

# Offline Reinforcement Learning for Traffic Signal Control

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## Abstract

Traffic signal control is an important problem in urban mobility with a significant potential of economic and environmental impact. While there is a growing interest in Reinforcement Learning (RL) for traffic signal control, the work so far has focussed on learning through simulations which could lead to inaccuracies due to simplifying assumptions. Instead, real experience data on traffic is available and could be exploited at minimal costs. Recent progress in *offline* or *batch* RL has enabled just that. Model-based offline RL methods, in particular, have been shown to generalize from the experience data much better than others.

We build a model-based learning framework which infers a Markov Decision Process (MDP) from a dataset collected using a cyclic traffic signal control policy that is both commonplace and easy to gather. The MDP is built with pessimistic costs to manage out-of-distribution scenarios using an adaptive shaping of rewards which is shown to provide better regularization compared to the prior related work in addition to being PAC-optimal. Our model is evaluated on a complex signalized roundabout showing that it is possible to build highly performant traffic control policies in a data efficient manner.

## 1 Introduction

Road traffic signal control has attracted substantial interest as an application of reinforcement learning (RL) Wei et al. (2019); Yau et al. (2017). However most published work in the area is unlikely to be applied in practice as trial and error methods for interacting with the environment are not feasible in the real world. Similarly trying to infer an RL policy using a simulator does not take advantage of the fact that real experience data about traffic is available from transportation management operators.

A more appropriate and realistic set up is to use offline RL training to learn from static experience data Lange et al. (2012). A typical data set will consist of a set of tuples of the form  $\{s_i, a_i, r_i, s_{i+1}\}$ , *i.e.*, when the system was in state  $s_i$ , action  $a_i$  was taken, which resulted in a reward  $r_i$  and the system then transitioned into a new state  $s_{i+1}$ . From the experience data the objective is to learn a policy, *i.e.*, a mapping from state to action which maximizes the long term expected cumulative reward. In a traffic signal control setting, the state captures the distribution of traffic on the road network, the action space consists of different phases (red, green, amber) on signalized intersections and the reward is a metric of traffic efficiency.

Compared to the traditional (*i.e.*, online or off-policy) RL approaches, offline (or batch) RL shifts the focus of learning from data exploration to data-driven policy building. The offline policy building is challenging due to deviation in the state-action visitation by the policy being learned and the policy that logged the static dataset Fujimoto et al. (2019b); Levine et al. (2020). A number of different solution frameworks are proposed for offline RL that are either model-free or derive a Markov Decision Process (MDP) model from the dataset (see Section 7). We focus on model-based RL approaches which have been shown to offer better regularization in presence of uncertain data. Such approaches are characterized by a mechanism to penalize under-explored or under-represented transitions, a notion referred to as *pessimism under uncertainty* Fujimoto et al. (2019b).

We build on the DAC framework Shrestha et al. (2020), which derives a finite Markov Decision Process (MDP) from a static dataset and solves it using an optimal planner. The MDP derivation uses empirical averages to interpolate contributions from nearby transitions seen in the dataset. We incorporate an adaptive reward shaping function into the MDP building which makes the learning both robust to dataset properties

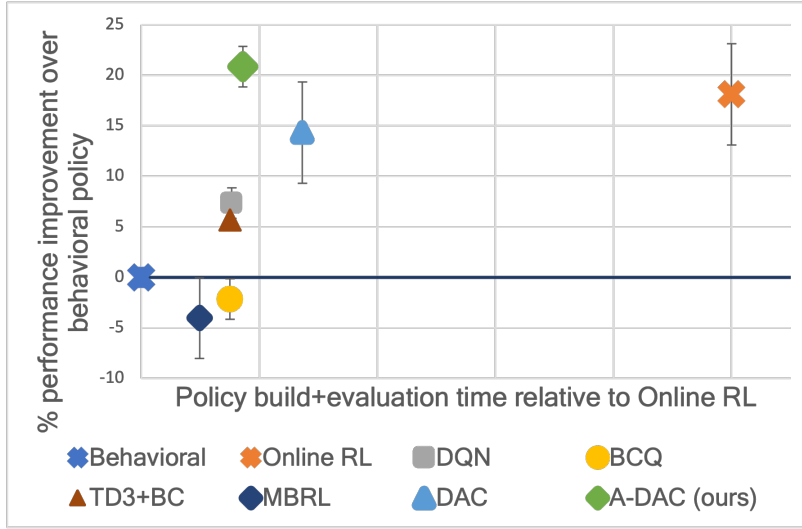


Figure 1: A-DAC, our model-based offline RL approach for traffic control achieves the best performance out-of-the-box compared to three model-free (DQN Mnih et al. (2016), BCQ Fujimoto et al. (2019b), and TD3+BC Fujimoto and Gu (2021)), one model-based RL (MBRL Kidambi et al. (2020)), and our predecessor (DAC Shrestha et al. (2020)) RL, predecessor of A-DAC. The dataset is collected by a behavioral policy that cycles through each traffic signal change action. DQN is greedy and prone to exploration errors. BCQ, TD3+BC and MBRL fail to generalize despite built-in pessimism. DAC is sensitive to its hyperparameters and needs multiple online evaluations. An Online RL baseline is included for comparison which is matched or bettered by our approach in a fraction of time.

and significantly faster by eliminating online interactions for policy evaluation. Figure 1 illustrates our contribution, dubbed Adaptive(A)-DAC, evaluated on a real traffic control setup. A-DAC finds a policy significantly better than a common behavioral (or data collection) policy in a small fraction of time compared to online learning and does so out-of-the-box. A key insight of our approach is that data collected from cyclic policies that are oblivious to rate of traffic arrival and is often the norm in many traffic signal scenarios, can be leveraged to infer superior policies which improve overall traffic efficiency.

### Contributions:

- We formulate traffic signal control as an offline RL problem. While RL has recently been proposed for offline optimization, to the best of our knowledge, it has not been used for the traffic signal control before.
- We extend a recent model-based offline RL framework, DAC, to our problem and improve it by employing an adaptive reward penalty mechanism that enables the best trade-offs in the performance and the policy building overheads. We provide PAC guarantees for A-DAC under more relaxed and realistic assumptions on the Q-function.
- We propose a methodology for data collection at a traffic intersection using macro statistics provided by traffic authorities. We evaluate our approach on a complex signalized roundabout where traffic is coordinated using eleven phases.

**Outline:** The rest of the paper is structured as follows. In Section 2, we introduce a small idealized traffic control problem as a working example. A primer on offline RL methods and their applicability to traffic control is provided in Section 3. In Section 4, our proposed approach, A-DAC, is introduced and elaborated upon. Section 5 reasons about suitability of our approach to the traffic control problem. Section 6, then, evaluates A-DAC and other baselines using a complex signalized roundabout. The most relevant related work is presented in Section 7 and we conclude in Section 8 with a summary of the work.

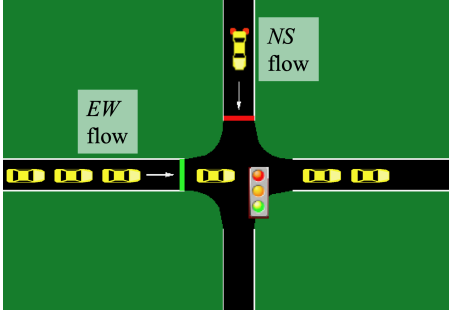


Figure 2: A simple signalized intersection with two traffic flows: North-South (*NS*) and East-West (*EW*).

Table 1: A small experience dataset collected using *Cyclic* traffic signal control policy applied on the intersection in Figure 2.

State $s$	Action $a$	State $s'$	Reward $r$
(1,5)	<i>EW</i>	(3,3)	2
(3,3)	<i>NS</i>	(1,5)	2
(6,1)	<i>NS</i>	(2,3)	4
(2,3)	<i>EW</i>	(6,1)	2
(0,5)	<i>EW</i>	(2,3)	2
(2,3)	<i>NS</i>	(0,5)	2

Table 2: Derived rewards for the core states identified from the experience dataset in Table 1.

State $s$	Averagers ( $C = 0$ )		DAC @ $C = 1$		DAC @ $C = 2$		A-DAC	
	$\bar{R}(s, NS)$	$\bar{R}(s, EW)$	$\bar{R}(s, NS)$	$\bar{R}(s, EW)$	$\bar{R}(s, NS)$	$\bar{R}(s, EW)$	$\bar{R}(s, NS)$	$\bar{R}(s, EW)$
(2,3)	2.67	2	2.41	1.77	1.85	1.25	1.58	1.53
(6,1)	2.67	2	2.29	1.16	1.46	-0.7	1.17	0.32
(3,3)	2.67	2	2.45	1.66	1.98	0.89	1.82	1.31
(1,5)	2.67	2	2.14	1.85	0.96	1.52	0.55	1.70
(0,5)	2.67	2	2.03	1.82	0.63	1.43	0.14	1.65

## 2 Basic Traffic Control

We start with a simple scenario of traffic signal control first presented in Rizzo et al. (2019) where an optimal model for traffic light phase duration is derived based on simplistic assumptions. When these assumptions are relaxed in a realistic setting, more complex machinery is required for optimal control which we present later.

Consider a traffic signal at an intersection which controls traffic in only two directions: either from north to south (*NS*) or from east to west (*EW*) (See Figure 2). Suppose the traffic follows a Poisson process with the rate of traffic arrival being  $\lambda_1$  and  $\lambda_2$  respectively for the two traffic flows. The traffic starts arriving from time  $t = 0$  and the total cycle time is  $T$ . The number of vehicles entering any incoming traffic arms is uniformly distributed by definition of the Poisson process and the expected number at time  $t$  is given by  $\lambda t$ . The optimal setting for the duration of the green phase for *NS* can be derived by minimizing the average delay faced by a vehicle. It evaluates to:  $\frac{\lambda_1}{\lambda_1 + \lambda_2} T$ . The equation states that the green phase duration should be proportional to the arrival rate of the vehicles. It can be easily generalized to a case of  $n$  exclusive traffic flows where the optimal green phase duration for an edge  $i$  is  $\frac{\lambda_i}{\sum_{i=0}^n \lambda_i} T$ .

We relax the assumption of the known arrival rate. Instead, we assume an experience dataset of vehicle movement at the intersection exists where the signal is controlled by a cyclic policy. The *Cyclic* policy simply alternates green phase between the *NS* flow and the *EW* flow at every time step, mimicking a commonly observed scenario. The idea is to use the dataset to devise a smarter control policy. We study some of the recent developments in offline RL towards this problem scenario before introducing a real signalized roundabout we target. The techniques we develop are general enough to apply directly to the more complex test environment.

## 3 Offline Reinforcement Learning

In Reinforcement learning (RL) Sutton and Barto (2018), an agent interacts with an environment, assumed to be a Markov Decision Process (MDP), in order to learn an optimal control (action selection) policy. The MDP is given by a tuple  $(\mathcal{S}, \mathcal{A}, P, R, \gamma)$ , with a state space  $\mathcal{S}$ , an action space  $\mathcal{A}$ , transition dynamics  $P(s, a, s')$ , a stochastic reward function  $R(s, a)$  and a discount factor  $\gamma \in [0, 1)$ . The agent aims to learn a policy function

$\pi : \mathcal{S} \rightarrow \mathcal{A}$  which maximizes the expected sum of discounted rewards. Formally, the objective of RL is given by the following.

$$\max_{\pi} \mathbb{E}_{\substack{a_t \sim \pi(\cdot|s_t) \\ s_{t+1} \sim P(\cdot|s_t, a_t)}} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \right] \quad (1)$$

The policy  $\pi$  has a Q-function  $Q^{\pi}(s, a)$  giving the expected infinite-horizon discounted reward starting with state-action pair  $(s, a)$ . The optimal policy  $\pi^*$  maximizes the Q-function over all policies and state-action pairs. The maximum Q-values are computed by repeated application of the Bellman backup operator  $B$  stated below.

$$B[Q](s, a) = R(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s, a)} \left[ \max_a Q(s', a) \right] \quad (2)$$

RL strives to discover a near-optimal policy by exploring actions in the environment. In an offline setting, the environment is replaced by a static dataset collected apriori. The dataset  $\mathcal{D}$  is made up of tuples  $\{(s_i, a_i, r_i, s'_i)\}$  where each tuple takes action  $a_i$  from state  $s_i$  to transition to state  $s'_i$  while giving the reward  $r_i$ . The dataset is collected from multiple episodes/ trajectories of the form  $(s_1, a_1, r_1, s_2, \dots, s_H)$  where  $H$  is the trajectory length.

**Example:** Table 1 presents a sample experience dataset collected by a policy cycling between the actions *NS* and *EW*. It includes three trajectories, each contributing two tuples. Duplicates are removed to make the example concise.

In the basic traffic control setup outlined in Section 2, an action corresponds to activating green phase for one of the traffic flows for a fixed time duration, called *observation period*. Therefore,  $a_i \in \{NS, EW\}$ . A state is given by a vector of the number of vehicles arriving from each incoming traffic arm, making it 2-dimensional in our case. The reward is a non-negative integer denoting the number of vehicles that cross the signal during the observation period.

### 3.1 Model-free Learning

The first solution approach we consider is to adapt a popular *off-policy* Q-learning approach Deep Q Network (DQN) Mnih et al. (2015) to the offline setting. The offline setting often causes extrapolation errors in Q-learning which result from mismatch between the dataset and the state-action visitation of the policy under training. Fujimoto et al. (2019b) proposes a batch-constrained Q learning (BCQ) approach to minimize distance between the selected actions and the dataset. BCQ trains a state-conditioned generative model of the experience dataset. In discrete settings, the model  $G_{\omega}$  gives a good estimate of the behavioral policy  $\pi_b$  used to collect data. That means, we can constrain the selected actions to data using a threshold  $\tau \in [0, 1)$ :

$$\pi(a|s) = \arg \max_{a' | G_{\omega}(a'|s) / \max_a G_{\omega}(\hat{a}|s) > \tau} Q(s, a') \quad (3)$$

While the BCQ is effective at pruning the under-explored transitions, its benefits are limited when the behavioral policy tends to a uniform distribution which holds true in our case: The behavior policy  $\pi_b$  we aim to utilize is *Cyclic* for which  $\pi_b(a|s) = \pi_b(a) = \frac{1}{|\mathcal{A}|}$ .

In another offline RL approach, a minimalistic change to classic TD3 algorithm is proposed in Fujimoto and Gu (2021) which modifies the TD3 policy update with a behavioral cloning (BC) term:

$$\pi \leftarrow \arg \max_{\pi} \mathbb{E}_{(s, a) \sim D} \left[ \lambda Q(s, \pi(s)) - (\pi(s) - a)^2 \right] \quad (4)$$

Here,  $\lambda$  is a normalizing scalar that is set to a value inverse of the average critic ( $Q$ ) function value. This approach is termed TD3+BC and is regarded as the current state-of-the-art model-free offline RL algorithm.

### 3.2 Derived MDP-based Learning

A contrastive approach to constraining the RL to the dataset is to derive an MDP from the data and either solve it optimally or use model-based policy optimization (MBPO). This approach provides better generalization since each transition gets more supervision compared to the off-policy approaches. There exist multiple recent approaches built on this principle called Model-based (MB-)RL. MBRL primarily learn an approximate transition model and (optionally) a reward model by supervising data followed by a phase of uncertainty quantification to deal with out-of-distribution visitations Yu et al. (2020); Kidambi et al. (2020); Argenson and Dulac-Arnold (2020). We employ a simple instantiation, called DAC-MDP Shrestha et al. (2020), which is based on Averagers’ framework Gordon (1995). The idea is to learn transitions and rewards as empirical averages over the nearest neighbors in the state-action space. It lends to a natural approximation and enables an intuitive uncertainty quantifier. Furthermore, the DAC-MDP has a finite structure which makes the computation very efficient despite an infinite continuous state space. We contribute an adaptive reward penalty mechanism to the DAC framework which works as an effective uncertainty quantifier. Before outlining our contributions, we formalize the DAC framework.

**Assumption 3.1.** We assume a continuous state space  $\mathcal{S}$  and a finite action space  $\mathcal{A}$ . We are given a nearest neighbor function  $NN(s, a, k, \alpha)$  that finds at most  $k$  nearest neighbors to a state-action pair  $(s, a)$  with an optional maximum distance threshold  $\alpha$ .  $NN$  uses a metric function  $d$ , such as *Euclidean*, that keeps the pairs with different actions infinitely distant while the distance between the pairs with the same action is evaluated on the state metric space: For example,  $d(s_i, a_i, s_j, a_j) = \|s_i - s_j\|$  if  $a_i = a_j$ ,  $\infty$  otherwise.

We use shorthand  $d_{ij}$  for distance between pairs  $(s_i, a_i)$  and  $(s_j, a_j)$ . Notation  $d'_{ij}$  indicates a normalized version of distance  $d_{ij}$ . Given a smoothness parameter  $k$  and a distance threshold  $\alpha$ , the derived MDP  $\tilde{M}$  is defined below.

**Definition 3.2.** MDP  $\tilde{M} = (\mathcal{S}, \mathcal{A}, \tilde{R}, \tilde{P}, \gamma)$  shares the state space and the action space of the underlying MDP  $M$ . For a state-action pair  $(s, a)$ , let  $kNN = NN(s, a, k, \alpha)$  be its nearest neighbors from  $\mathcal{D}$ . The reward and transition functions are then defined as:

$$\tilde{R}(s, a) = \frac{1}{k} \sum_{i \in kNN} r_i, \quad \tilde{P}(s, a, s') = \frac{1}{k} \sum_{i \in kNN} I[s' = s'_i]$$

DAC modifies the reward function to penalize transitions to an under-explored region with an additive penalty parameterized by a cost parameter  $C \geq 0$ :

$$\tilde{\tilde{R}}(s, a) = \frac{1}{k} \sum_{i \in kNN} (r_i - C * d(s, a, s_i, a_i)) \quad (5)$$

It should be noted that while the reward shaping in model-based online RL acts as a way to incorporate additional incentive based on domain knowledge to an otherwise sparse reward function Ng et al. (1999), the offline setting uses it as a means to incorporate pessimism to the MDP.

**Example:** Table 2 compares the rewards derived with different cost penalties.  $k = 3$  throughout and the distances are normalized to the maximum distance for ease of presentation. As an example,  $\tilde{\tilde{R}}_{C=0}((2, 3), NS) = \frac{1}{3}(r[1] + r[2] + r[5]) = \frac{1}{3}(4 + 2 + 2) = 2.67$ , where  $r[\cdot]$  is indexes Table 1. State  $(2, 3)$ , for which the reward  $r$  from dataset is equal for both actions, is assigned a higher reward for action  $NS$  due to influence from a high-reward neighbor  $(6, 1)$ .

The DAC framework builds a finite MDP by focusing on *core* states which are formed by the union of the destination states in dataset  $\mathcal{D}$ . The MDP built over the core states is solved using any standard tabular solver such as value iteration to compute values  $\tilde{V}$  for the core states. We can then compute  $\tilde{Q}$  for a non-core state using a 1-step lookup:

$$\tilde{Q}(s, a) = \tilde{\tilde{R}}(s, a) + \gamma \sum_{s' \in \tilde{P}(s, a)} \tilde{P}(s, a, s') \cdot \tilde{V}(s') \quad (6)$$

**Example:** Consider a new state  $(1, 4)$ . Its  $Q$ -values on MDP with no cost penalties and  $\gamma = 0.99$  evaluate to  $\tilde{Q}_{C=0}((1, 4), NS) = 7.92$  and  $\tilde{Q}_{C=0}((1, 4), EW) = 7.25$ . The policy  $\tilde{\pi}$  would, therefore, choose action  $NS$  which is suboptimal since the state indicates more traffic on the  $EW$  lane. Cost penalties help us in this instance: The MDP derived with  $C = 2$  gives  $\tilde{Q}_{C=2}((1, 4), NS) = 32.6$  and  $\tilde{Q}_{C=2}((1, 4), EW) = 32.9$ . The setting  $C = 2$  is not arbitrary: it is the lowest  $C$  value for which the  $EW$  action gets chosen. We discuss this choice further in the next section.

## 4 A-DAC MDP Derivation

We have just shown through example that building offline solutions for traffic control is not trivial even in the simplest of the scenarios. We build an adaptive approach for reward shaping that retains the optimality guarantees while improving the efficiency of the DAC framework significantly; the resulting framework is called as A-DAC.

**Definition 4.1.** A-DAC automatically adjusts the penalties built into the reward function of the derived MDP  $\tilde{M}$  in the following manner:

$$\text{Given } kNN = NN(s, a, k, \alpha) \text{ and } r_{max} = \max_{i \in kNN} r_i$$

$$\tilde{R}(s, a) = \frac{1}{k} \sum_{i \in kNN} r_i - r_{max} * d'(s, a, s_i, a_i)$$

It should be noted that  $d'$  is a max-normalized version of  $d$  and it brings the penalty term to the units of rewards. The intuition behind using the max reward in the neighborhood is to penalize the under-explored but highly rewarding transitions more heavily. Let's understand with an example.

**Example:** We saw earlier that for state  $(2, 3)$  from Table 2, action  $NS$  brings a higher reward in DAC. This is due to influence from a high-reward-getter neighbor  $(6, 1)$ . It can be noticed from the same table that A-DAC's adaptive rewards make both the actions equally rewarding. Moreover,  $Q$ -values for a non-core state  $(1, 4)$  evaluate to  $\tilde{Q}_{A-DAC}((1, 4), 'NS') = 31.4$  and  $\tilde{Q}_{A-DAC}((1, 4), 'EW') = 32.5$  selecting the action  $EW$  automatically.

We present a canonical use case in Figure 3 to illustrate how the rewards are shaped in A-DAC. Of the  $k$  neighbors considered, one neighbor is kept floating to simulate different types of neighborhood. e.g.  $r_{max} = 1$  gives the most homogeneous configuration, while a high  $r_{max}$  replicates an under-represented region. Figure 4 shows how a global cost parameter  $C$  would shape the reward for  $(s, a)$ . While a low  $C$  gives high rewards for an under-represented region, a high  $C$  is detrimental to the homogeneous configurations. A-DAC can be seen to offer a good balance.

**Optimality.** The policy learned by solving the A-DAC  $\tilde{M}$ , denoted  $\tilde{\pi}$ , can potentially be arbitrarily sub-optimal in true MDP  $M$ . We obtain a lower bound on the values obtained by policy  $V^{\tilde{\pi}}$  in relation to the values  $V^*$  provided by the optimal policy  $\pi^*$  in  $M$  under a "smoothness" assumption.

**Assumption 4.2.** A-DAC assumes local Lipschitz continuity for  $Q$ -function  $Q$ : For a state-action pair  $(s_i, a_i)$  and another pair  $(s_j, a_j)$  in its neighborhood, i.e.,  $d'_{ij} < \alpha$  for  $\alpha \in [0, 1]$ , there exists a local constant  $L_Q(i, \alpha) \geq 0$  such that  $|Q(s_i, a_i) - Q(s_j, a_j)| \leq L_Q(i, \alpha)d'_{ij}$ .

The local continuity is a much weaker assumption compared to the global smoothness assumed in the DAC framework. Further, it is found to be practical based on an analysis of our traffic control setup presented in Section 5. In addition, we assume that the rewards are bounded to  $[0, R_{max}]$  which holds for most practical applications including ours.  $\tilde{\pi}$ , then, provides the following PAC guarantee.

**Theorem 4.3.** Given a static dataset  $\mathcal{D}$  with its sample complexity indicated by covering number (see Definition A.2)  $\mathcal{N}_{SA}(\alpha)$  and an A-DAC MDP  $\tilde{M}$  built on  $\mathcal{D}$  with parameters  $k$  and  $\alpha$ , if  $\frac{\bar{Q}_{max}^2}{\epsilon_s^2} \ln\left(\frac{2\mathcal{N}_{SA}(\alpha)}{\delta}\right) \leq k \leq \frac{2\mathcal{N}_{SA}(\alpha)}{\delta}$ , then

$$V^{\tilde{\pi}} \geq V^* - \frac{2\epsilon_s + \bar{d}_{max}R_{max}}{1 - \gamma}, \text{ w.p. } 1 - \delta$$

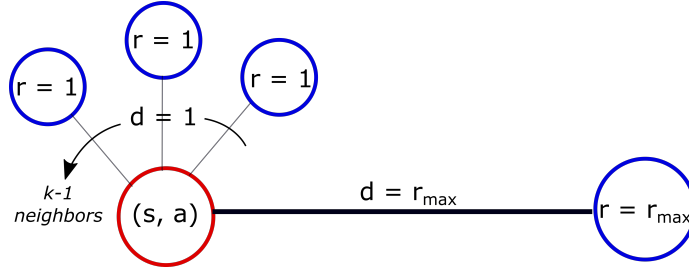


Figure 3: A configuration of  $k$ -nearest neighbors for state  $(s, a)$  where  $k - 1$  neighbors are at distance 1 each with reward 1. The remaining neighbor floats at distance  $r_{max}$  bringing in reward  $r_{max}$  where  $r_{max} > 1$ .

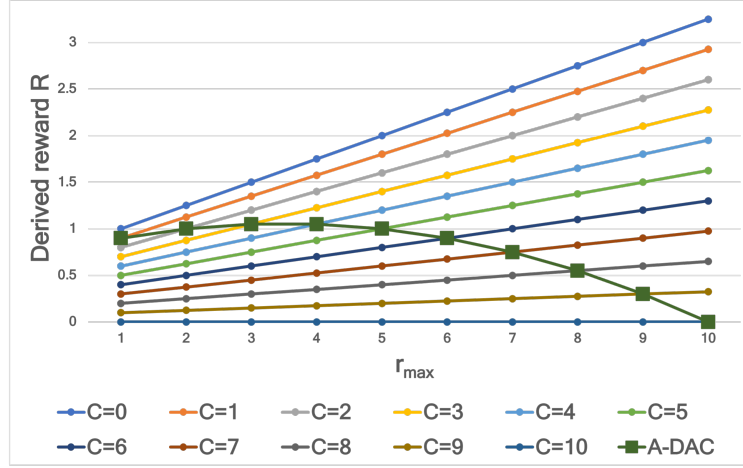


Figure 4: Comparison of rewards derived by DAC with different settings of cost  $C$  to those obtained by A-DAC using the configuration in Figure 3 controlled by variable  $r_{max}$ . A-DAC penalizes the configurations with under-explored regions more while keeping the rewards high for homogeneous configurations.

The proof is presented in Appendix A. The first component  $\epsilon_s$  denotes the maximum sampling error caused by using a finite number of neighbors; it helps us set a lower bound on  $k$ . The second component denotes the estimation error due to neighbors being at non-zero distance: Quantity  $\bar{d}_{max}$  gives the worst case average distance which is upper bounded by  $\alpha$  and gets lower with dataset size.

## 5 A-DAC's Feasibility to Traffic Control

We use the simple two action single intersection model in Figure 2 to analyze A-DAC's feasibility to traffic control.

We assume a fixed rate of arrival for each of the two incoming lanes where  $\lambda_{EW} = 3 * \lambda_{NS}$ . The starting state is assumed to be  $(NS, EW) = (1, 3)$ . A maximum capacity ( $R_{max}$ ) of 4 is allowed in order to maintain a steady flow. On this setup, two experience trajectories are collected using *Cyclic* control policy as illustrated in Figure 5. It can be easily seen that this policy moves  $3T$  vehicles in  $T$  timesteps. Next, A-DAC is trained on this toy dataset. Figure 6 shows a rollout with the same starting state. It can be noticed that a cumulative reward of  $4T$  is achieved, giving 33% improvement over the behavioral policy (and matching the optimal policy). It shows that A-DAC generalizes well. Inspect state (2,5) as an instance: The only action observed from it in the dataset is  $NS$ , but our policy has learned to take action  $EW$  instead.

Finally, using the same model, we can show that for the Q-function, local Lipschitz continuity is the right condition to enforce. For example, suppose the arrival rate on each lane is  $\lambda_1$  and  $\lambda_2$  respectively. Assume,

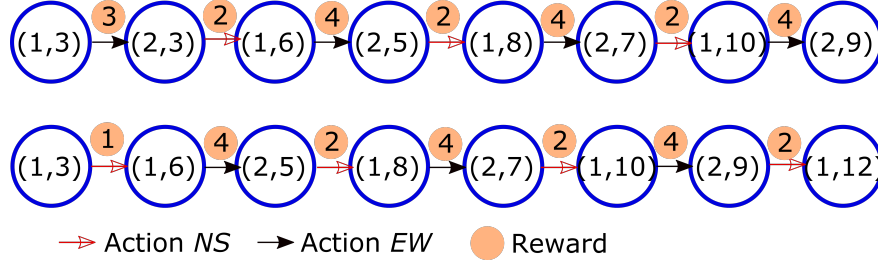


Figure 5: Two experience trajectories from the traffic intersection in Figure 2 taken using *Cyclic* behavior policy under a fixed traffic load assumption.

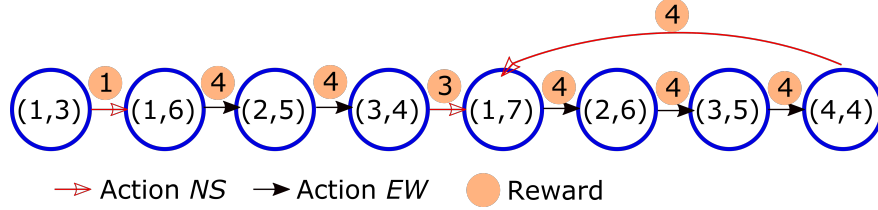


Figure 6: Policy learned by A-DAC using the dataset in Figure 5 improves the behavior policy by 33%.

we start the cycle from the  $\lambda_1$  lane, then the optimal return is:

$$\begin{aligned}
 J(\pi^*) &= \frac{\lambda_1^2}{\lambda_1 + \lambda_2} + \gamma * \frac{\lambda_2^2}{\lambda_1 + \lambda_2} + \gamma^2 * \frac{\lambda_1^2}{\lambda_1 + \lambda_2} + \dots \\
 &= \frac{1}{1 - \gamma^2} * \frac{\lambda_1^2}{\lambda_1 + \lambda_2} + \frac{\gamma}{1 - \gamma^2} * \frac{\lambda_2^2}{\lambda_1 + \lambda_2}
 \end{aligned}$$

Now if we assume that the  $\lambda_1 > 0$  and  $\lambda_2 > 0$  and  $\lambda_1 + \lambda_2 \geq \delta$ , the above expression is upper bounded by a convex function  $c * (\lambda_1^2 + \lambda_2^2)$  for some  $c > 0$  which is locally Lipschitz (as a function of  $\lambda_1$  and  $\lambda_2$ ).

## 6 Evaluation

This section addresses the following questions:

1. *How well is the data collected from simple cyclic behavioral policy exploited by offline learners?* (§6.3)
2. *Does a partially trained behavior policy offer any added benefits to offline learning?* (§6.4)
3. *How do different latent state representations used in A-DAC compare?* (§6.5)
4. *How does A-DAC’s efficiency and hyperparameter sensitivity compare to prior work?* (§6.6)

Before delving into these questions, we describe a real complex traffic roundabout environment used in evaluation (§6.1) and the details of our experimental setup (§6.2).

### 6.1 Environment for a Signalized Roundabout

We model a signalized roundabout shown in Figure 7. It is a complex intersection containing multiple lanes in each traffic arm and 10 traffic signals controlling the area. We model the state of traffic as a 68-dimensional vector, each dimension providing the number of vehicles seen at a designated location. An action corresponds to a phase of traffic that covers a certain configuration of the traffic signals. Details on engineering the states, actions, and rewards for this intersection are provided in Appendix B. The appendix also provides details on creating an experience batch from a micro-simulator bootstrapped with macro traffic statistics when access to the real experience trajectories data is limited.



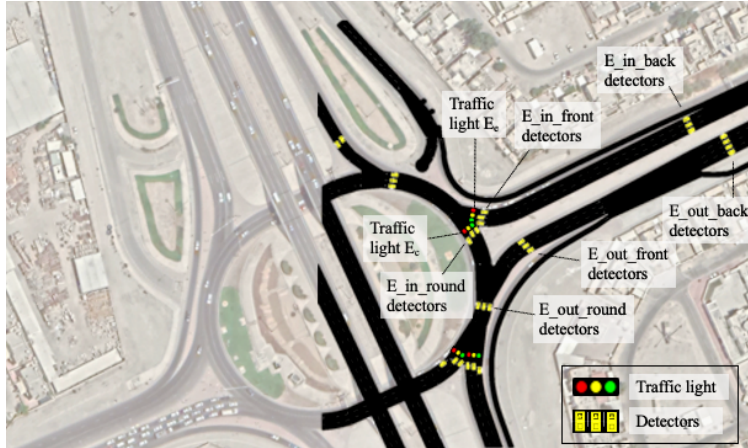


Figure 7: A signalized roundabout A-DAC optimizes. 68 loop detector devices installed in and around the junction report the traffic they observe which forms the state.

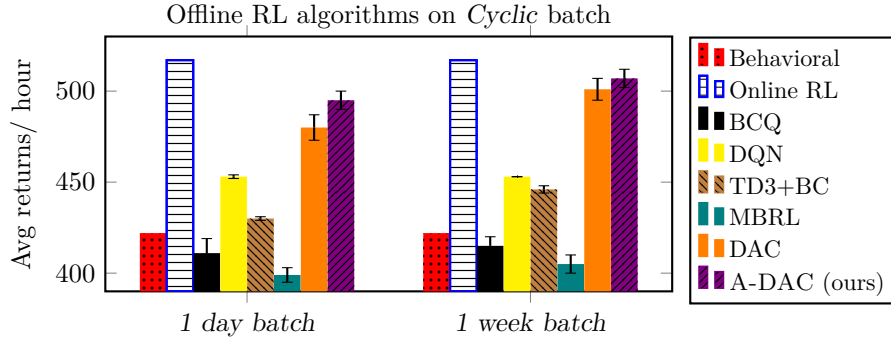


Figure 8: Comparison of offline RL algorithms on *Cyclic* batch. Error bars indicate the min-max interval obtained after 5 runs with different seeds under the best model settings. BCQ, TD3+BC and MBRL fail to improve the behavioral policy. Both DAC and A-DAC improve the policy significantly, even matching the Online RL performance with the larger batch, While DAC needs a large evaluation overhead, A-DAC works out-of-the-box.

## 6.2 Experimental Setup

We use the signalized roundabout detailed in Section 6.1 for evaluation. Each episode lasts an hour with 360 time steps. O-D matrix data is available for each hour of a typical weekday which allows us to create a single day batch. A random noise is added to the matrix when creating a larger batch. Two batches are used: (a) *1 day batch* giving  $\approx 10k$  timesteps, and (b) *1 week batch* giving  $\approx 70k$  timesteps.

**Behavioral policies.** We study two data collection policies:

1. *Cyclic*: Cycles through all actions. Represents a scenario commonly found across traffic intersections.
2. *Partial-RL*: An RL policy is partially trained via online interactions. A noisy version of the policy is then used to control data. This policy has been shown to be suitable for offline learning previously Fujimoto et al. (2019a).

Batch collection and evaluation is carried out using SUMO microsimulator Lopez et al. (2018). A fixed five hour workload, that corresponds to five RL episodes, is used for evaluation throughout: it includes two hours of light traffic, one hour of medium traffic, and two hours of peak traffic. Each single hour episode measures the cumulative discounted rewards. The average return across the five hours is reported as our performance measure.

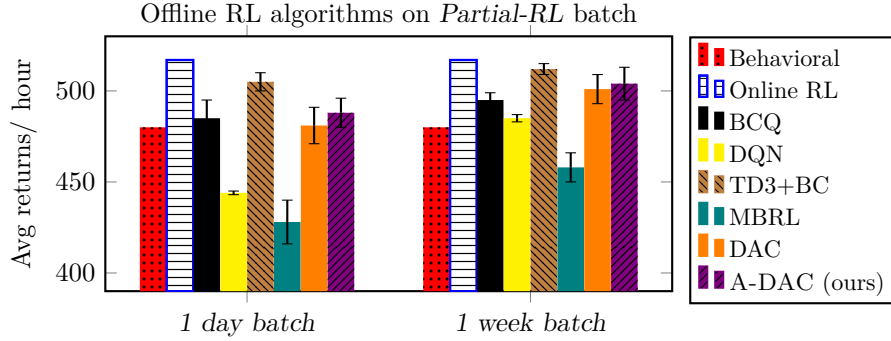


Figure 9: Comparison of offline RL algorithms on *Partial-RL* batch. Error bars indicate the min-max interval obtained after 5 runs with different seeds under the best model settings. Failures of DQN and MBRL exemplify the deadly triad issue in RL. Both DAC and A-DAC, on account of a more principled approximation of the dynamics manage to improve the behavior policy, the magnitude being higher on the larger batch. TD3+BC provides the best improvement where data from a partially trained policy aids the combination of TD-learning and BC the best.

MDP derivation in DAC requires a nearest neighbor index: We use a memory-mapped fast approximate nearest neighbor search index ann. Distances are max-normalized by diameter of the state space before deriving MDP. We use a sampling-based fast approximate algorithm to estimate the diameter. Derived MDP is solved optimally with a value iteration algorithm `mdp` which can exploit sparseness in transition matrix for efficiency gains.

**Baselines.** We compare A-DAC against recent approaches to offline learning. Firstly, we use three model-free offline RL baselines, namely, DQN, BCQ, and TD3+BC. Next, we use MOREL Kidambi et al. (2020) as a representative of a general MBPO approach that covers both a classical (naïve) MBRL and a Pessimistic MDP-based MBRL. Except for Figure 1, we only report the results on the Pessimistic MDP which we observed to perform better than the naïve version. Finally, we analyze the DAC-MDP framework. An online RL policy is also included in the evaluation which uses an off-policy DQN fully trained for close to 100k timesteps.

### 6.3 Cyclic Policy Batch

Figure 8 compares the algorithms on the *Cyclic* dataset. Approaches that employ pessimism, *viz.*, BCQ and MBRL, fail to improve the batch as the *Cyclic* policy does not offer much insight that the deep function approximators (employed either to learn policy or Q-values) can easily exploit. DAC-based policies provide considerable performance improvements due to the nearest neighbor-based dynamics approximation specialized for finite spaces like ours. For DAC, we only report results from the best hyperparameters found after 6 online evaluations. As seen in Figure 1, the performance is highly sensitive to these settings. A-DAC, on the other hand, does not need extra online evaluations.

### 6.4 Partial RL Policy Batch

Figure 9 compares the offline RL algorithms on the *Partial-RL* dataset. BCQ manages to exploit this batch better with its constrained Q-value approximation. Surprisingly, pessimistic MBRL approach fails to even match the behavioral policy performance. It, along with DQN, suffers from the issue of “the deadly triad of deep RL” Sutton and Barto (2018) which exemplifies the inherent difficulties in planning with learned deep-network models. The incorporation of behavioral cloning into TD3 policy updates in TD3+BC makes the good actions stand out early during training on this dataset and, as an effect, it works much better here than on *Cyclic* dataset. DAC framework simplifies the dynamics derivation process thus significantly reducing the reliance on the learned models. Between DAC and A-DAC, A-DAC, once again, offers better robustness to hyperparameters.

Table 3: Comparison of the state representations used in A-DAC. Error bars indicate the min-max interval obtained after 5 runs with different seeds. The *Cyclic* policy is improved significantly using native state representation. The *Partial-RL* batch is better optimized using the learned state representations.

Batch type	Batch size	A-DAC state representation		
		Loop Counts	BCQ	DQN
<i>Cyclic</i>	<i>1 day</i>	<b>495</b> $\pm$ 5	473 $\pm$ 10	459 $\pm$ 12
	<i>1 week</i>	487 $\pm$ 7	<b>507</b> $\pm$ 5	460 $\pm$ 10
<i>Partial-RL</i>	<i>1 day</i>	477 $\pm$ 8	460 $\pm$ 10	<b>488</b> $\pm$ 8
	<i>1 week</i>	481 $\pm$ 5	<b>504</b> $\pm$ 9	491 $\pm$ 4

Table 4: Comparison of offline RL algorithms based on the overhead required to build and evaluate policy given a *Cyclic* experience batch. DAC and A-DAC use the state representation from BCQ. An Online RL algorithm is also included for completeness.

Algorithm	Time (minutes)	Number of timesteps
Online RL	660	100k
BCQ	99	24k
DQN	101	25k
TD3+BC	100	20k
MBRL	65	18k
DAC	180	28k
A-DAC	113	24k

## 6.5 State Representations in A-DAC

Results for A-DAC (and for DAC) presented so far, did not discuss the latent state representations used by the model. We analyze the following three state representations:

- 1. Loop Counts:** 68 dimensional native representation, corresponding to counts from the loop detector devices.)
- 2. BCQ:** Deep representation learned by BCQ algorithm. Penultimate layer of a learned Q-network is used as a surrogate high dimensional state.
- 3. DQN:** Deep representation learned by DQN algorithm. The original state vector is mapped to a high dimensional space in a manner similar to BCQ.

Table 6.5 shows that the native state representation can exploit simplistic *Cyclic* batch really well. For a *Partial-RL* batch, however, a representation learned by a strong offline exploration algorithm, such as BCQ, offers more benefits provided a sufficiently large sized batch is available. DQN uses a highly discriminative model which leads to a largely unchanged state representation with increasing data size. Results point at the flexibility of DAC framework to use state representations learned from other batch learning approaches and improve them further.

## 6.6 Efficiency and Hyperparameter Sensitivity

DAC results presented so far are obtained from the best policy from 6 policies trained with different hyperparameters based on a guideline given in Shrestha et al. (2020). It should be recalled that DAC requires two hyperparameters: a smoothness parameter  $k$ , and a cost penalty  $C$ . A-DAC uses the parameter  $k$  as well as a distance threshold  $\alpha$  for smoothness while the cost penalty is not required. We compare sensitivity of the algorithms to each parameter in Appendix C. The main takeaways are presented next.

DAC has been shown to be robust to the smoothness parameter  $k$ ; It holds for A-DAC’s smoothness parameters too. We set a high value of  $\alpha$ , such as 0.8, and use a low value of  $k$ , 5 by default, which enables efficient computation while achieving robust results. DAC is highly sensitive to parameter  $C$ . A significant time overhead is required for online evaluations in order to tune this parameter. A-DAC offers robustness guarantees that are experimentally verified.

We compare the total time overhead for our baselines in Table 3 using conservative settings for evaluation stopping criteria or for the amount of policy optimization. In addition to the policy evaluation, we have to use significant computation overhead for building the MDP and solving it optimally. (See Appendix C for details.) With DAC, this overhead multiplies by the number of MDPs it derives in order to explore the best policy. While computational advances in future can potentially reduce this overhead, it is unlikely to be a non-trivial component of the overall optimization time.

## 7 Related work

### 7.1 Traffic Signal Control

Classical transportation engineering approaches such as SCATS or Max-pressure rely on classical model-based control theory or rule-based signal control plans Wei et al. (2019). RL approaches built over the years develop different models for the state and the reward functions in order to optimize objectives such as throughput, waiting time, or carbon emissions. Recent deep learning RL approaches such as deep policy gradient or graph convolutions have found success on more complex problems such as coordinated multi-agent control. We cite two survey papers: one on classical RL approaches Yau et al. (2017) and another covering a more comprehensive literature Wei et al. (2019). Shi et al. (2021) is the only work to the best of our knowledge that uses offline RL for traffic modeling but it is targeted at autonomous driving. Our work should be treated as an important first step towards exploiting the big potential of offline RL for traffic signal control.

### 7.2 Offline Model-free RL

Off-policy model-free RL methods Mnih et al. (2016) adapted to batch settings have been shown to fail in practice due to extrapolation errors Fujimoto et al. (2019b). Various regularizers are applied ranging from KL-divergence Kumar et al. (2019); Jaques et al. (2019), uncertainty quantifiers Agarwal et al. (2020), or simply behavior cloning Fujimoto and Gu (2021) leading to a mixed success.

### 7.3 Offline Model-based RL

Compared to model-free, the model-based offline approaches have proven to be more data-efficient while also benefiting from more supervision Levine et al. (2020); Yu et al. (2020). Our work builds on this recent success of MBRL to offline batches. Uncertainty quantifiers are critical for generalization of the model Argenson and Dulac-Arnold (2020). Reward penalties act as a strong regularizer Yu et al. (2020); Kidambi et al. (2020) and fits naturally to the nearest neighbor approximation used in our MDP model Shrestha et al. (2020). The strong approximation of the dynamics and optimal planning on account of finite problem structure are key to our approach avoiding the deadly triad of deep RL Sutton and Barto (2018). Further, our adaptive mechanism for shaping the reward penalties makes the MDP more robust to input data.

## 8 Conclusion

We have modeled traffic signal control as an offline RL problem and learnt a policy from a static batch of data without interacting with a real or simulated environment. The offline RL is a more realistic set up as it is practically infeasible to learn a policy by interacting with a real environment. Similarly in a simulator it is not clear how to integrate real data that is often available through traffic signal operators.

We have introduced a model-based learning framework, A-DAC, that uses the principle of *pessimism under uncertainty* to adaptively modify or shape the reward function to infer a Markov Decision Process (MDP). Due to the adaptive nature of the reward function, A-DAC works out of the box while the nearest competitor requires substantial hyperparameter tuning to achieve comparable performance. An evaluation is carried out on a complex signalized roundabout showing a significant potential to build high performance policies in a data efficient manner using simplistic *cyclic* batch collection policies. In future, we would like to explore other applications in the traffic domain which can benefit from offline learning.

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## A Proof for Optimality Guarantee

In this section, we present the proof for Theorem 4.3 using the construction proposed in Pazis and Parr (2013) for Probably Approximately Correct (PAC) exploration in continuous state MDPs, with some additional modification due to our pessimistic setting.

We assume that the rewards lie in  $[0, R_{max}]$ . We further assume local Lipschitz continuity of the  $Q$ -function as defined in Assumption 4.2. The local Lipschitz continuity assumption allows us to use samples from nearby state-actions as required by A-DAC model and approximate  $Q$ -function without much error.

**Definition A.1.** For a state-action pair  $(s_i, a_i)$ , the pessimistic  $Q$ -value function used in A-DAC MDP  $\tilde{M}$  is defined as:

$$\tilde{Q}(s_i, a_i) = \frac{1}{k} \sum_{j \in NN(s_i, a_i, k, \alpha)} (\max\{0, r_j + \gamma \tilde{V}(s'_j) - r_{max} d'_{ij}\}) \quad (7)$$

where  $d'_{ij} := d(s_i, a_i, s_j, a_j)$  max-normalized by the diameter of the state-action sample space,  $\alpha$  is the distance threshold used in the nearest neighbor function, and  $r_{max} = \max_{j \in NN(s_i, a_i, k, \alpha)} r_j$ .

The number of samples required to get a good approximation depends on the covering number of the state-action space  $\mathcal{N}_{\mathcal{SA}}(\alpha)$  defined next.

**Definition A.2.** The covering number  $\mathcal{N}_{\mathcal{SA}}(\alpha)$  of a state-action space is the size of the largest minimal set  $C$  of state-action pairs such that for any  $(s_i, a_i)$  reachable from the starting state, there exists  $(s_j, a_j) \in C$  such that  $d'_{ij} \leq \alpha$ .

Let  $\tilde{\pi}$  be the optimal policy of A-DAC MDP  $\tilde{M}$ . Our goal is to bound the value  $V^{\tilde{\pi}}(s)$  of  $\tilde{\pi}$  in the true MDP  $M$  in terms of the optimal value  $V^*(s)$  for any state  $s$ . The following lemma suggests that it is sufficient to bound the Bellman error  $\tilde{Q}(s, a) - B[\tilde{Q}](s, a)$  across all  $(s, a)$  pairs with respect to the true MDP.

**Lemma A.3.** [Theorem 3.12 from Pazis and Parr (2013)] Let  $\epsilon_- \geq 0$  and  $\epsilon_+ \geq 0$  be constants such that  $\forall (s, a) \in (\mathcal{S}, \mathcal{A})$ ,  $-\epsilon_- \leq Q(s, a) - B[Q](s, a) \leq \epsilon_+$ . Any greedy policy  $\pi$  over a  $Q$ -function  $Q$  then satisfies:

$$\forall s \in \mathcal{S}, V^\pi(s) \geq V^*(s) - \frac{\epsilon_- + \epsilon_+}{1 - \gamma}$$

In order to use this lemma, we want to bound the quantity  $Q(s, a) - B[Q](s, a)$  for a fixed point solution  $\tilde{Q}$  to the pessimistic  $Q$ -function in Definition A.1<sup>1</sup>. For a locally Lipschitz continuous value function (Assumption 4.2), the value of a state-action pair can be expressed in terms of any other state-action pair in its neighborhood as  $Q(s_i, a_i) = Q(s_j, a_j) + \xi_{ij} L_Q(i, \alpha) d'_{ij}$ , where  $\xi_{ij}$  is a fixed but possibly unknown constant in  $[-1, 1]$ . For a sample  $(s_j, a_j, r_j, s'_j)$ , let

$$x_{ij} = r_j + \gamma V(s'_j) + \xi_{ij} L_Q(i, \alpha) d'_{ij}$$

Then:

$$\begin{aligned} E_{s'_j}[x_{ij}] &= E_{s'_j}[r_j + \gamma V(s'_j)] + \xi_{ij} L_Q(i, \alpha) d'_{ij} \\ &= Q(s_j, a_j) + \xi_{ij} L_Q(i, \alpha) d'_{ij} \end{aligned}$$

Consider a state-action pair  $(s_0, a_0)$  and its  $k$ -nearest neighbors given by  $NN(s_0, a_0, k, \alpha)$ , we can estimate the  $Q$ -value for the pair by averaging over the predicted values of its neighbors:

$$\hat{Q}(s_0, a_0) = \frac{1}{k} \sum_{i \in NN(s_0, a_0, k, \alpha)} x_{0i} \quad (8)$$

Let us define a new Bellman operator  $\hat{B}$  corresponding to the definition of  $\hat{Q}$  above. While  $B$  denotes the exact Bellman operator,  $\hat{B}$  denotes the approximate Bellman operator for the pessimistic value function in Definition A.1.

<sup>1</sup>It can be easily proven that  $\tilde{Q}$  has a unique fixed point by showing that the Bellman operator  $\tilde{B}$  is a contraction in maximum norm.

The Bellman error can be decomposed into two parts: (a) the maximum sampling error  $\epsilon_s$  caused by using a finite number of neighbors, and (b) the estimation error  $\epsilon_d$  due to using neighbors at non-zero distance. The following lemma from Pazis and Parr (2013) bounds the minimum number of neighbors  $k$  required to guarantee certain  $\epsilon_s$  with probability  $1 - \delta$ :

**Lemma A.4.** [Lemma 3.13 from Pazis and Parr (2013)] If  $\frac{\tilde{Q}_{max}^2}{\epsilon_s^2} \ln\left(\frac{2\mathcal{N}_{SA}(\alpha)}{\delta}\right) \leq k \leq \frac{2\mathcal{N}_{SA}(\alpha)}{\delta}$ ,

$$\forall(s, a), |\hat{B}[\tilde{Q}](s, a) - B[\tilde{Q}](s, a)| \leq \epsilon_s, \quad w.p. \quad 1 - \delta$$

The proof (not included here) applies Hoeffding’s inequality to bound the difference in the true expectation (given by operator  $B$ ) and its estimation using mean over  $k$  samples (given by operator  $\hat{B}$ ).

The second piece of the Bellman error requires us to bound the term  $\epsilon_d = \tilde{B}[\tilde{Q}](s, a) - \hat{B}[\tilde{Q}](s, a)$ .

**Lemma A.5.** For all known state-action pairs  $(s, a)$

$$0 \leq \tilde{B}[\tilde{Q}](s, a) - \hat{B}[\tilde{Q}](s, a) \leq \bar{d}_{max} R_{max}$$

*Proof.* We can simplify the estimation error  $\epsilon_d$  by using the definitions (7) and (8).

$$\begin{aligned} \epsilon_d &= \tilde{B}[\tilde{Q}](s_i, a_i) - \hat{B}[\tilde{Q}](s_i, a_i) \\ &= \frac{1}{k} \sum_{j \in NN(s_i, a_i, k, \alpha)} (-r_{max} - \xi_{ij} L_Q(i, \alpha)) * d'_{ij} \end{aligned}$$

We can set  $\xi_{ij} = -\frac{R_{max}}{L_Q(i, \alpha)}$  which will bound the quantity inside the bracket from  $[0, R_{max}]$ ,  $\forall(s, a)$  since  $0 \leq r_{max} \leq R_{max}$ . The worse case average distance is defined as  $\bar{d}_{max}$ , therefore ensuring that  $\epsilon_d \leq \bar{d}_{max} R_{max}$ .  $\square$

Finally, to prove the PAC bound in Theorem 4.3, we need to bound the quantity  $\tilde{Q}(s, a) - B[\tilde{Q}](s, a)$ . To achieve that, we combine the above two lemmas and apply the operators on the fixed point solution  $\tilde{Q}$ , giving us :

$$\text{If } \frac{\tilde{Q}_{max}^2}{\epsilon_s^2} \ln\left(\frac{2\mathcal{N}_{SA}(\alpha)}{\delta}\right) \leq k \leq \frac{2\mathcal{N}_{SA}(\alpha)}{\delta}, \forall(s, a), -\epsilon_s \leq \tilde{Q}(s, a) - B[\tilde{Q}](s, a) \leq \epsilon_s + \bar{d}_{max} R_{max}$$

Putting these bounds in Lemma A.3 gives us the final result.  $\square$

## B Modeling Environment for a Signalized Roundabout

We model a signalized roundabout (Figure 7) used previously to learn an online RL policy using policy gradient Rizzo et al. (2019). It consists of three types of lanes or traffic arms: (a) approaching/ incoming lanes, (b) outgoing lanes, and (c) circulatory lanes that enable traffic flow redirection. Each traffic arm has multiple lanes. It is important to consider each lane separately because the way the incoming traffic ‘weaves in’ to the circulatory lanes impacts the wait time of vehicles. Movement in or out of the circulatory lanes is controlled by traffic signals numbering 10 in total.

**States.** A state corresponds to the number of vehicles counted by the loop detector devices installed on every lane. For each approaching or outgoing lanes, there are two devices per lane, one close to the roundabout and another several meters farther. The detectors number 68 in total.

**Actions.** Actions correspond to traffic control phases activated for a fixed time duration. A phase is provided for each set of non-conflicting flows. For example, traffic moving from north to south and from south to north does not conflict and therefore constitutes a single phase. In all, we use a discrete set of 11 actions as modeled in Rizzo et al. (2019).



**Rewards.** Traffic signal control typically serves a dual objective: maximize throughput and avoid long traffic queues. We optimize only the first objective here and leave the the later for a future demonstration. For throughput maximization, the rewards are modeled as the cumulative capacity: The cumulative capacity  $C(t)$  at time  $t$  is the number of vehicles that left the roundabout from time 0 to  $t$ . Reward at time step  $t$  is then defined as  $R(t) = C(t) - C(t - 1)$ .

**O-D Driven Traffic Simulation:** Access to real experience trajectories data is often very limited and/ or allows only a specific behavioral policy (e.g. *Cyclic* in our case) offering little room for experimentation. We describe the process employed to augment the batch collection process in our environment.

A micro-simulator is set up using real network configuration with traffic signals and loop detector devices correctly placed. Traffic is generated using an Origin-Destination (O-D) matrix provided by traffic authorities. The O-D matrix corresponds to macro statistics for a relatively small, but significantly larger than the traffic phase duration, period of the day. It enumerates the number of vehicles that move between each pair of traffic zones positioned in the close vicinity of the roundabout. The O-D data is fed to our simulator which generates vehicles following the provided source, destination, and frequency requirements. The routes are generated using a random trip generator.

## C Additional Evaluation

### C.1 Hyperparameter Sensitivity of DAC

We analyze the sensitivity of each hyperparameter individually. A-DAC retains DAC’s robustness to smoothness parameter  $k$ ; Figure 10 provides the evidence. For a small value of  $k$  (ranging between 2 – 10), we find that parameter  $\alpha$  has a minimal role. For instance, Figure 11 studies the impact of  $\alpha$  when  $k$  is set to 5. Except for very low values of  $\alpha$ , the performance remains unaffected. It should be noted that having a large distance threshold does not harm since our adaptive reward computation penalizes distant neighbors. Based on this, we set  $\alpha$  to 0.8 by default.

When it comes to parameter  $C$ , Figure 12 shows that DAC is highly sensitive to the parameter. The values for  $C$  are varied between the minimum and the maximum rewards observed in the dataset. The robustness offered by A-DAC by adapting rewards based on local neighborhood ensures that A-DAC does not need to spend expensive cycles on hyperparameter tuning.

### C.2 Computational Overhead

A breakdown of computation time overhead is presented in Figure 13. All numbers are obtained from a server running a 16-core 2nd generation Intel Xeon processor with 128GB RAM. Our implementation uses fast approximate solutions to nearest neighbor searches and diameter computations. But the MDP build and MDP solve operations suffer as they process a transition matrix growing quadratically with the number of core states. Designing a distributed GPU based implementation for optimal planning is left as a future work.

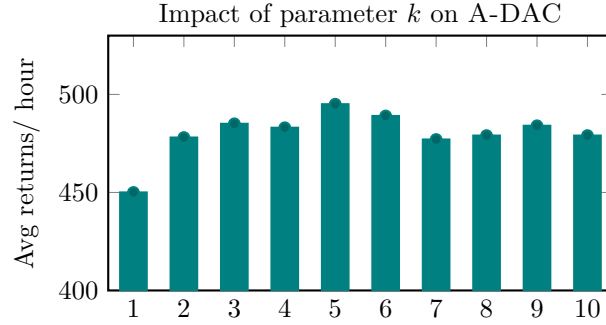


Figure 10: Analyzing the impact of smoothness parameter  $k$  in A-DAC. The setting  $k = 1$  makes the MDP deterministic and incapable of exploiting the different transitions observed in the experience data. Performance is not much sensitive to values of  $k > 1$ .

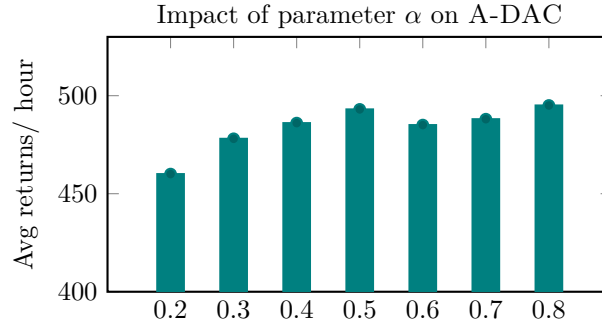


Figure 11: Analyzing the impact of smoothness parameter  $\alpha$  in A-DAC. Low settings result in insufficient neighbors used in approximation that has an adverse effect. High settings are more robust.

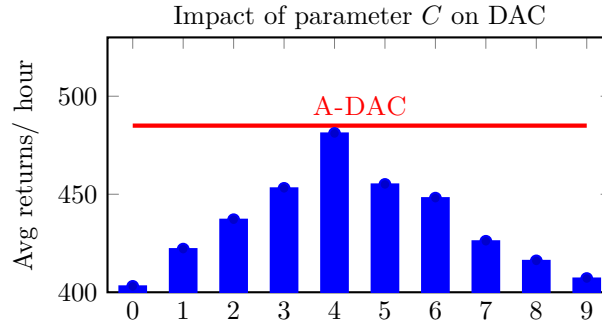


Figure 12: Comparing the impact of cost penalty  $C$  in DAC to the adaptive reward penalties in A-DAC. DAC is highly sensitive to  $C$  and requires a careful tuning. A-DAC manages to match or better the performance of the best  $C$  setting in DAC out-of-the-box.

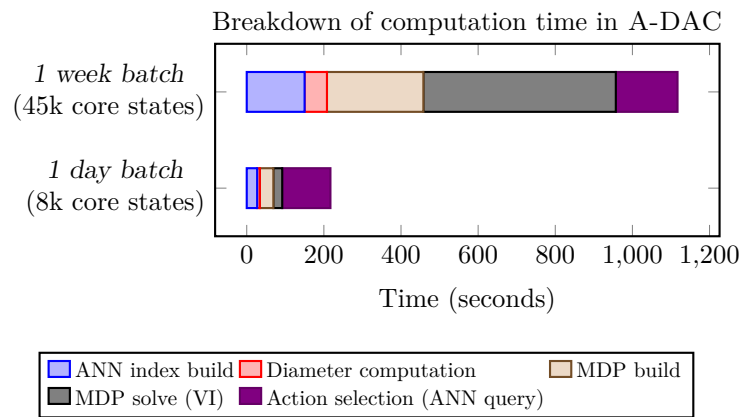


Figure 13: Computation time breakdown between different processes in A-DAC. The nearest neighbor querying and the diameter computation use fast approximate algorithms and scale well. But MDP build and solve stages suffer from quadratic time complexity.