Self-Consistent Flow: Unifying Velocity and Endpoint Prediction for Rectified Flow Models

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Abstract

In rectified-flow–based generative models, the neural network can be trained to predict two different targets, such as the instantaneous velocity or the data endpoint, to perform denoising. Although prior work shows that these parameterizations lead to different empirical behaviors, the mechanisms underlying their respective advantages remain to be underexplored, and how to combine them effectively is still unclear. In this work, we analyze how learning errors from different parameterizations affect the generation performance. We show that predicting the data endpoint has a clear training signal that stabilizes training, whereas predicting the velocity maintains stable sampling dynamics near the data manifold. Motivated by these insights, we propose Self-Consistent Flow (SC-Flow), a new method that unifies the benefits of both parameterizations. By employing a lightweight consistency loss, SC-Flow jointly trains a single network to predict *both* the local velocity and the data endpoint, and the consistency between the two predictions improves the model's performance. The method requires no major architectural changes and adds minimal computational overhead. Extensive experiments on image generation tasks demonstrate that SC-Flow substantially stabilizes optimization and improves the straightness of generation paths, leading to significant gains in generation quality over standard rectified-flow baselines.

1 Introduction

Generative models based on ordinary differential equations (ODEs) (Lipman et al., 2022; Liu et al., 2022), have emerged as a powerful alternative to traditional diffusion models, enabling high-quality synthesis with significantly fewer sampling steps. The neural network learns the vector field of an ODE that transforms a simple prior distribution into the data distribution.

In a flow matching model (Lipman et al., 2022; Liu et al., 2022), the generative model needs to predict the vector field to learn the ODE that defines the model distribution. Besides predicting the velocity field at a given noisy sample, the underlying neural network can also be parameterized to predict the clean sample or the noise (Li & He, 2025). Different parameterizations also appear in different diffusion-based generation (Song et al., 2020b), where the neural network can learn the score function in the reverse SDE (Song & Ermon, 2019), predict the noise (Ho et al., 2020; Rombach et al., 2022), or predict the clean sample (Song et al., 2020a). While these different predicting targets are mathematically equivalent, they do yield distinct empirical performances. Previous studies that develop specific methods often consider that one parameterization is superior over another one depending on the empirical performance. Recent research (Li & He, 2025) compares different flow parameterization in a systematic study and favors x-flow based on an argument of data manifold. Given that different parameterizations are equivalent in the optimal case, we believe that the learning error is the root reason for their performance difference. However, there still lacks a systematic analysis of learning errors in these different parameterizations.

In this work, we relate prediction targets to learning errors and develop a dual-target predictive model to reduce the error. We introduce Self-Consistent Flow (**SC-Flow**) to combine the strengths of the two learning targets. SC-Flow is a novel framework that trains a single model to simultaneously predict two complementary targets: the instantaneous velocity (v) and the likely destination of the ODE path (x). Our key insight is

that by receiving training signals in both forms, the neural network gains extra power in learning the vector field, bridging the optimization benefits of data prediction with the inference stability of velocity prediction. SC-Flow uses a single network backbone to compute the two fitting targets, with a binary mode flag to indicate the type of the target. The design adds negligible overhead to a baseline rectified flow model and incurs the same level of inference costs in the generation procedure.

Experiments on ImageNet show that SC-Flow substantially outperforms baseline methods, demonstrating the advantage of combining flow targets in a unified model. We also conduct an extensive empirical study to verify our theoretical analysis and demonstrate that the performance gain is due to our new design.

Our contributions can be summarized as follows:

- **Dual-Target Framework:** We propose SC-Flow, a novel training framework for rectified flows that supervises a single model on two complementary targets: the instantaneous velocity and the likely endpoint. The consistency loss enforces the analytical relationship between the two predictions. This improves optimization and ensures the learned vector fields are coherent.
- **Analysis of learning errors:** We rigorously analyze the variance and error dynamics of flow estimators. We show that direct endpoint prediction is less affected by sampling variance, whereas the velocity target is more stable near the data manifold.
- **Empirical verification:** SC-Flow substantially outperforms the standard rectified flow baseline on ImageNet 256×256, achieving significant FID improvements with the same sampling cost and negligible additional training overhead.

2 Related Works

Modern denoising-based generative models are formulated through three closely related perspectives: score-based modeling (Song & Ermon, 2019), reverse diffusion processes (Song et al., 2020b; Ho et al., 2020), and flow matching (Liu et al., 2022; Lipman et al., 2022). Across these formulations, the neural networks are often parameterized differently to perform the denoising step. In score-based methods (Song & Ermon, 2019), the neural network is trained to learn the score function at different noise levels. The learning target is also equivalent to learning the score function in the reverse SDE (Song et al., 2020b). In DDPM (Ho et al., 2020), the network is trained to predict the noise (ϵ-prediction). In DDIM (Song et al., 2020a), the network directly predicts the clean sample conditioned on the noisy sample.

In the framework of Flow Matching (Lipman et al., 2022) and Rectified Flows (Liu et al., 2022), the network learns a continuous-time ODE vector field to map a simple prior to the data distribution. With a noisy sample as the input, the output of the network should predict velocity (v-prediction) of the vector field. Compared against the diffusion-based formulation, the velocity is essentially the sum of the score function and the drift in the reverse SDE. With the assumption of rectified flow, the network can also predict the real data sample (x-flow) or the noise (ϵ-flow), and then derive the velocity prediction. Therefore, x-flow and ϵ-flow share a similar spirit respectively as DDPM and DDIM.

The different parameterizations are systematically investigated by Ma et al. (2024); Li & He (2025), which compared noise, score, and velocity predictions within the stochastic interpolant framework, demonstrating that generative performance heavily depends on the chosen target. In particular, Ma et al. (2024) observes a significant performance increase with weighted score and velocity (they are equivalent) in the latent diffusion model; while the study by Li & He (2025) favors x-flow on the original pixel space with an argument that the data is on a manifold and easy to predict. However, the argument is only supported by a small example. In this work, we deepen the understanding from a theoretical perspective and investigate how learning error affects the generation quality.

In diffusion-based generative models, Benny & Wolf (2022) have explored multiple parameterizations and dual outputs and let the neural network to both noise and the clean sample using a naive shared structure and a learned mixing strategy. However, this approach only relies on a shared network trunk and encourages the network to learn information from both targets. In our work, we develop a new algorithm from rectified

flow, which is very different from diffusion-based models. Furthermore, we restrict the model to be consistent with the two flows.

3 Background

Rectified Flow (RF) learns a neural ODE to transform an initial noise distribution p_0 into the target data distribution p_1, with a time-dependent vector field $v_t(\mathbf{x}_t) : \mathbb{R}^d \to \mathbb{R}^d$ that guides the evolution of state $\mathbf{x}_t$:

$$\mathrm{d}\mathbf{x}_t = v_t(\mathbf{x}_t)\mathrm{d}t, \quad \mathbf{x}_0 \sim p_0, \quad t \in [0, 1]. \tag{1}$$

Starting from $\mathbf{x}_0 \sim p_0$ (typically a standard Gaussian distribution), RF integrates the ODE from $t = 0$ to $t = 1$, yielding a sample $\mathbf{x}_1 \sim p_1$. The vector field $v_t(\mathbf{x}_t)$ is learned by a neural network $\mathbf{v}_\theta(\mathbf{x}_t, t)$, where θ represents the trainable parameters.

In an RF model, the vector field is given by a *conditional* flow. RF starts with a coupling distribution $p(\mathbf{x}_0, \mathbf{x}_1)$ such that the marginal $p(\mathbf{x}_0)$ and $p(\mathbf{x}_1)$ are respectively the noise distribution and the data distribution. Then the conditional flow is defined by the straight path between $(\mathbf{x}_0, \mathbf{x}_1) \sim p(\mathbf{x}_0, \mathbf{x}_1)$. Let $\mathbf{x}_t = t\mathbf{x}_1 + (1-t)\mathbf{x}_0$, the conditional velocity at x_t is:

$$u(\mathbf{x}_t|\mathbf{x}_0, \mathbf{x}_1) = \mathbf{x}_1 - \mathbf{x}_0 = \frac{\mathbf{x}_1 - \mathbf{x}_t}{1-t}. \tag{2}$$

The model $\mathbf{v}_\theta(\mathbf{x}_t, t)$ is then trained with conditional flow loss by:

$$\mathcal{L}(\phi) = \mathbb{E}_{t\sim\mathcal{U}[0,1],\mathbf{x}_0,\mathbf{x}_1}\left[\|(\mathbf{x}_1 - \mathbf{x}_0) - \mathbf{v}_\theta(\mathbf{x}_t, t)\|_2^2\right]. \tag{3}$$

Latent space diffusion/flow models achieve both high generation quality and efficiency compared to the pixel space models. In the latent space model, the RGB image is encoded to a latent space sample $\mathbf{x}$ using an encoder E. The reconstructed image is decoded from the latent $\mathbf{x}$ by the decoder D. In this work, we focus on learning the data distribution $p(\mathbf{x}_1)$ and assume the encoder-decoder is fixed.

4 SC-Flow Model

Motivation. SC-Flow is motivated by re-examining the flow-matching objective. Let $p(\mathbf{x}_0, \mathbf{x}_1) = p_0(\mathbf{x}_0)p_1(\mathbf{x}_1)$ denote the independent coupling between the prior and the target data distribution. Here, $\mathbf{x}_1 \sim p_1$ is a clean sample drawn from the empirical data distribution, and $\mathbf{x}_0 \sim p_0 = \mathcal{N}(\mathbf{0}, \mathbf{I})$ is a random noise sample drawn from a standard Gaussian prior.

By the definition of the straight path $\mathbf{x}_t = (1-t)\mathbf{x}_0 + t\mathbf{x}_1$, we can algebraically rewrite the instantaneous velocity as $\mathbf{x}_1 - \mathbf{x}_0 = \frac{\mathbf{x}_1 - \mathbf{x}_t}{1-t}$. Substituting this directly into the flow-matching objective, for $t \in (0, 1)$, the training loss becomes:

$$\begin{aligned}\mathbb{E}_p\left[\|\mathbf{v}_\theta(\mathbf{x}_t, t) - (\mathbf{x}_1 - \mathbf{x}_0)\|_2^2\right] &= \mathbb{E}_p\left[\left\|\mathbf{v}_\theta(\mathbf{x}_t, t) - \frac{\mathbf{x}_1 - \mathbf{x}_t}{1-t}\right\|_2^2\right] \\ &= \frac{1}{(1-t)^2}\mathbb{E}_p\left[\|((1-t)\mathbf{v}_\theta(\mathbf{x}_t, t) + \mathbf{x}_t) - \mathbf{x}_1\|_2^2\right].\end{aligned} \tag{4}$$

Because the network $\mathbf{v}_\theta(\mathbf{x}_t, t)$ makes its predictions conditioned only on the intermediate state $\mathbf{x}_t$ and time t, the pointwise optimal predictor that minimizes this squared error is given by the conditional expectation. Therefore, the optimal value for $\mathbf{v}_\theta$ satisfies:

$$((1-t)\mathbf{v}_\theta(\mathbf{x}_t, t) + \mathbf{x}_t) = \mathbb{E}_p[\mathbf{x}_1|\mathbf{x}_t].$$

Noting that the left-hand side only involves $(\mathbf{x}_t, t)$, we can define a new function $\mathbf{m}_\theta(\mathbf{x}_t, t)$ to replace the left-hand side. We then train by directly regressing $\mathbf{m}_\theta(\mathbf{x}_t, t)$ toward the data endpoint $\mathbf{x}_1$. The relationship

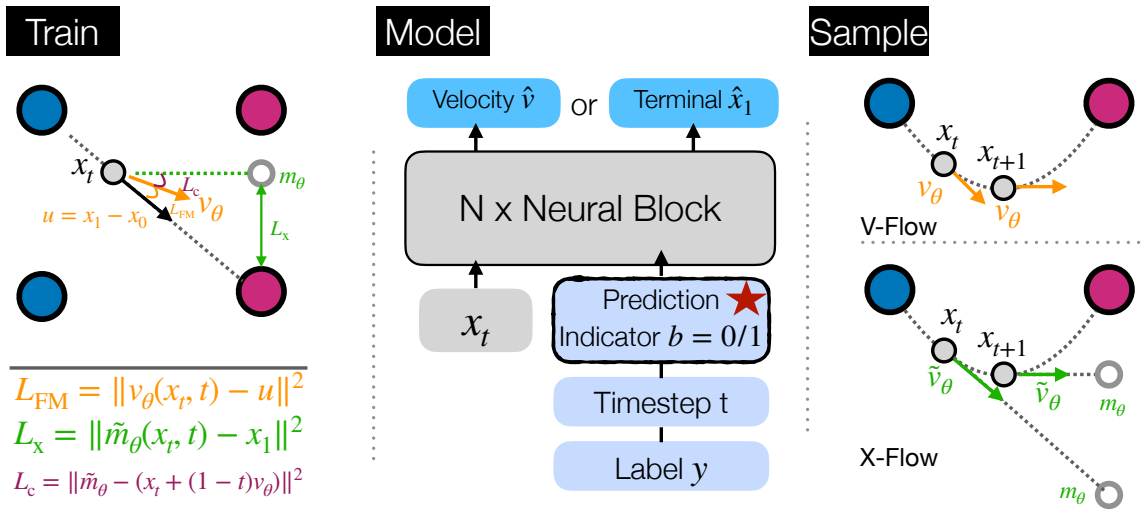


Figure 1: **SC-Flow overview: (A) Training.** Linear rectified path $x_t = (1-t)x_0 + tx_1$ with instantaneous velocity $u = x_1 - x_0$; Training minimizes $L_{\text{FM}} = \|\mathbf{v}_\theta - u\|^2$, $L_{\text{x}} = \|\tilde{\mathbf{m}}_\theta - x_1\|^2$, and the consistency term $L_{\text{c}} = \|\tilde{\mathbf{m}}_\theta - (x_t + (1-t)\mathbf{v}_\theta)\|^2$. **(B) Model.** Single network $\text{nn}_\theta(x_t, t, y, b)$ with summed embeddings $E_t(t)+E_y(y)+E_b(b)$; mode b=0 outputs $\mathbf{v}_\theta$ (velocity) and b=1 outputs $\tilde{\mathbf{m}}_\theta$ (endpoint). **(C) Sampling.** Sampling integrates the probability-flow ODE with a per-step drift chosen from $\mathbf{v}_\theta$ and $(\tilde{\mathbf{m}}_\theta - x_t)/(1-t)$ (Equation (10)); any selection/blending policy may be used without retraining.

between $\mathbf{m}_\theta$ and $\mathbf{v}_\theta$ is:

$$\mathbf{m}_\theta = (1-t)\mathbf{v}_\theta(\mathbf{x}_t, t) + \mathbf{x}_t, \qquad \mathbf{v}_\theta = \frac{\mathbf{m}_\theta(\mathbf{x}_t, t) - \mathbf{x}_t}{1-t} \tag{5}$$

The SC-Flow method. We introduce another neural function $\tilde{\mathbf{m}}_\theta$ to directly learn $\mathbf{x}_1$. The learning objective for $\tilde{\mathbf{m}}_\theta$ is:

$$L_{\text{x}}(\theta) = \|\tilde{\mathbf{m}}_\theta(\mathbf{x}_t, t) - \mathbf{x}_1\|_2^2 \tag{6}$$

Here we use the same θ to denote the network parameters, since later both $\mathbf{v}_\theta$ and $\tilde{\mathbf{m}}_\theta$ are parameterized by the same network. Compared with equation 4, this objective removes the leading factor $1/t^2$ to improve training stability; empirically, we find that this change has a negligible impact on performance.

Since $\mathbf{v}_\theta$ and $\tilde{\mathbf{m}}_\theta$ are computed from different neural functions, we need to make them consistent with each other. We include another consistency term in the training objective:

$$L_{\text{c}} = \|\tilde{\mathbf{m}}_\theta(\mathbf{x}_t, t) - \mathbf{m}_\theta(\mathbf{x}_t, t)\|_2^2 = \|\tilde{\mathbf{m}}_\theta(\mathbf{x}_t, t) - ((1-t)\mathbf{v}_\theta(\mathbf{x}_t, t) + \mathbf{x}_t)\|_2^2 \,. \tag{7}$$

The entire training objective is:

$$\mathcal{L}(\theta) = w_v L_{\text{FM}}(\theta) + w_x L_x(\theta) + w_c L_c(\theta) \tag{8}$$

Parameterization. While $\mathbf{v}_\theta$ and $\tilde{\mathbf{m}}_\theta$ play different roles, their required information is similar. In this work, we train a *single* network nn_θ to compute both functions. In particular, we use a binary flag $b \in \{0, 1\}$ to indicate which function it computes:

$$\mathbf{v}_\theta(\mathbf{x}_t, t) := \text{nn}_\theta(\mathbf{x}_t, t, b{=}0), \qquad \tilde{\mathbf{m}}_\theta(\mathbf{x}_t, t) := \text{nn}_\theta(\mathbf{x}_t, t, b{=}1). \tag{9}$$

From $\tilde{\mathbf{m}}_\theta$, we derive the velocity:

$$\tilde{\mathbf{v}}_\theta = \frac{\tilde{\mathbf{m}}_\theta(\mathbf{x}_t, t) - \mathbf{x}_t}{1-t} \tag{10}$$

We refer to the original flow $\mathbf{v}_\theta$ as **V-Flow** and to our new flow $\tilde{\mathbf{v}}_\theta$ as **X-Flow**.

We also consider conditional generation. Let y denote a conditioning variable (e.g., a class label). We augment the inputs with y, so the network becomes $\mathrm{nn}_\theta(\mathbf{x}_t, t, b, y)$.

In the implementation, we embed the time t, the indicator b, and the condition y with vectors and send their sum into the neural network, $e(t, y, b) = e_t(t) + e_y(y) + e_b(b)$. Here we assume y is a condition with a simple form.

Sampling. In the generative procedure, we have two vector fields $\mathbf{v}_\theta(\mathbf{x}_t, t)$ and $\tilde{\mathbf{v}}_\theta(\mathbf{x}_t, t)$ for sampling. We can use either of them for sampling with path integration. We can also blend them in the sampling procedure.

$$\dot{\mathbf{x}}_t \;=\; \begin{cases} \tilde{\mathbf{v}}_\theta(\mathbf{x}_t, t), & t \le \tau, \\ \mathbf{v}_\theta(\mathbf{x}_t, t), & t > \tau, \end{cases} \tag{11}$$

In our experiment, we use τ=0.5. The blending strategy slightly improves generation performance in the experiment, though $\mathbf{v}_\theta$ and $\tilde{\mathbf{v}}_\theta$ do not have significant differences because of the consistent term. The strategy is justified by the stability during inference. Detailed analysis can be found in analysis section.

Computation. Compared with a baseline that predicts $\mathbf{v}_\theta$ with a single network, our model introduces only a minimal number of additional parameters –the embedding of the binary switch b. Training time increases only slightly because the computations for $\mathbf{v}_\theta$ and $\mathbf{m}_\theta$ are largely shared. At sampling time, the runtime is nearly identical to that of the baseline flow-matching model: the compute budget is essentially unchanged, and we simply toggle b to select $\mathbf{v}_\theta$ or $\tilde{\mathbf{v}}_\theta$.

Algorithm 1: SC-Flow training (single head)

Input: (w_v, w_x, w_c), clamp ε, optimizer
for minibatches (x_0, x_1, y) in dataset **do**
  Draw $t \sim \mathcal{U}([\varepsilon,\ 1{-}\varepsilon])$
  $x_t \leftarrow (1{-}t)x_0 + tx_1, \quad u \leftarrow x_1 - x_0$
  $\mathbf{v}_\theta \leftarrow \mathrm{nn}_\theta(x_t, t, y, b{=}0)$
  $\tilde{\mathbf{m}}_\theta \leftarrow \mathrm{nn}_\theta(x_t, t, y, b{=}1)$
  Compute $L_{\mathrm{FM}}, L_{\mathrm{x}}, L_{\mathrm{c}}$
  $\mathcal{L}(\theta) \leftarrow w_v L_{\mathrm{FM}}(\theta) + w_x L_x(\theta) + w_c L_c(\theta)$
  Update θ with $\nabla_\theta L$
end for

Algorithm 2: SC-Flow sampling (ODE)

Input: prior $x_0 \sim p_0$, condition y, clamp ε
Choose per-step drift rule using equation 11
for t from 0 to 1 with an ODE solver **do**
  $\mathbf{v}_\theta \leftarrow \mathrm{nn}_\theta(x_t, t, y, b{=}0)$
  $\tilde{\mathbf{m}}_\theta \leftarrow \mathrm{nn}_\theta(x_t, t, y, b{=}1)$
  $\tilde{\mathbf{v}}_\theta \leftarrow (\hat{x}_1 - x_t)/\max(1{-}t, \varepsilon)$
  $\dot{x}_t \leftarrow \mathrm{mix}(\mathbf{v}_\theta, \tilde{\mathbf{v}}_\theta, t)$
  Advance x_t one ODE step using $\dot{x}_t$
end for
return $x_{t=1}$

5 Analysis of SC-Flow

In this section, we analyze learning errors from fitting velocity and data endpoints and consider their impact on generation performance.

We have two reasons to prefer X-Flow: the learning target has a low variance, and the optimal velocity stays in a low-dimensional latent space. With our previous notation, $p(\mathbf{x}_0, \mathbf{x}_1)$ is a coupling of $\mathbf{x}_0$ and $\mathbf{x}_1$ that defines a distribution of paths via the linear interpolation $\mathbf{x}_t = (1-t)\mathbf{x}_0 + t\mathbf{x}_1$ for $t \in (0, 1)$. Given $(\mathbf{x}_t, t)$, let $p(\mathbf{x}_0, \mathbf{x}_1 | \mathbf{x}_t, t)$ be the conditional distribution of paths passing through $\mathbf{x}_t$. The conditional velocity is $\mathbf{u} = \frac{\mathbf{x}_1 - \mathbf{x}_t}{1-t} = \mathbf{x}_1 - \mathbf{x}_0$. Then:

$$\mathrm{Var}[\mathbf{x}_1 | \mathbf{x}_t, t] = (1-t)^2 \mathrm{Var}[\mathbf{u} | \mathbf{x}_t, t] \tag{12}$$

We can see that $\mathbf{x}_1$ has a lower variance than $\mathbf{u}$. The finite-sample estimation of the true target $\mathbb{E}[\mathbf{x}_1 | \mathbf{x}_t, t]$ has lower variance than the velocity target $\mathbb{E}[\mathbf{u} | \mathbf{x}_t, t]$.

Li & He (2025) advocates X-Flow and argues that the $\mathbf{x}_1$ is from a low-dimensional manifold and thus is easy to fit. They support the argument with a small learning problem. Here, we consider a formal setting where the observed data $\mathbf{x} \in \mathbb{R}^h$ lie on a low-dimensional manifold. Following (Zhang et al., 2025), we consider a

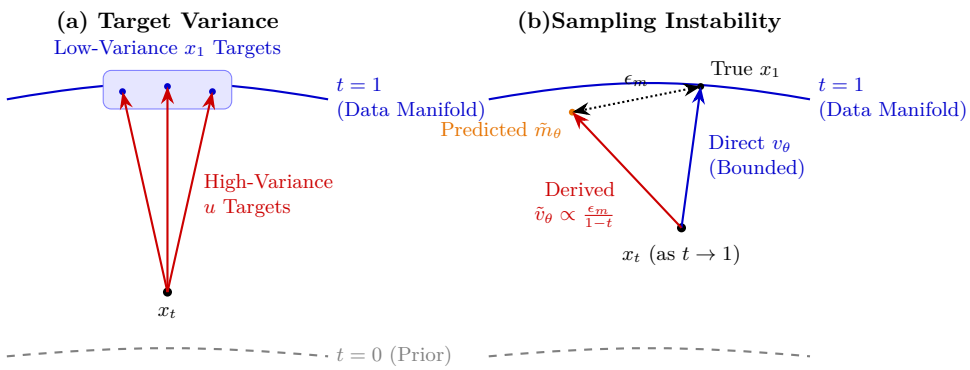


Figure 2: **Conceptual illustration of SC-Flow's theoretical advantages. (a) Training Variance:** For a given intermediate state x_t, possible ground-truth endpoints x_1 form a tight, bounded region on the data manifold. Conversely, the velocity targets u diverge significantly, creating a high-variance target space. Endpoint prediction stabilizes this. **(b) Sampling Instability:** Near the end of the generative process ($t \to 1$), a small endpoint prediction error ϵ_m results in a highly divergent derived velocity $\tilde{v}_\theta$ relative to the true target. SC-Flow solves this by directly predicting a bounded local velocity v_θ.

latent space $\mathbb{R}^l$ with $l < h$ and assume that the latent representation $\mathbf{z} \in \mathbb{R}^l$ can fully reconstruct the data point $\mathbf{x} = Q\mathbf{z} \in \mathbb{R}^h$ through a column-orthonormal matrix $Q \in \mathbb{R}^{h\times l}$ ($Q^\top Q = I_l$). Therefore, the latent space $\mathbb{R}^l$ contains the latent data manifold as a subset.

The vector field on $\mathbf{x}$ entails a vector field on $\mathbf{z}$. Given the conditional flow $\mathbf{x}_t = t\mathbf{x}_1 + (1-t)\mathbf{x}_0$, the projected conditional flow is $\mathbf{z}_t = t\mathbf{z}_1 + (1-t)\mathbf{z}_0$, where $\mathbf{z}_0 = Q^\top \mathbf{x}_0$. If the optimal velocity estimator in the data and latent spaces are respectively $\mathbf{v}_h^*(\mathbf{x}_t) = \mathbb{E}[\mathbf{x}_1 - \mathbf{x}_0|\mathbf{x}_t]$ and $\mathbf{v}_l^*(\mathbf{z}_t) = \mathbb{E}[\mathbf{z}_1 - \mathbf{z}_0|\mathbf{z}_t]$. Because of the noise term $\mathbf{x}_0$ in the high-dimensional space, the optimal velocity $\mathbf{v}_h^*$ is still in the high-dimensional space. In particular, the two vector fields have the following relationship (Zhang et al., 2025). We put the proof in appendix A.

$$\mathbf{v}_h^*(\mathbf{x}_t) = Q\mathbf{v}_l^*(Q^\top \mathbf{x}_t) - \frac{1}{1-t}(I - QQ^\top)\mathbf{x}_t. \tag{13}$$

In contrast, with the assumption of linear construction, the predicting target of X-Flow stays in an l-dimensional subspace. Let $\mathbf{m}_h^*(\mathbf{x}_t) = \mathbb{E}[\mathbf{x}_1|\mathbf{x}_t]$ and $\mathbf{m}_l^*(\mathbf{x}_t) = \mathbb{E}[\mathbf{z}_1|\mathbf{z}_t]$ be the optimal predicting targets of X-Flow, respectively, in the data and latent spaces. We can separate the linear components of $\mathbf{x}_t$ by $\mathbf{x}_t = (1-t)\mathbf{x}_0 + tQ\mathbf{z}_1 = (1-t)(I-QQ^\top)\mathbf{x}_0 + Q\mathbf{z}_t$. Let $\mathbf{s}_t = (1-t)(I-QQ^\top)\mathbf{x}_0$. With the setup of the learning problem: 1) the isotropic Gaussian distribution of $\mathbf{x}_0$ and 2) the independent coupling, $\mathbf{s}_t$ is independent of $\mathbf{z}_0 = Q^\top \mathbf{x}_0$ and $\mathbf{z}_1 = Q^\top \mathbf{x}_1$. Then we have the relationship between the two optimal predicting targets:

$$\mathbf{m}_h^*(\mathbf{x}_t) = \mathbb{E}[Q\mathbf{z}_1|\mathbf{x}_t] = \mathbb{E}[Q\mathbf{z}_1|\mathbf{z}_t, \mathbf{s}_t] = Q\mathbb{E}[\mathbf{z}_1|\mathbf{z}_t] = Q\mathbf{m}_l^*(\mathbf{z}_t). \tag{14}$$

This result indicates that the network in the X-Flow setup can predict targets in a low-dimensional latent space; therefore, it is more likely to reduce the learning error than the network that directly predicts velocity values. For these two reasons, we argue that it is easier to train the neural network in the X-Flow setting.

However, the learning error of $\tilde{\mathbf{m}}_\theta$ will be scaled up by $\frac{1}{1-t}$. With the conditional distribution $p(\mathbf{x}_1, \mathbf{x}_0|\mathbf{x}_t, t)$, let $\delta = \|\mathbf{v}_\theta(\mathbf{x}_t, t) - \mathbb{E}[\mathbf{u}]\|_2^2$ be the learning error of the V-Flow model. Let $\epsilon = \|\tilde{\mathbf{m}}_\theta(\mathbf{x}_t, t) - \mathbb{E}[\mathbf{x}_1]\|_2^2$ be the learning error of the X-Flow model, then the error of the velocity $\tilde{\mathbf{v}}_\theta$ computed from $\tilde{\mathbf{m}}_\theta$ is $\tilde{\delta} = \frac{\epsilon}{(1-t)^2}$, which is scaled up because of the factor $\frac{1}{1-t}$.

At the early stage of the path, when $(1-t)$ is significantly greater than 0, it is hard to argue whether $\mathbf{v}_\theta$ or $\tilde{\mathbf{v}}_\theta$ has smaller error for reasons from both sides. However, the analysis does indicate that X-Flow has an advantage when the data dimension is much higher the compact latent dimension. The previous work by Li & He (2025) studies parameterization methods in the pixel space and shows that some V-Flow networks $\mathbf{v}_\theta$ cannot even converge while the X-Flow model $\tilde{\mathbf{m}}_\theta$ can. The finding is consistent with our theoretical analysis

above. However, at the late stage when $1-t$ is small, and the error ϵ of the X-Flow model is significantly amplified by the factor $\frac{1}{1-t}$, then X-Flow is a worse choice than V-Flow.

Nevertheless, we can compare the learning errors on a validation set by checking their learning objectives. The two errors δ and $\tilde{\delta}$ can be compared by:

$$\delta = \mathbb{E}_{\mathbf{x}_t}[\|(\mathbf{v}_\theta - \mathbb{E}[\mathbf{u}|\mathbf{x}_t])\|_2^2] = \mathbb{E}[\|\mathbf{v}_\theta - \mathbf{u}\|_2^2] - \mathbb{E}_{\mathbf{x}_t}[\mathrm{tr}(\mathrm{Var}(\mathbf{u}))], \tag{15}$$

$$\tilde{\delta} = \mathbb{E}_{\mathbf{x}_t}[\|(\tilde{\mathbf{v}}_\theta - \mathbb{E}[\mathbf{u}|\mathbf{x}_t])\|_2^2] = \mathbb{E}[\|\tilde{\mathbf{v}}_\theta - \mathbf{u}\|_2^2] - \mathbb{E}_{\mathbf{x}_t}[\mathrm{tr}(\mathrm{Var}(\mathbf{u}))]. \tag{16}$$

The second term is a constant and does not affect the comparison. Our experiment later compares learning errors through the learning objectives computed from different learned velocities on the validation set.

6 Experiments

We empirically evaluate SC-Flow on image generation tasks. We first check the performance of SC-Flow and then examine the effect of the proposed training method with a series of ablation studies.

Datasets and evaluation metrics. The experiment is conducted on the two well-known datasets, CIFAR-10 (Krizhevsky, 2009) and ImageNet 256×256 (Deng et al., 2009). The evaluation metrics are Fréchet Inception Distance (FID) (Heusel et al., 2017), sFID (Siarohin et al., 2019), Inception Score (IS) (Salimans et al., 2016), and Precision (Kynkäänniemi et al., 2019). Lower FID/sFID and higher IS and Precision correspond to better performance.

Our primary baseline for performance comparison is Rectified Flow. All methods share the same neural backbone architecture, with the only difference being the additional switch b introduced in SC-Flow. SC-Flow supports three sampling strategies: V-Flow, X-Flow, and a mixed approach defined in equation 11. We denote the three strategies with suffixes -V, -X, and -Mix, respectively.

6.1 Evaluation with CIFAR-10

For our CIFAR-10 experiments, we use a U-Net backbone with an architecture similar to the one used in DDPM++ (Nichol & Dhariwal, 2021), which is a standard for this dataset. Both the baseline and SC-Flow models are trained for 250K iterations. For evaluation, we generate samples using Heun's method with 250 steps to ensure stable ODE integrations.

Table 1: Main results on CIFAR-10 (250K iterations, 250 sampling steps). SC-Flow consistently outperforms the Rectified Flow baseline.

Model	**FID**↓	**sFID**↓	**IS**↑	**Precision**↑
RectifiedFlow	3.12	3.34	9.47	0.72
SC-Flow-V	2.49	2.93	9.61	0.73
SC-Flow-X	**2.41**	2.88	9.64	0.73
SC-Flow-Mix (τ=0.5)	**2.41**	**2.83**	**0.65**	0.73

As shown in Table 1, SC-Flow demonstrates a significant performance improvement over the standard rectified flow baseline on CIFAR-10. The SC-Flow-Mix and SC-Flow-X variants achieve the best FID score of **2.41**, a substantial 22.8% relative improvement over the baseline's 3.12. All SC-Flow sampling modes achieve better performance than the baseline method, indicating that the new training scheme also improves the V-flow $\mathbf{v}_\theta$. This strong performance with a convolutional U-Net architecture (Ronneberger et al., 2015) (as opposed to a Transformer (Vaswani et al., 2017)) highlights the general applicability and benefit of our proposed method.

6.2 Evaluation with ImageNet 256x256

Our experiments on ImageNet use the DiT-L/4 and DiT-XL/2 (Peebles & Xie, 2023) backbones as the foundation for our models. Here L and XL are mode sizes defined in the original paper. The number are patch sizes used by the generative model. We use "-L" and "-XL" to indciate these two settings.

Our standard Rectified Flow baselines are trained following the SiT (Ma et al., 2024). The SC-Flow model employs the same backbone architecture, except for the input of the binary switch b. The L/4 models are trained for 500K iterations and the XL/2 models for 800K iterations, both with a constant learning rate of 1×10^{-4}. For evaluation, all models are sampled using the Dopri5 ODE solver (Dormand & Prince, 1980) with 250 steps.

In this experiment, we include DiT as a baseline. DiT uses the same network backbone but is a diffusion-based generative method. We also train a flow matching model with only X-flow: we train $\tilde{\mathbf{m}}_\theta$ with the objective L_{x} and generate images with the derived X-flow $\tilde{\mathbf{v}}_\theta$. We denote this method as RectifiedFlow-L-X or RectifiedFlow-XL-X. All experiments are conducted with classifier free guidance (cfg) (Ho & Salimans, 2022) during sampling.

Table 2: Performances on ImageNet 256×256. SC-Flow with all three sampling methods has significant improvements over the baseline model.

(a) L/4 (cfg=4.0, 500K training steps, 250 sampling steps)

Model	Training Steps	**FID**↓	**sFID**↓	**IS**↑	**Precision**↑
DiT-L	400K	11.84	12.22	116.50	0.63
SiT-L	400K	11.53	12.06	110.75	0.65
SiT-L-X	400K	11.48	12.02	112.63	0.65
SC-Flow-L-V	400K	9.92	10.04	143.09	**0.68**
SC-Flow-L-X	400K	10.63	11.90	137.86	0.67
SC-Flow-L-Mix (τ=0.5)	400K	**9.85**	**10.02**	**145.61**	**0.68**

(b) XL/2 (cfg=1.5, 800K training steps, 250 sampling steps)

Model	Training Steps	**FID**↓	**sFID**↓	**IS**↑	**Precision**↑
DiT-XL	800K	6.41	6.67	175.01	0.73
DiT-XL	7M	2.27	4.60	278.24	0.83
SiT-XL	800K	6.22	6.65	157.46	0.73
SiT-XL-X	800K	6.19	6.72	155.25	0.74
SiT-XL	7M	2.06	4.60	258.09	0.81
SC-Flow-XL-V	800K	4.21	4.68	184.97	**0.79**
SC-Flow-XL-X	800K	4.28	4.68	183.48	**0.79**
SC-Flow-XL-Mix (τ=0.5)	800K	**4.19**	**4.65**	**185.01**	**0.79**
SC-Flow-XL-Mix (τ=0.5)	4M	**1.86**	**4.21**	**285.93**	**0.85**

Table 2 presents our main results on ImageNet. SC-Flow substantially outperforms the Rectified Flow baselines across all metrics. On the L/4 model, our mixed-inference strategy (SC-Flow-L-Mix) improves the FID score of Rectified Flow from 11.53 to **9.85**, a relative improvement of 14.6%. The gains are even more pronounced on the larger DiT-XL/2 model, where SC-Flow-XL-Mix reduces the FID from 6.22 to **4.19**, a 32.6% relative improvement. Notably, SC-Flow also achieves a significantly higher Inception Score (IS) and Precision, indicating that the generated samples are not only more realistic but also more faithful to the training data. SC-Flow using the other two sampling methods also consistently outperforms the baseline, with the X-flow performing slightly better than the V-flow.

Training Efficiency. To evaluate the impact of dual-target supervision on optimization, we compare the training convergence of SC-Flow against standard SiT and DiT baselines. As illustrated in Figure 3, SC-Flow exhibits a markedly superior learning curve. By supervising both the local velocity and the global destination, the model benefits from the variance reduction properties established in our theoretical analysis. Specifically, SC-Flow reaches an FID of 2.0 in approximately 2M iterations, whereas the SiT baseline requires nearly 7M iterations to achieve comparable quality. This 3.5× acceleration in training efficiency highlights that the consistency loss stabilizes the gradient flow, allowing the network to capture the underlying data manifold more effectively in the early stages of training.

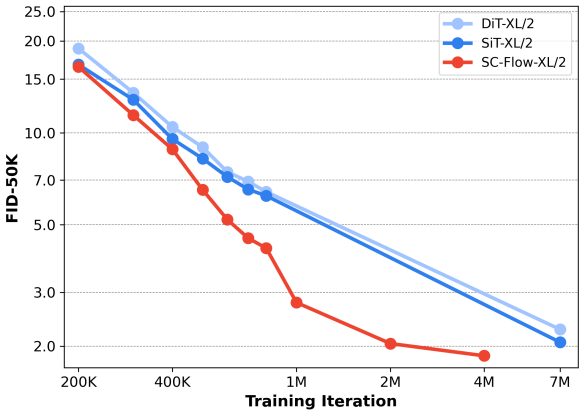
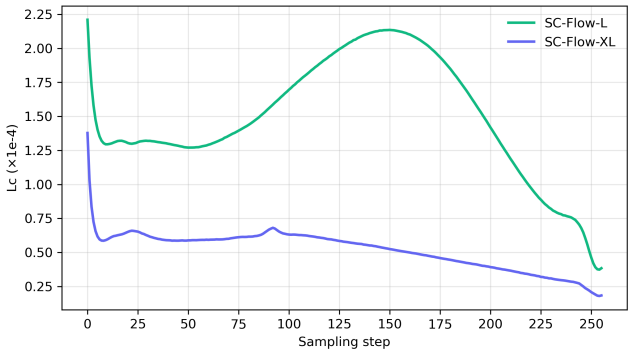


Figure 3: **Training Convergence.** FID-50K vs. training iterations on ImageNet-21k. SC-Flow achieves significantly faster convergence, reaching the performance of the SiT baseline in 3.5× fewer iterations.

6.3 Ablation Studies.

(a) The scale of L_c versus sampling steps.



(b) Measurement of path straightness.

Model	**Max-dev↓**	**Mean-cos↑**
RectifiedFlow-L	0.256	0.828
SC-Flow-L-V	0.250	0.833
SC-Flow-L-X	0.248	0.833

(c) FID versus consistency weight w_c.

w_c	**0.0**	**0.1**	**0.2**	**0.3**
SC-Flow-L-Mix	11.14	9.85	9.86	9.91

Figure 4: Analysis of the effect of the consistency term in SC-Flow. **(a)** The consistency loss L_c measured during a 250-step ODE solver. The gap between $\mathbf{m}_\theta$ and $\tilde{\mathbf{m}}_\theta$ is largest at early steps and converges to zero. **(b)** The evaluation of path straightness shows SC-Flow generates more direct paths than Rectified Flow. **(c)** A sweep over the consistency weight w_c shows that $w_c > 0$ is critical.

To validate our design choices and understand how SC-Flow improves the generation performance, we conduct a series of ablation studies focusing on the role of the consistency loss (L_c).

The X-flow or the V-flow does not work well separately. To isolate the effect of the X-flow training from minimizing $L_{\mathrm{x}}(\theta) = \|\tilde{\mathbf{m}}_\theta(\mathbf{x}_t, t) - \mathbf{x}_1\|_2^2$, we can train a Rectified Flow model by minimizing $L_{\mathrm{x}}(\theta)$ only and then draw samples from $\tilde{\mathbf{v}}_\theta$ derived from $\tilde{\mathbf{m}}_\theta$. As we mentioned before, this method is labeled as RectifiedFlow-X, and its performance is shown in Table 2. Its performance is nearly identical to the performance of the baseline Rectified Flow. Note that the V-flow $\mathbf{v}_\theta$ is trained against a corrupted image $\mathbf{x}_1 - \mathbf{x}_0$, while the X-flow is trained against a clean image $\mathbf{x}_1$. This result demonstrates that the different training targets do not bring a clear performance difference.

The consistency loss is necessary. In Figure 4(c), we vary the the weight w_c of the consistency term $L_{\mathrm{c}}(\theta)$. The best performance is achieved with $w_c \in \{0.1, 0.2\}$. When $w_c = 0$ eliminates the consistency constraint, SC-Flow only has a slight performance improvement over the baseline Rectified Flow (11.14 vs.

11.53 FID). We hypothesize that the improvement is from the network sharing, which is the focus of the next experiment. This result reveals that the consistency loss L_c is essential for unlocking the full potential of our method. Our qualitative study later in Figure 7 also shows that the consistency loss improves the quality of the generated images.

The shared neural architecture brings performance gain. Sharing the same neural architecture clearly saves computation. At the same time, they learn targets that are quite related, so sharing the architecture should also be a reasonable choice for performance consideration. To verify this hypothesis, we train two independent DiT-L models for the V-flow and the X-flow. The training objective is the same as SC-Flow models above. We denote this new setup with the suffix "-2net". The performance of the two models under this setup is shown in Table 3. The performance still improves upon the baseline Flow Matching model. The improvement is solely due to the consistency term because that's the only difference from the baseline. However, their performance cannot match that of SC-Flow models with shared networks, which indicates the performance benefit of learning related targets with a single network backbone.

Table 3: The performance of SC-Flow models using two separate networks for the V-Flow and X-Flow.

Model	**FID**↓	**sFID**↓
SC-Flow-L-V-2net	10.92	12.15
SC-Flow-L-X-2net	10.57	10.77

Consistency helps to overcome randomness in training. As we analyzed in Section 3, the V-flow and the X-flow should be equivalent when there are infinite training examples, and the consistency term would not be useful. We hypothesize that the consistency between the V-flow and the X-flow is very beneficial when the model is trained with limited data and noise. To verify this hypothesis, we increase the amount of random noise examples to 4 times the training samples in *each* training batch, and therefore reduce the random noise in the training process.

We conducted this experiment with the CIFAR-10 dataset. The performance of the trained models is shown in Table 4. The new training strategy improves the performance of both models because it essentially increases the training batches of the both models. However, this new setup shows less performance improvement with SC-Flow than the standard setup: the reduction of FID is (2.57 - 2.28) from the previous reduction (3.12 - 2.41). This provides strong evidence supporting our hypothesis.

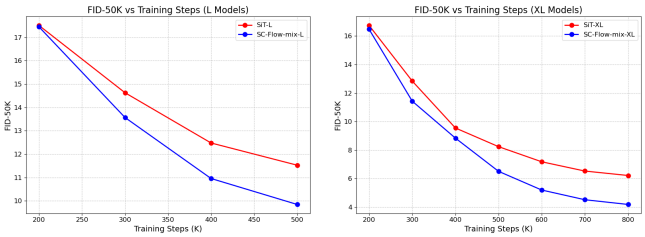
Table 4: Performance of models trained with the 1:4 ratio of training samples and noise samples on CIFAR-10.

Model	**FID**↓	**sFID**↓
RectifiedFlow (1:4)	2.57	2.94
SC-Flow-Mix (1:4)	2.28	2.73

SC-Fow slightly straightens sampling paths We compare SC-Flow and Rectified Flow in terms of the straightness of sampling paths. The straightness is measured by Max-dev (the maximum deviation from the chord connecting the two ends of the path) and Mean-cos (average cosine similarity between each step and the chord). Lower Max-dev or higher Mean-cos indicates more straight paths. The results are shown in Figure 4(b). SC-Flow achieves slightly straighter paths. One possible explanation is that the X-flow aims to predict the destination and thus has a better chance to learn straight paths. We also plot the consistency loss L_c in Figure 4(a) at each step of the ODE path. For both L and XL models, the difference is very small (at the scale of 1e-4), although the two models share a similar pattern. The inconsistency is highest near $t = 0$ where the path is most uncertain, and converges towards zero as $t \to 1$.

Computation efficiency. Figure 5 demonstrates that SC-Flow's significant performance gains are achieved with high efficiency. The plots on the left show the FID-50K score as a function of training steps for both L and XL models. At every evaluation checkpoint, SC-Flow maintains a consistent and substantial advantage over the standard Rectified Flow baseline, indicating not only a better final result but also faster convergence. The table on the right quantifies the computational cost. The training time per epoch for SC-Flow is only marginally higher than the baseline. This is due to our efficient single-head design, which requires only one additional forward pass through the shared network backbone to compute both targets. Crucially, the sampling time per step is nearly identical, as both methods require just one model evaluation per ODE step. Taken together, these results confirm that SC-Flow is a "plug-and-play" upgrade, offering superior

(a) FID-50K vs. Training Steps. Left: L Models. Right: XL Models.



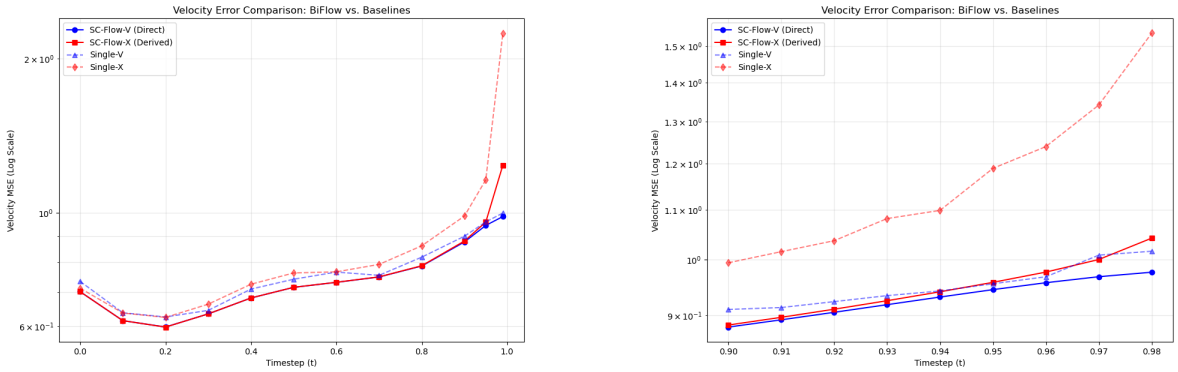
(b) Training and Sampling Time Comparison (seconds) on ImageNet

Model	Tr./epoch	Spl./step
SiT-L	0.80	0.09
SC-Flow-V-L	1.01	0.10
SiT-XL	0.78	0.51
SC-Flow-V-XL	0.81	0.56

Figure 5: Efficiency and Training Dynamics. **(a)** SC-Flow has lower FID score than the baseline at every checkpoint. **(b)** The training overhead is negligible and sampling cost is identical.

sample quality and faster training convergence with a negligible increase in training cost and no change to the sampling budget.

Stability during inference. To empirically validate the asymptotic behavior described in Theorem 3, we analyze the precision of the learned vector fields near the data manifold. This analysis demonstrates how different supervision signals affect the numerical stability of the ODE solver.



(a) Full Range $t \in [0, 1]$

(b) Endpoint Region $t \in [0.9, 1.0]$

Figure 6: **Velocity Error Comparison.** We compare the Mean Squared Error (MSE) of velocity predictions against ground truth for SC-Flow and single-target baselines. As predicted by **Theorem 3**, the derived velocity from endpoint predictors (red lines) spikes exponentially as $t \to 1$ due to the $(1-t)^{-1}$ error magnification. SC-Flow (blue solid) maintains the lowest overall error, demonstrating how joint supervision and consistency regularize the vector field against asymptotic instability. The right panel provides a focused view of this behavior near the data manifold.

We evaluate the velocity prediction error using 2,000 images randomly sampled from the ImageNet validation set. From these 2,000 validation samples, we compute the learning objective $\|\mathbf{v}_\theta(\mathbf{x}_t, t) - (\mathbf{x}_1 - \mathbf{x}_0)\|_0^2$ with $\mathbf{v}$-s derived from different models: $\mathbf{v}_\theta$ from SC-Flow with $b = 0$ (SC-Flow-V), $\tilde{\mathbf{v}}_\theta$ derived from SC-Flow with $b = 1$ (SC-Flow-X), $\mathbf{v}_\theta$ from the baseline trained with velocity (Single-V), and $\tilde{\mathbf{v}}_\theta$ derived from the baseline trained with endpoints (Single-X). As shown in Figure 6, the empirical results show that X-Flow and V-Flow are not very distinguishable at the first stage of the path. When $t > 0.8$, the error of X-Flow begins to increase quickly because of the amplification effect in our discussion. SC-Flow clearly decreases the learning error throughout all time t. Even for SC-Flow, X-Flow still tends to have a large error at the end of the sampling stage. Our mixed sampling strategy avoids such error by using V-Flow in the latter part of the sampling procedure.

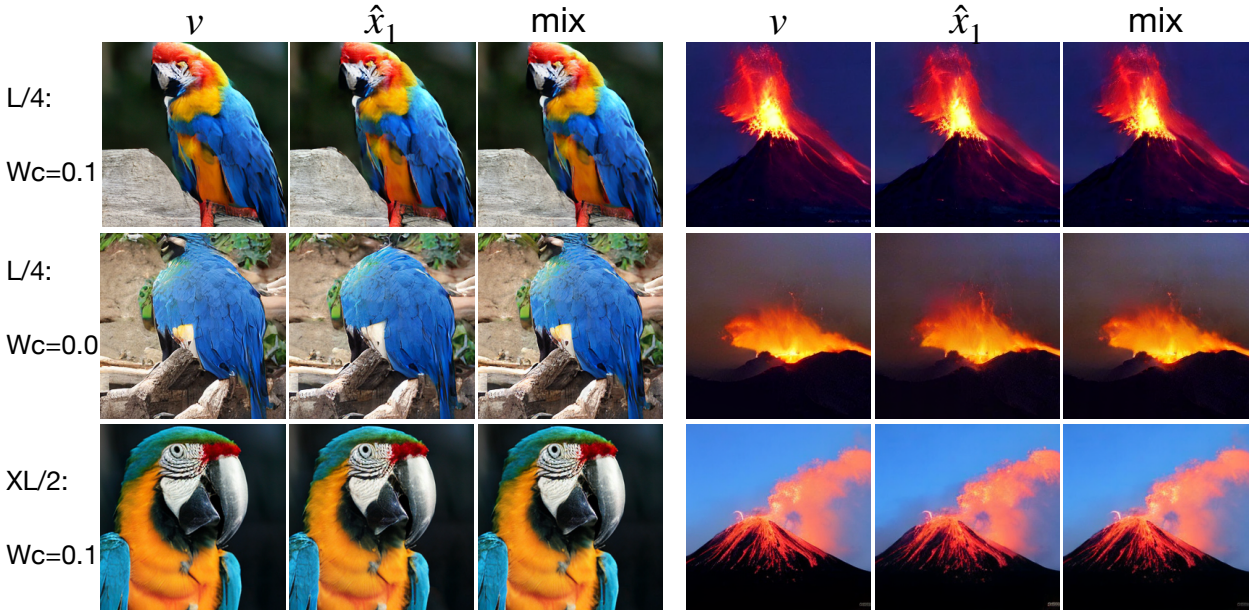


Figure 7: **Qualitative Comparison on ImageNet** 256×256**.** We compare samples from the L/4 and XL/2 SC-Flow models using V-Flow $\mathbf{v}_\theta$, X-Flow $\tilde{\mathbf{v}}_\theta$, and mixed inference. The bottom row (XL/2) shows higher fidelity than the L/4 model. The middle row ablates our consistency loss ($w_c = 0.0$), revealing a clear visual divergence between the velocity and endpoint predictions. The top row ($w_c = 0.1$) demonstrates that our proposed consistency loss forces the two predictions into alignment, producing more coherent and higher-quality samples.

Qualitative Analysis. Figure 7 provides a qualitative analysis of SC-Flow. As expected, the larger XL/2 model (bottom row) generates samples with noticeably higher fidelity and finer detail than the L/4 model (top row), consistent with the quantitative metrics. The critical comparison is between the model trained with our consistency loss ($w_c = 0.1$, top row) and the ablation without it ($w_c = 0.0$, middle row). When $w_c = 0.0$, the model's two predictions are unconstrained and learn inconsistent vector fields. This is visually apparent: the samples generated respectively from the V-flow and the X-flow exhibit clear discrepancies in texture, lighting, and fine details. For example, the parrot's perch has noticeable artifacts in the V-flow sample that are absent in the X-flow. By introducing the consistency loss, these two predictions are forced into agreement. The resulting images from both flows become nearly identical and have reduced artifacts. This visually confirms the significance of the consistent loss, which enforces the entire model to learn a coherent generation flow.

7 Conclusion

In this work, we theoretically and empirically investigate the distinct behaviors of different prediction targets within the rectified flow framework. We reveal a fundamental trade-off: predicting the data endpoint provides a low-variance, on-manifold signal that stabilizes training, while predicting the instantaneous velocity ensures bounded integration error for stable sampling near the data manifold. To unify these complementary strengths, we introduced **SC-Flow** (Self-Consistent Flow). By employing a lightweight algebraic consistency penalty, SC-Flow trains a single network with a binary control bit to concurrently predict both the local velocity and the global destination. At inference, this dual-target parameterization enables a zero-overhead switching policy that seamlessly transitions from X-Flow to V-Flow. Extensive experiments on CIFAR-10 and ImageNet 256×256 demonstrate that explicitly coupling local motion with global awareness significantly stabilizes optimization, straightens generation paths, and improves overall sample quality. Ultimately, SC-Flow offers a principled, plug-and-play approach to building stronger continuous-time generative models without requiring architectural upheaval or additional sampling compute.

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A Appendix

Appendix A.1 Proof

Proof. Let $P = QQ^\top$ denote the orthogonal projection matrix onto the data manifold $\mathcal{M}$, and $P^\perp = I - QQ^\top$ denote the projection onto its orthogonal complement. We can decompose the optimal ambient velocity target into its intrinsic and orthogonal components:

$$\mathbf{x}_1 - \mathbf{x}_0 = P(\mathbf{x}_1 - \mathbf{x}_0) + P^\perp(\mathbf{x}_1 - \mathbf{x}_0). \tag{17}$$

First, we analyze the orthogonal component. Because the data $\mathbf{x}_1 = Q\mathbf{z}_1$ resides strictly on the manifold, it has no orthogonal component ($P^\perp \mathbf{x}_1 = 0$). Applying $P^\perp$ to both sides of the flow trajectory $\mathbf{x}_t = t\mathbf{x}_1 + (1-t)\mathbf{x}_0$ yields:

$$P^\perp \mathbf{x}_t = (1-t)P^\perp \mathbf{x}_0 \quad \Longrightarrow \quad P^\perp \mathbf{x}_0 = \frac{1}{1-t} P^\perp \mathbf{x}_t. \tag{18}$$

Consequently, the orthogonal component of the velocity target is deterministic given $\mathbf{x}_t$:

$$\mathbb{E}[P^\perp(\mathbf{x}_1 - \mathbf{x}_0)|\mathbf{x}_t] = -\frac{1}{1-t} P^\perp \mathbf{x}_t = -\frac{1}{1-t}(I - QQ^\top)\mathbf{x}_t. \tag{19}$$

Next, we evaluate the intrinsic manifold component. Using $\mathbf{x}_1 = Q\mathbf{z}_1$ and the definition of the projected noise $\mathbf{z}_0 = Q^\top \mathbf{x}_0$, we have:

$$P(\mathbf{x}_1 - \mathbf{x}_0) = Q(Q^\top \mathbf{x}_1 - Q^\top \mathbf{x}_0) = Q(\mathbf{z}_1 - \mathbf{z}_0). \tag{20}$$

Assuming the ambient noise $\mathbf{x}_0$ is isotropic Gaussian, its orthogonal projection $P^\perp \mathbf{x}_0$ and intrinsic projection $Q^\top \mathbf{x}_0$ are statistically independent. Therefore, conditioning on the full ambient state $\mathbf{x}_t$ for the intrinsic variables is mathematically equivalent to conditioning strictly on the projected intrinsic state $\mathbf{z}_t = Q^\top \mathbf{x}_t$. Taking the conditional expectation yields:

$$\mathbb{E}[P(\mathbf{x}_1 - \mathbf{x}_0)|\mathbf{x}_t] = Q\mathbb{E}[\mathbf{z}_1 - \mathbf{z}_0|\mathbf{z}_t] = Q\mathbf{v}_l^*(\mathbf{z}_t). \tag{21}$$

Finally, by the linearity of expectation, summing the intrinsic and orthogonal conditional expectations completes the proof:

$$\begin{aligned} \mathbf{v}_h^*(\mathbf{x}_t) &= \mathbb{E}[P(\mathbf{x}_1 - \mathbf{x}_0)|\mathbf{x}_t] + \mathbb{E}[P^\perp(\mathbf{x}_1 - \mathbf{x}_0)|\mathbf{x}_t] \\ &= Q\mathbf{v}_l^*(Q^\top \mathbf{x}_t) - \frac{1}{1-t}(I - QQ^\top)\mathbf{x}_t. \end{aligned} \tag{22}$$

□

Appendix A.2 Limitations

First, our training objective assumes a linear rectified path and ODE-based sampling. Extending it to other types of conditional paths would require some careful analysis. Second, SC-Flow needs to be validated at generation tasks with high-resolution images and other modalities (e.g., text-to-image, video). Third, SC-Flow slightly increases training time.

Appendix A.3 ImageNet Experiment Details

This section documents the full training and evaluation setup used for ImageNet 256×256 with DiT-L/4 and DiT-XL/2 backbones. Unless stated, SC-Flow and the Rectified-Flow baselines share identical settings.

Data and Latent Encoding

Dataset and preprocessing. We train on ImageNet-1K (train split) with class-conditional labels. Images are center-cropped and resized to 256×256, followed by random horizontal flip and normalization to $[-1, 1]$.

Latent space (VAE). All models operate in the latent space of a frozen, pretrained VAE encoder (no finetuning). We use `diffusers AutoencoderKL.from_pretrained("stabilityai/sd-vae-ft-ema")`. Latents are scaled by the standard factor 0.18215 (as in Stable Diffusion).

Table 5: Latent-space and encoder specifics.

Item	Encoder	Trainable	Image Size	Downsample	Latent Size	Latent Channels
VAE	SD-VAE-FT-{ema|mse}	No (frozen)	256×256	×8	32×32	4

Model, Conditioning, and Mode Bit

Backbones. We use DiT-L/4 and DiT-XL/2 transformers (patch sizes 4 and 2 respectively). Exact widths/depths follow DiT; we summarize placeholders below for completeness.

Table 6: Backbone summary (filled from the provided model definitions).

Model	Input Size	In Ch.	Patch	Tokens	Depth	Hidden	Heads	PosEnc
DiT-L/4	32	4	4×4	8×8=64	24	1024	16	2D sin/cos (frozen)
DiT-XL/2	32	4	2×2	16×16=256	28	1152	16	2D sin/cos (frozen)

Conditioning streams and SC-Flow mode. We sum three embeddings into the adaLN-Zero conditioning vector at every block:

$$e(t, y, b) \;=\; E_t(t) \;+\; E_y(y) \;+\; E_b(b).$$

Time uses a sinusoidal projection (`frequency_embedding_size`= 256) followed by an MLP to the hidden size. Class labels use an embedding table of size `num_classes + 1` when classifier-free guidance (CFG) dropout is enabled. The SC-Flow *mode bit* $b \in \{0, 1\}$ selects which prediction the single head should output (local motion when b=0, global endpoint when b=1).

Table 7: Embedding dimensions and fusion.

Stream	Source	Dim	Notes	Fusion
Time	sinusoidal → MLP	d_{hid}	freq emb size 256	
Class	embedding table	d_{hid}	(`num_classes`+1) if CFG	$E_t + E_y + E_b$
Mode bit b	embedding table (2 entries)	d_{hid}	b=0: local motion; b=1: endpoint	

Optimization and Training Schedule

We train with AdamW (constant learning rate 1×10^{-4}, no weight decay) and exponential moving average (EMA) of weights. TensorFloat-32 (TF32) is enabled on A100/H200.

Sampling and Evaluation

We integrate the probability-flow ODE using Dopri5 with a fixed budget of 250 steps. For SC-Flow we keep compute parity by evaluating only the active branch per step (one forward pass per step). We report FID-50K, sFID, IS, and Precision.

Table 8: Optimizer, precision, and regularization.

Setting	Optimizer	LR	Weight Decay	Betas	Eps	EMA Decay	AMP
All	AdamW	1×10^{-4}	0	(0.9, 0.999)	1×10^{-8}	0.9999	`False`

Table 9: Sampling configuration and guidance.

Model	Solver	Steps	Switch τ	CFG Scale
DiT-L/4	Dopri5	250	0.5	4.0
DiT-XL/2	Dopri5	250	0.5	1.5

Path Straightness Metrics: `mean_cos` and `max-dev`

Setup and notation. For a batch of discrete sampling paths $\{x_k^{(b)}\}_{k=0}^{S}$ with $b = 1, \dots, B$ and $x_k^{(b)} \in \mathbb{R}^D$ (flattened images), define the *chord*

$$c^{(b)} \;=\; x_S^{(b)} - x_0^{(b)}, \qquad u^{(b)} \;=\; \frac{c^{(b)}}{\|c^{(b)}\|} \;\text{(unit chord)}. \tag{23}$$

Let the *step vector* be

$$s_k^{(b)} \;=\; x_{k+1}^{(b)} - x_k^{(b)}, \qquad k = 0, \dots, S-1. \tag{24}$$

Mean cosine to the chord (`mean_cos`). The per–time-step cosine between the step and the chord direction is

$$\gamma_k^{(b)} \;=\; \frac{\langle s_k^{(b)},\, u^{(b)}\rangle}{\|s_k^{(b)}\|} \;=\; \frac{\big\langle x_{k+1}^{(b)} - x_k^{(b)},\, \frac{x_S^{(b)} - x_0^{(b)}}{\|x_S^{(b)} - x_0^{(b)}\|}\big\rangle}{\|x_{k+1}^{(b)} - x_k^{(b)}\|}. \tag{25}$$

We average over steps and the batch:

$$\texttt{mean_cos} \;=\; \frac{1}{BS}\sum_{b=1}^{B}\sum_{k=0}^{S-1}\gamma_k^{(b)}. \tag{26}$$

Values closer to 1 indicate that steps remain well aligned with the global chord (straighter paths).

Maximum lateral deviation (`max-dev`). For each intermediate point, decompose $x_k^{(b)}$ into chord-parallel progress and orthogonal residual. Define the signed progress along the chord,

$$d_k^{(b)} \;=\; \big\langle x_k^{(b)} - x_0^{(b)},\, u^{(b)}\big\rangle, \tag{27}$$

the orthogonal projection onto the chord,

$$p_k^{(b)} \;=\; x_0^{(b)} + d_k^{(b)}\, u^{(b)}, \tag{28}$$

and the residual (lateral) displacement

$$r_k^{(b)} \;=\; x_k^{(b)} - p_k^{(b)}, \qquad \delta_k^{(b)} \;=\; \|r_k^{(b)}\|. \tag{29}$$

The per-path maximum lateral deviation (normalized by chord length) is

$$\texttt{max-dev}^{(b)} \;=\; \frac{1}{\|c^{(b)}\|}\,\max_{0\le k\le S}\,\delta_k^{(b)}. \tag{30}$$

We report the batch average:

$$\texttt{max-dev} \;=\; \frac{1}{B}\sum_{b=1}^{B}\texttt{max-dev}^{(b)}. \tag{31}$$

Smaller values indicate paths that hug the straight line more tightly (fewer detours).

Implementation notes. All vectors $x_k^{(b)}$ are flattened before inner products; norms are Euclidean. For numerical stability, denominators are clamped, e.g. $\|c^{(b)}\| \leftarrow \max(\|c^{(b)}\|, \varepsilon)$ and $\|s_k^{(b)}\| \leftarrow \max(\|s_k^{(b)}\|, \varepsilon)$ with $\varepsilon \approx 10^{-8}$.

Appendix A.3 Pseudo-code of SC-Flow

Algorithm 1 SC-Flow Training: PyTorch-like Pseudo-code

```
class SC-FlowTrainer(nn.Module):
    def __init__(self, model, vae, transport, lr=1e-4, ema_decay=0.9999):
        super().__init__()
        self.net = model # single-head, b in {0,1}
        self.vae = vae.eval() # frozen encoder
        self.transport = transport # provides path plan, time sampling
        self.opt = torch.optim.AdamW(self.net.parameters(), lr=lr, weight_decay=0.0)
        self.ema = copy.deepcopy(self.net).eval()
        self.ema_decay = ema_decay
        torch.backends.cuda.matmul.allow_tf32 = True
        torch.backends.cudnn.allow_tf32 = True

    @torch.no_grad()
    def _update_ema(self):
        for p_ema, p in zip(self.ema.parameters(), self.net.parameters()):
            p_ema.mul_(self.ema_decay).add_(p, alpha=1.0 - self.ema_decay)

    def training_step(self, x_img, y, w_v=1.0, w_x=1.0, w_c=0.1, time_weight=False):
        with torch.no_grad():
            z = self.vae.encode(x_img).latent_dist.sample().mul_(0.18215) # (B,4,32,32)

        # sample t, x0, x1, and path (xt, ut) from transport
        t, x0, x1 = self.transport.sample(z) # shapes match z
        t, xt, ut = self.transport.path_sampler.plan(t, x0, x1)

        b0 = torch.zeros(xt.size(0), device=xt.device, dtype=torch.long) # velocity branch
        b1 = torch.ones (xt.size(0), device=xt.device, dtype=torch.long) # endpoint branch
        v_pred = self.net(xt, t, y=y, b=b0) # local motion
        x1_pred = self.net(xt, t, y=y, b=b1) # global endpoint

        L_v = ((v_pred - ut) ** 2).mean(dim=(1,2,3)).mean()
        if time_weight:
            tw = 0.5 + t.view(-1,1,1,1)
            L_x1 = (tw * (x1_pred - x1) ** 2).mean(dim=(1,2,3)).mean()
        else:
            L_x1 = ((x1_pred - x1) ** 2).mean(dim=(1,2,3)).mean()

        t_exp = t.view(-1,1,1,1)
        x1_from_v = xt + (1 - t_exp) * v_pred
        L_c = ((x1_pred - x1_from_v) ** 2).mean(dim=(1,2,3)).mean()

        loss = w_v * L_v + w_x * L_x1 + w_c * L_c
        return loss, {'L_v': L_v, 'L_x1': L_x1, 'L_c': L_c}

    def train_loop(self, loader, epochs, log_every=100):
        self.net.train()
        for ep in range(epochs):
            for i, (x_img, y) in enumerate(loader):
                x_img, y = x_img.cuda(non_blocking=True), y.cuda(non_blocking=True)
                loss, logs = self.training_step(x_img, y)
                self.opt.zero_grad(set_to_none=True)
                loss.backward()
                self.opt.step()
                self._update_ema()
                if (i + 1) % log_every == 0:
                    _ = (loss.item(), logs['L_v'].item(), logs['L_x1'].item(), logs['L_c'].item())
```

Algorithm 2 SC-Flow-mix Sampling: PyTorch-like Pseudo-code

```
class SC-FlowSampler:
    def __init__(self, model, vae, transport, cfg_scale=1.0, tau=0.5):
        self.net = model.eval()
        self.vae = vae.eval() # frozen decoder (optional for image output)
        self.transport = transport
        self.cfg_scale = cfg_scale
        self.tau = tau # switch between endpoint-induced vs direct motion

    @torch.no_grad()
    def _drift(self, x, t, y):
        if t[0] <= self.tau:
            b = torch.ones(x.size(0), device=x.device, dtype=torch.long) # endpoint branch
            x1_pred = self.net(x, t, y=y, b=b)
            t_safe = torch.clamp(1 - t, min=1e-3)
            v = (x1_pred - x) / path.expand_t_like_x(t_safe, x) # endpoint-induced motion
        else:
            b = torch.zeros(x.size(0), device=x.device, dtype=torch.long) # velocity branch
            v = self.net(x, t, y=y, b=b)
        return v

    @torch.no_grad()
    def _drift_cfg(self, x, t, y):
        if self.cfg_scale <= 1.0:
            return self._drift(x, t, y)
        half = x[: len(x)//2]
        xin = torch.cat([half, half], dim=0)
        vin = self._drift(xin, t, y)
        eps, rest = vin[:, :3], vin[:, 3:]
        cond, uncond = torch.chunk(eps, 2, dim=0)
        guided = torch.cat([uncond + self.cfg_scale * (cond - uncond)]*2, dim=0)
        return torch.cat([guided, rest], dim=1)

    @torch.no_grad()
    def sample(self, B, y, steps=250, method='euler'):
        z = torch.randn(B, 4, 32, 32, device=y.device)
        t0, t1 = self.transport.check_interval(self.transport.train_eps, self.transport.sample_eps, sde=
             False, eval=True)
        if method == 'euler':
            dt = (t1 - t0) / steps
            for k in range(steps):
                t_scalar = t0 + k * dt
                t = torch.full((B,), t_scalar, device=z.device)
                v = self._drift_cfg(z, t, y)
                z = z + v * dt
        else:
            solver = ODESolver(self._drift_cfg, t0=t0, t1=t1, steps=steps) # placeholder
            z = solver.integrate(z, y)
        x = self.vae.decode(z / 0.18215) # optional decode
        return x, z
```

Appendix A.4 Additional Visual Results for CIFAR-10

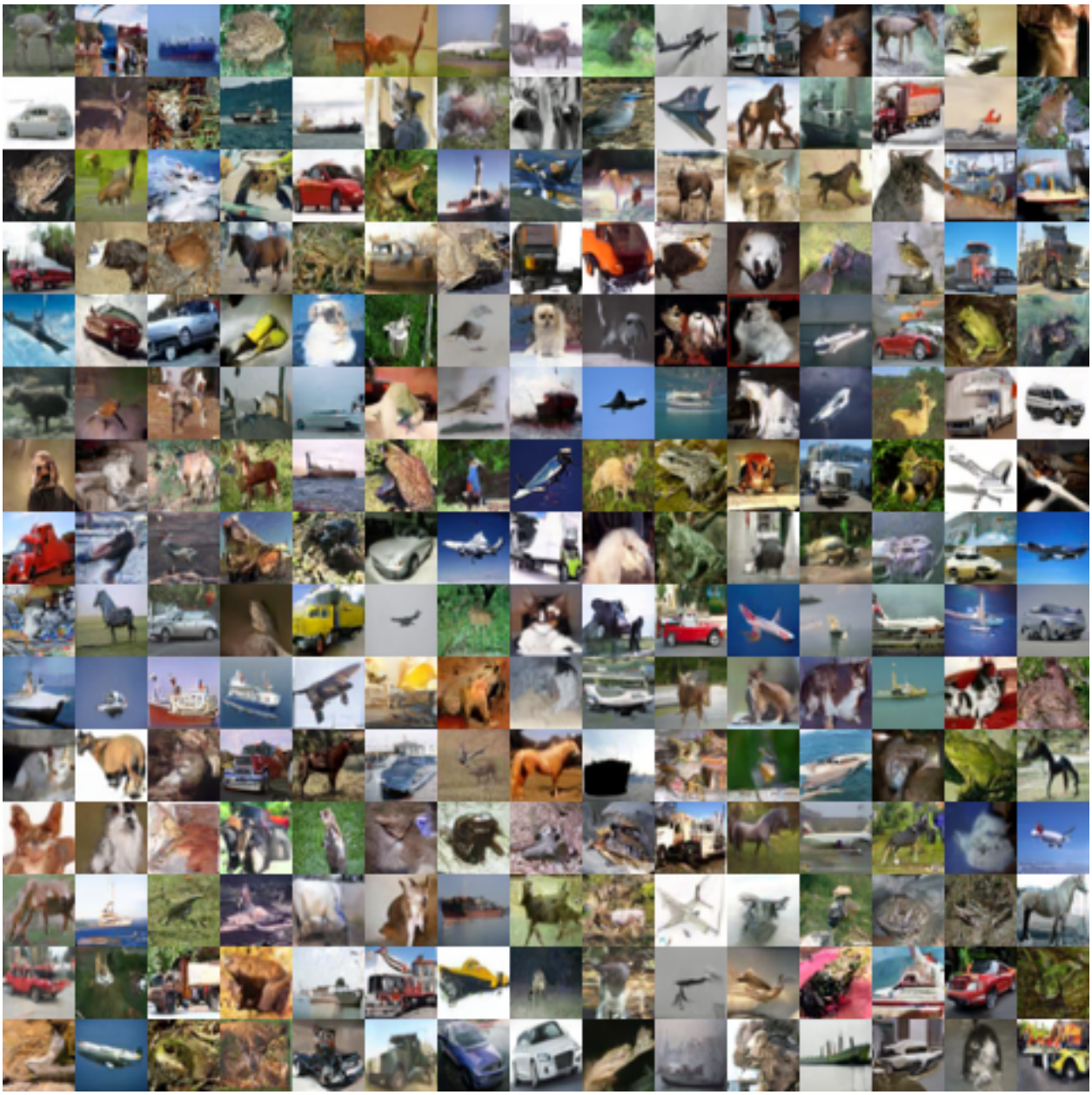

Figure 8: Samples for CIFAR-10.

Appendix A.5 Additional Visual Results for Imagenet

All samples are from SC-Flow-mix-XL, with cfg=4.0.

Figure 9: Samples for ImageNet class 88 (macaw).

Figure 10: Samples for ImageNet class 207 (golden retriever).

Figure 11: Samples for ImageNet class 980 (volcano).

Figure 12: Samples for ImageNet class 279 (arctic fox).

Figure 13: Samples for ImageNet class 33 (loggerhead turtle).

Figure 14: Samples for ImageNet class 417 (balloon).

Figure 15: Samples for ImageNet class 387 (red panda).

Figure 16: Samples for ImageNet class 974 (geyser).