000 001 002 003 004 FEDCOMLOC: COMMUNICATION-EFFICIENT DIS-TRIBUTED TRAINING OF SPARSE AND QUANTIZED **MODELS**

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ABSTRACT

Federated Learning (FL) has garnered increasing attention due to its unique characteristic of allowing heterogeneous clients to process their private data locally and interact with a central server, while being respectful of privacy. A critical bottleneck in FL is the communication cost. A pivotal strategy to mitigate this burden is *Local Training*, which involves running multiple local stochastic gradient descent iterations between communication phases. Our work is inspired by the innovative Scaffnew algorithm of [Mishchenko et al.](#page-10-0) [\(2022\)](#page-10-0), which has considerably advanced the reduction of communication complexity in FL. We introduce FedComLoc (Federated Compressed and Local Training), integrating practical and effective compression into Scaffnew to further enhance communication efficiency. Extensive experiments, using the popular TopK compressor and quantization, demonstrate its prowess in substantially reducing communication overheads in heterogeneous settings.

1 INTRODUCTION

030 032 Privacy concerns and limited computing resources on edge devices often make centralized training impractical, where all data is gathered in a data center for processing. In response to these challenges, Federated Learning (FL) has emerged as an increasingly popular framework [\(McMahan](#page-10-1) [et al.,](#page-10-1) [2016;](#page-10-1) [Kairouz et al.,](#page-9-0) [2019\)](#page-9-0). In FL, multiple clients perform computations locally on their private data and exchange information with a central server. This process is typically framed as an empirical risk minimization problem [\(Shalev-Shwartz & Ben-David,](#page-10-2) [2014\)](#page-10-2):

$$
\min_{x \in \mathbb{R}^d} \left[f(x) := \frac{1}{n} \sum_{i=1}^n f_i(x) \right],
$$
\n(ERM)

037 038 039 040 where f_i represents the local objective for client i, n is the total number of clients, and x is the model to be optimized. Our primary objective is to solve the problem [\(ERM\)](#page-0-0) and deploy the optimized global model to all clients. For instance, x might be a neural network trained in an FL setting. However, a considerable bottleneck in FL are communication cost, particularly with large models.

⁰⁴¹ 042 043 044 045 046 047 To mitigate these costs, FL often employs *Local Training* (LT), a strategy where local parameters are updated multiple times before aggregation [\(Povey et al.,](#page-10-3) [2014;](#page-10-3) [Moritz et al.,](#page-10-4) [2016;](#page-10-4) [McMahan](#page-10-5) [et al.,](#page-10-5) [2017;](#page-10-5) [Li et al.,](#page-10-6) [2020;](#page-10-6) [Haddadpour & Mahdavi,](#page-9-1) [2019;](#page-9-1) [Khaled et al.,](#page-9-2) [2019;](#page-9-2) [2020;](#page-9-3) [Karimireddy](#page-9-4) [et al.,](#page-9-4) [2020;](#page-9-4) [Gorbunov et al.,](#page-9-5) [2020;](#page-9-5) [Mitra et al.,](#page-10-7) [2021\)](#page-10-7). However, there is a lack of theoretical understanding regarding the effectiveness of LT methods. The recent introduction of Scaffnew by [Mishchenko et al.](#page-10-0) [\(2022\)](#page-10-0) marked a substantial advancement, as this algorithm converges to the exact solution with accelerated complexity, in convex settings.

⁰⁴⁸ 049 050 051 052 053 Another approach to reducing communication costs is through compression [\(Haddadpour et al.,](#page-9-6) [2021;](#page-9-6) [Condat et al.,](#page-9-7) [2022;](#page-9-7) [Yi et al.,](#page-10-8) [2024\)](#page-10-8). In centralized training, one often aims to learn a sparsified model for faster training and communication efficiency [\(Dettmers & Zettlemoyer,](#page-9-8) [2019;](#page-9-8) [Kuznedelev](#page-10-9) [et al.,](#page-10-9) [2023\)](#page-10-9). Dynamic pruning strategies like gradual magnitude pruning [\(Gale et al.,](#page-9-9) [2019\)](#page-9-9) and RigL [\(Evci et al.,](#page-9-10) [2020\)](#page-9-10) are common. But in FL, the effectiveness of these model sparsification methods based on thresholds remains unclear. The work by [Babakniya et al.](#page-8-0) [\(2023\)](#page-8-0) considers FL sparsity mask concepts, showing promising results.

054 055 056 Quantization is another efficient model compression technique [\(Han et al.,](#page-9-11) [2021;](#page-9-11) [Bhalgat et al.,](#page-8-1) [2020;](#page-8-1) [Shin et al.,](#page-10-10) [2023\)](#page-10-10), though its application in heterogeneous settings is limited. [Gupta et al.](#page-9-12) [\(2022\)](#page-9-12) introduced FedAvg with Kurtosis regularization [\(Chmiel et al.,](#page-8-2) [2020\)](#page-8-2) in FL.

057 058 059 060 Furthermore, studies such as [Haddadpour et al.](#page-9-6) [\(2021\)](#page-9-6); [Condat et al.](#page-9-7) [\(2022\)](#page-9-7) have theoretical convergence guarantees for unbiased estimators with restrictive assumptions. As this work employs the biased TopK compressor these are unsuitable in this case.

061 Thus, we tackle the following question:

062 063 064 *Is it possible to design an efficient algorithm combining accelerated local training with compression techniques, such as quantization and Top-K, and validate its efficiency empirically on popular FL datasets?*

065 066 067 068 069 Our method for investigating this question consists of two steps. Firstly, we design an algorithm, termed FedComLoc, which integrates general compression into ScaffNew, an accelerated local training algorithm. Secondly, we empirically validate FedComLoc for popular compression techniques (Top K and quantization) on popular FL datasets (FedMNIST and FedCIFAR10).

- **070** We were able to answer this question affirmatively with the following contributions:
	- We have developed a communication-efficient method FedComLoc for distributed training, specifically designed for heterogeneous environments. This method integrates general compression techniques and is motivated by previous theoretical insights.
	- We proposed three variants of our algorithm addressing several key bottlenecks in FL: FedComLoc-Com addresses communication costs from client to server, FedComLoc-Global addresses communication costs from server to client and FedComLoc-Local addresses limited computational resources on edge devices.
	- We conducted detailed comparisons and ablation studies, validating the effectiveness of our approach. These reveal a considerable reduction in communication and, in certain cases, an enhancement in training speed in number of communication rounds. Furthermore, we demonstrated that our method outperforms well-established baselines in terms of training speed and communication costs.
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2 RELATED WORK

086 2.1 LOCAL TRAINING

088 089 090 091 092 093 094 095 096 The evolution of LT in FL has been profound and continuous, transitioning through five distinct generations, each marked by considerable advancements from empirical discoveries to reductions in communication complexity. The pioneering FedAvg algorithm [\(McMahan et al.,](#page-10-5) [2017\)](#page-10-5) represents the first generation of LT techniques, primarily focusing on empirical evidence and practical applications [\(Povey et al.,](#page-10-3) [2014;](#page-10-3) [Moritz et al.,](#page-10-4) [2016;](#page-10-4) [McMahan et al.,](#page-10-5) [2017\)](#page-10-5). The second generation of LT methods consists in solving [\(ERM\)](#page-0-0) based on homogeneity assumptions such as bounded gradients^{[1](#page-1-0)} [\(Li et al.,](#page-10-6) [2020\)](#page-10-6) or limited gradient diversity^{[2](#page-1-1)} [\(Haddadpour & Mahdavi,](#page-9-1) [2019\)](#page-9-1). However, the practicality of such assumptions in real-world FL scenarios is debatable and often not viable [\(Kairouz et](#page-9-0) [al.,](#page-9-0) [2019;](#page-9-0) [Wang et al.,](#page-10-11) [2021\)](#page-10-11).

097 098 099 100 101 102 103 Third-generation methods made fewer assumptions, demonstrating sublinear [\(Khaled et al.,](#page-9-2) [2019;](#page-9-2) [2020\)](#page-9-3) or linear convergence up to a neighborhood [\(Malinovsky et al.,](#page-10-12) [2020\)](#page-10-12) with convex and smooth functions. More recently, fourth-generation algorithms like Scaffold [\(Karimireddy et al.,](#page-9-4) [2020\)](#page-9-4), S-Local-GD [\(Gorbunov et al.,](#page-9-5) [2020\)](#page-9-5), and FedLin [\(Mitra et al.,](#page-10-7) [2021\)](#page-10-7) have gained popularity. These algorithms effectively counteract client drift and achieve linear convergence to the exact solution under standard assumptions. Despite these advances, their communication complexity mirrors that of GD, i.e. $\mathcal{O}(\kappa \log \epsilon^{-1})$, where $\kappa = L/\mu$ denotes the condition number.

104 105 The most recent Scaffnew algorithm, proposed by [Mishchenko et al.](#page-10-0) [\(2022\)](#page-10-0), revolutionizes the field with its accelerated communication complexity $\mathcal{O}(\sqrt{\kappa} \log \epsilon^{-1})$. This seminal development estab-

¹There exists $c \in \mathbb{R}$ s.t. $\|\nabla f_i(x)\| \leq c$ for $1 \leq i \leq d$.

²There exists $c \in \mathbb{R}$ s.t. $\frac{1}{n} \sum_{i=1}^{n} ||\nabla f_i(x)||^2 \leq c ||\nabla f(x)||^2$.

108 109 Algorithm 1 FedComLoc

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135 136 137 138 139 140 141 142 143 lishes LT as a communication acceleration mechanism for the first time, positioning Scaffnew at the forefront of the fifth generation of LT methods with accelerated convergence. Further enhancements to Scaffnew have been introduced, incorporating aspects like variance-reduced stochastic gradients [\(Malinovsky et al.,](#page-10-13) [2022\)](#page-10-13), personalization [\(Yi et al.,](#page-10-14) [2023\)](#page-10-14), partial client participation [\(Condat](#page-9-13) [et al.,](#page-9-13) [2023\)](#page-9-13), asynchronous communication [\(Maranjyan et al.,](#page-10-15) [2022\)](#page-10-15), and an expansion into a broader primal–dual framework (Condat & Richtárik, [2023\)](#page-9-14). This latest generation also includes the 5GCS algorithm [\(Grudzien et al.](#page-9-15), 2023), with a different strategy where the local steps are part of an inner loop to approximate a proximity operator. Our proposed FedComLoc algorithm extends Scaffnew by incorporating pragmatic compression techniques, such as sparsity and quantization, resulting in even faster training measured by the number of bits communicated.

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2.2 MODEL COMPRESSION IN FEDERATED LEARNING

147 148 149 150 151 152 153 154 Model compression in the context of FL is a burgeoning field with diverse research avenues, particularly focusing on the balance between model efficiency and performance. [Jiang et al.](#page-9-16) [\(2022\)](#page-9-16) innovated in global pruning by engaging a single, powerful client to initiate the pruning process. This strategy transitions into a collaborative local pruning phase, where all clients contribute to an adaptive pruning mechanism. This involves not just parameter elimination, but also their reintroduction, integrated with the standard FedAvg framework [\(McMahan et al.,](#page-10-1) [2016\)](#page-10-1). However, this approach demands substantial local memory for tracking the relevance of each parameter, a constraint not always feasible in FL settings.

155 156 157 158 159 160 161 Addressing some of these challenges, [Huang et al.](#page-9-17) [\(2022\)](#page-9-17) introduced an adaptive batch normalization coupled with progressive pruning modules, enhancing sparse local computations. These advancements, however, often do not fully address the constraints related to computational resources and communication bandwidth on the client side. Our research primarily focuses on magnitudebased sparsity pruning. Techniques like gradual magnitude pruning [\(Gale et al.,](#page-9-9) [2019\)](#page-9-9) and RigL [\(Evci et al.,](#page-9-10) [2020\)](#page-9-10) have been instrumental in dynamic pruning strategies. However, their application in FL contexts remains relatively unexplored. The pioneering work of [Babakniya et al.](#page-8-0) [\(2023\)](#page-8-0) extends the concept of sparsity masks in FL, demonstrating noteworthy outcomes.

162 163 164 165 Quantization is another vital avenue in model compression. Seminal works in this area include [Han](#page-9-11) [et al.](#page-9-11) [\(2021\)](#page-9-11), [Bhalgat et al.](#page-8-1) [\(2020\)](#page-8-1), and [Shin et al.](#page-10-10) [\(2023\)](#page-10-10). A major advance has been made by [Gupta](#page-9-12) [et al.](#page-9-12) [\(2022\)](#page-9-12), who combined FedAvg with Kurtosis regularization [\(Chmiel et al.,](#page-8-2) [2020\)](#page-8-2). We are looking to go even further by integrating accelerated LT with quantization techniques.

166 167 168 169 170 171 172 However, a gap exists in the theoretical underpinnings of these compression methods. Research by [Haddadpour et al.](#page-9-6) [\(2021\)](#page-9-6) and [Condat et al.](#page-9-7) [\(2022\)](#page-9-7) offers theoretical convergence guarantees for unbiased estimators, but these frameworks are not readily applicable to common compressors like Top-K sparsifiers. In particular, CompressedScaffnew [\(Condat et al.,](#page-9-7) [2022\)](#page-9-7) integrates an unbiased compression mechanism in Scaffnew, that is based on random permutations. But due to requiring shared randomness it is not practical. Linear convergence has been proved when all functions f_i are strongly convex.

173 174 175 176 177 178 179 180 To the best of our knowledge, no other compression mechanism has been studied in Scaffnew, either theoretically or empirically, and even the mere convergence of Scaffnew in nonconvex settings has not been investigated either. Our goal is to go beyond the convex setting and simplistic logistic regression experiments and to study compression in Scaffnew in realistic nonconvex settings with large datasets such as Federated CIFAR and MNIST. Our integration of compression in Scaffnew is heuristic but backed by the findings and theoretical guarantees of CompressedScaffnew in the convex setting, which shows a twofold acceleration with respect to the conditioning κ and the dimension d, thanks to LT and compression, respectively.

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3 PROPOSED ALGORITHM FedComLoc

3.1 SPARSITY AND QUANTIZATION

186 Let us define the sparsifying $TopK(\cdot)$ and quantization $Q_r(\cdot)$ operators.

Definition 3.1. Let $x \in \mathbb{R}^d$ and $K \in \{1, 2, ..., d\}$. We define the sparsifying compressor TopK: $\mathbb{R}^d \to \mathbb{R}^d$ as:

$$
\text{Top}K(x) \coloneqq \arg\min_{y \in \mathbb{R}^d} \{ \|y - x\| \, \mid \, \|y\|_0 \le K \},
$$

191 192 where $||y||_0 = |\{i : y_i \neq 0\}|$ denotes the number of nonzero elements in the vector $y =$ $(y_1, \dots, y_d)^\intercal \in \mathbb{R}^d$. In case of multiple minimizers, TopK is chosen arbitrarily.

Definition 3.2. For any vector $x \in \mathbb{R}^d$, with $x \neq 0$ and a number of bits $r > 0$, its binary quantization $Q_r(x)$ is defined componentwise as

$$
Q_{r}(x) = (||x||_2 \cdot \text{sgn}(x_i) \cdot \xi_i(x, 2^r))_{1 \leq i \leq d},
$$

where $\xi_i(x, 2^r)$ are independent random variables. Let $y_i := \frac{|x_i|}{||x||}$ $\frac{|x_i|}{||x||_2}$. Then their probability distribution is given by

$$
\xi_i(x, 2^r) = \begin{cases} \left\lceil 2^r y_i \right\rceil / 2^r & \text{with proba. } 2^r y_i - \lfloor 2^r y_i \rfloor \, ; \\ \left\lfloor 2^r y_i \right\rfloor / 2^r & \text{otherwise.} \end{cases}
$$

203 If $x = 0$, we define $Q_r(x) = 0$.

> The distributions of the $\xi_i(x, r)$ minimize variance over distributions with support $\{0, 1/r, \ldots, 1\}$, ensuring unbiasedness, i.e. $\mathbb{E}[\xi_i(x,r)] = |x_i|/||x||_2$. This definition is based on an equivalent one in [Alistarh et al.](#page-8-3) [\(2017\)](#page-8-3).

3.2 INTRODUCTION OF THE ALGORITHMS

210 211 212 FedComLoc (Algorithm [1\)](#page-2-0) is an adaptation of Scaffnew, with modifications for compression. $TopK(\cdot)$ is used as the default compression technique for simplicity, although quantization is equally applicable. We examine three distinct variants in our ablations:

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• FedComLoc-Com: This variant addresses the communication bottleneck. It focuses on compressing the uplink network weights transmitted from each client to the server. This setup is adopted as our default setting.

Table 1: Test accuracy for various Top-K ratios. Table 2: Test accuracy score for various Dirichlet factors α and sparsity ratios.

Figure 1: Performance outcomes for various Top-K ratios.

Figure 2: Training loss and test accuracy for a density ratio of $K = 10\%$.

10%.

• FedComLoc-Local: Here, the local model is compressed during each training step. This addresses limited computational power and resources available to each client.

100%.

• FedComLoc-Global: Here, the server model is compressed before sending it to the clients. This variant is tailored for FL situations where downloading the model from the server is costly e.g. due to network bandwidth constraints.

4 EXPERIMENTS

256 Baselines. Our evaluation comprises three distinct aspects. Firstly, we conduct experiments to assess the impact of compression on communication costs. FedComLoc is assessed for varying sparsity and quantization ratios. Secondly, we compare FedComLoc-Com with FedComLoc-Local and FedComLoc-Global. Thirdly, we explore the efficacy of FedComLoc against non-accelerated local training methods, including FedAvg [\(McMahan et al.,](#page-10-1) [2016\)](#page-10-1), its Top-K sparsified counterpart sparseFedAvg, and Scaffold [\(Karimireddy et al.,](#page-9-4) [2020\)](#page-9-4).

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258 259 260 261 262 263 Datasets. Our experiments are conducted on FedMNIST [\(LeCun,](#page-10-16) [1998\)](#page-10-16) and FedCI-FAR10 [\(Krizhevsky et al.,](#page-10-17) [2009\)](#page-10-17) with the data processing framework FedLab [\(Zeng et al.,](#page-10-18) [2023\)](#page-10-18). For FedMNIST, we employ MLPs with three fully-connected layers, each coupled with a ReLU activation function. For FedCIFAR10, we utilize CNNs with two convolutional layers and three fully-connected layers. Comprehensive statistics for each dataset, details on network architecture and training specifics can be found in Appendix [A.](#page-12-0)

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265 266 267 268 269 Heterogeneous Setting. We explore different heterogeneous settings. Similar to [\(Zhang et al.,](#page-10-19) [2023;](#page-10-19) [Yi et al.,](#page-10-8) [2024\)](#page-10-8), we create heterogeneity in data by using a Dirichlet distribution, which assigns each client a vector indicating class preferences. This vector guides the unique selection of labels and images for each client until all data is distributed. The Dirichlet parameter α indicates the level of non-identical distribution. We also include a visualization of this distribution for the CIFAR10 dataset in Appendix [B.1.1.](#page-12-1)

Figure 3: CNN Performance on the FedCIFAR10 Dataset. For the left most columns, the step size was optimized for each density ratio K. For the two rightmost columns, a fixed stepsize of 0.01 is used. This is the maximum feasible step size which ensures convergence across all configurations.

289 290 291 292 293 294 295 Default Configuration. In the absence of specific clarifications, we adopt the Dirichlet factor $\alpha = 0.7$. To balance both communication and local computation costs, we use $p = 0.1$, resulting in an average of 10 local iterations per communication round. The learning rate is chosen by conducting a grid search over the set $\{0.005, 0.01, 0.05, 0.1, 0.5\}$. With communication costs being of most interest, our study employs FedComLoc-Com as the default strategy. The experiments are run for 2500 communication rounds for the CNN on FedCIFAR10 and 500 rounds for the MLP on FedMNIST. Furthermore, the dataset is distributed across 100 clients from which 10 are uniformly chosen to participate in each global round.

296 297 298 Furthermore, in our Definition [3.1](#page-3-0) of $Top K$, K is the number of nonzero parameters. However, we will rather specify the enforced density ratio, i.e. the ratio of nonzero parameters. For instance, specifying $K = 30\%$ means retaining 30% of parameters.

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4.1 TOP-K SPARSITY RATIOS

302 303 304 305 306 307 308 This section investigates the effects of different sparsity rations by investigating $TopK$ ratios on FedMNIST. The outcomes can be found in Table [1.](#page-4-0) Notably, $K = 10\%$ in TopK yields an accuracy of 0.9374, merely 3.94% lower than the 0.9758 unsparsified baseline. Remarkably, a 70% sparsity level $(K = 30\%)$ attains commendable performance, with only a 1.07% accuracy reduction, alongside a 70% reduction in communication costs. Furthermore, from the communication bits depicted in Figure [1](#page-4-0) it is evident that sparsity yields faster convergence, the more so with increased sparsity (smaller K).

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4.2 DATA HETEROGENEITY/DIRICHLET FACTORS

311 312 313 314 315 316 317 318 This subsection aims to assess the impact of varying degrees of data heterogeneity on FedMNIST. Hence, an analysis of the Dirichlet distribution factor α is presented, exploring the range of values $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9, 1.0\}$. Remember that a lower α means increased heterogeneity. Alongside, we examine the influence of different $\text{Top}K$ factors, specifically 10%, 50% and 100%. The results are shown in Table [2.](#page-4-0) Figure [2](#page-4-0) reports training loss and test accuracy for a sparsity ratio of 90% ($K = 10\%$). Additionally, round-wise visualizations for $K = 50\%$ and $K = 100\%$ (non-sparse) are presented in Figure [11](#page-13-0) in the Appendix.

319 Key observations from our study include:

320 321 322 323 a) When examining the effects of the heterogeneity degree α (as seen in each column of Table [2\)](#page-4-0), we observe that sparsity performance is influenced by heterogeneity degrees. For instance, $\alpha = 0.1$ results in a relative performance drop of 9.79% from an unsparsified to a sparsified model with $K = 10\%$. In contrast, for $\alpha = 0.3$, this drop is 5.80%, and for $\alpha = 1.0$, it is 3.63%. Interestingly, for commonly used heterogeneity ratios in literature ($\alpha = 0.3, 0.5, 0.7$), the performance drop does

Figure 5: FedComLoc employing $Q_r(\cdot)$. The number of quantization bits r is set to $r \in$ $\{4, 8, 16, 32\}.$

Figure 6: Data heterogeneity ablations for Fed-ComLoc utilizing $Q_r(\cdot)$ with number of bits r either 8 or 16. The same results plotted over the number of communication rounds can be found in Figure [13](#page-14-0) in the Appendix.

347 348 349 not decrease substantially when moving from $\alpha = 0.3$ to $\alpha = 0.5$, or from $\alpha = 0.5$ to $\alpha = 0.7$, unlike the shift from $\alpha = 0.1$ to $\alpha = 0.3$.

350 351 352 353 b) Focusing on the rows of Table [2,](#page-4-0) we find that lower sparsity ratios are more sensitive to heterogeneous distributions. In particular, observe that with $K = 10\%$, the absolute performance improvement from $\alpha = 0.1$ to $\alpha = 1$ is 7.01%. However, for $K = 50\%$, this improvement is only 1.22%.

354 355 356 357 358 359 c) It should be noted that each method was run with a fixed learning rate without scheduling, and the maximum communication round was set to 1000. Previous studies suggest that higher sparsity ratios require more communication rounds in centralized settings [\(Kuznedelev et al.,](#page-10-9) [2023\)](#page-10-9). This phenomenon was also observed in our FL experiments. Therefore, there is the potential for performance enhancement through sufficient model rounds and adaptive learning rate adjustments, especially for methods with higher sparsity.

361 4.3 CNNS ON FEDCIFAR10

362 363 364 365 366 367 368 369 370 371 372 This section repeats the experiments for CIFAR10 and a Convolutional Neural Network (CNN). We explored a range of stepsizes ($\gamma \in \{0.005, 0.01, 0.05, 0.1\}$). Further information is provided in Appendix [A.](#page-12-0) The CIFAR10 results, which involve optimizing a Convolutional Neural Network (CNN), are presented in Figure [3](#page-5-0) for both tuned and a fixed step size. Observe the accelerated convergence of sparsified models in terms of communicated bits when the step size is tuned. Interestingly, a sparsity of 90% ($K = 10\%$) shows faster convergence in terms of communication rounds (as shown in the first column), suggesting the potential for enhanced training efficiency in sparsified models. For a fixed step size (the two rightmost columns of Figure [3\)](#page-5-0) and $K = 10\%$, one can observe slower convergence compared to other configurations. This indicates that sparsity training requires more data and benefits from either increased communication rounds or a larger initial stepsize. This aligns with recent similar findings in the centralized setting [\(Kuznedelev et al.,](#page-10-9) [2023\)](#page-10-9).

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4.4 QUANTIZATION

376 377 This section explores using quantization Q_r for compression with the number of bits, r, set to $r \in \{4, 8, 16, 32\}$ on FedMNIST. This approach aligns with the methodologies outlined in [Alis](#page-8-3)[tarh et al.](#page-8-3) [\(2017\)](#page-8-3). The results after 1000 communication rounds are illustrated in Figure [5.](#page-6-0) Our

Figure 7: Loss and test accuracy over communication rounds and total costs. Total costs are a combined measurement of both communication costs and local computation cost. A communication round has unit cost while a local training round has cost τ . In a realistic FL system, τ is typically much less than 1, as the primary bottleneck is often communication and hence we set $\tau = 0.01$.

Figure 8: Comparison among FedAvg, Scaffold, FedDyn, FedComLoc-Local, FedComLoc-Com, and FedComLoc-Global. First column: $K = 50\%$; second column: $K = 100\%$ (no sparsity).

Figure 9: Sparsity ablation studies of FedComLoc-Local, FedComLoc-Com, and FedComLoc-Global on FedCIFAR10 and tuned stepsizes.

analysis reveals that quantization offers superior performance compared to $TopK$ -style sparsity. For instance, with 16-bit quantization corresponding to a 50% reduction in communication cost, the performance decrease is a mere 0.14%, Furthermore, Figure [6](#page-6-0) shows outcomes for different degrees of data heterogeneity. These findings demonstrate that quantization reduces communication at minor performance tradeoff while also exhibiting only minor sensitivity to data heterogeneity. The Appendix gives further results for both FedMNIST (section [B.2.1\)](#page-13-1) and FedCIFAR10 (section [B.2.2\)](#page-13-2).

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4.5 NUMBER OF LOCAL ITERATIONS

427 428 429 430 431 This section explores the performance impact of varying the expected number of local iterations on FedMNIST. The expected number of local iterations is $1/p$ where p is the communication probability. Hence, we investigate the influence of p ranging from $p \in \{0.05, 0.1, 0.2, 0.3, 0.5\}$. Furthermore, $K = 30\%$ is used. The results are presented in Figure [7.](#page-7-0) A key finding is that more local training rounds (i.e. smaller p) not only accelerate convergence but can also improve the final performance.

432 433 4.6 FEDCOMLOC VARIANTS

434 435 436 437 438 439 440 441 442 443 444 In this section, we compare FedComLoc-Local, FedComLoc-Com, and FedComLoc-Global on Fed-CIFAR10. The findings are illustrated in Figure [9.](#page-7-1) Observe that at high levels of sparsity (indicated by a small K in Top K), FedComLoc-Com underperforms the other algorithms. This could be attributed to the heterogeneous setting of our experiment: each client's model output is inherently skewed towards its local dataset. When this is coupled with extreme $TopK$ sparsification, more bias is introduced, which adversely affects performance. Conversely, at low sparsity (e.g. $K = 90\%$), FedComLoc-Com surpasses FedComLoc-Global. In addition, we observe that sparsity during local training (i.e. FedComLoc-Local) tends to yield better results. One possible explanation is that due to the local data bias the communication bandwidth between client and server might be crucial. Remember that FedComLoc-Local had no communication compression while both FedComLoc-Com and FedComLoc-Global do. Further FedMNIST results are shown in the Appendix.

4.7 FEDAVG AND SCAFFOLD

447 448 449 450 451 452 453 454 455 In this section, the performance of FedComLoc is compared with baselines in form of Fe-dAvg [\(McMahan et al.,](#page-10-1) [2016\)](#page-10-1) and Scaffold [\(Karimireddy et al.,](#page-9-4) [2020\)](#page-9-4) on FedCIFAR10. Furthermore, a sparsified version of FedAvg is employed, termed as sparseFedAvg. For sparseFedAvg a learning rate of 0.1 is used, whereas for FedComLoc, a lower rate of 0.05 is utilized. The outcomes of this analysis are depicted in Figure 8. The left part illustrates the performance of compressed models. We observe notably faster convergence for FedComLoc-type methods in comparison with sparseFedAvg despite the lower learning rate. The right part of the figure compares FedAvg with Scaffold, devoid of sparsity, using an identical learning rate of 0.005. This uniform rate ensures that each method achieves satisfactory convergence over a sufficient number of epochs. Here again, faster convergence is demonstrated with FedComLoc.

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5 CONCLUSION AND FUTURE WORK

459 460 461 462 463 464 This paper advances the field of FL by tackling one of its main challenges, namely its high com-munication cost. Building on the accelerated Scaffnew algorithm [\(Mishchenko et al.,](#page-10-0) [2022\)](#page-10-0), we introduced FedComLoc. This novel approach blends the practical compression techniques of model sparsity and quantization into the efficient local training framework. Our extensive experimental validation shows that FedComLoc preserves computational integrity while notably cutting communication costs.

465 466 467 468 469 470 Future research could explore the reduction of internal variance in stochastic gradient estimation, akin to the approach described in [Malinovsky et al.](#page-10-13) [\(2022\)](#page-10-13). The FedComLoc-Global algorithm we propose offers potential for obtaining a sparsified model suitable for deployment. Additionally, investigating the development of an efficiently sparsified deployed model extensively presents an intriguing avenue for further study.

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CONTENTS

648 649 A EXPERIMENTAL DETAILS

A.1 DATASETS AND MODELS

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652 Our research primarily focuses on evaluating the effectiveness of our proposed methods and various baselines on widely recognized FL datasets. These include Federated MNIST (FedMNIST) and Federated CIFAR10 (FedCIFAR10), which are benchmarks in the field. The use of the terms FedMNIST and FedCIFAR10 is intentional to distinguish our federated training approach from the centralized training methods typically used with MNIST and CIFAR10. The MNIST dataset consists of 60,000 samples distributed across 100 clients using a Dirichlet distribution. For this dataset, we employ a three-layer Multi-Layer Perceptron (MLP) as our default model. CIFAR10, also comprising 60,000 samples, is utilized in our experiments to conduct various ablation studies. The default setting for our FedCIFAR10 experiments is set with 10 clients. The model chosen for CIFAR10 is a Convolutional Neural Network (CNN) consisting of 2 convolutional layers and 3 fully connected layers (FCs). The network architecture is chosen in alignment with [\(Zeng et al.,](#page-10-18) [2023\)](#page-10-18).

A.2 TRAINING DETAILS

670 671 Our experimental setup involved the use of NVIDIA A100 or V100 GPUs, allocated based on their availability within our computing cluster. We developed our framework using PyTorch version 1.4.0 and torchvision version 0.5.0, operating within a Python 3.8 environment. The FedLab framework [\(Zeng et al.,](#page-10-18) [2023\)](#page-10-18) was employed for the implementation of our code. For the FedMNIST dataset, we established the default number of global iterations at 500, whereas for the FedCIFAR10 dataset, this number was set at 2500. We conducted a comprehensive grid search for the optimal learning rate, exploring values within the range of [0.005, 0.01, 0.05, 0.1]. Our intention is to make the code publicly available upon the acceptance of our work.

Figure 10: Data distribution with different Dirichlet factors on CIFAR10 distributed over 100 clients

B COMPLEMENTARY EXPERIMENTS

B.1 EXPLORING HETEROGENEITY

B.1.1 VISUALIZATION OF HETEROGENEITY

700 701 The Dirichlet non-iid model serves as our primary means to simulate realistic FL scenarios. Throughout this paper, we extensively explore the effects of varying the Dirichlet factor α and examine how our algorithms perform under different degrees of data heterogeneity. In Figure [10,](#page-12-6) we

702 703 704 705 706 707 present a visualization of the class distribution in the FedCIFAR10 dataset. We visualize the first 10 clients. This illustration clearly demonstrates that a smaller α results in greater data heterogeneity, with $\alpha = 1000$ approaching near-homogeneity. To further our investigation, we conduct thorough ablation studies using values of α in the range of [0.1, 0.3, 0.5, 0.7, 0.9, 1.0]. It is important to note that an α value of 1.0, while on the higher end of our test spectrum, still represents a heterogeneous data distribution.

B.1.2 INFLUENCE OF HETEROGENEITY WITH NON-COMPRESSED MODELS

712 713 714 715 716 717 In our previous analyses, the impact of sparsified models with a sparsity factor $K = 10\%$ was illustrated in Figure [2,](#page-4-0) and the effects of quantized models were depicted in Figure [6.](#page-6-0) Extending this line of inquiry, we now present additional experimental results that explore the influence of data heterogeneity on models with $K = 50\%$ and those without compression, as shown in Figure [11.](#page-13-0) Our findings indicate that while model compression can result in slower convergence rates, it also potentially reduces the total communication cost, thereby enhancing overall efficiency. Notably, a Dirichlet factor of $\alpha = 0.1$ creates a highly heterogeneous setting, impacting both the speed of convergence and the final accuracy, with results being considerably inferior compared to other degrees of heterogeneity.

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735 736 737 738 739 Figure 11: Exploration of variations in loss and accuracy across diverse sparsity ratios, communication rounds, and communicated bits is depicted through our figures. The first set of four figures on the left showcases results obtained with a sparsity ratio of $K = 50\%$. In contrast, the corresponding set on the right, consisting of another four figures, represents scenarios where $K = 100\%$, indicative of scenarios without model compression.

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B.2 COMPLEMENTARY QUANTIZATION RESULTS

B.2.1 ADDITIONAL QUANTIZATION RESULTS ON FEDMNIST

In Figure [6,](#page-6-0) we presented the quantization results in terms of communicated bits. For completeness, we also display the results with respect to communication rounds in Figure [13.](#page-14-0)

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B.2.2 QUANTIZATION ON FEDCIFAR10

752 753 754 755 Previously, in Figure [5,](#page-6-0) we detailed the outcomes of applying quantization to the FedMNIST dataset. This section includes an additional series of experiments conducted on the FedCIFAR10 dataset. The results of these experiments are depicted in Figure [14.](#page-14-1) Consistent with our earlier findings, we observe that quantization considerably reduces communication costs with only a marginal decline in performance.

⁰ 50 100 150 200 Communication rounds 0.0 0.5 $1.0 +$ 1.5 ვ
9 2.0 2.5 3.0 **SS**
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Fraining $\alpha = 0.1$ $\alpha = 0.3$ $\alpha = 0.5$ ® = 0.7 $\alpha = 0.9$ $\alpha = 1.0$ 0 50 100 150 200 Communication rounds $\frac{5}{6}$ 0.2 e.
ding $0.6 +$ င္ပဲ ၀.၊ 1.0 Testing Accuracy
Testing 0.4
0.2 $\alpha = 0.1$ 0.3 ® = 0.5 $= 0.7$ $\alpha = 0.9$ $\alpha = 1.0$ 0.0 0.5 1.0 1.5 2.0 Communication Bits1e8 0.5 1.0 $\overline{2}$ 1.5 Training Loss
Training
1.0
1.0 2.5 $3.0 +$ $\alpha = 0.1$ 0.3 ® = 0.5 ® = 0.7 α c $\alpha = 1.0$ 0.0 0.5 1.0 1.5 2.0 Communication Bits1e8 $0.2 + 1$ $0.4 +$ 0.6 0.8 Testing Accuracy $_{0.1}$ ® = 0.3 $\alpha = 0.5$ $\alpha = 0.7$ α $\alpha = 1.0$ 0 50 100 150 Communication rounds 0.0 0.5 1.0 1.5 $\frac{8}{9}$ 2.0 2.5 SS
0
Training 1.5
F $\alpha = 0.1$ $\alpha = 0.3$ 0.5 $\alpha = 0.7$ \circ 0.9 $\alpha = 1.0$ 0 50 100 150 Communication rounds $0.2 0.4 +$ 0.6 င္မွာ ၀.8
၉ 1.0 Testing Accuracy
Testing 0.4
0.2 $\alpha = 0.1$ 0.3 0.5 0.7 $\alpha = 0.9$ $\alpha = 1.0$ 0.0 0.5 1.0 1.5 Communication Bits1e8 0.5 1.0 1.5 $2₀$ $\overline{2}$. Training Loss $\alpha = 0.1$ $\alpha = 0.3$ $\alpha = 0.5$ $\alpha = 0.7$ $\alpha = 0.9$ $\alpha = 1.0$ 0.0 0.5 1.0 1.5 Communication Bits1e8 $0.2 +$ $0.4 +$ $0.6 +$ $0.8 +$ Testing Accuracy $\alpha = 0.1$ 0.3 0.5 ® = 0.7 $\alpha = 0.9$ $\alpha = 1.0$

Tegence 15. Fedocomboth dimension $Q_{\text{F}}(\cdot)$ with a fixed 7 value of 0 (as shown in the feat figure) and 10 (in the right figure) with respected to communication rounds. We conduct ablations across various α . Figure 13: FedComLoc utilizing $Q_r(\cdot)$ with a fixed r value of 8 (as shown in the left figure) and 16 α.

Figure 14: FedComLoc with $Q_r(\cdot)$ on CIFAR10.

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1.0

1.0 Training Loss

