PRISM: Refracting Large-Batch Model Editing through Jacobian-Inferred FFN Subspaces

Abstract

Model editing updates factual knowledge in large language models (LLMs) without retraining, but simultaneous large-batch edits can induce non-local side effects. We study this problem through a multi-scale analysis of feed-forward (FFN) and multi-head self-attention (MHSA) geometry. Weak FFN–MHSA token-level alignment can coexist with non-trivial distributional coupling, while Jacobian-inferred FFN local-response geometry provides a distinct proxy for locally supported response directions. Based on this distinction, we propose **PRISM**, which replaces unconstrained residual-target fitting with controlled residual correction within edit-conditioned, Jacobian-inferred local-response subspaces. On the standard CounterFact large-batch benchmark, PRISM improves the baseline's Score by 1.3–1.7 points and Generalization by 1.7–1.8 points at 10k edits, with larger Generalization gains in beyond-10k scaling tests at 20k edits while maintaining competitive Specificity.

1 Introduction

Large Language Models (LLMs) store substantial factual knowledge (Brown et al., 2020; Wei et al., 2022; Petroni et al., 2019), yet that knowledge is **static**: deployed models can become outdated or retain incorrect facts (Huang et al., 2025; Ji et al., 2023). Retraining or fine-tuning is expensive and can interfere with previously learned behavior (McCloskey and Cohen, 1989; Kirkpatrick et al., 2017).

Model editing objectives at scale. Model editing seeks targeted factual updates without retraining and is commonly evaluated by **Efficacy**, **Generalization**, and **Specificity** (Wang et al., 2024). We focus on simultaneous large-batch editing, where thousands of edits are written in one run rather than introduced sequentially over time. In this regime, the central difficulty is more specific than a generic three-way trade-off: large-batch editing is not simply single-edit difficulty repeated many times, because interactions among edits accumulate and increasingly govern how transfer and locality degrade. For strong editors, efficacy on the edited prompts is often already high once the target fact is written; the harder problem is to preserve generalization under these accumulating interactions without collapsing specificity. This scale-conditioned tension drives our analysis and method design, although later results also show simultaneous gains on all three core metrics in several canonical 10k settings.

An open question in FFN-side editing. ROME rewrites FFN associations by rank-one updates, MEMIT extends this idea to large-batch editing, and PMET separates FFN- and MHSA-side target optimization while still writing back to FFN weights (Meng et al., 2022, 2023; Li et al., 2024). This FFN focus is not arbitrary: prior editing and mechanistic studies suggest that factual associations are often mediated by middle-layer FFN computations and can be effectively modified through FFN-side interventions (Meng et al., 2022, 2023; Geva et al., 2022). The practical question is therefore not only whether an FFN-side update can write the target fact, but whether its direction can sustain transfer without inducing off-target drift as edit interactions densify. A plausible failure path is residual-stream mixing: FFN-side updates may partially align with directions shared with MHSA, allowing side effects to spread beyond the intended scope. This motivates asking whether token-level targets align with locally supported FFN response directions, or instead mix in prompt-specific components that are weakly supported by the FFN's local response geometry.

A multi-scale geometric motivation. We study this question geometrically on WSC (Levesque et al., 2012) through three non-equivalent objects: token-level directional relations, distributional sub-

Figure 1: Motivating view of PRISM's Jacobian-inferred constraints on FFN writeback.

spaces estimated from tokens, and local-response subspaces estimated from JVP responses to random probe directions. Weak token-level alignment can coexist with non-trivial coupling between distributional subspaces, whereas local-response directions are consistently less aligned than the corresponding distributional subspaces. We therefore use distributional geometry as a diagnostic of cross-component coupling and local-response geometry as a proxy for directions that the local FFN can actually support. A matched-norm intervention test later links stronger distributional coupling to larger target-side response magnitudes and larger off-target drift, but we use this evidence as design motivation rather than as a causal proof.

PRISM: controlled residual correction in locally supported directions. Motivated by the separation between distributional coupling and local-response geometry, we propose **PRISM** (Projected Residual into Inferred Subspaces of Models). Existing FFN-side editors typically follow a target-then-writeback pattern: they fit layer-local residual targets and write them into FFN weights. Rather than fitting unconstrained layer-local targets, PRISM keeps the closed-form writeback but restricts each layerwise correction to an edit-conditioned, Jacobian-inferred local-response subspace of the FFN, then restores capacity through in-subspace refinement. Figure 1 summarizes this constrained target-construction view.

Contributions.

- **Multi-scale FFN–MHSA Geometry.** We distinguish FFN–MHSA token-level alignment, FFN–MHSA distributional coupling, and Jacobian-inferred local-response geometry, showing that weak token-level alignment can coexist with non-trivial distributional coupling while local-response geometry provides a separate proxy for locally supported FFN response directions.

- **Constrained Target Construction.** We introduce PRISM, which constructs layerwise FFN residual targets by selecting and refining directions inside edit-conditioned, Jacobian-inferred local-response subspaces while retaining the standard closed-form writeback.

- **Scale-Stable Editing Quality.** On Counter-Fact and zsRE, PRISM improves large-batch editing quality relative to strong baselines, with especially consistent gains in generalization and slower degradation as edit scale increases while maintaining competitive specificity.

2 Related Work

Knowledge editing at scale. Knowledge editing seeks targeted updates that improve the desired prediction while preserving the rest of the model (Wang et al., 2024). Sequential editing studies interference across repeated rewrites, whereas our focus is the complementary large-batch regime, where many edits are written in one run and the key issue is how generalization and locality degrade as edit interactions accumulate (Meng et al., 2023; Li et al., 2024; Gupta et al., 2024). Recent continuous and lifelong editors study a related but different regime in which edits arrive over time and later updates may interfere with earlier ones (Li and Chu, 2025; Xu et al., 2025). Beyond direct rewriting, prior families also include learned editors such as MEND and MALMEN (De Cao et al., 2021; Mitchell et al., 2022a; Tan et al., 2024) and memory-based editors such as SERAC (Mitchell et al., 2022b).

Direct rewriting and geometry-aware constraints. ROME, MEMIT, and PMET edit factual associations through FFN-side writeback or FFN-centered target construction (Meng et al., 2022, 2023; Li et al., 2024). Recent constrained editors, especially for sequential or continual editing regimes, use regularization, null-space projection, or orthogonalization to reduce parameter-level or

history-dependent interference (Fang et al., 2025; Xu et al., 2025). Our contribution is not another writeback solver, but a target-construction rule for layerwise residuals within the same fixed edited-layer set: PRISM explicitly constrains the residual target before writeback to edit-conditioned, Jacobian-inferred FFN response subspaces, prioritizing locally realizable corrections and de-emphasizing weakly supported components that may mix with MHSA, rather than relying on preserved-example null spaces or past-edit caches. This is motivated by mechanistic and geometric work showing that transformer components are not a uniform update space, including FFNs as key-value memories, neuron-level factual units, attention-head specialization, and anisotropic contextual geometry (Geva et al., 2021; Dai et al., 2022; Olsson et al., 2022; Ethayarajh, 2019; Valeriani et al., 2023).

3 Preliminaries

Notation. We denote an autoregressive Transformer by f_θ with L layers and hidden size d. Given an edit set $\mathcal{E} = \{e_i\}_{i=1}^{|\mathcal{E}|}$ and edited layers $\mathcal{L} \subseteq \{1, \dots, L\}$, each edit request is a factual triple $e_i = (s_i, r_i, o_i)$, realized by a prompt family $\mathcal{X}_i = \{x_{i,c} : c \in \mathcal{C}\}$, where c indexes prompt/context realizations for edit i; the fact is edited at the triple level, while optimization and evaluation operate on its textual realizations.

Transformer blocks and component outputs. Let $h_t^{\ell-1} \in \mathbb{R}^d$ be the residual stream at token position t entering layer ℓ. Using a pre-norm decomposition, we write the MHSA and FFN pre-residual outputs as

$$a_t^\ell = \text{Attn}^\ell(\text{LN}(h_t^{\ell-1})), \ m_t^\ell = \text{MLP}^\ell(\text{LN}(\bar{h}_t^\ell)),$$

where $\bar{h}_t^\ell = h_t^{\ell-1}$ for parallel blocks and $\bar{h}_t^\ell = h_t^{\ell-1} + a_t^\ell$ for sequential blocks, so that $h_t^\ell = h_t^{\ell-1} + a_t^\ell + m_t^\ell$ in either case. This notation tracks residual increments only; it does not equate the two block organizations. Throughout, a_t^ℓ and m_t^ℓ denote the component outputs immediately before residual addition. We write the FFN output as $m_t^\ell = W^\ell z_t^\ell$, where W^ℓ denotes the second FFN linear map, i.e., the output projection edited throughout the paper.

Task objective and layer-local targets. At the task level, we follow the standard prediction-plus-KL locality objective (Wang et al., 2024): the edited model should increase $p_{\theta'}(o_i \mid x_{i,c})$ while

remaining close to the original model on unrelated prompts. Closed-form editors then replace this global objective by tractable layer-local targets. In our notation, a layer-local intervention adds a residual δ to the FFN pre-residual output at the edited site, and layerwise targets T^ℓ collect such corrections across edits. At this stage, T^ℓ is an output-space target; how its direction is chosen is separate from how it is written back into parameters.

Keys, covariances, and closed-form writeback. Let k_i^ℓ be the realized input to the edited FFN output projection for edit e_i at layer ℓ, and let $K^\ell = [k_1^\ell, \dots, k_{|\mathcal{E}|}^\ell]$. Together with the corresponding second-moment matrix C^ℓ, these quantities are used only by the writeback solver: once T^ℓ has been specified, they translate it into a parameter update on W^ℓ but do not determine its direction. In the generic notation C^ℓ may be layer-specific; in our implementation, following MEMIT, we reuse a single precomputed second-moment matrix across edited layers. We then apply the standard MEMIT-style writeback

$$\Delta^\ell = T^\ell A^{\ell\top}, \ A^\ell = (\lambda C^\ell + K^\ell K^{\ell\top})^{-1} K^\ell \quad (1)$$

Here, λ balances new and old associations in the closed-form writeback. Our contribution lies upstream, in how the targets T^ℓ are constructed before this solver is applied.

Jacobians and Jacobian-vector products. For a component operator F^ℓ, Jacobian-vector products (JVPs) $J_F^\ell(x)v$ provide local linear responses without explicitly forming the full Jacobian. Stacking normalized probe responses yields a low-dimensional proxy for dominant local-response directions near x; the probe budget controls the quality-cost trade-off of this estimate.

4 Geometric Analysis

Token-level disagreement can mask subspace structure, so we compare token-level relations, *distributional* subspaces from retained tokens, and *local-response* subspaces from JVP responses. The first captures instantaneous cross-component alignment, the second captures which directions co-vary most under the observed token distribution, and the third is closer to directions that the component can locally realize and amplify around the current state. Accordingly, we use the distributional view as a diagnostic of shared variation and the local-response

view as the proxy that motivates the constrained update space.

Extraction points and semantic token slices. We use WSC and Quoref as diagnostic datasets because their localized semantic anchors reduce prompt-formatting noise in FFN–MHSA comparisons. Following Section 3, we record the same pre-residual MHSA and FFN component outputs at each layer. The geometry diagnostics use all 804 WSC examples and a fixed 1,000-example subset of Quoref validation prompts. To reduce prompt-formatting confounds, we restrict analysis to task-relevant semantic spans: antecedent/pronoun regions for WSC and annotated answer spans for Quoref, with a small local token window around each anchor. The same extraction interface is used for both parallel and sequential blocks, with schematics deferred to Appendix A.

Token-level FFN–MHSA alignment. We center MHSA and FFN outputs within each prompt and layer over the analyzed span, compute signed and absolute cosine similarities for all retained tokens, and summarize each layer by a token-level mean angle derived from the mean absolute cosine. The main text uses GPT-J / WSC as the representative case, while Appendix A summarizes the remaining settings.

Figure 2: GPT-J on WSC: token-level FFN–MHSA cosine similarity distributions.

In Figure 2, each box summarizes all retained tokens in one layer, with outliers suppressed for readability. Signed cosines stay near zero in many layers but remain clearly depth-dependent, and the absolute cosines yield a high across-layer mean

token-level angle of $86.67°$. Table 4 shows the same weak but non-uniform picture in the other settings, so token-level statistics alone do not rule out lower-dimensional cross-component overlap.

Distributional Subspaces. To test for shared statistical structure, we form centered token matrices for MHSA and FFN outputs and compare their principal subspaces by principal angles. We use a Gaussian-noise rank rule to focus the main comparison on structure above the matched noise floor, and summarize each layer by the mean of its smallest five available principal angles; alternative rank and top-k choices are treated as robustness checks in Appendix A.4. In the representative GPT-J / WSC case (Figure 3, top), the across-layer mean of this distributional summary is $40.53°$, far below the token-level mean angle, indicating non-trivial overlap between the dominant distributional subspaces despite weak token-level alignment.

Local-Response Subspaces. Distributional overlap does not identify the directions each component locally amplifies, and the FFN's true functional update space is not directly observable from activations alone. We therefore use JVP-based local-response subspaces as an operational proxy for directions the component can locally realize near the current operating point. For the main comparison in Figure 3, we use the same Gaussian-noise rank rule and summarize each layer by its top-5 principal-angle mean.

Figure 3: GPT-J on WSC: cross-component distributional (top) and local-response (bottom) geometry.

For GPT-J / WSC, the across-layer mean rises from $40.53°$ for the distributional summary to

57.68° for the local-response summary, and this upward shift appears in all 28 layers. Appendix A confirms the same ordering across settings, broader summaries, and control analyses. Taken together, token-level relations, distributional overlap, and local-response geometry are distinct objects; the stability controls provide additional evidence for using local-response geometry as an operational proxy for locally supported update directions in PRISM.

Intervention Test: Is Distributional Coupling Associated with Side-Effect Susceptibility? As a separate controlled diagnostic, we connect this geometry to behavior through a weight-free inference-time intervention on GPT-J / CounterFact, using 300 relation-balanced intervention cases and 300 matched reference prompts. We define high- and low-coupling layer groups as the layers with the smallest (high coupling) and largest (low coupling) distributional top-5 angles from a pooled semantic-anchor analysis over CounterFact rewrite prompts. Layer groups are fixed from these coupling statistics before perturbation evaluation, so intervention outcomes do not enter the selection. We then inject equal-norm Gaussian perturbations at the FFN output module before residual addition. We measure two outcomes: *target effect*, the magnitude of the target-margin change on intervention cases, and *reference KL*, off-target drift on matched reference prompts.

Figure 4: Matched-norm intervention on GPT-J.

In the canonical Gaussian-noise / top-5 setting, high-coupling layers produce larger target-side changes on average than low-coupling layers by 0.1003 and also larger off-target drift by 0.00555. Figure 4 shows this aggregate paired-difference signal for the 300 intervention and 300 reference cases. The effect is directionally consistent across relations, with sensitivity checks deferred to Table 8. We therefore read the intervention as controlled diagnostic evidence: stronger distributional coupling is associated with greater side-effect susceptibility under matched-norm FFN perturbations, while the local-response analysis remains the direct motivation for PRISM's constrained update space.

5 Methodology

PRISM turns the geometric distinction into a constrained editor: distributional coupling remains diagnostic, while the update space is built from Jacobian-inferred directions that better approximate what the local FFN can actually support at the current operating point. The matched-norm intervention test links stronger distributional coupling to greater side-effect susceptibility.

Rationale: Controllability–Plasticity Tension. Unconstrained residual updates are plastic but can include prompt-specific or weakly supported components; overly tight constraints improve control but can underfit. PRISM resolves this tension with four stages: a first-order proposal, an edit-conditioned Jacobian basis for the local FFN, one in-subspace refinement step, and standard closed-form writeback. For each edit $e_i = (s_i, r_i, o_i)$, layer $\ell \in \mathcal{L}$, and token position $t^\star$, the optimization variable is a layer-local residual intervention δ. Figure 5 summarizes Steps 2–4 after the initial first-order proposal.

Step 1: First-Order Direction Estimation. Let $\widetilde{L}_i^\ell(\delta)$ denote the layer-local surrogate obtained by intervening at $(\ell, t^\star)$. For compactness, write $p_{i,c}^{\ell,\delta} = p_\theta^{(\ell,t^\star,\delta)}(o_i \mid x_{i,c})$ and $q_{i,\ell}^\delta = p_\theta^{(\ell,t^\star,\delta)}(\cdot \mid x_i^{\mathrm{KL}})$. We use the same surrogate throughout Steps 1 and 3:

$$
\begin{aligned}
\widetilde{L}_i^\ell(\delta) = \beta_{\mathrm{NLL}} &\sum_{x_{i,c} \in \mathcal{X}_i} \left[-\log p_{i,c}^{\ell,\delta} \right] \\
&+ \beta_{\mathrm{KL}} D_{\mathrm{KL}}\left(q_{i,\ell}^0 \,\|\, q_{i,\ell}^\delta \right).
\end{aligned} \quad (2)
$$

Here β_{NLL} and β_{KL} weight the rewrite and locality terms. Step 1 does not solve this objective to convergence; it extracts a single normalized first-order proposal

$$
g_{i,\ell} = \nabla_\delta \widetilde{L}_i^\ell(0), \quad \delta_{i,\ell}^{\mathrm{raw}} = -\eta_\delta \frac{g_{i,\ell}}{\|g_{i,\ell}\|_2 + \epsilon}. \quad (3)
$$

Figure 5: Overview of the PRISM large-batch model editing method for Steps 2–4.

Here η_δ is the normalized Step 1 step size, and ϵ is a numerical stabilizer. This yields one high-plasticity proposal per edit–layer pair with a single backward pass, which is key to scalability in large edit batches. Because the surrogate aggregates multiple prefix contexts, the gradient is less sensitive to per-prompt idiosyncrasies; nevertheless, at scale these raw proposals can still contain prompt-specific or weakly supported components. We therefore use $\delta_{i,\ell}^{\mathrm{raw}}$ only as a proposal to be filtered and refined in Steps 2–3; Appendix C.1 verifies that these raw directions are structured rather than arbitrary gradient noise.

Step 2: Estimating the FFN Local-Response Subspace. Step 2 estimates an edit-conditioned FFN basis that captures dominant local-response directions and filters off-subspace, weakly supported components. For each edit–layer pair, we collect N_{probe} JVP responses, stack them into

$$M_{i,\ell}^{\mathrm{bundle}} = \left[r_{i,\ell}^{(1)}, \ldots, r_{i,\ell}^{(N_{\mathrm{probe}})} \right] = U\Sigma V^\top, \quad (4)$$

and retain the leading columns of U until the explained-energy threshold τ_σ is reached, yielding an edit-conditioned basis U_i^ℓ. This explained-energy truncation is part of the editor construction; it is separate from the Gaussian-noise rank rule used only for the diagnostic geometry in Section 4. Smaller subspaces impose stronger control but can suppress transfer; larger ones recover more plasticity at the risk of off-target drift. In practice, the probe budget N_{probe} is the main quality-cost knob, with downstream effects reported in Appendix C.3.

Step 3: In-Subspace Refinement. Given the orthonormal basis U_i^ℓ, Step 3 reparameterizes the residual as $\delta = U_i^\ell \alpha$. We initialize the subspace coordinates by orthogonally projecting the raw proposal:

$$\alpha_{i,\ell}^{(0)} = U_i^{\ell\top} \delta_{i,\ell}^{\mathrm{raw}}, \; \delta_{i,\ell}^{\mathrm{proj}} = U_i^\ell \alpha_{i,\ell}^{(0)}. \quad (5)$$

We then restrict the same surrogate to this coordinate space and compute the subspace gradient:

$$\begin{aligned} \phi_{i,\ell}(\alpha) &= \widetilde{L}_i^\ell(U_i^\ell \alpha), \\ g_{i,\ell}^\alpha &= \nabla_\alpha \phi_{i,\ell}(\alpha)\big|_{\alpha=\alpha_{i,\ell}^{(0)}}. \end{aligned} \quad (6)$$

The refined update is obtained by one normalized gradient step in subspace coordinates, where η_α is the Step 3 step size:

$$\begin{aligned} \alpha_{i,\ell}^{(1)} &= \alpha_{i,\ell}^{(0)} - \eta_\alpha \frac{g_{i,\ell}^\alpha}{\|g_{i,\ell}^\alpha\|_2 + \epsilon}, \\ \delta_{i,\ell}^{\mathrm{sub}} &= U_i^\ell \alpha_{i,\ell}^{(1)}. \end{aligned} \quad (7)$$

Here $\delta_{i,\ell}^{\mathrm{proj}}$ is the projection-only target used in the ablation, while $\delta_{i,\ell}^{\mathrm{sub}}$ is the final target. Projection removes off-subspace components that are more likely to induce off-target drift, and refinement restores capacity without leaving the same safety boundary.

Step 4: Closed-Form Writeback (Layerwise Allocation). Finally, we form $T^\ell = [\delta_{1,\ell}^{\mathrm{sub}}, \ldots, \delta_{|\mathcal{E}|,\ell}^{\mathrm{sub}}]$ and apply the MEMIT-style ridge-regularized writeback from Section 3. Step 4 is therefore only the realization stage: the key change in PRISM lies upstream, in how each layer's target residual is constructed before writeback. Unlike fixed residual spreading, our layerwise targets are data-dependent: they are first semantically directed, then filtered to locally supported FFN directions, and finally refined within the same subspace.

6 Experiments

The empirical sections connect geometry to editing: distributional coupling diagnoses risky FFN directions, local-response geometry supplies the constrained space, interventions test behavioral relevance, and ablations separate projection from refinement–projection sets the locality boundary while in-subspace refinement restores capacity.

Method	Score	Efficacy	Generalization	Specificity	Fluency	Consistency
Base model: GPT-J (6B), 10k edits						
Base model	22.4	15.2 (0.7)	17.7 (0.6)	83.5 (0.5)	622.4 (0.3)	29.4 (0.2)
FT-W	67.6	99.4 (0.1)	77.0 (0.7)	46.9 (0.6)	293.9 (2.4)	15.9 (0.3)
MEND	23.1	15.7 (0.7)	18.5 (0.7)	83.0 (0.5)	618.4 (0.3)	31.1 (0.2)
ROME	50.3	50.2 (1.0)	50.4 (0.8)	50.2 (0.6)	589.6 (0.5)	3.3 (0.1)
MEMIT	85.8	98.9 (0.2)	88.6 (0.5)	**73.7 (0.5)**	619.9 (0.3)	40.1 (0.2)
PMET	86.2	99.5 (0.1)	92.8 (0.4)	71.4 (0.5)	620.0 (0.3)	**40.6 (0.2)**
PRISM	**87.5**	**99.7 (0.1)**	**94.6 (0.4)**	73.1 (0.5)	**622.6 (0.3)**	40.3 (0.2)
Base model: GPT-NeoX (20B), 10k edits						
Base model	23.7	16.8 (1.9)	18.3 (1.7)	81.6 (1.3)	620.4 (0.6)	29.3 (0.5)
MEMIT	82.0	97.2 (0.8)	82.2 (1.6)	70.8 (1.4)	606.4 (1.0)	36.9 (0.6)
PMET	84.3	98.4 (0.2)	89.4 (0.5)	70.3 (0.5)	598.1 (0.6)	38.9 (0.2)
PRISM	**86.0**	**99.5 (0.1)**	91.1 (0.5)	**72.6 (0.5)**	617.1 (0.3)	39.1 (0.2)

Table 1: CounterFact results for 10k edits on GPT-J (6B) and GPT-NeoX (20B).

Method	Efficacy	Generalization	Specificity
Base model: GPT-J (6B), 10k edits			
Base model	26.4 (0.6)	25.8 (0.5)	27.0 (0.5)
FT-W	69.6 (0.6)	64.8 (0.6)	24.1 (0.5)
MEND	19.4 (0.5)	18.6 (0.5)	22.4 (0.5)
ROME	21.0 (0.7)	19.6 (0.7)	0.9 (0.1)
MEMIT	96.7 (0.3)	89.7 (0.5)	26.6 (0.5)
PMET	96.9 (0.3)	90.6 (0.2)	26.7 (0.2)
PRISM	**98.2 (0.2)**	**91.1 (0.5)**	**27.1 (0.5)**
Base model: GPT-NeoX (20B), 10k edits			
Base model	30.2 (0.6)	29.5 (0.6)	31.1 (0.5)
MEMIT	91.7 (0.4)	85.8 (0.5)	28.9 (0.5)
PMET	Unavailable (Out of memory)		
PRISM	**95.5 (0.3)**	**89.0 (0.5)**	**31.3 (0.5)**

Table 2: zsRE results for 10k edits on GPT-J (6B) and GPT-NeoX (20B).

Figure 6: CounterFact scale stability.

6.1 Experimental Protocol

We evaluate on GPT-J (6B) and GPT-NeoX (20B), comparing primarily against MEMIT and PMET, with FT-W (weight fine-tuning), MEND, and ROME reported where available. Editing benchmarks are reported on CounterFact and zsRE, with large-batch stability as the main focus. Following the MEMIT/PMET mass-editing protocol, Counter-Fact uses the standard 17-point log-scale schedule from 1 to 10k edits, with 15k/20k as stress tests. The geometric diagnostics use WSC and a matched-norm CounterFact intervention only to motivate and validate the coupling statistic, not to benchmark weight-updating editors. To isolate direction selection, we keep the edited-layer sets fixed to those used by MEMIT for each backbone, so our contribution is in target construction within a fixed layer set rather than in layer selection.

Under this fixed-layer protocol, we report Efficacy, Generalization, Specificity, and their harmonic mean Score, with Fluency and Consistency as auxiliary diagnostics. Parenthesized entries in subsequent result tables are 95% CI half-widths. Unless stated otherwise, Step 2 uses $N_{probe} =$ 1024 JVP probes with $\tau_\sigma = 0.99$, computed by automatic differentiation in evaluation mode with deterministic seeds. All experiments use NVIDIA H100 cloud instances; Appendix C.2 reports runtime cost and cached-subspace performance, and Appendix C.3 reports the probe-budget sweep.

6.2 10k Large-Batch Editing Results

Table 1 shows that PRISM attains the strongest overall results among the compared editors on both base models. On GPT-J, where Efficacy is already near saturation for the strongest editors, the main gain comes from Generalization: PRISM reaches 94.6 versus 92.8 for PMET and 88.6 for MEMIT, while keeping Specificity above PMET and close to MEMIT. On GPT-NeoX, the gain is broader: PRISM attains the best Score and improves all three core metrics over the strongest baselines.

As shown in Table 2, the transfer benefit of PRISM also extends to zsRE. On GPT-J, it achieves the best values on all three core metrics; on GPT-NeoX, where PMET is unavailable, it consistently improves over MEMIT on Efficacy, Generalization, and Specificity. Together with GPT-NeoX / CounterFact results, this shows simultaneous gains on

all three core metrics in multiple 10k settings.

6.3 Scale Stability Tests

Figure 6 characterizes *scale stability*: how editing quality changes as the edit budget n increases and interference becomes denser. These are dedicated scaling runs, so the 10k values can differ slightly from the multi-run aggregates in Table 1. The trend is nevertheless clear: PRISM is close to PMET at small edit budgets, overtakes it as n grows, and by 10k edits reaches the best Score (87.49), mainly through slower Generalization degradation while keeping Specificity above PMET and close to MEMIT. The same ordering persists at 15k and 20k in Figure 7, where PRISM remains strongest overall with the best Score, Efficacy, and Generalization while retaining competitive Specificity.

Figure 7: Beyond-10k CounterFact stress tests on GPT-J at 15k and 20k edits.

6.4 Ablation Study

All ablations use CounterFact with 10k edits while fixing dataset, budget, and schedule. We isolate Jacobian projection and in-subspace refinement here; the Step 2 probe-budget sweep is deferred to Appendix C.3.

Projection, refinement, and stage-wise direction structure. Table 3 shows the core interaction. The unconstrained variant is highly plastic but weak on Specificity. Projection improves Specificity but underfits, whereas in-subspace refinement restores Efficacy and much of the lost Generalization while keeping Specificity above the unconstrained variant, producing the best overall Score. Thus, projection supplies the locality boundary and refinement recovers expressivity within it; auxiliary generation metrics vary only slightly.

To relate this metric interaction to direction structure, we compare δ^{proj} and δ^{sub} after grouping edits by relation, Figure 8 shows that projection

Table 3: Controlled ablation on CounterFact with 10k edits.

Metric	Unconstrained	+Projection	+Refinement
Score	86.11	82.48	**87.49**
Efficacy	**99.70 (0.1)**	92.72 (0.5)	**99.70 (0.1)**
Generalization	**97.20 (0.3)**	79.84 (0.7)	94.62 (0.4)
Specificity	68.86 (0.5)	**76.55 (0.5)**	73.05 (0.5)
Consistency	**40.81 (0.3)**	37.60 (0.2)	40.25 (0.2)
Fluency	617.29 (0.4)	621.12 (0.3)	**622.63 (0.3)**

Figure 8: Stage-wise direction structure on CounterFact.

reduces inter-relation overlap but weakens within-relation coherence, while refinement partially restores coherence without fully undoing the separation gain. We treat these statistics as structural diagnostic evidence rather than a causal explanation of the metric gains. Appendix C.1 verifies that raw proposals already contain relation-level structure above shuffled controls.

Jacobian probe ablation. The probe budget controls transfer–locality: 1024–1536 probes give the best balance on CounterFact 10k, while larger budgets improve Generalization at the cost of Specificity; see Appendix C.3.

7 Conclusion

Large-batch knowledge editing must sustain transfer as edit interactions densify without sacrificing locality. PRISM addresses this by constructing FFN targets in Jacobian-inferred local-response subspaces: distributional coupling diagnoses side-effect-prone directions, interventions connect the geometry to behavior, and ablations show that projection sets the locality boundary while in-subspace refinement restores capacity. On CounterFact and zsRE, this design improves scale-stable generalization while maintaining competitive specificity.

Limitations

Our method also has clear limitations. Our evidence is limited to the evaluated models, benchmarks, and edit budgets; the current results do not establish safety under denser editing regimes, longer contexts, or broader distribution shifts. Our evaluation follows the standard large-batch editing protocol based on efficacy, generalization, and specificity; it does not exhaustively test broader post-edit general capabilities or ripple effects over related facts, which are complementary evaluation axes studied in recent work (Gu et al., 2024; Cohen et al., 2024). These remain practical boundaries for deployment and natural directions for future work.

Ethical Considerations

More controlled knowledge editing can be useful for localized factual repair without full retraining, but the same capability can also be used to alter model behavior at scale in ways that are difficult to audit. For that reason, stronger editing methods should be paired with careful logging, downstream evaluation, and access controls rather than treated as inherently benign.

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A Additional Geometry Results

To keep the main text focused, §4 centers on the representative parallel-block GPT-J / WSC case under the Gaussian-noise rank policy. This appendix checks whether the same geometry persists across extraction interfaces, additional model/dataset settings, and rank/control choices. Unless stated otherwise, all analyses use the same pre-residual MHSA/FFN outputs, semantic-span token pool, Gaussian-noise / top-5 principal-angle summary, and token-level mean-angle baseline as in the main text.

A.1 Extraction Interface

(a) Parallel block.

(b) Sequential block.

Figure 9: Extraction points for MHSA and FFN component outputs in parallel and sequential Transformer blocks.

Figure 9 shows the component-output extraction interface used for the geometry diagnostics. For both parallel and sequential blocks, we record MHSA and FFN outputs after their output projections and before dropout/residual addition, matching the notation in Section 3.

A.2 Cross-setting Geometry Summary

Table 4: Cross-setting geometry summary under the Gaussian-noise rank policy.

Setting	Token angle	Dist. top-5 angle	Local top-5 angle	Layers local> dist.
GPT-J / WSC	86.67°	40.53°	57.68°	28/28
GPT-J / Quoref	86.96°	41.12°	58.00°	28/28
GPT-2 XL / WSC	83.00°	32.88°	47.80°	48/48
GPT-2 XL / Quoref	83.10°	31.53°	43.34°	45/48

The columns in Table 4 report the token-level mean angle, the across-layer means of the distributional and local-response top-5 angles, and the number of layers where the local-response angle exceeds the distributional one. Across all four settings, local-response angles are consistently larger than distributional angles, supporting the main-text claim that the two geometries capture distinct structure.

A.3 Layerwise Geometry on Additional Settings

The remaining three settings are summarized by a single token-level appendix figure, Figure 10, which collects the absolute-cosine distributions for GPT-J / Quoref, GPT-2 XL / WSC, and GPT-2 XL / Quoref. Their corresponding layerwise geometry plots appear in Figures 11 to 13. To keep the appendix compact, these token-level replications retain only the absolute-cosine distributions, which are the quantities used to form the token-level mean-angle baselines in the geometry plots. These plots are layerwise replications of the main-text geometry view under the same Gaussian-noise / top-5 protocol.

A.4 Rank and Control Robustness

This subsection tests whether the main ordering depends on the Gaussian-noise rank policy or on same-layer comparisons alone. We compare the default Gaussian-noise rank policy with two alternatives: 90% EV retains the minimum number of singular directions whose cumulative squared singular values explain at least 90% of the total energy, while Fixed-$k = 32$ keeps the top 32 singular directions regardless of explained energy. Rows labeled Noise use the default Gaussian-noise rank policy: the retained rank is the number of empirical singular values exceeding the corresponding threshold from same-shape Gaussian matrices scaled to the empirical standard deviation. Alternative rank rules and top-10 summaries are sensitivity checks; since they preserve the same distributional–local-response ordering, the tables report the top-5 summaries.

The rank-policy sensitivity checks and control comparisons appear in Tables 5 and 6, and the corresponding effective-rank and split-half summaries appear in Table 7. Here, *random-null* compares each observed MHSA–FFN pair against same-rank random orthonormal bases in the same ambient dimension, asking whether the observed overlap is stronger than chance given rank alone. *Cross-layer* is a distance-matched, rank-matched control: for each layer, the MHSA subspace is compared with FFN subspaces from other layers within the same

Figure 10: Token-level absolute-cosine distributions on the three additional settings.

Figure 11: GPT-J on Quoref: distributional and local-response geometry.

Figure 12: GPT-2 XL on WSC: distributional and local-response geometry.

Figure 13: GPT-2 XL on Quoref: distributional and local-response geometry.

layer-distance buckets, so that the reported gap reflects same-layer specificity rather than arbitrary cross-layer overlap.

In Tables 5 and 6, 'Obs. angle' denotes the across-layer mean of the observed top-5 angles, 'Rand.-null' and 'Cross-layer' denote the corresponding control means, 'Frac. $< q05$' reports the fraction of layers below the random-null 5th percentile, and 'Frac. $<$ cross' reports the fraction

below the cross-layer mean. This random-null percentile is a robustness threshold only; it is distinct from the Gaussian-noise rank rule used to select subspace dimensions in the main geometric analysis. In Table 7, the effective-rank columns use entropy effective rank computed from normalized squared singular values, and the split-half columns report mean subspace cosines under random half-splits of the retained-token or JVP-response pools.

Table 5: Distributional geometry robustness controls.

Setting / policy	Obs. angle	Rand.-null	Cross-layer	Frac. < q05	Frac. < cross
GPT-J / WSC / Noise	40.53	56.08	53.85	1.000	0.922
GPT-J / WSC / 90% EV	31.70	42.64	47.97	1.000	0.926
GPT-J / WSC / Fixed-k32	64.27	81.41	73.72	1.000	0.932
GPT-J / Quoref / Noise	41.12	58.33	59.74	1.000	0.888
GPT-J / Quoref / 90% EV	29.28	41.60	51.29	1.000	0.870
GPT-J / Quoref / Fixed-k32	62.69	81.41	72.88	1.000	0.936
GPT-2 XL / WSC / Noise	32.88	51.28	45.93	1.000	0.921
GPT-2 XL / WSC / 90% EV	20.16	30.35	36.78	0.979	0.921
GPT-2 XL / WSC / Fixed-k32	53.51	76.22	62.08	1.000	0.883
GPT-2 XL / Quoref / Noise	31.53	56.19	48.60	1.000	0.886
GPT-2 XL / Quoref / 90% EV	18.67	33.13	39.59	0.979	0.885
GPT-2 XL / Quoref / Fixed-k32	47.79	76.22	55.61	1.000	0.933

Table 6: Local-response geometry robustness controls.

Setting / policy	Obs. angle	Rand.-null	Cross-layer	Frac. < q05	Frac. < cross
GPT-J / WSC / Noise	57.68	65.56	82.66	0.964	0.920
GPT-J / WSC / 90% EV	44.92	47.20	85.96	0.821	0.917
GPT-J / WSC / Fixed-k32	71.22	81.41	76.52	1.000	0.909
GPT-J / Quoref / Noise	58.00	65.72	82.63	0.929	0.928
GPT-J / Quoref / 90% EV	43.70	46.01	85.89	0.821	0.894
GPT-J / Quoref / Fixed-k32	68.81	81.41	75.49	1.000	0.904
GPT-2 XL / WSC / Noise	47.80	61.56	76.80	1.000	0.921
GPT-2 XL / WSC / 90% EV	39.69	42.46	82.43	0.979	0.913
GPT-2 XL / WSC / Fixed-k32	57.46	76.22	66.51	1.000	0.904
GPT-2 XL / Quoref / Noise	43.34	60.41	74.43	1.000	0.909
GPT-2 XL / Quoref / 90% EV	37.88	40.54	82.19	0.958	0.897
GPT-2 XL / Quoref / Fixed-k32	52.27	76.22	63.12	1.000	0.928

Table 7: Effective-rank and split-half summaries.

Analysis	Setting	Eff. rank (MHSA)	Eff. rank (FFN)	Split-half (MHSA)	Split-half (FFN)
Distributional	GPT-J / WSC	268.7	646.7	0.858	0.770
Distributional	GPT-J / Quoref	387.1	691.0	0.807	0.767
Distributional	GPT-2 XL / WSC	164.4	415.3	0.896	0.804
Distributional	GPT-2 XL / Quoref	175.7	346.8	0.867	0.815
Local-response	GPT-J / WSC	11.6	1000.0	0.999	0.918
Local-response	GPT-J / Quoref	11.8	1071.7	0.999	0.908
Local-response	GPT-2 XL / WSC	13.1	548.8	0.999	0.952
Local-response	GPT-2 XL / Quoref	13.9	615.9	0.998	0.942

Together, these tables support the main-text geometry claims under alternative rank rules and explicit stability controls.

B Intervention Sensitivity Checks

This appendix reports sensitivity checks for the main-text matched-norm diagnostic experiment. Table 8 gives paired high-minus-low differences under alternative top-k summaries and rank rules. Across all variants, high-coupling layers produce larger target-side changes than low-coupling layers; the reference-KL gap remains positive but varies in magnitude, so we use the intervention as directional evidence rather than as a calibrated effect-size estimate.

For all rows in Table 8, the high- and low-coupling groups are formed from the three most and three least coupled valid layers after excluding layers whose usable rank is insufficient for the chosen top-k summary. The reported ΔTarget values are paired over intervention cases and use the absolute target-margin change, whereas ΔRefKL first averages over matched reference prompts within each coupling group and is then paired at the

reference-prompt level.

Table 8: Intervention sensitivity checks.

Setting	ΔTarget	ΔRefKL
Noise / top-5	0.1003	0.00555
Noise / top-10	0.0434	0.000610
90% EV / top-5	0.0883	0.00543
90% EV / top-10	0.0912	0.00549

C   Additional Ablation and Efficiency Details

This section complements the main ablation by defining the direction-structure diagnostics, analyzing the Step 2 probe budget, and quantifying runtime cost and cached-subspace performance.

C.1   Direction-Structure Diagnostics

Figure 14: Raw-direction relation signal on Counter-Fact.

The stage-wise analysis in Figure 8 uses CounterFact edits grouped by relation ID at each edited layer. For every stage (δ^{raw}, δ^{proj}, δ^{sub}), we first row-normalize the edit directions within each relation group. The raw-direction signal in Figure 14 is measured by the explained-variance ratio of the top principal components (PCs) within each group, with split-half reproducibility computed against a shuffled matched-size baseline. For the stage-wise trade-off plots, *within-relation similarity* is the mean pairwise cosine within each relation group, aggregated across groups by relation-group size, and *inter-relation overlap* is the mean absolute cosine between normalized relation-group centroids. We exclude zero or near-zero raw directions and skip any relation group with fewer than 100 remaining examples at a layer, so that the directional statistics are computed only where the group structure is well-defined. For the raw-direction signal, PC1

signal is roughly 1.9–2.7$\times$ the shuffled baseline across edited layers, PC10 signal remains about 1.5–1.8$\times$ higher, and top-10 split-half stability is around 0.53–0.65 versus 0.33–0.43 for shuffled groups. For the stage-wise trade-off in Figure 8, projection reduces the average within-relation similarity from 0.0386 to 0.0174 and inter-relation overlap from 0.362 to 0.297; the refined stage restores within-relation similarity to 0.0231 while keeping overlap below the raw stage on five of six edited layers.

C.2   Runtime Cost and Cached-Subspace Performance

Figure 15: Wall-clock time across the 17-point Counter-Fact edit schedule on GPT-J.

Figure 16: Score under the cached-subspace approximation on the same schedule.

The main additional cost of PRISM comes from the edit-conditioned JVP probes used to estimate local-response subspaces in Step 2. We report wall-clock time on GPT-J / CounterFact across the standard 17-point edit schedule under the same experimental setup as the main editing results. To separate the cost of subspace estimation from the benefit of subspace-constrained targets, we also evaluate PRISM-Cache, an efficiency-oriented approximation that estimates one layerwise projection matrix from a disjoint 10k-sample calibration set and reuses it for all edit requests. Because these cached bases are no longer edit-conditioned,

Figure 17: Jacobian probe-count sweep on CounterFact.

PRISM-Cache is not our default method; it is a diagnostic approximation for the runtime–editing-quality trade-off.

Figures 15 and 16 show that cached subspaces reduce a substantial fraction of the additional runtime overhead while preserving most of the editing quality at moderate edit budgets. Averaged across the schedule, PRISM-Cache reduces runtime by 20.8% relative to the full edit-conditioned variant; at 10k edits, wall-clock time decreases from 25.9k s to 18.9k s (-27.0%). From 3 edits onward, the cached variant is also faster than PMET, although MEMIT remains the fastest method.

The editing-quality trade-off becomes more pronounced as the edit batch grows. In the low-to-medium regime, cached subspaces remain competitive and are slightly better at 1k edits (92.33 vs. 91.76 Score). At 10k edits, however, the full edit-conditioned variant is stronger (87.49 vs. 86.19 Score), suggesting that request-specific local-response geometry becomes more important as edit interactions densify. These results support using cached subspaces as a practical runtime approximation, while retaining edit-conditioned subspaces as the default choice when large-batch transfer–locality control is the primary objective.

C.3 Probe-Budget Sweep

To complement the main ablation in Section 6.4, we vary the Step 2 JVP probe budget while keeping the rest of the pipeline fixed.

Figure 17 shows a stable transfer–locality trade-off: 1024–1536 probes provide the best overall balance on CounterFact 10k, whereas larger budgets continue to improve Generalization at the cost of Specificity.

D Artifact Statement

We use third-party scientific artifacts, including pre-trained language models, English-language benchmarks, diagnostic datasets, and baseline implementations. We also provide an accompanying software artifact of PRISM. We do not collect new human-subject data or release a new dataset. Table 9 summarizes the main artifacts used or provided in this work.

Table 9: Scientific artifacts used or provided.

Artifact	License / terms
GPT-J-6B (Wang and Komatsuzaki, 2021)	Apache-2.0
GPT-NeoX-20B (Black et al., 2022)	Apache-2.0
GPT-2 XL (Radford et al., 2019)	MIT-style
CounterFact (Meng et al., 2022)	Research release
zsRE (Levy et al., 2017)	Research release
WSC (Levesque et al., 2012)	CC-BY
Quoref (Dasigi et al., 2019)	CC-BY-4.0
WikiText (Merity et al., 2017)	CC-BY-SA
ROME code (Meng et al., 2022)	MIT
MEMIT code (Meng et al., 2023)	MIT
PMET code (Li et al., 2024)	No explicit license
PRISM	MIT

The pretrained models are used as editing backbones or for geometry diagnostics; the editing benchmarks are CounterFact and zsRE, and WSC and Quoref support the geometry diagnostics. CounterFact follows the standard 17-point schedule

from 1 to 10k edits, with 15k/20k stress tests; zsRE is evaluated at 10k edits. The diagnostic datasets use all 804 WSC examples and a fixed 1,000-example Quoref validation subset. WikiText is used only to estimate the precomputed second-moment matrix for the writeback solver. The accompanying software artifact contains our PRISM implementation, evaluation scripts, configurations, reproduction documentation, and the included ROME and MEMIT baseline implementations. PMET is used as an external public implementation for baseline comparison only; because its repository does not specify an explicit license at the time of writing, it is not redistributed with our software artifact.

Intended use and redistribution. Third-party artifacts are used for research evaluation and analysis only, consistently with their public benchmark, model-release, or research-code purposes. The accompanying software artifact is intended for research reproduction and analysis. We do not redistribute modified versions of the original datasets or pretrained models. The software artifact documents external artifact dependencies and will not package third-party datasets or pretrained models whose licenses require separate access.

Privacy and content. The datasets may contain naturally occurring entity names because factual editing and coreference benchmarks require named entities. We do not scrape new text, introduce new personal data, infer protected attributes, or report individual-level results. No additional anonymization is applied to benchmark examples because entity names are part of the task definition; results are reported only in aggregate.

Documentation and coverage. We document both the data artifacts used in the experiments and the accompanying software artifact. All language data used in this work is English. CounterFact and zsRE provide factual-editing examples, WSC and Quoref provide coreference-style diagnostic examples, and WikiText is used only for covariance estimation. The subset sizes used in our experiments are reported above and in the main experimental protocol. The software artifact includes a README, quick-start instructions, a code-structure overview, dependency requirements, configuration files, and scripts for reproducing the main experiments.