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Observer-based H_∞ Sliding Mode Control for Networked Systems with Stochastic Communication Protocol and Packet Loss

1st Pengtao Song
School of Automation Science and Engineering
Xi'an Jiaotong University
Xi'an 710049, China
songpengtao@stu.xjtu.edu.cn

2nd Qingyu Yang*
School of Automation Science and Engineering, SKLMSE Laboratory
Xi'an Jiaotong University
Xi'an 710049, China
yangqingyu@mail.xjtu.edu.cn

3rd Donghe Li
School of Automation Science and Engineering
Xi'an Jiaotong University
Xi'an 710049, China
lidonghe2020@xjtu.edu.cn

4th Yuheng Wu
School of Automation Science and Engineering
Xi'an Jiaotong University
Xi'an 710049, China
wuyuheng_0401@stu.xjtu.edu.cn

***Abstract*—This paper investigates the robust control of discrete delayed networked systems with stochastic communication protocol (SCP), quantization and packet loss. Considering limited communication resources and network bandwidth, SCP is firstly designed to coordinate the network environment and avoid node conflicts. Then, the logarithmic quantifier is employed to quantize the signal transmitted over the network, and a Bernoulli random variable is constructed to characterize the packet loss with uncertain occurrence probability. By combining these factors, a comprehensive signal transmission model is established to approach the engineering practice and is stabilized by the designed observer-based H_∞ sliding mode controller (SMC). Sequentially, both the stability conditions and unknown gain matrices are rigorously derived in terms of linear matrix inequalities (LMIs). Simulation and experimental results demonstrate the effectiveness of the proposed scheme.**

***Index Terms*—Networked control system, sliding mode control, stochastic communication protocol (SCP), quantization, packet loss**

I. Introduction

Networked control systems (NCSs), as the mainstream framework of modern industrial automation, have been widely applied in various engineering areas, significantly improving industrial production efficiency and reliability [1]–[3]. In NCSs, the information interaction between each device is conducted via a bus or wireless network, so some network-induced phenomena, such as transmission delay, packet loss, and packet jam, will inevitably occur. During the past two decades, extensive research has focused on the analysis and synthesis of NCSs, mainly including quantization control [4], [5], scheduling control [6], [7], security control [8], etc.

In NCSs, information needs to be packed into data packets with a specific size before transmission. Most existing results rely on the assumption of single-packet transmission [4], [5], [9], i.e., all sensor and controller signals from different nodes need to be packaged into a single packet before being sent over the network respectively. However, it is generally unrealistic in engineering practice where nodes are distributed in space and network bandwidth is limited [10]. When using the multi-packet mechanism, the transmission order of each node must be determined to avoid data conflict. Thus, some scheduling protocols have been proposed to arrange sending sequence and coordinate network resources, including event-triggered protocol [4], [13], round-robin protocol (RRP) [7], try-once-discard protocol (TODP) [11], and stochastic communication protocol (SCP) [10], [12]. Naturally, system synthesis and analysis under scheduling have become a hot issue. For example, in [7], a fuzzy PID controller is designed for nonlinear systems with RRP and multiple network-induced factors. In [11], the set-membership filtering problem for time-varying systems with TODP and mixed time-delay is studied, where a switch model is proposed to inscribe the impact of scheduling strategy. In [12], the output feedback problem for NCSs under SCP is investigated and the stability of closed-loop systems are analyzed rigorously.

In addition, due to the complex working environment and limited network bandwidth, data quantization and packet loss are also tricky challenges for NCSs. Both of them will cause signal transmission errors and thus degrade the system performance, and in severe cases, make the system completely unstable [13]–[15]. Since most network factors can be characterized as bounded disturbances, linear control with H_∞ technique has been widely used for anti-disturbance control. In [5] and [13], the state feedback and output feedback are designed respectively to suppress network uncertainty. In [7], a PID-based H_∞ controller is constructed to further improve

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system robustness. However, as these schemes focus mainly on stability and are derived by LMIs, the closed-loop system may suffer from poor response performance.

Sliding mode control (SMC), owning the characteristics of fast response and strong robustness, has attracted extensive attention in recent years, especially in NCSs [16]. Some exploratory works on stability analysis and synthesis of NCSs with network induced factors, such as delay, packet loss and quantization, have been investigated in [17]–[21]. For instance, Hu *et al.* [17] proposed a robust adaptive SMC for discrete singular systems subject to randomly mixed time-delays under uncertain occurrence probabilities. In [18], a robust SMC was designed for a class of discrete nonlinear systems with mixed-delays and random packet losses. Considering that the system state cannot be obtained directly, observer-based SMC schemes have been studied in [19]–[21], where the state of observer was employed to construct the sliding surface and control law. However, there is still a gap for the analysis and synthesis of observer-based SMC for discrete delayed systems with disturbance, scheduling protocol, quantization and packet loss, which motivates this work.

Based on the above discussion, this paper aims to investigate the robust control of discrete delayed NCSs subject to SCP, quantization and packet loss. Three challenges need to be addressed: 1) how to characterize the SCP and network-induced factors; 2) how to combine multiple factors and construct an appropriate observer to track the system state; 3) how to design a robust controller to ensure the system is asymptotically stable. Based on these challenges, the main contributions are as follows.

- A novel augmented system, including distributed sensors, quantizer, observer, communication network and scheduling protocol, is established to approach the engineering practice, and its stability analysis and synthesis problems are investigated.
- An observer-based H_∞ sliding mode controller is designed to stabilize the system, which can use compromised signals to generate control actions and be robust against multiple uncertainties.
- Both the stability criterion and gain matrices are rigorously derived in terms of LMIs. Moreover, it is theoretically proved that the designed controller satisfies the improved reaching condition.

II. Problrm Setup and Preliminaries

Consider the following discrete delayed systems with disturbance:

$$\begin{cases} x_{k+1} = Ax_k + A_d x_{k-\tau_k} + Bu_k + Dw_k \\ y_k = Cx_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control input, $w_k \in \mathbb{R}^q$ is the external disturbance belonging to $l_2[0, \infty)$, and $y_k \in \mathbb{R}^p$ denotes the system output. The delay τ_k is time-varying and bounded satisfying $\tau_m \le \tau_k \le \tau_M$ with τ_m and τ_M being the lower and upper bounds, respectively. A, A_d, B, C and D are constant matrices with appropriate dimensions.

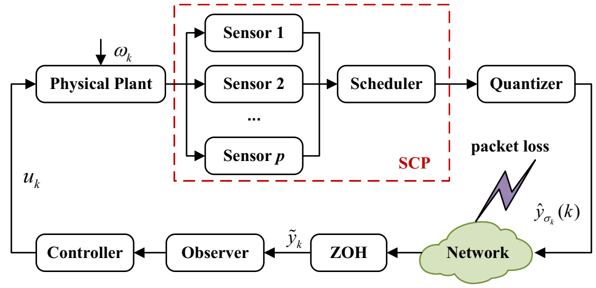

Fig. 1. Networked control system with stochastic communication protocol.

A. Signal Transmission Model Construction

The whole framework is shown in Fig. 1, where the following devices are considered for control synthesis: distributed sensors, quantizers, zero-order holders (ZOH), observer and controller. They achieve signal transmission and communication over a network. In view of the resource constraints, at each sampling instant, only one sensor node can get the permission to transmit output signals over the network, which is determined by the stochastic scheduling protocol.

Define the random variable $\sigma_k \in \{1, 2, ..., p\}$ to denote the authorized node at time k. Then, the scheduling process can be characterized by a discrete-time Markov chain [10], and the transition probability from node i at time k to node j at time $k+1$ is expressed as

$$\text{Prob}\{\sigma_{k+1} = j \,|\sigma_k = i\} = \pi_{ij} \tag{2}$$

where $\sum_{j=1}^{p} \pi_{ij} = 1$. Under the SCP scheduling, the actual output signal at time k is

$$\bar{y}_{\sigma_k}(k) = \phi_{\sigma_k} y_k \tag{3}$$

where $\phi_{\sigma_k} = \text{diag}\{\delta(1-\sigma_k)I, \delta(2-\sigma_k)I, ..., \delta(p-\sigma_k)I\}$, and $\delta(a)$ is a Kronecker function satisfying $\delta(a) = 1$ for $a = 0$, otherwise $\delta(a) = 0$.

Considering the limited network bandwidth, the logarithmic quantizer is introduced to quantize the output signal $\bar{y}_{\sigma_k}(k)$ for further encoding and transmission. To simplify the analysis, the sector bound [22] is applied to estimate the quantization error. Then, the signal actually transmitted through the network is given by

$$\hat{y}_{\sigma_k}(k) = f(\bar{y}_{\sigma_k}(k)) = (I + \Delta_f)\bar{y}_{\sigma_k}(k) \tag{4}$$

where Δ_f is the quantization uncertainty with $\Delta_f \in [-\delta_f, \delta_f]$, and δ_f is the preset quantization constant.

Furthermore, the Bernoulli variable α_k is employed to characterize the packet loss

$$\text{Prob}(\alpha_k = 1) = \theta + \Delta\theta \tag{5}$$

where $\alpha_k = 1$ indicates the data $\hat{y}_{\sigma_k}$ is successfully transmitted, and $\mathbb{E}\{\alpha_k\} = \theta + \Delta\theta$ holds, where θ is a given scalar, and $\Delta\theta$ is unknown but bounded with $|\Delta\theta| \le \varepsilon$. Then, the available signals of the i-th sensor satisfies

$$\tilde{y}_i(k) = \begin{cases} \hat{y}_{\sigma_k}(k), \text{ if } \sigma_k = i \text{ and } \alpha_k = 1 \\ \tilde{y}_i(k-1), \text{ otherwise} \end{cases} \tag{6}$$

By combining (3)–(6), the actual signal received by the observer is given by

$$\tilde{y}_k = \alpha_k(I+\Delta_f)\phi_{\sigma_k} y_k + (I-\alpha_k\phi_{\sigma_k})\tilde{y}_{k-1} \tag{7}$$

Note that a tricky challenge in this paper is to design an efficient observer to estimate the actual system state by using the compromised signal $\tilde{y}_k$.

B. Observer Design

Based on the output signal $\tilde{y}_k$, the following observer is constructed

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + A_d\hat{x}_{k-\tau_k} + Bu_k + L(\tilde{y}_k - f(\hat{y}_k)) \\ \hat{y}_k = C\hat{x}_k \end{cases} \tag{8}$$

where $\hat{x}_k \in \mathbb{R}^n$ denotes the observer state, $\hat{y}_k \in \mathbb{R}^p$ is the observer output, and $L \in \mathbb{R}^{n\times p}$ is the gain matrix to be designed. For simplicity, we use $f(\hat{y}_k)$ to replace $\hat{y}_k$, which can be easily achieved by adding a quantizer after the observer. Then, together with (1)–(8), the error dynamics of the observer is derived as

$$\begin{cases} e_{k+1} = A_e e_k + A_d e_{k-\tau_k} + A_{ex}\hat{x}_k + A_{ey}\tilde{y}_{k-1} + Dw_k \\ y_e(k) = Ce_k \end{cases} \tag{9}$$

where e_k is the tracking error, and

$$A_e = A - L(I+\Delta_f)\alpha_k\phi_{\sigma_k}C,\ A_{ey} = -L(I-\alpha_k\phi_{\sigma_k})$$
$$A_{ex} = L(I+\Delta_f)(I-\alpha_k\phi_{\sigma_k})C$$

C. Sliding Surface Design

To improve the robustness of the system, the following discrete integral-like sliding mode function is designed

$$s_k = G\hat{x}_k - G(A+BK)\hat{x}_{k-1} \tag{10}$$

where $G \in \mathbb{R}^{m\times n}$ is needed to be chosen such that GB is nonsingular. Based on the quasi-sliding mode, the following condition holds

$$s_{k+1} = s_k = 0 \tag{11}$$

Then, from (8), (10) and (11), we can deduce the equivalent control law as

$$\begin{aligned} u_{eq}(k) = K\hat{x}_k &- (GB)^{-1}GA_d\hat{x}_{k-\tau_k} \\ &-(GB)^{-1}GL(\tilde{y}_k - f(\hat{y}_k)) \end{aligned} \tag{12}$$

Subsequently, the observer dynamics is rewritten as

$$\hat{x}_{k+1} = A_x\hat{x}_k + \tilde{G}A_d\hat{x}_{k-\tau_k} + \bar{A}_{xe}e_k + \bar{A}_{xy}\tilde{y}_{k-1} \tag{13}$$

where $\tilde{G} = I - B(GB)^{-1}G$, and

$$A_x = A + BK - \tilde{G}L(I+\Delta_f)(I-\alpha_k\phi_{\sigma_k})C$$
$$A_{xe} = \tilde{G}L(I+\Delta_f)\alpha_k\phi_{\sigma_k}C,\ A_{xy} = \tilde{G}L(I-\alpha_k\phi_{\sigma_k})$$

By defining $\xi_k = \begin{bmatrix}\hat{x}_k^T & e_k^T & \tilde{y}_{k-1}^T\end{bmatrix}^T$ and combining (7), (9) and (13), the augmented closed-loop system is given by

$$\xi_{k+1} = \bar{A}\xi_k + \bar{A}_d\xi_{k-\tau_k} + \bar{D}w_k \tag{14}$$

with

$$\bar{A} = \begin{bmatrix} A_x & A_{xe} & A_{xy} \\ A_{ex} & A_e & A_{ey} \\ A_{yx} & A_{ye} & A_y \end{bmatrix},\ A_{yx} = A_{ye} = \alpha_k(I+\Delta_f)\phi_{\sigma_k}C$$

$$\bar{A}_d = \begin{bmatrix} \tilde{G}A_d & 0 & 0 \\ 0 & A_d & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \bar{D} = \begin{bmatrix} 0 \\ D \\ 0 \end{bmatrix},\ A_y = I - \alpha_k\phi_{\sigma_k}$$

Remark 1: Note that the augmented system (14) considers the effects of multiple network-induced factors and simultaneously characterizes the dynamics of both the observer and plant, which can better simulate the signal transmission process in real engineering.

D. Problem Statement

The whole signal transmission process is shown in Fig. 1, where distributed sensors are employed to measure the system output. Before transmission over the network, the SCP determines the authorized node and quantizes the output signal. With the help of ZOH, the observer is constructed by using the compromised signal $\tilde{y}_k$ to estimate the system state. Finally, an observer-based SMC is designed to stabilize the closed-loop system.

Based on the above statements, the main purposes of this article are threefold.

- Construct an efficient observer to estimate the actual system state by using the compromised signal $\tilde{y}_k$.
- Design an observer-based SMC to ensure the robustness of the closed-loop system under multiple network-induced factors.
- Give a solution for the unknown gain matrix and analyze the stability of the closed-loop system.

III. Main Results

In this section, we will give the design scheme to solve the formulated problem. Before starting this section, the following lemmas are given to facilitate the subsequent proof.

Lemma 1: [23] Given a symmetric matrix R, arbitrary matrices M, N and $F(k)$. The inequality $R + MF(k)N + N^TF^T(k)M^T < 0$ holds for any matrix $F(k)$ satisfying $F^T(k)F(k) \le I$, if and only if there exists scalar $\varepsilon > 0$ such that $R + \varepsilon MM^T + \varepsilon^{-1}N^TN < 0$.

Lemma 2: [24] Given matrices X and Y, there exists an invertible matrix L such that $2X^TY \le X^TLX + Y^TL^{-1}Y$ holds.

Lemma 3: [25] Given matrices P, W, Q and S with appropriate dimensions. When the following inequality holds

$$\begin{bmatrix} W & Q \\ * & -P^{-1} \end{bmatrix} < 0,$$

then we can obtain

$$\begin{bmatrix} W & QS \\ * & sym\{-S\} + P \end{bmatrix} < 0.$$

A. Stability Analysis

Theorem 1: Given scalars $\gamma>0$, $\theta>0$, $\varepsilon>0$, $\tau_M>\tau_m\geq 0$, gain matrices $L\in\mathbb{R}^{n\times p}$ and $K\in\mathbb{R}^{m\times n}$, if there exist scalar $\mu>0$, positive-definite matrices $P_i\in\mathbb{R}^{\bar{n}\times\bar{n}}$ $(i=1,2,...,p)$, $Q_j\in\mathbb{R}^{\bar{n}\times\bar{n}}$ $(j=1,2,3)$, $R\in\mathbb{R}^{\bar{n}\times\bar{n}}$ and $X_l\in\mathbb{R}^{n\times n}$ $(l=1,2)$ satisfying (15), then the augmented system (14) is asymptotically stable with a given H_∞ performance γ.

$$\Xi^{(i)}=\begin{bmatrix}\Sigma_{11}^{(i)} & \Sigma_{12}\\ * & \Sigma_{22}\end{bmatrix}<0,\quad (i=1,\ 2,\ ...,\ p) \tag{15}$$

where

$$\bar{n}=6n+p,\ \Sigma_{11}^{(i)}=\Phi_1^{(i)}+\Phi_2+\Phi_3+\Phi_4+\Psi_1^{(i)}+\Psi_2^{(i)}$$
$$\Phi_1^{(i)}=e_1^T\bar{P}_je_1-e_2^TP_ie_2,\ \Phi_3=de_2^TRe_2-e_4^TRe_4$$
$$\Phi_2=e_2^TQ_1e_2-e_3^T(Q_1-Q_2)e_3-e_4^T(Q_2-Q_3)e_4-e_5^TQ_3e_5$$
$$\Phi_4=e_1^T\bar{C}^T\bar{C}e_1-\gamma^2e_6^Te_6,\ d=\tau_M-\tau_m+1$$
$$\Psi_1^{(i)}=\text{sym}\{\bar{I}\bar{\Pi}_1^{(i)}\},\ \bar{I}=\begin{bmatrix}I & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$$
$$\bar{\Pi}_1^{(i)}=\begin{bmatrix}-X & \lambda_1^{(i)} & 0 & \lambda_2 & 0 & \lambda_3\end{bmatrix},\ \Psi_2^{(i)}=\text{sym}\{\bar{I}\bar{\Pi}_2^{(i)}\}$$
$$\bar{\Pi}_2^{(i)}=\begin{bmatrix}0 & \lambda_4^{(i)} & 0 & 0 & 0 & 0\end{bmatrix},\ \bar{\theta}=\theta+\varepsilon,\ \bar{\phi}_i=I-\bar{\theta}\phi_i$$
$$\lambda_1^{(i)}=\begin{bmatrix}X_1(A+BK) & 0 & 0\\ 0 & X_2A & 0\\ \bar{\theta}\phi_iC & \bar{\theta}\phi_iC & \bar{\phi}_i\end{bmatrix},\lambda_2=\begin{bmatrix}X_1\tilde{G}A_d & 0 & 0\\ 0 & X_2A_d & 0\\ 0 & 0 & 0\end{bmatrix}$$
$$\lambda_3=\begin{bmatrix}0\\ X_2D\\ 0\end{bmatrix},\lambda_4^{(i)}=\begin{bmatrix}-X_1\tilde{G}L\bar{\phi}_iC & \bar{\theta}X_1\tilde{G}L\phi_iC & X_1\tilde{G}L\bar{\phi}_i\\ X_2L\bar{\phi}_iC & -\bar{\theta}X_2L\phi_iC & -X_2L\bar{\phi}_i\\ 0 & 0 & 0\end{bmatrix}$$
$$\Sigma_{12}=\begin{bmatrix}M & \mu N^T\end{bmatrix},\ \Sigma_{22}=\text{diag}\{-\mu,\ -\mu\}$$
$$M=col\{\bar{M}^T,0,0,0,0,0\},\bar{M}=col\{(X_1\tilde{G}L)^T,-(X_2L)^T,0\}$$
$$N=\begin{bmatrix}0 & \bar{N} & 0 & 0 & 0 & 0\end{bmatrix},\ \bar{N}=\begin{bmatrix}-\delta_f\bar{\phi}_iC & \delta_f\bar{\theta}\phi_iC & 0\end{bmatrix}$$

Proof: Consider the following Lyapunov–Krasovskii functional

$$V_k=V_{1k}+V_{2k}+V_{3k} \tag{16}$$

where $V_{1k}=\xi_k^TP_i\xi_k$, $V_{3k}=\sum_{j=-\tau_M+1}^{-\tau_m+1}\sum_{s=k-1+j}^{k-1}\xi_s^TR\xi_s$, and

$$V_{2k}=\sum_{j=k-\tau_m}^{k-1}\xi_j^TQ_1\xi_j+\sum_{j=k-\tau_k}^{k-\tau_m-1}\xi_j^TQ_2\xi_j+\sum_{j=k-\tau_M}^{k-\tau_k-1}\xi_j^TQ_3\xi_j$$

where P_i $(i=1,2,...,p)$, Q_j $(j=1,2,3)$ and R are positive-definite matrices with appropriate dimensions.

Assume that the node i obtains the network communication license at time k, and node j is selected to transmit signal over the network at time $k+1$, i.e., $\sigma_k=i$ and $\sigma_{k+1}=j$. To simplify the derivation, the following augmented vector is defined

$$\eta_k=\begin{bmatrix}\xi_{k+1}^T & \xi_k^T & \xi_{k-\tau_m}^T & \xi_{k-\tau_k}^T & \xi_{k-\tau_M}^T & w_k^T\end{bmatrix}^T \tag{17}$$

Then, the difference of V_k is

$$\Delta V_k=V_{k+1}-V_k=\mathbb{E}\{\Delta V_{1k}+\Delta V_{2k}+\Delta V_{3k}\} \tag{18}$$

with

$$\mathbb{E}\{\Delta V_{1k}\}=\eta_k^T(e_1^T\bar{P}_je_1-e_2^TP_ie_2)\eta_k=\eta_k^T\Phi_1^{(i)}\eta_k \tag{19}$$

$$\begin{aligned}\mathbb{E}\{\Delta V_{2k}\}&=\xi_k^TQ_1\xi_k-\xi_{k-\tau_m}^T(Q_1-Q_2)\xi_{k-\tau_m}\\&\quad-\xi_{k-\tau_k}^T(Q_2-Q_3)\xi_{k-\tau_k}-\xi_{k-\tau_M}^TQ_3\xi_{k-\tau_M}\\&=\eta_k^T\Phi_2\eta_k\end{aligned} \tag{20}$$

$$\begin{aligned}\mathbb{E}\{\Delta V_{3k}\}&=\mathbb{E}\left\{\sum_{j=-\tau_M+1}^{-\tau_m+1}(\xi_k^TR\xi_k-\xi_{k-1+j}^TR\xi_{k-1+j})\right\}\\&=\mathbb{E}\left\{(\tau_M-\tau_m+1)\xi_k^TR\xi_k-\sum_{j=k-\tau_M}^{k-\tau_m}\xi_j^TR\xi_j\right\}\\&\leq\eta_k^T\left[(\tau_M-\tau_m+1)e_2^TRe_2-e_4^TRe_4\right]\eta_k\\&=\eta_k^T\Phi_3\eta_k\end{aligned} \tag{21}$$

where $\bar{P}_j=\sum_{j=1}^p\pi_{ij}P_j$, e_i $(i=1,2,...,6)$ is a row vector with the ith element as I and other elements as 0, i.e., $e_1=\begin{bmatrix}I & 0 & 0 & 0 & 0 & 0\end{bmatrix}$.

Moreover, consider the following free weighting matrix

$$2\eta_k^T\Lambda\left[-\xi_{k+1}+\bar{A}\xi_k+\bar{A}_d\xi_{k-\tau_k}+\bar{D}w_k\right]=0 \tag{22}$$

According to (19)–(22), we have

$$\mathbb{E}\{\Delta V_k\}\leq\eta_k^T(\mathbb{E}\left\{\text{sym}\left\{\Lambda\Pi^{(i)}\right\}\right\}+\Phi_1^{(i)}+\Phi_2+\Phi_3)\eta_k \tag{23}$$

where $\Pi^{(i)}=\begin{bmatrix}-I & \bar{A} & 0 & \bar{A}_d & 0 & \bar{D}\end{bmatrix}$, and we choose $\Lambda=\begin{bmatrix}X^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$ with $X=\text{diag}\{X_1,X_2,I\}$, then the following inequality holds

$$\mathbb{E}\left\{\text{sym}\left\{\Lambda\Pi^{(i)}\right\}\right\}\leq\Psi_1^{(i)}+\Psi_2^{(i)}+\text{sym}\left\{\bar{I}\bar{\Pi}_3^{(i)}\right\} \tag{24}$$

where $\Psi_1^{(i)}$, $\Psi_2^{(i)}$ and $\bar{I}$ are defined below (15), and

$$\bar{\Pi}_3^{(i)}=\begin{bmatrix}0 & \lambda_5^{(i)} & 0 & 0 & 0 & 0\end{bmatrix}$$
$$\lambda_5^{(i)}=\begin{bmatrix}-X_1\tilde{G}L\Delta_f\bar{\phi}_iC & \bar{\theta}X_1\tilde{G}L\Delta_f\phi_iC & 0\\ X_2L\Delta_f\bar{\phi}_iC & -\bar{\theta}X_2L\Delta_f\phi_iC & 0\\ 0 & 0 & 0\end{bmatrix}$$

By Lemma 1, there exist scalar $\mu>0$ such that

$$\mathbb{E}\left\{\text{sym}\left\{\Lambda\Pi^{(i)}\right\}\right\}\leq\Psi_1^{(i)}+\Psi_2^{(i)}+\mu^{-1}MM^T+\mu N^TN \tag{25}$$

Then, together with (23)–(25), we have

$$\begin{aligned}\Delta V_k\leq\eta_k^T(&\Psi_1^{(i)}+\Psi_2^{(i)}+\mu^{-1}MM^T+\mu N^TN\\&+\Phi_1^{(i)}+\Phi_2+\Phi_3)\eta_k\end{aligned} \tag{26}$$

When the external disturbances $w_k\neq 0$, let the following inequality hold

$$\Delta V_k+y_e^T(k)y_e(k)-\gamma^2w_k^Tw_k<0 \tag{27}$$

By integrating (27) from 0 to $+\infty$, we have

$$\sum_{k=0}^{\infty}y_e^T(k)y_e(k)-\gamma^2\sum_{k=0}^{\infty}w_k^Tw_k\leq V(0)-V(\infty)\leq 0 \tag{28}$$

Obviously, the inequality $\|y_e(k)\|_2 \le \gamma \|w_k\|_2$ holds for any $w_k \in L_2[0,\infty)$. Thus, from (27), we have

$$\Psi_1^{(i)} + \Psi_2^{(i)} + \mu^{-1}MM^T + \mu N^T N + \Phi_1^{(i)} + \sum_{j=2}^{4} \Phi_j < 0 \quad (29)$$

where $\Phi_4 = e_1^T \bar{C}^T \bar{C} e_1 - \gamma^2 e_6^T e_6$ and $\bar{C} = \begin{bmatrix} 0 & C & 0 \end{bmatrix}$. With the aid of schur complement, (15) can be easily obtained. This completes the proof. □

Remark 2: The delay-dependent Lyapunov functional is constructed in Theorem 1, where the upper and lower bounds of time-varying delays are considered in V_{2k} and V_{3k}. Meanwhile, to simplify the analysis, the variable ξ_{k+1} is directly used to replace the expansion terms of (14). Moreover, the free weighting matrix is employed to deal with the conservatism issue of the results.

B. Solution of Gain Matrix

Theorem 2: Given scalars $\gamma > 0$, $\theta > 0$, $\varepsilon > 0$, $\tau_M > \tau_m \ge 0$, if there exist scalar $\mu > 0$, positive-definite matrices $P_i \in \mathbb{R}^{\bar{n}\times\bar{n}}$ $(i = 1, 2, ..., p)$, $Q_j \in \mathbb{R}^{\bar{n}\times\bar{n}}$ $(j = 1, 2, 3)$, $R \in \mathbb{R}^{\bar{n}\times\bar{n}}$ and $X_l \in \mathbb{R}^{n\times n}$ $(l = 1, 2)$, constant matrices W, U and Θ with appropriate dimensions satisfying (30), then the augmented system (14) is asymptotically stable with a given H_∞ performance index γ.

$$\begin{bmatrix} \tilde{\Xi}^{(i)} & \tilde{\Sigma}_{12} & E_2^T U^T B^T B & 0 \\ * & \tilde{\Sigma}_{22} & 0 & 0 \\ * & * & \mathrm{sym}\left\{-B^T B W\right\} & B^T X_1^T - W^T B^T \\ * & * & * & -\Theta \end{bmatrix} < 0 \quad (30)$$

where $i = 1, 2, ..., p$, and

$$\tilde{\Xi}^{(i)} = \begin{bmatrix} \bar{\Sigma}_{11}^{(i)} & \bar{\Sigma}_{12} \\ * & \Sigma_{22} \end{bmatrix}, \ \bar{\Sigma}_{11}^{(i)} = \Phi_1^{(i)} + \Phi_2 + \Phi_3 + \Phi_4 + \bar{\Psi}_1^{(i)}$$

$$\bar{\Psi}_1^{(i)} = \mathrm{sym}\left\{\bar{I}\tilde{\Pi}_1^{(i)}\right\}, \ \tilde{\Pi}_1^{(i)} = \begin{bmatrix} -X & \tilde{\lambda}_1^{(i)} & 0 & \lambda_2 & 0 & \lambda_3 \end{bmatrix}$$

$$\tilde{\lambda}_1^{(i)} = \begin{bmatrix} X_1 A + BU & 0 & 0 \\ \bar{L}\bar{\phi}_i C & X_2 A - \bar{\theta}\bar{L}\phi_i C & -\bar{L}\bar{\phi}_i \\ \bar{\theta}\phi_i C & \bar{\theta}\phi_i C & \bar{\phi}_i \end{bmatrix}$$

$$\bar{\Sigma}_{12} = \begin{bmatrix} \tilde{M} & \mu N^T \end{bmatrix}, \ \tilde{M} = \begin{bmatrix} \hat{M}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\hat{M} = \begin{bmatrix} 0 & -\bar{L}^T & 0 \end{bmatrix}^T, \ \tilde{\Sigma}_{12} = \begin{bmatrix} Y^T & \tilde{Z}^T & E_1\Theta \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ Y_1 = \begin{bmatrix} \tilde{G}^T X_1^T & 0 & 0 \end{bmatrix}^T$$

$$\tilde{Z} = X_2 Z, \ E_1 = \begin{bmatrix} \tilde{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \ \tilde{I} = \begin{bmatrix} I & 0 & 0 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0 & \tilde{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tilde{\Sigma}_{22} = \mathrm{diag}\left\{-X_2, -X_2, -\Theta\right\}$$

Then, the gain matrices can be given by

$$K = W^{-1}U, \ L = X_2^{-1}\bar{L}$$

Proof: Note that (15) is equivalent to

$$\tilde{\Xi}^{(i)} + \mathrm{sym}\{Y^T Z\} + \mathrm{sym}\left\{E_1 J E_2\right\} < 0 \quad (31)$$

where $J = (X_1 B - BW)(W^{-1}U)$, $\tilde{\Xi}^{(i)}$, Y, E_1 and E_2 are defined below (30), and

$$Z = \begin{bmatrix} 0 & Z_1 & 0 & 0 & 0 & 0 & L & 0 \end{bmatrix}, \ Z_1 = \begin{bmatrix} -L\bar{\phi}_i C & L\bar{\theta}\phi_i C & L\bar{\phi}_i \end{bmatrix}$$

Based on Lemma 2, there exist invertible matrices Υ and Θ such that the following inequality holds

$$\tilde{\Xi}^{(i)} + Y^T \Upsilon^{-1} Y + Z^T \Upsilon Z + E_\Theta < 0 \quad (32)$$

where $E_\Theta = E_1 \Theta E_1^T + E_2^T J^T \Theta^{-1} J E_2$. Define $\Upsilon = X_2$ and based on schur complement, (32) can be further expressed as

$$\begin{bmatrix} \tilde{\Xi}^{(i)} & \tilde{\Sigma}_{12} & E_2^T (W^{-1}U)^T \\ * & \tilde{\Sigma}_{22} & 0 \\ * & * & \Sigma_{33} \end{bmatrix} < 0, \ i = 1, 2, ..., p \quad (33)$$

where

$$\tilde{\Sigma}_{12} = \begin{bmatrix} Y^T & \tilde{Z}^T & E_1\Theta \end{bmatrix}, \ \tilde{\Sigma}_{22} = \mathrm{diag}\left\{-X_2, -X_2, -\Theta\right\}$$

$$\Sigma_{33} = -[(X_1 B - BW)^T \Theta (X_1 B - BW)]^{-1}, \ Z = X_2 Z$$

Subsequently, by Lemma 3, we can get that

$$\begin{bmatrix} \tilde{\Xi}^{(i)} & \tilde{\Sigma}_{12} & E_2^T U^T B^T B \\ * & \tilde{\Sigma}_{22} & 0 \\ * & * & \mathrm{sym}\left\{-B^T B W\right\} + \Sigma_{33} \end{bmatrix} < 0 \quad (34)$$

Finally, by applying schur complement to (34), the inequality (30) can be obtained. This completes the proof. □

C. Design of Sliding Mode Controller

As in [26], the following reaching condition is introduced for the augmented system (14)

$$\begin{cases} \Delta s_k \le -\alpha s_k - \beta e^{-\nu k}\mathrm{sgn}(s_k), & \text{if } s_k > 0 \\ \Delta s_k \ge -\alpha s_k - \beta e^{-\nu k}\mathrm{sgn}(s_k), & \text{if } s_k < 0 \end{cases} \quad (35)$$

where $0 < \alpha < 1$, $\beta > 0$ and $v \ge 0$. Then, the design scheme of SMC is given by the following theorem.

Theorem 3: Consider the system (1) with the observer (8) and sliding surface (10), then the reaching condition (35) can be ensured if the controller is designed as

$$\begin{aligned} u_k = {} & K(\hat{x}_k - \hat{x}_{k-1}) + (GB)^{-1}[G\hat{x}_k - GA\hat{x}_{k-1} \\ & - GA_d \hat{x}_{k-\tau_k} - \alpha s_k - \beta e^{-vk}\mathrm{sgn}(s_k) - \Gamma \mathrm{sgn}(s_k)] \end{aligned} \quad (36)$$

where $\Gamma = \|GL\tilde{y}_k\| + \|GLf(\hat{y}_k)\|$. The gain matrices K and L is solved by Theorem 2.

Proof: By combining the sliding surface (10), the state observer (8) and sliding mode controller (36), we have

$$\begin{aligned} \Delta s_k = s_{k+1} - s_k = {} & GL(\tilde{y}_k - f(\hat{y}_k)) - \Gamma \mathrm{sgn}(s_k) \\ & -\alpha s_k - \beta e^{-vk}\mathrm{sgn}(s_k) \end{aligned} \quad (37)$$

Note that when $s_k > 0$, the following inequality holds

$$\begin{aligned} & GL(\tilde{y}_k - f(\hat{y}_k)) - \Gamma \mathrm{sgn}(s_k) \\ & \le \|GL(\tilde{y}_k - f(\hat{y}_k))\| - (\|GL\tilde{y}_k\| + \|GLf(\hat{y}_k)\|) \\ & \le 0 \end{aligned} \quad (38)$$

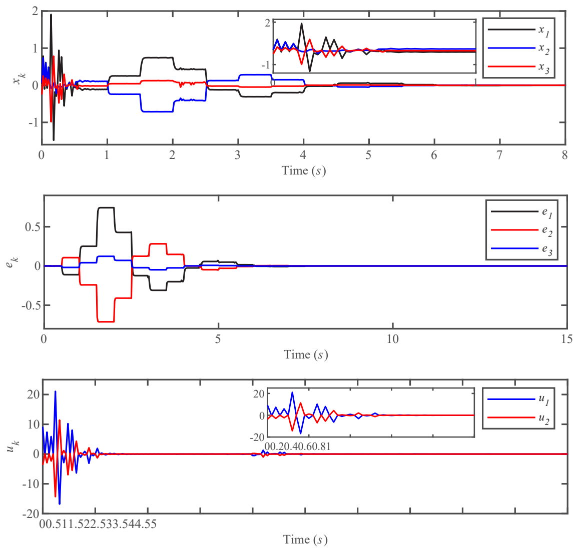

Fig. 2. Response curves of the closed-loop system.

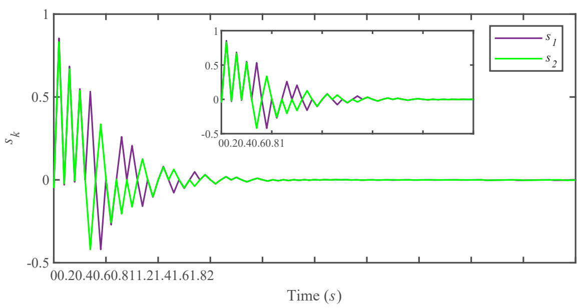

Fig. 3. Trajectories of sliding variable s_k.

Obviously, the reaching condition (35) is satisfied. Similar results can be yielded when $s_k < 0$, we omit it for simplicity. Then, under the proposed controller (36), the system state can be driven into the prescribed neighborhood of sliding surface within a finite time. This completes the proof. □

IV. Numerical Example

In this section, we will verify the validity of the proposed method with a numerical example.

Example: Consider a discrete-time delayed system with the following parameter matrices

$$A = \begin{bmatrix} 0.15 & -0.25 & 0 \\ 0 & 0.13 & 0.01 \\ 0.03 & 0 & -0.05 \end{bmatrix}, \ D = \begin{bmatrix} -0.08 \\ 0.011 \\ -0.019 \end{bmatrix}$$

$$A_d = \begin{bmatrix} -0.02 & 0.06 & 0.04 \\ 0.04 & 0.01 & -0.03 \\ 0.01 & 0.02 & 0.05 \end{bmatrix}, \ B = \begin{bmatrix} 0.08 & -0.017 \\ 0.2 & 0.3 \\ 0.13 & 0.26 \end{bmatrix}$$

$$C = \operatorname{diag}\{0.04, 0.04, 0.06\}, \ w_k = 17k^3 \cos k / e^k$$

Set the sampling period $T = 0.02$, initial state $x(0) = \hat{x}(0) = \begin{bmatrix} 1 & 0.5 & -1 \end{bmatrix}^T$, quantization parameter $\delta = 0.10$, matrix $G = (B^T B)^{-1} B^T$, and prescribed scalars $\theta = 0.78$, $\Delta\theta = 0.02$, $\tau_m = 1$, $\tau_M = 3$, $\gamma = 1.7$. Moreover, the transition probability matrix is given by

$$P = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.4 & 0.1 & 0.5 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

Then, by using Theorem 2, we can obtain

$$K = \begin{bmatrix} -0.610 & 0.304 \\ 0.884 & -0.636 \\ -0.035 & 0.045 \end{bmatrix}^T, L = \begin{bmatrix} 0.049 & -0.001 & -0.001 \\ -0.001 & 0.050 & 0.003 \\ -0.001 & 0.004 & 0.045 \end{bmatrix}$$

Based on the proposed SMC scheme, the response curves of the closed-loop system are shown in Fig. 2. One can see that the error dynamics of the observer is asymptotically stable, so the designed observer can efficiently estimate the system state from the compromised signal $\tilde{y}_k$. Due to the involvement of SCP and multiple network-induced factors, the system overshoot occurs in the dynamic phase, but the proposed controller ensures that the state of closed-loop system converges to a small error band, which further proves the validity of Theorem 2.

In the discrete-time domain, only quasi-sliding mode (QSM) can be performed due to finite sampling frequency. The sliding mode signal s_k is displayed in Fig. 3, which demonstrates that the reaching condition (35) holds under the solved gain matrices. Fig. 4 shows the transmission sequence of the output signal $\hat{y}_{\sigma_k}(k)$ under SCP, where we consider three sensor nodes and take i ($i = 1, 2, 3$) to denote the ith node that

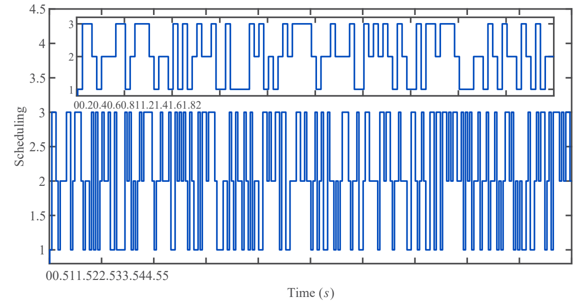

Fig. 4. Transmission sequence of output signal $\hat{y}_{\sigma_k}(k)$ under SCP.

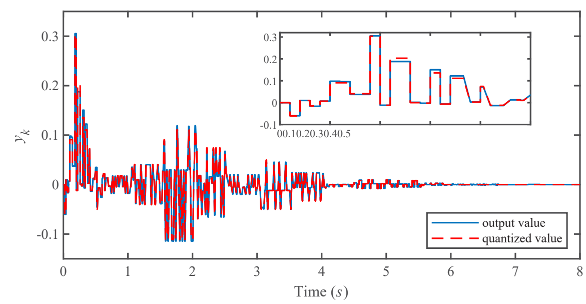

Fig. 5. Comparison of output signal $\bar{y}_{\sigma_k}(k)$ before and after quantization.

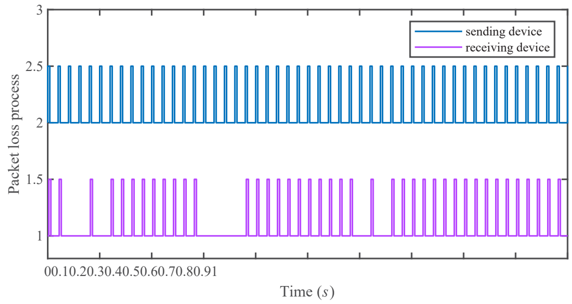

Fig. 6. Packet loss of signal during network transmission.

gets access to network communication. With the designed SCP scheduling, more communication resources can be saved for some real-time tasks and to avoid node conflicts. Fig. 5 displays the comparison of output signal $\bar{y}_{\sigma_k}(k)$ before and after quantization. It is clear that there are differences between the signals before and after quantization. The packet loss of signals during network transmission is shown in Fig. 6, which can be seen that the receiving device fails to receive any information at some time due to packet loss. Although packet loss and quantisation can degrade the system performance, the designed control scheme ensures that the closed-loop system is asymptotically stable. All the simulation results confirm the effectiveness of the proposed scheme.

V. Conclusion

In this paper, an observer-based H_∞ SMC is designed for a class of discrete delayed NCSs subject to SCP, quantization and packet loss. The discrete-time Markov chain is firstly introduced to characterize the scheduling behavior of SCP. Then, the logarithmic quantifier and a Bernoulli random variable are employed to model the quantization error and packet loss, respectively. On this basis, a novel augmented system model is proposed and its stability is analyzed rigorously. Moreover, some sufficient conditions are given to solve the unknown gain matrices. Experimental results demonstrate that the proposed scheme is robust to multiple uncertainties and the asymptotic stability of the closed-loop system can be guaranteed.

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