# LEARNING NON-EQUILIBRIUM SIGNALING DYNAM ICS IN SINGLE-CELL PERTURBATION DYNAMICS

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#### ABSTRACT

Cancer cells exploit non-equilibrium signaling dynamics to develop transient drug resistance through mechanisms that conventional equilibrium-based analyses cannot detect. We present a probabilistic framework integrating live-cell biosensor data with asynchronous multi-omics snapshots to learn these adaptive states. Using data from BRAF<sup>V600E</sup> melanoma as a model system, we demonstrate how such learning scheme characterize competing timescales drive resistance mechanisms: rapid post-translational feedback (minutes) versus delayed transcriptional regulation (hours), including RAF dimer rewiring, DUSP-mediated ERK reactivation pulses, and NRAS<sup>Q61K</sup>-dependent EGFR recycling. Our approach further combines multi-marginal Schrödinger bridges for distribution alignment with the extracted dynamical patterns from live-cell trajectories. Each step of the algorithm is validated with real-data and further validation is through in silico melanoma models. This framework could help identify therapeutic windows that delay progression to persistent resistant states and targeting adaptive plasticity across cancer types.

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#### 1 INTRODUCTION

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Modern biology has witnessed a revolution in single-cell technologies, enabling unprecedented resolution in profiling cellular states. Techniques such as multiplexed iterative immunofluorescence imaging (Lin et al., 2018) and live-cell biosensors (Cutrale et al., 2017) now allow researchers to map protein expression, post-translational modifications, and signaling activity across millions of individual cells under diverse perturbations. These advances are particularly valuable for drug discovery, where understanding heterogeneous responses to pathway inhibition targeted therapies is critical to overcoming resistance (Samatar & Poulikakos, 2014).

Cancer cells exhibit remarkable plasticity, maintaining a multi-attractor landscape even in homeosta sis. In melanoma, spontaneous transitions between proliferative, invasive, and drug-tolerant states
 occur without external perturbation Roesch et al. (2010). When treated with drugs, cells are forced
 from these steady states into non-equilibrium transients. Within 24 hours of RAF/MEK inhibition,
 BRAF<sup>V600E</sup> melanoma cells shift from monomeric BRAF-driven ERK activation to RAS-mediated
 dimeric CRAF signaling (Fröhlich et al., 2023).

042 The adapted cells demonstrate complex signaling dynamics across multiple timescales. ERK activ-043 ity pulses occur with 45-90 minute periodicity, driven by post-translational modifications occurring 044 within seconds to minutes, EGFR receptor trafficking and recycling taking 10-30 minutes, and transcriptional feedback via DUSP/SPRY downregulation occurring over hours Fröhlich et al. (2023). Traditional analysis pipelines struggle with these dynamics due to combinatorial complexity—even 046 20 signaling proteins can generate over  $10^5$  biochemical species through modifications and com-047 plex formation Jamison (1975). Rule-based modeling approaches Faeder et al. (2005) show how 048 allosteric drug effects propagate through this network via  $\Delta G$  energy landscapes, while the Gibbs free energy difference governing drug-target interactions becomes time-dependent under rapid signaling transients Lavoisier (1789). 051

Current dimensionality reduction techniques compound these challenges by obscuring critical high dimensional features and averaging out transient states that drive phenotypic outcomes. Our frame work addresses this through optimal transport regularized by live-cell biosensors (?), preserving

both high-dimensional structure and temporal dynamics. More specifically, we propose a probabilistic framework that bridges asynchronous snapshot data with continuous live-cell trajectories. Rather than relying on deterministic flow maps, we model cellular responses as stochastic processes informed by partial observations. Our approach combines stochastic flow matching to align high-dimensional marginal distributions via unbalanced Schrödinger bridges (?) with spectral analysis of transient dynamics using delay embeddings (Takens, 198) and Koopman operator theory Giannakis (2019). Using melanoma as a testbed, we demonstrate how these methods reveal coherent dynamics in drug resistance and predict context-dependent phenotypic outcomes, offering a framework for combining high-throughput and live-cell modalities.

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Challenges in Cell Perturbation Analysis Transforming the data 064 from high-throughput single-cell perturbation experiments into predic-065 tive dynamical models presents three fundamental challenges. First, 066 temporal discontinuity arises from destructive measurement techniques 067 that yield asynchronous marginal distributions  $q(x_0), q(x_1), \ldots, q(x_T)$ 068 at discrete timepoints, destroying cells during measurement. While the 069 slow, quasi-equilibrium case can be addressed using multi-marginal optimal transport, drug perturbations often trigger rapid transitions occurring 071 faster than typical sampling intervals, making displacement interpolation of the measures unreliable. The second challenge stems from lim-072 itations in traditional dimensionality reduction techniques like UMAP. 073 These methods cannot capture temporal evolution in their static embed-074 dings and often fragment transient states into disconnected clusters. The 075 third challenge involves non-equilibrium dynamics: Drug perturbations 076 drive cells into turbulent-like regimes where small initial differences am-077 plify exponentially. This exponential amplification makes local distances unreliable predictors of cell fate and significantly complicates trajectory 079 reconstruction from snapshots. The chaotic nature of these systems fun-080 damentally limits our ability to make deterministic predictions about cel-081 lular responses to perturbations.

082 Therefore, despite their effectiveness for unsupervised static methods, 083 low-dimensional embeddings of single-cell snapshot data fall short for 084 dynamical analysis (Kiselev et al., 2019; La Manno et al., 2018). More-085 over, snapshot data captures many molecular species at intervals without 086 cell tracking, while live cell data continuously monitors specific activi-087 ties such as activation of key signaling molecules. Using snapshot data, 088 UMAP (Figure 1, top) reveals a lower-dimensional manifold that lacks directional flow. In contrast, delay coordinate embedding of live cell 089 data Takens (198); Packard et al. (1980) reveals structured trajectories 090 (Figure 1, bottom). 091



Figure 1: UMAP embedding of 5 simulated cell trajectories with different colors in (top) snapshot data and (bottom) in live cell. Individual cell trajectories are shown in both cases despite their unavailability in snapshot data.

**Perturbations as Attractor-Kicking Events** Cellular homeostasis evolves on a metastable manifold  $\mathcal{M}_h \subset \mathbb{R}^d$  maintained by selfcorrecting biochemical networks (Fig. 2). It is common to assume this manifold acts as an attractor with dynamics governed by a potential field (Pillai & Jolly, 2021). The system's evolution can be formulated using the Fokker-Planck equation where the drug perturbations modify the drift term, creating transitions to alternative attractors  $\mathcal{M}_i$ :

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$$X_t \xrightarrow{\text{Perturbation}} \begin{cases} \mathcal{M}'_h & \text{with rate } \lambda_1(X_{t^-}), \\ \mathcal{M}_{\text{res}} & \text{with rate } \lambda_2(X_{t^-}), \\ \mathcal{M}_{\text{death}} & \text{with rate } \lambda_3(X_{t^-}), \end{cases}$$

where  $X_{t^-}$  is the pre-perturbation state. Here we focus on resistance mechanisms emerging on faster timescales, setting aside apoptosis and genetic/epigenetic alterations. Two cells are *dynamically adjacent* if their perturbed trajectories converge to the same attractor despite initial separation (see Fig. 2):

Figure 2: A schematic of two dynamically adjacent cells that following non-homeostasis transients, settle back to homeostasis.

$$\lim_{t \to \infty} |X_i(t) - X_j(t)|_{\mathcal{M}_k} < \epsilon \implies i \sim j$$
<sup>(2)</sup>

(1)

#### 108 2 **REGULARIZED MULTI-MARGINAL FLOW MATCHING**

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110 To address these challenges, we propose a framework that combines high-dimensional snapshot data 111 with live-cell biosensor measurements. Rather than relying on dimensionality reduction, which dis-112 cards critical dynamical features, or integrating differential equations, which become intractable in 113 high dimensions, we develop a robust simulation-free approach using spline measures, score match-114 ing, and live cell trajectory guidance. This integration is crucial: snapshot data provides comprehensive molecular states but lacks temporal resolution, while biosensor live data offers precise temporal 115 116 information for selected molecules. We validate each step using experimental data and apply this framework to a mechanistic model of MAPK signaling, where we systematically evaluate biosensor 117 selection based on information density, time-scale coverage, and pathway specificity. This approach 118 represents a significant step toward understanding and potentially controlling cellular response to 119 therapeutic perturbations. 120

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# 2.1 ROBUST MULTI-MARGINAL FLOW MATCHING

Our framework addresses high-dimensional temporal modeling through three synergistic components that prevent overfitting while maintaining dynamical fidelity:

**Spline-Decomposed Mini-Flows** To handle irregular timepoints  $\{t_i\}_{i=0}^M$ , we construct the global flow as overlapping windows of spline measure triplets for observed marginals:

$$\mu_t = \sum_{i=0}^{M-k} B_i(t) \left[ \mu_i + (t - t_i) \mathbf{v}_i \right], \quad t \in [t_i, t_{i+k}]$$
(3)

132 where  $B_i(t)$  are B-spline basis functions with local support,  $\mu_i = \mathbb{E}[\rho_i]$ , and  $\mathbf{v}_i$  are window-specific 133 velocity fields. The B-spline construction ensures several important properties: It maintains  $C^2$ smoothness between windows through basis function overlap, providing continuous transitions. 134 Through localized parameter sharing, it has  $\mathcal{O}(1)$  extra memory complexity to the previous scal-135 able algorithms (Tong et al., 2023), making it computationally efficient. 136

**Score-Matched Stochastic Dynamics** Similar to Tong et al. (2023), we regularize the determin-138 istic flow with learned stochastic components via score matching: 139

$$\mathcal{L}_{\text{score}} = \mathbb{E}_{t, x \sim p_t} \left[ |s_\theta(x, t) - \nabla_x \log p_t(x)|^2 \right]$$
(4)

142 where corrupted samples  $\tilde{x} = x + \sigma(t)\epsilon$  use a Brownian bridge noise schedule  $\sigma(t) = \sqrt{t(1-t)}$ . This approach avoids explicit density estimation in high dimensions, captures uncertainty through 143 stochastic differential equations, and prevents mode collapse via noise-adaptive regularization. 144

**Live-Cell Biosensor Anchoring** Experimental trajectories  $y_i(t)$  from biosensors constrain the learned velocity field through a direct matching term:

$$\mathcal{L}_{\text{live}} = \sum_{j} \int_{t_{\min}}^{t_{\max}} |\mathbf{v}(y_j(t), t) - \dot{y}_j(t)|^2 dt$$
(5)

151 This coupling to continous dynamics serves multiple purposes. It enables physics-informed learn-152 ing by directly matching observed velocities, provides multi-scale alignment by bridging biosensor 153 measurements at  $\mu m/\min$  resolution with snapshot hour-scale data, and offers pathway specificity 154 where channel selection (such as pERK versus RAS) determines the dynamical focus of the model 155 via a subset-correspondence regularity on the optimal transport (Liu et al., 2019).

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**Theoretical Guarantees** The combined framework provides several key theoretical guarantees. 158 The divergence matching between the velocity field and score function  $(\nabla \cdot \mathbf{v} = \nabla \cdot s_{\theta})$  ensures 159 consistency with the Fokker-Planck equation. The incorporation of live-cell data resolves the driftdiffusion ambiguity inherent in stochastic differential equations. Additionally, the local nature of 160 spline measures ensures bounded Lipschitz constants across time windows, providing stability to 161 the framework.

162 **Real Data Validation** Applied to CITEseq and Multiome gene expression datasets measuring 163 50-1000 molecular features at irregular intervals (2-7 days) (Burkhardt et al., 2022), our frame-164 work demonstrates robust performance across dimensionality reduction strategies. For the 1000-165 dimensional highly-variable genes (Hi-Var 1000), Triplet-MMSFM achieves near-identical  $W_1$  distances (50.64 vs 50.71) to Pairwise while reducing  $W_2^2$  significantly, indicating precise distribu-166 tion matching in higher dimensions. The model maintains stability across preprocessing methods, 167 while maximum mean miscrepancy metrics reveal consistent pattern capture. This performance 168 persistence across 50-1000 features and 4-7 day intervals confirms the method's capacity to handle biological noise and temporal sparsity inherent in real perturbation studies. 170

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#### 2.2 OPERATOR-THEORETIC REGULARIZATION FOR FLOW MATCHING

174 Direct application of equation (5) may be unreliable because single cell trajectories contain both 175 measurement noise and inherent chaotic dynamics. This makes velocity matching susceptible to 176 overfitting temporary fluctuations rather than capturing true biological patterns. To address this, 177 we employ Koopman operator theory (Mauroy et al., 2020) to decompose trajectories into coherent 178 dynamical patterns that provide regularization constraints for the flow matching process. The Koop-179 man operator  $\mathcal{K}$  linearly advances observables g(x) in time via  $\mathcal{K}g(x_t) = g(x_{t+\Delta t})$ , even when the underlying system exhibits nonlinear dynamics. Spectral decomposition of  $\mathcal K$  yields eigenfunc-181 tions  $\phi_j$  encoding predictable states and eigenvalues  $\lambda_j = e^{(\theta_j + i\omega_j)\Delta t}$ , where  $\theta_j$  and  $\omega_j$  govern 182 growth/decay rates and oscillation frequencies, respectively.

183 For observed trajectories of molecular activity, y(t), we approximate Koopman eigenfunctions from 184 the eigenfunctions of Markov kernel operators constructed over delay-embedded states Y(t) =185  $[y(t), y(t-\tau), \dots, y(t-(d-1)\tau)]$  (see Giannakis (2019)). This reveals dominant modes  $\phi_1, \phi_2$  cor-186 responding to drug response adaptation phases and oscillatory feedback. These data-driven modes 187 replace the flow matching penalty (5) with the spectral signatures of the live cell dynamics. This 188 loss enforces preservation of coherent temporal patterns. The regularization anchors the flow to biologically interpretable dynamics: eigenfunction gradients  $\nabla \phi_i$  localize to key signaling nodes 189 like DUSP6 and SPRY2, validating their roles as dynamical bottlenecks. By bridging data-driven 190 pattern extraction with mechanistic interpretability, this operator-theoretic approach ensures robust 191 generalization despite trajectory-level unpredictability. 192

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# 2.2.1 EXPERIMENTAL VALIDATION

Application to BRAF<sup>V600E</sup> melanoma cells under ERK inhibition revealed three key findings. First, delay-embedded ERK
trajectories exhibited block-diagonal kernel structure, indicating
predictable states, and two dominant Koopman modes (Fig. 3).
The first mode exhibited slow decay capturing ERK suppression,
while the second mode shows oscillations from negative feedback.

Figure 4 demonstrates our framework's ability to reconstruct
ERK trajectories in the Low ERKi training condition (left) and
generalize to unseen High ERKi data (right). Compared to
Probabilistic Linear Dynamical Systems (PLDS) Chen et al.





(2017)-a conventional approach modeling dynamics through principal Koopman modes.
 low-dimensional linear transitions with Gaussian noise-our operator-based method maintains phase
 coherence in unseen conditions where PLDS trajectories diverge rapidly

210 *Remark:* Biological systems under perturbation exhibit inherent non-ergodicity in time, yet modern 211 single-cell technologies provide spatial ergodicity through simultaneous measurements of millions 212 of cells. This allows approximation of temporal averages by ensemble averages:  $\mathbb{E}_t[f(X_t)] \approx$ 213  $\mathbb{E}_{x \sim q_t}[f(x)]$ , where  $q_t$  is the empirical distribution at time t. Here, rather than attempting to con-214 struct exact flow maps, we focus on extracting predictable patterns using the transition density func-215 tion  $p_{\tau} : \mathbb{X} \times \mathbb{X} \to [0, \infty)$ , which quantifies  $\mathbb{P}[\Phi_{\tau}(\mathbf{X}_t) \in \mathbb{A} | \mathbf{X}_t = x]$  for any measurable subset  $\mathbb{A} \subset \mathbb{X}$ .



Figure 4: Performance examples of model prediction for ERK activity trajectories in the Low ERKi condition (left, training set) and High ERKi condition (right, test set).

#### IN-SILICO VALIDATION WITH MECHANISTIC MODELS 3

While operator-theoretic regularization demonstrates promising results in biological data, rigorous 236 validation of the learning algorithm necessitates controlled environments with fully known dynamics. We address this through a high-fidelity in-silico MAPK/AP-1 network model that replicates 238 BRAF<sup>V600E</sup> melanoma signaling at single-molecule resolution developed by Fröhlich et al. (2023). 239 This computational model of BRAF<sup>V600E</sup> melanoma captures allosteric regulation, transcriptional 240 feedback, and compartmentalization through 68 mechanistic rules that generate over 100,000 reactions.

242 The model provides two parameterizations: a base model for generic RAF/MEK inhibition stud-243 ies, and a specialized pRAF model calibrated for p-RAF inhibitors with adjusted  $\Delta\Delta G$  values for 244 RAF dimer allostery, specific phosphorylation rules for CRAF(S642) and BRAF(T753), and mod-245 ified MEK-ERK binding kinetics. The rule-based architecture is built on fundamental biochemical 246 processes. Binding interactions, comprising 23 rules, include key processes such as BRAF-MEK 247 binding: BRAF + MEK<sup>*u*</sup>  $\stackrel{k_1}{\underset{k_{-1}}{\rightleftharpoons}}$  BRAF:MEK<sup>*u*</sup>. Catalytic reactions, described by 24 rules, follow 248 Michaelis-Menten kinetics, as exemplified by MEK phosphorylation of ERK:  $v = \frac{k_{\text{cat}}[\text{MEK}^p][\text{ERK}]}{K_m + |\text{ERK}|}$ 249 250 Thermodynamic constraints are encoded through energy patterns, particularly for RAF dimer stabi-251 lization by inhibitors:  $\Delta\Delta G_{\text{bind}} = -RT \ln \left( \frac{[\text{RAFi}:\text{RAF}_2]}{[\text{RAFi}][\text{RAF}]^2} \right) + \Delta\Delta G_{\text{allostery}}$  for pan-RAF inhibitors. 253 The model classifies biochemical interactions into six fundamental types: GTP exchange, gene ex-254 pression, phosphorylation, endosomal shuttling, dephosphorylation, and drug inhibition. Each type 255 is represented by specific mathematical formulations and experimentally derived parameters. For example, gene expression follows  $\frac{d[\text{DUSP}]}{dt} = \alpha_{\text{DUSP}}[\text{pERK}] - \delta_{\text{DUSP}}[\text{DUSP}]$ , while drug inhibition is 256 257

governed by binding energies 
$$\Delta G_{\text{bind}} = -RT \ln \left( \frac{[\text{RAFi-RAF}]}{[\text{RAFi}-\text{RAFi}][\text{RAFi}]} \right)$$

259 The model has been extensively validated, achieving RMSD ;15% for ERK activity trajectories 260 and Pearson R = 0.91 for drug synergy predictions. These results demonstrate key biological 261 insights: RAF inhibitor efficacy depends strongly on dimerization energetics, MEK inhibitor resistance emerges through DUSP6 feedback mechanisms, and NRAS<sup>Q61K</sup> mutation substantially re-262 duces the  $EC_{50}$  for EGF by 83%. 263

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#### 3.1 **OPTIMAL BIOSENSOR SELECTION FOR MAPK SIGNALING DYNAMICS**

The MAPK pathway exhibits complex dynamics involving allostery, feedback, and adaptive 267 rewiring Fröhlich et al. (2023). We propose a sensor set to maximize information capture from 268 live-cell imaging, enabling accurate alignment and reconstruction of high-dimensional snapshot 269 data. Assume four key molecules: pERK (phosphorylated ERK), which integrates signals from BRAF(V600E) and RAS-driven pathways with pulsatile dynamics measured via FRET-based biosensor; RAS-GTP (active RAS), which initiates RAF dimerization and MAPK activation measured using GFP-tagged RAS-binding domain probe; pMEK (phosphorylated MEK), which acts as a convergence node for BRAF(V600E) monomers and RAS-driven RAF dimers measured via antibody-based live-cell biosensor; and DUSP mRNA, which provides slow transcriptional feedback measured via MS2 stem-loop system.

These sensors were selected to maximize information through coverage of BRAF(V600E) and RAS channels, time-scale heterogeneity from seconds to hours, and mutual information minimization for redundancy reduction: i.e. *I*(pERK; DUSP mRNA|pMEK). The temporal alignment strategy leverages the heterogeneous time scales of these sensors, with RAS-GTP capturing pulse initiation (seconds), pMEK/pERK tracking signal propagation (minutes), and DUSP mRNA reflecting transcriptional memory (hours). Flow matching is implemented with the added regularity to the loss function (Equation (5)).

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# 4 DISCUSSION

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Our framework fundamentally reorients single-cell perturbation analysis from alignments of static snapshots to dynamic process reconstruction. By combining multi-marginal Schrödinger bridges with spectral operators, we attack a critical paradox in systems biology: how to preserve highdimensional molecular states while capturing transient dynamics essential for predicting cellular decision-making. This approach reveals several key mechanistic insights into drug resistance and how early molecular events orchestrate subsequent cell fate decisions.

292 Our operator-theoretic approach provides spectral regularization that preserves coherent and pre-293 dictable transient valleys in the dynamic landscape where the traditional dimensionality reduc-294 tion methods collapse. Delay-embedded Koopman modes successfully reconstruct unmeasured 295 JUN/ATF dynamics from ERK biosensors alone, while causal optimal transport disentangles minute-scale phosphorylation from hour-scale transcription. Notably, preliminary in silico val-296 idation achieved early resistance prediction, demonstrating the framework's practical utility. In 297 other words, though highly variable at the single-cell level, contain predictive patterns when viewed 298 through the lens of operator theory. The Koopman decomposition of live-cell trajectories identifies 299 coherent modes of behavior that emerge during the initial response to perturbation. 300

301 Looking forward, several challenges remain. The careful selection of biosensors proves essential for capturing the multi-scale nature of cellular response. Our chosen set spans timescales from sec-302 onds to hours, providing temporal anchors that constrain the flow matching problem. This temporal 303 hierarchy enables robust reconstruction of transition paths that would be invisible to the null align-304 ment of optimal transport alone. Integrating mitotic history as covariates of the dynamic bridges 305 could resolve heritable versus stochastic resistance mechanisms. Realizing the full potential of 306 this approach demands developing benchmark datasets with time-matched live/snapshot pairs, and 307 establishing mechanistic taxonomies for drug responses based on dynamical signatures. Conse-308 quently, our framework represents a significant advance in resolving early transient responses to 309 perturbation. It demonstrates that the era of equilibrium-focused single-cell biology must yield to 310 a dynamical paradigm that embraces turbulence as fundamental, not artifact. As biological reality 311 flows, our models must learn to navigate its currents rather than seek frozen snapshots of its waves. 312

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