvement heuristics and construction methods have proved valid in some specific problems. In contrast, this paper aims to solve general MIP problems by learning improvement heuristic policies.

Learning to solve general MIP problems: Dual and primal are two main perspectives to improve solving efficiency for general MIP problems. Specifically, dual view aims to improve inner policies of Branch and Bound, like variable selection (Gasse et al., 2019; Zarpellon et al., 2021; Gupta et al., 2020), node selection (He et al., 2014) and cut selection (Tang et al., 2020; Paulus et al., 2022b;a). With a better decision at each node, the overall solving process can be greatly simplified.

In the primal perspective, the algorithms aim to find better feasible solutions by prediction or learning-based heuristics. For example, Ding et al. (2020) learned a tripartite graph-based deep

Table 7: Description of the tripartite graph features.

Tensor	Feature Description
$\mathcal{V}$	variable type (binary, integer, continuous).
	objective coefficient.
	lower and upper bound.
	reduced cost.
	solution value fractionality.
	(dynamic) solution value in incumbent.
	(dynamic) average solution value.
	(dynamic) best solution value.
	(Branching Only) LNS decisions at current step.
$\mathcal{C}$	cosine similarity with objective.
	tightness indicator in LP solution.
	dual solution value.
	bias value, normalized with constraint coefficients
$\mathcal{O}$	average states of related variables.
$\mathcal{V}$ - $\mathcal{C}$	constraint coefficient per variable.
$\mathcal{V}$ - $\mathcal{O}$	objective coefficient per variable.
$\mathcal{C}$ - $\mathcal{O}$	constraint right-hand-side (RHS) coefficients.

Table 8: Average variable/constraints of instances

Num of	Training						Generalization					
	SC	MIS	CA	MC	SC2	MIS2	CA2	MC2	SC4	MIS4	CA4	MC4
Variables	1000	1500	4000	2975	2000	3000	8000	5975	4000	6000	16000	11975
Constraints	5000	5939	2674	4950	5000	11933	5344	9950	5000	23905	10717	19950

Table 9: Training, Validation and Test accuracy for graph-based branching network.

	Local Branching						Global Branching					
	SC	MIS	CA	MC	Item	AMIPLIB	SC	MIS	CA	MC	Item	AMIPLIB
Train%	89.5	84.9	79.6	86.3	85.5	77.5	86.9	87.3	81.5	88.5	83.4	75.9
Validation%	84.8	83.5	75.1	82.1	82.8	74.9	83.7	84.9	80.9	87.0	81.8	75.1
Test%	82.5	81.6	72.9	80.5	81.5	74.2	83.1	82.6	80.1	84.5	80.7	73.8

neural network to generate partial assignments for binary variables, and to deal with the general integer variables, Nair et al. (2020b) proposed a bound tightening mechanism and learned partial assignments for each bit, respectively. Nevertheless, they were only applied in neural diving, and directly fixing may also lead to performance degradation or even infeasibility. To obtain broader applicability, learning-based primal heuristics, like large neighborhood search (Huang et al., 2023b; Song et al., 2020; Sonnerat et al., 2021; Nair et al., 2020a), and local branching (Liu et al., 2022), gradually catch more attention.

In this paper, we mainly focus on large neighborhood search heuristics, which have achieved remarkable progress in recent years. For example, Hendel (2022) designed an adaptive approach to combine multiple existing LNS heuristics to enhance the performance of a single policy, while it is largely limited by the rule-based heuristics and requires hand-crafted hyperparameters. To make it further, learning a better neighborhood function became popular recently. Sonnerat et al. (2021) and Song et al. (2020) both utilized imitation learning to select variable subsets to optimize at each step. However, the equal-size subsets make it inflexible and dramatically limit the performance. In this respect, Wu et al. (2021a) factorized the LNS policy into elementary actions on each variable and trained an RL-based policy to select variable subsets dynamically. However, current studies on LNS mainly focus on binary variables and are often susceptible to local optima even in the training phase due to the problem's complexity. In this respect, we propose a binarized-tightening branch and search approach to learn more efficient LNS policies for general problems.

A.2 Detail for the Experiments

Tripartite graph-based features: We describe in Table 7 the variable, constraint, objective, and multi-source edge features of the tripartite graph utilized in both the LNS and branching policy

Table 10: Performance comparison on MIPLIB2017 instances that contained unbounded variables.

Instance	SCIP	U-LNS	R-LNS	FT-LNS	**BTBS-LNSw/o ubd**	**BTBS-LNS**	Gurobi
gen-ip054	6858.879	6858.879	6852.733	6858.879	6852.733	6852.733	6840.966*
gen-ip002	-4783.733*	-4772.597	-4772.597	-4768.253	-4783.733*	-4783.733*	-4783.733*
neos-3046615-murg	1610	1670	1651	1651	1610	1607	1600*
buildingenergy	42652.34	42652.34	42652.34	42652.34	34243.89	33324.73	33283.85*

learning in detail. Then we will clarify the features and connections for the encoded substitute variables. They are also characterized by static and dynamic features. For static features, the variable type and bounds are set to binary and 1/0, respectively, while other features are directly inherited from the original integer variables. For dynamic features, the solution value for each substitute variable is determined based on the encoded results. For example, for a general integer variable with a range of [0, 7], if the current solution value is 5, the solution values of the three substitute variables would be 1, 0, 1. Additionally, the connections between the substitute variables and other nodes are directly inherited from the original integer variables.

Except for the dynamic solving status, all the other features are collected at the root node of the search tree, and the dynamic features are collected along with the optimization process.

Sizes for the generated instances with different difficulties: The average variable and constraint size used in our experiments are listed in Table 8, which consists of small-scale training instances and some hard instances to evaluate the generalization ability.

Accuracy for the imitation learning based branching policy: Table 9 reports the training, validation, and testing accuracy of the global and local branching variants.

We compare our proposed **BTBS-LNS** with various baselines, which are explained as follows in detail:

1) SCIP (v7.0.3), Gurobi (v9.5.0): state-of-the-art open source and commercial solver, and were fine-tuned with the aggressive mode to focus on improving the objectives.

2) U-LNS: an LNS version that uniformly samples variables at a fixed subset size. Note that for U-LNS, R-LNS and FT-LNS, we perform the same settings as Wu et al. (2021a).

3) R-LNS: an LNS version (Song et al., 2020) that randomly groups variables into equal subsets and re-optimizes them.

4) DINS (Ghosh, 2007), **GINS** (Maher et al., 2017), **RINS** (Danna et al., 2005) and **RENS** (Berthold, 2014): heuristic-based LNS policies.

5) FT-LNS (Song et al., 2020): an LNS approach that applies imitation learning to learn the best R-LNS policies.

6) RL-LNS (Wu et al., 2021a): Reinforcement learning LNS approach for variable subset optimization, while mainly focused on binary variable optimization.

7) Branching (Sonnerat et al., 2021): An imitation learning-based LNS approach that learned from local branching constraints.

8) LNS-TG: A variant of our method, where we replace the tripartite graph with the widely used bipartite graph.

10) LNS-Branch: A variant of our method, where we remove the extra branching policy.

11) LNS-IBT: A variant of our method, where the general integer variables are equally treated as binary variables.

12) LNS-IT: A variant of our method, where we remove the "Tightening" technique and fix the integer variable to its current solution when either bit is fixed.

13) LNS-ATT: A variant of our method, where we replace our attention-based graph attention network with the widely used GAT.

14) BTBS-LNS-F: A variant of our method, where we replace our bound tightening mechanism with that proposed by Nair et al. (2020b).

Table 11: Experiments with Gurobi as the baseline for binary Integer Programming (IP)

Methods	SC		MIS		CA		MC	
	Gap%	PI	Gap%	PI	Gap%	PI($\times 10^3$)	Gap%	PI
U-LNS	2.59	18820	0.41	635.32	3.78	2690.5	4.07	5633.8
R-LNS	3.01	18925	0.34	545.71	4.85	2999.0	3.75	5189.5
FT-LNS	3.38	19521	0.73	462.45	4.40	2856.4	3.79	5214.7
RL-LNS	1.57	16911	0.09	179.94	1.37	2029.1	3.52	4812.5
BTBS-LNS	**0.28**	**15987**	**0**	**165.24**	**0.27**	**1710.6**	**0.38**	**426.89**
Gurobi	0.75	16796	**0**	173.15	1.44	2075.4	0.62	842

Table 12: Generalization to large-scale binary integer programming (IP) instances with Gurobi as the baseline

Methods	SC2		MIS2		CA2		MC2	
	Gap%	PI	Gap%	PI	Gap%	PI($\times 10^3$)	Gap%	PI
U-LNS	2.48	13599	0.32	1551.6	3.06	5442.1	3.76	13713
R-LNS	2.73	14052	0.55	1845.2	2.60	5112.0	3.75	13359
FT-LNS	3.29	14338	0.28	1485.2	3.72	5823.5	4.04	13753
BTBS-LNS	**0.28**	**12275**	**0**	**462.38**	**0.47**	**4125.1**	**0.01**	**350.45**
Gurobi	0.71	12528	0.01	495.88	3.60	5723.5	1.01	2195.6
Methods	SC4		MIS4		CA4		MC4	
	Gap%	PI	Gap%	PI	Gap%	PI($\times 10^3$)	Gap%	PI
U-LNS	2.56	14150	0.64	5515.7	4.02	15712	4.95	46965
R-LNS	2.36	14112	0.52	4846.3	3.95	15275	4.84	46380
FT-LNS	3.34	14515	0.54	4915.0	3.68	14588	4.78	45795
BTBS-LNS	**0.27**	**13424**	**0.01**	**2051.8**	**0.67**	**10025**	**0**	**11034**
Gurobi	1.22	13795	0.04	2215.7	12.61	21959	5.38	51298

Note that the work by Sonnerat et al. (2021) doesn't have open-source code and some hyperparameters are difficult to fine-tune in different problems. However, to further evaluate our proposed framework with pure local branching based methods, we try to reproduce them, with the following details:

1) For a fair comparison, we replace the neural diving in Sonnerat et al. (2021) with an initial feasible solution generated by SCIP, the same as our approach.

2) In data collection, the desired Hamming radius η_t are selected as 50, the same as our branching policy.

3) The model structure was the same as its descriptions, where we use the code provided by Gasse et al. (2019), and additionally use a fixed-size window (3 in the paper) of past variable assignments as variable features.

4) In the inference phase with the learned policy, we performed the same action sampling mechanism as in Sonnerat et al. (2021). As for the adaptive neighborhood size, we start with 10% of the integer variable size, and the dynamic factor a was tuned from 1.01 to 1.05. Best-performing parameters will be selected for comparison in each problem. As a result, on SC and MIS, a was set as 1.02, and $a = 1.03$ can deliver the best performance on other problems.

A.3 DETAILED ANALYSIS ON MIPLIB2017

Table 13: Evaluation by Gurobi as baseline solver (MIP).

Methods	Item			AMIPLIB
	Obj	Gap%	PI	Gap%
U-LNS	17.64	36.08	3004.3	6.44
R-LNS	16.62	31.94	2788.6	6.01
FT-LNS	15.64	27.31	2519.4	5.45
BTBS-LNS	**12.27**	**4.56**	**1823.7**	**0.47**
Gurobi	12.67	6.73	1895.6	0.81

In Sec. 4.5, we evaluated our approach on the whole MIPLIB2017 benchmark set, which showed superior performance. To further evaluate the effectiveness of our novel virtual bound technique specifically for unbounded integer variables (see Alg. 1), we conducted an extensive analysis across all instances featured in the MIPLIB2017 benchmark set. Notably, there are 19 and 4 instances that contained unbounded integer variables before and after the presolve, respectively. In this section, we compared our **BTBS-LNS** with a variant **BTBS-LNSw/o ubd**, where the special handling for unbounded integer variables (see Line 2-6 in Alg. 1) is removed. In other words, unbounded variables were free to optimize at each step. The comparison results on the four instances that still contain unbounded variables after presolve are gathered in Table 10.

Table 14: Average Standard Deviations for our proposed **BTBS-LNS** on different problems.

Methods	SC		SC2		SC4	
	Obj	Gap%	Obj	Gap%	Obj	Gap%
BTBS-LNS	547.88 ± 0.59%	0.47 ± 0.88%	293.56 ± 0.77%	0.51 ± 0.68%	169.80 ± 0.68%	0.84 ± 1.01%
Methods	MIS		MIS2		MIS4	
	Obj	Gap%	Obj	Gap%	Obj	Gap%
BTBS-LNS	-685.86 ± 0.74%	0.05 ± 0.78%	-1372.66 ± 0.51%	0.04 ± 0.21%	-2747.04 ± 0.32%	0.07 ± 0.19%
Methods	CA		CA2		CA4	
	Obj	Gap%	Obj	Gap%	Obj	Gap%
BTBS-LNS	-112864 ± 0.32%	2.18 ± 0.29%	-222590 ± 0.39%	1.67 ± 0.41%	-439431 ± 0.33%	1.39 ± 0.49%
Methods	MC		MC2		MC4	
	Obj	Gap%	Obj	Gap%	Obj	Gap%
BTBS-LNS	-909.17 ± 0.48%	1.99 ± 0.52%	-1831.00 ± 0.66%	1.45 ± 0.58%	-3664 ± 0.73%	1.52 ± 0.84%
Methods	Item		AMIPLIB		MIPLIB2017	
	Obj	Gap%	Obj	Gap%	Obj	Gap%
BTBS-LNS	13.82 ± 1.09%	16.82 ± 0.96%	/	4.19 ± 1.51%	/	1.75 ± 1.62%

As can be seen, our proposed **BTBS-LNS**, outperforms the variant **BTBS-LNSw/o ubd** on two instances and achieves parity on the other two. These findings underscore the potent effectiveness of our proposed bound-tightening technique, substantiating its value in enhancing solution quality and optimization efficiency. We will continue the experimentation on more unbounded MIP problems in the future.

A.4 Gurobi version of our **BTBS-LNS**

To evaluate the performance of different approaches with Gurobi as the baseline solver, we perform extensive experiments on MIP problems, four binary integer programming problems, and their scale-transfer instances.

The hyperparameters remain unchanged from those in SCIP counterparts. The results of four binary integer programming problems and their scale-transfer instances are gathered in Table 11 and Table 12. The comparison results on MIP problems are reported in Table 13. As can be seen, our **BTBS-LNS** consistently outperforms Gurobi across all the problems with different sizes, indicating the effectiveness and generalization ability to different solvers.

A.5 Experiments with CPU vs GPU

All the experiments presented in Sec. 4 were performed on the Intel(R) Xeon(R) E5-2678 v3 2.50GHz CPU with 4 physical cores, and it achieved competitive performance even compared with the leading commercial solver. In this section, we will further evaluate the GPU version (NVIDIA GeForce RTX 2080) of our proposed **BTBS-LNS** on the balanced item placement problem.

Fig. 5 depicts the anytime primal gap comparison between the CPU and GPU versions in detail within the 200s time limit. As can be seen, compared with CPU implementation, GPU version **BTBS-LNS** delivers slightly better performance almost at any time, in which the overall primal gap and primal integral improve by 0.83% and 0.99%, respectively. In other words, our proposed **BTBS-LNS** may achieve even better performance when implemented in a GPU environment.

A.6 Stability Analysis of our approach

To make a fair comparison between different competing approaches, all the experiments in Sec. 4 were conducted with three different seeds. The average standard deviations for our proposed **BTBS-LNS** on different problems are gathered in Table 14. As can be seen, it is fairly robust to different seeds, with average standard deviations lower than 2% even on hard and heterogeneous problems, like MIPLIB2017.

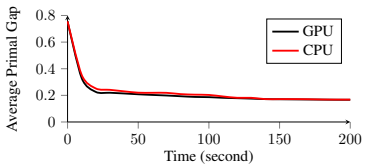

Figure 5: Anytime Performance comparison (GPU **vs** CPU).

A.7 Detailed Anytime performance on Integer Programming Problems

To further evaluate the anytime performance among the competing approaches, we plot the anytime primal gap curves on four binary integer programming problems, Set Covering (SC), Maximal Independent Set (MIS), Combinatorial Auction (CA), and Maximum Cut (MC), respectively. The results are gathered in Figure 6, 7, 8, 9, respectively.

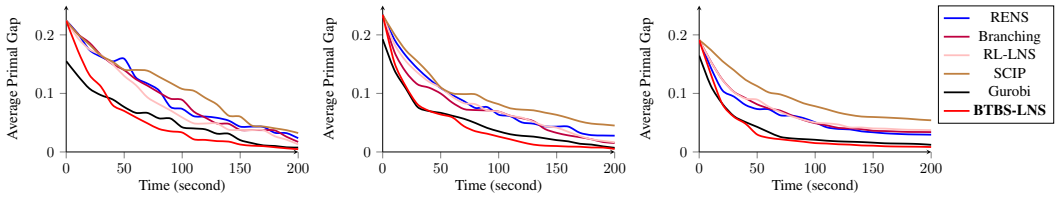

Figure 6: Anytime Performance on Set Covering (SC) problem and its scale-transfer instances. **From left to right**: Performance comparison on instances from SC, SC2, SC4. (see Table 8 for detail).

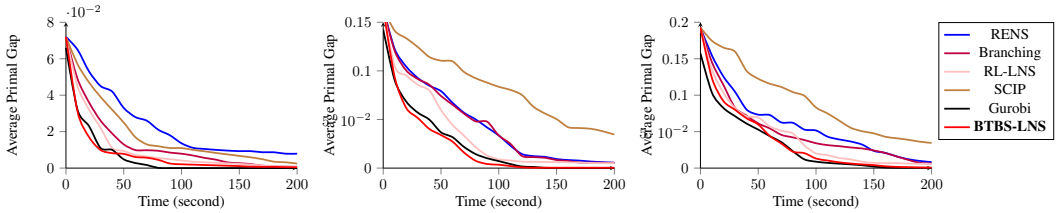

Figure 7: Anytime Performance on Maximal Independent Set (MIS) problem and its scale-transfer instances. **From left to right**: Performance comparison on instances from MIS, MIS2, MIS4. (see Table 8 for detail).

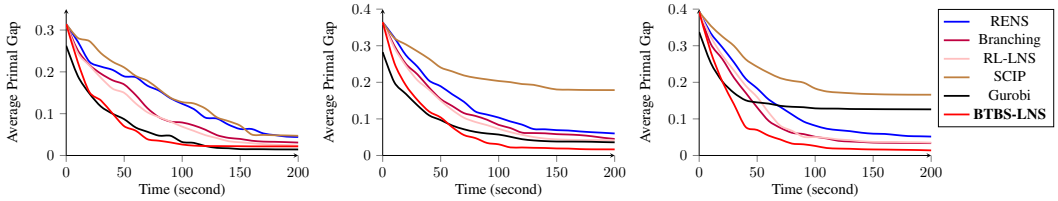

Figure 8: Anytime Performance on Combinatorial Auction (CA) problem and its scale-transfer instances. **From left to right**: Performance comparison on instances from CA, CA2, CA4. (see Table 8 for detail).

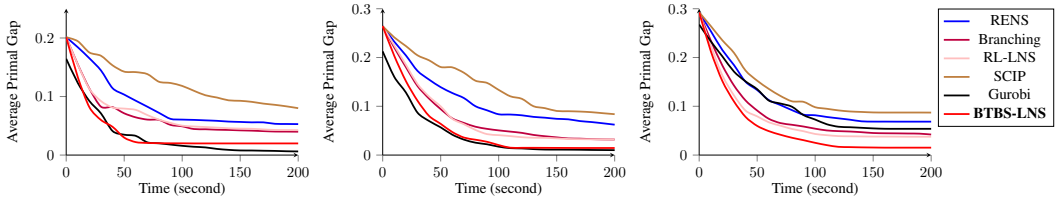

Figure 9: Anytime Performance on Maximum Cut (MC) problem and its scale-transfer instances. **From left to right**: Performance comparison on instances from MC, MC2, MC4. (see Table 8 for detail).

As seen from the results, our **BTBS-LNS** delivers consistently superior performance over the competing LNS baselines almost at any point, demonstrating its efficiency and effectiveness. More surprisingly, the proposed approach can achieve superior performance over the leading commercial solver in some cases, especially on the scale-transfer instances, purely by the learned policy on small-scale instances, with SCIP as the off-the-shelf solver.

A.8 Per-instance performance comparison on MIPLIB2017

Considering that the results on MIPLIB2017 instances may deliver high variances due to the significantly different problem distributions across instances, showing only the average gap may not be sufficient. In this respect, we report the detailed per-instance performance within the given time limit on the competing approaches, and the results are gathered in Table 15.

We report **218**/240 instances from the MIPLIB2017 benchmark set. The following instances were removed, as no feasible solution can be found for them within the pre-defined time limit by the off-the-shelf solver SCIP:

1) Instances that are infeasible (6):

- bnatt500
- cryptanalysiskb128n5obj14
- fhnw-binpack4-4
- neos-2075418-temuka
- neos-3988577-wolgan
- neos859080

2) Instances that cannot generate feasible solution by the baseline solver within timelimit (16):

- cryptanalysiskb128n5obj16
- gfd-schedulen180f7d50m30k18
- highschool1-aigio
- irish-electricity
- neos-1354092
- neos-3402454-bohle
- neos-4532248-waihi
- neos-5104907-jarama
- neos-5114902-kasavu
- ns1116954
- ns1952667
- peg-solitaire-a3
- physiciansched3-3
- rail02
- supportcase19
- supportcase22

A.9 A SMALL NUMERICAL EXAMPLE FOR ALG. 1

Consider a general integer variable x_0 with a $[0, 15]$ range and a current solution of 3. This variable is encoded using 4 substitution binary variables: $x_{0,1}$, $x_{0,2}$, $x_{0,3}$, and $x_{0,4}$. At each LNS iteration t, distinct actions $a^t_{0,1}$, $a^t_{0,2}$, $a^t_{0,3}$, and $a^t_{0,4}$ are taken for each substitution variable, controlling the range of the original variable at different levels of significance. Specifically, the decision actions $a^t_{0,j}$ for all j are evaluated sequentially, and the upper and lower bounds are tightened around the current solution whenever $a^t_{0,j} = 0$, as described in Line 11-12 of Alg. 1. Here are some examples:

1. If $a^t_{0,1} = 0$ and the others are 1, the updated bounds for x_0 will be $[0, 10]$.
2. If $a^t_{0,1} = a^t_{0,2} = 0$ and the others are 1, the updated bounds for x_0 will be $[0, 8]$.
3. ...
4. If all decision actions are 0, the updated bounds for x_0 will be $[0, 6]$, with the current solution precisely at the midpoint of the variable bounds. Similarly, when the current solution is 12, the updated bounds will be $[9, 15]$.

In each LNS iteration, our bound-tightening method refines the variable bounds around the current solution, guided by the LNS decisions, to balance computational complexity and exploration efficiency. Importantly, this bound-tightening process is conducted independently at each step, always initiating from the original variable bounds.

Table 15: Per-instance performance comparison on MIPLIB2017

Instance	SCIP	SCIP(600s)	SCIP(900s)	U-LNS	R-LNS	FT-LNS	BTBS-LNS	BTBS-LNS-F	Gurobi	Optimal Solution
30n20b8	**302**	**302**	**302**	353	**302**	**302**	**302**	**302**	**302**	302
50v-10	3340.37	3316.92	3313.18	3324.38	3340.37	3334.01	**3311.18**	3315.24	**3311.18**	3311.18
academictimetablesmall	228	228	228	228	228	228	**6**	45	**6**	0
air05	**26374**	**26374**	**26374**	26439	**26374**	26441	**26374**	**26374**	**26374**	26374
app1-1	**-3**	**-3**	**-3**	**-3**	-2	**-3**	**-3**	**-3**	**-3**	-3
app1-2	-23	**-41**	**-41**	-24	-23	-29	**-41**	**-41**	**-41**	-41
assign1-5-8	**212**	**212**	**212**	214	**212**	**212**	**212**	**212**	**212**	212
atlanta-ip	98.01	93.01	93.01	93.01	95.01	93.01	**90.01**	**90.01**	**90.01**	90.01
b1c1s1	27031.16	27004.55	25571.02	27540.75	27031.16	26379.4	**24544.25**	**24544.25**	**24544.25**	24544.25
bab2	-354064.7	-354064.7	-354064.7	-354064.7	-354064.7	-354091	-354092.9	-354092.9	**-357525.96**	-357544.312
bab6	-279121.2	-279121.2	-279121.2	-280546.4	-280546.4	-280546.4	-280546.4	-280546.4	**-284248.23**	-284248.23
beasleyC3	**754**	**754**	**754**	759	759	755	**754**	**754**	**754**	754
binkar10_1	**6742.2**	**6742.2**	**6742.2**	6746.76	6747.78	6743.24	**6742.2**	**6742.2**	**6742.2**	6742.2
blp-ar98	6565.99	6303.11	6243.77	6565.99	6584.43	6565.99	**6205.21**	**6205.21**	**6205.21**	6205.21
blp-ic98	4744.08	4719.1	4641.77	4719.1	4963.66	4963.66	**4491.45**	**4491.45**	**4491.45**	4491.45
bnatt400	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	1
bppc4-08	**53**	**53**	**53**	56	56	54	**53**	**53**	**53**	53
brazil3	102	102	102	102	102	102	**24**	41	**24**	24
buildingenergy	42652.34	34250.38	34250.38	42652.34	42652.34	42652.34	33324.73	34250.38	**33283.85**	33283.85
cbs-cta	**0**	**0**	**0**	43.16	43.16	**0**	**0**	**0**	**0**	0
chromaticindex1024-7	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	4
chromaticindex512-7	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	4
cmflsp50-24-8-8	57921400	57921400	57921400	57921400	57921400	57921400	**55789390**	**55789390**	**55789390**	55789390
CMS750_4	261	254	**252**	261	269	253	**252**	**252**	**252**	252
co-100	11833720	11833720	11833720	11833720	11833720	11833720	**2639942.06**	**2639942.06**	**2639942.06**	2639942.06
cod105	**-12**	**-12**	**-12**	-11	-11	-8	**-12**	**-12**	**-12**	-12
comp07-2idx	823	148	148	148	148	78	**6**	23	**6**	6
comp21-2idx	250	179	142	250	250	225	**75**	88	88	74
cost266-UUE	25222800	**25148941**	**25148941**	25222800	25222800	25164070	**25148941**	**25148941**	**25148941**	25148941
csched007	362	**351**	**351**	362	356	354	**351**	**351**	**351**	351
csched008	**173**	**173**	**173**	176	178	174	**173**	**173**	**173**	173
cvs16r128-89	-93	-95	-95	-86	-80	-84	**-97**	**-97**	-96	-97
dano3_3	**576.345**	**576.345**	**576.345**	577.475	576.52	576.52	**576.345**	**576.345**	**576.345**	576.345
dano3_5	**576.925**	**576.925**	**576.925**	581.725	581.725	577.316	**576.925**	**576.925**	**576.925**	576.925
decomp2	**-160**	**-160**	**-160**	-152	-152	-133	**-160**	**-160**	**-160**	-160
drayage-100-23	**103334**	**103334**	**103334**	**103334**	**103334**	**103334**	**103334**	**103334**	**103334**	103333.874
drayage-25-23	**101283**	**101283**	**101283**	106897	101344	101344	**101283**	**101283**	**101283**	101282.647
dws008-01	56691.23	46179.85	38873.46	56691.23	56691.23	56691.23	**37412.6**	**37412.6**	**37412.6**	37412.6

Instance	SCIP	SCIP(600s)	SCIP(900s)	U-LNS	R-LNS	FT-LNS	BTBS-LNS	BTBS-LNS-F	Gurobi	Optimal Solution
eil33-2	**934.008**	**934.008**	**934.008**	987.674	987.674	**934.008**	**934.008**	**934.008**	**934.008**	934.008
eilA101-2	1313.47	995.77	995.77	1443.53	1443.53	1443.53	**880.92**	**880.92**	923.01	880.92
enlight_hard	**37**	**37**	**37**	**37**	**37**	**37**	**37**	**37**	**37**	37
ex10	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	100
ex9	**81**	**81**	**81**	**81**	**81**	**81**	**81**	**81**	**81**	81
exp-1-500-5-5	**65887**	**65887**	**65887**	**65887**	**65887**	**65887**	**65887**	**65887**	**65887**	65887
fast0507	**174**	**174**	**174**	176	175	**174**	**174**	**174**	**174**	174
fastxgemm-n2r6s0t2	**230**	**230**	**230**	**230**	236	**230**	**230**	**230**	**230**	230
fhnw-binpack4-48	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0
fiball	140	**138**	**138**	**138**	140	**138**	**138**	**138**	**138**	138
gen-ip002	**-4783.73**	**-4783.73**	**-4783.73**	-4772.6	-4772.6	-4768.25	**-4783.73**	-4772.33	**-4783.73**	-4783.73
gen-ip054	6858.88	**6840.97**	**6840.97**	6858.88	6852.73	6858.88	6852.73	6858.58	**6840.97**	6840.97
germanrr	48440630	48096190	48096190	48440630	48440630	48440630	48440630	48440630	**47135500**	47095869.6
glass-sc	**23**	**23**	**23**	25	25	24	**23**	**23**	**23**	23
glass4	**1200012600**	**1200012600**	**1200012600**	**1200012600**	**1200012600**	**1200012600**	**1200012600**	**1200012600**	**1200012600**	1200012600
gmu-35-40	-2406458	-2406458	-2406458	-2406458	-2406458	-2406458	**-2406733**	-2406458	**-2406733**	-2406733.37
gmu-35-50	-2606871	-2606930	-2606930	-2605465	-2605387	-2606930	**-2607958.3**	**-2607958.3**	-2607922.7	-2607958.33
graph20-20-1rand	**-9**	**-9**	**-9**	-8	-8	**-9**	**-9**	**-9**	**-9**	-9
graphdraw-domain	**19686**	**19686**	**19686**	19848	19848	19772	**19686**	19688	**19686**	19686
h80x6320d	**6382.1**	**6382.1**	**6382.1**	6416.61	**6382.1**	**6382.1**	**6382.1**	**6382.1**	**6382.1**	6382.1
hypothyroid-k1	**-2851**	**-2851**	**-2851**	**-2851**	**-2851**	**-2851**	**-2851**	**-2851**	**-2851**	-2851
ic97_potential	3945	3945	3945	3952	3945	3952	**3942**	3952	**3942**	3942
icir97_tension	6392	6382	**6375**	**6375**	6382	6376	**6375**	**6375**	**6375**	6375
irp	**12159.49**	**12159.49**	**12159.49**	12160.2	12161.5	12161.5	**12159.49**	**12159.49**	**12159.49**	12159.49
istanbul-no-cutoff	**204.08**	**204.08**	**204.08**	214.797	212.961	214.797	**204.08**	**204.08**	**204.08**	204.08
k1mushroom	-204	-293	**-3288**	-204	-204	-204	-3144	-3144	**-3288**	-3288
lectsched-5-obj	48	41	39	46	48	44	**24**	27	**24**	24
leo1	419655200	412600400	410709200	488216000	443395100	429182100	404989400	404989400	**404227536**	404227536
leo2	436700200	426090600	424958900	438115100	436700200	436444000	405531200	405531200	**404077441**	404077441
lotsize	1557868	1484323	1483960	1626587	1557868	1495682	**1480195**	**1480195**	1494101	1480195
mad	0.067	0.038	0.0352	0.067	0.0772	0.0392	**0.0268**	**0.0268**	0.028	0.0268
map10	-480	**-495**	**-495**	-468	-410	-472	**-495**	**-495**	**-495**	-495
map16715-04	-78	-109	**-111**	-82	-78	-83	**-111**	**-111**	**-111**	-111
markshare_4_0	**1**	**1**	**1**	3	**1**	**1**	**1**	**1**	**1**	1
markshare2	31	28	28	31	36	31	**11**	**11**	**11**	1
mas74	**11801.19**	**11801.19**	**11801.19**	**11801.19**	**11801.19**	**11801.19**	**11801.19**	**11801.19**	**11801.19**	11801.1857
mas76	**40005.05**	**40005.05**	**40005.05**	**40005.05**	**40005.05**	**40005.05**	**40005.05**	**40005.05**	**40005.05**	40005.05
mc11	**11689**	**11689**	**11689**	11720	11731	11896	**11689**	**11689**	**11689**	11689
mcsched	**211913**	**211913**	**211913**	212874	212874	212911	**211913**	**211913**	**211913**	211913
mik-250-20-75-4	**-52301**	**-52301**	**-52301**	**-52301**	**-52301**	**-52301**	**-52301**	**-52301**	**-52301**	-52301
milo-v12-6-r2-40-1	**326481.1**	**326481.1**	**326481.1**	326820.6	326820.6	**326481.1**	**326481.1**	**326481.1**	**326481.1**	326481.1
momentum1	372399.4	282447.1	134897	365944	365944	372399.4	**109143.5**	**109143.5**	**109143.5**	109143.5
mushroom-best	**0.0553**	**0.0553**	**0.0553**	0.0869	0.0869	**0.0553**	**0.0553**	**0.0553**	**0.0553**	0.0553
mzzv11	**-21718**	**-21718**	**-21718**	-21678	-21668	-21678	**-21718**	**-21718**	**-21718**	-21718
mzzv42z	**-20540**	**-20540**	**-20540**	**-20540**	**-20540**	-20400	**-20540**	**-20540**	**-20540**	-20540

Instance	SCIP	SCIP(600s)	SCIP(900s)	U-LNS	R-LNS	FT-LNS	BTBS-LNS	BTBS-LNS-F	Gurobi	Optimal Solution
n2seq36q	52600	**52200**	**52200**	52800	52600	52400	**52200**	**52200**	**52200**	52200
n3div36	**130800**	**130800**	**130800**	**130800**	131400	**130800**	**130800**	**130800**	**130800**	130800
n5-3	**8105**	**8105**	**8105**	8405	**8105**	**8105**	**8105**	**8105**	**8105**	8105
neos-1122047	**161**	**161**	**161**	**161**	**161**	**161**	**161**	**161**	**161**	161
neos-1171448	**-309**	**-309**	**-309**	-307	-305	**-309**	**-309**	**-309**	**-309**	-309
neos-1171737	-190	-192	-192	-173	-190	-190	**-195**	**-195**	**-195**	-195
neos-1445765	**-17783**	**-17783**	**-17783**	**-17783**	**-17783**	**-17783**	**-17783**	**-17783**	**-17783**	-17783
neos-1456979	186	184	184	207	184	186	**176**	178	**176**	176
neos-1582420	**91**	**91**	**91**	**91**	**91**	**91**	**91**	**91**	**91**	91
neos-2657525-crna	7.23	7.23	7.23	7.23	8.06	7.23	**1.81075**	7.23	**1.81075**	1.81075
neos-2746589-doon	2099.6	2099.6	2099.6	2099.6	2099.6	2099.6	**2008.2**	2099.6	**2008.2**	2008.2
neos-2978193-inde	**-2.388**	**-2.388**	**-2.388**	-2.197	**-2.388**	**-2.388**	**-2.388**	**-2.388**	**-2.388**	-2.38806169
neos-2987310-joes	**-607702988**	**-607702988**	**-607702988**	**-607702988**	**-607702988**	**-607702988**	**-607702988**	**-607702988**	**-607702988**	-607702988
neos-3004026-krka	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0
neos-3024952-loue	126520	97446	71336	81469	97446	126520	**26756**	27349	**26756**	26756
neos-3046615-murg	1610	1610	1607	1670	1651	1651	1607	1611	**1600**	1600
neos-3083819-nubu	**6307996**	**6307996**	**6307996**	**6307996**	**6307996**	**6307996**	**6307996**	**6307996**	**6307996**	6307996
neos-3216931-puriri	151160	151160	151160	141275	151160	141275	141275	141275	**71320**	71320
neos-3381206-awhea	**453**	**453**	**453**	**453**	454	**453**	**453**	454	**453**	453
neos-3402294-bobin	**0.06725**	**0.06725**	**0.06725**	0.08775	**0.06725**	0.08175	**0.06725**	**0.06725**	**0.06725**	0.06725
neos-3555904-turama	**-34.7**	**-34.7**	**-34.7**	**-34.7**	**-34.7**	**-34.7**	**-34.7**	**-34.7**	**-34.7**	-34.7
neos-3627168-kasai	989301.6	989301.6	989301.6	990006.8	989301.6	989301.6	**988585.62**	**988585.62**	**988585.62**	988585.62
neos-3656078-kumeu	-11067.1	-11067.1	-11067.1	-11067.1	-11067.1	-11067.1	-13127	-13120	**-13171**	-13171
neos-3754480-nidda	13832.17	13639.97	13639.97	13832.17	13832.17	13832.17	**12940.5**	**12940.5**	12941.69	12940.5
neos-4300652-rahue	7.4454	2.8193	2.7595	6.1813	7.4454	6.1813	**2.1416**	**2.1416**	**2.1416**	2.1416
neos-4338804-snowy	1477	1474	1473	1482	1479	1479	**1471**	1473	**1471**	1471
neos-4387871-tavua	35.14	35.14	35.14	35.14	35.14	35.14	**33.38**	**33.38**	**33.38**	33.38
neos-4413714-turia	**45.37**	**45.37**	**45.37**	51.94	51.94	**45.37**	**45.37**	**45.37**	**45.37**	45.37
neos-4647030-tutaki	27268.48	27268.48	27268.48	27268.48	27268.48	27268.48	**27265.71**	**27265.71**	**27265.71**	27265.71
neos-4722843-widden	25438.44	25210.88	25210.88	27707.88	26277.44	26277.44	**25009.7**	25309.66	**25009.7**	25009.7
neos-4738912-atrato	285010500	283680800	283680100	285662900	285662900	285010500	**283627957**	**283627957**	**283627957**	283627957
neos-4763324-toguru	6760.735	6760.735	6760.735	6760.735	6760.735	6760.735	**1613.039**	**1613.039**	**1613.039**	1613.039
neos-4954672-berkel	2678506	2627560	2624735	2678506	2678506	2678506	**2612710**	**2612710**	2614881	2612710
neos-5049753-cuanza	636	636	636	636	636	600	**562**	600	**562**	562
neos-5052403-cygnet	293	293	184	293	293	293	**182**	**182**	**182**	182
neos-5093327-huahum	6686	6686	6686	6686	6960	6686	**6260**	**6260**	6270	6260
neos-5107597-kakapo	4248	3744	3654	4194	4293	4158	**3645**	**3645**	**3645**	3645
neos-5188808-nattai	0.11257	0.11257	0.11207	0.11257	0.11257	0.11257	**0.11029**	**0.11029**	**0.11029**	0.11029
neos-5195221-niemur	0.00406	**0.00384**	**0.00384**	0.00418	0.00418	0.00406	**0.00384**	**0.00384**	**0.00384**	0.00384
neos-631710	214	214	214	214	214	214	**203**	**203**	**203**	203
neos-662469	245034.5	184745.5	184679.5	224993.5	225044	245034.5	**184380**	184390	**184380**	184380
neos-787933	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	30
neos-827175	**112.002**	**112.002**	**112.002**	**112.002**	**112.002**	**112.002**	**112.002**	**112.002**	**112.002**	112.002
neos-848589	12359660	2359.54	2359.54	12359660	12359660	12359660	**2358.43**	**2358.43**	3206.12	2358.43
neos-860300	**3201**	**3201**	**3201**	**3201**	3267	**3201**	**3201**	**3201**	**3201**	3201

Instance	SCIP	SCIP(600s)	SCIP(900s)	U-LNS	R-LNS	FT-LNS	BTBS-LNS	BTBS-LNS-F	Gurobi	Optimal Solution
neos-873061	122.92	122.72	122.72	125.93	122.92	123.66	**113.656**	**113.656**	**113.656**	113.656
neos-911970	**54.76**	**54.76**	**54.76**	54.83	54.83	**54.76**	**54.76**	**54.76**	**54.76**	54.76
neos-933966	2388	320	320	2389	2388	2388	**318**	**318**	**318**	318
neos-950242	**4**	**4**	**4**	**4**	5	**4**	**4**	**4**	**4**	4
neos-957323	**-237.76**	**-237.76**	**-237.76**	-234.76	-235.76	-235.76	**-237.76**	**-237.76**	**-237.76**	-237.76
neos-960392	0	**-238**	**-238**	0	0	-234	**-238**	**-238**	**-238**	-238
neos17	**0.15**	**0.15**	**0.15**	0.171	0.167	0.151	**0.15**	**0.15**	**0.15**	0.15
neos5	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	15
neos8	**-3719**	**-3719**	**-3719**	**-3719**	**-3719**	**-3719**	**-3719**	**-3719**	**-3719**	-3719
net12	**214**	**214**	**214**	**214**	255	**214**	**214**	**214**	**214**	214
netdiversion	4900438	4900438	263	4900438	4900438	263	**242**	244	**242**	242
nexp-150-20-8-5	300	234	**231**	771	771	237	**231**	**231**	239	231
ns1208400	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	2
ns1644855	-1419.67	**-1524.33**	**-1524.33**	-1486.67	-1486.67	-1419.67	**-1524.33**	**-1524.33**	**-1524.33**	-1524.33
ns1760995	-429.36	-429.36	-429.36	-429.36	-429.36	-429.36	**-548.02**	**-548.02**	-516.07	-549.214385
ns1830653	**20622**	**20622**	**20622**	23622	23622	21622	**20622**	**20622**	**20622**	20622
nu25-pr12	**53905**	**53905**	**53905**	**53905**	**53905**	**53905**	**53905**	**53905**	**53905**	53905
nursesched-medium-hint03	8080	8080	7906	8080	8080	8080	**117**	997	152	115
nursesched-sprint02	**58**	**58**	**58**	**58**	67	**58**	**58**	**58**	**58**	58
nw04	**16862**	**16862**	**16862**	16876	16876	16876	**16862**	**16862**	**16862**	16862
opm2-z10-s4	-29112	-33062	-33062	-26538	-26538	-26538	**-33269**	**-33269**	-33139	-33269
p200x1188c	**15078**	**15078**	**15078**	**15078**	**15078**	**15078**	**15078**	**15078**	**15078**	15078
pg	**-8674.34**	**-8674.34**	**-8674.34**	-8662.84	-8662.84	**-8674.34**	**-8674.34**	**-8674.34**	**-8674.34**	-8674.34
pg5_34	-14324.46	-14324.81	-14325.83	-14310.96	-14324.46	-14324.81	**-14339.4**	**-14339.4**	**-14339.4**	-14339.4
physiciansched6-2	**49324**	**49324**	**49324**	**49324**	**49324**	**49324**	**49324**	**49324**	**49324**	49324
piperout-08	**125055**	**125055**	**125055**	133707	**125055**	**125055**	**125055**	**125055**	**125055**	125055
piperout-27	**8124**	**8124**	**8124**	**8124**	**8124**	**8124**	**8124**	**8124**	**8124**	8124
pk1	**11**	**11**	**11**	12	**11**	**11**	**11**	**11**	**11**	11
proteindesign121hz512p9	2609	2609	2609	2609	2609	2609	2609	2609	**1477**	1473
proteindesign122trx11p8	2916	2916	2916	2916	2916	2916	**1748**	1762	**1748**	1747
qap10	**340**	**340**	**340**	**340**	**340**	**340**	**340**	**340**	**340**	340
radiationm18-12-05	19527	18874	18874	19853	19527	19202	**17566**	17569	17567	17566
radiationm40-10-02	235396	155354	155354	235396	235396	209796	**155330**	156939	155331	155328
rail01	-69.09	-69.09	-69.09	-69.89	-69.89	-69.09	-69.89	-69.89	**-70.57**	-70.57
rail507	**174**	**174**	**174**	178	175	**174**	**174**	**174**	**174**	174
ran14x18-disj-8	3715	3714	**3712**	3798	3798	3715	**3712**	**3712**	3736	3712
rd-rplusc-21	179751.8	179751.8	179751.8	179836.5	179751.8	179751.8	**165395.3**	**165395.3**	**165395.3**	165395.3
reblock115	-36721080	-36799530	-36800600	-36777270	-36799530	-36799530	**-36800603**	**-36800603**	**-36800603**	-36800603
rmatr100-p10	**423**	**423**	**423**	442	424	457	**423**	**423**	**423**	423
rmatr200-p5	5489	5489	**4521**	5489	5489	5489	**4521**	**4521**	**4521**	4521
rocI-4-11	**-6020203**	**-6020203**	**-6020203**	-5040303	-5040303	**-6020203**	**-6020203**	**-6020203**	**-6020203**	-6020203
rocII-5-11	-4.65	-5.66	-5.67	-4.65	-5.66	-4.65	**-6.68**	**-6.68**	-5.68	-6.68
rococoB10-011000	19988	19879	19534	19701	19879	19988	**19449**	**19449**	19497	19449
rococoC10-001000	11530	**11460**	**11460**	11576	11472	**11460**	**11460**	**11460**	**11460**	11460
roi2alpha3n4	-61.37	-63.17	-63.17	-62.41	-62.41	-63.17	**-63.21**	**-63.21**	**-63.21**	-63.21

Instance	SCIP	SCIP(600s)	SCIP(900s)	U-LNS	R-LNS	FT-LNS	BTBS-LNS	BTBS-LNS-F	Gurobi	Optimal Solution
roi5alpha10n8	-44.89	-45.15	-45.15	-44.36	-44.89	-44.89	**-52.28**	**-52.28**	-50.59	-52.3222744
roll3000	**12890**	**12890**	**12890**	12902	**12890**	**12890**	**12890**	**12890**	**12890**	12890
s100	0	0	0	0	0	0	**-0.16966**	**-0.16966**	-0.03945	-0.169723527
s250r10	-0.1437	-0.1698	-0.1708	-0.1437	-0.1437	-0.1437	**-0.17178**	**-0.17178**	**-0.17178**	-0.17178
satellites2-40	49	49	49	49	49	49	**-19**	**-19**	**-19**	-19
satellites2-60-fs	28	28	27	27	27	27	**-19**	**-19**	**-19**	-19
savsched1	31846.3	31846.3	31846.3	45875.9	45875.9	31846.3	3265	3265	**3218**	3218
sct2	-230.91	**-230.99**	**-230.99**	-230.78	-230.85	-230.91	**-230.99**	**-230.99**	**-230.99**	-230.99
seymour	427	425	**423**	427	428	427	**423**	**423**	**423**	423
seymour1	**410.76**	**410.76**	**410.76**	**410.76**	**410.76**	**410.76**	**410.76**	**410.76**	**410.76**	410.76
sing326	7833336	7765711	7765711	7833336	7833336	7833336	**7753675**	**7753675**	7753676	7753675
sing44	8175655	8174767	8174767	8177833	8163698	8175655	**8128831**	**8128831**	8130643	8128831
snp-02-004-104	586912700	586816300	586804500	586829700	587089300	586821500	**586803239**	**586803239**	**586803239**	586803239
sorrell3	-11	-15	-15	-11	-15	-15	**-16**	**-16**	**-16**	-16
sp150x300d	**69**	**69**	**69**	**69**	**69**	70	**69**	**69**	**69**	69
sp97ar	688832800	682989900	681332100	681332100	673491900	679524100	**660705646**	660834000	**660705646**	660705646
sp98ar	537245600	533010800	532905600	533010800	533455300	532891300	**529740623**	529905800	**529740623**	529740623
splice1k1	-73	-121	-394	-121	-121	-121	**-394**	**-394**	-338	-394
square41	26	26	26	26	21	21	**15**	17	16	15
square47	29	29	29	21	21	21	**18**	20	20	16
supportcase10	19	19	19	9	19	19	**8**	**8**	**8**	7
supportcase12	-7430.15	-7437.1	-7475.67	-7351.97	-7436.17	-7449.13	-7543.26	-7543.26	**-7559.2419**	-7559.2419
supportcase18	49	49	49	50	51	49	**48**	**48**	49	48
supportcase26	1781.003	1747.033	1747.033	1755.845	1768.264	1768.264	1755.525	1755.525	**1745.124**	1745.124
supportcase33	**-345**	**-345**	**-345**	-340	**-345**	**-345**	**-345**	**-345**	**-345**	-345
supportcase40	24478.86	24465.78	24465.78	24465.78	24478.86	24294.09	**24256.31**	**24256.31**	**24256.31**	24256.31
supportcase42	8.0904	8.0019	7.7683	7.7678	7.7685	7.7811	**7.7586**	7.7713	**7.7586**	7.7586
supportcase6	51921.76	51921.76	51921.76	51921.76	51921.76	**51906.48**	**51906.48**	**51906.48**	**51906.48**	51906.48
supportcase7	**-1132.223**	**-1132.223**	**-1132.223**	-1129.28	**-1132.223**	**-1132.223**	**-1132.223**	**-1132.223**	**-1132.223**	-1132.223
swath1	**379.07**	**379.07**	**379.07**	**379.07**	381.51	**379.07**	**379.07**	**379.07**	**379.07**	379.07
swath3	**397.76**	**397.76**	**397.76**	399.33	**397.76**	**397.76**	**397.76**	**397.76**	**397.76**	397.76
tbfp-network	131.88	**24.16**	**24.16**	25.12	24.91	**24.16**	**24.16**	**24.16**	**24.16**	24.16
thor50dday	59310	59310	40432	40432	40432	40432	**40417**	**40417**	**40417**	40417
timtab1	**764772**	**764772**	**764772**	766166	766345	**764772**	**764772**	**764772**	**764772**	764772
tr12-30	**130596**	**130596**	**130596**	130608	**130596**	130608	**130596**	**130596**	**130596**	130596
traininstance2	79180	77420	77420	79180	84090	79180	**71820**	72950	**71820**	71820
traininstance6	29420	28460	28460	**28290**	28460	29250	**28290**	29250	**28290**	28290
trento1	25255630	18223810	18223810	18223810	7282245	15981790	**5189487**	5191562	**5189487**	5189487
triptim1	25.5	25.5	25.5	**22.87**	25.5	**22.87**	**22.87**	**22.87**	**22.87**	22.87
uccase12	**11507.41**	**11507.41**	**11507.41**	11507.42	11507.48	**11507.41**	**11507.41**	**11507.41**	**11507.41**	11507.41
uccase9	463233.3	48328.09	15347.75	48328.09	48328.09	20176.81	11052.31	11052.31	**10994.13**	10993.1314
uct-subprob	315	**314**	**314**	317	315	**314**	**314**	**314**	**314**	314
unitcal_7	19635620	**19635558**	**19635558**	**19635558**	**19635558**	**19635558**	**19635558**	**19635558**	**19635558**	19635558
var-smallemery-m6j6	**-149.375**	**-149.375**	**-149.375**	-147.031	-146.312	**-149.375**	**-149.375**	**-149.375**	**-149.375**	-149.375
wachplan	**-8**	**-8**	**-8**	**-8**	**-8**	**-8**	**-8**	**-8**	**-8**	-8