# GVFI: LEARNING 3D GAUSSIAN VELOCITY FIELDS FROM DYNAMIC VIDEOS

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# ABSTRACT

In this paper, we aim to model 3D scene geometry, appearance, and physical information just from dynamic multi-view videos in the absence of any human labels. By leveraging physics-informed losses as soft constraints or integrating simple physics models into neural networks, existing works often fail to learn complex motion physics, or doing so requires additional labels such as object types or masks. In this paper, we propose a new framework named GVFi to model the motion physics of complex dynamic 3D scenes. The key novelty of our approach is that, by formulating each 3D point as a rigid particle with size and orientation in space, we choose to directly learn a translation rotation dynamics system for each particle, explicitly estimating a complete set of physical parameters to govern the particle's motion over time. Extensive experiments on three existing dynamic datasets and two newly created challenging synthetic and real-world datasets demonstrate the extraordinary performance of our method over baselines in the task of future frame extrapolation. A nice property of our framework is that multiple objects or parts can be easily segmented just by clustering the learned physical parameters. Our datasets and code will be released at https://github.com/.

# 1 INTRODUCTION

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Regarding our daily dynamic 3D scenes such as falling balls, rotating fans, and folding chairs, precisely modeling their geometry, appearance, and physical properties, and further predicting their future states are crucial for emerging applications in robotics, mixed reality, and embodied AI. With the advancement of recent 3D representations such as NeRF (Mildenhall et al., 2020) and 3DGS (Kerbl et al., 2023), a plethora of works (Pumarola et al., 2021; Yang et al., 2024; Wu et al., 2024) have been proposed to model various dynamic 3D scenes, achieving excellent performance in interpolating novel views within the observed time. However, they often fail to extrapolate future frames, fundamentally because they cannot learn the underlying physics priors of complex 3D scenes.

037 To learn physics priors, existing methods mainly consist of two categories: 1) physics-informed 038 neural network (PINN) based methods (Raissi et al., 2019) which integrate the governing partial differential equations (PDEs) into loss functions to drive neural networks to learn physically plau-040 sible dynamic 3D scenes such as floating smoke (Chu et al., 2022) and simple moving objects (Li 041 et al., 2023b). Although demonstrating promising results in modeling 3D geometry and physics 042 such as velocity and viscosity, these methods usually need boundary constraints such as accurate 043 object/foreground masks which may not always be available in practice. In addition, adding PINN 044 losses is not a free lunch, but significantly sacrificing the efficiency in training and accuracy at boundary regions. 2) Physics model based methods (Jonathan et al., 2020; Zhong et al., 2024; Whitney et al., 2024) which encode various physics systems into neural networks to model elastic objects, 046 fluids, etc.. Thanks to the explicit physics priors, these methods obtain impressive results in physical 047 properties learning and simulation. Nevertheless, they are often limited to specific types of objects, 048 materials, or motions due to the lack of generality of encoded physics priors, thus being unable to 049 predict future motions of complex dynamic 3D objects and scenes. 050

In this paper, we aim to introduce a new framework to model dynamic 3D scenes just from multiview RGB videos, without needing any additional human labels such as object types or masks, ultimately being able to predict future frames viewing from arbitrary angles. Among various physical properties of a dynamic 3D scene, following the recent work NVFi (Li et al., 2023a), we also

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Figure 1: An illustration of the overall framework.

061 choose to learn a velocity field as it directly governs 3D scene movement. However, to accurately 062 learn the physical velocity from RGB videos is extremely challenging, essentially due to the lack of 063 sufficient physics constraints from raw color pixels. This problem is even harder when multiple ob-064 jects or parts are undergoing rather different motion patterns. For example, regarding two adjacent 065 objects moving in opposite directions in 3D space, the velocity of neighboring 3D surface points at 066 the intersection region tends to have particularly distinct patterns. This means that the latent representation of per-point dynamics in 3D space could be discrete in nature. Therefore, it is more 067 desirable to model per-point dynamics independently, thus every point having its unique motion. 068 For generality, we regard each 3D point in space as a rigid particle with its size and orientation. If 069 its size is zero, the rigid particle degenerates to a point.

071 With this insight, for each rigid particle in space, we propose to learn an independent dynamics system that includes a complete set of physical parameters to govern its motion over time. According 072 to the laws of classical mechanics, for a specific rigid particle traversing 3D space over time, its 073 motion can always be regarded as a rotational movement about a rotation center which has its own 074 translation. Given this general foundation, we choose to learn a translation rotation dynamics system 075 for each rigid particle, allowing its future motion to be derived accordingly. Alongside learning the 076 core dynamics, we must also model the geometry and appearance of 3D scenes. In this paper, 077 we naturally choose the recent 3D Gaussian Splatting (3DGS) (Kerbl et al., 2023) as our scene representation, thanks to its unprecedented fidelity in reconstruction and its particle (a Gaussian 079 kernel) based representation in nature, which shares the basic concept of our defined rigid particle.

As illustrated in Figure 1, our framework consists of two major components: 1) a **3D scene representation module** to learn dynamic scene geometry and appearance at a canonical timestamp, which is implemented by a vanilla 3DGS (Kerbl et al., 2023), though other variants can be adopted as well; 2) a **translation rotation dynamics system module** to learn a full set of physical parameters for each input rigid particle, which is just realized by multilayer perceptrons (MLPs). Based on these system parameters, the rigid particle's velocity is then derived according to the laws of classical mechanics, without needing additional physics priors such as PINN (Raissi et al., 2019) in training.

The key to our framework is the second module which simply regards each 3D Gaussian kernel as a rigid particle and takes it as input into MLPs. Nevertheless, we empirically find that it is hard to optimize this module due to the inaccuracy and instability of Gaussian kernels regressed at early training epochs. To tackle this issue, we simply train an auxiliary deformation field in parallel with our second module using an existing work such as (Yang et al., 2024) and (Wu et al., 2024).

Different from current works for modeling dynamic scenes, including NeRF-based methods, *e.g.*, D-NeRF(Pumarola et al., 2021)/TiNeuVox(Fang et al., 2022)/HexPlane(Cao & Johnson, 2023), and 3DGS-based methods such as DefGS (Yang et al., 2024), 4DGS (Wu et al., 2024), and E-D3DGS (Bae et al., 2024), our core novelty is the introduced translation rotation dynamics system together with its effective optimization strategy, which allows us to truly learn physical parameters, ultimately achieving future frame extrapolation. By comparison, all those existing methods fail to do so, though they perform well for past frame interpolation, as extensively verified in Tables 1&2.

Our method, named **GVFi**, leverages 3D Gaussians to model scene geometry and appearance, while learning velocity fields via estimating translation rotation dynamics systems. Our contributions are:

- We introduce a new framework to model motion physics of complex dynamic 3D scenes, without needing prior knowledge of object shapes, types, or masks.
- We propose to learn a translation rotation dynamics system for each 3D rigid particle, thus allowing the velocity field to be derived without needing additional physics constraints in training.
- We demonstrate superior results in future frame extrapolation on three existing datasets, and two newly collected synthetic and real-world datasets with extremely challenging dynamics.

# 108 2 RELATED WORKS

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**3D Shape Representations:** Static 3D objects and scenes are traditionally represented by voxels, 111 point clouds, meshes, etc., but they usually have limited representation capabilities due to the nature 112 of discretization. Recently, implicit representations have been developed in the literature, includ-113 ing occupancy fields (OF) (Mescheder et al., 2019; Chen & Zhang, 2019), un/signed distance fields 114 (U/SDF) (Park et al., 2019; Chibane et al., 2020), and radiance fields (NeRF) (Mildenhall et al., 115 2020). Although demonstrating excellent performance in novel view synthesis and shape recon-116 struction, they are time-consuming to render 2D images or extract 3D shapes due to the integration of their continuous coordinate-based representations. To tackle this issue, the very recent 3D Gaus-117 sian Splatting (Kerbl et al., 2023) turns to represent a 3D shape as a set of explicit Gaussian ker-118 nels with various properties, achieving real-time rendering speed. In our framework, we adopt this 119 particle-based representation, as it is amenable to our particle-based physics learning framework. 120

121 **Dynamic 3D Reconstruction:** Recent advances in dynamic 3D reconstruction primarily follow the 122 development of static 3D techniques such as SDF, NeRF, and Gaussian Splatting. To model the temporal relationship, existing works (Tretschk et al., 2021; Li et al., 2021; Barron et al., 2021; 123 Gao et al., 2021; Tian et al., 2023; Liu et al., 2023; Cao & Johnson, 2023; Fridovich-Keil et al., 124 2023; Cai et al., 2022; Fang et al., 2022; Li et al., 2022; Park et al., 2021; You & Hou, 2023; Du 125 et al., 2021; Park et al., 2023; Xian et al., 2021; Wang et al., 2021; Liu et al., 2024b) usually add 126 the time dimension into static 3D representations to learn a motion or deformation field for rigid or 127 deformable objects and scenes. Despite achieving excellent performance in novel view synthesis, 128 especially when integrating 3DGS as the backbone (Wu et al., 2024; Yang et al., 2024; Li et al., 129 2024; Lei et al., 2024; Lin et al., 2024; Lu et al., 2024), these works can only interpolate 2D views 130 within the observed time, instead of predicting physically meaningful future frames. Basically, this 131 is because the commonly learned motion or deformation field does not encode physics priors in 132 nature, but just fits the correlation between pixels. In this paper, the key difference between these 133 works and us is that we separately learn translation rotation dynamics systems for 3D rigid particles, 134 thus enabling us to estimate physically meaningful future frames, whereas they cannot.

135 3D Physics Learning: To learn various physical properties for 3D objects and scenes, the recent 136 physics-informed neural networks (PINN) (Raissi et al., 2019; Mishra & Molinaro, 2023; Raissi 137 et al., 2020; Hao et al., 2023; Baieri et al., 2023; Chalapathi et al., 2024; Wang et al., 2024; Zhao 138 et al., 2024) are widely applied to convert PDEs into loss functions as soft constraints, driving neural 139 networks to learn physically meaningful targets. However, it is often inefficient to train PINNs due 140 to the large amount of data samples needed to regularize, and the soft constraints are usually not 141 sufficient to obtain satisfactory results. In this paper, we do not rely on such inefficient PINN losses to incorporate physics priors to train neural networks. Another line of works (Qiao et al., 2022; Deng 142 et al., 2023; Xue et al., 2023; Franz et al., 2023; Whitney et al., 2024) integrate explicit physics 143 systems such as springs, graphs, etc., into the learning process to model elastic objects (Zhong et al., 144 2024; Zhang et al., 2024; Liu et al., 2024a), fluids (Jonathan et al., 2020; Lienen et al., 2024), etc., 145 achieving impressive results in physics learning and simulation. In this paper, we also opt to learn 146 physics systems. However, the core difference is that we learn a translation rotation dynamics system 147 which is applicable to common deformation and transformation dynamics, whereas existing works 148 often learn a spring or fluid system only applicable to elastic objects or fluids. 149

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# 3 GVFI

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153 Our framework mainly comprises two modules together with an auxiliary deformation field to model 154 3D geometry, appearance, and physics. Given dynamic multi-view RGB videos with known camera 155 poses and intrinsics, the 3D scene representation module aims to learn a set of 3D Gaussian kernels 156 to represent the 3D scene geometry and appearance in a canonical space. The auxiliary deformation 157 field is designed to predict the translation and distortion of each Gaussian kernel given the current 158 training time t. For these two components, we simply follow the design of existing works (Kerbl 159 et al., 2023; Yang et al., 2024) briefly elaborated in Section 3.1. Notably, the deformation field alone cannot extrapolate frames beyond the training time. Our core module of the translation rotation 160 dynamics system aims to learn a set of physical parameters for each 3D rigid particle, governing its 161 motion dynamics over time, which is detailed in Section 3.2.

# 162 3.1 PRELIMINARY

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For the input multi-view RGB videos, T represents the greatest timestamp in training and N the total number of cameras. For training stability, we first use all frames  $\{I_0^1 \cdots I_0^n \cdots I_0^N\}$  at time t = 0 to train a reasonable static 3DGS model as an initialization of the 3D scene geometry and appearance, and then use the remaining frames to jointly optimize our translation and rotation dynamics system and the auxiliary deformation field.

**Canonical 3D Gaussians:** Following the vanilla 3DGS (Kerbl et al., 2023), we employ a set of learnable 3D Gaussian kernels  $G_0$  to represent the canonical scene geometry and appearance at t =0. Each kernel is parameterized by a 3D position  $x_0$ , covariance matrix obtained from quaternion  $r_0$ , scaling  $s_0$ , opacity  $\sigma$ , and color c computed from spherical harmonics (SH). Following prior works (Yang et al., 2024; Wu et al., 2024), we assume the opacity  $\sigma$  and color c of each Gaussian will not be updated, but constantly associated with the kernel and transported over time.

Given the N images at timestamp t = 0, we either initialize all canonical 3D Gaussian kernels G<sub>0</sub> randomly or based on sparse points created by SfM (Schonberger & Frahm, 2016). To train all kernels, we exactly follow the process of 3DGS (Kerbl et al., 2023) by 1) projecting Gaussian kernels into camera space, 2) rendering the projected kernels into image space, and 3) optimizing all kernel parameters via  $\ell_1$  and  $\ell_{ssim}$  losses used in 3DGS as follows.

$$\underbrace{\{\cdots(\boldsymbol{x}_0, \boldsymbol{r}_0, \boldsymbol{s}_0, \sigma, \boldsymbol{c})\cdots\}}_{\boldsymbol{G}_0} \xleftarrow{\text{project+render}}_{\ell_1 + \ell_{\text{ssim}}} \left\{ \boldsymbol{I}_0^1 \cdots \boldsymbol{I}_0^n \cdots \boldsymbol{I}_0^N \right\}$$
(1)

183 Auxiliary Deformation Field: To aid the learning of our translation and rotation dynamics system, 184 we leverage an existing deformation field (Yang et al., 2024), but we are also amenable to other 185 deformable Gaussian methods such as 4DGS (Wu et al., 2024), as demonstrated in our experiments in Section 4.1. In particular, the 3D position  $x_0$  of each canonical Gaussian kernel and the current 187 timestamp t are fed into an MLP-based deformation network, denoted as  $f_{defo}$ , directly predicting 188 the corresponding position displacement  $\delta x$ , and the change of quaternion  $\delta r$  and scaling  $\delta s$  from 189 timestamp 0 to t. All Gaussians  $G_t$  at time t can be easily computed as follows, where the operations 190  $\circ$  and  $\odot$  follow (Yang et al., 2024). 191

$$\underbrace{\left\{\cdots\left((\boldsymbol{x}_{t}=\boldsymbol{x}_{0}+\delta\boldsymbol{x}),(\boldsymbol{r}_{t}=\boldsymbol{r}_{0}\circ\delta\boldsymbol{r}),(\boldsymbol{s}_{t}=\boldsymbol{s}_{0}\odot\delta\boldsymbol{s}),\sigma,\boldsymbol{c}\right)\cdots\right\}}_{\boldsymbol{G}_{t}},(\delta\boldsymbol{x},\delta\boldsymbol{r},\delta\boldsymbol{s})=f_{defo}(\boldsymbol{x}_{0},t)$$
(2)

All these deformed Gaussians will be projected and optimized by visual images at timestamp t in a later stage as clarified in Section 3.3, where the deformation net  $f_{defo}$  will be optimized from scratch. All details are provided in Appendix A.1 and A.2.

# 3.2 TRANSLATION ROTATION DYNAMICS SYSTEM

This module aims to learn physical parameters that govern the motion of 3D scenes. However, the dynamics of an entire space are extremely complex. Here, we simplify this problem and formulate it into just learning per rigid particle dynamics, where we treat each (canonical or deformed) Gaussian kernel as a rigid particle with size and orientation. According to the laws of classical mechanics, for a specific rigid particle  $P \in \mathbb{R}^3$ , its motion in a 3D world coordinate system can be regarded as a rotational movement about a rotation center which has its own translation. To this end, we aim to learn the following two groups of physical parameters for each 3D rigid particle P:

- Group #1 Rotation Center Parameters including: 1) the center's position  $P_c \in \mathcal{R}^3$ , 2) the center's velocity  $v_c \in \mathcal{R}^3$ , and 3) acceleration  $a_c \in \mathcal{R}^3$  in the world coordinate system.
- Group #2 Rigid Particle Rotational Parameters including: 1) the rigid particle's rotation vector  $w_p \in \mathbb{R}^3$  with regard to its center  $P_c$ , and 2) the rigid particle's angular acceleration  $\epsilon_p$ .

As illustrated in Figure 2, our translation rotation dynamics system module, denoted as  $f_{trd}$ , takes a rigid particle P as input, directly predicting the physical parameters of its rotation center and its own rotational information. Then, this rigid particle will be naturally driven by its learned physical parameters, forming its motion dynamics, as illustrated by the trajectory in Figure 2. This module is implemented by simple MLPs:

$$\{(\boldsymbol{P}_c, \boldsymbol{v}_c, \boldsymbol{a}_c), (\boldsymbol{w}_p, \boldsymbol{\epsilon}_p)\} = f_{trd}(\boldsymbol{P})$$
(3)



Figure 2: The proposed translation rotation dynamics system for a specific rigid particle. The rigid particle will be driven by its learned physical parameters over time, forming a trajectory in 3D space.

Notably, for an input rigid particle P, the elegance of this module  $f_{trd}$  is that it only needs to learn this full translation rotation dynamics system at its canonical timestamp, *i.e.*, t = 0, and that particle's future motion will be governed by the learned dynamics system when t > 0 according to the laws of mechanics. We will now derive the rigid particle's future motion as follows.

For a specific rigid particle P, now we have its estimated translation rotation dynamics parameters  $\{(P_c, v_c, a_c), (w_p, \epsilon_p)\}$ . Given a future timestamp t, we now calculate its updated parameters for both the rotation center and the particle itself as follows:

$$\boldsymbol{P}_{c}^{t} = \boldsymbol{P}_{c} + \boldsymbol{v}_{c}t + \frac{1}{2}\boldsymbol{a}_{c}t^{2}, \quad \boldsymbol{v}_{c}^{t} = \boldsymbol{v}_{c} + \boldsymbol{a}_{c}t, \quad \boldsymbol{a}_{c}^{t} = \boldsymbol{a}_{c}, \quad \boldsymbol{w}_{p}^{t} = \left(\|\boldsymbol{w}_{p}\| + \epsilon_{p}t\right)\frac{\boldsymbol{w}_{p}}{\|\boldsymbol{w}_{p}\|}, \quad \epsilon_{p}^{t} = \epsilon_{p} \quad (4)$$

Theoretically, our above updating scheme can be naturally extended to higher orders or reduced to lower orders with regard to future time t. Intuitively, a higher order relationship from time 0 to t is expected to capture extremely complex dynamics such as a rolling ball suddenly breaking up into pieces due to unknown explosives inside, whereas a much lower order relationship tends to only capture static or constant speed scenes, thus being oversimplified. In this paper, we opt to the above second-order scheme to update dynamics parameters from time 0 to t for two primary reasons:

- In many applications such as robot manipulation, the need for future prediction usually involves a relatively short interval, *i.e.*, |t 0| is rather small, *e.g.*, in milliseconds. In this case, a second-order relationship is usually sufficient to achieve decent approximations. In addition, a simple sliding window based approach can be applied to continuously and incrementally predict future frames given the newest visual observations from sensors.
- In our daily life, the majority of common physical movements such as rolling balls or moving cars can be generally described by a second-order relationship. In fact, both Newton's First and Second Law of Motion can be captured. Notably, since the whole 3D scene comprises a large number of rigid particles, each particle has up to second-order dynamics, *i.e.*, with a constant acceleration between  $0 \sim t$ . Therefore, the compounded dynamics for the entire 3D scene can be rather complex, including various deformations and transformations in our daily lives.

Nevertheless, it is still interesting yet non-trivial to learn much higher-order relationships and we leave it for future exploration. More implementation details of this module are in Appendix A.3.

# 3.3 TRAINING

With our translation rotation dynamics module and the auxiliary deformation field, we now discuss how to connect and train them together, such that physical parameters can be truly learned.

For two timestamps t' and t, where t is usually sampled from the training set and  $\Delta t = t - t'$  is predefined to be small enough, we can easily obtain Gaussians  $G_{t'}$  from the deformation field  $f_{defo}$ .

Having our translation rotation dynamics module  $f_{trd}$  at hand, we naturally regard the transportation of all kernels from t' to t is governed by the corresponding physical parameters estimated by  $f_{trd}$  at time t'. From Equations 4, at time t', the physical parameters of a rigid particle P are:

$$\{(\boldsymbol{P}_{c}^{t'}, \boldsymbol{v}_{c}^{t'}, \boldsymbol{a}_{c}^{t'}), (\boldsymbol{w}_{p}^{t'}, \boldsymbol{\epsilon}_{p}^{t'})\} \xleftarrow{t'} \{(\boldsymbol{P}_{c}, \boldsymbol{v}_{c}, \boldsymbol{a}_{c}), (\boldsymbol{w}_{p}, \boldsymbol{\epsilon}_{p})\} = f_{trd}(\boldsymbol{P})$$
(5)

Now we can easily compute the kernel's orientation change  $\Delta r$  from  $r_{t'}$  to  $r_t$  as follows:

$$\Delta \boldsymbol{r} = \left(\cos\frac{\Delta\theta}{2}, \sin\frac{\Delta\theta}{2} \cdot \frac{\boldsymbol{w}_p}{\|\boldsymbol{w}_p\|}\right), \quad \text{where } \Delta\theta = \left(\|\boldsymbol{w}_p\| + \epsilon_p t'\right)(t - t') \tag{6}$$

Table 1: Quantitative results of all methods for both future frame extrapolation and novel view interpolation on Dynamic Object Dataset and Dynamic Indoor Scene Dataset.

< 1 Z	interpolation on Dynamic Object Dataset and Dynamic Indoor Seene Dataset.													
272			Dynamic Object Dataset					Dynamic Indoor Scene Dataset						
213		In	Interpolation			Extrapolation			Interpolation			Extrapolation		
274		PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	
775	T-NeRF(Pumarola et al., 2021)	13.163	0.709	0.353	13.818	0.739	0.324	24.944	0.742	0.336	22.242	0.700	0.363	
275	D-NeRF(Pumarola et al., 2021)	14.158	0.697	0.352	14.660	0.737	0.312	25.380	0.766	0.300	20.791	0.692	0.349	
276	TiNeuVox(Fang et al., 2022)	27.988	0.960	0.063	19.612	0.940	0.073	29.982	0.864	0.213	21.029	0.770	0.281	
077	T-NeRF <sub>PINN</sub>	15.286	0.794	0.293	16.189	0.835	0.230	16.250	0.441	0.638	17.290	0.477	0.618	
277	HexPlane <sub>PINN</sub>	27.042	0.958	0.057	21.419	0.946	0.067	25.215	0.763	0.389	23.091	0.742	0.401	
278	NSFF(Li et al., 2021)	-	-	-	-	-	-	29.365	0.829	0.278	24.163	0.795	0.289	
	NVFi(Li et al., 2023a)	29.027	0.970	0.039	27.594	0.972	0.036	<u>30.675</u>	0.877	0.211	29.745	0.876	0.204	
279	DefGS(Yang et al., 2024)	<u>37.865</u>	0. <u>994</u>	0.007	19.849	0.949	0.045	29.926	<u>0.916</u>	<u>0.130</u>	21.380	0.819	0.188	
280	$DefGS_{nvfi}$	37.316	0.994	0.008	28.749	0.984	0.013	30.170	0.915	0.133	31.096	0.945	0.077	
	E-D3DGS(Bae et al., 2024)	28.075	<mark>0.963</mark>	<mark>0.049</mark>	18.526	<mark>0.923</mark>	<mark>0.087</mark>	<mark>29.267</mark>	<mark>0.874</mark>	0.222	<mark>20.374</mark>	0.772	<mark>0.307</mark>	
281	4DGS(Wu et al., 2024)	<mark>37.285</mark>	<mark>0.986</mark>	<mark>0.020</mark>	<mark>20.354</mark>	<mark>0.950</mark>	<mark>0.052</mark>	<mark>29.381</mark>	<mark>0.889</mark>	<mark>0.212</mark>	<mark>21.107</mark>	<mark>0.793</mark>	<mark>0.274</mark>	
282	GVFi <sub>4dgs</sub> (Ours)	35.961	<mark>0.985</mark>	0.021	28.316	<mark>0.978</mark>	0.023	<mark>27.932</mark>	<mark>0.860</mark>	0.252	<u>31.590</u>	<mark>0.909</mark>	0.194	
	GVFi (Ours)	38,788	0.995	0.006	28,758	0.982	0.011	32.202	0.928	0.089	34.556	0.964	0.046	

Table 2: Quantitative results of all methods for future frame extrapolation optionally with novel view interpolation on NVIDIA Dynamic Scene Dataset, Dynamic Multipart Dataset, and GoPro Dataset.

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		NVIDI.	A Dynam	ic Scene	Dataset			Dyn	amic Mu	ltipart Da	ataset		Go	Pro Data	aset
	In	terpolati	ion	Ex	trapolat	ion	In	terpolati	on	Ex	trapolat	ion	E)	trapolat	ion
	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	<mark>PSNR↑</mark>	<mark>SSIM↑</mark>	LPIPS↓
T-NeRF(Pumarola et al., 2021)	23.078	0.684	0.355	21.120	0.707	0.358	9.833	0.567	0.550	10.064	0.576	0.537	-	-	-
D-NeRF(Pumarola et al., 2021)	22.827	0.711	0.309	20.633	0.709	0.327	13.279	0.747	0.378	13.344	0.767	0.340	-	-	-
TiNeuVox(Fang et al., 2022)	28.304	0.868	0.216	24.556	0.863	0.215	29.957	0.966	0.067	20.804	0.923	0.090	20.323	0.738	0.318
T-NeRF <sub>PINN</sub>	18.443	0.597	0.439	17.975	0.605	0.428	-	-	-	-	-	-	-	-	-
HexPlane <sub>PINN</sub>	24.971	0.818	0.281	24.473	0.818	0.279	-	-	-	-	-	-	-	-	-
NVFi(Li et al., 2023a)	27.138	0.844	0.231	28.462	0.876	0.214	27.516	0.960	0.052	25.235	0.955	0.046	19.879	0.736	0.415
DefGS(Yang et al., 2024)	26.662	0.893	0.127	24.240	0.895	0.140	34.635	0.990	0.019	20.664	0.930	0.067	<mark>21.193</mark>	0.842	0.185
DefGS <sub>nvfi</sub>	26.972	0.890	0.128	27.529	0.927	0.102	34.637	0.990	0.018	28.455	0.979	0.017	25.469	0.882	<u>0.141</u>
E-D3DGS(Bae et al., 2024)	20.848	0.541	0.532	20.301	0.565	0.522	26.180	0.955	0.062	18.615	0.904	0.114	-	-	-
4DGS(Wu et al., 2024)	<mark>19.411</mark>	<mark>0.462</mark>	<mark>0.532</mark>	<mark>22.510</mark>	<mark>0.703</mark>	<mark>0.408</mark>	<mark>37.021</mark>	<mark>0.992</mark>	<u>0.014</u>	<mark>20.564</mark>	<mark>0.935</mark>	<mark>0.067</mark>	-	-	-
GVFi <sub>4dqs</sub> (Ours)	18.995	<mark>0.448</mark>	<mark>0.544</mark>	22.706	0.714	<mark>0.400</mark>	<u>36.542</u>	0.991	0.015	<b>30.801</b>	<mark>0.983</mark>	<u>0.016</u>	-	-	-
GVFi (Ours)	26.943	<u>0.891</u>	0.102	29.388	0.938	0.067	34.807	<u>0.991</u>	0.011	30.721	0.986	0.012	<mark>26.276</mark>	<mark>0.890</mark>	0.131

Then, we compute the kernel's position translation  $\Delta x$  from  $x_{t'}$  to  $x_t$ , which consists of two parts: 1) the translation of its rotation center, and 2) the displacement caused by the kernel's rotation with regard to its center. In particular, they are:

$$\Delta \boldsymbol{x} = \left[\boldsymbol{v}_{c}^{t'}(t-t') + \frac{1}{2}\boldsymbol{a}_{c}^{t'}(t-t')^{2}\right] + \left[(\Delta \mathbf{R} - \boldsymbol{I})(\boldsymbol{x}_{t'} - \boldsymbol{P}_{c}^{t'})\right]$$
(7)

where  $\Delta \mathbf{R}$  is a 3 × 3 rotation matrix converted from quaternion  $\Delta \mathbf{r}$ . Since the rotation change  $\Delta \mathbf{r}$  will update both the orientation and position of a Gaussian kernel, so it is also used in Equation 7.

With the above relationships, we optimize all learnable parameters of canonical Gaussian kernels  $G_0$ , the deformation field  $f_{defo}$  and our translation rotation dynamics module  $f_{trd}$  as follows:

- Step #1: We sample two close timestamps t' and t, where t is a timestamp appears in training dataset and t' can be greater or smaller than t, but  $\Delta t = |t t'|$  is appropriately small.
- Step #2: We get  $\{(\boldsymbol{P}_{c}^{t'}, \boldsymbol{v}_{c}^{t'}, \boldsymbol{a}_{c}^{t'}, \boldsymbol{w}_{p}^{t'}, \epsilon_{p}^{t'}), (\boldsymbol{P}_{c}, \boldsymbol{v}_{c}, \boldsymbol{a}_{c}, \boldsymbol{w}_{p}, \epsilon_{p})\} \leftarrow f_{trd}(\boldsymbol{P})$ , and then calculate  $\Delta \boldsymbol{x}$  and  $\Delta \boldsymbol{r}$  based on  $(\boldsymbol{x}_{t'}, \boldsymbol{r}_{t'}, \boldsymbol{\sigma}_{c}, \boldsymbol{c}) \leftarrow f_{defo}(\boldsymbol{x}_{0}, t')$ , according to Equations 6&7.
- Step #3: We obtain the kernel information at time t:  $(x_{t'} + \Delta x, r_{t'} \circ \Delta r, s_{t'}, \sigma, c)$  which is transported by our physicals parameters from time t', where  $\circ$  represents quaternion multiplication.
- Step #4: Lastly, we render all the above kernels at time t to 2D image space following 3DGS, comparing with the training images at time t. All parameters are supervised by  $\ell_1$  and  $\ell_{ssim}$ :

$$(\mathbf{G}_0, f_{defo}, f_{trd}) \longleftarrow (\ell_1 + \ell_{ssim})$$
(8)

# 4 EXPERIMENTS

Datasets: Our method is designed to learn meaningful physical information of 3D dynamic scenes, aiming at accurately predicting future motions, instead of just fitting observed video frames. In this regard, the closest work to us is the recent NVFi (Li et al., 2023a). Following NVFi, we primarily evaluate our method on its three dynamic datasets: 1) Dynamic Object dataset. It consists of 6 dynamic objects. Each object displays a unique motion pattern belonging to either rigid or deformable movement. 2) Dynamic Indoor Scene dataset. It has 4 complex indoor scenes. Each scene has multiple objects undergoing different rigid body motions. 3) NVIDIA Dynamic Scene dataset (Yoon et al., 2020). It consists of two real-world dynamic 3D scenes.

Upon a closer look at the above three datasets, we find that their dynamics captured are relatively simple. In our daily life, the majority of objects and scenes consist of multiple parts undergoing radically different motions over time, showing extremely challenging physical patterns to learn. To further evaluate the effectiveness of our design, we collect a new synthetic dataset, named **4**) **Dynamic Multipart dataset**, and a new real-world dataset by 20 GoPros, named **5**) **GoPro dataset**.

Our new synthetic dataset comprises 4 objects. Each has 2 to 5 distinct motion patterns on different object parts. Following (Li et al., 2023a), for each object, we collect RGBs at 15 different viewing angles over 1 (virtual) second after normalization, where each viewing angle has 60 frames captured. We reserve the first 46 frames at randomly picked 12 viewing angles as the training split, *i.e.*, 552 frames, while leaving the 46 frames at the remaining 3 viewing angles for testing *interpolation* ability, *i.e.*, 138 frames for novel view synthesis within the training time period, and keeping the last 14 frames at all 15 viewing angles for evaluating future frame *extrapolation*, *i.e.*, 210 frames.

Our new real-world dataset captures 4 dynamic scenes with 20 GoPro cameras. For each dynamic scene, we select 89 frames from each view, and resize images to be a resolution of  $960 \times 540$ . We reserve the first 67 frames at 17 picked viewing angles as the training split, *i.e.*, 1139 frames, while leaving the 67 frames at the remaining 3 viewing angles for evaluating *novel view interpolation* within the training time period, *i.e.*, 201 frames. We keep the last 22 frames at all 20 viewing angles for evaluating *future frame extrapolation*, *i.e.*, 440 frames in total. More details are in Appendix A.6.

- 343 Baselines: We select the following baselines: 1) NVFi (Li et al., 2023a): This is the closest work 344 to us, but differs from us in two folds. First, NVFi relies on PINN losses to learn physics priors, but we directly learn physical parameters. Second, NVFi adopts NeRF as a backbone, being short in 3D 345 scene geometry and appearance modeling, but our method is amenable to and adopts the powerful 346 3DGS in nature. 2) T-NeRF (Pumarola et al., 2021). 3) D-NeRF (Pumarola et al., 2021). 4) NSFF 347 (Li et al., 2021). 5) TiNeuVox (Fang et al., 2022). The latter four methods are based on NeRF and 348 designed for novel view interpolation. Therefore they are expected to be rather weak for future frame 349 extrapolation. For a fair and extensive comparison, we also include the following two baselines. 6) 350 DefGS (Yang et al., 2024), 7) 4DGS (Wu et al., 2024), and 8) E-D3DGS (Bae et al., 2024). These 351 very recent deformable 3D Gaussians methods are particularly strong to model dynamic 3D scenes 352 for novel view synthesis using 3DGS as a backbone. 9) **DefGS**<sub>nvfi</sub>. We build this baseline by 353 combining DefGS with the velocity field proposed by NVFi. This baseline has the powerful 3DGS 354 as a backbone as well as the current state-of-the-art NVFi learning strategy. It is trained with exactly the same settings as our method. To demonstrate the flexibility of our framework, we also adopt 355 4DGS as our auxiliary deformation field, denoted as GVFi<sub>4das</sub>. 356
- Metrics: The standard metrics PSNR, SSIM, and LPIPS are reported for RGB view synthesis in two tasks: interpolation and future frame extrapolation.
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# 4.1 MAIN RESULTS OF FUTURE FRAME EXTRAPOLATION

All methods are trained in a scene-specific fashion. Since NSFF (Li et al., 2021) is not suitable for white-background images, it is not compared on our new Dynamic Multipart dataset. When training our method, we set  $\Delta t$  to be 2 divided by the training set frame rate. The time t' for our auxiliary deformation field  $f_{defo}$  is set to be 0.7 in the extrapolation task, and target time t is chosen as the frame time. The time difference  $\Delta t$  is dynamically computed.

Our primary goal is to extrapolate meaningful future frames as a continuum of the last training observations. In our evaluation, we follow Steps  $\#1 \sim \#4$  in Section 3.3 to extrapolate future frames from the last timestamp of training frames. For benchmarking, we also compare novel view (past frame) interpolation with baselines, but this is less important to us. Particularly, we also follow Steps  $\#1 \sim \#4$ to render past frames. Though this can also be achieved by progressively querying our translation rotation dynamics module  $f_{trd}$ , it is inferior due to accumulated errors as detailed in Appendix A.9.

**Results & Analysis:** Tables 1&2 compare all methods on the five datasets. It can be seen that:

Compared with NeRF and 3DGS based dynamic scene modeling methods such as T-NeRF/ D-NeRF/ TiNeuVox/ NSFF/ DefGS/ 4DGS/ E-D3DGS, both versions of our method achieve about 10 points higher on PSNR for future frame extrapolation. This means that, without explicitly

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383	Dynamic Object Dataset Dynamic Multipart Dataset Dynamic Indoor Scene Dataset
384	Figure 3: Qualitative results of clustering translation rotation physics parameters. Gaussian kernels
385	can be autonomously grouped into meaningful objects or parts according to their motion patterns.
386	learning physical information like us, these dynamic methods completely fail to predict the future,
387	highlighting the core value of our method.
388	• Compared with the closest and also strongest baselines NVFi/DefGS <sub><math>nvfi</math></sub> , our method is still con-
389	stantly better than them on all datasets for future frame extrapolation. Notably, on the Dynamic
390	Indoor Scene dataset and our newly collected Dynamic Multipart dataset, there are much more
391	complex motion dynamics such as different objects or parts moving in distinct directions, but our
392	best results are constantly about 3 points higher on PSNR than them. Fundamentally, this is be-
393	cause both NVFi and DefGS <sub>nv fi</sub> rely on PINN losses as soft constraints to incorporate physics
394	priors, whereas we directly integrate hard physics by learning translation rotation dynamics system
395	parameters, thus being more effective in learning dynamics.
396	<ul> <li>Lestly, our framework is indeed amonable to avisting deformation fields such as DefGS and 4DGS.</li> </ul>
397	• Lastly, our mane work is indeed amenable to existing deformation needs such as Deros and 4DOS,
398	Figure 4 shows qualitative results. More results of the total <b>5</b> detects are in Appendix A 10 / A 11 /
399	Figure 4 shows qualitative results. While results of the total $\frac{5}{2}$ datasets are in Appendix A.107 A.117
400	A.127 A.15/A.14. we also report the training/test time, memory cost, etc., in Appendix A.4.
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# 4.2 ANALYSIS OF DYNAMICS PARAMETERS

403 Our core translation rotation dynam-404 ics system module is designed to learn 405 per rigid particle's physical parameters. 406 Ideally, for those rigid particles under-407 going the same motion pattern such as 408 all surface points of a single rigid part, they should have the same or similar 409 physical parameters. Given this, multi-410

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Table 3:	Quantitative results of motion segmentation re-
sults on I	Dynamic Indoor Scene dataset.

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	AP↑	PQ↑	F1↑	Pre↑	Rec↑	mIoU↑
M2F(Cheng et al., 2022)	65.37	73.14	78.29	<u>94.83</u>	68.88	64.42
D-NeRF(Pumarola et al., 2021)	57.26	46.15	59.02	56.55	62.94	46.58
NVFi(Li et al., 2023a)	91.21	<u>78.74</u>	<u>93.75</u>	93.76	<u>93.74</u>	<u>67.64</u>
DefGS(Yang et al., 2024)	51.73	57.60	66.43	63.21	70.07	54.46
$DefGS_{nvfi}$	55.26	62.75	69.83	69.39	72.91	56.82
GVFi (Ours)	95.82	93.28	97.90	96.21	99.86	79.55

ple dynamic objects or parts with distinct motions can be automatically segmented based on the similarity of learned physical parameters. By comparison, the prior work NVFi (Li et al., 2023a) can hardly achieve this autonomous dynamic segmentation by its own design, unless an external motion grouping method is applied. To further evaluate this nice property of our method, we conduct the following steps to analyze the learned dynamics parameters.

First, after training our method on the Dynamic Object dataset, Dynamic Multipart dataset, and
Dynamic Indoor Scene dataset, for each dynamic scene, we have a set of well-trained canonical 3D
Gaussians, an auxiliary deformation field, and our translation rotation dynamics parameters.

Then, we use the auxiliary deformation field  $f_{defo}$  to deform the canonical 3D Gaussians  $G_0$  to time t = 0.7, which is the maximum time the deformation field can query in our training. At this timestamp, the motions of different objects and parts normally achieve a steady state.

Lastly, we query all the physical parameters at this time, *i.e.*,  $\{(P_c^t, v_c^t, a_c^t), (w_p^t, \epsilon_p^t)\}$ . We choose ( $\|v_c^t\|, v_c^t/\|v_c^t\|, \|w_p^t\|, w_p^t/\|w_p^t\|$ ) as the features to cluster Gaussian particles via a simple K-means algorithm. As shown in Figure 3, all Gaussian particles can be grouped into physically meaningful objects or parts according to their actual motion patterns. More results are in Appendix A.17.

We further quantitatively evaluate our motion grouping results on Dynamic Indoor Scene Dataset. In particular, we follow Gaussian Grouping (Ye et al., 2024) to render 2D object segmentation masks for all 30 views over 60 timestamps on all 4 scenes, *i.e.*, 7200 images in total. We compare with **D-NeRF**, **NVFi**, **DefGS** and **DefGS**<sub>nvfi</sub>. We follow NVFi to obtain segmentation results of D-NeRF and NVFi. For the 3DGS-based baselines, we also adopt OGC (Song & Yang, 2022) to segment Gaussians based on scene flows induced from their learned deformation fields. All implementation details are in Appendix. Additionally, we include a strong image-based 2D object seg mentation method, Mask2Former (Cheng et al., 2022) pre-trained by human annotations on COCO
 dataset (Lin et al., 2014) as a fully-supervised baseline.

As shown in Table 3, our method achieves almost perfect object segmentation results on all metrics,
 significantly outperforming all baselines. This shows that our learned physical parameters correctly
 model object physical motion patterns and can be easily leveraged to identify individual objects
 according to their motions, without needing any human annotations.

441 4.3 ABLATION STUDY

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Our framework mainly comprises 3DGS as the backbone and our core translation rotation dynamics
system module, together with an auxiliary deformation field. To verify different choices of our
method, we conduct the following three groups of ablation experiments.

(1) Different choices of time difference  $\Delta t$  in training stage: Given the time interval between two consecutive frames in the training set as  $\delta t$ , we compare three choices of the time difference  $\Delta t$  in training stage: { $\delta t$ ,  $2\delta t$ ,  $3\delta t$ }. We choose  $\Delta t = 2\delta t$  in our main experiments.

(2) Removing the auxiliary deformation field  $f_{defo}$ : In particular, we feed the Gaussian particles at time t' = 0 directly into  $f_{trd}$ , and use the output physical parameters to directly move particles to a target future timestamp t. Note that, here  $\Delta t$  is meaningless.

(3) Learning time-dependent physical parameters at t': Instead of using Equation 4 to derive physical parameters at time t', we directly learn them by a 6-layer MLPs as:  $f_{trd'}(P, t')$ . Theoretically, such a complex function can learn higher order relationships to approximate arbitrary motions than our second-order Equation 4. Nevertheless, it would be more challenging to learn and unable to guarantee physical parameters of the same motion pattern to be consistent over time.

**Results & Analysis:** Table 4 shows all ablation results 458 for future frame extrapolation on our new Dynamic Mul-459 tipart dataset. It can be seen that: 1) The greatest im-460 pact is caused by the removal of the deformation field 461  $f_{defo}$ . Although this deformation field itself is unable to 462 learn physics, it significantly aids our core translation ro-463 tation dynamics system module to learn physical parame-464 ters given the motion information. 2) The choice of time 465 difference  $\Delta t$  is also important. Once it is as small as the 466 interval between two consecutive frames, the performance

Table 4: Quantitative results of ablationstudies on Dynamic Multipart dataset.

			Extrapolation				
	$f_{defo}$	$f_{trd}$	PSNR↑	SSIM↑	LPIPS↓		
(1) $\delta t$	1	1	29.441	0.984	0.013		
(1) $2\delta t$	1	1	30.721	0.986	0.012		
(1) $3\delta t$	1	1	30.246	0.985	0.012		
(2) -	X	1	27.081	0.981	0.018		
$(3) 2\delta t$	1	×	29.986	0.985	0.012		

drops apparently, because the motion in short intervals is too subtle to be distinguished. For example, a rotation may be learned as a translation. However, if  $\Delta t$  is too large, the appearance fitting could be sacrificed, so the performance is slightly weaker. 3) If physical parameters are learned but not derived, the lack of physics consistency will influence motion learning.

Detailed ablation settings and results are in Appendix A.7. More ablations of  $\Delta t$  on four datasets, and more ablations of using 1st-/2nd-order relationships in Equation 4 are in Appendix A.8.

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# 5 CONCLUSION

476 In this paper, we have demonstrated that complex motion dynamics can be explicitly learned just 477 from multi-view RGB videos without needing additional human labels such as object types and 478 masks. This is achieved by a new generic framework that simultaneously models 3D scene geome-479 try, appearance and physics by extending the appealing 3D Gaussian Splatting technique. In contrast 480 to existing works which usually rely on PINN losses as soft constraints to learn physics priors, we 481 instead directly learn a complete set of physical parameters to govern the motion pattern of each 3D 482 rigid particle in space via our core translation rotation dynamics system module. Extensive experi-483 ments on three public dynamic datasets and a newly created dynamic multipart dataset have shown the extraordinary performance of our method in the challenging task of future frame extrapolation 484 over all baselines. In addition, the learned physical parameters can be directly used to segment 485 objects or parts according to the similarity of parameters.



Figure 4: Qualitative results of RGB view synthesis for interpolation and extrapolation tasks.

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#### 702 APPENDIX А 703

#### 704 The appendix includes: 705

705	
706	• Rendering equation and preliminary for vanilla 3DGS.
707	• Implementation details of Auxiliary Deformation Field.
708	• Implementation datails of translation and ratation system
709	• Implementation details of translation and rotation system.
710	<ul> <li>Training and evaluation resources for the model.</li> </ul>
711	• Additional incremental learning experiments for self-propelled objects.
712	Additional details of datasets.
713	
714	• Additional quantitative results for ablation study in the main context.
715	• Additional ablation studies.
716	• Analysis of different ways for interpolation period
717	Analysis of different ways for interpolation period.
718	<ul> <li>Additional quantitative results for GoPro dataset.</li> </ul>
719	• Additional quantitative & qualitative results for future frame extrapolation.
720	• Additional qualitative results for motion segmentation.
721	• Additional qualitative results for extrapolation beyond dataset time span.
723	
724	A.1 PRELIMINARY FOR VANILLA 3DGS
725	
726	3D Gaussian Splatting (Kerbl et al., 2023) represents a 3D scene by a set of colored 3D Gaussian
727	kernels. Specifically, each Gaussian kernel is parameterized by a 3D position $P \in \mathcal{R}^3$ , an orienta-
700	tion represented by a quaternion $r$ , and a scaling $s$ . By transforming the orientation $r$ and scaling
120	s into the rotation matrix R and scaling matrix S, a 3D covariance matrix $\Sigma$ can be composed as
729	$\Sigma = \mathbf{RSS}^T \mathbf{R}^T$ . Then the Gaussian kernel can be evaluated at any location $\mathbf{x} \in \mathcal{R}^3$ in the 3D space:
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$$G(\boldsymbol{x}) = e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{P})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{P})}.$$
(9)

732 Besides, each Gaussian kernel has an opacity  $\sigma$  indicating its influence in rendering, and a color c 733 computed from spherical harmonics (SH) for view-dependent appearance. 734

The rendering of Gaussian kernels on the image consists of two steps. Firstly, The Gaussian kernels 735 are projected onto the image plane, following the differentiable rasterization pipeline proposed in 736 (Zwicker et al., 2001). The 3D position P and covariance matrix  $\Sigma$  of each Gaussian kernel are 737 projected into 2D position P' = JWP and covariance matrix  $\Sigma' = JW\Sigma W^T J^T$  respectively, 738 where J denotes the Jacobian of the approximated projective transformation and W denotes the 739 transformation from the world to camera coordinates. Secondly, the color of a pixel  $\mu$  on the image 740 can be rendered by  $\alpha$ -blending as follows: 741

$$\boldsymbol{C}(\boldsymbol{\mu}) = \sum_{i} T_{i} \alpha_{i} \boldsymbol{c}_{i}, \quad T_{i} = \prod_{j=1}^{i-1} (1 - \alpha_{j}), \quad (10)$$

745 where  $\alpha_i$  is obtained by evaluating the projection of the Gaussian kernel  $G_i$  on the pixel  $\mu$ , *i.e.*, 746  $\alpha_i = \sigma_i e^{-\frac{1}{2}(\boldsymbol{\mu} - \boldsymbol{P'})^T \Sigma'^{-1}(\boldsymbol{\mu} - \boldsymbol{P'})}$ . By adjusting the parameters of Gaussian kernels mentioned above 747 and adaptively controlling the Gaussian density, a high-fidelity representation of a 3D scene can be 748 obtained from multi-view images. We refer readers to (Kerbl et al., 2023) for more details. 749

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# A.2 IMPLEMENTATION DETAILS OF AUXILIARY DEFORMATION FIELD

752 We leverage an existing deformation field introduced in (Yang et al., 2024) as our auxiliary defor-753 mation field. In particular, the 3D position  $x_0$  of each canonical Gaussian kernel and the current timestamp t are fed into an MLP-based deformation network, denoted as  $f_{defo}$ . The implementa-754 tion of this MLP is directly adapted from (Yang et al., 2024), *i.e.*, an MLP with 8 layers in total 755 and 256 hidden sizes for each layer, plus a ResNet layer at layer 4. At the input layer, an 8-degree

756 positional embedding is applied onto the 3D position  $x_0$  and a 5-degree positional embedding onto 757 time t. 758

Mathematically, the deformation field  $f_{defo}(\boldsymbol{x},t): \mathcal{R}^4 \to \mathcal{R}^{10}$  is defined as 759 760 761

$$\delta \boldsymbol{x}, \delta \boldsymbol{r}, \delta \boldsymbol{s}) = f_{defo}(\boldsymbol{x}_0, t), \tag{11}$$

where  $\delta x$  represents the translation of the center of Gaussian kernel,  $\delta r$  represents the rotation for the pose of Gaussian kernel in quaternion representation, and  $\delta s$  is the difference of Gaussian sizes. 764 Note a Gaussian size vector is parametrized by  $z = \log(s)$ , so the difference can be defined as 765  $\delta \boldsymbol{s} = \exp(\delta \boldsymbol{z}).$ 766

767 After applying the deformation field onto Gaussian kernels, we can deform the Gaussian kernels from canonical time to time t as: 768

$$\boldsymbol{x}_t = \boldsymbol{x}_0 + \delta \boldsymbol{x} \tag{12}$$

$$\boldsymbol{r}_t = \delta \boldsymbol{r} \circ \boldsymbol{r}_0 \tag{13}$$

$$\boldsymbol{s}_t = \exp(\boldsymbol{z}_0 + \delta \boldsymbol{z}) = \boldsymbol{s}_0 \odot \delta \boldsymbol{s}, \tag{14}$$

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where  $\circ$  is quaternion multiplication and  $\odot$  is element-wise multiplication.

# A.3 IMPLEMENTATION DETAILS OF TRANSLATION AND ROTATION SYSTEM

779 As discussed in the main context, we learn the following two groups of physical parameters for each 3D particle P as function  $f_{trd}(x) : \mathcal{R}^3 \to \mathcal{R}^{13}$ : 780

$$\{(\boldsymbol{P}_c, \boldsymbol{v}_c, \boldsymbol{a}_c), (\boldsymbol{w}_p, \boldsymbol{\epsilon}_p)\} = f_{trd}(\boldsymbol{P}), \tag{15}$$

783 where angular velocity is defined as  $\omega = \| \boldsymbol{w}_p \|_2$  around rotation axes direction  $\hat{\boldsymbol{k}} = \boldsymbol{w}_c / \omega$  following 784 right-hand rule.

785 This module is implemented by a simple  $6 \times 128$  MLPs with 6 layers in total and 128 hidden sizes 786 for each layer. In addition, a 5-degree positional embedding is applied onto the input 3D position x, 787 and relu is chosen as the activation functions. 788

#### A.4 TRAINING AND EVALUATION RESOURCES FOR THE MODEL

As the complexity of different scenes varies, the total number of Gaussians learned for each scene varies from 40k to 1.6M. In general, our training time is 1.05 times longer than DefGS (or 4DGS 793 if built on it). For example, on the bat of Dynamic Object Dataset, DefGS/4DGS need 25 minutes, 794 while we need 27 minutes, with a slight training cost addition. Since our additional module is a tiny MLPs, we only need 367.4kB larger storage. Our rendering speed is 0.85 times slower than DefGS (or 0.8 times slower than 4DGS if built on it). For example, on the *bat* of Dynamic Object Dataset, they achieve 40 fps and ours 32 fps. We train all our models on a single NVIDIA 3090 24G GPU.

A.5 INCREMENTAL LEARNING FOR SELF-PROPELLED OBJECTS

801 We include an additional incremental learning experiment to show that our framework can easily 802 adapt to new observations when internal forces change for self-propelled objects. We choose three 803 self-propelled objects from the Dynamic Object Dataset for this experiment. We keep the same 804 viewing angles in training and testing split, and incrementally train the network.

To be specific, we first feed time  $t = 0 \sim 0.15$  to train the network, and evaluate novel view 806 interpolation on  $t = 0 \sim 0.15$ , future frame extrapolation on  $t = 0.15 \sim 0.30$ . Next, we include 807  $t = 0.15 \sim 0.30$  to train, and evaluate novel view interpolation on  $t = 0 \sim 0.30$ , future frame 808 extrapolation on  $t = 0.30 \sim 0.45$ . We keep adding a time interval of 0.15 till we train from 809  $t = 0 \sim 0.75$ , and extrapolate from  $t = 0.75 \sim 0.9$ .

810 We compare our performance with DefGS. Table 5 shows quantitative results. It can be seen that 811 DefGS suffers from overfitting the previous timestamps and results in a decrease in its interpolation 812 ability, while our model can stably adapt to new observations and achieve excellent past and future 813 frame predictions. This means that even though the internal forces are changing for self-propelled 814 objects, our model can easily adapt to new observations. 815

Table 5: Quantitative results (PSINR) of incremental learning.								
Interpolation	$0.15 \rightarrow 0.30$	$0.30 \rightarrow 0.45$	$0.45 \rightarrow 0.60$	$0.60 \rightarrow 0.75$	$0.75 \rightarrow 0.90$	Average		
DefGS(Yang et al., 2024)	39.386	38.745	35.818	34.531	27.904	35.277		
GVFi (Ours)	40.032	40.706	41.013	40.466	39.971	40.438		
Extrapolation	0.15  ightarrow 0.30	$0.30 \rightarrow 0.45$	$0.45 \rightarrow 0.60$	0.60  ightarrow 0.75	$0.75 \rightarrow 0.90$	Average		
DefGS(Yang et al., 2024)	23.438	21.360	19.989	19.670	17.629	20.417		
GVFi (Ours)	29.958	32.260	31.384	29.527	28.958	30.417		

### A.6 ADDITIONAL DETAILS OF NEW DATASETS

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**Dynamic Multipart dataset:** This dataset comprises 4 distinct objects <sup>1</sup>, including a variety of challenging motions. Details of the 4 dynamic objects are:

- Foldingchair: A folingchair is given. This chair is composed of three parts. The whole motion is unfolding this chair, so all three parts are undergoing different rotating motions.
- Hypoerbolic Slot: This is an extremely hard case, where a stick is rotating through a hypoerbolic slot. Note that, only the stick in this hyperbolic shape is dynamic, this introduces more challenges in motion extrapolation.
- Satellite: This object is a satellite with two wing doors opening and one main door opening, all rotating in different directions.
- **Stove:** A home stove is given. The motion is mainly closing its top cover plate.

# **GoPro Dataset**: This dataset includes 4 challenging real-world dynamic scenes.

- Scene #1: Box. This scene contains a drawer-like box, and a person is trying to close it. The difficulty lies in a tight combination of the moving part and the static part of the box, especially in the future.
- Scene #2: Hammer. This scene contains a hammer moving on the topside of a box. The difficulty lies in the direct contact of moving objects and static objects, which requires sharp separation of diverse motion patterns in order to keep the right static/moving states in the future.
  - Scene #3: Collision. This scene contains a cube and a cup moving towards each other. The difficulty is the different directions of two motions. It is hard to keep the shapes of these two objects in the future.
    - Scene #4: Wrist Rest. A person is trying to bend a wrist rest. The difficulty is that the object is deformable and the motion is thus not rigid or part-wise rigid.
- A.7 ADDITIONAL QUANTITATIVE RESULTS FOR ABLATION STUDY FOR THE MAIN CONTEXT

854 We first elaborate how we implement the ablation study (3). Our original design of the translation 855 and rotation dynamics system module  $f_{trd}(x)$  is only relevant to space, but not time. To obtain the 856 corresponding physics parameters at time t, we use Equation 4 to derive the queried physics parameters. In our ablation study (3), we aim to keep the physics parameters changing over time, thus 858 making it more complex in theory. Particularly, we use the same network architecture of  $f_{trd}$ , except 859 changing the input from  $f_{trd}(x)$  to  $f_{trd}(x,t)$ , to force the change of physics parameters. 860

Here we show the total results for the ablation study in Table 6, both for interpolation and extrapo-861 lation. 862

<sup>&</sup>lt;sup>1</sup>All objects are purchased from SketchFab, licensed under the SketchFab Standard License: https://sketchfab.com/licenses, and are all allowed for AI generation model usage

			Interpolation			Extrapolation			
	$f_{defo}$	$f_{trd}$	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	
(1) $\delta t$	1	1	35.128	0.991	0.011	29.441	0.984	0.013	
(1) $2\delta t$	1	1	34.807	0.991	0.011	30.721	0.986	0.012	
(1) $3\delta t$	1	1	35.223	0.991	0.011	30.246	0.985	0.012	
(2) -	X	1	32.266	0.987	0.016	27.081	0.981	0.018	
(3) $2\delta t$	1	X	35.225	0.991	0.011	29.986	0.985	0.012	

Table 6: Quantitative results of ablation studies on Dynamic Multipart dataset.

# A.8 ADDITIONAL ABLATION STUDIES

Since the range of motion between two consecutive frames is different across different datasets, we conduct extensive ablations about different choices of  $\Delta t$  on all 4 datasets, and the results are listed in Table 7. We observe that  $3\delta t$  works better in extrapolation on three datasets (Dynamic Object/ Dynamic Indoor Scene/ NVIDIA Dynamic Scenes). The basic rule to select an appropriate  $\delta t$  is based on the motion range. If the motion changes fast, so the motion between two consecutive frames is apparent enough, then a smaller  $\delta t$  is good enough. Otherwise, if the motion is rather slow, then a larger  $\delta t$  is preferred.

Table 7: Quantitative results of ablation studies for  $\delta t$  on all four datasets.

set trapolation						
trapolation						
SCIM+ I PIPC						
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0.981 0.013						
0.982 0.011						
<b>0.983</b> 0.012						
NVIDIA Dynamic Scenes Dataset						
Interpolation Extrapolation						
SSIM↑ LPIPS↓						
0.934 0.070						
0.938 0.067						
0.938 0.067						

The updating scheme of our translation rotation dynamics system is chosen as a second-order relationship in Equation 4, *i.e.*, each rigid particle can have a constant acceleration. We also evaluate the third-order scheme (acceleration of acceleration) and first-order scheme (no acceleration) on Dynamic Multipart Dataset and Dynamic Object Dataset. Table 8 shows the results. We can see that, in Dynamic Object Dataset which has several self-propelled objects whose internal forces tend to change over time, not surprisingly, the third-order variant performs better. Nevertheless, due to the inherent over-parametrization, the third-order scheme tends to learn excessive rotation information to represent simple acceleration motions, thus incurring inferior performance on the Dynamic Multipart Dataset which does not have self-propelled objects.

Table 8: Quantitative results of ablation studies about 3 orders of Taylor expansion in Equation 4 on Dynamic Multipart Dataset and Dynamic Object Dataset.

		Dyn	amic Mu	ltipart Da	taset		Dynamic Object Dataset								
	Interpolation			Extrapolation			Interpolation			Extrapolation					
	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓			
1 <sup>st</sup> -order	34.776	0.990	0.013	26.729	0.976	0.018	38.892	0.995	0.005	28.536	0.983	0.012			
2 <sup>nd</sup> -order	34.807	0.991	0.011	30.721	0.986	0.012	38.788	0.995	0.006	28.758	0.982	0.011			
$3^{rd}$ -order	35.268	0.991	0.012	30.503	0.985	0.013	39.164	0.995	0.005	29.378	0.983	0.011			

### 918 A.9 ANALYSIS OF DIFFERENT WAYS FOR INTERPOLATION PERIOD

Our method has three possible strategies for interpolation rendering: (1) directly using  $f_{defo}$  to pre-dict the deformation at the given time  $t_{1}$  (2) progressively calculating the Gaussian deformation at the given time t from time 0 using the motion parameters predicted by  $f_{trd}$ , or (3) following the steps described in Section 3.3. We choose the third one in our main experiments. Here, we evaluate the other two strategies in Table 9. We can see that, the first and the third strategies are not strictly consistent, but achieve very similar performance. However, for the second strategy, the performance clearly decreases, we hypothesize that this is due to the accumulated errors in the autoregressive process.

Table 9: Quar	ntitative results of dif	ferent interpolation st	rategies of our metho	d on all four datasets.		
	Multipart	Object	Indoor	NVIDIA		
	PSNR↑ SSIM↑ LPIPS↓	PSNR↑ SSIM↑ LPIPS↓	PSNR↑ SSIM↑ LPIPS↓	PSNR↑ SSIM↑ LPIPS↓		
(1) C	35 0 40 0 001 0 011	20 407 0.005 0.005	22 50 0 020 0 000	AC 051 0.001 0.103		

		winipart			Object			muoor		INVIDIA		
	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓
$(1)f_{defo}$	35.040	0.991	0.011	38.406	0.995	0.005	32.569	0.930	0.088	26.951	0.891	0.102
$(2)f_{trd}$	30.310	0.984	0.017	33.527	0.991	0.009	31.776	0.926	0.092	25.899	0.875	0.118
$(3)f_{defo} + f_{trd}$	34.807	0.991	0.011	38.788	0.995	0.006	32.202	0.928	0.089	26.943	0.891	0.102

# A.10 ADDITIONAL QUANTITATIVE RESULTS ON DYNAMIC OBJECT DATASET

Here we show the total per-scene results on Dynamic Object Dataset in Table 10 and qualitative results in Figures 8, 9&10, both for interpolation and extrapolation.

# A.11 ADDITIONAL QUANTITATIVE RESULTS ON DYNAMIC INDOOR SCENE DATASET

Here we show the total per-scene results in Dynamic Indoor Scene Datasets in Table 11 and qualitative results in Figures 10&11, both for interpolation and extrapolation.

# A.12 ADDITIONAL QUANTITATIVE RESULTS ON NVIDIA DYNAMIC SCENE DATASET

Here we show the total per-scene results on NVIDIA Dynamic Scene Dataset in Table 12 and qualitative results in Figure 13, both for interpolation and extrapolation.

# A.13 ADDITIONAL QUANTITATIVE RESULTS ON DYNAMIC MULTIPART DATASET

Here we show the total per-scene results on our Dynamic Multipart Dataset in Table 13 and qualitative results in Figure 12, both for interpolation and extrapolation.

A.14 ADDITIONAL QUANTITATIVE & QUALITATIVE RESULTS FOR GOPRO DATASET

Here we show the total results on our GoPro Dataset in Table 14 and qualitative results in Figures 14,15,&16, both for interpolation and extrapolation.

# A.15 ADDITIONAL QUALITATIVE RESULTS FOR EXTRAPOLATION BEYOND DATASET TIME SPANS

We list some meaningful longer extrapolation results from each dataset here in Figure 21. In our dataset, the training period lasts from t = 0 to t = 0.75 and the extrapolation period lasts from t = 0.75 to t = 1.0. Here we show the qualitative results till t = 1.5, which is already twice the training period. We can see that our method can still obtain physically meaningful future frame prediction in particularly high quality.

969 A.16 Additional Qualitative Results for Object/Part Segmentation

Figures 5, 6 and 7 show more qualitative results for the autonomous object or part segmentation based on the learned physical parameters via the simple K-means clustering algorithm.

		seene	Fallin	ig Ball	iosuit				B	at	. 1.	•
Methods	In DCMD4	terpolati	lon	E: DOMDA	xtrapolat	I DIDC		terpolati	.on	Ex DOMDA	trapolat	10n
T NePE(Pumarola et al. 2021)	PSINK	0.782	0.326	PSINK	0.703	0 308	PSINK	0.836	0.234	13 807	0.834	0.23
D-NeRF(Pumarola et al., 2021)	15 548	0.762	0.320	15 116	0.793	0.308	14 087	0.830	0.234	15.697	0.834	0.22
TiNeuVox(Fang et al., 2022)	35.458	0.974	0.052	20.242	0.959	0.067	16.080	0.908	0.108	16.952	0.930	0.11
T-NeRF <sub>PINN</sub>	17.687	0.775	0.368	17.857	0.829	0.265	16.412	0.903	0.197	18.983	0.930	0.13
HexPlane <sub>PINN</sub>	32.144	0.965	0.065	20.762	0.951	0.081	23.399	0.958	0.057	21.144	0.951	0.06
NVFi(Li et al., 2023a)	35.826	0.978	0.041	31.369	0.978	0.041	23.325	0.964	0.046	25.015	0.968	0.04
DefGS(Yang et al., 2024)	37.535	0.995	0.009	20.442	0.976	0.033	38.750	0.997	0.004	17.063	0.936	0.07
DefGS <sub>NVFi</sub>	38.606	0.996	0.010	24.873	0.985	0.015	38.075	0.997	0.004	28.950	0.980	0.01
GVFi (Ours)	38.071	0.995	0.008	35.949	0.995	0.004	39.626	0.997	0.003	24.352	0.973	0.02
		. 1.	F	an	. 1.		, , , , , , , , , , , , , , , , , , ,	. 1.0	Teles	scope	. 1.	
Methods	In DENID+	serpolati	I DIDC	DCND+	seimt	I DIDC		sermet		EX DCNDA	ctrapolat	10n
T-NeRFPumarola et al. (2021)	8 001	0.308	0.646	8 494	0.302	0 593	13.031	0.615	$\frac{1}{0.472}$	13.802	0.670	0.4
D-NeRFPumarola et al. (2021)	7 915	0.308	0.640	8 624	0.392	0.595	13 295	0.609	0.472	14 967	0.070	0.4
TiNeuVoxFang et al. (2022)	24 088	0.930	0.090	20.932	0.935	0.023	31 666	0.982	0.402	20.456	0.921	0.00
T-NeRF <sub>PINN</sub>	9 233	0.541	0.508	9 828	0.606	0.443	14 293	0.739	0.366	15 752	0.804	0.2
HexPlane	22.822	0.921	0.079	19.724	0.919	0.080	25.381	0.948	0.066	23.165	0.932	0.0
NVFiLi et al. (2023a)	25.213	0.948	0.049	27.172	0.963	0.037	26.487	0.959	0.048	27.101	0.963	0.04
DefGSYang et al. (2024)	35.858	0.985	0.017	20.932	0.948	0.038	37.502	0.996	0.003	20.684	0.927	0.04
$DefGS_{NVFi}$	35.217	0.984	0.019	26.648	0.972	0.023	37.568	0.996	0.003	34.096	0.994	0.0
GVFi (Ours)	35.577	0.986	0.013	29.533	0.979	0.012	40.614	0.998	0.002	29.744	0.983	0.0
			Sh	ark					WI	nale		
Methods	In	terpolati	on	E	xtrapolat	ion	Ir	terpolati	on	Ex	trapolat	ion
	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPI
T-NeRFPumarola et al. (2021)	13.813	0.853	0.223	15.325	0.882	0.193	16.141	0.860	0.212	15.880	0.860	0.2
D-NeRFPumarola et al. (2021)	17.727	0.903	0.150	19.078	0.936	0.092	16.373	0.898	0.154	14.771	0.883	0.1
TiNeuVoxFang et al. (2022)	23.178	0.971	0.059	19.463	0.950	0.050	37.455	0.994	0.016	19.624	0.943	0.0
$T-NeRF_{PINN}$	17.315	0.878	0.177	18.739	0.921	0.115	16.778	0.927	0.141	15.974	0.919	0.1
HexPlane <sub>PINN</sub>	28.874	0.976	0.040	22.330	0.961	0.047	29.634	0.981	0.035	21.391	0.961	0.0
NVFiLi et al. (2023a)	32.072	0.984	0.024	28.874	0.982	0.021	31.240	0.986	0.025	26.032	0.978	0.0
DefGSYang et al. (2024)	37.802	0.994	0.006	19.924	0.957	0.034	39.740	0.997	0.004	20.048	0.951	0.0
DefGS <sub>NVFi</sub>	37.327	0.994	0.006	29.240	0.987	0.007	37.101	0.996	0.005	28.686	0.986	0.0
GVFI (Ours)	40.404	0.997	0.004	20.080	0.979	0.009	30.370	0.997	0.003	20.288	0.982	0.01
Table 11: F	Per-sce	ne qu	antitati	ive res	ults or	ı Dyna	mic In	door S	Scene o	lataset	•	
			Gnome	e House					Chess	sboard		
Methods	In	terpolati	on	E	xtrapolat	ion	Ir	terpolati	on	Ex	ctrapolat	ion
	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPII
		· · · · · ·	0.202	23 485	0.643	0.419	25.517	0.796	0.294	20.228	0.708	0.3
T-NeRF(Pumarola et al., 2021)	26.094	0.716	0.585	23.103	0.015	0.0/7	0.1050			10 155	0.708	0.0
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021)	26.094 27.000	0.716	0.385	21.714	0.641	0.367	24.852	0.774	0.308	19.455	0.675	0.3
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022)	26.094 27.000 30.646	0.716 0.745 0.831	0.319	21.714 21.418	0.641	0.367	24.852 33.001	0.774	0.308	19.455 19.718	0.675	0.3
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> UserPlane	26.094 27.000 30.646 15.008	0.716 0.745 0.831 0.375	0.385 0.319 0.253 0.668	21.714 21.418 16.200	0.641 0.699 0.409	0.367 0.326 0.651	24.852 33.001 16.549	0.774 0.917 0.457	0.308 0.177 0.621	19.455 19.718 17.197 21.518	0.708 0.675 0.765 0.472	0.3 0.3 0.6
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSEE(Li et al. 2021)	26.094 27.000 30.646 15.008 23.764 31.418	0.716 0.745 0.831 0.375 0.658 0.821	0.383 0.319 0.253 0.668 0.510 0.204	21.714 21.418 16.200 22.867 25.802	0.641 0.699 0.409 0.658 0.750	0.367 0.326 0.651 0.510 0.327	24.852 33.001 16.549 24.605 32.514	0.774 0.917 0.457 0.778	0.308 0.177 0.621 0.412 0.201	19.455 19.718 17.197 21.518 21.501	0.768 0.675 0.765 0.472 0.748	0.3 0.3 0.6 0.4
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVE(Li et al., 2022)	26.094 27.000 30.646 15.008 23.764 31.418 30.667	0.716 0.745 0.831 0.375 0.658 0.821 0.824	0.383 0.319 0.253 0.668 0.510 0.294 0.277	21.714 21.418 16.200 22.867 25.892 30.408	0.641 0.699 0.409 0.658 0.750 0.826	0.367 0.326 0.651 0.510 0.327 0.273	24.852 33.001 16.549 24.605 32.514 30.394	0.774 0.917 0.457 0.778 0.810 0.888	0.308 0.177 0.621 0.412 0.201 0.215	19.455 19.718 17.197 21.518 21.501 27.840	0.708 0.675 0.765 0.472 0.748 0.805 0.872	0.3 0.3 0.6 0.4 0.2
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) Def(SC(Yang et al. 2024)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918	0.383 0.319 0.253 0.668 0.510 0.294 0.277 0.132	21.714 21.418 16.200 22.867 25.892 30.408 21.703	0.615 0.641 0.699 0.409 0.658 0.750 0.826 0.775	0.367 0.326 0.651 0.510 0.327 0.273 0.207	24.852 33.001 16.549 24.605 32.514 30.394 27 355	0.774 0.917 0.457 0.778 0.810 0.888 0.912	$\begin{array}{c} 0.308 \\ 0.177 \\ 0.621 \\ 0.412 \\ 0.201 \\ 0.215 \\ 0.147 \end{array}$	19.455 19.718 17.197 21.518 21.501 27.840 20.032	0.708 0.675 0.765 0.472 0.748 0.805 0.872 0.808	0.3 0.3 0.6 0.4 0.2 0.2 0.2
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS(Yang et al., 2024)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b>	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919	0.383 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132	21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730	0.708 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS(V <i>i</i> ang et al., 2024) GVFi (Ours)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b>	0.383 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.101	21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b>	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 <b>0.962</b>	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 <b>0.055</b>	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b>	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 <b>0.960</b>	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 <b>0.060</b>	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b>	0.708 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 <b>0.966</b>	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2 0.1 <b>0.0</b>
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> <b>GVFi (Ours)</b>	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b>	0.383 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 <b>0.101</b>	21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 <b>0.962</b>	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 <b>0.055</b>	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b>	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 <b>0.960</b>	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 <b>0.060</b> Dining	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> 2 Table	0.763 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 <b>0.966</b>	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.1 <b>0.0</b>
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> GVFi (Ours) Methods	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati	0.383 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.101 Fac	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 <b>0.962</b>	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 <b>0.055</b>	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b>	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.907 0.960	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> 7 Table	0.765 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 <b>0.966</b>	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.1 <b>0.0</b>
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> <b>GVFi (Ours)</b> Methods	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 In PSNR↑	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati SSIM↑	0.383 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.101 Faction	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E: PSNR↑	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 0.962 xtrapolat SSIM↑	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.077 0.055	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> Ir PSNR↑	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.900 tterpolati	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining on LPIPS↓	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> 5 Table Ex PSNR↑	0.703 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 0.966 ctrapolat SSIM↑	0.3 0.6 0.4 0.2 0.2 0.1 0.0 0.0
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFP <sub>INN</sub> MSFF(Li et al., 2021) NVFi(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> <b>GVFi (Ours)</b> Methods T-NeRFPumarola et al. (2021)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> <b>32.881</b> <b>32.698</b> In PSNR↑	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati SSIM↑ 0.741	0.383 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.101 Fac on LPIPS↓ 0.328	21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E: PSNR↑ 24.276	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 0.962 xtrapolat SSIM↑ 0.722	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.077 0.055	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> Ir PSNR↑ 21.699	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.907 0.900 tterpolati SSIM↑ 0.716	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining on LPIPS↓ 0.338	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> Table Ex PSNR↑ 20.977	0.605 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 0.966 ttrapolat SSIM↑ 0.725	0.3 0.6 0.4 0.2 0.2 0.2 0.1 0.0 0.0 1 0.0
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFPINN HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> <b>GVFi (Ours)</b> Methods T-NeRFPumarola et al. (2021) D-NeRFPumarola et al. (2021)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 In PSNR↑ 26.467 28.818	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati SSIM↑ 0.741 0.818	0.383 0.383 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.101 Factor 0.328 0.252	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E. PSNR↑ 24.276 22.959	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 0.962 xtrapolat SSIM↑ 0.722 0.746	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.077 0.075 0.075 0.344 0.303	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> Ir PSNR↑ 21.699 20.851	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.960 terpolati SSIM↑ 0.716 0.725	0.308 0.177 0.621 0.201 0.215 0.147 0.156 0.060 Dining on LPIPS↓ 0.338 0.319	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> g Table Ex PSNR↑ 20.977 19.035	0.605 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 0.966 ttrapolat SSIM↑ 0.725 0.705	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2 0.1 0.0 0.1 0.0 0.3 0.3
$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 In PSNR↑ 26.467 28.818 32.684	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 0.921 terpolati SSIM↑ 0.741 0.818 0.909	0.363 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.148	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E: <b>PSNR</b> ↑ 24.276 22.959 22.622	0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 0.953 0.962 xtrapolat SSIM↑ 0.722 0.746 0.810	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.077 0.075 0.075 0.344 0.303 0.229	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> Ir PSNR↑ 21.699 20.851 23.596	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.960 terpolati SSIM↑ 0.716 0.725 0.798	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining on LPIPS↓ 0.338 0.319 0.274	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> § Table Ex PSNR↑ 20.977 19.035 20.357	0.703 0.675 0.765 0.765 0.472 0.748 0.805 0.872 0.808 0.917 0.966 €trapolat SSIM↑ 0.725 0.705 0.804	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.1 0.0 <b>0.0</b> <b>0.0</b> <b>0.0</b> <b>0.3</b> 0.3 0.2
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFP <sub>INN</sub> MSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> <b>GVFi (Ours)</b> Methods T-NeRFPumarola et al. (2021) D-NeRFPumarola et al. (2021) TiNeuVoxFang et al. (2022) TiNeuVoxFang et al. (2022)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.898</b> <b>In</b> <b>PSNR</b> ↑ 26.467 28.818 32.684 16.634	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 0.921 terpolati SSIM↑ 0.741 0.818 0.909 0.446	0.363 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.132 0.101 Fac 0.328 0.252 0.148 0.624	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E PSNR↑ 24.276 22.959 22.622 17.546	0.641 0.649 0.409 0.409 0.658 0.750 0.826 0.775 0.953 0.962 ttrapolat SSIM↑ 0.722 0.746 0.810 0.480	$\begin{array}{c} 0.367\\ 0.326\\ 0.651\\ 0.510\\ 0.327\\ 0.273\\ 0.207\\ 0.077\\ \hline 0.055\\ \hline \\ \hline \\ 0.005\\ \hline \\ 0.005\\ \hline \\ 0.303\\ 0.229\\ 0.609\\ \hline \end{array}$	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> Ir PSNR↑ 21.699 20.851 23.596 16.807	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.960 terpolati SSIM↑ 0.716 0.725 0.798 0.486	0.308 0.177 0.621 0.215 0.215 0.125 0.156 0.060 Dining 0n LPIPS↓ 0.319 0.274 0.324	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 33.685 g Table Ex PSNR↑ 20.977 19.035 20.357 18.215	0.755 0.765 0.765 0.772 0.748 0.805 0.872 0.808 0.917 0.966 trapolat SSIM↑ 0.725 0.705 0.804 0.548	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.1 <b>0.0</b> <b>0.0</b> <b>0.0</b> <b>0.0</b> <b>0.3</b> 0.3 0.2 0.5
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFP <sub>INN</sub> NSFF(Li et al., 2023) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> GVFi (Ours) T-NeRFPumarola et al. (2021) D-NeRFPumarola et al. (2021) TiNeuVoxFang et al. (2022) T-NeRFP <sub>INN</sub> HexPlane <sub>PINN</sub>	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 In PSNR↑ 26.467 28.818 32.684 16.634 16.634	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati SSIM↑ 0.741 0.818 0.909 0.846 0.909 0.446 0.826	0.365 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.328 0.328 0.328 0.328 0.522 0.148 0.624 0.283	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E: PSNR↑ 24.276 22.622 21.7.546 24.998	0.641 0.649 0.409 0.409 0.658 0.750 0.826 0.775 0.953 0.962 xtrapolat SSIM↑ 0.722 0.746 0.810 0.480 0.480 0.792	$\begin{array}{c} 0.367\\ 0.326\\ 0.651\\ 0.510\\ 0.327\\ 0.273\\ 0.207\\ 0.077\\ \hline 0.077\\ \hline 0.055\\ \hline \hline 0.055\\ \hline \hline 0.344\\ 0.303\\ 0.229\\ 0.609\\ 0.312\\ \end{array}$	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> Irr PSNR↑ 21.699 20.851 23.596 16.807 25.291	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.907 0.960 tterpolati SSIM↑ 0.716 0.725 0.798 0.486 0.788	0.308 0.177 0.621 0.215 0.215 0.125 0.156 0.060 Dining 0.156 0.060 Dining 0.319 0.274 0.640 0.350	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> <b>5</b> Table Ex PSNR↑ 20.977 19.035 20.357 20.357 18.215 22.979	0.703 0.675 0.765 0.748 0.805 0.805 0.805 0.808 0.917 0.966 trapolat SSIM↑ 0.725 0.705 0.804 0.548 0.771	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFP <sub>INN</sub> HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> <b>GVFi (Ours)</b> T-NeRFPumarola et al. (2021) D-NeRFPumarola et al. (2021) TiNeuVoxFang et al. (2022) T-NeRFPINN HexPlane <sub>PINN</sub> NSFFLi et al. (2021)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 In PSNR↑ 26.467 28.818 32.684 16.634 27.200 <b>33.975</b>	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 0.921 terpolati SSIM↑ 0.741 0.818 0.909 0.446 0.826 0.919	0.365 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.252 0.328 0.252 0.48 0.252 0.48 0.252 0.424 0.283 0.522	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E PSNR↑ 24.276 22.622 17.546 24.959 22.622 17.546 24.998 26.647	0.641 0.641 0.699 0.409 0.658 0.750 0.826 0.775 0.953 0.962 trapolat SSIM↑ 0.722 0.746 0.810 0.746 0.8480 0.480 0.480	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.055 0.055 0.344 0.303 0.229 0.609 0.312 0.196	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> Ir PSNR↑ 21.699 20.851 23.596 16.807 25.291 19.552	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.960 terpolati SSIM↑ 0.716 0.725 0.798 0.486 0.486 0.486	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining on LPIPS↓ 0.338 0.319 0.274 0.454 0.350 0.464	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> 5 Table Ex PSNR <sup>4</sup> 20.977 19.035 20.357 18.215 22.979 22.612	0.675 0.675 0.765 0.765 0.472 0.748 0.802 0.872 0.808 0.917 0.966 0.705 0.705 0.705 0.804 0.548 0.771 0.770	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2 0.1 0.2 0.2 0.1 0.3 0.3 0.3 0.2 0.5 0.3 0.3 0.3
$ \begin{array}{c} $T$-NeRF(Pumarola et al., 2021) \\ $D$-NeRF(Pumarola et al., 2021) \\ $T$-NeRF(Pumarola et al., 2022) \\ $T$-NeRF_{PINN} \\ $HexPlane_{PINN} \\ NSFF(Li et al., 2021) \\ $NVFi(Li et al., 2023) \\ $DefGS(Yang et al., 2024) \\ $T$-NeRFPumarola et al. (2021) \\ $T$-NeRFPumarola et al. (2022) \\ $T$-NeRFPumarola et al. (2021) \\ $T$-NeRFPUmarola et al. (20$	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 In PSNR↑ 26.467 28.818 32.684 16.634 27.200 <b>33.975</b> 32.460	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati SSIM↑ 0.741 0.818 0.909 0.446 0.826 0.829 0.912	0.365 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.101 Fac 0.328 0.252 0.148 0.624 0.624 0.624 0.624 0.52 0.152 0.151	21.714 21.418 16.200 22.867 35.892 30.408 21.703 33.630 <b>36.578</b> tory E PSNR↑ 24.276 24.276 24.2959 22.622 17.546 24.998 26.647 31.719	0.641 0.641 0.699 0.409 0.409 0.750 0.826 0.775 0.953 0.962 0.755 0.953 0.962 0.746 0.810 0.722 0.746 0.810 0.792 0.746 0.810 0.792 0.792 0.855 0.908	0.367 0.326 0.651 0.510 0.327 0.273 0.273 0.277 0.077 0.077 0.055 0.075 0.344 0.303 0.229 0.609 0.312 0.196 0.154	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> <b>I</b> r PSNR↑ 21.699 20.851 23.596 16.807 25.291 19.552 <b>29.179</b>	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 <b>0.960</b> terpolati SSIM↑ 0.716 0.725 0.798 0.486 0.788 0.4885 0.885	0.308 0.177 0.621 0.201 0.215 0.147 0.156 0.060 Dining 0n LPIPS↓ 0.338 0.319 0.274 0.464 0.350 0.464 0.199	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> ₹ Table Ex PSNR↑ 20.977 19.035 20.357 18.215 22.979 22.612 29.011	0.703 0.675 0.765 0.765 0.765 0.772 0.748 0.805 0.872 0.808 0.917 0.966 0.705 0.705 0.705 0.705 0.804 0.771 0.770 0.898	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFPINN HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> <b>GVFi (Ours)</b> T-NeRFPumarola et al. (2021) D-NeRFPumarola et al. (2022) T-NeRFPINN HexPlane <sub>PINN</sub> NSFFLi et al. (2021) NVFiLi et al. (2022) DefGSYang et al. (2024)	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 In PSNR↑ 26.467 28.818 32.684 16.634 27.200 <b>33.975</b> 32.460 33.629	0.716 0.745 0.831 0.375 0.658 0.821 0.919 0.919 0.921 terpolati SSIM↑ 0.741 0.818 0.909 0.446 0.819 0.912 0.912 0.912	0.385 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.253 0.254 0.254 0.253 0.254 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.148 0.254 0.254 0.254 0.151 0.254 0.254 0.152 0.152 0.152 0.152 0.152 0.152 0.152 0.152 0.152 0.152 0.152 0.152 0.152 0.155 0.254 0.255 0.155 0.055 0.155 0.055 0	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 36.578 tory E: PSNR↑ 24.276 22.959 22.622 17.546 24.998 26.647 31.719 22.820	0.641 0.641 0.699 0.409 0.409 0.750 0.826 0.775 0.953 0.962 0.746 0.810 0.7480 0.810 0.7480 0.810 0.8480 0.792 0.8480 0.8480 0.792 0.8480 0.8480 0.8480 0.8480 0.8480 0.752 0.783 0.783 0.783 0.783 0.783 0.783 0.983 0.983 0.983 0.983 0.983 0.983 0.983 0.983 0.983 0.983 0.983 0.983 0.983 0.783 0.983 0.983 0.783 0.983 0.983 0.983 0.833 0.855 0.994 0.833 0.855 0.995 0.855 0.955	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.077 0.055 0.077 0.055 0.344 0.344 0.303 0.229 0.609 0.312 0.154 0.154 0.154	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> In PSNR↑ 21.699 20.851 23.596 16.807 25.291 19.552 <b>29.179</b> 27.680	0.774 0.917 0.457 0.778 0.810 0.907 0.907 0.900 tterpolati SSIM↑ 0.716 0.725 0.798 0.486 0.725 0.798 0.486 0.788 0.665 0.885 0.885	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining on LPIPS↓ 0.319 0.274 0.338 0.319 0.274 0.640 0.350 0.274	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> § Table Ex PSNR↑ 20.977 19.035 20.357 18.215 22.979 22.612 29.011 20.965	0.675 0.675 0.765 0.765 0.805 0.805 0.808 0.8072 0.808 0.917 0.966 trapolat SSIM↑ 0.705 0.804 0.548 0.771 0.798 0.898 0.855	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFP <sub>INN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023a) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> GVFi (Ours) Methods T-NeRFPumarola et al. (2021) D-NeRFPumarola et al. (2021) TiNeuVoxFang et al. (2022) T-NeRF <sub>PINN</sub> HexPlane <sub>PINN</sub> NSFFLi et al. (2023a) DefGSYang et al. (2024) DefGSYang et al. (2024) DefGSYang et al. (2024) DefGSYang et al. (2024) DefGSYang et al. (2024)	26.094 27.000 30.646 15.008 23.764 31.418 32.698 32.698 132.698 10.508	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 0.921 terpolati SSIM↑ 0.741 0.818 0.909 0.446 0.826 0.919 0.912 0.943 0.943	0.365 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.132 0.328 0.328 0.328 0.328 0.552 0.148 0.624 0.283 0.152 0.151 0.096 0.097	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> Experimental State	0.641 0.649 0.669 0.750 0.826 0.750 0.826 0.755 0.953 0.953 0.962 xtrapolat \$SIM↑ 0.722 0.746 0.810 0.792 0.746 0.810 0.792 0.480 0.792 0.485 0.993 0.954 0.954	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.077 0.055 0.077 0.055 0.344 0.303 0.229 0.302 0.609 0.312 0.154 0.169 0.062	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> II PSNR↑ 21.699 20.851 23.596 16.807 25.291 19.552 <b>29.179</b> 27.680 27.957	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.960 0.907 0.960 0.716 0.716 0.725 0.798 0.486 0.788 0.486 0.788 0.665 0.885 0.890 0.881	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining 00 LPIPS↓ 0.338 0.319 0.274 0.640 0.350 0.464 0.145 0.145 0.145	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> ₹ Table Ex PSNR↑ 20.977 19.035 20.357 18.215 22.979 22.612 29.011 20.965 30.975	0.675 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 0.966 trapolat SSIM↑ 0.725 0.705 0.804 0.548 0.771 0.790 0.804 0.548 0.771 0.790 0.805 0.855 0.855	0.3 0.3 0.6 0.4 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> 32.698 <b>In</b> <b>PSNR↑</b> 26.467 28.818 32.684 46.634 47.200 <b>33.975</b> 32.460 33.629 33.643 33.623	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati SSIM↑ 0.741 0.818 0.909 0.446 0.826 0.919 0.943 <b>0.943</b> 0.941	$\begin{array}{c} 0.385\\ 0.319\\ 0.253\\ 0.668\\ 0.510\\ 0.294\\ 0.277\\ 0.132\\ 0.132\\ 0.132\\ 0.101\\ \hline Fac\\ 0.328\\ 0.252\\ 0.148\\ 0.624\\ 0.283\\ 0.152\\ 0.151\\ 0.096\\ 0.097\\ \hline 0.076\\ \end{array}$	21.714 21.418 16.200 22.867 30.408 21.703 33.630 <b>36.578</b> tory E: <b>PSNR</b> <sup>4</sup> 24.276 22.959 22.622 17.546 24.998 26.647 31.719 22.820 33.049 <b>34.906</b>	0.641 0.649 0.6658 0.750 0.826 0.755 0.953 0.953 0.953 0.962 trapolat SSIM <sup>4</sup> 0.722 0.746 0.810 0.792 0.855 0.908 0.792 0.855 0.908 0.839 0.954 0.954 0.963	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.077 0.055 0.077 0.055 0.0344 0.303 0.229 0.312 0.609 0.312 0.196 0.154 0.169 0.062 0.045	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> <b>7</b> <b>1</b> <b>7</b> <b>1</b> <b>7</b> <b>1</b> <b>7</b> <b>1</b> <b>7</b> <b>1</b> <b>7</b> <b>1</b> <b>7</b> <b>1</b> <b>1</b> <b>7</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b>	0.774 0.917 0.457 0.778 0.810 0.912 0.907 <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.960</b> <b>0.9716</b> <b>0.9716</b> <b>0.9716</b> <b>0.9716</b> <b>0.9716</b> <b>0.9716</b> <b>0.9716</b> <b>0.9716</b> <b>0.988</b> <b>0.9612</b> <b>0.988</b> <b>0.9612</b> <b>0.988</b> <b>0.9612</b> <b>0.988</b> <b>0.966</b> <b>0.716</b> <b>0.788</b> <b>0.985</b> <b>0.985</b> <b>0.985</b> <b>0.985</b> <b>0.985</b> <b>0.9891</b> <b>0.9891</b> <b>0.9891</b> <b>0.9891</b>	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining 00 LPIPS↓ 0.338 0.319 0.274 0.640 0.350 0.464 0.350 0.145 0.145 0.118	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> 5 Table Ex PSNR <sup>↑</sup> 20.977 19.035 20.357 18.215 22.979 22.612 29.011 20.965 30.975 <b>33.056</b>	0.675 0.675 0.765 0.765 0.765 0.765 0.872 0.808 0.917 0.966 trapolat SSIM↑ 0.725 0.705 0.705 0.705 0.705 0.804 0.771 0.770 0.898 0.875 0.955 0.955	0.1 0.1 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	26.094 27.000 30.646 15.008 23.764 31.418 32.698 32.698 132.698 132.698 132.698 16.634 27.200 33.643 32.643 33.643 33.643	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 <b>0.921</b> terpolati SSIM↑ 0.741 0.818 0.909 0.446 0.826 0.912 <b>0.943</b> <b>0.943</b>	0.365 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.252 0.148 0.624 0.283 0.151 0.096 0.097 0.076	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> Expression 2010 24.276 22.959 22.622 17.546 24.998 26.647 31.719 22.820 33.049 <b>34.906</b>	0.641 0.649 0.668 0.750 0.826 0.750 0.953 0.962 xtrapolat \$SIM↑ 0.722 0.746 0.810 0.792 0.746 0.810 0.792 0.480 0.792 0.480 0.480 0.792 0.908 0.908 0.909 0.954 0.908	0.367 0.326 0.651 0.510 0.327 0.207 0.077 0.077 0.055 0.077 0.055 0.344 0.303 0.229 0.609 0.312 0.609 0.312 0.154 0.154 0.169 0.062 0.045	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> <b>I</b> <b>I</b> <b>PSNR</b> ↑ 21.699 20.851 23.596 16.807 25.291 19.552 <b>29.179</b> <b>27.680</b> 27.957 <b>27.547</b>	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.960 terpolati SSIM↑ 0.716 0.725 0.798 0.486 0.788 0.665 0.885 0.890 0.8891 0.891	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining 0n LPIPS↓ 0.338 0.319 0.274 0.640 0.350 0.455 0.145 0.145	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 33.685 5 Table E PSNR↑ 20.977 19.035 20.357 18.215 22.979 22.612 29.011 20.965 30.975 <b>33.056</b>	0.675 0.675 0.765 0.472 0.748 0.805 0.872 0.808 0.917 0.966 0.966 trapolat SSIM↑ 0.705 0.804 0.771 0.705 0.804 0.548 0.771 0.795 0.804 0.548 0.771 0.795 0.804 0.548 0.805 0.805 0.805	0 0 0 0 0 0 0 0 0 0
T-NeRF(Pumarola et al., 2021) D-NeRF(Pumarola et al., 2021) TiNeuVox(Fang et al., 2022) T-NeRFPINN HexPlane <sub>PINN</sub> NSFF(Li et al., 2021) NVFi(Li et al., 2023) DefGS(Yang et al., 2024) DefGS <sub>NVFi</sub> GVFi (Ours) T-NeRFPumarola et al. (2021) TiNeuVoxFang et al. (2021) TiNeuVoxFang et al. (2022) T-NeRFPINN HexPlane <sub>PINN</sub> NSFFLi et al. (2021) NVFiLi et al. (2022) DefGSYang et al. (2024) DefGS <sub>NVFi</sub> GVFi (Ours) ATA ADDITIONAL	26.094 27.000 30.646 15.008 23.764 31.418 30.667 32.041 <b>32.881</b> <b>32.698</b> <b>In</b> <b>PSNR↑</b> 26.467 28.818 32.684 16.634 27.200 <b>33.975</b> 33.643 <b>33.975</b> 33.643 <b>33.9423</b>	0.716 0.745 0.831 0.375 0.658 0.821 0.824 0.918 0.919 0.919 0.919 0.921 terpolati SSIM↑ 0.741 0.818 0.909 0.446 0.818 0.909 0.446 0.812 0.912 0.912 0.912 0.943 0.941	0.365 0.319 0.253 0.668 0.510 0.294 0.277 0.132 0.132 0.132 0.132 0.132 0.132 0.132 0.252 0.148 0.252 0.148 0.252 0.148 0.252 0.151 0.096 0.097 0.076	21.714 21.714 21.418 16.200 22.867 25.892 30.408 21.703 33.630 <b>36.578</b> tory E: PSNR↑ 24.276 22.959 22.622 17.546 24.998 26.647 31.719 22.820 33.049 <b>34.906</b>	0.641 0.649 0.409 0.409 0.409 0.750 0.826 0.775 0.953 0.962 0.746 0.722 0.746 0.810 0.7480 0.792 0.746 0.810 0.480 0.792 0.746 0.810 0.792 0.993 0.792 0.998 0.839 0.954 0.963 0.965	0.367 0.326 0.651 0.510 0.327 0.273 0.207 0.077 0.055 0.077 0.055 0.077 0.344 0.303 0.229 0.344 0.303 0.229 0.609 0.312 0.609 0.312 0.154 0.154 0.165 0.154 0.165 0.154	24.852 33.001 16.549 24.605 32.514 30.394 27.355 26.200 <b>35.138</b> In PSNR↑ 21.699 20.851 23.596 16.807 25.291 19.552 <b>29.179</b> 27.680 27.957 27.547	0.774 0.917 0.457 0.778 0.810 0.888 0.912 0.907 0.960 0.907 0.960 0.716 0.716 0.725 0.798 0.486 0.788 0.486 0.788 0.485 0.885 0.8890 0.891 0.891	0.308 0.177 0.621 0.412 0.201 0.215 0.147 0.156 0.060 Dining on LPIPS↓ 0.319 0.274 0.338 0.319 0.274 0.640 0.350 0.464 0.145 0.145 0.145	19.455 19.718 17.197 21.518 21.501 27.840 20.032 26.730 <b>33.685</b> 21.501 <b>33.685</b> 20.977 19.035 20.977 19.035 20.357 20.357 20.357 20.357 30.975 <b>33.056</b>	0.675 0.675 0.765 0.765 0.805 0.805 0.808 0.8072 0.808 0.917 0.966 0.966 0.705 0.705 0.705 0.804 0.548 0.771 0.725 0.804 0.548 0.771 0.770 0.898 0.855 0.955 0.955	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0

Figures 17, 18, 19, &20 shows qualitative results for the rendered mask on Dynamic Indoor Scene dataset.

1	0	2	6
1	n	2	7

Table 12: Quantitative results of our method and baselines on the NVIDIA Dynamic Scene dataset.

1021		Truck							Skating					
1031		In	terpolati	ion	E E	xtrapolati	ion	In	terpolati	on	Ex	ion		
1032		<b>PSNR</b> ↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	
1000	T-NeRF(Pumarola et al., 2021)	18.673	0.548	0.447	18.176	0.567	0.447	27.483	0.820	0.263	24.063	0.846	0.269	
1033	D-NeRF(Pumarola et al., 2021)	17.660	0.554	0.431	16.905	0.544	0.445	27.994	0.869	0.187	24.361	0.873	0.208	
1034	TiNeuVox(Fang et al., 2022)	27.230	0.846	0.229	24.887	0.848	0.209	29.377	0.889	0.202	24.224	0.878	0.220	
1005	T-NeRF <sub>PINN</sub>	15.241	0.413	0.540	14.959	0.395	0.552	21.644	0.780	0.338	20.990	0.814	0.303	
1035	HexPlane <sub>PINN</sub>	25.494	0.768	0.337	24.991	0.768	0.325	24.447	0.867	0.225	23.955	0.868	0.232	
1036	NVFi(Li et al., 2023a)	27.276	0.840	0.235	28.269	0.855	0.220	26.999	0.848	0.227	28.654	0.896	0.208	
	DefGS(Yang et al., 2024)	28.327	0.885	0.115	24.947	0.875	0.131	24.997	0.900	0.138	23.532	0.914	0.148	
1037	$DefGS_{nvfi}$	28.169	0.884	0.114	28.481	0.922	0.088	25.774	0.896	0.141	26.577	0.931	0.115	
1038	GVFi (Ours)	27.977	0.880	0.097	29.655	0.931	0.063	25.909	0.901	0.106	29.120	0.944	0.071	
1039														

Table 13: Per-scene quantitative results on Dynamic Multipart dataset.

1047	Table 13: Per-scene quantitative results on Dynamic Multipart dataset.													
				Folding	g Chair					Hyperb	olic Slot			
1048	Methods	In	terpolati	on	Ex	trapolati	ion	In	terpolati	on	Ex	trapolati	ion	
1049		PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	
1010	T-NeRF(Pumarola et al., 2021)	10.146	0.598	0.537	10.260	0.586	0.548	7.437	0.424	0.749	7.098	0.404	0.739	
1050	D-NeRF (Pumarola et al., 2021)	11.681	0.717	0.437	13.177	0.765	0.357	7.279	0.485	0.714	7.547	0.468	0.695	
1051	TiNeuVox(Fang et al., 2022)	34.160	0.984	0.039	13.391	0.808	0.199	28.637	0.955	0.083	25.436	0.973	0.040	
1051	NVFi(Li et al., 2023a)	27.748	0.962	0.049	23.433	0.940	0.063	25.487	0.944	0.057	25.757	0.956	0.039	
1052	DefGS(Yang et al., 2024)	37.319	0.995	0.009	13.682	0.820	0.169	31.780	0.983	0.030	25.631	0.981	0.020	
1052	$DefGS_{NVFi}$	37.269	0.994	0.009	25.404	0.962	0.022	32.506	0.985	0.025	29.351	0.988	0.012	
1055	GVFi (Ours)	37.910	0.995	0.005	27.869	0.978	0.015	31.740	0.985	0.018	34.185	0.993	0.007	
1054		Satellite					Stove							
1055	Methods	In	terpolati	on	Еx	trapolati	on	Interpolation			E۶	Extrapolation		
1055		PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	
1056	T-NeRF(Pumarola et al., 2021)	14.614	0.754	0.307	14.468	0.751	0.328	7.134	0.490	0.605	8.429	0.562	0.531	
1057	D-NeRF(Pumarola et al., 2021)	17.991	0.930	0.100	17.252	0.926	0.102	16.165	0.856	0.262	15.400	0.908	0.205	
1057	TiNeuVox(Fang et al., 2022)	33.061	0.983	0.035	28.627	0.978	0.032	23.969	0.943	0.109	15.760	0.934	0.087	
1058	NVFi(Li et al., 2023a)	29.644	0.973	0.029	30.075	0.975	0.027	27.186	0.959	0.072	21.675	0.950	0.054	
1050	DefGS(Yang et al., 2024)	36.832	0.993	0.007	27.622	0.979	0.016	32.607	0.989	0.029	15.721	0.941	0.063	
1059	DefGS <sub>NVFi</sub>	36.640	0.993	0.007	34.282	0.990	0.007	21.134	0.988	0.029	24.781	0.977	0.027	
1060	GVFi (Ours)	36.687	0.994	0.006	31.383	0.987	0.009	32.892	0.990	0.016	29.446	0.985	0.017	

Table 14: Quantitative results for both novel view interpolation and future frame extrapolation on GoPro Dataset. 

1071		GoPro Dataset								
1071		In	terpolati	on	Ех	trapolati	ion			
1072		<b>PSNR</b> ↑	SSIM↑	LPIPS↓	<b>PSNR</b> ↑	<b>SSIM</b> ↑	LPIPS↓			
1073	TiNeuVox(Fang et al., 2022)	15.306	0.588	0.516	20.323	0.738	0.318			
1074	NVFi(Li et al., 2023a)	14.229	0.568	0.569	19.879	0.736	0.415			
1075	DefGS(Yang et al., 2024)	20.018	0.838	0.167	21.193	0.842	0.185			
1076	$DefGS_{nvfi}$	20.254	0.838	0.167	25.469	0.882	0.141			
1077	GVFi (Ours)	20.124	0.834	0.168	26.276	0.890	0.131			
1077										
1078										











Under review as a conference paper at ICLR 2025







Dynamic Multipart dataset.



Figure 13: Qualitative results of RGB view synthesis for interpolation and extrapolation tasks on "Skating" scene of NVIDIA Dynamic Scene dataset.



Figure 14: Qualitative results of RGB view synthesis for interpolation and extrapolation tasks on "Box" scene of GoPro dataset.



Figure 15: Qualitative results of RGB view synthesis for interpolation and extrapolation tasks on "Hammer" scene of GoPro dataset.



Figure 16: Qualitative results of RGB view synthesis for interpolation and extrapolation tasks on "Collision" scene of GoPro dataset.













Figure 21: Qualitative results of RGB view synthesis for longer extrapolation from our method.