Learning to Image in Slepian-Pollack Basis – Achieving Super-resolution of Nanostructures

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1. Introduction

The diffraction limit of optical instruments is determined by loss of information in the transfer of the optical Fourier components from near- to far-field, operated by the imaging system. However, when some prior knowledge about the object and coherent measurements are available, sub-wavelength information can be robustly recovered from far-field data.
By expressing far-field measurements of, size-limited, sub-diffraction structures in Slepian–Pollak (SP) basis—a set of eigenmodes that optimally represents fields within finite field of view—we show that a learning model trained on physics-consistent examples can exploit correlations between accurately measured low-order SP modes and noise-sensitive high-order modes. This enables robust recovery of sub-diffraction geometric features in our nanostructures.

Using this approach, we achieve imaging resolution of λ/13. Importantly, this performance is achieved without increasing the collection numerical aperture, employing structured-light illumination, or relying on nonlinear fluorescent contrast mechanisms.

2. Methods

In a proof of principle experiment, nanosamples on a sapphire cube (single-crystal, 7.5 mm edge; two c-normal faces) are illuminated using total internal reflection (TIR) at λ = 638 nm from a 20mW frequency-stabilized diode laser with linewidth < 0.1 pm. The s-polarized laser beam enters the cube at ~45°, yielding ~66° incidence at the sample (n_0 = 1.765).
The beam is weakly focused to place its waist near the object plane. Scattered light is collected using a 0.9-NA objective on an inverted microscope. A digital mirror device (DMD), positioned near the conjugate back focal plane via a 2f relay, implements complex-amplitude masks using 4×4-pixel super-pixels. A piezo stage provides pre-positioning and drift correction

For each TIR direction, eight Fourier-plane images with different DMD masks are recorded using a cooled sCMOS camera. These measurements retrieve the magnitude and phase of the 17 strongest SP coefficients; higher-order coefficients are limited by residual parasitic scattering. The measurements are repeated for four orthogonal TIR illumination directions with phase compensation [1].

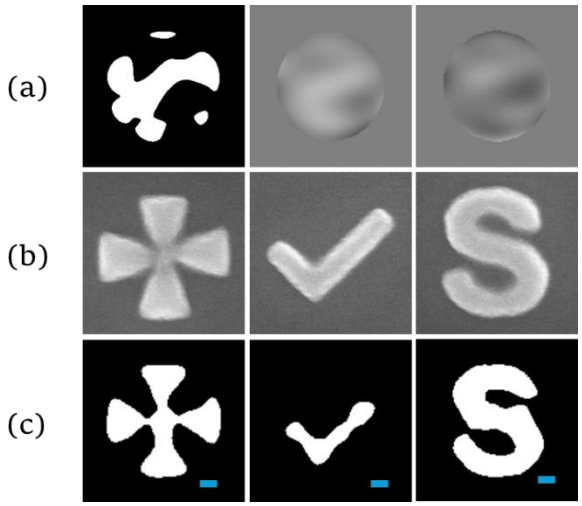

Fig. 1: (a) left - Example of Fourier-cutoff random shape training sample; middle and right – real and imaginary coherent summed images of SP coefficients at one of the four illumination directions. (b) Scanning electron microscope image of nanosamples being imaged. (c) Images reconstructions from learning on the SP coefficients images. Blue bar denotes the length scale of λ/10.

A dataset of image–SP-coefficients pairs is generated using 1,000 random nanoshapes created using a Fourier-cutoff random-shape model, with randomly sampled spatial frequency cutoff f_c. This produces objects with feature sizes ranging from near-diffraction to deep sub-diffraction scales. After inverse transforming, objects are thresholded and apodized with an error-function window to ensure compactness within the field of view. This synthesis strategy avoids overfitting to specific shape families and produces a balanced distribution across

low- and high-order SP modes. For each object, complex SP-basis fields from the four TIR directions are coherently summed. The network input consists of the real and imaginary parts of these images (eight channels total) (Fig. 1a).

Image reconstruction is performed using a 2D residual U-Net with seven encoder–decoder levels, average pooling in the encoder, and deconvolution-based upsampling in the decoder. The network outputs a single-channel nanosample image, for every eight channels inputs. Training uses the AdamW optimizer and mean-square error loss; the model with the best validation loss is retained for testing.

3. Results and Discussion

To quantify resolution, we compute an effective numerical aperture (NA_{eff}). Starting from the ground-truth nanosample image, we take its centered Fourier transform, apply a circular k-space mask $|k| \leq NA \cdot k_0$, and let NA vary. The inverse-transformed image represents a diffraction-limited surrogate for that NA.
This surrogate is binarized using a scalar threshold, normalized, and compared with the reconstructed image using a normalized correlation metric. NA_{eff} is defined as the NA giving the highest correlation. Resolution is reported using the Abbe criteria, as $\lambda/(2\ NA_{eff})$.

On an unseen test set of 100 nanosamples, we obtain a median resolution of **λ/13**, with an interquartile range corresponding to **λ/11 to λ/14** (Fig. 2). Representative examples show clear recovery of junctions, corners, and curved features consistent with the extracted NA_{eff} (Fig. 1b-c).

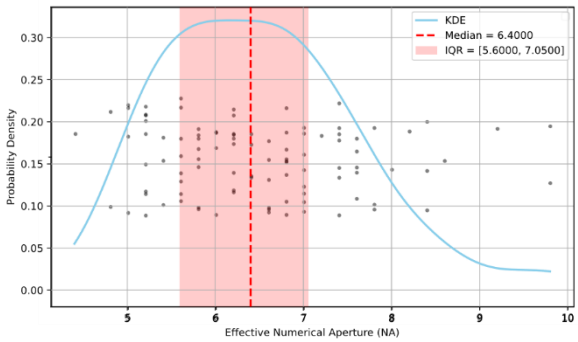

Fig. 2: Effective numerical aperture on unseen test nanosamples.

4. Conclusion

We demonstrated far-field reconstruction of sub-diffraction nanostructures by measuring SP coefficients under TIR illumination and using a deep-learning model to reconstruct images from these coefficients. Using a correlation-matched NA_{eff} metric, we achieve a median resolution of λ/13 within a field of view < 0.8λ, without increasing the collection NA, using nonlinear contrast, or employing structured illumination. The role of photon-limited noise in the measured SP coefficients—and its effect on achievable resolution—will be discussed further.

Acknowledgments

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References

[1] T. Chang, G. Adamo, J. So, E. Chan, N. Papasimakis, Y. Shen, N. Zheludev, Experimental realization of the Slepian-Pollack tomographic imaging: Super-resolution for limited-size objects. In proceedings on 18th International Congress on Artificial Materials for Novel Wave Phenomena – Metamaterials, Crete, Greece (2024)