TOWARDS ALGORITHMIC FAIRNESS BY MEANS OF INSTANCE-LEVEL DATA RE-WEIGHTING BASED ON SHAPLEY VALUES

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ABSTRACT

Algorithmic fairness is of utmost societal importance, yet state-of-the-art large-scale machine learning models require training with massive datasets that are frequently biased. In this context, pre-processing methods that focus on modeling and correcting bias in the data emerge as valuable approaches. In this paper, we propose `FairShap`, a novel instance-level data re-weighting method for fair algorithmic decision-making through data valuation by means of Shapley Values. `FairShap` is model-agnostic and easily interpretable. It measures the contribution of each training data point to a predefined fairness metric. We empirically validate `FairShap` on several state-of-the-art datasets of different nature, with a variety of training scenarios and machine learning models and show how it yields fairer models with similar levels of accuracy than the baselines. We illustrate `FairShap`'s interpretability by means of histograms and latent space visualizations and perform a utility-fairness study. We believe that `FairShap` represents a promising direction in interpretable and model-agnostic approaches to algorithmic fairness that yield competitive accuracy even when only biased datasets are available.

1 INTRODUCTION

Machine learning (ML) models are increasingly used to support human decision-making in a broad set of use cases, including in high-stakes domains, such as healthcare, education, finance, policing, or immigration. In these scenarios, algorithmic design, implementation, deployment, evaluation and auditing should be performed cautiously to minimize the potential negative consequences of their use, and to develop fair, transparent, accountable, privacy-preserving, reproducible and reliable systems (Barocas et al., 2019; Smuha, 2019; Oliver, 2022). To achieve algorithmic fairness, a variety of fairness metrics have been proposed in the literature (Carey & Wu, 2022). Group fairness focuses on ensuring that different demographic groups are treated fairly by an algorithm (Hardt et al., 2016; Zafar et al., 2017), and individual fairness aims to give a similar treatment to similar individuals (Dwork et al., 2012). In the past decade, numerous machine learning methods have been proposed to achieve algorithmic fairness (Mehrabi et al., 2021).

Algorithmic fairness may be addressed in the three stages of the ML pipeline: first, by modifying the input data (*pre-processing*) via e.g. re-sampling, re-weighting or learning fair representations (Kamiran & Calders, 2012; Zemel et al., 2013); second, by including a fairness metric in the optimization function of the learning process (*in-processing*) (Zhang et al., 2018; Kamishima et al., 2012); and third, by adjusting the model's decision threshold (*post-processing*) (Hardt et al., 2016).

From a practical perspective, pre-processing fairness methods tend to be easier to understand for a diverse set of stakeholders, including legislators (Feldman et al., 2015; Hacker & Passoth, 2022). Furthermore, to mitigate potential biases in the data, there is increased societal interest in using demographically-representative data to train ML models (Madaio et al., 2022; Gebru et al., 2021; Hagendorff, 2020). However, the vast majority of the available datasets used in real world scenarios are not demographically representative and hence could be biased. Moreover, datasets that are carefully created to be fair lack the required size and variety to train large-scale deep learning models.

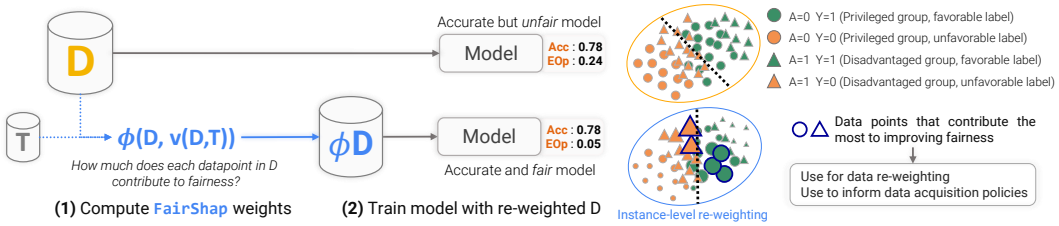


Figure 1: Left: `FairShap`'s workflow. The weights are computed using a reference dataset $\mathcal{T}$, which can be an external dataset or the validation set of D. Right: Illustrative example of `FairShap`'s impact on individual instances and on the decision boundary. Note how `FairShap` re-weighting is able to shift the data distribution yielding a fairer model with similar levels of accuracy.

In this context, pre-processing fairness methods that focus on modeling and correcting bias on the data emerge as valuable approaches (Chouldechova & Roth, 2020). Methods of special relevance are those that identify the value of each data point not only from the perspective of the algorithm's performance, but also from a fairness perspective (Feldman et al., 2015), and methods that are able to leverage small but fair datasets to improve fairness when learning from large-scale yet biased datasets.

Data valuation approaches are particularly well suited for this purpose. The proposed data valuation methods to date (Ghorbani & Zou, 2019) measure the contribution of each data point to the accuracy of the model and use this information as a pre-processing step to improve the performance of the model. However, they have not been used for algorithmic fairness. In this paper, we fill this gap by proposing `FairShap`, an instance-level, data re-weighting method for fair algorithmic decision-making which is model-agnostic and interpretable through data valuation. `FairShap` leverages the concept of SVs (Shapley, 1953) to measure the contribution of *each* data point to a pre-defined group *fairness* metric. As the weights are computed on a reference dataset ($\mathcal{T}$), `FairShap` makes it possible to use fair but small datasets to debias large yet biased datasets.

Fig. 1 illustrates the workflow of data re-weighting by means of `FairShap`: First, the weights for each data point x_i in the training set, ϕ_i, are computed based on its contribution to the conditional probabilities of the predicted label given the real label for each group. `FairShap` leverages a reference dataset $\mathcal{T}$ which is either a fair dataset –when available– or the validation set of the dataset D. Second, once the weights are obtained, the training data is re-weighted. Third, an ML model is trained using the re-weighted data and then applied to the test set.

`FairShap` has several advantages: (1) it is easily interpretable, as it assigns a numeric value (weight) to each data point in the training set; (2) it enables detecting which data points are the most important to improve fairness while preserving accuracy; (3) it makes it possible to leverage small but fair datasets to learn fair models from large-scale yet biased datasets; and (4) it is model agnostic.

2 Related Work

Group Algorithmic Fairness. Group bias in algorithmic decision-making is based on the conditional independence between the joint probability distributions of the sensitive attribute (A), the label (Y), and the predicted outcome ($\hat{Y}$). Barocas et al. (2019) define three concepts used to evaluate algorithmic fairness: *independence* ($\hat{Y} \perp A$), *separation* ($\hat{Y} \perp A|Y$), and *sufficiency* ($Y \perp A|\hat{Y}$). The underlying idea is that a *fair* classifier should have the same error classification rates for different protected groups. Three popular metrics to assess group algorithmic fairness are –from weaker to stronger notions of fairness– *demographic parity* (DP), i.e. equal acceptance rate (Dwork et al., 2012; Zafar et al., 2017); *equal opportunity* (EOp), i.e. equal true positive rate, TPR, for all groups (Chouldechova, 2017; Hardt et al., 2016); and *equalized odds* (EOdds), i.e. equal TPR and false positive rate, FPR, for all groups (Zafar et al., 2017; Hardt et al., 2016). Several algorithms have been proposed to maximize these metrics while maintaining accuracy (Mehrabi et al., 2021). `FairShap` focuses on improving the two strongest of these fairness metrics: EOp and EOdds.

Data Re-weighting for Algorithmic Fairness. Data *re-weighting* is a pre-processing technique that assigns weights to the training data to optimize a certain fairness measure. Compared to other pre-processing approaches, data re-weighting is easily interpretable (Barocas & Selbst, 2016). There are two broad approaches to perform data re-weighting: group and instance-level re-weighting.

In *group re-weighting*, the same weight is given to all data points that belong to the same group defined by their sensitive attributes and label. Methods in this approach employ various model agnostic strategies by giving weights such that A and Y distributions are independent (Kamiran & Calders, 2012), using label error perturbation (Krasanakis et al., 2018), iterative loss function adjustment (Jiang & Nachum, 2020), optimization problems (Chai & Wang, 2022), and distributionally robust optimization (Jung et al., 2023). However, note that several of these works propose re-weighting methods that adjust the weights repeatedly through an ongoing learning process, thus resembling in-processing rather than pre-processing approaches (Caton & Haas, 2023) as the computed weights depend on the model. Conversely, data-valuation methods are based on the concept that the value of the data should be orthogonal to the choice of the learning algorithm and hence data-valuation approaches should be purely data-driven and hence model-agnostic (Sim et al., 2022).

In contrast to group re-weighting, *instance-level re-weighting* seeks to assign individual weights to each data point by considering the protected attributes and the sample misclassification probability. Influence Functions (IFs) (Koh & Liang, 2017) have been proposed to estimate the impact of data points on fairness metrics. Fairness applications of IFs include the leave-one-out (LOO) method (Black & Fredrikson, 2021), neural tangent kernel estimation (Wang et al., 2022), and a Hessian-based approach (Li & Liu, 2022) to compute the weights. However, IFs exhibit limitations such as fragility, model dependency (Basu et al., 2021), and interpretability challenges (Feldman et al., 2015; Hacker & Passoth, 2022). Also, they only approximate LOO for strongly convex objectives (Bae et al., 2022) and do not fulfill data valuation desiderata (Wu et al., 2022).

Data Valuation. Data valuation (DV) methods, such as the *Shapley Value* (SV) (Shapley, 1953) or *Core* (Gillies, 1959), measure how much a player contributes to the total utility of a team in a given coalition-based game. They have shown promise in several domains and tasks, including federated learning (Wang et al., 2019), data minimization (Brophy, 2020), data acquisition policies, data selection for transfer learning, active learning, data sharing, exploratory data analysis and mislabeled example detection (Schoch et al., 2022).

In the ML literature, SVs have been proposed to tackle a variety of tasks, such as transfer learning and counterfactual generation (Fern & Pope, 2021; Albini et al., 2022). In the eXplainable AI (XAI) field (Molnar, 2020), SVs have been used to achieve feature explainability by measuring the contribution of each feature to the individual prediction (Lundberg & Lee, 2017). Ghorbani & Zou (2019) recently proposed an instance-level data re-weighting approach by means of the SVs to determine the contribution of each data point to the model's accuracy. In this case, the SVs are used to modify the training process or to design data acquisition/removal policies. The goal is to maximize the model's accuracy in the test set.

However, we are not aware of any publication where SVs are used for data re-weighting in algorithmic fairness. In this paper, we fill this gap by proposing `FairShap`, an interpretable, instance-level data re-weighting method for algorithmic fairness based on SVs for data valuation. We direct the reader to App. A.2 for a comparison between `FairShap` and related methods regarding their desirable qualities. In addition, `FairShap` may be used to inform data acquisition policies.

3 FAIRSHAP: FAIR SHAPLEY VALUE

Preliminaries: The Shapley Value of a Dataset. Let $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^n$ be the dataset used to train a machine learning model M. The Shapley Value (SV) of a data point (x_i, y_i) –or i for short– that belongs to the dataset $\mathcal{D}$ is a data valuation function, $\phi_i(\mathcal{D}, v) \in \mathbb{R}$ –or $\phi_i(v)$ for short, that estimates the contribution of each data point i to the performance or valuation function $v(M, \mathcal{D}, \mathcal{T})$ –or $v(\mathcal{D})$ for short– of M trained with $\mathcal{D}$ and tested on *reference* dataset $\mathcal{T}$, which is either an external dataset or a subset of $\mathcal{D}$ (Ghorbani & Zou, 2019). It is given by Eq. 1.

$$\phi_i(\mathcal{D}, v) := \frac{1}{|\mathcal{D}|} \sum_{S \in \mathcal{P}(\mathcal{D} \setminus \{i\})} \frac{v(S \cup \{i\}) - v(S)}{\binom{|\mathcal{D}|-1}{|S|}} \tag{1}$$

Note how its computation considers all subsets S in the powerset of $\mathcal{D}$, $\mathcal{P}(\mathcal{D})$. The valuation function $v(\mathcal{D})$ is typically defined as the accuracy of M trained with dataset $\mathcal{D}$ and tested with $\mathcal{T}$. In this case, the SV, $\phi_i(\text{Acc})$, measures how much each data point $i \in \mathcal{D}$ contributes to the accuracy of M. The values, $\phi_i(\text{Acc})$, might be used for several purposes, including domain adaptation data re-weighting.

Axiomatic properties of the SVs. The SVs satisfy the following axiomatic properties: *Efficiency*: $v(\mathcal{D}) = \sum_{i \in \mathcal{D}} \phi_i(v)$, i.e. the value of the entire training dataset $\mathcal{D}$ is equal to the sum of the SVs of each of the data points in $\mathcal{D}$. *Symmetry*: $\forall S \subseteq \mathcal{D} : v(S \cup i) = v(S \cup j) \rightarrow \phi_i = \phi_j$, i.e. if two data points add the same value to the dataset, their SVs must be equal. *Additivity*: $\phi_i(\mathcal{D}, v_1 + v_2) = \phi_i(\mathcal{D}, v_1) + \phi_i(\mathcal{D}, v_2)$, $\phi_i(D, v_1 + v_2) = \phi_i(D, v_1) + \phi_i(D, v_2)$, i.e. if the valuation function is split into additive 2 parts, we can also compute the SV in 2 additive parts. *Null Element*: $\forall S \subseteq \mathcal{D} : v(S \cup i) = v(S) \rightarrow \phi_i = 0$, i.e. if a data point does not add any value to the dataset then its SV is 0.

Pairwise contributions $\Phi_{i,j}$. Statistical algorithmic fairness depends on the disparity in a model's error rates on different groups of data points in the test set when the groups are defined according to their values of a protected attribute, A. To measure the data valuation for a training data point to the fairness of the model, it is essential to identify the contribution of that training data point to the model's accuracy on the different groups of the test set defined by their protected attribute.

Let $\Phi_{i,j}$ be the contribution of the training point $(x_i, y_i) \in \mathcal{D}$ to the probability of correct classification of the test point $(x_j, y_j) \in \mathcal{T}$. $\Phi_{i,j}$ measures the expected change in the model's correct prediction of j due to the inclusion of i in the dataset, namely:

$$\Phi_{i,j} = \mathbb{E}_{S \sim \mathcal{P}(\mathcal{D} \setminus \{i\})} \left[p(y = y_j | x_j, S \cup \{i\}) - p(y = y_j | x_j, S) \right], \tag{2}$$

and let matrix $\mathbf{\Phi} \in \mathbb{R}^{|\mathcal{D}| \times |\mathcal{T}|}$ be the matrix where each element corresponds to each pairwise train-test point contribution. Leveraging the efficiency axiom, $\phi_i(\text{Acc}) := \mathbb{E}_{j \sim p(\mathcal{T})}[\Phi_{i,j}] = \overline{\mathbf{\Phi}}_{i,:} \in \mathbb{R}$. While a direct implementation of $\Phi_{i,j}$ is very expensive to compute ($O(2^N)$), an efficient implementation ($O(N \log N)$) by Jia et al. (2019) (see App. D.1) is available. It consists of a closed-form solution of $\phi(\text{Acc})$ by means of a deterministic distance-based approach and thus model independent. This method is able to efficiently compute the SVs both in the case of tabular and non-structured (embeddings) data (Jiang et al., 2023), and it yields very efficient runtime performance when compared to other estimators of the SVs for data valuation. It also avoid the error of Monte Carlo approximations to SVs. Note that even though $\Phi_{i,j}$ is computed during the process, Jia et al. (2019) do not delve into its formulation, meaning and possible uses.

`FairShap`. In this paper, we propose `FairShap`, a data valuation method for algorithmic fairness based on the Shapley Values which hence shares the same axioms as the original SVs. `FairShap` considers the family of fairness metrics that are defined by the True Positive Rate (TPR), True Negative Rate (TNR), False Positive Rate (FPR), False Negative Rate (FNR) and their A-Y conditioned versions, namely Equalized Odds (EOdds) and Equal Opportunity (EOp). To obtain fair data valuations, `FairShap` computes $\phi(\mathcal{D}, v)$ based on $\Phi_{i,j}$ and on a reference dataset, $\mathcal{T}$, which can be a small and fair external dataset or a partition of $\mathcal{D}$. Furthermore, no model is required to compute the fair data valuations and thus `FairShap` is model agnostic. In the following, we derive the expressions to compute the weights of a dataset according to `FairShap` in a binary classification case (i.e., Y is a binary variable) and with binary protected attributes. The extension to non-binary protected attributes and multi-class scenarios is provided in App. D.5.

Equalized Odds (EOdds) and Equal Opportunity (EOp) are the two group fairness metrics that `FairShap` uses as valuation functions. Given that TPR and TNR are their building blocks, let $\phi_i(\text{TPR})$ and $\phi_i(\text{TNR})$ be two valuation functions that measure the contribution of training point i to the TPR and TNR, respectively. Note that $\text{TPR} = \text{Acc}|_{Y=1}$ and $\text{TNR} = \text{Acc}|_{Y=0}$. Therefore, $\phi_i(\text{TPR})$ corresponds to the expected change in the model's probability of correctly predicting the positive class when point i is included in the training dataset $\mathcal{D}$, considering all possible training dataset subsets and the distribution of the reference dataset.

$$\begin{aligned} \phi_i(\text{TPR}) :=& \mathbb{E}_{j \sim p(\mathcal{T}|Y=1)} \left[\mathbb{E}_{S \sim \mathcal{P}(\mathcal{D} \setminus \{i\})} \left[p(y = y_j | x_j, S \cup \{i\}) - p(y = y_j | x_j, S) \right] \right] \\ =& \mathbb{E}_{j \sim p(\mathcal{T}|Y=1)}[\Phi_{i,j}] = \overline{\mathbf{\Phi}}_{i,:|Y=1} \in \mathbb{R} \end{aligned} \tag{3}$$

The value for the entire dataset is $\boldsymbol{\phi}(\text{TPR}) = [\phi_0(\text{TPR}), \cdots, \phi_n(\text{TPR})] \in \mathbb{R}^{|\mathcal{D}|}$. $\boldsymbol{\phi}(\text{TNR})$ is obtained similarly but for $Y = y = 0$. In addition, $\phi_i(\text{FNR}) = 1/|\mathcal{D}| - \phi_i(\text{TPR})$ and $\phi_i(\text{FPR}) = 1/|\mathcal{D}| - \phi_i(\text{TNR})$. These four functions fulfill the SV axioms. More details about these metrics are shown in App. B. Intuitively, $\boldsymbol{\phi}(\text{TPR})$ and $\boldsymbol{\phi}(\text{TNR})$ quantify how much the examples in the training set contribute to the correct classification when $y = 1$ and $y = 0$, respectively. To illustrate $\boldsymbol{\phi}(\text{TPR})$ and $\boldsymbol{\phi}(\text{TNR})$, Fig. 7 in App. E.2 depicts the $\boldsymbol{\phi}(\text{TPR})$ and $\boldsymbol{\phi}(\text{TNR})$ of a simple synthetic example with two normally distributed classes. Once $\phi_i(\text{TPR})$, $\phi_i(\text{TNR})$, $\phi_i(\text{FPR})$ and $\phi_i(\text{FNR})$ have been obtained, we can compute the `FairShap` weights for a given dataset. However, there are two scenarios to consider, depending on whether the sensitive attribute (A) and the target variable or label (Y) are the same or not.

`FairShap` weights when $A = Y$ In this case, the group fairness metrics (EOp and EOdds) collapse to measure the disparity between TPR and TNR or FPR and FNR for the different values of the actual label (Berk et al., 2021), Y, which, in a binary classification case, may be expressed as the Equal Opportunity measure computed as EOp $:=$ TPR $-$ FPR $\in [-1, 1]$ or its scaled version EOp $=$ (TPR $+$ TNR)$/2 \in [0, 1]$. Thus, the $\phi_i(\text{EOp})$ of data point i may be expressed as

$$\phi_i(\text{EOp}) := \frac{\phi_i(\text{TPR}) + \phi_i(\text{TNR})}{2} \tag{4}$$

For more details on the equality of the group fairness metrics when $A = Y$ and how to obtain $\phi_i(\text{EOp})$, we refer the reader to App. D.3.

`FairShap` weights when $A \neq Y$ This is the most common scenario. In this case, EOp and EOdds use true/false positive/negative rates conditioned not only on Y, but also on A. Therefore, we define $\text{TPR}_{|A=a} = \text{Acc}_{|Y=y,A=a}$, or TPR_a for short, and thus

$$\phi_i(\text{TPR}_a) := \mathbb{E}_{j\sim p(\mathcal{T}|Y=1,A=a)}[\Phi_{i,j}] = \overline{\mathbf{\Phi}}_{i,:|Y=1,A=a} \tag{5}$$

where the value for the entire dataset is $\boldsymbol{\phi}(\text{TPR}_a) = [\phi_0(\text{TPR}_a), \cdots, \phi_n(\text{TPR}_a)]$. Intuitively, $\phi_i(\text{TPR}_a)$ measures the contribution of the training point i to the TPR of the testing points belonging to a given protected group ($A = a$). $\phi_i(\text{TNR}_a)$ is obtained similarly but for $y = 0$. Given EOp $:=$ $\text{TPR}_{|A=a} - \text{TPR}_{|A=b}$ and EOdds $:= \frac{(\text{FPR}_{A=a} - \text{FPR}_{A=b}) + (\text{TPR}_{A=a} - \text{TPR}_{A=b})}{2}$, then $\phi_i(\text{EOp})$ is given by

$$\phi_i(\text{EOp}) := \phi_i(\text{TPR}_a) - \phi_i(\text{TPR}_b) \tag{6}$$

and $\phi_i(\text{EOdds})$ is expressed as

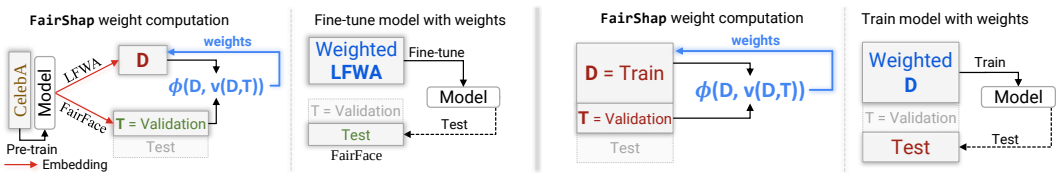
$$\phi_i(\text{EOdds}) := \frac{(\phi_i(\text{FPR}_a) - \phi_i(\text{FPR}_b)) + (\phi_i(\text{TPR}_a) - \phi_i(\text{TPR}_b))}{2} \tag{7}$$

where their corresponding $\boldsymbol{\phi}(\text{EOp})$ and $\boldsymbol{\phi}(\text{EOdds})$ vectors. A detailed view on the complete formula and a step-by-step derivation of the equations above can be found in App. B and D.4. Additionally, App. E.2 present a synthetic example showing the impact of $\phi(\cdot)$ on the decision boundaries and metrics. Algorithm 1 provides the pseudo-code to compute the data weights according to `FairShap`.

Re-weighting with `FairShap`. The definition of a data valuation function states that the larger the value assigned to a data point, the larger the point's contribution to the measure. Yet, it does not necessarily mean that a larger value is desirable: it depends on the value function of choice. In the case of accuracy, a larger value denotes a larger contribution to accuracy (Ghorbani & Zou, 2019). In the case of fairness, we prioritize points with high $-\phi(\text{EOp}) = \phi_i(\text{TPR}_B) - \phi_i(\text{TPR}_A)$, where B is the discriminated group (i.e. $\text{TPR}_A > \text{TPR}_B$). Therefore, assigning larger a weight to data points with a positive $-\phi_i(\text{EOp})$ contributes to increasing the TPR of the discriminated group, balancing the difference in TPR between groups and thus yielding a smaller EOp and a fairer model. In the experimental section, we refer the re-weighting $-\phi(\text{EOp})$ as $\phi(\text{EOp})$ for simplicity and the same for $\phi_i(\text{EOdds})$.

4 Experiments

In this section, we present the experiments performed to evaluate `FairShap`. We report results on a variety of benchmark datasets for $A = Y$ and $A \neq Y$, and with fair and biased reference datasets $\mathcal{T}$.



(a) $A = Y$ and $\mathcal{T}$ is fair. (b) $A \neq Y$ and $\mathcal{T}$ is biased.

Figure 2: Pipelines for the two scenarios described in the experiments.

SCENARIO 1: A=Y AND A FAIR $\mathcal{T}$

In this scenario, the task is to predict the sensitive attribute, i.e., $A = Y$, and the reference dataset $\mathcal{T}$ is fair. We perform a sex classification task from facial images by means of a deep convolutional network (Inception Resnet V1) (Szegedy et al., 2017) using `FairShap` for data re-weighting. Sex (male/female) is both the protected attribute (A) and the target variable (Y).

Datasets. We leverage three publicly available face datasets: CelebA, LFWA (Liu et al., 2015) and FairFace (Karkkainen & Joo, 2021), where LFWA is the training set $\mathcal{D}$ (large-scale and biased) and FairFace is the reference dataset $\mathcal{T}$ (small but fair). The test split in the FairFace dataset is used for testing. CelebA is used to pre-train the Inception Resnet V1 model to obtain the LFWA and FairFace embeddings that are needed to compute the SVs efficiently in the embedding space. In the three datasets, sex is a binary variable with two values: male, female.

Pipeline. The pipeline to obtain the `FairShap`'s weights in this scenario is depicted in Fig. 2a and proceeds as follows: (1) Pre-train an Inception Resnet V1 model with the CelebA dataset; (2) Use this model to obtain the embeddings of the LFWA and FairFace datasets; (3) Compute the weights on the LFWA training set ($\mathcal{D}$) using as reference dataset ($\mathcal{T}$) the FairFace validation partition. (4) Fine-tune the pretrained model using the re-weighted data in the LFWA training set according to $\boldsymbol{\phi}$; and (5) Test the resulting model on the test partition of the FairFace dataset. The experiment's training details and hyper-parameter setting are described in App. E.3.

FairShap Re-weighting. In this case, the group fairness metrics are equivalent and thus we report results using $\phi_i(\text{EOp})$: $\phi_i(\text{EOp})$ quantifies the contribution of the ith data point (image) in LFWA to the fairness metric (Equal Opportunity) of the model tested on the FairFace dataset.

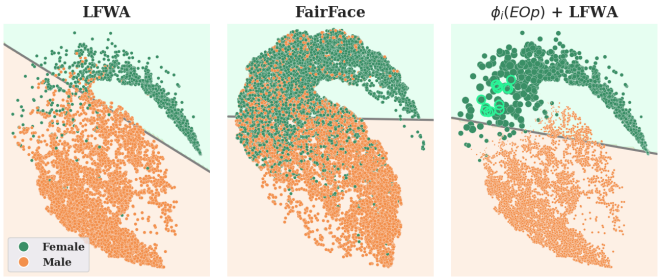
Baselines. We compare `FairShap` with three baselines: the pre-trained model using CelebA; the fine-tuned model using LFWA without re-weighting; and a data re-weighting approach using $\boldsymbol{\phi}(\text{Acc})$ from Ghorbani & Zou (2019). We report the accuracy of the models in correctly classifying the sex in the images and EOP. A summary of the experimental setup for this scenario is depicted in Fig. 2a.

Results. The results of this experiment are summarized in Tab. 1. Note how both re-weighting approaches ($\boldsymbol{\phi}(\text{Acc})$ and `FairShap`) significantly improve the fairness metrics while *increasing the accuracy* of the model. `FairShap` yields the best results **both in fairness and accuracy**. Regarding EOP, the model trained with data re-weighted according to `FairShap` yields improvements of **88%** and **66%** when compared to the model trained without re-weighting (LFWA) and the model trained with weights according to $\boldsymbol{\phi}(\text{Acc})$, respectively. In sum, data re-weighting with `FairShap` is able to leverage complex models trained on biased datasets and improve both their fairness and accuracy. To gain a better understanding of the behavior of `FairShap` in this scenario, Fig. 3b (bottom) depicts a histogram of the $\boldsymbol{\phi}(\text{EOp})$ values on the LFWA training dataset. As seen in the Figure, $\boldsymbol{\phi_i}(\text{EOp})$ are mostly positive for the examples labeled as *female* and mostly zero or negative for the examples labeled as *male*. This result makes intuitive sense given that the original model is biased against females, Fig. 3b (top) depicts the five images with the largest $\phi_i(\text{EOp})$: they all belong to the female category and depict faces with a variety of poses, different facial expressions and from diverse races.

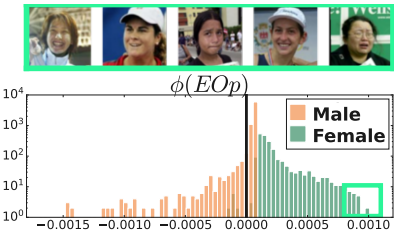
Table 1: Performance of the Inception Resnet V1 model tested on the FairFace dataset without and with re-weighting and with A=Y=sex.

Training Set	Acc↑	TPR_W | TPR_M	EOp ↓
FairFace	0.909	0.906 | 0.913	0.007
CelebA	0.759	0.580 | 0.918	0.34
LFWA	0.772	0.635 | 0.896	0.26
$\boldsymbol{\phi}(\text{Acc})$	0.793	0.742 | 0.839	0.09
$\boldsymbol{\phi}(\text{EOp})$	**0.799**	0.782 | 0.813	**0.03**

Note that in this case `FairShap` behaves like a distribution shift method. Fig. 3a shows how $\phi_i(\text{EOp})$ shifts the distribution of $\mathcal{D}$ (LFWA) to be as similar as possible to the distribution of the reference dataset $\mathcal{T}$ (FairFace). Therefore, biased datasets (such as $\mathcal{D}$) may be debiased by re-weighting their data according to $\phi_i(\text{EOp})$, yielding models with competitive performance both in terms of accuracy and fairness. Fig. 3a illustrates how the group fairness metrics impact individual data points: critical data points are those near the decision boundary. This finding is consistent with recent work that has proposed using SVs to identify counterfactual samples (Albini et al., 2022).



(a) LFWA, FairFace and $\phi_i(\text{EOp})$+LFWA embeddings.



(b) Images with largest $\phi_i(\text{EOp})$ and histogram of $\phi_i(\text{EOp})$ on the LFWA dataset.

Figure 3: (a) Image embeddings for LFWA (Left), FairFace (Middle) and LFWA with data point sizes $\propto |\phi_i(\text{EOp})|$ (Right). Points with the largest $\phi_i(\text{EOp})$ are highlighted in green. (b) 5 Images with the largest $\phi_i(\text{EOp})$ and histogram of $\phi_i(\text{EOp})$ on the LFWA dataset.

SCENARIO 2: $A \neq Y$ AND BIASED $\mathcal{T}$

In this section, we consider a common real-life scenario where the target variable Y is not a protected attribute and a single biased dataset $\mathcal{D}$ is used for training, validation, and testing. Thus, the validation set $\mathcal{T}$ is obtained from $\mathcal{D}$ according to the pipeline illustrated in Fig. 2b. Given that $A \neq Y$, `FairShap` considers two different valuation functions: $\phi(\text{EOp})$ and $\phi(\text{EOdds})$ as per Eq. (6) and Eq. (7), respectively. Note that in this case the weights assigned to each data point, w_i, are obtained by normalizing $\phi(\text{EOp})$ and $\phi(\text{EOdds})$ following a methodology similar to that described in Chai & Wang (2022): $w_i = {\phi_i}/{(\sum_i \phi_i')}|D|$ where $\phi_i' = {(\phi_i - \min(\boldsymbol{\phi}))}/{(\max(\boldsymbol{\phi}) - \min(\boldsymbol{\phi}))}$.

Datasets. We test `FairShap` on three commonly used datasets in the algorithmic fairness literature: (1) the German Credit (Kamiran & Calders, 2009) dataset, (2) the Adult Income dataset (Kohavi et al., 1996),and (3) the COMPAS (Angwin et al., 2016) dataset (see App. E.8 for details).

Pipeline. The model in all experiments is a Gradient Boosting Classifier (GBC) (Friedman, 2001). The pipeline in this set of experiments is depicted in Fig. 2b. Here, the reference dataset $\mathcal{T}$ is the validation set of $\mathcal{D}$. The reported results correspond to the average values of running the experiment 50 times with random splits stratified by sensitive group and label: 70% of the original dataset used for training ($\mathcal{D}$), 15% for the reference set ($\mathcal{T}$) and 15% for the test set. Train, reference and test set are stratified by A and Y such that they have the same percentage of A - Y samples as in $\mathcal{D}$.

Baselines. To the best of our knowledge, `FairShap` is the only interpretable, instance-level model-agnostic data re-weighting approach for group algorithmic fairness (see Tab. 4). We compare its performance with 6 state-of-the-art algorithmic fairness methods that only *partially* satisfy `FairShap`'s properties: *1. Group RW*: A group-based re-weighting method (Kamiran & Calders, 2012); *2. Post-processing*: A post-processing approach proposed by (Hardt et al., 2016); *3. LabelBias:* An in-processing re-weighting technique by (Jiang & Nachum, 2020); *4. Opt-Pre:* A feature and label transformation-based approach by (Calmon et al., 2017); *5. IFs:* An Influence Function (IF)-based approach described in (Li & Liu, 2022); and *6.* $\phi(\text{Acc})$: A data re-weighting method by means of accuracy-based Shapley Values (Ghorbani & Zou, 2019). An extended explanation of the methods and the hyperparameters used in the experiments can be found in App. E.4.

Results. The metrics used for evaluation are accuracy (Acc); Macro-F1 (M-F1), EOp and EOdds. Tab. 2 summarizes the results for the Adult dataset. The arrows indicate if the optimal result is 0 (↓) or 1 (↑). Fig. 4 summarizes the results on the German and Compas datasets. The complete table for all results on the three datasets is shown in Tab. 6 in App. E.5.

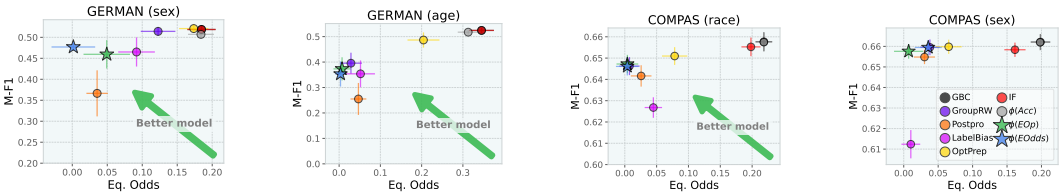


Figure 4: Utility vs fairness analysis. The models trained with `FairShap` re-weighting (★ in the plots) improve fairness while maintaining a competitive level of accuracy compared to the baselines.

Table 2: Results on the Adult dataset. **Bold** denote the best model and *italic* the second-best. Statistically significant differences with the best model are denoted by ‡ for p <0.01 and † for p <0.05.

Adult (Sex|Race)	Acc ↑	M-F1 ↑	EOp ↓	EOdds ↓	Acc ↑	M-F1 ↑	EOp ↓	EOdds ↓
GBC	*.803*±.001	‡.680±.002	‡.451±.004	‡.278±.003	**.803**±.001	‡.682±.002	‡.164±.010	‡.106±.006
Group RW	‡.790±.001	**.684**±.002	*.002*±.009	*.001*±.005	**.803**±.001	‡.683±.002	.010±.009	.010±.005
Postpro	‡.791±.001	†.679±.004	‡.056±.013	‡.034±.007	.802±.001	**.688**±.002	‡.061±.011	‡.042±.006
LabelBias	‡.781±.001	‡.681±.002	‡.065±.011	‡.049±.006	‡.800±.001	*.686*±.002	‡.118±.013	‡.074±.007
OptPrep	‡.789±.001	‡.676±.004	‡.064±.029	‡.037±.017	‡.800±.001	†.685±.002	‡.044±.015	‡.029±.009
IF	‡.787±.002	†.681±.003	‡.159±.037	‡.092±.022	‡.797±.002	†.685±.002	‡.042±.020	‡.031±.012
ϕ(Acc)	**.804**±.001	‡.681±.002	‡.452±.005	‡.279±.003	**.803**±.001	‡.681±.002	‡.161±.011	‡.104±.007
ϕ(EOp)	‡.790±.001	**.684**±.002	*.002*±.009	**3e-4**±.005	.802±.001	‡.683±.002	*.009*±.010	*.009*±.005
ϕ(EOdds)	‡.790±.001	.683±.002	**8e-4**±.009	*.001*±.005	.802±.001	‡.683±.002	**.007**±.009	**.007**±.005

Data re-weighting by means of `FairShap` (ϕ(EOdds) and ϕ(EOp)) generally yields significantly better results in the fairness metrics than the baselines while keeping competitive levels of accuracy. This improvement is notable when compared to the performance of the model built without data re-weighing (GBC). For example, in the German (Sex) dataset, the model's EOdds metric is **93x** smaller when re-weighting via `FairShap` (ϕ(EOdds)) than the baseline model (GBC) and **18x** better than the most competitive baseline (PostPro). Interestingly, the variance in the accuracy of the PostPro is significantly larger than that of other methods. Note that a simple method (Group RW) delivers very competitive results, even better than more sophisticated, recent approaches. Finally, accuracy is not an appropriate metric due to the imbalance of the datasets, being M-F1 a more suitable metric.

To shed further light on the behavior of data re-weighting with `FairShap`, Fig. 5 depicts the histograms of ϕ(EOp) and ϕ(Acc) for the German Credit dataset with sex as protected attribute. Note how the distribution of ϕ(Acc) is similar for males and females, even though the dataset is highly imbalanced: examples with good credit, irrespective of their sex, receive larger weights than those with bad credit. Conversely, the ϕ(EOp) values are larger for female applicants with good credit their male counterparts; and ϕ(EOp) are larger for male applicants with bad credit than their female counterparts. These distributions of ϕ(EOp) compensate for the imbalances in the raw dataset (both in terms of sex and credit risk), yielding fairer classifiers, as reflected in the results reported in Tab. 2.

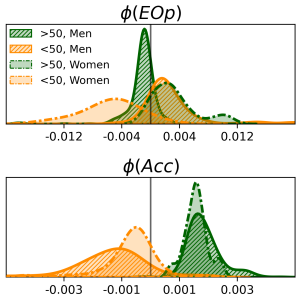


Figure 5: ϕ_i(EOp) (left) and ϕ_i(Acc) (right) for the German Credit dataset with A = sex.

Accuracy vs Fairness To further illustrate the impact of `FairShap`'s re-weighting, Fig. 6 depicts the utility-fairness trade-off curves on the three benchmark datasets. We define a parameter α that controls the contribution to the weights of each data point according to `FairShap`, ranging from $\alpha = 0$ (no data re-weighting) to $\alpha = 1$ (weights as given by `FairShap`). Thus, the weights of each data point i are computed as $w'_i = (1-\alpha)\mathbf{1}_{|\mathcal{D}|} + \alpha w_i$ where $\mathbf{1}_n = (1, 1, \ldots, 1) \in \mathbb{R}^n$ is the constant vector and w_i are the weights according to `FairShap`. As shown in the Fig. 6, the larger the importance of `FairShap`'s weights, the better the model's fairness. In some scenarios, such as on the German (age) dataset, we observe a utility-fairness Pareto front where the fairest models correspond to α = 1 and the best performing models correspond to α = 0. Conversely, on the COMPAS (sex) dataset, larger values of α significantly increase the fairness of the model while keeping similar levels of utility (M-F1 and Accuracy).

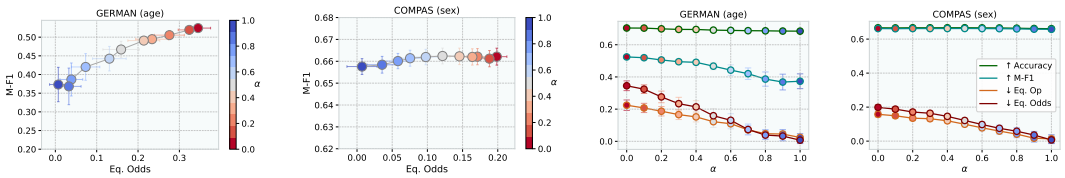


Figure 6: Utility vs fairness trade-off using Φ(EOp) re-weighting. Left graphs show the MF1-EOdds and the right graphs illustrate the Accuracy, M-F1, EOp and EOdds for increasing values of α.

5 CONCLUSION

In this paper, we have proposed `FairShap`, an instance-level, model-agnostic data re-weighting approach to achieve group fairness via data valuation using Shapley Values. We have empirically validated `FairShap` with several state-of-the-art datasets in different scenarios and using two different types of models (deep neural networks and GBCs). In our experimental results, the models trained with data re-weighted according to `FairShap` delivered competitive accuracy and fairness results. Our experiments also highlight the value of using fair reference datasets ($\mathcal{T}$) for data valuation. We have illustrated the interpretability of `FairShap` by means of histograms and a latent space visualization. We have also studied the utility vs fairness trade-off. From our experiments, we conclude that data re-weighting by means of `FairShap` could be a valuable approach to achieve algorithmic fairness. Furthermore, from a practical perspective, `FairShap` satisfies interpretability desiderata proposed by legal stakeholders and upcoming regulations.

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A PRELIMINARIES

A.1 NOTATION

Table 3: Notation.

Symbol	Description
$\mathcal{D} = \{(x_i, y_i)\}_{i=0}^{n}$	Training dataset
$\mathcal{T} = \{(x_j, y_j)\}_{i=0}^{m}$	Reference dataset
$S \subseteq \mathcal{D}$	Subset of a dataset $\mathcal{D}$
A	Set of variables that are protected attributes.
$\text{TPR}_{A=a}$	True positive rate for test points with values in the protected attribute equal to a. Also TPR_a if the protected attribute is known. The same logic applies to FPR, TNR, and FNR.
$p(y\|x, \mathcal{D})$	Predictive distribution of data point x when trained with $\mathcal{D}$.
$p(y = y_j\|x_j, \mathcal{D})$	Likelihood of correct classification of data point x when trained with $\mathcal{D}$.
$\phi_i(\mathcal{D}, v)$	SV for data point i in the training dataset $\mathcal{D}$ according to the performance function v
$\boldsymbol{\phi}(\mathcal{D}, v)$	Vector with all the SVs of the entire dataset $\in \mathbb{R}^{\|\mathcal{D}\|}$.
$v(S, T)$	Value of dataset S w.r.t a reference dataset $\mathcal{T}$. E.g., the accuracy of a model trained with S tested on $\mathcal{T}$ ($v = \text{Acc}$) or the value of Equal Opportunity of of a model trained with S tested on $\mathcal{T}$ ($v = \text{EOp}$)
$\boldsymbol{\Phi} \in \mathbb{R}^{\|\mathcal{D}\| \times \|\mathcal{T}\|}$	Matrix where $\Phi_{i,j}$ is the contribution of the training point $i \in \mathcal{D}$ to the correct classification of $j \in \mathcal{T}$ according to D.1
$\overline{\boldsymbol{\Phi}}_{i,:}$	Mean of row i
$\overline{\boldsymbol{\Phi}}_{i,:\|A=a}$	Mean of row i conditioned to columns where $A = a$
$\mathbf{1}$	Vector of ones := $[1, 1, ..., 1]$

A.2 DESIDERATA

Table 4: Comparative properties of related algorithmic fairness methods

Method	**D1** Data Val.	**D2** Interpretable	**D3** Pre-processing	**D4** Model agnostic	**D5** Data RW	**D6** Instance-level
`FairShap`	✓	✓	✓	✓	✓	✓
Group-RW	✗	✓	✓	✓	✓	✗
Influence Functions	✓	✗	✗	✗	✓	✗
Inpro-RW (LabelBias)	✗	✓	✗	✗	✓	✗
Massaging (OptPre)	✗	✓	✓	✓	✗	✗
Post-pro	✗	✓	✗	✓	✗	✗

As previously noted in Section 2, the closest methods to `FairShap` in the literature are *Influence Functions* (Wang et al., 2022; Li & Liu, 2022), *In-processing reweighting* (e.g. LabelBias) (Krasanakis et al., 2018; Jiang & Nachum, 2020; Chai & Wang, 2022), *Group reweighting* (Kamiran & Calders, 2012) and *Massaging* (e.g. OptPre) (Feldman et al., 2015; Calmon et al., 2017).

D1 - Data valuation method. Our aim is to propose a novel fairness-aware data valuation approach. Thus, the first desired property concerns whether the method performs data valuation or not (Hammoudeh & Lowd, 2022). Data valuation methods compute the contribution or influence of a given data point to a target function, typically by analyzing the interactions between points (LOO, pair-wise or all the subsets in the data powerset).

D2 - Interpretable. The method should be easy to understand by a broad set of technical and non-technical stakeholders when applied to a variety of scenarios and purposes, including for data minimization, data acquisition policies, data selection for transfer learning, active learning, data sharing, mislabeled example detection and federated learning.

D3 - Pre-processing. The method should provide data insights that can be applicable to train a wide variety of ML learning methods.

D4 - Model agnostic. The computation of model-weights, data valuation values, data insights or data transformations should not rely on learning a model iteratively, to enhance flexibility, computational efficiency, interpretability and mitigate uncertainty. Therefore, this follows the guidelines to make data valuation models data-driven (Sim et al., 2022).

D5 - Data Re-weighting. The data insights drawn from applying the method should be in the form of weights to be applied to the data, which can be used to rebalance the dataset.

D6 - Instance-Level. Different insights or weights are given to each of the data points.

A.3 Clarification of the concept of fairness

Note that the concept of fairness in the definition of the Shapley Values (SVs) is different from algorithmic fairness. The former relates to the desired quality of the SVs to be proportional to how much each data point contributes to the model's performance. Formally, this translates to the SVs fulfilling certain properties (e.g. efficiency, symmetry, additivity...) to ensure a fair payout. The latter refers to the concept of fairness used in the machine learning literature, as described in the introduction. `FairShap` uses SVs for data valuation in a pre-processing approach with the objective of mitigating bias in machine learning models. As `FairShap` is based on the theory of SVs, it also fulfills their four axiomatic properties.

A.4 Algorithmic Fairness Definitions

As aforementioned, the fairness metrics used as valuation functions in `FairShap` depend on the *conditioned* true/false negative/positive rates, depending on the protected attribute A:

$$\text{TPR}_{A=a} := \mathbb{P}[\hat{Y} = 1 | Y = 1, A = a], \;\; \text{TNR}_{A=a} := \mathbb{P}[\hat{Y} = 0 | Y = 0, A = a]$$
$$\text{FPR}_{A=a} := \mathbb{P}[\hat{Y} = 1 | Y = 0, A = a], \;\; \text{FNR}_{A=a} := \mathbb{P}[\hat{Y} = 0 | Y = 1, A = a]$$

Note that different fairness metrics are defined by forcing the equality in true/false negative/positive rates between different protected groups. For instance, in a binary classification scenario with a binary sensitive attribute, Equal Opportunity (EOp) and Equalized Odds (EOdds) are defined as follows:

$$\text{EOp} := \mathbb{P}[\hat{y} = 1 | Y = 1, A = a] = \mathbb{P}[\hat{Y} = 1 | Y = 1]$$
$$\text{EOdds} := \mathbb{P}[\hat{y} = 1 | Y = i, A = a] = \mathbb{P}[\hat{Y} = 1 | Y = i], \forall i \in \{0, 1\}$$

In practical terms, the metrics above are relaxed and computed as the difference for the different groups:

$$\text{EOp} := \text{TPR}_{A=a} - \text{TPR}_{A=b}, \;\; \text{EOdds} := \frac{1}{2}((\text{TPR}_{A=a} - \text{TPR}_{A=b}) + (\text{FPR}_{A=a} - \text{FPR}_{A=b}))$$

The proposed Fair SVs include as their valuation function these group fairness metrics.

B Shapley Values proposed in FairShap

`FairShap` proposes $\phi(\text{EOp})$ and $\phi(\text{EOdds})$ as the data valuation functions to compute the SVs of individual data points in the training set. These functions are computed from the $\phi(\text{TPR})$, $\phi(\text{FPR})$, $\phi(\text{TNR})$ and $\phi(\text{FNR})$ functions, leveraging the Efficiency axiom of the SVs, and the decomposability properties of fairness metrics.

Accuracy (Jia et al. (2019)):

$\phi_i(\text{Acc}) := \frac{1}{|\mathcal{T}|} \sum_{j \in \mathcal{T}} \Phi_{i,j} = \overline{\mathbf{\Phi}}_{i,:} = \mathbb{E}_{j \sim p(\mathcal{T})}[\Phi_{i,j}]$

True/False Positive/Negative rates (Our contribution):

$$\begin{aligned}
\phi_i(\text{TPR}) &:= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=1)}\left[\mathop{\mathbb{E}}_{S\sim\mathcal{P}(\mathcal{D}\setminus\{i\})}[p(y=1|x_j,S\cup\{i\})-p(y=1|x_j,S)]\right] \\
&= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=1)}[\Phi_{i,j}] = \frac{\sum_{j\in\mathcal{T}}\Phi_{i,j}\mathbb{I}[y_j=1]}{|\{x:x\in\mathcal{T}\,|y=1\}|} \\
\phi_i(\text{TNR}) &:= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=0)}\left[\mathop{\mathbb{E}}_{S\sim\mathcal{P}(\mathcal{D}\setminus\{i\})}[p(y=0|x_j,S\cup\{i\})-p(y=0|x_j,S)]\right] \\
&= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=0)}[\Phi_{i,j}] = \frac{\sum_{j\in\mathcal{T}}\Phi_{i,j}\mathbb{I}[y_j=0]}{|\{x:x\in\mathcal{T}\,|y=0\}|} \\
\phi_i(\text{FNR}) &:= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=1)}\left[\mathop{\mathbb{E}}_{S\sim\mathcal{P}(\mathcal{D}\setminus\{i\})}[p(y=0|x_j,S\cup\{i\})-p(y=0|x_j,S)]\right] = \frac{1}{|\,\mathcal{D}\,|}-\phi_i(\text{TPR}) \\
\phi_i(\text{FPR}) &:= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=0)}\left[\mathop{\mathbb{E}}_{S\sim\mathcal{P}(\mathcal{D}\setminus\{i\})}[p(y=1|x_j,S\cup\{i\})-p(y=1|x_j,S)]\right] = \frac{1}{|\,\mathcal{D}\,|}-\phi_i(\text{TNR})
\end{aligned}$$

Conditioned True/False Positive/Negative rates (Our contribution):

$\phi_i(\text{TPR}_a) := \mathbb{E}_{j\sim p(\mathcal{T}\,|Y=1,A=a)}[\Phi_{i,j}] = \frac{\sum_{j\in\mathcal{T}}\Phi_{i,j}\mathbb{I}[y_j=1,A_j=a]}{|\{x:x\in\mathcal{T}\,|y=1,A=a\}|} = \overline{\mathbf{\Phi}}_{i,:|Y=1,A=a}$

$\phi_i(\text{TPR}_b) := \mathbb{E}_{j\sim p(\mathcal{T}\,|Y=1,A=b)}[\Phi_{i,j}] = \frac{\sum_{j\in\mathcal{T}}\Phi_{i,j}\mathbb{I}[y_j=1,A_j=b]}{|\{x:x\in\mathcal{T}\,|y=1,A=b\}|} = \overline{\mathbf{\Phi}}_{i,:|Y=1,A=b}$

$\phi_i(\text{TNR}_a) := \mathbb{E}_{j\sim p(\mathcal{T}\,|Y=0,A=a)}[\Phi_{i,j}] = \frac{\sum_{j\in\mathcal{T}}\Phi_{i,j}\mathbb{I}[y_j=0,A_j=a]}{|\{x:x\in\mathcal{T}\,|y=0,A=a\}|} = \overline{\mathbf{\Phi}}_{i,:|Y=0,A=a}$

$\phi_i(\text{TNR}_b) := \mathbb{E}_{j\sim p(\mathcal{T}\,|Y=0,A=b)}[\Phi_{i,j}] = \frac{\sum_{j\in\mathcal{T}}\Phi_{i,j}\mathbb{I}[y_j=0,A_j=b]}{|\{x:x\in\mathcal{T}\,|y=0,A=b\}|} = \overline{\mathbf{\Phi}}_{i,:|Y=0,A=b}$

$\phi_i(\text{FPR}_a) := \frac{1}{|\,\mathcal{D}\,|} - \phi_i(\text{TNR}_a)$

$\phi_i(\text{FPR}_b) := \frac{1}{|\,\mathcal{D}\,|} - \phi_i(\text{TNR}_b)$

$\phi_i(\text{FNR}_a) := \frac{1}{|\,\mathcal{D}\,|} - \phi_i(\text{TPR}_a)$

$\phi_i(\text{FNR}_b) := \frac{1}{|\,\mathcal{D}\,|} - \phi_i(\text{TPR}_b)$

When A=Y (Our contribution):

$\phi_i(\text{EOp}) := \phi_i(\text{EOp}) = \phi_i(\text{TPR}) + \phi_i(\text{TNR}) - \frac{1}{|\,\mathcal{D}\,|}$

or its scaled version $\phi_i(\text{EOp}) = \frac{\phi_i(\text{TPR})+\phi_i(\text{TNR})}{2}$.

See App. D.3 for more details on how to derive these formulas.

When A≠Y (Our contribution):

$$\begin{aligned}
\phi_i(\text{EOp}) :=& \phi_i(\text{TPR}_a) - \phi_i(\text{TPR}_b) \\
&= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=1,A=a)}[\Phi_{i,j}] - \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=1,A=b)}[\Phi_{i,j}] \qquad (8) \\
&= \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=1,A=a)}\left[\mathop{\mathbb{E}}_{S\sim\mathcal{P}(\mathcal{D}\setminus\{i\})}[p(y=1|x_j,\mathcal{D}\cup\{i\})-p(y=1|x_j,\mathcal{D})]\right] \\
&- \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}\,|Y=1,A=b)}\left[\mathop{\mathbb{E}}_{S\sim\mathcal{P}(\mathcal{D}\setminus\{i\})}[p(y=1|x_j,\mathcal{D}\cup\{i\})-p(y=1|x_j,\mathcal{D})]\right]
\end{aligned}$$

$$\phi_i(\text{EOdds}) := \frac{1}{2}\left((\phi_i(\text{FPR}_a) - \phi_i(\text{FPR}_b)) + (\phi_i(\text{TPR}_a) - \phi_i(\text{TPR}_b))\right) \tag{9}$$

$$\begin{aligned} = \frac{1}{2}\Bigg(\Bigg(\left(\frac{1}{|\mathcal{D}|} - \phi_i(\text{TNR}_a)\right) - \left(\frac{1}{|\mathcal{D}|} - \phi_i(\text{TNR}_b)\right)\Bigg) \\ + \left(\mathop{\mathbb{E}}_{j\sim p(\mathcal{T}|Y=1,A=a)}[\Phi_{i,j}] - \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}|Y=1,A=b)}[\Phi_{i,j}]\right)\Bigg) \end{aligned} \tag{10}$$

$$\begin{aligned} = \frac{1}{2}\Bigg(\Bigg(\left(\frac{1}{|\mathcal{D}|} - \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}|Y=0,A=a)}[\Phi_{i,j}]\right) - \left(\frac{1}{|\mathcal{D}|} - \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}|Y=0,A=b)}[\Phi_{i,j}]\right)\Bigg) \\ + \left(\mathop{\mathbb{E}}_{j\sim p(\mathcal{T}|Y=1,A=a)}[\Phi_{i,j}] - \mathop{\mathbb{E}}_{j\sim p(\mathcal{T}|Y=1,A=b)}[\Phi_{i,j}]\right)\Bigg) \end{aligned}$$

We refer to the reader to App. D.4 for more details on how to obtain these formulas from the algorithmic fairness definitions.

C FAIRSHAP ALGORITHM

Algorithm 1 Data re-weighting for algorithmic fairness via SVs, $A \neq Y$

Input Training set $\mathcal{D}$, reference set $\mathcal{T}$, protected groups A, parameter k
procedure CALCULATEFAIRSHAPLEYVALUES($\mathcal{D}$, $\mathcal{T}$, k)
  Initialize $\mathbf{\Phi}$ as a matrix of zeros with dimensions $|\mathcal{D}| \times |\mathcal{T}|$
  for j in $\mathcal{T}$ **do**
    Order $i \in \mathcal{D}$ according to the L_2 distance to $j \in \mathcal{T} \rightarrow (x_1, x_2, \ldots, x_N)$
    Compute $\Phi_{N,j} = \frac{\mathbb{I}[y_{x_N} = y_j]}{N}$
    for i from $N-1$ to 1 **do**
      ▷ *How much does i contribute to j's likelihood of correct classification? (i.e., $\Phi_{i,j}$)* ◁
      $\Phi_{i,j} = \Phi_{i+1,j} + \frac{\mathbb{I}[y_i = y_j] - I[y_{i+1} = y_j]}{\max(k,i)}$
  $\boldsymbol{\phi}(\text{TPR}_a) = [\phi_i(\text{TPR}_a) = \mathbb{E}_{j\sim p(\mathcal{T}|Y=1,A=a)}[\Phi_{i,j}] : \forall i \in \mathcal{D}]\ \forall a \in A$ ▷ *Eq.* (5)
  $\boldsymbol{\phi}(\text{FPR}_a) = [\phi_i(\text{FPR}_a) = \frac{1}{|\mathcal{D}|} - \phi_i(\text{TNR}_a) :\ \forall i \in \mathcal{D}]\ \forall a \in A$
  $\boldsymbol{\phi}(\text{EOp}) = [\phi_i(\text{EOp}) = \phi_i(\text{TPR}_a) - \phi_i(\text{TPR}_b) : \forall i \in \mathcal{D}]$ ▷ *Eq.* (6)
  $\boldsymbol{\phi}(\text{EOdds}) = [\phi_i(\text{EOdds}) = \frac{(\phi_i(\text{FPR}_a) - \phi_i(\text{FPR}_b)) + (\phi_i(\text{TPR}_a) - \phi_i(\text{TPR}_b))}{2} : \forall i \in \mathcal{D}]$ ▷ *Eq.* (7)
  Output:
  SV matrix $\mathbf{\Phi} \in \mathbb{R}^{|\mathcal{D}| \times |\mathcal{T}|}$
  `FairShap` arrays $\boldsymbol{\phi}(\text{EOp}) \in \mathbb{R}^{|\mathcal{D}|}$ and $\boldsymbol{\phi}(\text{EOdds}) \in \mathbb{R}^{|\mathcal{D}|}$

D METHODOLOGY

D.1 EFFICIENT k-NN SHAPLEY VALUE

Jia et al. (2019) propose an efficient, exact calculation of the SVs by means of a recursive k-NN algorithm with complexity $O(N \log N)$. The proposed method yields a matrix $\mathbf{\Phi} \in \mathbb{R}^{|\mathcal{D}| \times |\mathcal{T}|}$ with the contribution of each training point to the accuracy of each point in the reference data set $\mathcal{T}$. Therefore, $\Phi_{i,j}$ defines how much data point i in the training set contributes to the probability of correct classification of data point j in $\mathcal{T}$. The intuition behind is that $\Phi_{i,j}$ quantifies to which degree a training point i helps in the correct classification of j. The k-NN-based recursive calculation is at follows.

For each j in $\mathcal{T}$:

- Order $i \in \mathcal{D}$ according to the distance to $j \in \mathcal{T} \rightarrow (x_1, x_2, ..., x_N)$
- Calculate $\Phi_{i,j}$ recursively, starting from the furthest point:

$$\Phi_{N,j} = \frac{I[y_{x_N} = y_j]}{N}$$

$$\Phi_{i,j} = \Phi_{i+1,j} + \frac{I[y_i = y_j] - I[y_{i+1} = y_j]}{\max K, i}$$

- $\mathbf{\Phi}$ is a $|\mathcal{D}| \times |\mathcal{T}|$ matrix given by:

$$\mathbf{\Phi} = \begin{bmatrix} \Phi_{00} & \cdots & \Phi_{0|\mathcal{T}|} \\ \vdots & \ddots & \vdots \\ \Phi_{|\mathcal{D}|0} & \cdots & \Phi_{|\mathcal{D}||\mathcal{T}|} \end{bmatrix} \in \mathbb{R}^{|\mathcal{D}| \times |\mathcal{T}|}$$

where $\Phi_{i,j}$ is the contribution of training point i to the accuracy of the model on point j in $\mathcal{T}$. Thus, the overall SV of a training point i with respect to $\mathcal{T}$ is the average of all the values of row i in the SV matrix:

$$\phi_i(\text{Acc}) = \frac{1}{m} \sum_{j=0}^{m} \Phi_{i,j} = \overline{\mathbf{\Phi}}_{i,:} \in \mathbb{R}$$

Note that the mean of a column j in $\mathbf{\Phi}$ is the accuracy of the model on that test point. The vector with the SV of every training data point is computed as:

$$\boldsymbol{\phi}(\text{Acc}) = [\phi_0, \cdots, \phi_n] \in \mathbb{R}^{|\mathcal{D}|}$$

In addition, given the efficiency axiom of the SV, the sum of $\boldsymbol{\phi}$ is the accuracy of the model on the training set.

$$V(\mathcal{D}) = \sum_{i=0}^{n} \phi_i = \sum_{i=0}^{n} \frac{1}{m} \sum_{j=0}^{m} \Phi_{i,j} = \text{Acc}$$

Technically speaking, the process may be parallelized over all points in $\mathcal{T}$ (columns of the matrix) since the computation is independent, reducing the practical complexity from $O(N \log N)$ to $O(N)$.

D.2 Threshold independence

Computing $\boldsymbol{\phi}(\cdot)$ according to the original SV implementation entails evaluating the performance function $v(S)$ on each data point, which requires testing the model trained with S. As the group fairness metrics are based on different classification errors, they depend on the classification threshold t, such that TP $= |\{\hat{Y} > t | Y = 1\}|$, TN $= |\{\hat{Y} < t | Y = 0\}|$, FP $= |\{\hat{Y} > t | Y = 0\}|$ and FN $= |\{\hat{Y} < t | Y = 1\}|$.

However, the previously described efficient method (App. D.1) is threshold independent since it calculates the accuracy as the average of the probability of correct classification for all test points.

D.3 $\phi_i(\text{EOp})$ derivation when $A = Y$

When $A = Y$ in a binary classification task, TPR and TNR are the accuracies for each protected group, respectively. In this case, DP collapses to $\mathbb{P}(\hat{Y} = 1 | A = a) \rightarrow \mathbb{P}(\hat{Y} = 1 | Y = a)$. In this case, EOp measures the similarity of TPRs between groups.

As a result, when $A = Y$ in a binary classification scenario, the group fairness metrics measure the relationship between TPR, TNR, FPR and FNR not conditioned on the protected attribute A, since these metrics already depend on Y and $A = Y$. As an example, Equal opportunity is defined in this case as $(\text{TPR} + \text{TNR})/2 \in [0, 1]$ (Hardt et al., 2016):

$$\text{EOp} = \frac{\text{TPR} - \text{FPR} + 1}{2} = \frac{\text{TPR} - (1 - \text{FNR}) + 1}{2} = \frac{\text{TPR} + \text{TNR}}{2} \in [0, 1]$$

Consequently, $\phi_i(\text{EOp}) \in [0, 1]$ when $A = Y$ can be obtained as follows:

$$\text{EOp} = \frac{\sum_{i \in \mathcal{D}} \phi_i(\text{TPR}) + \sum_{i \in \mathcal{D}} \phi_i(\text{TNR})}{2} = \sum_{i \in \mathcal{D}} \frac{\phi_i(\text{TPR})}{2} + \sum_{i \in \mathcal{D}} \frac{\phi_i(\text{TNR})}{2}$$

$$\phi_i(\text{EOp}) = \frac{\phi_i(\text{TPR}) + \phi_i(\text{TNR})}{2}$$

D.4 $\phi_i(\text{EOp})$ AND $\phi_i(\text{EOdds})$ DERIVATION WHEN $A \neq Y$

We derive $\boldsymbol{\phi}(\text{EOp})$ and $\boldsymbol{\phi}(\text{EOdds})$ when $A \neq Y$ using the definitions for EOdds and EOp given by:

$$\text{EOp} = \text{TPR}_{A=a} - \text{TPR}_{A=b}$$
$$\text{EOdds} = \frac{1}{2}((\text{TPR}_{A=a} - \text{TPR}_{A=b}) + (\text{FPR}_{A=a} - \text{FPR}_{A=b}))$$

Leveraging the Efficiency property of SVs, $\boldsymbol{\phi}(\text{EOp})$ is computed as:

$$\text{EOp} = \sum_{i \in \mathcal{D}} \phi_i(\text{TPR}_{A=a}) - \sum_{i \in \mathcal{D}} \phi_i(\text{TPR}_{A=b})$$
$$\text{EOp} = \sum_{i \in \mathcal{D}} (\phi_i(\text{TPR}_{A=a}) - \phi_i(\text{TPR}_{A=b})) \rightarrow \phi_i(\text{EOp}) = \phi_i(\text{TPR}_{A=a}) - \phi_i(\text{TPR}_{A=b})$$

Similarly, $\boldsymbol{\phi}(\text{EOdds})$ can be obtained as follows:

$$\begin{aligned}
\text{EOdds} &= \frac{1}{2}((\text{TPR}_{A=a} - \text{TPR}_{A=b}) + (\text{FPR}_{A=a} - \text{FPR}_{A=b})) \\
&= \frac{(\sum_{i \in \mathcal{D}} \phi_i(\text{TPR}_{A=a}) - \sum_{i \in \mathcal{D}} \phi_i(\text{TPR}_{A=b})) + (\sum_{i \in \mathcal{D}} \phi_i(\text{FPR}_{A=a}) - \sum_{i \in \mathcal{D}} \phi_i(\text{FPR}_{A=b}))}{2} \\
&= \frac{\sum_{i \in \mathcal{D}} (\phi_i(\text{TPR}_{A=a}) - \phi_i(\text{TPR}_{A=b})) + \sum_{i \in \mathcal{D}} \left(\phi_i(\text{FPR}_{A=a}) - \sum_{i \in \mathcal{D}} \phi_i(\text{FPR}_{A=b})\right)}{2} \\
&= \frac{\sum_{i \in \mathcal{D}} ((\phi_i(\text{TPR}_{A=a}) - \phi_i(\text{TPR}_{A=b})) + (\phi_i(\text{FPR}_{A=a}) - \phi_i(\text{FPR}_{A=b})))}{2} \rightarrow \\
\rightarrow \phi_i(\text{EOdds}) &= \frac{(\phi_i(\text{TPR}_{A=a}) - \phi_i(\text{TPR}_{A=b})) + (\phi_i(\text{FPR}_{A=a}) - \phi_i(\text{FPR}_{A=b}))}{2}
\end{aligned}$$

D.5 EXTENSION TO MULTI-LABEL AND CATEGORICAL SENSITIVE ATTRIBUTE SCENARIOS

As in the binary setting, the group fairness metrics are computed from TPR, TNR, FPR and FNR. Taking as an example TPR, the main change consists of replacing $y = 1$ or $y = 0$ for y_j=y:

$$\phi_i(\text{TPR}_{|Y=y}) = \mathbb{E}_{j \sim p(\mathcal{T}|Y=y)}[\Phi_{i,j}] = \frac{\sum_{j \in \mathcal{T}} \Phi_{i,j} \mathbb{I}[y_j = y]}{|\{x : x \in \mathcal{T} \,|y = y\}|} = \overline{\boldsymbol{\Phi}}_{i,:|Y=y}$$

The conditioned version $\phi_i(\text{TPR}_a)$ may be obtained as:

$$\phi_i(\text{TPR}_{|Y=y,A=a}) = \mathbb{E}_{j \sim p(\mathcal{T}|Y=y,A=a)}[\Phi_{i,j}] = \frac{\sum_{j \in \mathcal{T}} \Phi_{i,j} \mathbb{I}[y_j = y, A_j = a]}{|\{x : x \in \mathcal{T} \,|y = y, A = a\}|} = \overline{\boldsymbol{\Phi}}_{i,:|Y=y,A=a}$$

where y and a can be categorical variables. In the scenario where a is not a binary protected attribute, EOp is calculated as $\text{EOp}_a = |\text{TPR} - \text{TPR}_{A=a}| \forall a \in A$, and then the maximum difference is selected as the unique EOp for the model $\text{EOp} = \max_{\forall a \in A} \text{EOp}_a$, i.e. the EOp for the group that most differs from the TPR of the entire dataset. Therefore, $\phi_i(\text{EOp})$ for each data point is computed as $\phi_i(\text{EOp}) = \phi_i(\text{TPR}_a) - \phi_i(\text{TPR})$ being a the value of the protected attribute with maximum EOp. The same procedure applies to EOdds. In other words, $\phi_i(\text{EOp}) = \mathbb{E}_{j \sim p(\mathcal{T}|Y=1,A=a)}[\Phi_{i,j}] - \mathbb{E}_{j \sim p(\mathcal{T}|Y=1)}[\Phi_{i,j}]$.

E EXPERIMENTS

The code is publicly available at `https://github.com/AdrianArnaiz/fair-shap/`.

E.1 IMPACT OF BIASED DATASETS ON THE MODELS' EVALUATION

It is crucial to be aware that models trained on biased datasets may perform well in terms of accuracy and fairness when tested against themselves. However, when evaluated against fair datasets, their

performance can deteriorate significantly. It is widely recognized that biased datasets can lead to biased machine learning models, which can perpetuate and exacerbate societal inequities. These models can reinforce existing biases and stereotypes, leading to unfair and discriminatory outcomes for certain groups, especially underrepresented or marginalized communities. Therefore, it is essential to develop fair reference datasets to ensure that machine learning models are tested in a way that accounts for the potential impact of bias and promotes fairness. In light of this, we present here the results of our experiments that illustrate the performance and fairness of a model trained and tested on three different dataset combinations: large yet biased datasets (LFWA and CelebA) and a smaller and unbiased dataset (FairFace). As illustrated in Tab. 5, the performance of the models trained on biased datasets (LFWA and CelebA) and tested on fair datasets (FairFace) is significantly worse than when tested on the biased datasets.

Table 5: Sex classification results reported as Accuracy ↑ | Accuracy Disparity ↓ for an Inception Resnet V1 model trained and tested on different datasets. The protected attribute A is sex. Note the degradation in performance when training on a biased dataset and evaluating on a fair dataset (marked in red font in the Table).

Train \ Test	FairFace	LFWA	CelebA
FairFace	90.9 | 0.01	95.7 | 0.03	96.7 | 0.09
LFWA	77.2 | 0.49	96.6 | 0.08	98.3 | 0.02
CelebA	76.1 | 0.61	96.9 | 0.09	98.2 | 0.01

E.2 Experiments on synthetic datasets

$\phi(\text{TPR})$ and $\phi(\text{TNR})$ In this section, we present a visual analysis of $\phi(\text{TPR})$ and $\phi(\text{TPR})$ using a synthetic binary classification dataset featuring two Gaussian distributions.

Fig. 7 illustrates the extent to which each data point contributes positively or negatively to the True Positive Rate (TPR) and True Negative Rate (TNR). Points with larger $\phi(\text{TPR})$ correspond to instances from the positive class located on the correct side near the decision boundary whereas points with smaller $\phi(\text{TPR})$ represent positive class points placed on the wrong side of the decision boundary. The same logic applies to $\phi(\text{TNR})$ with respect to the negative class points, providing intuitive insights related to the contributions to TPR or TNR of different data points.

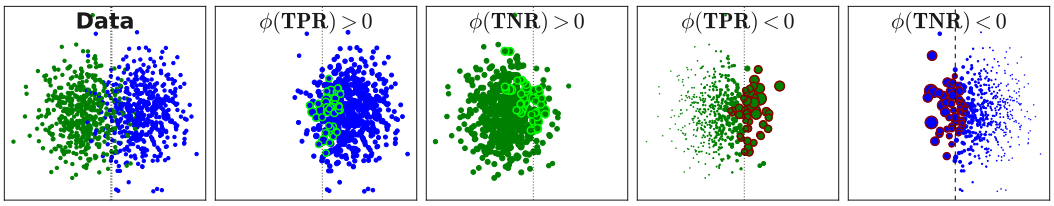
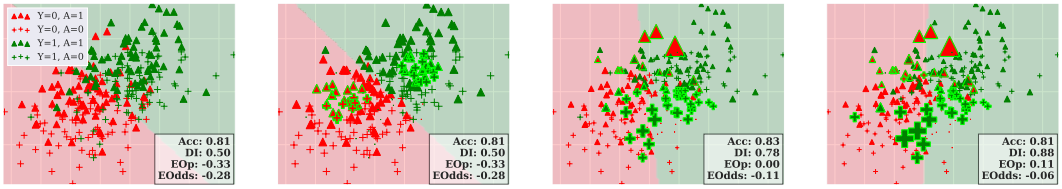


Figure 7: Synthetic example with positive (Y=1, blue) and negative (Y=0,green) classes. Data points with the 50 largest (Y=0, green) and smallest (Y=1, red) ϕ_i are highlighted. Size $\propto |\phi_i|$.

A≠Y In this scenario, we generate synthetic data where the protected attribute A and the label Y are slightly correlated. Specifically, we employ Case I from Zafar et al. (2017) as a reference, where the disparity between the False Negative Rate (FNR) and the False Positive Rate (FPR) exhibits a distinct sign: larger FPR for the privileged group and larger FNR for the disadvantaged group. Consequently, the mean overlap occurs between the unfavorable-privileged and the favorable-disadvantaged classes.

Fig. 8 visualizes data instances of this scenario as points, where the size of each point is proportional to its $|\phi(\cdot)|$. Additionally, we highlight in green the top-50 points based on their $\phi(\cdot)$. The label $Y = 1$ corresponds to the favorable outcome (colored in green), and the privileged group is defined by $A = 1$ (represented as triangles). The label $Y = 0$ corresponds to the unfavorable outcome (colored in red), and the disadvantaged group is defined by $A = 0$ (represented as crosses). We train unconstrained Logistic Regression models on various versions of the data and assess their performance using the same test split.



(a) Original Data (b) $\phi(\text{Acc})$ (c) $\phi(\text{EOp})$ (d) $\phi(\text{EOdds})$

Figure 8: Synthetic example where the group FPR and the group FNR differences have different signs (Case I in Zafar et al. (2017)). The size of each data point is proportional to its $|\phi(\cdot)|$. The top-50 points, according to each $\phi(\cdot)$, are highlighted in green. The label $Y = 1$ corresponds to the favorable outcome (colored in **green**), and the privileged group is defined by $A = 1$ (represented as triangles ▲). The label $Y = 0$ corresponds to the unfavorable outcome (colored in **red**), and the disadvantaged group is defined by $A = 0$ (represented as crosses +). Logistic Regression models are trained on different re-weighted versions of the data and evaluated using the same test split. Decision regions are shaded.

The experimental results shown in Fig. 8 illustrate significant changes in the decision boundaries of the models when trained using weights given by $\phi(\text{EOp})$ or $\phi(\text{EOdds})$, yielding fairer models while maintaining comparable or improved levels of accuracy. Moreover, the analysis reveals that both $\phi(\text{EOp})$ and $\phi(\text{EOdds})$ predominantly prioritize instances in the unfavorable-privileged (red triangles) and favorable-disadvantaged groups (green crosses).

E.3 Computer vision training set-up

In the experiment described in Section 4, the Inception Resnet V1 model was initially pre-trained on the CelebA dataset and subsequently fine-tuned on LFWA. Binary cross-entropy loss and the Adam optimizer were used in both training phases. The learning rate was set to 0.001 for pre-training and reduced to 0.0005 for fine-tuning, each lasting 100 epochs. Training batches consisted of 128 images with an input shape of (160x160), and a patience parameter of 30 was employed for early stopping, saving the model with the highest accuracy on the validation set. The classification threshold for this model was set at 0.5.

E.4 Description of the baselines used in the experiments (Section 4)

Group RW (Kamiran & Calders, 2012)]: A group-based re-weighting method that assigns the same weights to all samples from the same category or group according to the protected attribute. Weights are assigned to compensate that the expected probability if A and Y where independent on $\mathcal{D}$ is higher than the observed probability value.

$$w_i(a_i, y_i) = \frac{|\{X \in \mathcal{D}\,|X(A) = a_i\}| \times |\{X \in \mathcal{D}\,|X(Y) = y_i\}|}{|\{\mathcal{D}\}| \times |\{X \in \mathcal{D}\,|X(A) = a_i, X(Y) = y_i\}|}$$

Group RW does not require any additional parameters for its application. We use the implementation from `AIF360` (Bellamy et al., 2019).

Post-pro (Hardt et al., 2016): A post-processing algorithmic fairness method that assigns different classification thresholds for different groups to equalize error rates. The method applies a threshold to the predicted scores to achieve this balance.

In our experiments, we adopt an enhanced implementation of this method, provided by the authors and based on the `error-parity` library (Cruz & Hardt, 2024). This implementation makes its predictions using an ensemble of randomized classifiers instead of relying on a deterministic binary classifier. A randomized classifier is one that lies within the convex hull of the classifier's ROC curve at a specific target ROC point. This approach enhances the method's ability to satisfy the equality of error rates.

LabelBias (Jiang & Nachum, 2020): This model learns the weights in an iterative, in-processing manner based on the model's error. Consequently, this method is neither a pre-processing nor a model-agnostic approach.

We use an implementation based on the `google-research/label_bias` repository, which is the official implementation of the original work. We apply the settings described in Jiang & Nachum (2020) and use a learning rate of $\mu = 1$ with a fixed number of 100 iterations.

Opt-Pre (Calmon et al., 2017): A model-agnostic pre-processing approach for algorithmic fairness based on feature and label transformations solving a a convex optimization.

We use the pre-defined hyperparameters provided by both the authors (see Calmon et al., 2017, Supplementary 4.1-4.3) and the `AIF360` library: the discrimination parameter $\epsilon = 0.05$; distortion constraints of [0.99, 1.99, 2.99], which are distance thresholds for individual distortions; and finally we use probability bounds of [.1, 0.05, 0] for each threshold in the distortion constraints (Calmon et al., 2017, Eq. 5). We use the implementation from `AIF360` (Bellamy et al., 2019).

IFs (Li & Liu, 2022): An Influence Function (IF)-based approach, where the influence of each training sample is modeled with regard to a fairness-related quantity and predictive utility. This is an in-processing, and re-training approach, as follows. First, a model is trained. Second, the Hessian vector product is computed for every sample $\mathbf{H}^{-1}_{\hat{\theta}(\mathbf{1})} \nabla^2_\theta \ell(x_i; \hat{\theta}(\mathbf{1}))$, where the Hessian is defined as $\mathbf{H}_{\hat{\theta}(\mathbf{1})} = \sum_{i=1} \nabla^2_\theta \ell(x_i; \hat{\theta}(\mathbf{1}))$ and $\hat{\theta}(\mathbf{1})$ is the empirical risk minimization with equal sample weights. Third, the influence functions for every sample are obtained based on the vector products. Fourth, a linear problem based on these influence functions is solved to compute the weights. Finally, the model is re-trained with the new weights. Notably, this method exhibits behavior resembling hard removal re-weighing –as observed in our experiments– where the weights are either 0 or 1 for all samples, with no in-between values. This pattern aligns with the observations made by the authors themselves. While the method is theoretically categorized as individual re-weighting, in practice, it works as a sampling method.

We set the hyperparameters to the values reported by the authors for each dataset. Namely, for the German dataset: $\alpha = 1$, $\beta = 0$, $\gamma = 0$ and $\text{l2reg} = 5.85$. For the Adult dataset: $\alpha = 1$, $\beta = 0.5$, $\gamma = 0.2$ and $\text{l2reg} = 2.25$. Finally, for the COMPAS dataset: $\alpha = 1$, $\beta = 0.2$, $\gamma = 0.1$ and $\text{l2reg} = 37$. We use an implementation from the `influence-fairness` repository by Brandeis ML, which needs the request and installation of a Gurobi license.

$\phi(\textbf{Acc})$ (Ghorbani & Zou, 2019): A method based on data re-weighting by means of an accuracy-based data valuation function without any fairness considerations. This method is explained in detail on App. D.1. We use our own efficient implementation using the Numba python library.

E.5 Results on Additional Datasets

Tab. 6 shows the results of the experiments with the 3 different datasets: German, Adult and COMPAS. Utility is measured with accuracy and Macro F1 and fairness is measured with Equal Opportunity (EOp) and Equalized Odds (EOdds).

In addition, Fig. 9 depicts the effect of `FairShap` on the accuracy-fairness tradeoff in 2 more datasets than in Section 4: German (Sex) and Adult (Race). Results on these datasets are consistent with the previous results. Larger values of α significantly increase the fairness of the model while keeping similar levels of utility.

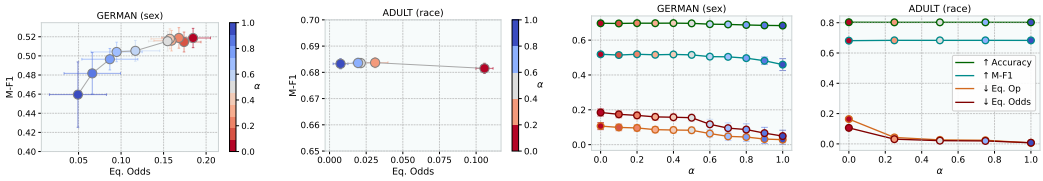


Figure 9: Utility vs fairness trade-off. $\mathbf{\Phi}(\text{EOp})$ is used to re-weight the German dataset, and $\mathbf{\Phi}(\text{EOdds})$ to re-weight in the Adult dataset. Left graphs show the MF1-EOdds and the right graphs illustrate the Accuracy, M-F1, EOp and EOdds for increasing values of α.

Table 6: Performance of GBC without and with data re-weighting on all datasets. Best results marked in **bold** and second-best in *italic*. Statistically significant differences with the best performing model are denoted by ‡ for $p < 0.01$ and † for $p < 0.05$.

	Sex				Age			
German	Accuracy ↑	M-F1 ↑	EOp ↓	EOdds ↓	Accuracy ↑	M-F1 ↑	EOp ↓	EOdds ↓
GBC	*.697*±.006	*.519*±.010	‡.107±.020	‡.185±.020	†*.704*±.005	**.524**±.010	‡.224±.032	‡.345±.030
Group RW	.695±.006	.514±.010	‡.062±.019	‡.123±.025	‡.684±.004	‡.396±.041	‡.040±.025	‡.029±.026
Postpro	‡.691±.005	‡.366±.055	*.013*±.014	†*.036*±.015	‡.686±.005	‡.255±.063	‡.044±.022	‡.047±.019
LabelBias	.695±.006	‡.465±.035	‡.051±.019	‡.092±.026	‡.690±.004	‡.354±.053	‡.052±.029	‡.052±.035
OptPrep	.694±.006	**.521**±.010	‡.104±.022	‡.174±.021	‡.693±.007	‡.487±.030	‡.130±.031	‡.204±.039
IF	*.697*±.006	*.519*±.010	‡.107±.020	‡.185±.020	†*.704*±.005	**.524**±.010	‡.224±.032	‡.345±.030
ϕ(Acc)	**.700**±.005	†.507±.009	‡.097±.018	‡.184±.018	**.706**±.005	‡.517±.010	‡.193±.025	‡.313±.025
ϕ(EOp)	‡.683±.006	‡.460±.034	.029±.026	†.049±.033	‡.685±.004	‡.373±.046	*.024*±.023	*.007*±.021
ϕ(EOdds)	‡.686±.006	‡.477±.009	**.002**±.025	**.002**±.031	‡.681±.005	‡.353±.049	**.019**±.020	**.003**±.013

	Sex				Race			
Adult	Accuracy ↑	M-F1 ↑	EOp ↓	EOdds ↓	Accuracy ↑	M-F1 ↑	EOp ↓	EOdds ↓
GBC	*.803*±.001	‡.680±.002	‡.451±.004	‡.278±.003	**.803**±.001	‡.682±.002	‡.164±.010	‡.106±.006
Group RW	‡.790±.001	**.684**±.002	*.002*±.009	*.001*±.005	**.803**±.001	‡.683±.002	.010±.009	.010±.005
Postpro	‡.791±.001	†.679±.004	‡.056±.013	‡.034±.007	.802±.001	**.688**±.002	‡.061±.011	‡.042±.006
LabelBias	‡.781±.001	‡.681±.002	‡.065±.011	‡.049±.006	‡.800±.001	*.686*±.002	‡.118±.013	‡.074±.007
OptPrep	‡.789±.001	‡.676±.004	‡.064±.029	‡.037±.017	‡.800±.001	†.685±.002	‡.044±.015	‡.029±.009
IF	‡.787±.002	†.681±.003	‡.159±.037	‡.092±.022	‡.797±.002	†.685±.002	‡.042±.020	‡.031±.012
ϕ(Acc)	**.804**±.001	‡.681±.002	‡.452±.005	‡.279±.003	**.803**±.001	‡.681±.002	‡.161±.011	‡.104±.007
ϕ(EOp)	‡.790±.001	**.684**±.002	*.002*±.009	**3e-4**±.005	.802±.001	‡.683±.002	*.009*±.010	*.009*±.005
ϕ(EOdds)	‡.790±.001	.683±.002	**8e-4**±.009	*.001*±.005	.802±.001	‡.683±.002	**.007**±.009	**.007**±.005

	Sex				Race			
COMPAS	Accuracy ↑	M-F1 ↑	EOp ↓	EOdds ↓	Accuracy ↑	M-F1 ↑	EOp ↓	EOdds ↓
GBC	*.666*±.004	**.662**±.004	‡.158±.014	‡.199±.014	**.663**±.004	**.658**±.004	‡.184±.013	‡.218±.013
Group RW	.664±.004	.660±.004	.020±.016	‡.038±.014	‡.649±.004	‡.646±.004	‡.028±.015	.007±.016
Postpro	‡.660±.003	‡.655±.003	.017±.017	†.030±.015	‡.647±.005	‡.642±.005	**1e-4**±.015	‡.026±.016
LabelBias	‡.639±.005	‡.612±.007	**.006**±.013	*.010*±.014	‡.645±.004	‡.627±.005	‡.030±.011	‡.045±.014
OptPrep	.664±.003	.660±.003	‡.045±.020	‡.065±.019	‡.655±.004	†.651±.004	‡.044±.020	‡.078±.020
IF	.663±.003	.658±.003	‡.129±.016	‡.161±.015	.660±.004	.655±.004	‡.165±.017	‡.198±.015
ϕ(Acc)	**.667**±.004	**.662**±.004	‡.156±.014	‡.198±.013	**.663**±.004	*.657*±.004	‡.184±.013	‡.218±.013
ϕ(EOp)	†.661±.003	†.658±.004	*.013*±.024	**.007**±.021	‡.650±.004	‡.647±.004	‡*.027*±.016	**.004**±.017
ϕ(EOdds)	.663±.004	.659±.004	.019±.021	†.036±.020	‡.648±.004	‡.646±.004	‡.036±.017	**.004**±.018

E.6 Ablation study of the impact of the reference dataset's size

In this section, we examine the influence of the size of the reference dataset, $\mathcal{T}$, and the impact of the alignment between $\mathcal{T}$ and the test set on the effectiveness of `FairShap`'s re-weighting. To do so, we perform an ablation study. We partition the three benchmark datasets (German, Adult and COMPAS) into training (60%, $\mathcal{D}$), validation (20%), and testing (20%). We select subsets from the validation dataset –ranging from 5% to 100% of its size– and use them as $\mathcal{T}$. For each subset, we compute `FairShap`'s weights on $\mathcal{D}$ with respect to $\mathcal{T}$, train a Gradient Boosting Classifier (GBC) model and evaluate its performance on the test set. This process is repeated 10 times with reported results comprising both mean values and standard deviations shown in Fig. 10.

As shown in the Figures, the size of the validation dataset has a discernible impact on the variance of the evaluation metrics, both in terms of accuracy and fairness. Increasing the size of the reference dataset T leads to a notable reduction in the variability of the outcomes. However, averages for all the metrics remain stable across different sizes.

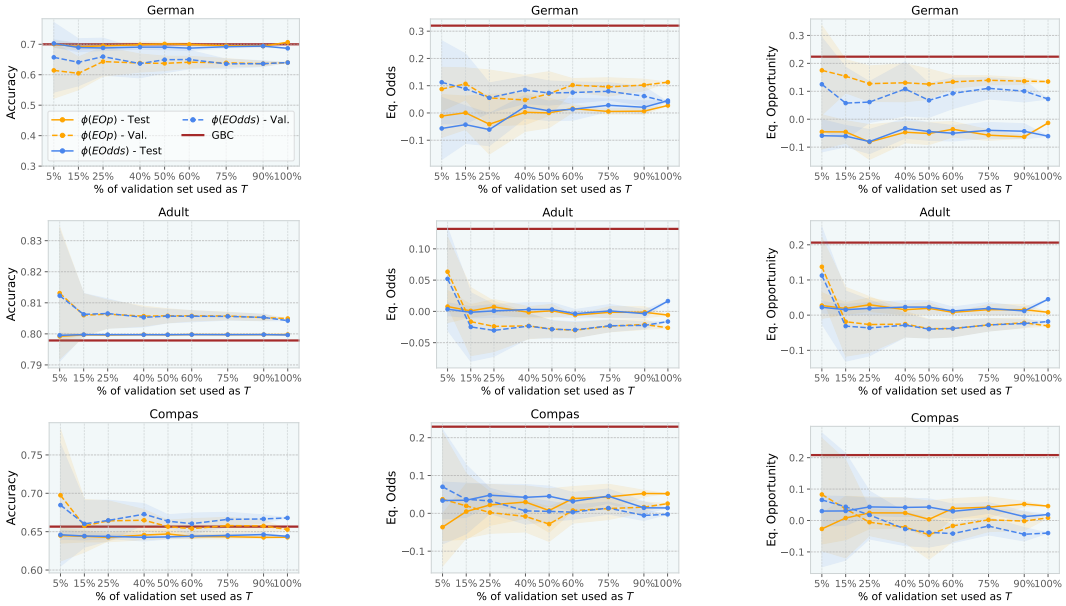


Figure 10: Accuracy and fairness metrics when applying data re-weighting via `FairShap` (ϕ(EOp) and ϕ(EOdds)) as the size of the validation sets $\mathcal{T}$ increases, evaluated on both validation (- -) and test sets (—). The performance of the baseline GBC without re-weighting is shown as a red line. From top to bottom, the rows correspond to the German, Adult and COMPAS datasets, respectively. From left to right, the columns depict the Accuracy, Equalized Odds and Equal Opportunity, respectively.

E.7 Utility metrics

The reported experiments with the tabular datasets (i.e. German, Adult and COMPAS) include different utility metrics to evaluate the performance of the models.

In imbalanced datasets, where one class significantly outweighs the other in terms of the number of examples, accuracy does not provide a good assessment of a model's performance, given that high accuracies might be obtained by a simple model that predicts the majority class. In these cases, the F1 metric is more appropriate, defined as $2 \times \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}}$, where precision is given by $\frac{\text{TP}}{\text{TP+FP}}$ and recall by $\text{TPR} = \frac{\text{TP}}{\text{TP+FN}}$.

However, the F1 metric is only meaningful when the positive class is the minority class. Otherwise, i.e. if the positive class is the majority class, a constant classifier that always predicts the positive class can achieve a high F1 values. For example, in a scenario where the positive class has 100 examples and the negative class only 20, a simple model that always predicts the positive class will get an accuracy of 0.83 and a F1 score of 0.91. However, the F1 score for the negative class would be 0 in this case.

The Macro-F1 metric arises as a solution to this scenario. Unlike the standard F1 score, the Macro-F1 computes the average of the F1 scores for each class. Thus, the Macro-F1 score can provide insight into the model's performance on every class for imbalanced datasets.

Thus, in our experiments with tabular data, we report the Macro-F1 scores.

E.8 Dataset statistics

Image Datasets Total number of images and male/female distribution from the CelebA, LFWA and FairFace datasets are shown on Tab. 7.

Fairness Benchmark Datasets The tables below summarize the statistics of the German, Adult and COMPAS datasets in terms of the distribution of labels and protected groups. Note that all the nomenclature regarding the protected attribute names and values is borrowed from the official documentation of the datasets.

Table 7: Face Datasets Statistics. Rows stand for `#male|#female`.

Dataset	Train	Validation	Test
CelebA	94,509|68,261	11,409|8,458	12,247|7,715
LFWA	7,439|2,086	2,832|876	–
FairFace	45,986|40,758	9,197|8,152	5,792|5,162

Tab. 8a shows the distributions of sex and label for the German dataset (Kamiran & Calders, 2009). It contains 1,000 examples with target binary variable the individual's *credit risk* and protected groups *age* and *sex*. We use 'Good Credit' as the favorable label (1) and 'Bad Credit' as the unfavorable one (0). Regarding *age* as a protected attribute, 'Age>25' and 'Age<25' are considered the favorable and unfavorable groups, respectively. When using *sex* as protected attribute, *male* and *female* are considered the privileged and unprivileged groups, respectively. Features used are the one-hot encoded credit history (delay, paid, other), one-hot encoded savings (>500, <500, unknown) and one-hot encoded years of employment (1-4y, >4y, unemployed).

Table 8: Tabular datasets statistics.

(a) German Credit

A\Y	Bad	Good	Total
Male	191	499	690 (69%)
Female	109	201	310 (31%)
Age>25	220	590	810 (81%)
Age<25	80	110	190 (19%)
Total	300 (30%)	700 (70%)	1,000

(b) Adult Income

A\Y	<50k	>50k	Total
White	31,155	10,607	41,762 (86%)
non-White	6,000	1,080	7,080 (14%)
Male	22,732	9,918	32,650 (67%)
Female	14,423	1,769	16,192 (33%)
Total	37,155 (76%)	11,687 (24%)	48,842

(c) COMPAS

A\Y	Recid	No Recid	Total
Male	2,110	2,137	4,247 (80%)
Female	373	658	1,031 (20%)
Caucasian	822	1,281	2,103 (40%)
non-Cauc.	1,661	1,514	3,175 (60%)
Total	2,483 (47%)	2,795 (53%)	5,278

Tab. 8b depicts the data statistics for the Adult Income dataset (Kohavi et al., 1996). This dataset contains 48,842 examples where the task is to predict if the *income* of a person is more than 50k per year, being >50k considered as the favorable label (1) and <50k as the unfavorable label (0). The protected attributes are *race* and *sex*. When *race* is the protected attribute, *white* refers to the privileged group and *non-white* to the unprivileged group. With *sex* as protected attribute, *male* is considered the privileged group and *female* the disadvantaged group. The features are the one-hot encoded age decade (10, 20, 30, 40, 50, 60, >70) and education years (<6, 6, 7, 8, 9, 10, 11, 12, >12).

Tab. 8c contains the statistics about the COMPAS (Angwin et al., 2016) dataset. This dataset has 5,278 examples with target binary variable *recidivism*. We use *Did recid* as the unfavorable label (0) and *No recid* as the favorable label (1). When *sex* is the protected attribute, *male* is the disadvantaged group and *female* as the privileged one. When using *race* as protected attribute, *caucasian* is the privileged group and *non-caucasian* the disadvantaged one. Regarding the features, we use one-hot encoded age (<25, 25-45, >45), one-hot prior criminal records of defendants (0, 1-3, >3) and one-hot encoded charge degree of defendants (Felony or Misdemeanor).

All datasets are pre-processed using AIF360 by Bellamy et al. (2019), which use the same pre-processing as in Calmon et al. (2017).

E.9 COMPUTATIONAL COST

We describe experiments to illustrate `FairShap`'s computational performance relative to other approaches by applying data re-weighting on synthetic datasets of varying sizes, ranging from 1k to 100k data points, each comprising 200 features. We compare the run time (in seconds) of computing the weights using `FairShap`, Group Re-weighting (Kamiran & Calders, 2012), OptPrep (Calmon et al., 2017), LabelBias (Jiang & Nachum, 2020) and IFs (Li & Liu, 2022). We leave the post-processing (Hardt et al., 2016) approach out of the comparison since its based on tweaking thresholds after a model is trained, such that the running time heavily depends on the training time of the model of choice. In these experiments, we allocate 80% of the data for training and 20% for validation. With 10 iterations for each configuration, we compute the mean and standard deviation of the run time on an Intel i7-1185G7 3.00GHz CPU. Results are reported in Fig. 11.

As seen in the Figure, instance level re-weighting via `FairShap` is computationally competitive for datasets with up to 30k data points. Group Re-weighting and LabelBias are computationally more efficient than `FairShap` on datasets with >30k data points.

Note that OptPrep (Calmon et al., 2017) and IFs (Li & Liu, 2022) require a hyperparameter search for each model and each dataset, yielding a significant increase on the computation time. In our experiments, we used the hyperparameters provided by the authors and hence did not have to tune them. Consequently, the actual running time for these methods would significantly increase depending on the number of hyperparameter configurations to be tested. For example, OptPrep consistently requires $\approx$ 10 seconds regardless of the dataset's size. However, a hyperparameter grid-search scenario with 20 different hyperparameter settings and 10-fold cross-validation, would increase the run time to 2,000 seconds (i.e., 10s/it x 20 x 10) or 20,000s for IFs on a dataset size of 60,000 samples (i.e., 100s/it x 20 x 10). These run times are significantly larger than those required to compute `FairShap`'s weights.

The Figure also depicts `FairShap`'s execution times (in seconds) with different numbers of features in datasets of increasing sizes. As seen in the Figure, three datasets with 60k, 80k, and 100k instances and feature dimension of 18, have runtimes of 14.7s, 29.1s, and 47.8s.

Finally, we provide an overview of `FairShap`'s run times for the experiments described in Section 4. On the German dataset, `FairShap` has an average execution time of 0.001s$\pm$0.002, where $|\mathcal{D}| = 700$, $|\mathcal{T}| = 150$, and there are 11 features. In the case of the Adult dataset, the execution time remains consistent at 12.7s$\pm$3, for a dataset with $|\mathcal{D}| = 34,189$, $|\mathcal{T}| = 7,326$, and 18 features. Finally, for the COMPAS dataset, the run time is 0.063s$\pm$0.004 on a dataset with $|\mathcal{D}| = 3,694$, $|\mathcal{T}| = 792$, and 10 features. These numbers are consistent with the run times reported in Fig. 11.

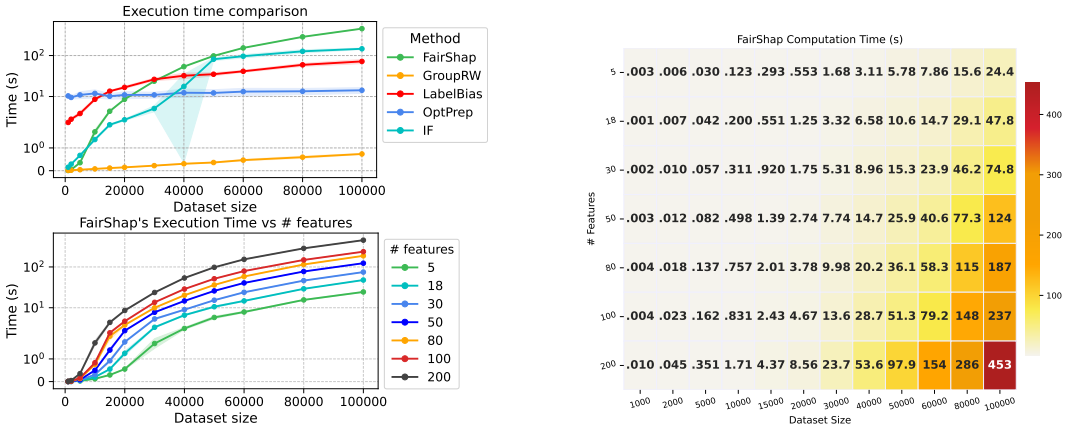


Figure 11: Left: Run time comparison of re-weighting via `FairShap` and baselines with respect to data set size and number of features. Datasets are split in 80% as D and 20% as T. We report mean and std run times for 10 iterations. Right: `FairShap`'s execution times (in seconds) on datasets with increasing numbers of features and sizes. In all experiments, the CPU is an Intel i7-1185G7 3.00GHz.