COLLABEDIT: TOWARDS NON-DESTRUCTIVE COL-LABORATIVE KNOWLEDGE EDITING

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Abstract

Collaborative learning of large language models (LLMs) has emerged as a new paradigm for utilizing private data from different parties to guarantee efficiency and privacy. Meanwhile, *Knowledge Editing (KE)* for LLMs has also garnered increased attention due to its ability to manipulate the behaviors of LLMs explicitly, yet leaves the collaborative KE case—in which knowledge edits of multiple parties are aggregated in a privacy-preserving and continual manner—unexamined. To this end, this manuscript dives into the first investigation of collaborative KE, in which we start by carefully identifying the unique three challenges therein, including *knowledge overlap, knowledge conflict*, and *knowledge forgetting*. We then propose a non-destructive collaborative KE framework, COLLABEDIT, which employs a novel model merging mechanism to mimic the global KE behavior while preventing the severe performance drop. Extensive experiments on two canonical datasets demonstrate the superiority of COLLABEDIT compared to other destructive baselines, and results shed light on addressing three collaborative KE challenges and future applications.

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1 INTRODUCTION

028 Large Language Models (LLMs) (Achiam et al., 2023; Qiao 029 et al., 2023) recently have emerged as the promising solution toward general artificial intelligence. However, deploy-031 ing LLMs in practice usually requires customizing LLMs with specific knowledge (Meng et al., 2022), where re-training 032 LLMs may be expensive and unacceptable (Jang et al., 2023). 033 Accordingly, Knowledge Editing (KE) (Meng et al., 2022; 034 Mitchell et al., 2022a; Tan et al., 2024; Zhang et al., 2023), 035 which allows efficient modification of knowledge stored in existing models, has been proposed as an alternative solution. 037

To explicitly update LLMs with knowledge from multiple parties or organizations—each possesses a distinct and private dataset (Ye et al., 2024; Wu et al., 2022; McMahan et al., 2017)—and meet individual demands, current KE

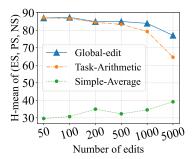
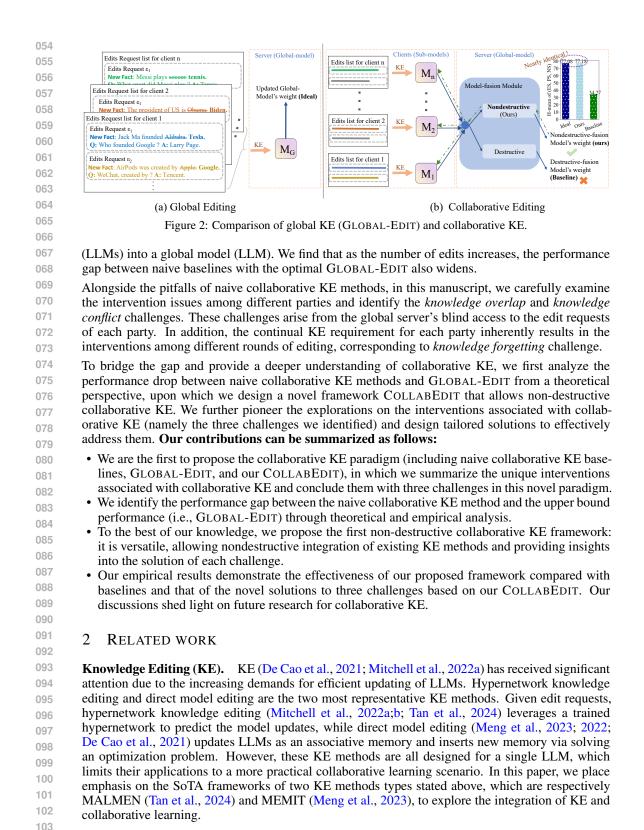


Figure 1: Limits of existing KE methods under the collaborative KE scenarios on the Multi-CounterFact dataset (Meng et al., 2022).

methods (Meng et al., 2022; 2023) first need to collect edit requests from these parties with violated privacy concerns: the edit request itself contains sensitive private information and thus becomes infeasible for sharing. It motivates resorting to the cross-silo collaborative learning paradigm (Wu et al., 2023; Kairouz et al., 2021)—by only communicating the locally-updated-models, rather than uploading a list of risky edit requests—namely collaborative KE for LLMs.

However, existing KE methods are all designed for the single-party single-model scenario (Meng et al., 2022; Mitchell et al., 2022a; Tan et al., 2024; Meng et al., 2023). Noting that model merging (MM) techniques (Ortiz-Jimenez et al., 2023; Chronopoulou et al., 2023; Yadav et al., 2023) allow a straightforward extension of KE methods to a collaborative KE scenario. Therefore as our (side)-contribution, we examine a naive combination of local KE and MM techniques, and compare them with the optimal global KE method (GLOBAL-EDIT): we can witness that all these naive collaborative KE methods are destructive. In detail, we conduct independent KE (i.e., MEMIT (Meng et al., 2023)) on each party locally, and then use model merging techniques like Simple-Average (Chronopoulou et al., 2023) and Task-Arithemetic (Ilharco et al., 2023) to merge local models



Collaborative learning and model merging. Collaborative learning (Kairouz et al., 2021; Wang et al., 2021; Fan et al., 2024; Mohtashami et al., 2023) allows multiple parties to jointly and continuously learn a machine learning model by sharing their updates to a global server for aggregation. Alongside the orthogonal techniques to address data heterogeneity issue (Karimireddy et al., 2020; Li et al., 2019), model aggregation/merging (Li et al., 2023b; Wortsman et al., 2022; Ortiz-Jimenez

108 et al., 2023; Yadav et al., 2023) has emerged as a promising research direction to collaborative 109 learning, which employs the global server to directly merge model updates in the weight space 110 without disclosing the training data of each party. The most commonly used model merging tech-111 niques themselves are Simple-Average (SA) (Chronopoulou et al., 2023; Wortsman et al., 2022) and Task-Arithmetic (TA) (Ortiz-Jimenez et al., 2023). Moreover, TIES-merging (Yadav et al., 2023) 112 has recently proposed to further enhance the merging performance by solving the symbol conflicts 113 among different models. However, all existing model merging techniques only achieve destructive 114 editing performance when used for collaborative KE, which inevitably results in knowledge loss 115 during the merging process. 116

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3 PRELIMINARIES OF COLLABORATIVE KNOWLEDGE EDITING

We first introduce the basics of KE in a single LLM. Then, we illustrate the naive approaches to conduct collaborative KE. Finally, we describe the inherent interventions within collaborative KE.

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3.1 INTRODUCTION TO KNOWLEDGE EDITING IN A SINGLE LLM

124 LLMs can answer natural-language queries about *facts* based on implicit knowledge encoded within 125 the parameters. Following Meng et al. (Meng et al., 2023), we define a fact f as "(*subject s, relation* 126 r, *object o*)", e.g., "(s = Danielle Darrieux, r = spoke the language, o = French)". Given a $127 sequence of facts <math>\mathcal{E} = \{f_i | f_i = (s_i, r_i, o_i)\}$ to edit (denoted as *edit requests*), knowledge editing 128 aims to maximize the likelihood that the updated LLM \mathcal{M}_{θ} predicts the desired object o_i for any 129 factual prompt $\mathbf{x} \oplus p(s_i, r_i)$, which involves a prefix \mathbf{x} and a templated prompt $p(s_i, r_i)$:

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$$\arg\min_{\mathcal{M}_{\boldsymbol{\theta}}} \frac{1}{|\mathcal{E}|} \sum_{i=1}^{|\mathcal{E}|} \mathbb{E}_{\mathbf{x}} \left[-\log \Pr_{\mathcal{M}_{\boldsymbol{\theta}}} \left[o_i | \mathbf{x} \oplus p(s_i, r_i) \right] \right].$$
(1)

The state-of-the-art knowledge editing methods (Meng et al., 2022; 2023; Tan et al., 2024) found that modifying a small sequence of MLP layers in the critical path of LLM is sufficient to edit its factual associations. In particular, linear operation \mathbf{W}^l in an MLP layer can operate as a key-value store for input keys \mathbf{K}^l and the memory/knowledge values \mathbf{M}^l , where input keys correspond to the intermediate feature vector of the model from a set of edit requests. Knowledge editing modifies each MLP layer such that it associates \mathbf{K}^l to the desired \mathbf{M}^l by solving $\mathbf{W}^l\mathbf{K}^l \approx \mathbf{M}^l$. For brevity, we will describe knowledge editing for a specific layer and omit *l* throughout the paper.

139 Given a set of facts \mathcal{E} to edit (i.e., edit requests), we first obtain their input keys $\mathbf{K} = [\mathbf{k}_1, \dots, \mathbf{k}_{|\mathcal{E}|}]$ to the layer l via a single feed-forward. We also obtain the desired memory values $\mathbf{M} =$ 140 $[\mathbf{m}_1,\ldots,\mathbf{m}_{|\mathcal{E}|}]$ of layer l that maximize $\Pr[o_i|\mathbf{x} \oplus p(s_i,r_i)]$. The goal of editing the layer l can 141 be formulated as optimizing the Δ such that the updated weight $W + \Delta$ associates the input keys K 142 to the desired memory values M. Note that the MLP layer also contains previously stored memories 143 of existing knowledge, which should be preserved during the knowledge editing. Therefore, we also 144 maintain the associations between input keys of existing knowledge \mathbf{K}_{init} and their memory values 145 (WK_{init}). Following MEMIT (Meng et al., 2023), we derive the closed form of Δ for a specific 146 layer *l* as: 147

$$\boldsymbol{\Delta} = \mathbf{R}\mathbf{K}^{\top}(\mathbf{C} + \mathbf{K}\mathbf{K}^{\top})^{-1},\tag{2}$$

where $\mathbf{C} = \mathbf{K}_{\text{init}} \mathbf{K}_{\text{init}}^{\top}$ is the covariance matrix of the input keys of existing knowledge, and $\mathbf{R} = \mathbf{M} - \mathbf{W} \mathbf{K}$ represents the residual error in the output space of layer *l*. See more details in Appendix A.

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3.2 DESTRUCTIVE MODEL MERGING ENCOUNTERS KNOWLEDGE EDITING

153 KE in practice involves editing the factual associations of LLM, such as correcting the hallucinations 154 or updating outdated information. This process often requires handling simultaneous edit requests, where multiple parties or clients access and collaboratively contribute to the same LLM service. 155 Though *Global KE* (GLOBAL-EDIT) illustrated in Figure 2(a) represents the ideal editing cases, 156 it also necessitates each client to directly share the edit requests with the server, which violates 157 the privacy constraints. Collaborative KE in Figure 2(b) instead allows each client to edit on its 158 local model and only rely on the server to aggregate the edit updates using the model merging 159 techniques (Wortsman et al., 2022; Ortiz-Jimenez et al., 2023; Yadav et al., 2023). 160

161 However, existing KE algorithms are all designed for a single client and cannot be trivially generalized to the collaborative KE scenario. As evidenced in Figure 1, naively extending existing editing methods or model merging methods yields a dramatic performance drop compared to that of the
 GLOBAL-EDIT (upper bound), especially when the number of edits increases. Given the limits of *de- structive collaborative KE* methods, we aim to develop a *non-destructive collaborative KE* method
 that can achieve a similar editing performance as GLOBAL-EDIT, even with a large number of edits.

167 3.3 INTERVENTIONS WITHIN COLLABORATIVE KNOWLEDGE EDITING

In addition to the performance drop, our proposed concept of collaborative KE also suffers from several key challenges, due to the unique characteristics of this scenario. By default, we assume a trustworthy and non-adversarial collaborative KE scenario. The collaborative KE employs a global server to aggregate the edits of local clients without disclosing their edit requests, while requiring each client to continually edit the global model by updating its local model in a multi-round manner. However, there still exist several unique challenges due to the interventions among different clients and different rounds of editing, warranting research in the future.

176 3.3.1 INTERVENTIONS AMONG DIFFERENT CLIENTS

177 In collaborative KE, multiple clients may use similar edit requests to update their local models and 178 send the updated models to the global server for aggregation. The interventions among clients are 179 then raised in the editing event $e := (s_1, r_1, o_1 \rightarrow o_2, t_1, m_1)$, which includes s_1 as a subject, r_1 as 180 a relationship, o_1 and o_2 as objects, t_1 as a editing timestamp and m_1 as client model.

181 **Knowledge conflict** indicates that edit requests from the same/different clients (in the same round¹) 182 share the same subject *s* and relation *r* but with different objects *o*. Such a conflict renders the 183 effectiveness of knowledge editing and may even compromise the overall KE performance. We 184 elaborate the general formulation of *conflict edit* below (detailed illustration can be found in the 185 Table 7 of Appendix):

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$$\begin{aligned}
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& (e_1 = (s_1, r_1, o_1 \to o_2, t_1, m_1) \\
& (e_2 = (s_1, r_1, o_1 \to o_3, t_2, m_2)
\end{aligned}$$
(3)

where local model m_1 and m_2 perform a conflicting editing for the same subject s_1 and relationship r_1 at timestamp t_1 and t_2 respectively, changing the same original object o_1 to different o_2 and o_3 .

Similar to the composite edit operations mentioned by Li et al. (2024), composite conflict and composite overlap arising from such operations may also occur in collaborative KE scenarios, with even more diverse and complex forms. Here we aim to briefly introduce the key concept of knowledge conflict, and a more detailed definition and investigation of this issue left for future work.

Knowledge overlap is a simplified case of knowledge conflict, where the object changing relationship (i.e. $o_1 \rightarrow o_2$ and $o_1 \rightarrow o_3$) in editing events of e_1 and e_2 becomes identical. Knowledge overlap is also closely related to the overfitting problem in machine learning, in which excessive overlapped editing requests can degrade the model's editing performance on other edit requests (excluding those repeated edit requests).

200 3.3.2 INTERVENTIONS AMONG DIFFERENT ROUNDS OF EDITING

The collaborative KE paradigm naturally requires multiple clients to continually update their local models in a multi-round manner and thus edit the global model with the latest knowledge. **Knowl**edge forgetting issue, therefore, arises given the continual arrival of a large number of new editing requests, alongside the existing knowledge and editing requests.

Assume that each client has a set of old edit requests \mathcal{E}_o , as well as m sets of new edit requests $\mathcal{E}_n = [\mathcal{E}_{n_1}, \mathcal{E}_{n_2}, \cdots, \mathcal{E}_{n_m}]$, where the new edit requests are irrelevant (i.e., their subjects s and relationships r are different) to the old edit requests. The model is initialized by updating the model with the old edit requests \mathcal{E}_o , and the local model of each client will be updated with the new edit requests \mathcal{E}_{n_i} at *i*-th round of editing, followed by the model aggregation step. The knowledge forgetting issue encountered after m rounds of local editing and global aggregation can then be defined as the editing performance on the old knowledge obtained from the old edit requests \mathcal{E}_0 .

In particular, we find that as the value of m increases, the evaluation performance of the model on old knowledge \mathcal{E}_o deteriorates, as evidenced in Section 5.3.

¹In cases of conflict between edits from different rounds, due to the overwriting nature of KE, the latter conflicting edit will overwrite the former, naturally resolving the conflict.

216 4 METHODOLOGY 217

218 COLLABEDIT: NON-DESTRUCTIVE COLLABORATIVE KE 4.1 219

220 To better understand the performance drop, we first explicitly model the relationship between the 221 weight updates Δ_G of the global model using GLOBAL-EDIT and that of each client model Δ_i using 222 local editing. For ease of presentation, we consider the collaborative KE scenario with N clients and each client model has M edit requests. We simplify the theoretical analysis to the single-round 224 editing case and demonstrate the effectiveness of COLLABEDIT for multi-round editing in Remark 2.

225 **Lemma 1** (The relationship between the weight updates from GLOBAL-EDIT and local editing). Take the KE method MEMIT as an example. Following the definitions in Section 3.1, we denote 226 C as an aggregated statistic over the previously stored keys of existing knowledge and use K_i to 227 represent the new keys derived from client i's edit. Then, the relationship between ${f \Delta}_G$ and ${f \Delta}_i$ is 228 measured as: 229

$$\boldsymbol{\Delta}_{G} = \sum_{i=1}^{N} \boldsymbol{\Delta}_{i} \cdot \left(\alpha_{i} := (\mathbf{C} + \mathbf{K}_{i} \mathbf{K}_{i}^{\top}) (\mathbf{C} + \sum_{i=1}^{N} \mathbf{K}_{i} \mathbf{K}_{i}^{\top})^{-1} \right).$$
(4)

See detailed proof in Appendix **B.1**.

Intuition: If we can estimate Δ_G using Δ_i , then we can merge $\{\Delta_i\}_{i=1}^N$ to obtain the same global 233 model as GLOBAL-EDIT and, therefore, obtain non-destructive collaborative KE. 234

235 **Details of COLLABEDIT:** Indeed Δ_G can be represented as the weighted sum of different local 236 weight updates Δ_i with coefficient α_i . However, the coefficient α_i relies on the value of \mathbf{K}_i of all 237 the clients: it breaks the privacy, given the fact that \mathbf{K}_i is an intermediate feature vector of the model 238 from a set of edit requests and any external party can easily reconstruct the edit requests if \mathbf{K}_i is 239 leaked. As a remedy, our COLLABEDIT instead proposes to directly communicate $K_i K_i^{\top}$, in which we prove in Section 6 that $\mathbf{K}_i \mathbf{K}_i^{\mathsf{T}}$ is non-trivial to attack. See our pseudo-code in Appendix D. 240

241 **Remark 1.** Currently, we consider two mainstream KE methods (Akyürek et al., 2023), namely 242 (1) locate and edit activations (same as "Direct model editing" mentioned in Section 2, e.g., 243 MEMIT (Meng et al., 2023) and ROME (Meng et al., 2022)); and (2) train an auxiliary model to di-244 rectly predict parameters (same as "Hypernetwork knowledge editing" mentioned in Section 2, e.g., MEND (Mitchell et al., 2022a) and MALMEN (Tan et al., 2024)). Our framework COLLABEDIT is 245 general enough to integrate many other KE methods, and we leave them for future work. 246

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Justifying the performance drop for destructive editing approaches. We further analyze the performance degradation for destructive editing approaches when the number of edits increases, as 249 illustrated in Figure 1. For the sake of simplicity, we take the TASK-ARITHMETIC (Ilharco et al., 250 2023) with MEMIT as an example. The drop can be explained by:

$$\boldsymbol{\Delta}_{G} - \boldsymbol{\Delta}_{G}^{\prime} = \sum_{i=1}^{N} \boldsymbol{\Delta}_{i} \left[(\mathbf{C} + \mathbf{K}_{i} \mathbf{K}_{i}^{\top}) (\mathbf{C} + \sum_{j=1}^{N} \mathbf{K}_{i} \mathbf{K}_{i}^{\top})^{-1} - \lambda \mathbf{I} \right],$$
(5)

where Δ_G and Δ'_G represent the weight updates derived from COLLABEDIT (our non-destructive collaborative KE) and a destructive collaborative KE using TASK-ARITHMETIC, respectively. We can see that the impact of new knowledge $\mathbf{K}_i \mathbf{K}_i^{\top}$ is negligible compared to existing knowledge \mathbf{C} when the number of edits is small², resulting in $(\mathbf{C} + \sum_{j=1}^{N} \mathbf{K}_i \mathbf{K}_i^{\top})^{-1} \approx \mathbf{C}$ and thus $\Delta_G \approx \Delta'_G$ when $\lambda = 1$. The gap becomes wider when the number of edits increases, contributing to the continuous decline in TASK-ARITHMETIC's performance in Figure 1 compared to GLOBAL-EDIT.

Remark 2 (COLLABEDIT is effective for multi-round editing). Collaborative KE involves mul-261 tiple clients continuously editing the local models and sharing the updated global model across 262 multiple rounds, and thus requires robust support to ensure seamless knowledge integration and consistent knowledge memorization. COLLABEDIT achieves non-destructive collaborative KE for 264 single-round editing—as an approximation of aggregating all edit requests of clients in a specific 265 round and applying global KE to update the global model—remains effective for multi-round editing. 266 Note that multi-round editing is equivalent to applying global KE to iteratively update a single LLM 267 multiple times under the reasonable editing budgets (Gupta et al., 2024).

²We randomly sample 100 edit requests to estimate the norm of $K_i K_i^T$. We observe that the average ℓ_2 -norm of $K_i K_i^T$ is approximately 0.0001% of that of C, which supports the claim.

4.2 REMEDY TOWARDS SOLVING INTERVENTION CHALLENGES IN COLLABORATIVE KNOWLEDGE EDITING: SOME CASE STUDIES

Interventions within collaborative KE scenarios are non-trivial, due to the challenges of explicitly modeling the impacts of editing requests from different clients. Our COLLABEDIT paves the path by mimicking the optimal GLOBAL-EDIT and allowing the non-destructive editing. This subsection case studies how our COLLABEDIT sheds insights on solving unique challenges caused by the interventions among different clients (spatial aspect) and editing rounds (temporal aspect), namely knowledge overlap, knowledge conflict, and knowledge forgetting.

4.2.1 EDITING RESIDUAL DETECTS KNOWLEDGE OVERLAP

COLLABEDIT simplifies the knowledge overlap challenge in collaborative KE scenarios into the over-fitting problem under the global KE scenarios. In other words, multiple clients edit the same piece of knowledge is equivalent to integrating several identical pieces of knowledge into the global model. In detail: performing KE in the model results in weights update Δ and residual \mathbf{R}_{old} , as determined by the input key K. In the case of editing the same knowledge (i.e., same K), we can get new residual $\mathbf{R}_{new} = \mathbf{R}_{old} - \Delta \mathbf{K}$, where the following equation can be leveraged to track the dynamics of KE:

$$\mathbf{R}_{\text{new}} := \mathbf{R}_{\text{old}} - \mathbf{\Delta}\mathbf{K} = \mathbf{R}_{\text{old}} - \mathbf{R}_{\text{old}}\mathbf{K}^{\top} (\mathbf{C} + \mathbf{K}\mathbf{K}^{\top})^{-1} \mathbf{K}.$$
 (6)

Intuitively, (6) explains that the residual should gradually approach $0 \cdot I$. If the residual R gradually approaches zero, then we can accurately detect the knowledge overlap by examining the residual R, as demonstrated in Section 5.3.

4.2.2 ADDRESSING KNOWLEDGE CONFLICT VIA DATA AUGMENTAION

Recall that in rare cases, edit requests from the same/different clients in the same round may share the same subject s and relation r but with different objects o, known as knowledge conflict. An ideal solution to the knowledge conflict should consist of two stages. In the first stage, the global server and clients need to collaboratively detect the conflict in a privacy-preserving manner. For example, when the knowledge conflict occurs, the global server produces poor editing performance on some edit requests. As a result, the clients (who contribute to the edits) could report the issue.

Once the conflict is identified, the server will determine which of the conflicting edit requests to retain for the global model based on the client's report and a predefined strategy (e.g., FCFS (Zhao & Stankovic, 1989) or FIFO (Morse & Richardson, 1983) strategy). The client whose edit request is selected for integration can apply data augmentation techniques, such as incorporating relevant knowledge (Li et al., 2024), to enhance the KE of the selected edit request and effectively resolve the knowledge conflict.

306 4.2.3 DYNAMIC COVARIANCE MATRIX ALLEVIATES KNOWLEDGE FORGETTING

The previously memorized knowledge may be forgotten by the LLM after a large number of edits, termed as *knowledge forgetting* issue. COLLABEDIT simplifies the analysis of this issue and we can witness from (2) that the covariance matrix C of existing knowledge is immutable, amplifying the forgetting as the number of edits increases. As a remedy, we propose using a dynamic version of C, i.e.,

$$\mathbf{C} = \beta_0 \mathbf{C}_0 + \beta_1 \mathbf{C}_1 = \beta_0 \mathbf{C}_0 + \beta_1 \sum_{i} \mathbf{K}_i \mathbf{K}_i^{\mathsf{T}}, \tag{7}$$

where β_0 and β_1 are hyper-parameters that balance the influences of existing knowledge and newly acquired knowledge. C_0 is the covariance matrix of existing knowledge and C_1 is the accumulated covariance matrix of new knowledge. K_i represents the input keys obtained from all the edit requests at the *i*-th round. The dynamic covariance matrix continuously updated for the new knowledge can effectively mitigate the knowledge forgetting issue, as verified in Section 5.3.

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5 EXPERIMENTS

320 5.1 EXPERIMENTAL SETUP

Datasets and models. Following the literature (Meng et al., 2022; 2023), we use Multi-CounterFact
 (MCF) (Meng et al., 2022) and zsRE (Levy et al., 2017) as datasets and evaluate the editing performance on GPT2-XL (Radford et al., 2019) and GPT-J (6B) (Wang & Komatsuzaki, 2021).

Table 1: Overall editing performance on GPT2-XL. GLOBAL-EDIT is 5000 × 1, which means we edit 5000
requests in one model (global model) at one time. GLOBAL-EDIT is an ideal situation. Others are merging methods (500 × 10) where we edit 10 models and each model will be edited by 500 requests. The line of GPT2-XL
means we directly evaluate 5000 requests without any editing operation to test the model's original performance. The "Score" serves as the overall metric for assessing the performance of each method on each dataset.

Method	MCF			zsRE				
	NS ↑	PS ↑	ES ↑	Score ↑	NA ↑	PA ↑	EA ↑	Score †
GPT2-XL	78.24	23.88	21.50	29.65	24.32	21.87	22.80	22.95
GLOBAL-EDIT	65.08	80.66	89.66	77.08	25.25	64.71	68.96	43.12
TIES-MERGING Task-Arithmetic Simple-Average	78.46 66.84 76.90	26.35 55.19 29.97	27.16 61.66 33.06	60.85	24.94 24.97 25.78	25.99 33.66 29.26	27.59 34.80 30.62	26.12 30.45 28.40
CollabEdit	65.26	80.67	89.70	77.18	25.21	64.27	68.40	42.95

Table 2: Overall editing performance on GPT-J (6B), based on MEMIT (Meng et al., 2023). The experimental setting is identical to GPT2-XL in Table 1. The "Score" serves as the overall metric.

Method	MCF				zsRE			
	NS ↑	PS ↑	ES ↑	Score ↑	NA ↑	PA ↑	EA ↑	Score ↑
GPT-J	83.45	17.17	14.78	21.75	26.99	26.25	27.04	26.75
GLOBAL-EDIT	57.20	96.13	99.26	79.03	28.05	88.79	92.05	51.92
Ties-Merging Task-Arithmetic Simple-Average	76.15 50.24 78.04	30.13 72.82 41.28	30.98 73.26 54.68	38.16 63.44 54.22	30.17 18.77 29.19	42.55 45.16 47.96	43.55 46.75 51.38	37.68 30.98 40.22
CollabEdit	57.12	96.03	99.06	78.91	28.26	88.78	92.19	52.17

Baselines. We compare COLLABEDIT with three naive collaborative KE methods, which apply standard KE algorithms (e.g., MEMIT (Meng et al., 2023) and MALMEN (Tan et al., 2024)) to update the local model and use the current model merging algorithms to merge local updates into the global model. In particular, we experiment with three most commonly used algorithms for model merging, including SIMPLE-AVERAGE (Chronopoulou et al., 2023), TASK-ARITHMETIC (Ortiz-Jimenez et al., 2023), and TIES-MERGING (Yadav et al., 2023).

358 Evaluation metrics. Unless otherwise mentioned, we utilize MEMIT as the backend KE algorithm 359 and adopt the same metrics as MEMIT to evaluate editing performance. Strictly following the lit-360 erature (Meng et al., 2022; 2023; Tan et al., 2024), we use Efficacy Score (ES), Paraphrase Score 361 (PS), Neighborhood Score (NS), N-gram Entropy (NE), Reference Score (RS), and Score (i.e., the harmonic mean of ES, PS, NS) as metrics for MCF; we use Neighborhood Accuracy (NA), Para-362 phrase Accuracy (PA), Efficacy accuracy (EA), and Score (i.e., the harmonic mean of NA, PA, and 363 EA) as metrics for zsRE. When using MALMEN (Tan et al., 2024) as the backend KE algorithm, 364 we adopt the same metrics as MALMEN for a fair comparison, including "editing success" (EA), "generalization success" (PA), and "locality success" (NA). See detailed descriptions in Appendix C. 366

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Evaluation benchmark for conflict knowledge editing scenarios. In order to conveniently simu-368 late potential scenarios of collaborative knowledge conflict and analyze the issues and impacts that 369 these scenarios may bring, we reconstruct two existing benchmarks to simulate knowledge conflict 370 situations through GPT-3.5-turbo. Initially, we attempt to explore the impact of knowledge conflict 371 on model performance using Multi-CounterFact (MCF) (Meng et al., 2022) due to its large scale. 372 For each data point (s, r, o) in the MCF dataset, we utilize GPT-3.5-turbo to generate a conflict ob-373 ject that is identical to s and r but differs in o. Section E in the Appendix showcases a concrete 374 example of the generated conflict object. To validate the effectiveness of our two-stage mechanism 375 to resolve knowledge conflict, we utilize the Easy dataset (Li et al., 2024) for the sake of simplicity. This dataset was constructed by creating several additional related knowledge edits for each edit 376 using Wikipedia as the source, which MCF does not include. Additionally, we also generated a 377 corresponding conflict object for each edit in the dataset using GPT-3.5-turbo.

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GPT2-XL (zsRE) GPT-J (zsRE) Method EA ↑ PA ↑ NA ↑ Score ↑ || EA ↑ PA ↑ NA 1 Score ↑ GLOBAL-EDIT 99.21 93.08 16.5 36.84 99.95 95.66 27.32 52.57 TIES-MERGING 15 52 14.85 18 68 16 18 27.86 2676 25.18 26 55 TASK-ARITHMETIC 50.37 45.79 27.59 27.76 4.28 10.89 29.6 26.3 SIMPLE-AVERAGE 52.39 46.03 4.58 11.57 71.15 53.96 4.82 12.49 COLLABEDIT 99.06 92.66 15.49 35.11 99.62 92.88 23.25 47.01

Table 3: Overall editing performance on GPT-J (6B) and GPT2-XL, based on MALMEN (Tan et al., 2024). We
 edit 8 models and each model will be edited by 125 requests of zsRE. The "Score" serves as the overall metric.

389 5.2 EXPERIMENTAL RESULTS OF KE PERFORMANCE

390 Superior collaborative knowledge editing performance. As shown in Table 1 and 2 when using 391 MEMIT (Tan et al., 2024) as the backend KE algorithm, our privacy-preserving solution COLLABE-392 DIT achieves on-par editing performance with that of GLOBAL-EDIT, and significantly outperforms 393 other naive model merging methods in terms of the "Score" on two datasets and two models. Additionally, our COLLABEDIT has nearly identical performance with GLOBAL-EDIT via combining 394 the weight updates of each client, which ensures both privacy protection and editing quality. Nev-395 ertheless, there exists a significant gap between the performance of baselines and GLOBAL-EDIT. 396 Table 3 additionally shows the editing performance of COLLABEDIT when using MALMEN as the 397 backend KE algorithm (Tan et al., 2024): COLLABEDIT is capable of performing nondestructive 398 collaborative KE across various mainstream KE methods. 399

400 **Discussion about the performance of baselines.** Though other baselines (Table 1 and 2) have 401 a relatively higher NS value compared to GLOBAL-EDIT and our COLLABEDIT, we conjecture 402 that it might be caused by the under-fitting phenomenon: these model merging methods are not 403 specifically designed for merging the weight updates from knowledge editing, which is reflected 404 by their low values of PS, ES, and Score. The results of two models GPT2-XL and GPT-J (6B) 405 (the first line) further confirms that the high NS of other baselines are largely due to the inherent high quality of the model itself, exhibiting their poor collaborative KE effects. Note that NS/NA 406 emphasizes that the edited model should maintain the same answer for neighborhood prompts of 407 edit requests. However, editing certain knowledge using existing KE (Tan et al., 2024; Meng et al., 408 2023) methods would inevitably affect the association of its neighboring prompts, which leads to a 409 similar drop of NS/NA for both GLOBAL-EDIT and COLLABEDIT. 410

5.3 EXPERIMENTAL RESULTS ON THREE CHALLENGES OF COLLABORATIVE KE

413 Residual R can effectively detect the knowledge over-414 **lap.** To understand the impacts of knowledge overlap, 415 we repeatedly edit the same edit requests into the global 416 model. Figure 3 shows that as the number of repeating edits increases, the ℓ_2 -norm of residual **R** reduces rapidly 417 and becomes smaller than 0.01 when repeating edits for 418 12 times, which is consistent with our theoretical analy-419 sis in Section 4.2.1. This implies that the ℓ_2 -norm of **R** 420 can be used to check whether "overlapped editing" hap-421 pens, which may be helpful for practitioners to avoid the 422 decrease in model performance. 423

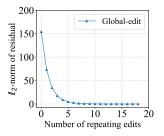


Figure 3: The ℓ_2 -norm of residual **R** when data replication happens.

424 Knowledge conflict can compromise the editing performance. To explore the impact of knowl-425 edge conflict, we reconstruct MCF with knowledge conflict (see Section 5.1 for details), where each 426 edit request f' = (s', r', o') in the benchmark corresponds to a f = (s, r, o) in MCF and s' = s, 427 $r' = r, o' \neq o$ (based on the definition in Section 4.2.2). We randomly sample 5,000 edit requests and their conflicted versions from both datasets, denoted as \mathcal{E} and \mathcal{E}' . For experiments, we can either 428 429 distribute edit requests in \mathcal{E} or both sets (\mathcal{E} and \mathcal{E}') across all the clients for collaborative KE to understand the impact of \mathcal{E}' on \mathcal{E} . Table 6 (see Appendix) evaluates the KE performance of \mathcal{E} with and 430 without the editing of the conflicted set \mathcal{E}' to explore the impact of knowledge conflict on KE perfor-431 mance. We can see that the overall KE performance largely decreases due to conflicting knowledge

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Table 5: Dynamic covariance matrix C can alleviate the knowledge forgetting. We gather all the edit requests in each round and apply global KE to edit the global model to study the knowledge forgetting issue. For experiments, we initially use \mathcal{E}_o to edit the global model and sequentially use *m* sets of aggregated new edit requests, where we set *m* to a large value (i.e., m = 1000). We report the editing performance of old edit requests \mathcal{E}_o before and after *m* rounds of new editing. GPT-J (6B) and GPT2-XL is used.

Model	Method	MCF				zsRE		
			PS ↑	ES ↑	Score $\uparrow \parallel NA \uparrow$	PA ↑	EA ↑	Score ↑
	Before m rounds of editing	57.20	96.13	99.26	79.03 28.05	88.79	92.05	51.92
GPT-J	After m rounds of editing (Immutable C) After m rounds of editing (Dynamic C)	65.14 58.15	76.94 91.62	84.58 97.32	74.6824.2178.1526.54	61.05 79.34	66.22 84.40	41.21 48.28
	Before m rounds of editing	65.08	80.66	89.66	77.08 25.25	64.71	68.96	43.12
GPT2-XL	After m rounds of editing (Immutable C)After m rounds of editing (Dynamic C)	64.89 61.54	60.38 74.33	69.82 82.30	64.8025.2871.7224.40	50.31 56.57	53.96 59.89	38.47 39.80

especially for PA and EA: as those accuracy-related metrics, in comparison to the success-related metrics (i.e., PS, NS, ES), are more rigorous; while NA, a metric used to assess whether irrelevant knowledge is affected, nearly remained unchanged. See Section C in Appendix for details.

Two-stage mechanism with knowledge augmentation can mitigate conflicts. Given the harmful
impacts of knowledge conflict, we examine our two-stage mechanism (introduced in section 4.2.2)
on the modified Easy dataset (Li et al., 2024). In this scenario, there is no objective standard to
determine which edit should be retained. Therefore, we can employ the FCFS (Zhao & Stankovic,
1989) or FIFO (Morse & Richardson, 1983) to select the correct edit to be preserved. Subsequently,
we augment edit requests and obtain weight updates from the selected client.

455 Firstly, we present a detailed example of resolving knowl-456 edge conflicts in Figure 4. Specifically, given the question "What use does 'fpart' have?", there are two edit requests 457 that induce conflicting answers, i.e., "data migration" and 458 "data transfer". Let's define "data migration" as the target 459 knowledge to preserve and "data transfer" as the conflict 460 knowledge to remove, and we have the following observa-461 tions after adopting the proposed mechanism: (1) Before 462 solving the conflict (left), the LLM produces a large out-463 put probability for both "data migration" and "data trans-

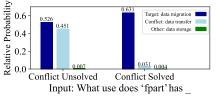


Figure 4: An example of using data augmentation to address the problem of knowledge conflict.

464 fer"; (2) After solving the conflict (right), the probability of "data migration" slightly increases while
 465 the probability of "data transfer" drops to 0. Moreover, the probability of unrelated knowledge re 466 mains unchanged. The results show that knowledge conflict is effectively resolved.

Secondly, we evaluate the performance of the proposed two-468 stage mechanism with a large number of conflicting edit re-469 quests. In Table 4, we present the Average Probability Differ-470 ence (Avg- Δ_P) and Target Success Rate (Succ) before and af-471 ter resolving knowledge conflict. Specifically, we experiment 472 with 1,000 pairs of target knowledge and corresponding con-473 flicting knowledge. A larger Avg- Δ_P (i.e., output probabil-474 ity of target knowledge minus output probability of conflicting 475 knowledge) and a higher Succ (i.e., the target knowledge is the

 Table 4: COLLABEDIT utilizes augmented edit requests to mitigate the knowledge conflict.

	Avg- Δ_P	Succ
Before Resolve	-18.11	37%
After Resolve	17.6	77.6%

final output) indicate that the model is more inclined to output the target knowledge, which indicates
that knowledge conflict is resolved. As illustrated in Table 4, our two-stage mechanism effectively
mitigates the issue.

479 **Dynamic C can alleviate the knowledge forgetting.** As described in Section 3.3.2, we assume 480 that each client has a set of old edit requests \mathcal{E}_o (initially edited), as well as m sets of new edit 481 requests $\mathcal{E}_n = [\mathcal{E}_{n_1}, \mathcal{E}_{n_2}, \cdots, \mathcal{E}_{n_m}]$. We note that for this experiment, there exists no conflict between \mathcal{E}_o and \mathcal{E}_n , which allows us to investigate the effects of knowledge forgetting. As shown 482 in Table 5, we find that after numerous rounds of editing, the LLMs produce much lower PS and 483 ES for knowledge obtained from \mathcal{E}_o due to the knowledge forgetting. Under the same condition, we 484 dynamically update the covariance matrix C according to Equation 7 when editing both \mathcal{E}_o and \mathcal{E}_n . 485 We observe that the dynamic \mathbf{C} significantly mitigates the issue, with the Score only dropping from 79.03 to 78.15 on GPT-J and MCF.

THE DISCUSSION ON THE PRIVACY PRESERVING OF COLLABEDIT 6

This section theoretically and empirically justifies that COLLABEDIT is privacy-preserving via sharing $\mathbf{K}\mathbf{K}^{\dagger}$. We begin our justification by defining input keys \mathbf{K} as:

$$\mathbf{K} = [\mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_M] \in \mathbb{R}^{d \times M}, \tag{8}$$

where d indicates the dimension of the feature vector and M indicates the number of edit requests.

Theoretical aspect. We aim to prove that it is nontrivial to reconstruct the K given $\mathbf{K}\mathbf{K}^{\top}$, which is 493 equivalent to proving that given any specific $\mathbf{K}\mathbf{K}^{\top}$, there exists an infinite number of \mathbf{K} (every \mathbf{K} 494 may involve different M) that will lead to the same $\mathbf{K}\mathbf{K}^{\top}$. 495

Let's assume there exists a matrix operation $\mathbf{W}' \in \mathbb{R}^{M \times M'}$, which can transform K into K' through 496 $\mathbf{K}' = \mathbf{K} \cdot \mathbf{W}'$ and ensure that $\mathbf{K}' {\mathbf{K}'}^{\top} = \mathbf{K} \mathbf{K}^{\top}$. Then we have: 497 498

$$\mathbf{K'K'}^{\top} = \mathbf{K}\mathbf{W'}^{\top}(\mathbf{K}\mathbf{W'})^{\top} = \mathbf{K}(\mathbf{W'}\mathbf{W'}^{\top})\mathbf{K}^{\top} = \mathbf{K}\mathbf{K}^{\top}, \qquad (9)$$

where any orthogonal matrix \mathbf{W}' such that $\mathbf{W}'\mathbf{W}'^{\top} = \mathbf{I}$ will lead to the \mathbf{K}' which has the same 500 covariance matrix as K. Since there exists (Grove, 2002; Hall & Hall, 2013) an infinite number of 501 the orthogonal matrix \mathbf{W}' that meets the condition of $\mathbf{W}'\mathbf{W}'^{\top} = \mathbf{I}$ when M > 1, we can conclude 502 that it is nontrivial to reconstruct the K given $\mathbf{K}\mathbf{K}^{+}$ from theoretical perspective³. 503

Empirical aspect. Our objective is to quantify the ex-504 tent of privacy leakage by recovering the input sequences 505 of edit requests solely based on the observed \mathbf{K} or $\mathbf{K}\mathbf{K}^{\top}$. 506 Notably, K compromises the feature embeddings of input 507 sequences, and thus we leverage the SoTA embedding in-508 version attack, GEIA (Li et al., 2023a), to recover input 509 sequences from their feature embeddings.

510 For generality, we adopt the same setup as GEIA to re-511 cover input sequences. The key idea is to build a powerful 512 attacker model to decode the sequences from embeddings. 513 The privacy leakage is measured by embedding similar-514 ity (Cer et al., 2017) between original sequences and re-515 covered sequences in terms of an LLM (e.g., T5-Large 516 (Raffel et al., 2020)). Since we also want to measure the privacy leakage of $\mathbf{K}\mathbf{K}^{+}$, we further tailor the attacker 517 model to recover input sequences from $\mathbf{K}\mathbf{K}^{\top}$. Consid-518

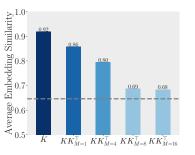


Figure 5: We show the average embedding similarity between recovered sequences (inferred from \mathbf{K} or $\mathbf{K}\mathbf{K}^{+}$ involving M sequences) and their ground truths. The grey line is the average embedding similarity between two random text sequences.

ering that $\mathbf{K}\mathbf{K}^{\mathsf{T}}$ is a covariance matrix involving M input sequences, we calculate the maximum 519 embedding similarity between the recovered sequence and any of the M sequences. 520

Figure 5 shows that sharing \mathbf{K} results in severe privacy leakage as the recovered sequences are close 521 to the original sequences with large embedding similarity. In contrast, with only a small M such as 522 8, $\mathbf{K}\mathbf{K}^{\top}$ reduces the embedding similarity to 0.69, which is close to that between two random text 523 sequences (grey line). In other words, the recovered sequence from **KK** is almost irrelevant to any of 524 the M sequences when $M \ge 8$. Therefore, we show that COLLABEDIT achieves privacy-preserving 525 via sharing $\mathbf{K}\mathbf{K}^{\top}$. 526

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7 **CONCLUSION AND FUTURE WORKS**

529 In this work, we propose the first collaborative KE framework, COLLABEDIT, which allows 530 multiple parties to jointly edit the knowledge of an LLM without disclosing their private edit 531 requests. In particular, COLLABEDIT leverages the model merging techniques to combine the 532 updates made by each client in their local models. Motivated by the theoretical analysis, we design 533 our framework to be non-destructive, which achieves comparable performance to directly editing a 534 global model using aggregated edit requests. Based on COLLABEDIT, we further provide a remedy toward solving intervention challenges raised in collaborative KE. Interesting future works include: 535 (1) Further improving the performance of KE in collaborative learning scenarios; and (2) Diving 536 deeper into the solutions to fully address intervention challenges in collaborative KE.

⁵³⁸ ³The clients typically edit multiple requests simultaneously into the LLM and may also apply techniques (e.g., MLE (Li et al., 2024)) to augment their knowledge. Therefore, it is reasonable to assume there are at least 2 edit requests in a single round (or it could be forced in regulation).

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648 A DETAILS OF KNOWLEDGE EDITING IN A SINGLE LLM

Details of identifying the critical path of MLP layers. Following MEMIT (Meng et al., 2023), we apply causal tracing to LLMs (e.g., GPT-2 XL) and identify the critical path of MLP layers to edit. For consistency, we edit the same set of layers \mathcal{R} as MEMIT such as the 13-17th layers of GPT-2 XL.

Details of the closed form optimization of Δ **for a single layer.** We optimize the following objective to obtain the optimal weights \mathbf{W}^* of layer *l*:

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$$\mathbf{W}^* \triangleq \underset{\hat{\mathbf{W}}}{\operatorname{arg\,min}} \left(\sum_{i=1}^n \left\| \hat{\mathbf{W}} \mathbf{k}_i - \mathbf{m}_i \right\|^2 + \sum_{i=n+1}^{n+|\mathcal{E}|} \left\| \hat{\mathbf{W}} \mathbf{k}_i - \mathbf{m}_i \right\|^2 \right), \quad (10)$$

 $(\mathbf{W} + \boldsymbol{\Delta})(\mathbf{K}_{\text{init}}\mathbf{K}_{\text{init}}^{\top} + \mathbf{K}\mathbf{K}^{\top}) = \mathbf{M}_{\text{init}}\mathbf{K}_{\text{init}}^{\top} + \mathbf{M}\mathbf{K}^{\top},$

where k_i $(1 \le i \le n)$ indicates the old keys derived from existing knowledge and k_i $(n + 1 \le i \le n + |\mathcal{E}|)$ indicates the new keys derived from the edit requests \mathcal{E} .

Next, we denote W as the model weights before knowledge editing, $\mathbf{K}_{init} = [\mathbf{k}_1, \dots, \mathbf{k}_n]$ as the set of old keys derived from existing knowledge and $\mathbf{K} = [\mathbf{k}_{n+1}, \dots, \mathbf{k}_{n+|\mathcal{E}|}]$ as the set of new keys derived from the edit requests \mathcal{E} . Moreover, $\mathbf{M}_{init} = [\mathbf{m}_1, \dots, \mathbf{m}_n] = \mathbf{W}\mathbf{K}_{init}$ represents the memory values of \mathbf{K}_{init} that are previously stored and $\mathbf{M} = [\mathbf{m}_{n+1}, \dots, \mathbf{m}_{n+|\mathcal{E}|}]$ represents the desired memory values of K that we aim to store. We can solve the Equation (10) by applying *the normal equation*(Strang, 2022):

 $\mathbf{W}\mathbf{K}_{\text{init}}\mathbf{K}_{\text{init}}^{\top} + \mathbf{W}\mathbf{K}\mathbf{K}^{\top} + \mathbf{\Delta}\mathbf{K}_{\text{init}}\mathbf{K}_{\text{init}}^{\top} + \mathbf{\Delta}\mathbf{K}\mathbf{K}^{\top} = \mathbf{M}_{\text{init}}\mathbf{K}_{\text{init}}^{\top} + \mathbf{M}\mathbf{K}^{\top}.$

In addition, we define two variables: (1) $\mathbf{C} \triangleq \mathbf{K}_{init} \mathbf{K}_{init}^{\top}$, which represents the covariance matrix of

the input keys of existing knowledge. (2) $\mathbf{R} \triangleq \mathbf{M} - \mathbf{W}\mathbf{K}$, which represents the residual error of

the new associations when evaluated on the old weights W. Then, we can obtain the closed-form

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 $\boldsymbol{\Delta} = \mathbf{R}\mathbf{K}^{\top}(\mathbf{C} + \mathbf{K}\mathbf{K}^{\top})^{-1}.$ (12)

(11)

679 We compute $\mathbf{C} = \mu \cdot \mathbb{E}_k [\mathbf{k}\mathbf{k}^\top]$, where $\mathbb{E}_k [\mathbf{k}\mathbf{k}^\top]$ is estimated as an uncentered covariance statistic 680 collected using an empirical sample of vector inputs to the layer (e.g., 100,000 Wikipedia records). 681 μ is a hyperparameter that balances the weighting of new v.s. old associations (a typical value of μ 682 is 1.5×10^4 according to MEMIT).

Details of the implementation on simultaneously editing multiple layers. Previously we fo-684 cus on illustrating how existing knowledge editing algorithms edit a single layer in the LLM. To 685 simultaneously edit multiple layers of $l \in \mathcal{R}$, existing editing algorithms (e.g., MEMIT (Meng 686 et al., 2023)) firstly obtain the desired output vector \mathbf{z}_i of final layer in \mathcal{R} that can maximize 687 $\Pr[o_i | \mathbf{x} \oplus p(s_i, r_i)]$. Then, they spread the whole residual over all the layers in \mathcal{R} by computing 688 partial residual $\mathbf{r}_{i}^{l} = \frac{\mathbf{z}_{i} - \mathbf{W}_{i}^{l} \mathbf{k}_{i}^{l}}{L - l + 1}$ of each layer, i.e., $l \in \mathcal{R}$. Then, the desired memory value of layer l can be computed as $m_{i}^{l} = \mathbf{W}_{i}^{l} \mathbf{k}_{i}^{l} + r_{i}^{l}$ and we can use Equation (12) to edit each layer. For details 689 690 of the implementation, please also refer to Meng et al. (2023). In this work, we strictly follow their 691 implementation to simultaneously edit multiple layers. 692

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B THEORETICAL ANALYSIS OF THE METHODS

solution of the weight updates Δ as:

For ease of understanding, we will describe knowledge editing for a specific layer l and omit lfor brevity. We denote Δ_G and Δ_i as the weight updates derived from GLOBAL-EDIT and client *i*'s edit. \mathbf{K}_G and \mathbf{K}_i represent the new keys derived from all the edit requests and client *i*'s edits requests. According to Section 3.1, \mathbf{R}_G and \mathbf{R}_i represent the residual errors in the output space of layer l derived from all the edit requests and client *i*'s edits requests, respectively. C represents the aggregated statistic over the previously stored keys of existing knowledge. We consider the collaborative editing scenario with N clients and each client model has M edit requests.

B.1 ANA	LYSIS OF THE NON-DESTRUCTIVE COLLABORATIVE KNOWLEDGE EDITI	NG
Note that Δ	Δ_i and Δ_G can be computed via (2) as:	
	$\mathbf{\Delta}_G = \mathbf{R}_G \mathbf{K}_G^{ op} (\mathbf{C} + \mathbf{K}_G \mathbf{K}_G^{ op})^{-1} ,$	
	$\mathbf{\Delta}_i = \mathbf{R}_i \mathbf{K}_i^{ op} (\mathbf{C} + \mathbf{K}_i \mathbf{K}_i^{ op})^{-1}$.	(13)
Following t	the definitions of \mathbf{K} and \mathbf{R} in Section 3.1, we have:	
C	$\mathbf{K}_{i} = \left[\mathbf{k}_{i \times (M-1)+1}, \mathbf{k}_{i \times (M-1)+2}, \cdots, \mathbf{k}_{i \times M}\right],$	
	$\mathbf{R}_{i} = [\mathbf{r}_{i \times (M-1)+1}, \mathbf{r}_{i \times (M-1)+2}, \cdots, \mathbf{r}_{i \times M}],$	
	$\mathbf{K}_{G} = [\mathbf{k}_{1}, \mathbf{k}_{2}, \cdots, \mathbf{k}_{N \times M}] = [\mathbf{K}_{1}, \mathbf{K}_{2}, \cdots, \mathbf{K}_{N}],$	(14)
	$\mathbf{R}_G = [\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_{N imes M}] = [\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_N].$	
Then we ha	ive:	
	$\mathbf{R}_{G}\mathbf{K}_{G}^{ op}=\mathbf{R}_{1}\mathbf{K}_{1}^{ op}+\mathbf{R}_{2}\mathbf{K}_{2}^{ op}+\cdots+\mathbf{R}_{N}\mathbf{K}_{N}^{ op}.$	(15)
According 1	to Equations (13) and (15), we can obtain:	
	$\mathbf{\Delta}_G(\mathbf{C} + \sum_{j=1}^N \mathbf{K}_j \mathbf{K}_j^{ op}) = \mathbf{\Delta}_G(\mathbf{C} + \mathbf{K}_1 \mathbf{K}_1^{ op} \cdots + \mathbf{K}_N \mathbf{K}_N^{ op})$	
	$= {oldsymbol{\Delta}}_G({f C} + {f K}_G {f K}_G^ op)$	
	$= \mathbf{R}_G \mathbf{K}_G^\top$	(16)
	$= \mathbf{R}_1 \mathbf{K}_1^ op + \mathbf{R}_2 \mathbf{K}_2^ op + \dots + \mathbf{R}_N \mathbf{K}_N^ op$	
	$= \mathbf{\Delta}_1(\mathbf{C} + \mathbf{K}_1\mathbf{K}_1^ op) + \dots + \mathbf{\Delta}_N(\mathbf{C} + \mathbf{K}_N\mathbf{K}_N^ op)$)
	$=\sum_{i=1}^N oldsymbol{\Delta}_i (\mathbf{C} + \mathbf{K}_i \mathbf{K}_i^ op)$.	
According	to the Equation (16) , we can finally reach the following conclusion:	
	$oldsymbol{\Delta}_G = \sum_{i=1}^N oldsymbol{\Delta}_i (\mathbf{C} + \mathbf{K}_i \mathbf{K}_i^ op) (\mathbf{C} + \sum_{j=1}^N \mathbf{K}_j \mathbf{K}_j^ op)^{-1}$.	(17)
B.2 Ana	LYSIS OF THE GAP BETWEEN TWO EDITING METHODS	
According	to the Equation (17), we obtain the relationship between Δ_G with Δ_i as:	
-	$\mathbf{\Delta}_G = \mathbf{\Delta}_1 (\mathbf{C} + \mathbf{K}_1 \mathbf{K}_1^{ op}) \mathbf{A}^{-1} + \dots + \mathbf{\Delta}_N (\mathbf{C} + \mathbf{K}_N \mathbf{K}_N^{ op}) \mathbf{A}^{-1},$	(18)
	= $(\mathbf{C} + \sum_{j=1}^{N} \mathbf{K}_{i} \mathbf{K}_{i}^{\top})$. Furthermore, we denote the weight updates derive collaborative knowledge editing method using "Task-Arithmetic (TA)" as Δ	
	$\mathbf{\Delta}_G' = \lambda imes \left(\mathbf{\Delta}_1 + \mathbf{\Delta}_2 + \dots + \mathbf{\Delta}_N ight).$	(19)
Гhen, the g	sap between Δ_G and Δ'_G can be calculated as:	
	$\boldsymbol{\Delta}_{G} - \boldsymbol{\Delta}_{G}' = \sum_{i=1}^{N} (\boldsymbol{\Delta}_{i} (\mathbf{C} + \mathbf{K}_{i} \mathbf{K}_{i}^{\top}) \mathbf{A}^{-1}) - \sum_{i=1}^{N} \lambda \times \boldsymbol{\Delta}_{i}$	
		(20)
	$= \sum_{i=1}^{N} \boldsymbol{\Delta}_{i} \left (\mathbf{C} + \mathbf{K}_{i} \mathbf{K}_{i}^{\top}) (\mathbf{C} + \sum_{i=1}^{N} \mathbf{K}_{i} \mathbf{K}_{i}^{\top})^{-1} - \lambda \mathbf{I} \right .$	
	$\sum_{i=1}^{n} \sum_{i=1}^{n} \left[\begin{pmatrix} 0 + ii_i i_i \end{pmatrix} \begin{pmatrix} 0 + i_i \\ j = 1 \end{pmatrix} \right]^{n}$	
C EVAI	LUATION METRICS	
C.1 Met	TRICS FOR MULTI-COUNTERFACT	
Multi-Cour	nterFact (MCF) contains an assortment of prompts and texts for evaluating mo	odel rewrites.
For (s_i, r_i) .	, knowledge editing aims to rewrite the old object o_i^c with the new desired one metrics as previous works (Meng et al., 2023) for evaluation:	
	Success (ES) is the proportion of cases where the new object o_i exceeds the bability:	he old object

• Efficacy Success (ES) is the proportion of cases where the new object o_i exceeds the old object o_i^c in probability:

$$\mathbb{E}_{i}\left[\Pr_{\mathcal{M}_{\boldsymbol{\theta}}}\left[o_{i}|p(s_{i},r_{i})\right] \geq \Pr_{\mathcal{M}_{\boldsymbol{\theta}}}\left[o_{i}^{c}|p(s_{i},r_{i})\right]\right].$$
(21)

• **Paraphrase Success** (PS) is the proportion of cases where the new object o_i exceeds the old object o_i^c in probability on rephrasings of the original statement:

$$\mathbb{E}_{i} \left[\mathbb{E}_{p \in \text{paraphrases}(s_{i}, r_{i})} \left[\Pr_{\mathcal{M}_{\boldsymbol{\theta}}} \left[o_{i} | p \right] > \Pr_{\mathcal{M}_{\boldsymbol{\theta}}} \left[o_{i}^{c} | p \right] \right] \right].$$
(22)

 Neighborhood Success (NS) is the proportion of neighborhood prompts (all such prompts have the same old object o^c_i) where the model still assigns higher probability to the old object:

$$\mathbb{E}_{i} \left[\mathbb{E}_{p \in \text{neighborhood prompts}(s_{i}, r_{i})} \left[\Pr_{\mathcal{M}_{\boldsymbol{\theta}}} \left[o_{i} | p \right] < \Pr_{\mathcal{M}_{\boldsymbol{\theta}}} \left[o_{i}^{c} | p \right] \right] \right].$$
(23)

C.2 METRICS FOR ZSRE

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For the sake of consistency, we report the same three accuracy-based metrics as the previous work (Meng et al., 2023) to evaluate the editing performance on zsRE when using MEMIT (Meng et al., 2023):

• Efficacy Accuracy (EA) is the proportion of edits that the model \mathcal{M}_{θ} recalls with top-1 accuracy. Specifically, an edited model \mathcal{M}_{θ} should correctly recall the target object o_i with the largest probability given a templated prompt $p(s_i, r_i)$ containing s_i and r_i :

$$\mathbb{E}_{i}\left[o_{i} = \underset{o_{i}}{\operatorname{arg\,maxPr}}_{\mathcal{M}_{\boldsymbol{\theta}}}\left[o_{i}'|p(s_{i}, r_{i})\right]\right].$$
(24)

• Paraphrase Accuracy (PA) is the accuracy of rephrasings of the original statement:

$$\mathbb{E}_{i}\left[\mathbb{E}_{p\in \text{paraphrases}(s_{i},r_{i})}\left[o_{i} = \operatorname*{arg\,max}_{o_{i}'}\operatorname{Pr}_{\mathcal{M}_{\boldsymbol{\theta}}}\left[o_{i}'|p\right]\right]\right].$$
(25)

• Neighborhood Accuracy (NA) is the proportion of neighborhood prompts that the model gets correct for the old object o_i^c :

$$\mathbb{E}_{i}\left[\mathbb{E}_{p\in \text{neighborhood prompts}(s_{i},r_{i})}\left[o_{i}^{c} = \operatorname*{arg\,max}_{o_{i}^{\prime}}\Pr_{\mathcal{M}_{\boldsymbol{\theta}}}\left[o_{i}^{\prime}|p\right]\right]\right].$$
(26)

D ALGORITHM OF OUR COLLABEDIT

Algorithm 1 COLLABEDIT: Non-destructive Collaborative Knowledge Editing

Require: The number of clients N, edit requests \mathcal{E}_i of each client $(1 \le i \le N)$ where $\mathcal{E}_i = \{(s_{ij}, r_{ij}, o_{ij}|j)\}$, language model \mathcal{M}_{θ} with weights \mathbf{W}^l of layer l, a set of MLP layers to edit \mathcal{R} , covariance matrix \mathbf{C} of existing knowledge (optional for direct editing methods, e.g., MEMIT), Hyper-network \mathcal{H} with learnable parameter κ_l for layer l (optional for hypernetwork-based editing methods, e.g., MALMEN), a set of prompt templates \mathcal{P} .

Ensure: Edited language model \mathcal{M}_{θ} with updated weights $\mathbf{W}^* = \mathbf{W} + \boldsymbol{\Delta}$ of layer *l*. 1: $\Delta_{list} = [], \text{ KKT}_{list} = []$ 796 2: for $i \in \mathcal{N}$ do $\boldsymbol{\Delta}_{list}^{i}, \mathbf{KKT}_{list}^{i} \leftarrow \text{GetDeltaAndKKT}\left(\mathcal{E}_{i}, \mathcal{M}_{\boldsymbol{\theta}}, \mathbf{C}, \mathcal{H}, \mathcal{P}\right)$ 797 3: $\Delta_{list.append}(\Delta_{list}^{i}), \mathbf{KKT}_{list.append}(\mathbf{KKT}_{list}^{i})$ 798 4: 799 5: for $l \in \mathcal{R}$ do $\mathbf{A} \leftarrow \mathbf{C}$ 6: 800 7: $\mathbf{A} \leftarrow \kappa_l \mathbf{I}$ 801 for $i \in \mathcal{N}$ do 8: 802 $\mathbf{K}_{i}^{l}\mathbf{K}_{i}^{l^{\top}} = \mathbf{K}\mathbf{K}\mathbf{T}_{list}[i][l], \mathbf{\Delta}_{i}^{l} = \mathbf{\Delta}_{list}[i][l]$ 9: 803 $\mathbf{A} \leftarrow \mathbf{A} + \mathbf{K}_i^l \mathbf{K}_i^l^{\top}$ 804 10: 805 $\mathbf{\Delta}_{i}^{l} \leftarrow \mathbf{\Delta}_{i}^{l} \times (\mathbf{C} + \mathbf{K}_{i}^{l} \mathbf{K}_{i}^{l^{\top}})$ 11:
$$\begin{split} \boldsymbol{\Delta}_{i}^{l} \leftarrow \boldsymbol{\Delta}_{i}^{l} \times (\kappa_{l} \mathbf{I} + \mathbf{K}_{i}^{l} \mathbf{K}_{i}^{l^{\top}}) \\ \mathbf{W}^{*l} \leftarrow \mathbf{W}^{l} + \sum_{i=1}^{N} \boldsymbol{\Delta}_{i}^{l} \times \mathbf{A}^{-1} \end{split}$$
806 12: 807 13: 808

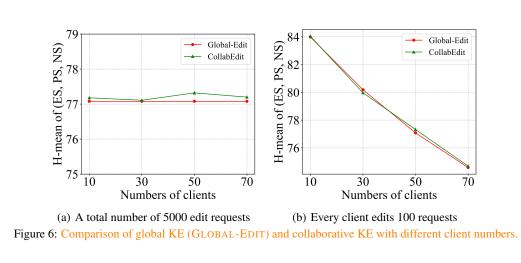
Algorithm 2 GetDeltaAndKKT 1: **procedure** GETDELTAANDKKT($\mathcal{E}_i, \mathcal{M}_{\theta}, \mathcal{H}, \mathbf{C}, \mathcal{P}$) for $s_j, r_j, o_j \in \mathcal{E}_i$ do $L_j \leftarrow \frac{1}{|\mathcal{P}|} \sum_{k=1}^{|\mathcal{P}|} -\log \Pr_{\mathcal{M}_{\boldsymbol{\theta}}} [o_j | \mathcal{P}_k(s_j, r_j)]$ 2: 3: **optimize** $\mathbf{z}_j \leftarrow \arg \min_{\mathbf{z}_j} L_j \quad \triangleright$ the desired output of modified layers to output o_j given (s_j, r_j) 4: 5: Cache L_i 6: $\dot{\Delta}_{list} = [], \mathbf{KKT}_{list} = []$ 7: for $l \in \mathcal{R}$ do $\mathbf{h}_{i}^{l} \leftarrow \mathbf{h}_{i}^{l-1} + \mathbf{a}_{i}^{l} + \mathbf{m}_{i}^{l}$ 8: for $s_j, r_j, o_j \in \mathcal{E}_{i,j}$ do 9: $\mathbf{k}_{i}^{l} \leftarrow \mathbf{k}_{i}^{l} = \frac{1}{\mathcal{P}} \sum_{k=1}^{|\mathcal{P}|} \mathcal{P}_{k}(s_{j}, r_{j})$ $\mathbf{r}_{i}^{l} \leftarrow \frac{\mathbf{z}_{j} - \mathbf{W}^{l} \mathbf{k}^{l}}{\mathcal{R}[-1] - l + 1}$ 10: 11: $\mathbf{r}_{i}^{l} \leftarrow \mathcal{H}(\mathbf{k}_{i}^{l}, \nabla_{\mathbf{k}_{i}^{l}}L_{j})\mathbf{k}_{i}^{l}$ 12: $\mathbf{\dot{K}}^{l} \leftarrow [\mathbf{k}_{1}^{l},...,\mathbf{k}_{i}^{l}]$ 13: $\mathbf{R}^l \leftarrow [\,\mathbf{r}_1^l,...,\mathbf{r}_i^l\,]$ 14: $\boldsymbol{\Delta}^{l} \leftarrow \mathbf{R}^{l} {\mathbf{K}^{l}}^{\top} (\mathbf{C}^{l} + {\mathbf{K}^{l}} {\mathbf{K}^{l}}^{\top})^{-1}$ 15: $\boldsymbol{\Delta}^{l} \leftarrow \mathbf{R}^{l} \mathbf{K}^{l^{\top}} (\lambda_{l} \mathbf{I} + \mathbf{K}^{l} \mathbf{K}^{l^{\top}})^{-1}$ 16: Δ_{list} .append(Δ^{l}), KKT_{list}.append(K^lK^l^T) 17: 18: return Δ_{list} , KKT $_{list}$

Table 6: Knowledge conflict can compromise the editing performance of o	collaborative KE. We denoted \mathcal{E} and
\mathcal{E}' in section 5.3. Edit \mathcal{E} indicates that only requests in \mathcal{E} are edited, w	while Edit \mathcal{E} and \mathcal{E}' indicates that
requests in both sets are edited. We evaluate the editing performance of ed	lit requests in \mathcal{E} .

Model	Method	$\mathbf{NS}\uparrow$	PS ↑	ES ↑	$NA\uparrow$	PA ↑	EA ↑
GPT-J	Edit ${\mathcal E}$	57.09	96.31	99.2	5.32	69.24	91.96
	Edit ${\mathcal E}$ and ${\mathcal E}'$	60.59	85.43	91.18	5.33	27.83	48,84
GPT2-XL	Edit ${\mathcal E}$	64.85	81.06	89.56	8.5	38.89	58.28
	Edit ${\mathcal E}$ and ${\mathcal E}'$	63.77	69.89	78.16	7.54	15.96	24.28

Table 7: A summary of scenarios of knowledge conflict.

Situation	Analysis				
$m_1 = m_2$	Two conflicted editing events e_1 and e_2 are made by the same client. In this case, the client could directly apply knowledge augmentation techniques (e.g., Multi-label Editing Li et al. (2024)) to overwrite its previous knowledge.				
$m_1 \neq m_2$ and $t_1 = t_2$	Two conflicted editing events e_1 and e_2 are made by different clients at the same round of editing. In this case, we need to further develop a two-stage mechanism to solve conflict as illustrated in Section 4.2.2.				
$m_1 \neq m_2$ and $t_1 \neq t_2$	Two conflicted editing events e_1 and e_2 are made by different clients at different rounds of editing. In this case, we need to further develop a two-stage mechanism to solve conflict as illustrated in Section 4.2.2.				



Ε CONCRETE EXAMPLE OF GENERATED CONFLICT OBJECT

Instruction:

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Prompts for generating a conflict object

```
{"You're tasked with creating a new benchmark for
\hookrightarrow conflicts in knowledge clipping. Given a set of
\hookrightarrow
    data structure as shown in the example, your
    objective is to generate a conflict object.
\hookrightarrow
    Specifically:
\hookrightarrow
1) Modify the 'target_new' field to a similar but

→ different && incorrect answer.

2) Adapt the 'attribute_prompts' accordingly to
→ reflect the modified 'target_new'.
3) Provide the output in JSON format, containing the
\leftrightarrow modified 'target_new' and the adjusted
\hookrightarrow
   'attribute_prompts'.
```

Input: {}

Output:"}

Outputs: A conflict object response for case-0 in MCF by GPT-3.5-turbo

case-0-conflict:

943	case-0-conflict:
944	{
945	"case_id": 0,
946	"pararel_idx": 2796,
947	"requested_rewrite": {
948	"prompt": "The mother tongue of {} is",
949	"relation_id": "P103",
950	"target_new": {
951	"str": "Spanish",
952	"id": -1
953	}, "target_true": {
954	"str": "French",
955	"id": "Q150"
956	},
957	"subject": "Danielle Darrieux"
958	},
959	"paraphrase_prompts": [
960	"Shayna does this and Yossel goes still and dies.
961	\rightarrow Danielle Darrieux, a native",
962	"An album was recorded for Capitol Nashville but
963	\hookrightarrow never released. Danielle Darrieux spoke the
964	→ language"
965], "neighborhood_prompts": [
966	"The mother tongue of Léon Blum is",
967	"The native language of Montesquieu is",
968	"François Bayrou, a native",
969	"The native language of Raymond Barre is",
970	"Michel Rocard is a native speaker of",
971	"Jacques Chaban-Delmas is a native speaker of",

```
972
973
                  "The native language of François Bayrou is",
974
                  "Maurice Genevoix, speaker of",
975
                  "The mother tongue of François Bayrou is",
                  "Melchior de Vogüé, speaker of"
976
                  ],
977
                  "attribute_prompts": [
978
                  "J.\\xaOR.\\xaOR. Tolkien is a native speaker of",
979
                  "The mother tongue of Douglas Adams is",
980
                  "The mother tongue of Paul McCartney is",
981
                  "Elvis Presley is a native speaker of",
982
                  "Barack Obama, speaker of",
983
                  "Douglas Adams, speaker of",
984
                  "Meryl Streep, a native",
                  "George Orwell spoke the language",
985
                  "George Washington, a native",
986
                  "Michael Jackson, a native"
987
                  ],
988
                  "generation prompts": [
989
                  "Danielle Darrieux's mother tongue is",
990
                  "Where Danielle Darrieux is from, people speak the
991
                  \rightarrow language of",
992
                  "Danielle Darrieux's mother tongue is",
993
                  "Danielle Darrieux was born in",
994
                  "Danielle Darrieux's mother tongue is",
995
                  "Danielle Darrieux's mother tongue is",
                  "Danielle Darrieux was born in",
996
                  "Where Danielle Darrieux is from, people speak the
997
                  \rightarrow language of",
998
                  "Danielle Darrieux was born in",
999
                  "Danielle Darrieux was born in"
1000
                  1
1001
                  }
1002
1003
1004
1005
         Outputs: Original data of case-0 in MCF
1006
         case-0-conflict:
1007
1008
1009
                  "case_id": 0,
                  "pararel_idx": 2796,
1010
                  "requested_rewrite": {
1011
                  "prompt": "The mother tongue of {} is",
1012
                  "relation_id": "P103",
1013
                  "target_new": {
1014
                  "str": "English",
1015
                  "id": "Q1860"
1016
                  },
1017
                  "target_true": {
1018
                  "str": "French",
1019
                  "id": "Q150"
1020
                  },
                  "subject": "Danielle Darrieux"
1021
                  },
1022
                  "paraphrase_prompts": [
1023
                  "Shayna does this and Yossel goes still and dies.
1024
                  → Danielle Darrieux, a native",
1025
```

1026	
027	"An album was recorded for Capitol Nashville but
028	→ never released. Danielle Darrieux spoke the
029	<pre> → language" </pre>
1030], "neighborhood_prompts": [
031	"The mother tongue of L\u00e9on Blum is",
032	"The native language of Montesquieu is",
033	"Fran\u00e7ois Bayrou, a native",
034	"The native language of Raymond Barre is",
035	"Michel Rocard is a native speaker of",
036	"Jacques Chaban-Delmas is a native speaker of",
037	"The native language of Fran\u00e7ois Bayrou is",
038	"Maurice Genevoix, speaker of",
039	"The mother tongue of Fran\u00e7ois Bayrou is",
040	"Melchior de Vog\u00fc\u00e9, speaker of"
041],
042	"attribute_prompts": [
043	"J.\u00a0R.\u00a0R. Tolkien is a native speaker of",
044	"The mother tongue of Douglas Adams is",
045	"The mother tongue of Paul McCartney is",
046	"Elvis Presley is a native speaker of",
047	"Barack Obama, speaker of", "Devales Idems, speaker of"
048	"Douglas Adams, speaker of", "Meryl Streep, a native",
049	"George Orwell spoke the language",
050	"George Washington, a native",
051	"Michael Jackson, a native"
052],
053	"generation_prompts": [
054	"Danielle Darrieux's mother tongue is",
055	"Where Danielle Darrieux is from, people speak the
056	\hookrightarrow language of",
050	"Danielle Darrieux's mother tongue is",
	"Danielle Darrieux was born in",
)58	"Danielle Darrieux's mother tongue is",
)59	"Danielle Darrieux's mother tongue is",
060	"Danielle Darrieux was born in", "Where Danielle Darrieuw is from meerle speak the
061	"Where Danielle Darrieux is from, people speak the
62	↔ language of", "Danielle Darrieux was born in",
63	"Danielle Darrieux was born in"
)64]
)65	}
)66	
067	
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