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Weak higher-order interactions enhance synchronization: a study on the Kuramoto model

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Extended Abstract

Synchronization is a fundamental phenomenon in complex systems, observed across a wide range of natural and engineered contexts [1]. The Kuramoto model [2] provides a foundational framework for understanding synchronization among coupled oscillators, traditionally assuming pairwise interactions. However, many real-world systems exhibit group and many-body interactions, which can be effectively modeled through hypergraphs [3]. Previous studies suggest that higher-order interactions make it harder to reach and potentially impairing synchronization, despite enriching the dynamics. In particular, Zhang et al. [4] showed that higher-order interactions make the attraction basin of the synchronous state smaller but more robust (deeper). These conclusions were further supported in [5], showing that the critical coupling for synchronization increases, facilitating desynchronization, and in [6], which demonstrated that once synchronization is achieved, it becomes harder to disrupt.

These recent findings motivated our first research goal, which is to quantify the effects of higher-order interactions on the synchronous state for general hypergraph topologies. To this end, we studied Kuramoto oscillators coupled through 2- and 3-body interactions and conducted a detailed numerical analysis on various random hypergraphs. Our study confirmed that, as expected, higher-order interactions enhance synchronization when the initial conditions are close to the synchronous state; however, when starting from incoherent states (i.e., far from synchronization), things become more interesting. In fact, while our simulations support the finding that higher-order interactions shrink the attraction basin of the synchronous state, they also reveal that weak higher-order interactions can, counterintuitively, enhance synchronization.

Our second research question was: given a limited amount of resources for connectivity of both pairwise and higher-order interactions, which is the optimal combination to enhance synchronization? With this investigation, we aimed to determine whether higher-order interactions can offer advantages over purely pairwise ones, and whether a mix of both can outperform structures relying exclusively on one type. Our analysis showed that, when the total budget for interactions (pairwise and higher-order) is limited, synchronization is enhanced by a combination of both types of interactions, regardless of the relative cost of higher-order interactions within that budget.

The two main findings of this work—that (i) weak higher-order interactions enhance synchronization, and that (ii) with a finite budget for connections, a combination of pairwise and higher-order interactions optimizes synchronization—are summarized in Fig. 1.

The insights gained in this study offer guidance for the design and control of complex systems with higher-order interactions and are relevant in both engineered and natural systems: in the former, it can guide resource allocation for building synchronizable systems; in the latter, it may help explain the interaction patterns that emerge in nature as evolved or self-organized solutions to synchronization demands. In the long term, we foresee possible applications in neuroscience, where interactions between neurons are higher-order [7], and synchronization is associated to pathological states, such as seizures and Parkinson's disease [8]. A deeper understanding of the interplay between structure and dynamics at a fundamental level may bring about more efficient and accessible treatments for patients suffering from those diseases. Lastly, the knowledge on the optimal topologies for synchronization can then be useful in the study of other phenomena, such as opinion dynamics, or spreading of infectious diseases.

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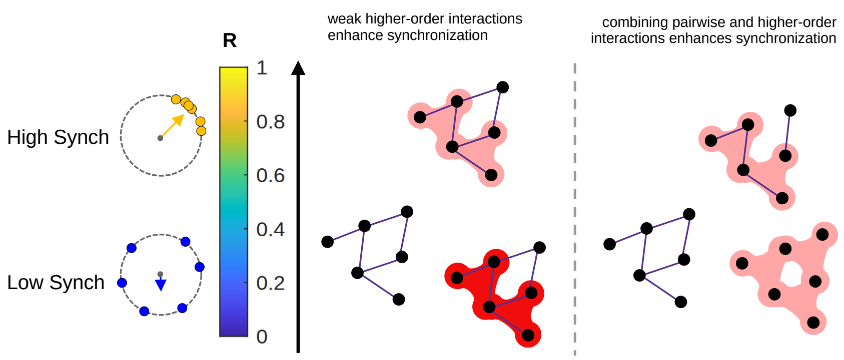

Figure 1: **Main findings of this work:** (1) While higher-order interactions generally hamper synchronization by making the attraction basin of the synchronous state "deeper but smaller", *weak* higher-order interactions enhance synchronization. (2) Under a finite budget for interactions, regardless of the relative cost of higher-order interactions, the optimal configuration for synchronization always involves a combination of pairwise and higher-order interactions. The colors of the hyperedges give a pictorial representation of the coupling strength, namely red indicates a strong coupling while pink denotes a weaker one.