Are Reflective Words in Large Reasoning Models a Sign of Genuine Capability or Memorized Patterns?

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Abstract

Recent Large Reasoning Models exhibit strong reasoning abilities in tasks like mathematics and logical inference, notably through human-like self-verification and reflection in their chain of thought. However, it remains unclear whether these reflective statements stem from genuine internal mechanisms or are merely memorized patterns. From a model interpretability perspective, this work investigates LRMs' representation space to determine whether specific features causally govern reflective capabilities. Using a difference-in-means approach, we extract Self-Reflection Features by contrasting model activations during self-reflection versus affirmative answering. Further causal analysis reveals that these features strongly influence knowledge parameters associated with reflection words, suggesting that such outputs are genuine manifestations of internal mechanisms rather than memorization. Finally, causal interventions demonstrate that modulating these features flexibly adjusts the model's self-reflective intensity.

1 Introduction

Recently, the emergence of Large Reasoning Models (LRMs) (OpenAI et al., 2024b; DeepSeek-AI et al., 2025; Team, 2025) optimized through reinforcement learning, has opened up new possibilities and room for advancement in the reasoning capabilities of language models. These advancements are particularly evident in tasks such as mathematics (Cobbe et al., 2021; Hendrycks et al., 2021), logical reasoning (Luo et al., 2024), and understanding scientific questions (Welbl et al., 2017). These Reasoning models excel at deconstructing complex problems into simpler, sequential sub-problems within their extensive chains of thought. Most impressively, they often adopt a human-like reasoning tone (Guo et al., 2025; Yang et al., 2025), seemingly engaging in self-verification and reflection (Gandhi et al., 2025), and evaluating their own proposed solutions before summarizing and recommending the most suitable option to the user.

Therefore, this raises a crucial question: do these reflective statements and verification words executed within the chain of thought represent **a genuine activation of the models' internal reflective capabilities, or are they simply reproductions of patterns memorized from their training data?**

In this work, from a model interpretability perspective, we delve into the representation space of LRMs. We aim to uncover whether specific existing features genuinely govern the deployment of these reflective capabilities, and to establish if a causal relationship exists between the activation of such features and the reflection words manifested in a reasoning model's chain of thought.

Specifically, in §3, we employ the *difference-in-means* technique (Marks and Tegmark, 2023; Rimsky et al., 2024) to extract **Self-Reflection Features** from four Large Reasoning Models (Guo et al., 2025; Team, 2025) across both mathematical and code datasets by contrasting the internal representations of the models when they engage in self-reflection versus when they provide affirmative answers. These features were subsequently visualized using Principal Component Analysis.

In §4, we conduct a causal analysis of Self-Reflection Features in LRMs from both internal and external perspectives. We identify knowledge parameters within the models that are highly correlated with reflection words and demonstrate that the presence of Self-Reflection Features amplifies the activation of these parameters in §4.1. This suggests that the reflection words in LRM chain-of-thought are genuine manifestations of these activated features, not just memorized patterns. Crucially, through causal intervention experiments detailed in §4.2, we further show that manipulating these extracted Self-Reflection Features allows for flexible modulation of the model's self-reflection intensity when answering questions. To conclude,

we uncovered and verified that the reflection words in Large Reasoning Models genuinely reflect the activation of their internal reflective capabilities.

2 Background and Related Work

2.1 Self-Reflection in Large Reasoning Models

The development of Large Reasoning Models (LRMs) (OpenAI et al., 2024b; Guo et al., 2025; Team, 2025) has opened up new prospects for enhancing the reasoning paradigms of language models. Most notably, they demonstrate impressive human-like self-reflection (Guo et al., 2025; Liu et al., 2025) and verification capabilities when engaged in the long chain of thoughts (Wei et al., 2023; Li et al., 2025).

Regarding the **human-like expressions** in the chain of thoughts (CoT) exhibited by LRMs, several studies have conducted preliminary investigations from the perspective of Reinforcement Learning training dynamics (Gandhi et al., 2025; Yang et al., 2025; Yu et al., 2025b). And in terms of the LRM's ability to **assess itself's uncertainty** or engage in self-reflection, existing research has explored both explicit and implicit ways to estimate the uncertainty. For explicit uncertainty, prior work proposed prompting strategies that guide LRMs to verbalize their confidence levels (Zeng et al., 2025). To study implicit uncertainty, researchers have trained probing classifiers on the model's internal representations to estimate its confidence (Zhang et al., 2025; Anthropic, 2025). However, there is still a lack of sufficient interpretability research exploring whether these explicit reflection patterns observed in COT genuinely correlate with the models' actual internal reflective capabilities.

2.2 Linear semantic features

Recent investigations in model interpretability have revealed that, for numerous cognitive behaviors observed in Large Language Models—including refusal to answer (Arditi et al., 2024), jailbreaking (Yu et al., 2025a), reasoning, and knowledge-recall (Hong et al., 2025)—the models encode corresponding linear semantic features within their activation space (Park et al., 2024). These linear semantic features have been discovered and extracted by contrasting inputs that differ primarily in the target semantic dimension (Marks and Tegmark, 2023). Once these features are pinpointed, they offer a mechanism for controlling model behavior through manipulation, which allows for targeted interventions in the generative process (Rimsky et al., 2024; Stickland et al., 2024). Our work extends this line of study by identifying linear features that determine models' engagement in self-reflection.

3 Self-Reflection Features Extraction

3.1 Methodology for Identifying Self-Reflection Features

For current Reasoning Models, given a question Q, we can decompose its output into multiple *Reasoning Segments*: $\{s_1, s_2, s_3, \ldots, s_n\}$. Each segment (except for s_1) represents the model's reflection on the previous segment's proposed approach and a new attempt at solving the target problem. The final segment, s_n, signifies the termination of reflection, and the model directly provides its final answer.

At each *Reasoning Segment*'s final token position during inference, the model can either select the current answer as its final output and terminate, or generate another segment to reflect, verify, and explore alternative solutions. Therefore, based on whether the model initiates a new reflection after a segment or directly provides the final answer, we can categorize the *Reasoning Segments* into two groups. The first group, where the model proposes a new reflection after the segment, we call $\mathcal{S}_{\textit{Check-point}}$. The second group, where the model directly gives the final answer after the segment, we call $\mathcal{S}_{\textit{Termination}}$.

Next, for both groups, we extract the hidden states from the last-token position of each segment s (excluding s_n) at the model's l-th layer[1], denoted as $h^{(l)}(s)$. Since this last-token position corresponds to where the model is about to generate the first token of the subsequent segment, we hypothesize that the hidden states at this crucial juncture store important information guiding the model's decision to either continue with reflection or proceed to termination in the next segment. Then, using the *difference-in-means* technique (Marks and Tegmark, 2023; Rimsky et al., 2024), we calculate the difference between the mean last-token hidden states for these two categories of *Reasoning Segments*:

$$\mathbf{f}^{(l)} = \frac{\sum_{s \in \mathcal{S}_{\textit{Check-point}}} \mathbf{h}^{(l)}(s)}{|\mathcal{S}_{\textit{Check-point}}|} - \frac{\sum_{s \in \mathcal{S}_{\textit{Termination}}} \mathbf{h}^{(l)}(s)}{|\mathcal{S}_{\textit{Termination}}|} \quad (1)$$

[1]Assuming the model has L layers, we conduct experiments on each individual layer of it.

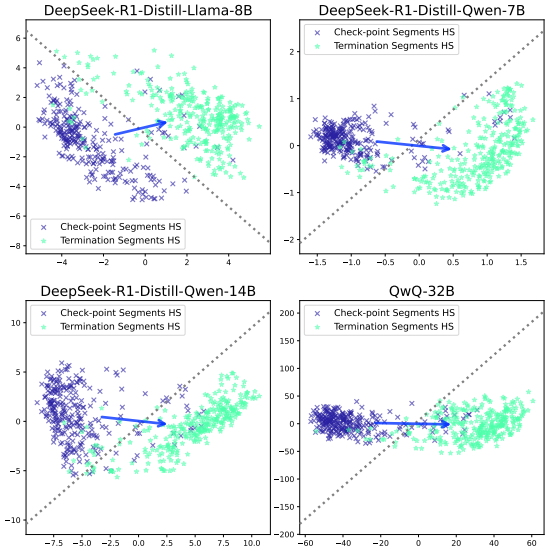

Figure 1: Visualization of the hidden states of four reasoning models on GSM8k dataset using 2-D PCA. The hidden states of datapoints in $\mathcal{S}_{Check\text{-}point}$ and $\mathcal{S}_{Termination}$ are positioned around the boundary (grey dashed line) fitted via logistic regression. The blue arrow approximately indicates the direction of the Self-Reflection Features. Results on other datasets are shown in §C.

The direction of the vector $\mathbf{f}^{(l)}$ represents the direction of the Self-Reflection Features that we extracted. The construction details of $\mathcal{S}_{Check\text{-}point}$ and $\mathcal{S}_{Termination}$ are provided in the next section.

3.2 Experimental Setups

Models We utilize two categories of reasoning models trained under different settings to investigate Self-Reflection Features. The first category includes DeepSeek-R1-Distill-Llama-8B, DeepSeek-R1-Distill-Qwen-7B, and DeepSeek-R1-Distill-Qwen-14B. These models are obtained by performing supervised fine-tuning on the base Llama-3.1 (Meta, 2024) or Qwen2.5 (Qwen et al., 2025) models using high-quality reasoning data generated by the DeepSeek-R1 model (Guo et al., 2025). The second category comprises the QwQ-32B model (Team, 2025), which is trained using reinforcement learning. The inference details are provided in §E.

Datasets We focus on analyzing LRMs on mathematical and coding tasks to facilitate the extraction of Self-Reflection Features and analyze their influence. For the mathematical tasks, we use GSM8k (Cobbe et al., 2021) and MATH-500 datasets (Lightman et al., 2023). For the coding tasks, we select MBPP dataset (Austin et al., 2021).

3.3 Visualization for Self-Reflection Features

Following the methodology outlined in §3.1, we first perform inference on the datasets using the target reasoning models to collect their responses. We then employ GPT-4o (OpenAI et al., 2024a) to automatically segment each response into multiple reasoning segments[2], where each segment independently represents an attempt by the model to solve the problem. Subsequently, based on the segmentation results, we categorize the segments into two groups, $\mathcal{S}_{Check\text{-}point}$ and $\mathcal{S}_{Termination}$. Detailed statistics for these two groups are presented in §B.

We then extract the hidden states from the corresponding positions and compute the Self-Reflection Features by applying Eq. (1). To more clearly visualize the direction of the Self-Reflection Features, we apply Principal Component Analysis (PCA) to the hidden states of data points in the $\mathcal{S}_{Check\text{-}point}$ and $\mathcal{S}_{Termination}$ sets. The results on GSM8k dataset are shown in Figure 1. From this, we observe that two groups of data points are clearly divisible into two clusters by a logistic regression line, explicitly revealing the presence of Self-Reflection Features.

4 Internal and External Causal Analysis of Self-Reflection Features in LRMs

In this section, we will investigate from both internal (model parameter activation) and external (runtime behavior) perspectives, to verify the genuine causal relationships connecting Self-Reflection Features with: (a) the presence of reflection words within chain-of-thought processes in §4.1, and (b) the intensity of the model's self-reflection during actual inference in §4.2.

4.1 Parameter Storing Human-like Reflection words

By applying the Logit Lens method (nostalgebraist, 2020)[3], we identified value vectors within the MLP module's value matrix of the large reasoning models that highly contain these reflection tokens. Specific examples are presented in Table 1. We can observe that the vector projections at corresponding positions in both models each contain a certain number of reflection tokens. Moreover, when the hidden states are in $\mathcal{S}_{Check\text{-}point}$ — that is, when they exhibit stronger self-reflection features — we ob-

[2]The exact prompts used, along with human verification results, are provided in §A of the appendix.

[3]More descriptions of this method and relevant background knowledge are provided in §D of the appendix.

Model	Example Vector	Top-scoring tokens	Activation values in $\mathcal{S}_{Termination}$	Activation values in $\mathcal{S}_{Check\text{-}point}$	Activation values $\mathcal{S}_{More\text{-}Reflection}$
DeepSeek-R1-Distill-Llama-8B	$\mathbf{v}^{31}_{10644}$	`Is, Let, OK, So, Next, If, Now, What,First, See, However, Like, Check, Right, Wait, Again`	0.15	**2.13** ↑2.0	**2.71** ↑2.6
DeepSeek-R1-Distill-Qwen-7B	$\mathbf{v}^{23}_{11862}$	`hi, well, its, hey, nah, its, Im, ye, alternative, _ok,, oh, Hello, notifies, thanks, Ye,waits, WAIT`	0.22	**1.94** ↑1.7	**2.40** ↑2.2

Table 1: Example value vectors identified in two LRMs via the Logit Lens method, showcasing top-scoring tokens related to Self-Reflection Features and their activation values in different reflection stages.

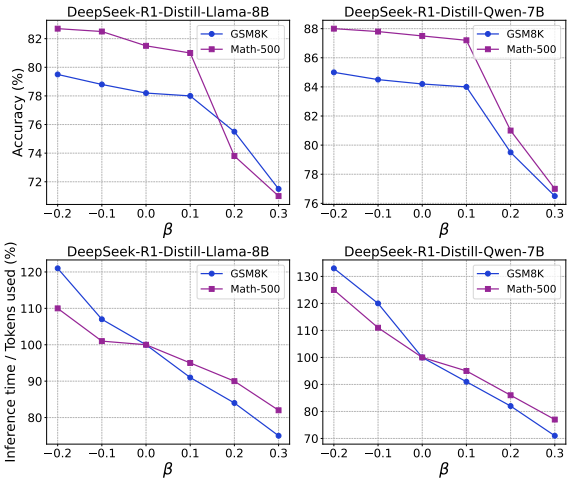

Figure 2: Accuracy and inference time on GSM8K and MATH-500 after intervening in the hidden states of DeepSeek-R1-Distill-Llama-8B and DeepSeek-R1-Distill-Qwen-7B using different β values in Eq. (2).

serve a significant increase in the activation[4] of the target value vector. When we follow Eq. (2) to further enhance the strength of self-reflection features in the model representations (i.e., transitioning to $\mathcal{S}_{More\text{-}Reflection}$ as shown in Table 2), we similarly observe a further increase in their activation.

This provides further support for the idea that the human-like reflection words appearing in the chain-of-thought processes of LRMs are not merely a result of memorizing training data. Instead, they are **a reflection of the genuine activation of Self-Reflection Features.**

4.2 Modulating Self-Reflection Intensity via Linear Feature Intervention

Building upon the Linear Reflection Features extracted from LRMs in §3, in this part we aim to modulate the intensity of model's self-reflection features within the model's representational space during inference for specific tasks, thereby addressing the potential issues of insufficient reflection (Aggarwal and Welleck, 2025) or "overthinking" (Cuadron et al., 2025; Zhang et al., 2025) that current LRMs may exhibit in practical scenarios.

Specifically, following Eq. (2) below, we attempt to control the extent of Self-Reflection ability in LRMs by intervening with Self-Reflection Features through adjusting the hyperparameter β in the model's hidden states:

$$\mathbf{h}'^{(l)}(s) \leftarrow \mathbf{h}^{(l)}(s) - \beta * \mathbf{f}^{(l)}, \tag{2}$$

where $\mathbf{h}^{(l)}(s)$ and $\mathbf{h}'^{(l)}(s)$ denote the hidden states in the l-th layer of the model before and after intervention, respectively, when performing inference on segment s. And $\mathbf{f}^{(l)}$ represents the feature direction we extracted using Eq. 1. Figure 2 shows the intervention effects on GSM8K and MATH-500 for both DeepSeek-R1-Distill-Llama-8B and DeepSeek-R1-Distill-Qwen-7B. Starting from $\beta = 0$, increasing the value of β leads to improved acceleration for the LRMs. However, accuracy does not immediately degrade. Once β reaches around 0.2, further acceleration comes at the cost of a noticeable drop in accuracy. Conversely, decreasing β—thereby increasing the influence of Self-Reflection Features—slightly improves accuracy at the expense of slower inference, which aligns with our hypothesis.

5 Discussion and Conclusion

This work aimed to determine if reflective language in LRMs reflects genuine internal processes or learned patterns. By extracting Self-Reflection Features from the representation space, we found a causal link to reflective words in their chain of thought. Crucially, manipulating these features allowed us to modulate LRM self-reflection intensity. These findings confirm LRMs' reflective statements stem from discernible, governable internal mechanisms, signifying true reflective activation.

[4]Activation refers to the coefficient corresponding to each value vector in Eq. (4).

Limitations

In this study, while we have identified the presence of Self-Reflection features within reasoning models, a comprehensive investigation into their **origins** was not conducted. Specifically, it remains to be clarified whether these features emerge primarily from the pre-training phase or are introduced during subsequent reinforcement learning post-training. Furthermore, the characteristics of the training data that facilitate the encoding of these Self-Reflection features into the model's representational space are yet to be identified. A deeper understanding of these aspects would provide a critical foundation for the future development of more robust and effective reasoning models. We plan to explore these questions in our future work.

Additionally, owing to resource constraints, we were unable to extend our experimental research to larger-scale reasoning models, such as DeepSeek-R1.

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A Prompt used for Reasoning Responses Segmentation

Table 2 presents the prompt we used to query GPT-4o to segment the model's responses into *Reasoning Segments*.

B Detailed Statistics of Segments Groups

We provide detailed statistics of the segment groups for the three datasets — GSM8K, MATH-500, and MBPP — as described in Table 3 below.

C Additional PCA Visualizations on MATH-500 and MBPP

Here, we present additional PCA results for the Self-Reflection Features on MATH-500 and MBPP in Figure 3.

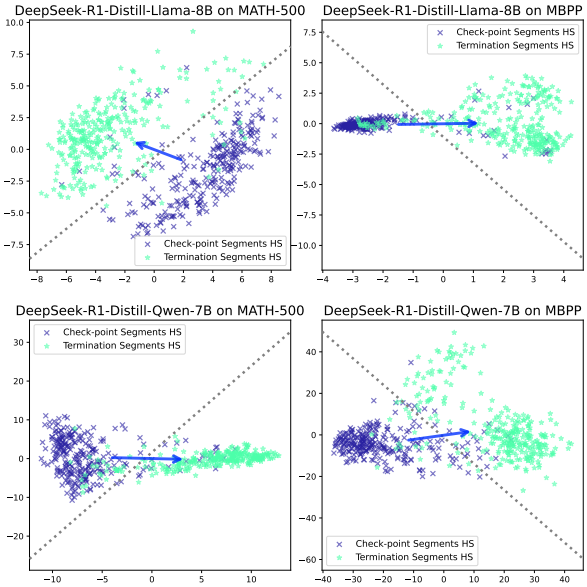

Figure 3: Visualization of the hidden states of two reasoning models on the MATH-500 and MBPP dataset using 2-dimensional PCA. The hidden states of datapoints in $\mathcal{S}_{Check\text{-}point}$ and $\mathcal{S}_{Termination}$ are positioned around the boundary (grey dashed line) fitted via logistic regression. The blue arrow approximately indicates the direction of the Self-Reflection Features. To make the image presentation clearer, we sampled 300 data points from each of $\mathcal{S}_{Check\text{-}point}$ and $\mathcal{S}_{Termination}$ for presentation. Results on other datasets are shown in §C of the Appendix.

D Foundations for Logit Lens Analysis

Here, we provide a brief introduction to the Logit Lens method and the background knowledge it involves.

D.1 MLP in Transformers

In transformer-based language models, the MLP is a crucial component for storing the model's factual knowledge, and its sub-layers can be viewed as key-value memories (Geva et al., 2021). To be specific, the first layer[5] of MLP sublayers can be viewed as a matrix W_K formed by key vectors $\{\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_n\}$, used to capture a set of patterns in the input sequence, and ultimately outputting the coefficient scores. The second layer can be viewed as a matrix W_V formed by value vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$, with each value vector containing the corresponding factual knowledge.

Formally, the output of the MLP in the transformer's ℓ-th layer, given an input hidden state $\mathbf{x}^\ell$, can be defined as:

$$\mathbf{M}^\ell = f\big(W_K^\ell \cdot \gamma(\mathbf{x}^\ell + \mathbf{A}^\ell)\big) W_V^\ell = \mathbf{m}^\ell W_V^\ell, \quad (3)$$

where $W_K^\ell, W_V^\ell \in \mathbb{R}^{n \times d}$. The function f and γ represent a non-linearity[6] and layer normalization, respectively. In the transformer's ℓ-th layer, $\mathbf{m}^\ell \in \mathbb{R}^n$ denotes the coefficient scores, and $\mathbf{A}^\ell$ represents the output of the attention component. The hidden state dimension is d, while the intermediate MLP has a dimension of n. Then, by denoting $\mathbf{v}_j^\ell$ as the j-th column (which will be called the value vector or parameter vector in the following sections) of W_V^ℓ and m_j^ℓ as the j-th element in the coefficients produced by the first layer of the MLP, we can view MLP's output $\mathbf{M}^\ell$ as a linear combination of the value vectors in W_V^ℓ, with their corresponding coefficients $\mathbf{m}^\ell$:

$$\mathbf{M}^\ell = \sum\nolimits_{j=1}^{n} m_j^\ell \mathbf{v}_j^\ell, \quad (4)$$

Each m_j^ℓ here also represents the activation value of the value vector we mentioned in Table 1. Fi-

[5] In most decoder-only models, such as GPT-2 (Radford et al., 2019) and GPT-J (?), the MLP component consists of two layers, whereas in LLaMA (Touvron et al., 2023), it comprises three layers. However, we can still regard LLaMA's first two layers collectively as the key matrices, with their output representing the coefficient scores.

[6] For brevity, the bias term is omitted.

Prompt

- **Analyze the model response and divide into reasoning segments. Return:**

  1. Labeled segments with independent solution attempts
  2. Each segment must include:
     - A new full solution pathway
     - Alternative interpretations (if applicable)
     - Verification/error-checking steps (if applicable)

  Format Requirements:

  - Use **Segment N** headers
  - Mutual exclusivity between segments
  - Avoid single-step fragmentation

  Examples:

  Problem: "Natalia sold clips to 48 of her friends in April, and then she sold half as many clips in May. How many clips did Natalia sell altogether in April and May?"

  Model Response: "[Full model response here...]"

  Segmentation:

  1. **Standard Calculation**:
     Segment 1: Direct arithmetic approach
     - April: 48 clips (given)
     - May: 48/2 = 24 clips
     - Total: 48 + 24 = 72
     - Verification: 40 + 20 + 8 + 4 = 72
  2. **Algebraic Reformulation**:
     Segment 2: Symbolic representation
     - Let $A = 48$ (April sales)
     - Define $M = A/2$ (May sales)
     - Total $T = A + M = 1.5A$
     - Compute $1.5 \times 48 = 72$
  3. **Semantic Analysis**:
     Segment 3: Ambiguity resolution
     - Challenge: "sold to friends" interpretation
     - Reject per-friend vs. total sales hypotheses
     - Confirm 48 = total clips (not friends count)

  Current Problem: {Problem}

  Current Model Response: {Response}

  Segmentation:

Table 2: Prompt for segmenting mathematical reasoning processes.

Dataset	Avg. Tokens in Termination Segments	Avg. Tokens in Check-point Segments
GSM8K	52.3	38.7
MATH-500	67.8	44.2
MBPP	41.6	29.5

Table 3: Detailed statistics of the segment groups for the three datasets — GSM8K, MATH-500, and MBPP.

nally, the hidden states at the ℓ-th layer of the language model can be defined as:

$$X^{\ell+1} = X^{\ell} + \mathbf{M}^{\ell} + \mathbf{A}^{\ell}, \tag{5}$$

where X^{ℓ}, $\mathbf{M}^{\ell}$ and $\mathbf{A}^{\ell}$ represent the hidden states, MLP's output, and the attention component's output in the transformer's ℓ-th layer, respectively.

D.2 Logit Lens

nostalgebraist (2020); Geva et al. (2021) proposed that the hidden states or module parameters of a transformer-based model can be directly decoded into the vocabulary space using the model's pretrained unembedding matrix, enabling an investigation into the information they encode:

$$Projection = E\mathbf{v}_j^{\ell}, \tag{6}$$

Here, E denotes the model's pretrained unembedding matrix, and the result of the projection, which lies in $\mathbb{R}^{|\mathcal{V}|}$, is a vector assigning a score to each token in the vocabulary $\mathcal{V}$. The set of the top-k highest-scoring tokens in this projection, denoted by $\mathcal{T}_{j,k}^{\ell}$, often reveals a clear pattern that corresponds to a specific knowledge being promoted by $\mathbf{v}_j^{\ell}$ during inference (Geva et al., 2022b,a).

E Implementation Details of the LRMs

For the inference settings of all four Large Reasoning Models, we use a temperature of 0.6, a top-p value of 0.95, and set the maximum generation length to 32,768 tokens, following the default settings.

All the experiments in this work were conducted on four 80GB NVIDIA A800 GPUs.