Skill-Targeted Adaptive Training

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Abstract

Language models often show little to no improvement (i.e., "saturation") when trained via vanilla supervised fine-tuning (SFT) on data similar to what they saw in their training set (e.g., MATH). We introduce a new fine-tuning strategy, STAT, to train such a student model by using the metacognition ability of a stronger large language model (LLM) as the teacher. The teacher uses the task dataset to create a list of skills needed for the task, and then labels each data point with its required skills [Didolkar et al., 2024]. By monitoring the student's answers, the teacher creates a `Missing-Skill-Profile` for the student, tracking how often they failed to apply each skill in their responses. We use this idea to build a modified training set in one of two ways. In *STAT-Sel*, the teacher uses an existing set of training examples but adaptively reweights them according to the `Missing-Skill-Profile`. In *STAT-Syn*, the teacher synthesizes additional examples involving missing skills. Across extensive experiments on Llama and Qwen models, our methods yield improvements of up to 7.5% on MATH, whereas SFT provides only limited gains. Furthermore, STAT enhances performance on out-of-distribution benchmarks (e.g., AIME24/25, AMC23, etc.) by an average of 4.6%. Crucially, we find that STAT is complementary to RL via GRPO [Shao et al., 2024]: after the model is improved using STAT to address skill gaps, GRPO continues to add further gains. We conclude that skill-targeted adaptive training should broadly improve current training pipelines.

1 Introduction

Language models trained with next-token prediction and fine-tuning achieve strong results on various tasks. However, this process is often inefficient and data hungry [Kaplan et al., 2020, Muennighoff et al., 2023, Zhang et al., 2024, Villalobos et al., 2024], with models quickly reaching a *saturation point* for a fixed dataset whereby further training does not help performance. Existing strategies to tackle this saturation aim at adapting the training data distribution using gradient or embedding features to target good validation loss [Xia et al., 2024, Yu et al., 2024b]. However, this strategy can fail when validation loss is coarse enough to capture generation mistakes of already saturated models. We propose to address the saturation problem by drawing inspiration from pedagogical practices rooted in cognitive science, which customize training to specially target the student's skill-deficiencies [Bandura and Walters, 1977, Hattie and Timperley, 2007].

> *How can we effectively use today's strong teacher models to design better training strategies to help small models overcome their saturation?*

We use growing line of research in LLM meta-cognition [Didolkar et al., 2024, Kaur et al., 2024], which leverages the predictive abilities of frontier LLMs to reason about the high-level skills required to solve a given task, as well as the skills actually being used in the student's answer. Thus, in principle,

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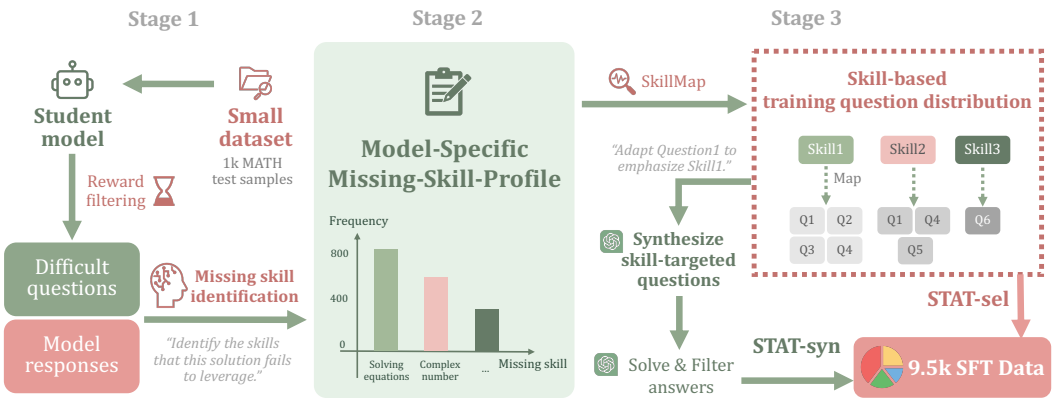

Figure 1: STAT is a three-stage skill-targeted data creation method for supervised fine-tuning (SFT). **Stage 1:** Identify difficult questions for each model using reward filtering on model responses. **Stage 2:** Use frontier LLMs to analyze the model responses and build a model-specific Missing-Skill-Profile. **Stage 3:** Use a pre-constructed SkillMap to map the missing skill distribution to a training question distribution, which constitutes the STAT-Sel data. STAT-Syn synthesizes new questions using frontier LLMs targeted to the missing skills.

frontier LLMs can act as the teacher who guides the training process of the student model, actively monitoring the student's competence on individual skills and adjusting their training examples.

Our three-stage pipeline profiles missing skills in difficult questions for an SLM using a frontier LLM, and creates targeted training dataset for the missing skills. On Llama-Instruct and Qwen models with MATH, both methods can achieve upto 7.5% gains over naive fine-tuning, with benefits extending out-of-distribution. Our proposed methods can further supplement performance with reinforcement-learning (RL). Extensive ablations show that models struggle with basic computational skills and our proposed methods are effective at addressing them.

2 Designing STAT

Let $\mathcal{Q}$ be the set of test set questions, out of which we use a subset $\mathcal{Q}^{val}$ as validation data and $\mathcal{Q}^{test} = \mathcal{Q} \setminus \mathcal{Q}^{val}$ as evaluation data. We also have access to a set of training questions $\mathcal{P}$. We aim to build a targeted training dataset $\mathcal{P}_{targeted}$ to train the model further. We use skill-aware strategies from [Didolkar et al., 2024], which we describe in this section. The strategy develops a set of skills $\mathcal{S}$ that are necessary to solve the questions in the task, enlisted from a large model like GPT-4o. Then, we define SkillMap $: \mathcal{S} \rightarrow \mathcal{P}$ to be a map between a skill and the set of questions from training dataset that require the skill to be solved, which we will also get by prompting the same LLM.

Stage 1: Detection of *difficult* questions via reward filtering. From $\mathcal{Q}^{val}$, we identify a model-specific set of difficult questions $\mathcal{Q}^{val}_{\text{difficult}}$ via a carefully designed reward-filtering method. Accordingly, we create $\mathcal{Q}^{test}_{\text{difficult}}$ from $\mathcal{Q}^{test}$. We only utilize $\mathcal{Q}^{val}_{\text{difficult}}$ in our data creation pipeline, while leaving $\mathcal{Q}^{test}_{\text{difficult}}$ exclusive for evaluation. More details are in Section B.

Stage 2: Constructing model-specific Missing-Skill-Profile. For each *difficult* question q in $\mathcal{Q}^{val}_{\text{difficult}}$, we use a frontier LLM (GPT-4o-mini) to predict the set of skills in $\mathcal{S}$ that are missing in the model's response. We pool all the missing skills to get a Missing-Skill-Profile. Each skill is repeated by the number of *difficult* questions that are linked with this skill as missing by the frontier LLM. We give all details of the prompt in Section D.

Stage 3: Selecting or synthesizing skill-based training data. We create $\mathcal{P}_{targeted}$ in this stage. For STAT-Sel, we use the training dataset $\mathcal{P}$ itself to subsample questions that are annotated with skills in the Missing-Skill-Profile. We loop through all skills from Missing-Skill-Profile and sample multiple questions from $\mathcal{P}$ that are mapped to that particular skill by referring to our SkillMap.

For STAT-Syn, we generate new synthetic questions using a frontier LLM. We loop through all skills from Missing-Skill-Profile. For each selected skill, we provide 3 in-context examples randomly sampled from $\mathcal{P}$ that have been mapped to the sampled skill in SkillMap, and ask the frontier LLM

Methods	MATH	MATH^{D}	MATH^2	GSM8K	AMC23	MATH-perturb		AIME		Avg.
						simple	hard	2024	2025	
Llama-3.2-3B-Instruct + SFT										
Base Model	44.0	18.2	21.9	73.0	21.7	33.7	12.2	33.3	16.7	30.5
MATH-Train	44.8	22.9	21.0	75.1	20.8	33.0	12.2	30.0	20.0	31.1
MATH-Augment	45.2	23.9	23.8	77.8	23.8	35.1	12.5	30.0	13.3	31.7
MATH-Hard	45.6	24.9	23.3	78.2	21.6	38.0	11.8	30.0	26.7	33.3
Embed-Sel	46.0	26.5	20.5	76.6	21.6	36.2	**14.7**	36.7	16.7	32.8
Embed-Syn	48.8	27.3	19.5	78.4	22.7	36.9	13.3	26.7	23.3	33.0
STAT-Sel	**51.5**	26.6	25.7	**80.2**	**24.7**	**39.8**	13.3	**43.3**	23.3	36.5
STAT-Syn	50.2	**31.7**	**26.2**	79.2	23.9	39.1	**14.7**	40.0	**30.0**	**37.2**
+ GRPO										
Base Model	45.4	24.4	23.3	77.4	25.8	38.4	11.8	33.3	3.3	31.8
MATH-Train	46.4	28.4	28.6	80.7	29.7	37.6	12.5	36.7	10.0	34.5
MATH-Augment	47.4	31.6	28.6	81.4	30.6	37.6	14.0	36.7	**33.3**	37.9
MATH-Hard	49.4	33.2	28.6	80.3	31.3	39.1	15.4	43.3	13.3	37.1
Embed-Sel	50.4	37.5	23.8	80.5	32.0	38.0	16.8	36.7	20.0	38.8
Embed-Syn	49.7	37.8	19.5	80.6	33.9	39.1	16.8	36.7	23.3	38.6
STAT-Sel	**52.2**	35.0	**32.4**	81.8	**34.2**	42.7	**17.6**	43.3	26.7	40.7
STAT-Syn	51.0	**39.1**	29.0	**82.0**	31.9	**43.0**	15.8	**46.7**	**33.3**	**41.3**

Table 1: Improvements on various math benchmarks from applying STAT. Results under '+SFT' show the performance of SFT models trained with each method, while '+GRPO' shows the performance after applying GRPO on top of the corresponding SFT models. Our methods, STAT-Sel and STAT-Syn, achieve an average gain of up to 6.7% over the base model, with strong OOD performances (AMC23 results reported on average@64, AIME on pass@64). Applying GRPO on top of fine-tuning with STAT further enhances these improvements by ~4%. Full results are provided for Qwen2.5-3B and Llama-3.2-1B-Instruct in Table 6, Section E.

to generate multiple questions along with 3 responses per question by referring to the provided in-context examples. We keep only those questions where the frontier LLM is consistent across at least 2 of its responses, and keep only those question-answer pairs in our training set. The algorithm for both methods is given in Section C.1.

3 Experiments

3.1 Experimental Setup

Datasets. All training data for STAT and the baselines are either selected or synthesized from the MATH dataset (7.5k train / 5k test) [Hendrycks et al., 2021], with the 5k test set randomly split into 1k validation and 4k test subsets (see Section 2). We also evaluate our method on extensive OOD benchmarks including GSM8K [Cobbe et al., 2021], MATH^2 [Shah et al., 2024], MATH-perturb [Huang et al., 2025], AMC23 [AI-MO, 2025], and AIME2024/2025 [HuggingFaceH4, n.d., HuggingFaceH5, n.d.]. We provide full details in Section D.1.

Model & Training Configuration. We focus on smaller models as a testbed, as their performance remains noticeably weaker on MATH. We employ GPT-4o-mini as the teacher model, and apply STAT on student models Llama-3.2-3B-Instruct, Llama-3.2-1B-Instruct [Meta AI, 2024], and Qwen2.5-3B [Qwen et al., 2025], and evaluate under 0-shot settings. We provide detailed hyperparameters in Section D.2, ablations on threshold sensitivity in Section F.1, and a discussion of teacher model variants in Section F.3.

Baselines. We compare skill-aware training against several baselines: MATH-Train [Hendrycks et al., 2021], MATH-Augment [TIGER-Lab, 2024], MATH-Hard [Sun et al., 2024], Embed-Sel [Li et al., 2025], and Embed-Syn [Jung et al., 2025]. Please find the detailed data creation procedure in Section D.3 and data synthesis prompts in Section D.4.

3.2 Evaluation Results

We present results for Llama-3.2-3B-Instruct in Table 1 and for Qwen2.5-3B and Llama-3.2-1B-Instruct in Table 6, Section E. Our findings can be summarized as follows.

Naive SFT provides little to no benefit. Both MATH-Train and MATH-Augment yield at most a 1–2% gain over the base model, and Qwen2.5-3B can even degrade under MATH-Train. Restricting

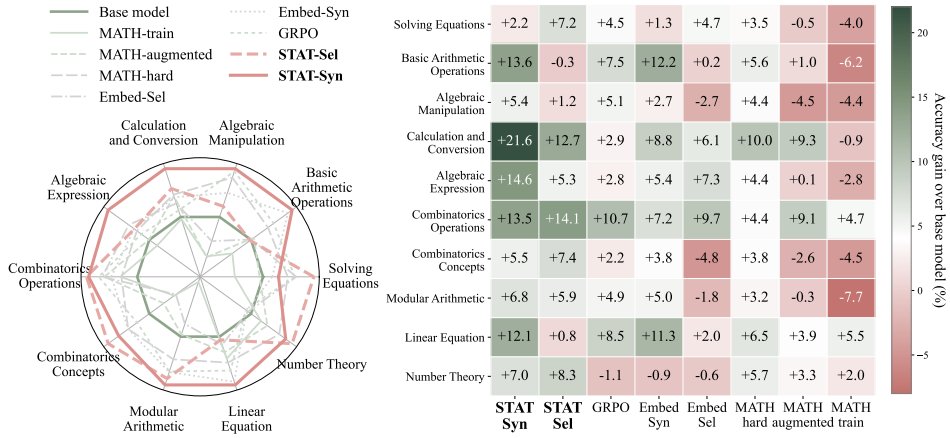

Figure 2: **Left:** Fine-tuned model performances on Top 10 frequent missing skills, across different training methods. For better visualization, accuracies are normalized per skill axis, with the base model drawn as a uniform circle and the highest-performing method on each skill placed at the outer edge. **Bottom right:** Model performances averaged across the Top 10 frequent missing skills, with different training methods. Our approaches STAT-Syn and STAT-Sel are most effective in enhancing model performance across nearly all the skills.

supervision to only the most difficult MATH questions (Levels 4–5), or selecting training questions semantically close to difficult validation examples, also fails to produce meaningful gains.

Skill-targeted adaptive training shows substantial improvements. STAT achieves average gains of up to 6.7% on Llama-3.2-3B-Instruct, 5.2% on Qwen2.5-3B, and 3.4% on Llama-3.2-1B-Instruct, over the performance of base model. On out-of-distribution benchmarks, we observe consistent improvements across 7 datasets. Specifically, on Llama-3.2-3B-Instruct, STAT-Sel and STAT-Syn improve averaged OOD performances by 5.3% and 5.8% respectively.

Compatibility with GRPO. For both Llama and Qwen, improvements from SFT on STAT carry over to subsequent GRPO, yielding average gains of up to 9.5% over GRPO on base model. Surprisingly, on Llama-3.2-1B-Instruct and Llama-3.2-3B-Instruct, where GRPO alone does not work well (improving $\leq 2.4\%$), STAT-Sel alone on STAT already produced better results than GRPO, and adding GRPO on top further boosts performance by $\sim 4\%$.

Continual learning on challenging benchmarks. We test whether models can adapt further to harder benchmarks while still using MATH-style data. Two variants, STAT-ConSel and STAT-ConSyn, trained on MATH data targeting missing MATH-perturb-hard skills, achieve 3–4% gains on MATH-perturb-hard, compared to only 1–2% from STAT-Sel and STAT-Syn. Full details are in Section E.2.

4 Discussion

We conduct ablation studies into the missing skills and how STAT improves the overall model's performance by targeting these skills. Additional ablations are in sections E.3, E.4 and E.7.

Missing-Skill-Profile: Figure 5 (appendix E) show the Top-10 missing skills of Llama-3.2-3B-Instruct and Llama-3.2-1B-Instruct at the end of Stage 2. All models struggle mainly with algebra-related skills (e.g., equation solving, manipulation, calculation) and also with conceptual areas like combinatorics, functions, and number theory. Llama-3.2-1B-Instruct makes mistakes on basic algebraic operations (e.g., solving equations), indicating more limited fundamental abilities.

STAT effectively addresses models' frequent missing skills. We take Llama-3.2-1B-Instruct as a case study to examine how different training strategies impact performance across skills. From its `Missing-Skill-Profile`, we select the 10 most frequently missing skills and build corresponding evaluation sets, each containing questions annotated via the `Skill-Map`. As shown in Figure 2 (Left), STAT consistently outperform all baselines across all 10 skills, whereas baseline models can even fall behind the base model on skills such as Algebraic Manipulation and Modular Arithmetic. Figure 2

(Right) provides a quantitative breakdown, showing that STAT can deliver over 10% accuracy gains on 5 skills, with the largest improvements on basic skills like Calculation Conversion, Algebraic Expression, and Combinatoric Expressions.

References

AI-MO. AIMO Validation AMC [dataset]. Hugging Face Datasets, May 2025. URL https://huggingface.co/datasets/AI-MO/aimo-validation-amc. Accessed: 2025-08-26.

Sanjeev Arora and Anirudh Goyal. A theory for emergence of complex skills in language models. *arXiv preprint arXiv:2307.15936*, 2023.

Albert Bandura and Richard H Walters. *Social learning theory*, volume 1. Prentice hall Englewood Cliffs, NJ, 1977.

Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, et al. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*, 2021.

Aniket Didolkar, Anirudh Goyal, Nan Rosemary Ke, Siyuan Guo, Michal Valko, Timothy Lillicrap, Danilo Jimenez Rezende, Yoshua Bengio, Michael C Mozer, and Sanjeev Arora. Metacognitive capabilities of llms: An exploration in mathematical problem solving. *Advances in Neural Information Processing Systems*, 37:19783–19812, 2024.

Kanishk Gandhi, Ayush Chakravarthy, Anikait Singh, Nathan Lile, and Noah D Goodman. Cognitive behaviors that enable self-improving reasoners, or, four habits of highly effective stars. *arXiv preprint arXiv:2503.01307*, 2025.

John Hattie and Helen Timperley. The power of feedback. *Review of educational research*, 77(1): 81–112, 2007.

Yinghui He, Abhishek Panigrahi, Yong Lin, and Sanjeev Arora. Adaptmi: Adaptive skill-based in-context math instruction for small language models. *arXiv preprint arXiv:2505.00147*, 2025.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset. *arXiv preprint arXiv:2103.03874*, 2021.

Kaixuan Huang, Jiacheng Guo, Zihao Li, Xiang Ji, Jiawei Ge, Wenzhe Li, Yingqing Guo, Tianle Cai, Hui Yuan, Runzhe Wang, et al. Math-perturb: Benchmarking llms' math reasoning abilities against hard perturbations. *arXiv preprint arXiv:2502.06453*, 2025.

HuggingFaceH4. aime_2024 [dataset]. Hugging Face Datasets, n.d. URL https://huggingface.co/datasets/HuggingFaceH4/aime_2024. Accessed: 2025-08-26.

HuggingFaceH5. aime_2025 [dataset]. Hugging Face Datasets, n.d. URL https://huggingface.co/datasets/math-ai/aime25. Accessed: 2025-08-26.

Jaehun Jung, Seungju Han, Ximing Lu, Skyler Hallinan, David Acuna, Shrimai Prabhumoye, Mostafa Patwary, Mohammad Shoeybi, Bryan Catanzaro, and Yejin Choi. Prismatic synthesis: Gradient-based data diversification boosts generalization in llm reasoning. *arXiv preprint arXiv:2505.20161*, 2025.

Jared Kaplan, Sam McCandlish, Tom Henighan, Tom B Brown, Benjamin Chess, Rewon Child, Scott Gray, Alec Radford, Jeffrey Wu, and Dario Amodei. Scaling laws for neural language models. *arXiv preprint arXiv:2001.08361*, 2020.

Simran Kaur, Simon Park, Anirudh Goyal, and Sanjeev Arora. Instruct-skillmix: A powerful pipeline for llm instruction tuning. *arXiv preprint arXiv:2408.14774*, 2024.

Jiazheng Li, Lu Yu, Qing Cui, Zhiqiang Zhang, Jun Zhou, Yanfang Ye, and Chuxu Zhang. Mass: Mathematical data selection via skill graphs for pretraining large language models, 2025. URL https://arxiv.org/abs/2503.14917.

Zehan Li, Xin Zhang, Yanzhao Zhang, Dingkun Long, Pengjun Xie, and Meishan Zhang. Towards general text embeddings with multi-stage contrastive learning, 2023. URL https://arxiv.org/abs/2308.03281.

Keming Lu, Hongyi Yuan, Zheng Yuan, Runji Lin, Junyang Lin, Chuanqi Tan, Chang Zhou, and Jingren Zhou. # instag: Instruction tagging for analyzing supervised fine-tuning of large language models. *arXiv preprint arXiv:2308.07074*, 2023.

Meta AI. Llama 3.2: Revolutionizing Edge AI and Vision with Open, Customizable Models, 2024. URL https://ai.meta.com/blog/llama-3-2-connect-2024-vision-edge-mobile-devices/.

Niklas Muennighoff, Alexander Rush, Boaz Barak, Teven Le Scao, Nouamane Tazi, Aleksandra Piktus, Sampo Pyysalo, Thomas Wolf, and Colin A Raffel. Scaling data-constrained language models. *Advances in Neural Information Processing Systems*, 36:50358–50376, 2023.

Xinzhe Ni, Yeyun Gong, Zhibin Gou, Yelong Shen, Yujiu Yang, Nan Duan, and Weizhu Chen. Exploring the mystery of influential data for mathematical reasoning. *arXiv preprint arXiv:2404.01067*, 2024.

OpenAI. Gpt-4o mini: advancing cost-efficient intelligence. https://openai.com/index/gpt-4o-mini-advancing-cost-efficient-intelligence/, 2024.

Qwen, :, An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, Fei Huang, Haoran Wei, Huan Lin, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Yang, Jiaxi Yang, Jingren Zhou, Junyang Lin, Kai Dang, Keming Lu, Keqin Bao, Kexin Yang, Le Yu, Mei Li, Mingfeng Xue, Pei Zhang, Qin Zhu, Rui Men, Runji Lin, Tianhao Li, Tianyi Tang, Tingyu Xia, Xingzhang Ren, Xuancheng Ren, Yang Fan, Yang Su, Yichang Zhang, Yu Wan, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, and Zihan Qiu. Qwen2.5 technical report, 2025. URL https://arxiv.org/abs/2412.15115.

Vedant Shah, Dingli Yu, Kaifeng Lyu, Simon Park, Jiatong Yu, Yinghui He, Nan Rosemary Ke, Michael Mozer, Yoshua Bengio, Sanjeev Arora, et al. Ai-assisted generation of difficult math questions. *arXiv preprint arXiv:2407.21009*, 2024.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, YK Li, Yang Wu, et al. Deepseekmath: Pushing the limits of mathematical reasoning in open language models. *arXiv preprint arXiv:2402.03300*, 2024.

Zhiqing Sun, Longhui Yu, Yikang Shen, Weiyang Liu, Yiming Yang, Sean Welleck, and Chuang Gan. Easy-to-hard generalization: Scalable alignment beyond human supervision, 2024. URL https://arxiv.org/abs/2403.09472.

TIGER-Lab. Math-plus. https://huggingface.co/datasets/TIGER-Lab/MATH-plus, 2024. Dataset available on Hugging Face.

Pablo Villalobos, Anson Ho, Jaime Sevilla, Tamay Besiroglu, Lennart Heim, and Marius Hobbhahn. Position: Will we run out of data? limits of llm scaling based on human-generated data. In *Forty-first International Conference on Machine Learning*, 2024.

Peiqi Wang, Yikang Shen, Zhen Guo, Matthew Stallone, Yoon Kim, Polina Golland, and Rameswar Panda. Diversity measurement and subset selection for instruction tuning datasets. *arXiv preprint arXiv:2402.02318*, 2024.

Xindi Wu, Dingli Yu, Yangsibo Huang, Olga Russakovsky, and Sanjeev Arora. Conceptmix: A compositional image generation benchmark with controllable difficulty. *Advances in Neural Information Processing Systems*, 37:86004–86047, 2024.

Mengzhou Xia, Sadhika Malladi, Suchin Gururangan, Sanjeev Arora, and Danqi Chen. Less: Selecting influential data for targeted instruction tuning. *arXiv preprint arXiv:2402.04333*, 2024.

Wei Xiong, Hanning Zhang, Nan Jiang, and Tong Zhang. An implementation of generative prm. https://github.com/RLHFlow/RLHF-Reward-Modeling, 2024.

Dingli Yu, Simran Kaur, Arushi Gupta, Jonah Brown-Cohen, Anirudh Goyal, and Sanjeev Arora. Skill-mix: A flexible and expandable family of evaluations for ai models. *arXiv preprint arXiv:2310.17567*, 2023a.

Longhui Yu, Weisen Jiang, Han Shi, Jincheng Yu, Zhengying Liu, Yu Zhang, James T Kwok, Zhenguo Li, Adrian Weller, and Weiyang Liu. Metamath: Bootstrap your own mathematical questions for large language models. *arXiv preprint arXiv:2309.12284*, 2023b.

Simon Yu, Liangyu Chen, Sara Ahmadian, and Marzieh Fadaee. Diversify and conquer: Diversity-centric data selection with iterative refinement. *arXiv preprint arXiv:2409.11378*, 2024a.

Zichun Yu, Spandan Das, and Chenyan Xiong. Mates: Model-aware data selection for efficient pretraining with data influence models. *Advances in Neural Information Processing Systems*, 37: 108735–108759, 2024b.

Biao Zhang, Zhongtao Liu, Colin Cherry, and Orhan Firat. When scaling meets llm finetuning: The effect of data, model and finetuning method. *arXiv preprint arXiv:2402.17193*, 2024.

Haoyu Zhao, Simran Kaur, Dingli Yu, Anirudh Goyal, and Sanjeev Arora. Can models learn skill composition from examples? *Advances in Neural Information Processing Systems*, 37: 102393–102427, 2024.

A Related Works

Recent line of works have shown that cognitive theories relevant for human learning can also be utilized to improve performance of language models. Arora and Goyal [2023] argue that language models learn to generalize beyond training data, by learning generalizable skills that connect the text tokens. This argument was later used by Wu et al. [2024], Yu et al. [2023a], Zhao et al. [2024] to create evaluation benchmarks to compare how different LLMs can generalize beyond training. Furthermore, Didolkar et al. [2024], He et al. [2025] utilize this framework to create effective instance specific in-context learning examples for language models. More related to our work is the work by Kaur et al. [2024] who create synthetic instruction following datasets using arbitrary combination of skills and show that small language models learn more efficiently from such synthetic tasks. Similarly, Gandhi et al. [2025] show that certain cognitive skills are necessary for models to explore during reinforcement learning, and one can similarly enforce such skills by targeted continual pretraining. In contrast, we show that we can use the skill-based framework to create targeted training datasets by analyzing the missing skills in model's responses after training and even unlock further gains.

Related to our work is a line of research that leverages model gradients to construct targeted training datasets [Xia et al., 2024, Yu et al., 2024b]. These approaches utilize gradients on both training and test sets to identify effective subsets of training data. However, a key limitation is their reliance on ground-truth solutions for the test data. In contrast, our work offers a complementary strategy: we employ a frontier LLM to analyze the model's responses and generate targeted training data based on its feedback.

We also compare our proposed method to embedding based data selection. Embedding based methods Wang et al. [2024], Yu et al. [2024a], Ni et al. [2024] have primarily been used to improve diversity of training dataset. As shown by our results, embedding similarity could also be utilized (albeit with a drop in performance) to get questions from training dataset that are similar to difficult questions.

Finally, we introduce STAT-Syn, an approach analogous to STAT-Sel, which synthesizes additional training data specifically targeted to the identified missing skills. Synthetic data generation has recently attracted significant attention as a practical alternative for augmenting real-world datasets, and has been shown to improve language model performance both in-distribution and out-of-distribution [Jung et al., 2025, Yu et al., 2023b, Lu et al., 2023, Li et al., 2023, Kaur et al., 2024]. A comprehensive comparison of STAT-Syn with prior work on synthetic data generation is left to future study.

B Detection of *difficult* questions via reward filtering.

In this stage, we will label a question $q \in \mathcal{Q}$ as *difficult* or not for the student model. We could simply define *difficult* questions as the set of questions that the model gets wrong after evaluation. However, this requires access to the ground truth labels. Instead, to make our technique more broadly applicable, we use a reward model to classify the responses of the student model. The reward model need not be a perfect reward model; we give more ablations in Section F.1. Given a question q, we use a reward model to score the response of the student model.

Reward filtering. As we primarily focus on math datasets, we assume that the model's response is composed of t steps for a question q and contains the answer in its final step. We will use the reward model to output reward scores for each step. For simplicity, we will refer to the scores of the reward model as $\{r_{q,1}, \cdots, r_{q,t}\}$. Then, we use thresholds τ_1, τ_2 to filter out *difficult* questions for the student model. We will refer to the threshold filtering function as $R : \mathcal{Q} \to \{0, 1\}$.

$$R(q) = \begin{cases} 0, & \text{(if) } r_{q,t} \leq \tau_1 \qquad \text{(final step has low reward)} \\ & \text{(or) } \frac{1}{t}\sum_{i=1}^{t} r_{q,i} \leq \tau_1 \qquad \text{(average low reward across all steps)} \\ & \text{(or) } \exists i < t \text{ s.t. } r_{q,i} \leq \tau_2 \qquad \text{(low reward at any step)} \\ 1, & \text{otherwise,} \end{cases} \tag{1}$$

Identifying difficult questions. We define $\mathcal{Q}_{\text{difficult}}$ as a model-specific subset of the MATH dataset, consisting of questions with low-reward model responses R.

To avoid training directly on the test data, we use two splits of $\mathcal{Q}_{\text{difficult}}$ as validation and test sets:

- $\mathcal{Q}^{\textbf{val}}_{\textbf{difficult}}$: Difficult questions in the validation set, given by $\mathcal{Q}_{\text{difficult}} \cap \mathcal{Q}^{\textbf{val}}$, are used to label missing skills in Stage 2.
- $\mathcal{Q}^{\textbf{test}}_{\textbf{difficult}}$: Difficult questions in the test set, given by $\mathcal{Q}_{\text{difficult}} \cap \mathcal{Q}^{\textbf{test}}$, are used for MATH^D evaluation in Table 1.

B.1 Stage 2: Constructing model-specific `Missing-Skill-Profile`.

For each *difficult* question q in $\mathcal{Q}^{val}_{\text{difficult}}$, we use a frontier LLM (GPT-4o-mini) to predict the set of skills in $\mathcal{S}$ that are missing in the model's responses. We call this map `Missing-Skill-Profile`: $\mathcal{Q}^{val}_{\text{difficult}} \rightarrow \mathcal{S}$. This map will be used to build STAT-Sel and STAT-Syn. See Section 4 for examples and an extensive analysis of `Missing-Skill-Profile` across models, and Section D.4 for prompts.

B.2 Stage 3: Selecting or synthesizing skill-based training data.

In this stage, we construct our skill-targeted training dataset, $\mathcal{P}_{targeted}$.

STAT-Sel. We create this set by directly sampling questions from the training dataset $\mathcal{P}$ according to the skills listed in the `Missing-Skill-Profile`. Specifically, for each question $q \in \mathcal{Q}^{val}_{\text{difficult}}$, we examine `Missing-Skill-Profile`(q) and, for every skill it contains, sample multiple questions from $\mathcal{P}$ that are linked to the same skill via the `Skill-Map`. Consequently, the frequency with which a skill contributes to the selection process is proportional to the number of questions associated with that skill in the `Missing-Skill-Profile`.

STAT-Syn. We generate new synthetic questions using the teacher model. For each question $q \in \mathcal{Q}^{val}_{\text{difficult}}$, we examine `Missing-Skill-Profile`(q). For each skill it contains, we randomly sample 3 questions from $\mathcal{P}$ that are linked to the same skill via the `Skill-Map`, and ask the teacher model to create new questions and responses by referring to the sampled questions. We keep only those questions where the teacher model is consistent across at least 2 of its responses, and keep only those question-answer pairs in our training set. Detailed procedures are given in Section C.

C Details of STAT data creation

C.1 Algorithm for constructing STAT-Sel and STAT-Syn data

Algorithm 1 outlines the procedure to construct $\mathcal{P}_{targeted}$ in Stage 3 (Section 2). For each question in the test set $\mathcal{Q}$, the algorithm first identifies the associated missing skills using the Missing-Skill Profile. For each missing skill, a small set of examples is retrieved from the Skill-Map, which links each skill to corresponding training data. In STAT-Sel, these retrieved examples are directly added to the target training set. Otherwise, the examples are used as seeds to prompt GPT-4o to generate new, skill-specific questions, which are then included instead. This process ensures that the resulting training set $\mathcal{P}_{targeted}$ is adaptively enriched with examples that directly address the model's weaknesses.

C.2 Training Data Creation Procedure of STAT

We now provide a detailed interpretation of our training data creation approach outlined Algorithm 1.

STAT-Sel. <u>4k unique questions, 9.5k QA pairs.</u> We begin by filtering 500 difficult questions from the validation set using our process reward model. For each such question, the teacher model identifies 2–3 missing skills in the student's response. As described in Section B.2, we then create the training set by selecting 5 questions for each missing skill in the question's `Missing-Skill-Profile`. We use 3 answers for each question and randomly sample a subset of 9.5k question-answer pairs as our training set.

STAT-Syn. <u>4k unique questions, 9.5k QA pairs.</u> We begin by filtering 500 difficult questions from the validation set using our process reward model. For each such question, the teacher model

Algorithm 1 Skill-based data selection/generation

Input: Test set $\mathcal{Q}$, `Skill-Map`: $\mathcal{S} \rightarrow \mathcal{P}$, `MissingSkillProfile`: $\mathcal{Q} \rightarrow \mathcal{S}$, STAT-Sel: bool
Output: $\mathcal{P}_{targeted}$

```
P_targeted ← []
for q in Q do
    skill_list ← MissingSkillProfile[q]
    if skill_list is not empty then
        for skill in skill_list do
            P_skill ← Skill-Map[skill]
            P_selected ← random_sample(P_skill, 3)
            if STAT-Sel then
                P_targeted ← P_targeted + P_selected
            else
                P_new ← GPT-4o(P_selected, skill, prompt="Propose a new question based on
                                                the given questions and the given skill.")
                P_targeted ← P_targeted + P_new
            end if
        end for
    end if
end for
return P_targeted
```

identifies 2–3 missing skills in the student's response. For each pair of `(difficult_question, missing_skill)`, we retrieve 3 questions from MATH training set. We input these 3 questions, along with the `missing_skill`, to the teacher model, prompting it to synthesize 2 new questions. The teacher further generates 3 solutions for each new question. We then filter the newly synthesized data by:

1. Compute consistency scores for each set of `(new_question, solution)` pairs, according to the number of solutions agreeing on the final answer. For example, a new question with 2 solutions agreeing on the final answer has a consistency score of 2.

2. Keep only the `new_question` with a consistency score of ≥ 2.

3. For each filtered question, keep only the `solution` that agrees on the final answer. [2]

This process enables our approach to generate diverse data, as we input 3 questions to the teacher model as references each time. The consistency-filtering step filters out both invalid questions and solutions, ensuring the quality of STAT-Syn.

[2] For STAT-Syn, after filtering teacher-generated answers using consistency, we obtained 9.5k valid question–answer pairs. To ensure comparability, we standardize the training data size to 9.5k pairs for all experiments.

D Experimental details

D.1 Datasets

Datasets. All training data for STAT and the baselines are either selected or synthesized from the MATH dataset (7.5k train / 5k test) [Hendrycks et al., 2021]. In addition to the original solutions provided in the dataset, we also collect three alternative versions of each answer by prompting the teacher model to re-write them three times. We further report performance of STAT and each baseline after continuing training with GRPO [Shao et al., 2024] on the same MATH training set. We randomly split the MATH test set into 1k validation and 4k test subsets, with both MATH and MATH^D evaluations drawn from the 4k test split. See Section 2 for design details.

We also evaluate our method on extensive OOD benchmarks including GSM8K [Cobbe et al., 2021], MATH^2 [Shah et al., 2024], MATH-perturb [Huang et al., 2025], AMC23 [AI-MO, 2025], and AIME2024/2025 [HuggingFaceH4, n.d., HuggingFaceH5, n.d.].

D.2 Model & Training Configurations

Model Settings. All inferences are under 0-shot settings, with temperature 0.1 for pass@1 sampling, and temperature 1.0 for average@64 or pass@64 sampling. For the process reward model in Stage 1 (Section 2), we use RLHFlow/Llama3.1-8B-PRM-Mistral-Data (Xiong et al. [2024]), an 8B process reward model fine-tuned from Llama-3.1-8B, with filtering thresholds $\tau_1 = 0.85, \tau_2 = 0.7$. We use seed=0 for all evaluations.

SFT configurations. For SFT, we adopt QLoRA with rank 16, scaling factor $\alpha = 32$, and dropout 0.05, applied to the attention and MLP projection modules. Models are trained in 4-bit NF4 quantization with bfloat16 compute, using the paged AdamW (8-bit) optimizer. We train for 3 epochs with a cosine learning rate schedule and a 3% warmup ratio. Peak learning rate is chosen separately for each method among 5e-6, 1e-5, and 5e-6, depending on the downstream performance. The effective batch size is 8 (per-device batch size of 2 with gradient accumulation of 4). We apply gradient clipping at 0.3, weight decay of 0.1, and enable group-by-length packing for efficiency. Other configurations follow the official code base from Llama[3] and Qwen[4].

GRPO configuration. We train for 6 epochs using a constant learning rate of 5e-7. The objective includes only the policy update loss, without any KL-divergence term, and the entropy coefficient is fixed at 0.0. Each batch contains 256 questions, with 4 rollouts generated per question. Responses are truncated at a maximum length of 2048 tokens. We set the PPO mini-batch size to 64, which implies that each batch of 256 questions is split into four mini-batches. The model performs four gradient updates before refreshing the reference model.

D.3 Training Data Creation Procedure of Baselines

We compare STAT-Sel and STAT-Syn with the following baseline models fine-tuned with various data selection/generation methods, to measure the effectiveness of skill-aware training:

- **MATH-Train:** 7.5k unique questions, 7.5k QA pairs. We naively train the model on all question from the training dataset, with a single answer from the original dataset for each question.
- **MATH-Augment:** 7.5k unique questions, 9.5k QA pairs. In order to make a fair comparison to our proposed methods, we pick 3 answers per question to create 22.5k question-answer pairs and then randomly sample a subset of 9.5k question answer pairs as our training set.
- **MATH-Hard:** 3k unique questions, 9.5k QA pairs. We include all questions from the levels 4 and 5 of the MATH dataset. We use 3 responses per question to create a pool of 12k question-answer pairs and then keep a random subset of 9.5k question answer pairs.

[3] https://github.com/meta-llama/llama-cookbook

[4] https://github.com/QwenLM/Qwen

Method	# Unique Questions	# QA Pairs	Synthetic Data	Training Data Description
MATH-Train [Hendrycks et al., 2021]	7.5k	7.5k	✗	MATH original training set.
MATH-Augment [TIGER-Lab, 2024]	7.5k	9.5k	✗	Augmented MATH training set with multiple teacher-rewritten solutions per question.
MATH-Hard [Sun et al., 2024]	3k	9.5k	✗	Subset of MATH-Augment with Level 4–5 MATH questions.
Embed-Sel [Li et al., 2025]	4k	9.5k	✗	Reweighted set of MATH-Augment via upweighting training questions similar to the *difficult* questions in embedding space.
Embed-Syn [Jung et al., 2025]	4k	9.5k	✓	Synthetic MATH-level QAs generated by the teacher, using training examples from Embed-Sel as references.
STAT-Sel (Ours)	4k	9.5k	✗	Reweighted set of MATH-Augment via upweighting training questions related to model's missing skills in solving the *difficult* questions.
STAT-Syn (Ours)	4k	9.5k	✓	Synthetic MATH-level QAs generated by the teacher, with training examples from STAT-Sel and their associated skills as references.

Table 2: Comparison of training data construction methods. We attach details of data construction procedure in Section C.2 and Section D.3.

- **Embed-Sel:** 4k unique questions, 9.5k QA pairs. Here, we compare the effectiveness of skill-based training data selection to embedding-based training data selection [5]. We use our *difficult* question set from stage 1 and for each question, we pick 5 similar questions from the training set using an embedding model based similarity score. We pick 3 answers per selected questions and keep a random subset of 9.5k question answer pairs.
- **Embed-Syn:** 4k unique questions, 9.5k QA pairs. For each question in the difficult set identified during stage 1, we retrieve 5 question–answer pairs from the training set $\mathcal{P}$ using an embedding-based similarity measure. The teacher model is then prompted to generate 5 new questions, each accompanied by 3 candidate responses, conditioned on different groups of 3 retrieved pairs as in-context examples. We retain only those generated questions for which the LLM produces at least 2 consistent responses, and add the corresponding consistent question–answer pairs to our training set. Finally, we keep a random subset of 9.5k question answer pairs to create our training set.

We give a summary and comparison of STAT and baseline data construction methods in Table 2.

[5] We use Alibaba-NLP/gte-Qwen2-7B-instruct as our embedding model [Li et al., 2023]

D.4 Prompts

D.4.1 Constructing Skill-Map on MATH

Statistics of skill lists. We adopt the list of mathematical skills obtained in Didolkar et al. [2024] using an LLM labeling→clustering pipeline. The skill list contains 128 skills in total, divided into 7 subsets across 7 subjects. Each subject includes ~18 skills.

Skill-Map construction procedure. To construct the `Skill-Map` (see Section 2), we follow Didolkar et al. [2024] to label skills on both the training and test sets of MATH using GPT-4o-mini [OpenAI, 2024]. We enlist all skills that we used to annotate the questions in MATH and dataset in Tables 4 and 5, which have been taken from Didolkar et al. [2024]. We ask the LLM to read the question and provide up to five skills required to solve this question, from the given existing skill list. We show an example prompt for annotating MATH Number Theory questions as follows.

Example skill annotation prompt for MATH Number Theory questions

[TASK]
You'll be given a math question. Your task is to output:
(1) < skill> list here up to five skill(s) that are required to solve this problem, seperated by commas </skill>.
(2) <reason> reason here why these skills are needed </reason>.

[SKILL LIST]
You should only choose the skills from this list:
[
"arithmetic_sequences",
"base_conversion",
"basic_arithmetic",
"division_and_remainders",
"exponentiation",
"factorization",
"greatest_common_divisor_calculations",
"modular_arithmetic",
"number_manipulation",
"number_theory",
"polynomial_operations",
"prime_number_theory",
"sequence_analysis",
"solving_equations",
"understanding_of_fractions"
]

[QUESTION]
{question}

[REASON AND SKILL(S)]

Table 3 shows some example MATH questions and their corresponding annotated skills. From the skill annotation, we construct a `Skill-Map` (see Section 2) that stores the required skills for each question.

Question	Annotated skills
What is the units digit of $3^1 + 3^3 + 3^5 + 3^7 + \ldots + 3^{2009}$?	exponentiation, modular arithmetic, sequence analysis
In the addition problem each letter represents a distinct digit. What is the numerical value of E? [Figure]	basic arithmetic, number manipulation, solving equations
In triangle ABC, $\tan(\angle CAB) = \frac{22}{7}$, and the altitude from A divides $\overline{BC}$ into segments of length 3 and 17. What is the area of triangle ABC?	geometry and space calculation, trigonometric calculations, arithmetic operations

Table 3: Example MATH questions, and the annotated skills generated by GPT-4o-mini.

Subject	**List of Skills**
Per subject split in MATH	
Algebra	`algebraic_expression_skills`, `algebraic_manipulation_skills`, `arithmetic_skills`, `calculation_and_conversion_skills`, `combinatorial_operations_and_basic_arithmetic`, `complex_number_skills`, `distance_and_midpoint_skills`, `exponent_and_root_skills`, `factoring_skills`, `function_composition_skills`, `function_skills`, `geometric_sequence_skills`, `graph_and_geometry_skills`, `inequality_skills`, `logarithmic_and_exponential_skills`, `number_theory_skills`, `polynomial_concepts`, `quadratic_equation_skills`, `ratio_and_proportion_skills`, `sequence_and_series_skills`, `solving_equations`
Counting and Probability	`calculating_and_understanding_combinations`, `combinatorial_mathematics`, `combinatorics_concepts`, `counting_principals`, `factorials_and_prime_factorization`, `number_theory_and_arithmetic_operations`, `permutation_and_combinations`, `probability_calculation_with_replacement`, `probability_concepts_and_calculations`, `probability_theory_and_distribution`, `combinatorics_operations`
Geometry	`3d_geometry_and_volume_calculation_skills`, `algebraic_skills`, `area_calculation_skills`, `circle_geometry_skills`, `combinatorics_and_probability_skills`, `coordinate_geometry_and_transformation_skills`, `other_geometric_skills`, `pythagorean_skills`, `quadrilateral_and_polygon_skills`, `ratio_and_proportion_skills`, `triangle_geometry_skills`, `trigonometry_skills`, `understanding_circle_properties_and_algebraic_manipulation`

Table 4: List of skills used for annotating questions in each subject in MATH dataset

Subject	List of Skills
Per subject split in MATH	
Intermediate Algebra	absolute_value_skills, algebraic_manipulation_and_equations, calculus_optimization_skills, complex_number_manipulation_and_operations, function_composition_and_transformation, graph_understanding_and_interpretation, inequality_solving_and_understanding, polynomial_concepts, properties_and_application_of_exponents, quadratic_equations_and_solutions, recursive_functions_and_sequences, sequence_and_series_analysis_skills, simplification_and_basic_operations, solving_inequalities, solving_system_of_equations, summation_and_analysis_of_series, understanding_and_application_of_functions, understanding_and_applying_floor_and_ceiling_functions, understanding_and_manipulation_of_rational_functions, understanding_and_utilizing_infininte_series, understanding_ellipse_properties, understanding_logarithmic_properties_and_solving_equations
Number Theory	arithmetic_sequences, base_conversion, basic_arithmetic, division_and_remainders, exponentiation, factorization, greatest_common_divisor_calculations, modular_arithmetic, number_manipulation, number_theory, polynomial_operations, prime_number_theory, sequence_analysis, solving_equations, understanding_of_fractions
Pre-algebra	average_calculations, basic_arithmetic_operations, circles, counting_and_number_theory, exponentiation_rules, fractions_and_decimals, geometry, multiples_and_zero_properties, multiplication_and_division, perimeter_and_area, prime_number_theory, probability_and_combinatorics, ratio_and_proportion, linear_equation
Pre-calculus	algebra_and_equations, basic_trigonometry, calculus, complex_number_operations, complex_numbers, coordinate_systems, determinant_calculation, geometric_relations, geometry_and_space_calculation, geometry_triangle_properties, matrix_operations, parametric_equations, sequences_series_and_summation, three_dimensional_geometry, trigonometric_calculations, vector_operations

Table 5: List of skills used for annotating questions in each subject of MATH dataset (continued from Table 4)

D.4.2 Missing skill Identification from Model Responses

In Stage 2 of STAT, we use GPT-4o-mini to label the skills that are missing from a model response. We ask the LLM to read the question along with the SLM response and provide the skills that the model fails to leverage in the response, from the given existing skill list. Below we show an example prompt for labeling missing skills for MATH Number Theory questions, as well as an example LLM output.

Example missing skill identification prompt for a MATH Number Theory question

[TASK]
You'll be given a math question and a step-by-step solution written by a Small Language Model. Your task is to output:
(1) <judge> judge here whether the solution is correct or incorrect </judge>
(2) <reason> if it's incorrect, reason here why the solution is incorrect </reason>,
(3) <skill> list here what skill(s) should the SLM enhance in order to answer correctly, seperated by commas </skill>.

[SKILL LIST]
{list of 15 number theory skills}

[QUESTION]
{question}

[MODEL_SOLUTION]
{model_response}

[YOUR REASON AND SKILL(S)]

Example GPT-4o-mini output for missing skill identification

(1) <judge> incorrect </judge>
(2) <reason> The solution incorrectly simplifies the expression and misapplies properties of modular arithmetic. The solution claims that the sum of the inverses from 1 to (p-1) is (-1 pmodp), which is incorrect. The correct approach involves recognizing that each term $\left(a^{-1} \cdot (a+1)^{-1}\right)$ should be evaluated individually, and the sum should be calculated directly without assuming incorrect simplifications. The solution also fails to correctly apply the properties of modular inverses and does not properly evaluate the sum. </reason>
(3) <skill> modular_arithmetic, number_theory, understanding_of_fractions </skill>

D.4.3 Proposing new questions from skill-based training question distribution

In Stage 3 of STAT-Syn, we use GPT-4o to propose new questions based on existing questions in the skill-based training question distribution. Below we show an example prompt.

Example prompt for proposing new questions using GPT-4o

[TASK]
You'll be given three math questions (e.g., [QUESTION 1]), with their solutions for reference. Your task is to output a new, novel math question that emphasizes the use of [SKILL].
Important Note: the new question should not be very similar to any of the given questions (e.g., naive adaptions like altering variable names or values from a given question is strictly prohibitted). Meanwhile, the new question should have similar difficulty with the given questions.
Output format:
(1) <reason> reason here how the given questions relates to the [SKILL] </reason>
(2) <draft> reason here how to design a new, novel question while emphasizing the [SKILL] </draft>
(3) <question> your newly constructed math question </question>

[QUESTION 1]
{train_set_question1}

[QUESTION 2]
{train_set_question2}

[QUESTION 3]
{train_set_question3}

[SKILL]
{missing_skill}

E Additional Results

E.1 Evaluation results on Llama-3.2-1B-Instruct

Table 6 shows the evaluation results on Llama-3.2-1B-Instruct. Similar to Table 1, STAT consistently outperforms both heuristic-based and embedding-based data augmentation baselines on in-distribution dataset and most OOD benchmarks. We presented more discussion in Section 3.2 and Section 4.

Models	MATH	MATH^{D}	MATH^{2}	GSM8K	AMC23	MATH-perturb		AIME		Avg.
						simple	hard	2024	2025	
Qwen2.5-3B + SFT										
Base model	55.8	45.3	34.8	80.9	26.4	43.7	24.0	23.3	20.0	39.4
MATH-Train	50.0	44.2	32.9	80.1	33.6	42.3	23.3	26.7	26.7	40.0
MATH-Augment	56.6	45.6	37.1	80.4	33.0	40.9	21.9	16.7	26.7	39.9
MATH-Hard	56.7	45.6	31.4	79.8	33.6	43.7	23.7	30.0	16.7	40.1
Embed-Sel	57.5	46.4	34.3	80.4	33.6	43.7	21.9	30.0	26.7	41.6
Embed-Syn	56.4	47.4	34.3	80.4	35.2	43.7	24.0	26.7	26.7	41.6
STAT-Sel	58.4	47.6	39.5	**82.3**	**35.5**	**45.9**	24.0	33.3	**30.0**	44.1
STAT-Syn	**59.4**	**49.2**	**40.5**	81.3	34.4	44.8	**25.1**	**36.7**	**30.0**	**44.6**
+ GRPO										
Base model	61.6	49.8	41.0	85.1	37.7	49.8	25.8	33.3	30.0	46.0
MATH-Train	61.6	51.1	34.8	84.8	36.9	**51.6**	26.5	33.3	30.0	45.6
MATH-Augment	61.0	48.2	40.5	84.0	36.3	48.7	26.2	36.7	26.7	45.4
MATH-Hard	59.0	51.1	35.7	84.2	37.7	49.8	26.5	33.3	23.3	44.5
Embed-Sel	59.7	48.9	41.0	84.3	38.4	46.6	25.8	26.7	**36.7**	45.3
Embed-Syn	61.4	52.3	40.0	83.7	38.8	47.7	28.0	26.7	30.0	45.4
STAT-Sel	**62.8**	52.1	**44.8**	84.8	38.8	48.7	**30.1**	36.7	33.3	48.0
STAT-Syn	61.8	**52.4**	41.9	**85.6**	**39.2**	50.9	26.9	**40.0**	**36.7**	**48.4**
Llama-3.2-1B-Instruct + SFT										
Base Model	26.0	15.1	9.1	40.7	11.1	17.2	6.5	20.0	10.0	17.3
MATH-Train	27.0	14.5	10.0	42.8	8.8	19.0	6.8	**26.7**	10.0	18.4
MATH-Augment	27.8	14.2	8.1	43.4	11.1	17.9	6.8	**26.7**	3.3	17.7
MATH-Hard	28.4	15.4	8.6	44.6	10.8	18.6	7.2	23.3	3.3	17.8
Embed-Sel	27.4	15.6	8.6	44.6	8.8	18.6	6.8	**26.7**	3.3	17.8
Embed-Syn	28.4	17.2	11.0	44.3	10.0	20.1	**7.9**	23.3	6.7	18.8
STAT-Sel	32.4	15.6	11.0	45.0	**12.0**	19.4	**7.9**	**26.7**	**16.7**	**20.7**
STAT-Syn	**34.5**	**18.3**	**12.4**	**45.6**	11.0	**20.8**	7.5	23.3	10.0	20.4
+ GRPO										
Base Model	31.8	14.4	9.5	49.7	13.3	23.3	8.2	20.0	6.7	19.7
MATH-Train	32.0	16.0	11.9	50.8	10.0	23.7	7.9	16.7	6.7	19.5
MATH-Augment	31.2	15.0	9.0	49.1	13.6	**24.7**	7.9	23.3	13.3	20.8
MATH-Hard	32.2	14.8	11.0	50.6	11.6	22.9	6.5	26.7	10.0	20.7
Embed-Sel	32.8	16.2	11.4	49.9	12.0	21.9	6.5	23.3	13.3	20.8
Embed-Syn	32.6	15.0	10.5	**51.0**	13.9	21.1	6.8	26.7	3.3	20.1
STAT-Sel	34.8	16.6	**13.8**	50.1	**14.8**	23.7	**9.0**	30.0	**16.7**	23.3
STAT-Syn	**35.2**	**21.1**	**13.8**	**51.0**	**14.8**	**24.7**	7.9	**33.3**	**16.7**	**24.3**

Table 6: Improvements on various math benchmarks from applying STAT. Results under '+SFT' show the performance of SFT models trained with each method, while '+GRPO' shows the performance after applying GRPO on top of the corresponding SFT models. Our methods, STAT-Sel and STAT-Syn, achieve an average gain of up to 3.4% over the base model, with strong OOD performances (AMC23 results reported on average@64, AIME on pass@64). Applying GRPO on top of fine-tuning with STAT further enhances these improvements. See Table 1 for results on Llama-3.2-3B-Instruct.

E.2 Continual learning on challenging benchmarks

As our earlier results show, STAT already generalizes strongly to a wide range of OOD tasks while using only MATH data for training. But in practice, models often face evaluation settings that grow

harder over time. A natural question then is: *can we continue adapting the model to these tougher benchmarks while still using similar questions as MATH?*

For our case study, we consider MATH-perturb-hard. We report performance for two model variants of Llama-3.2-3B-Instruct. **STAT-ConSel** takes a model trained with STAT-Sel, and trains further with a data creation pipeline identical to STAT-Sel, but with `Missing-Skill-Profile` built on validation questions from MATH-perturb-hard. **STAT-ConSyn** builds on STAT-Syn model with the same idea. In both cases, the evaluation benchmark only gives the skill profile, and the training examples still come from MATH.

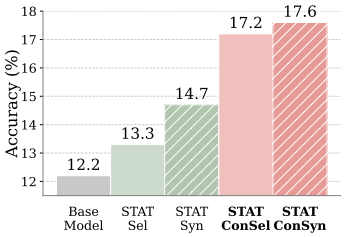

Figure 3: Continual learning results on MATH-perturb-hard. Further fine-tuning STAT models based on their missing skills on unseen data yields a 3–4% gain (STAT-ConSel/ConSyn).

As shown in Section E.2, STAT-Sel and STAT-Syn trained models show only 1–2% improvement on MATH-perturb-hard over the base model performance, which reflects the difficulty of this benchmark. However, continually trained models show a larger gain of 3–4%. This shows that our framework can be readily adapted to unseen test-time datasets by constructing `Missing-Skill-Profile` directly on them, while still using MATH training data. Thus, skill-aware training provides a flexible solution to adapt the models with more challenging evaluations while still relying on existing training datasets.

Figure 4 shows the snippets of model-specific Missing-Skill-Profile of Llama-3.2-3B-Instruct, Llama-3.2-1B-Instruct, and Qwen2.5-3B, obtained at the end of Stage 2 (see Section B.1). These profile snippets include the Top 10 frequent missing skills of the models. As discussed in Section 4, most of the frequent missing skills in both models are algebra-related, such as solving equations, manipulation, and calculation. In addition, both models also demonstrate noticeable weaknesses in conceptual and reasoning-oriented mathematical skills, including combinatorics, understanding and application of functions, and number theory. Compared to Llama-3.2-3B-Instruct, the missing skill profile of Llama-3.2-1B-Instruct concentrated more towards basic operations (e.g., solving equations), suggesting that smaller models have more pronounced limitations in fundamental computational abilities.

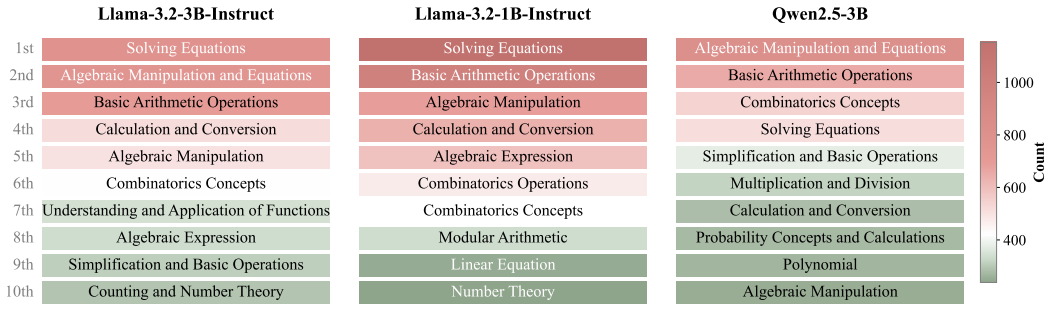

Figure 4: Top 10 missing skills of Llama-3.2-3B-Instruct, Llama-3.2-1B-Instruct, and Qwen2.5-3B. The models struggle most with fundamental mathematical skills such as solving equations and basic arithmetic operations.

E.3 Misalignment between baseline training data and missing skills

To investigate the reason behind the ineffectiveness of our baseline strategies, we adopt a skill-based evaluation by comparing the skill distribution of their training data with the model's missing skills in the `Missing-Skill-Profile`. Figure 5 highlights a clear misalignment between the model's actual missing skills (STAT-Sel) and the baselines: Neither MATH-Train nor Embed-Sel targeted the basic algebraic skills the model struggles with, even though Embed-Sel selects data directly via embedding similarity to difficult questions. They prioritize more advanced and conceptual areas such as polynomials, prime number theory, and trigonometric or matrix operations. We provide concrete question examples in Section E.5 to illustrate the distinct emphasis of each skill.

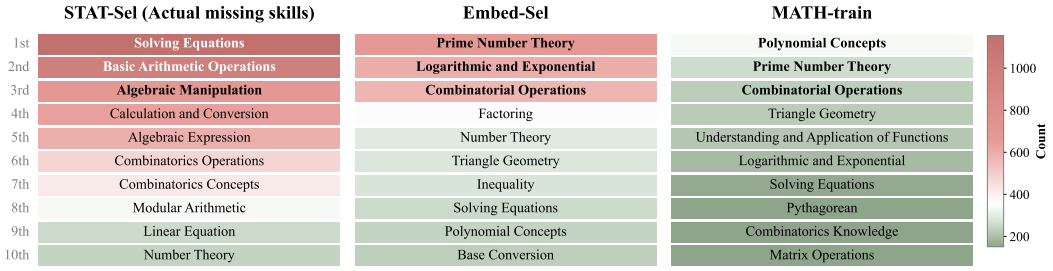

Figure 5: Comparison among the Top 10 frequent skills present in STAT-Sel, Embed-Sel, and MATH-Train questions selected on Llama-3.2-1B-Instruct. The skills emphasized in both baselines, MATH-Train and Embed-Sel, align poorly with the actual Top 10 missing skills of the model (i.e., skills in STAT-Sel).

E.4 Comparing STAT to GRPO

One of our interesting findings in Section 3.2 was that STAT could outperform GRPO training on Llama instruct models. Here, we compare these two approaches from a skill-based perspective. As shown in Figure 2 (Right), although GRPO on Llama-3.2-1B-Instruct also yields positive gains across nearly all the top skills, the overall effect remains less pronounced compared to STAT. A possible reason is that GRPO provides only coarse feedback to the model by contrasting correct and incorrect responses, whereas skill-targeted training pinpoints model weaknesses in a fine-grained way. In light of this, one future direction is to develop a GRPO variant that incorporates skill-based feedback into the reward.

E.5 Case study: example questions with different skills

We observe in Section 4 that the baseline training data distribution largely misaligns with model's actual missing skills, with baseline data emphasizing more advanced and conceptual skills (e.g., Prime Number Theory, Polynomial Concepts) while the model lacking more basic operational skills (e.g., Solving Equations). To better visualize this misalignment in data distribution, here we showcase three example QA pairs from the MATH training set, respectively associated with the three skills above.

Question with skill "Solving Equations"

Question: The inverse of $f(x) = \frac{2x-1}{x+5}$ may be written in the form $f^{-1}(x) = \frac{ax+b}{cx+d}$, where a, b, c, and d are real numbers. Find a/c.

Solution:
If we substitute $f^{-1}(x)$ into our expression for f, we get

$$f(f^{-1}(x)) = \frac{2f^{-1}(x) - 1}{f^{-1}(x) + 5}.$$

Since $f^{-1}(f(x)) = x$, we obtain

$$\begin{aligned} \frac{2f^{-1}(x) - 1}{f^{-1}(x) + 5} &= x \\ \Rightarrow \quad 2f^{-1}(x) - 1 &= x\left(f^{-1}(x) + 5\right) \\ \Rightarrow \quad 2f^{-1}(x) - 1 &= xf^{-1}(x) + 5x. \end{aligned}$$

Moving the terms involving $f^{-1}(x)$ to the left-hand side and the remaining terms to the right-hand side, we get

$$\begin{aligned} 2f^{-1}(x) - xf^{-1}(x) &= 5x + 1 \\ \Rightarrow \quad f^{-1}(x)(2 - x) &= 5x + 1 \\ \Rightarrow \quad f^{-1}(x) &= \frac{5x + 1}{-x + 2}. \end{aligned}$$

Now we can see that $(a, b, c, d) = (5, 1, -1, 2)$ for this representation of $f^{-1}(x)$, so

$$\frac{a}{c} = \frac{5}{-1} = \boxed{-5}.$$

Question with skill “Prime Number Theory"

Question: The positive integers A, B, $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is
- A. even
- B. divisible by 3
- C. divisible by 5
- D. divisible by 7
- E. prime

Express your answer using a letter, as A, B, C, D, or E.

Solution: The numbers $A - B$ and $A + B$ are both odd or both even. However, they are also both prime, so they must both be odd. Therefore, one of A and B is odd and the other even. Because A is a prime between $A - B$ and $A + B$, A must be the odd prime. Therefore, $B = 2$, the only even prime. So $A - 2$, A, and $A + 2$ are consecutive odd primes and thus must be 3, 5, and 7. The sum of the four primes 2, 3, 5, and 7 is the prime number 17, so the correct answer is $\boxed{\text{(E),}}$ prime.

Question with skill “Polynomial Concepts"

Question: The polynomial $P(x) = 2x^3 + ax^2 + bx + c$ has the property that the mean of its zeros, the product of its zeros, and the sum of the coefficients are all equal. The y-intercept of the graph of $y = P(x)$ is 8. What is b?

Solution: The y-intercept of the graph is the point at which $x = 0$. At that point, $P(x) = c$, which we are told is equal to 8. Thus, $c = 8$. The product of the roots of the given polynomial is $-\frac{c}{2} = -4$. The problem states that the mean of the zeros must also equal -4, so the sum of the three zeros (this is a cubic equation) is equal to $3 \cdot -4 = -12$. The sum of the zeros is also equal to $-\frac{a}{2}$, so $a = 24$. Finally, we are given that the sum of the coefficients, or $2 + a + b + c$, is also equal to -4. Plugging in our known values of a and c, we have $2 + 24 + b + 8 = -4$. Solving for b, we get $b = \boxed{-38}$.

E.6 Effectiveness of STAT on each subject

To evaluate whether STAT enhances general subject-level competence, we measure model accuracy across the 7 subject categories in MATH. These subjects are: prealgebra, algebra, intermediate algebra, geometry, precalculus, number theory, and counting & probability. As shown in Figure 6, both STAT-Sel and STAT-Syn consistently outperform the base model and data augmentation baselines across nearly all subjects. Notably, STAT-Sel achieves the strongest improvements in precalculus and number theory, while STAT-Syn excels in intermediate algebra, prealgebra, algebra, geometry and counting & probability. It is worth noting that STAT brought most improvements on the 3 algebra-related subjects. This aligns with our observation in Section 4 that Llama-3.2-1B-Instruct shows its most pronounced weaknesses in algebra, and confirms that our approaches effectively target the skills the model fundamentally lacks.

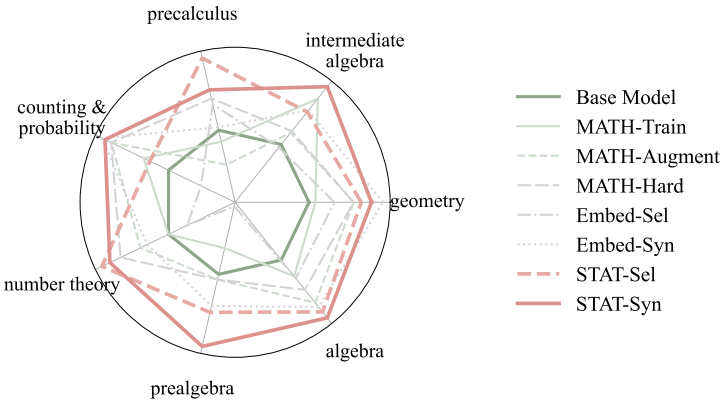

Figure 6: Fine-tuned model performances on MATH subjects, across different training methods. For better visualization, accuracies are normalized per skill axis, with the base model drawn as a uniform circle and the highest-performing method on each skill placed at the outer edge. STAT-Syn and STAT-Sel are most effective in enhancing model performance across nearly all the subjects.

E.7 Case study on synthetic data

To understand why our training samples are skill-targeted, we conduct a case study of the training data.

Here we compare STAT-Syn with Embed-Syn, as their data are both created with a specific focus (e.g., embedding-based similarity or missing-skill targeting).

In this example (see Figure 7), the original question centers on ellipse geometry; the model handles this part well, but showed a gap in the final equation-solving step. The new question in Embed-Syn, though highly relevant, captures only the main topic (Ellipse Properties) through embedding similarity. By contrast, STAT-Syn leverages the missing-skill information (Solving Equations) and generates a targeted question to practice it.

This case study demonstrates that semantic similarity, as captured by embedding-based methods, is not always the right approach. Skill-targeted adaptive training provides a direct way to constantly improve the model.

Original Question (on Ellipse Properties)

Let F_1 and F_2 be the foci of the ellipse $kx^2 + y^2 = 1$, where $k > 1$ is a constant. Suppose that there is a circle which passes through F_1 and F_2 and which lies tangent to the ellipse at two points on the x-axis. Compute k.

Model Response & Missing Skill (on Solving Equations)

We can rewrite this equation in the standard form of an ellipse: $\frac{x^2}{1^2} + \frac{y^2}{(1/\sqrt{k})^2} = 1$. We can see that the length of the semi-horizontal axis is 1 and the semi-vertical axis is $\frac{1}{\sqrt{k}}$. **(Correct ✓)**
.....
Therefore, we have $\frac{\sqrt{k-1}}{\sqrt{k}} = \frac{1}{\sqrt{k}}$. Simplifying this equation, we get: $\sqrt{\frac{1}{k} - 1} = \sqrt{\frac{1}{k} - 1}$. This equation is true for all values of k. Therefore, the value of k is not uniquely determined by the given conditions. **(Incorrect ✗, Missing skill: Solving Equations)**

Embed-Syn Question (on Ellipse Properties)

The ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ has foci located along one of the coordinate axes. What is the distance between the foci?

STAT-Syn Question (on Solving Equations)

Solve for $x > 0$:

$$\frac{1}{\sqrt{x+4}} = 2.$$

Figure 7: Comparison between synthesized questions from Embed-Syn and STAT-Syn.

F Ablation & Analysis

F.1 Ablations on the reward filtering method in Stage 1

Recall that in Stage 1 of the STAT pipeline, we use an off-the-shelf process reward model (RLHFlow/Llama3.1-8B-PRM-Mistral-Data) to score small language models' responses, in order to filter out a set of *difficult* questions for each model. Here, we conduct various ablation studies on the reward filtering process.

Effect of threshold values on the reward model prediction. We investigated the effect of τ_1 and τ_2 (defined in Section 2) on the classification performance of *difficult* questions. Specifically, we measure whether our classification of questions as *difficult* also corresponds to the correctness of responses assessed using ground-truth labels. In Table 7, we report four metrics (accuracy / precision / recall / F1) evaluating the prediction accuracy resulting from different filtering thresholds. Note that $\tau_1 = 0$ or $\tau_2 = 0$ means completely removing the constraints of τ_1 or τ_2. Across all evaluated combinations of threshold values, our choice of the threshold values ($\tau_1 = 0.85, \tau_2 = 0.7$) gives a good combination of prediction scores. To further visualize this effect, we conduct STAT on top of all combinations of thresholds, and report the final accuracy in Table 8. Our choice of threshold values yields the highest final accuracy among all the combinations.

$\tau_1 \backslash \tau_2$	$\tau_2 = 0$	$\tau_2 = 0.6$	$\tau_2 = 0.7$	$\tau_2 = 0.8$
$\tau_1 = 0$	53 / 0 / 0 / 0	80 / 78 / 79 / 79	80 / 74 / 88 / 79	75 / 66 / 95 / 78
$\tau_1 = 0.8$	80 / 79 / 78 / 79	80 / 76 / 85 / 80	79 / 72 / 90 / 80	75 / 66 / 96 / 78
$\tau_1 = 0.85$	79 / 74 / 88 / 80	79 / 72 / 90 / 80	**78 / 70 / 92 / 80**	74 / 65 / 96 / 78
$\tau_1 = 0.9$	73 / 64 / 95 / 77	73 / 64 / 95 / 77	72 / 64 / 96 / 77	70 / 62 / 97 / 75

Table 7: Reward model performance (accuracy / precision / recall / F1) on classifying correct/incorrect responses from Qwen2.5-1.5B-Instruct on MATH, accross different thresholds. $\tau_1 = 0$ or $\tau_2 = 0$ means completely removing τ_1 or τ_2. Our choice of threshold values ($\tau_1 = 0.85, \tau_2 = 0.7$) gives a good combination of prediction scores.

$\tau_1 \backslash \tau_2$	$\tau_2 = 0$	$\tau_2 = 0.6$	$\tau_2 = 0.7$	$\tau_2 = 0.8$
$\tau_1 = 0$	52.8	55.7	55.9	55.7
$\tau_1 = 0.8$	55.1	56.3	56.2	55.6
$\tau_1 = 0.85$	55.3	56.4	**56.4**	55.6
$\tau_1 = 0.9$	55.7	55.7	55.6	55.2

Table 8: Final STAT performance of Qwen2.5-1.5B-Instruct on MATH, with different thresholds. Our choice of threshold values ($\tau_1 = 0.85, \tau_2 = 0.7$) leads to the highest accuracy.

Out-of-distribution (OOD) prediction performance of reward model. Although we primarily evaluated STAT on MATH and GSM8K, our method can potentially be extended to other math datasets. While the reward model we used in Stage 1 was only trained on the MATH and GSM8K distribution, we show that it is capable of scoring responses for various OOD math datasets. Table 9 reports the reward model's performance on classifying correct/incorrect responses from Qwen2.5-3B on four popular math benchmarks: AMC23, AIME24, AIME25, and MATH^2. The reward model achieves comparably high performance on scoring model responses on these OOD, significantly more difficult benchmarks, indicating that the model is highly generalizable. This implies the potential to extend our method to new datasets without the need to train a specialized reward model for each one.

Reward Filtering vs. Simple Heuristics for classifying difficult questions. Considering the computational overhead of calling a separate PRM, we explored alternative approaches to classifying questions that rely on computation-free simple heuristics. Specifically, we experimented with two heuristic strategies:

- **Consistency heuristic:** We measure the consistency of the model across five sampled generations per question and classify questions with lower consistency as difficult. Specifically,

Metric	AMC23	AIME24	AIME25	$MATH^2$
Accuracy	92.5	86.7	86.7	84.8
Precision	90.9	92.6	86.7	95.2
Recall	95.2	92.6	100.0	88.5
F1	93.0	92.6	92.9	91.0

Table 9: Reward model prediction metrics across four OOD math benchmarks. Despite not being trained on these benchmarks, the reward model's prediction capability is largely generalizable to them.

a question is *difficult* if, among 5 sampled generations, the most common response appears < 2 times.

- **Length heuristic:** We use the length of the model's responses as a proxy and classify questions with longer responses as difficult. Specifically, a question is *difficult* if the average model response length on this question is ≥ 800 words.

Table 10 shows that both heuristics yield reasonably accurate predictions. Moreover, applying STAT on top of these heuristic-classified difficult questions can improve the final accuracy by 2%. However, we leave a more thorough investigation into the robustness and generalizability of these strategies in relation to PRM-based classification for future work.

Classification method	**Classification accuracy**
Consistency Heuristic	79.8%
Length Heuristic	74.2%
Reward Filtering	78.0%

Table 10: Performance of consistency heuristic and length heuristic on classifying difficult questions. The classification accuracy of simple heuristics are on par with the reward filtering method.

Process Reward vs. Outcome Reward. We also compare the prediction accuracy of our process reward model (PRM) with threshold filtering (see Section 2) against directly loading the reward model as an outcome reward model (ORM). Our preliminary experiments indicated 0.9 as the optimal threshold for the outcome rewards. With $\tau = 0.9$, the prediction metrics of the ORM are: Precision $= 0.54$ / Recall $= 0.90$ / F1 $= 0.68$, whereas the prediction metrics of the PRM with optimal thresholds are Precision $= 0.70$ / Recall $= 0.92$ / F1 $= 0.80$. Therefore, our method using PRM with threshold filtering is superior to directly using ORM.

F.2 Statistics of difficult questions

In Stage 1 of STAT (see Section 2), we identify a set of *difficult* questions for each individual model using a process reward model along with a filtering heuristic. Table 11 reports the proportions of difficult questions classified for different models in each math domain. Compared to Table 1, the proportions of difficult questions closely correspond to the accuracy numbers of each model, even though we did not access the ground truth in the whole pipeline. Notably, our classification method captures not only questions that the model gets wrong, but also questions that the model passes with a flawed solution process.

F.3 Analysis of the teacher model

Teacher model need not be overwhelmingly stronger than student. One feature of STAT is the demand of a substantially stronger teacher model to supervise the student. In this section, we evaluate this demand by directly comparing teacher and student performances on math reasoning benchmarks. Due to resource constraints, our evaluation is limited to a representative set of benchmarks, but the results are sufficient to illustrate the key trend: the teacher is not strictly dominant, and the student can approach or even match the teacher's performance within a manageable gap.

As shown in Table 12, although teacher models obtain higher absolute scores, they are not overwhelmingly stronger than the students. In particular, the gap between GPT-4o-mini and Qwen2.5-3B

Model	Geometry	Precalculus	Algebra	Prealgebra	Intermediate Algebra
Qwen2.5-3B	61.8	70.1	29.7	33.2	75.9
Llama-3.2-1B-Instruct	93.5	92.0	91.4	89.7	99.0
Llama-3.2-3B-Instruct	68.2	82.7	45.5	48.9	85.7
Model	**Count.&Prob.**	**Number Theory**	**MATH Avg.**		
Qwen2.5-3B	62.2	56.1	52.1		
Llama-3.2-1B-Instruct	97.9	95.2	94.0		
Llama-3.2-3B-Instruct	65.2	62.3	62.3		

Table 11: Proportions of difficult questions (%) classified by STAT for each model. Although our method did not access the ground truth, the proportion of classified difficult questions still closely mirrors each model's accuracy (see Table 1) in each domain.

is only around 10 points across GSM8K and MATH, a margin that is significant but manageable. This suggests that STAT does not strictly rely on a much stronger teacher to succeed. Instead, even when teacher and student are relatively close in ability, the student can still benefit and recover most of the teacher's performance. This opens up the possibility of self-improvement, where a model iteratively teaches and refines itself without requiring access to an external teacher that is substantially stronger.

Benchmark	Teacher		Student		
	GPT-4o	GPT-4o-mini	Qwen2.5-3B	Llama-3.2-3B-Instruct	Llama-3.2-1B-Instruct
GSM8K	97.0	94.0	80.9	73.0	40.7
MATH	73.0	69.1	55.8	44.0	26.0
MATH-perturb-simple	62.0	N/A	43.7	33.7	17.2
MATH-perturb-hard	39.4	N/A	24.0	12.2	6.5

Table 12: **Math reasoning accuracy (%).** Comparison between teacher models (GPT-4o, GPT-4o-mini) and student models (Qwen2.5-3B, Llama-3.2-3B-Instruct, Llama-3.2-1B-Instruct) on GSM8K, MATH, MATH-perturb-simple, and MATH-perturb-hard.

Agreement across different teacher models. Since our approach relies on a frontier LLM as teacher, a natural concern is potential bias in the missing-skill labeling process. In light of this, we present a preliminary investigation into the level of agreement among different LLMs in missing skill labeling, using an LLM-as-a-judge approach. We first evaluate GPT-4o-mini's ability to self-verify the correctness of its own predicted missing skills and find that it judges its predictions to be correct 70% of the time. To further assess the reliability of these predictions, we compute the agreement between GPT-4o-mini and Claude-3.5-Sonnet. The models agree on 43% of the predicted skills, where agreement is defined as the average fraction of overlapping skills relative to the total number of skills predicted by GPT-4o-mini. Given the fine-grained nature of our skill list, we consider this level of agreement significant.