000 FEDCOMLOC: COMMUNICATION-EFFICIENT DIS-001 TRIBUTED TRAINING OF SPARSE AND QUANTIZED 002 003 MODELS 004

Anonymous authors

005 006

007

008 009 010

011 012

013

014

015

016

017

018

019

021

024 025 026

027

031

034 035 Paper under double-blind review

ABSTRACT

Federated Learning (FL) has garnered increasing attention due to its unique characteristic of allowing heterogeneous clients to process their private data locally and interact with a central server, while being respectful of privacy. A critical bottleneck in FL is the communication cost. A pivotal strategy to mitigate this burden is Local Training, which involves running multiple local stochastic gradient descent iterations between communication phases. Our work is inspired by the innovative Scaffnew algorithm of Mishchenko et al. (2022), which has considerably advanced the reduction of communication complexity in FL. We introduce FedComLoc (Federated Compressed and Local Training), integrating practical and effective compression into Scaffnew to further enhance communication efficiency. Extensive experiments, using the popular TopK compressor and quantization, demonstrate its prowess in substantially reducing communication overheads in heterogeneous settings.

1 INTRODUCTION

028 Privacy concerns and limited computing resources on edge devices often make centralized train-029 ing impractical, where all data is gathered in a data center for processing. In response to these challenges, Federated Learning (FL) has emerged as an increasingly popular framework (McMahan et al., 2016; Kairouz et al., 2019). In FL, multiple clients perform computations locally on their private data and exchange information with a central server. This process is typically framed as an empirical risk minimization problem (Shaley-Shwartz & Ben-David, 2014): 033

$$\min_{\substack{\in \mathbb{R}^d}} \left[f(x) \coloneqq \frac{1}{n} \sum_{i=1}^n f_i(x) \right],$$
(ERM)

037 where f_i represents the local objective for client i, n is the total number of clients, and x is the model 038 to be optimized. Our primary objective is to solve the problem (ERM) and deploy the optimized global model to all clients. For instance, x might be a neural network trained in an FL setting. However, a considerable bottleneck in FL are communication cost, particularly with large models. 040

r

⁰⁴¹ To mitigate these costs, FL often employs Local Training (LT), a strategy where local parameters 042 are updated multiple times before aggregation (Povey et al., 2014; Moritz et al., 2016; McMahan 043 et al., 2017; Li et al., 2020; Haddadpour & Mahdavi, 2019; Khaled et al., 2019; 2020; Karimireddy 044 et al., 2020; Gorbunov et al., 2020; Mitra et al., 2021). However, there is a lack of theoretical understanding regarding the effectiveness of LT methods. The recent introduction of Scaffnew by 045 Mishchenko et al. (2022) marked a substantial advancement, as this algorithm converges to the exact 046 solution with accelerated complexity, in convex settings. 047

⁰⁴⁸ Another approach to reducing communication costs is through compression (Haddadpour et al., 2021; Condat et al., 2022; Yi et al., 2024). In centralized training, one often aims to learn a sparsified model for faster training and communication efficiency (Dettmers & Zettlemoyer, 2019; Kuznedelev 051 et al., 2023). Dynamic pruning strategies like gradual magnitude pruning (Gale et al., 2019) and RigL (Evci et al., 2020) are common. But in FL, the effectiveness of these model sparsification 052 methods based on thresholds remains unclear. The work by Babakniya et al. (2023) considers FL sparsity mask concepts, showing promising results.

Quantization is another efficient model compression technique (Han et al., 2021; Bhalgat et al., 2020; Shin et al., 2023), though its application in heterogeneous settings is limited. Gupta et al. (2022) introduced FedAvg with Kurtosis regularization (Chmiel et al., 2020) in FL.

Furthermore, studies such as Haddadpour et al. (2021); Condat et al. (2022) have theoretical convergence guarantees for unbiased estimators with restrictive assumptions. As this work employs the biased TopK compressor these are unsuitable in this case.

061 Thus, we tackle the following question:

Is it possible to design an efficient algorithm combining accelerated local training with compression
 techniques, such as quantization and Top-K, and validate its efficiency empirically on popular FL
 datasets?

Our method for investigating this question consists of two steps. Firstly, we design an algorithm, termed FedComLoc, which integrates general compression into ScaffNew, an accelerated local training algorithm. Secondly, we empirically validate FedComLoc for popular compression techniques (Top*K* and quantization) on popular FL datasets (FedMNIST and FedCIFAR10).

- We were able to answer this question affirmatively with the following contributions:
 - We have developed a communication-efficient method FedComLoc for distributed training, specifically designed for heterogeneous environments. This method integrates general compression techniques and is motivated by previous theoretical insights.
 - We proposed three variants of our algorithm addressing several key bottlenecks in FL: FedComLoc-Com addresses communication costs from client to server, FedComLoc-Global addresses communication costs from server to client and FedComLoc-Local addresses limited computational resources on edge devices.
 - We conducted detailed comparisons and ablation studies, validating the effectiveness of our approach. These reveal a considerable reduction in communication and, in certain cases, an enhancement in training speed in number of communication rounds. Furthermore, we demonstrated that our method outperforms well-established baselines in terms of training speed and communication costs.
- 082 083 084 085

071

073

075

076

077

078

079

081

2 RELATED WORK

087 2.1 LOCAL TRAINING

The evolution of LT in FL has been profound and continuous, transitioning through five distinct generations, each marked by considerable advancements from empirical discoveries to reductions in 090 communication complexity. The pioneering FedAvg algorithm (McMahan et al., 2017) represents 091 the first generation of LT techniques, primarily focusing on empirical evidence and practical appli-092 cations (Povey et al., 2014; Moritz et al., 2016; McMahan et al., 2017). The second generation of LT 093 methods consists in solving (ERM) based on homogeneity assumptions such as bounded gradients¹ 094 (Li et al., 2020) or limited gradient diversity² (Haddadpour & Mahdavi, 2019). However, the practicality of such assumptions in real-world FL scenarios is debatable and often not viable (Kairouz et 095 al., 2019; Wang et al., 2021). 096

⁰⁹⁷ Third-generation methods made fewer assumptions, demonstrating sublinear (Khaled et al., 2019; ⁰⁹⁸ 2020) or linear convergence up to a neighborhood (Malinovsky et al., 2020) with convex and smooth ⁰⁹⁹ functions. More recently, fourth-generation algorithms like Scaffold (Karimireddy et al., 2020), S-¹⁰⁰ Local-GD (Gorbunov et al., 2020), and FedLin (Mitra et al., 2021) have gained popularity. These ¹⁰¹ algorithms effectively counteract client drift and achieve linear convergence to the exact solution ¹⁰² under standard assumptions. Despite these advances, their communication complexity mirrors that ¹⁰³ of GD, i.e. $\mathcal{O}(\kappa \log \epsilon^{-1})$, where $\kappa := L/\mu$ denotes the condition number.

The most recent Scaffnew algorithm, proposed by Mishchenko et al. (2022), revolutionizes the field with its accelerated communication complexity $\mathcal{O}(\sqrt{\kappa}\log \epsilon^{-1})$. This seminal development estab-

106 107

¹There exists $c \in \mathbb{R}$ s.t. $\|\nabla f_i(x)\| \le c$ for $1 \le i \le d$.

²There exists $c \in \mathbb{R}$ s.t. $\frac{1}{n} \sum_{i=1}^{n} \|\nabla f_i(x)\|^2 \le c \|\nabla f(x)\|^2$.

Algorithm 1 FedComLoc

109 1: stepsize $\gamma > 0$, probability p > 0, initial iterate $x_{1,0} = \cdots = x_{n,0} \in \mathbb{R}^d$, initial control variates $h_{1,0}, \ldots, h_{n,0} \in \mathbb{R}^d$ on each client such that $\sum_{i=1}^n h_{i,0} = 0$, number of iterations $T \ge 1$, 110 111 compressor $C(\cdot) \in {Top}K(\cdot), Q_r(\cdot), \cdots$ 112 2: server: flip a coin, $\theta_t \in \{0, 1\}, T$ times, where $\operatorname{Prob}(\theta_t = 1) = p$ \diamond Decide when to skip 113 communication 114 3: send the sequence $\theta_0, \ldots, \theta_{T-1}$ to all workers 115 4: for $t = 0, 1, \dots, T - 1$ do 116 5: sample clients $S \subseteq \{1, 2, 3, \ldots, n\}$ 117 in parallel on all workers $i \in S$ do 6: 118 FedComLoc-Local: local compression $-g_{i,t}(x_{i,t}) = g_{i,t}(C(x_{i,t}))$ 7: 119 $\hat{x}_{i,t+1} = x_{i,t} - \gamma(g_{i,t}(x_{i,t}) - h_{i,t})$ \diamond Local gradient-type step adjusted via the local 8: 120 control variate $h_{i,t}$ 121 FedComLoc-Com: uplink compression – $\hat{x}_{i,t+1} = C(\hat{x}_{i,t+1})$ 9: 122 if $\theta_t = 1$ then 10: 123 $x_{i,t+1} = \frac{1}{n} \sum_{i=1}^{n} \hat{x}_{i,t+1}$ 124 \diamond Average the iterates (with small probability *p*) 11: 125 FedComLoc-Global: downlink compression – $x_{i,t+1} = C(x_{i,t+1})$ 12: 126 127 13: else $x_{i,t+1} = \hat{x}_{i,t+1}$ end if 14: ♦ Skip communication 128 15: 129 $h_{i,t+1} = h_{i,t} + \frac{p}{\gamma}(x_{i,t+1} - \hat{x}_{i,t+1})$ 16: \diamond Update the local control variate $h_{i,t}$ 130 end local updates 17: 131 18: end for 132

133 134

108

lishes LT as a communication acceleration mechanism for the first time, positioning Scaffnew at 135 the forefront of the fifth generation of LT methods with accelerated convergence. Further enhance-136 ments to Scaffnew have been introduced, incorporating aspects like variance-reduced stochastic gra-137 dients (Malinovsky et al., 2022), personalization (Yi et al., 2023), partial client participation (Condat 138 et al., 2023), asynchronous communication (Maranjyan et al., 2022), and an expansion into a broader 139 primal-dual framework (Condat & Richtárik, 2023). This latest generation also includes the 5GCS 140 algorithm (Grudzień et al., 2023), with a different strategy where the local steps are part of an inner 141 loop to approximate a proximity operator. Our proposed FedComLoc algorithm extends Scaffnew 142 by incorporating pragmatic compression techniques, such as sparsity and quantization, resulting in 143 even faster training measured by the number of bits communicated.

144 145

146

2.2 MODEL COMPRESSION IN FEDERATED LEARNING

147 Model compression in the context of FL is a burgeoning field with diverse research avenues, par-148 ticularly focusing on the balance between model efficiency and performance. Jiang et al. (2022) 149 innovated in global pruning by engaging a single, powerful client to initiate the pruning process. 150 This strategy transitions into a collaborative local pruning phase, where all clients contribute to an 151 adaptive pruning mechanism. This involves not just parameter elimination, but also their reintro-152 duction, integrated with the standard FedAvg framework (McMahan et al., 2016). However, this approach demands substantial local memory for tracking the relevance of each parameter, a con-153 straint not always feasible in FL settings. 154

Addressing some of these challenges, Huang et al. (2022) introduced an adaptive batch normalization coupled with progressive pruning modules, enhancing sparse local computations. These advancements, however, often do not fully address the constraints related to computational resources and communication bandwidth on the client side. Our research primarily focuses on magnitudebased sparsity pruning. Techniques like gradual magnitude pruning (Gale et al., 2019) and RigL (Evci et al., 2020) have been instrumental in dynamic pruning strategies. However, their application in FL contexts remains relatively unexplored. The pioneering work of Babakniya et al. (2023) extends the concept of sparsity masks in FL, demonstrating noteworthy outcomes. 162 Quantization is another vital avenue in model compression. Seminal works in this area include Han 163 et al. (2021), Bhalgat et al. (2020), and Shin et al. (2023). A major advance has been made by Gupta 164 et al. (2022), who combined FedAvg with Kurtosis regularization (Chmiel et al., 2020). We are 165 looking to go even further by integrating accelerated LT with quantization techniques.

166 However, a gap exists in the theoretical underpinnings of these compression methods. Research 167 by Haddadpour et al. (2021) and Condat et al. (2022) offers theoretical convergence guarantees for 168 unbiased estimators, but these frameworks are not readily applicable to common compressors like 169 Top-K sparsifiers. In particular, CompressedScaffnew (Condat et al., 2022) integrates an unbiased 170 compression mechanism in Scaffnew, that is based on random permutations. But due to requiring 171 shared randomness it is not practical. Linear convergence has been proved when all functions f_i are 172 strongly convex.

173 To the best of our knowledge, no other compression mechanism has been studied in Scaffnew, ei-174 ther theoretically or empirically, and even the mere convergence of Scaffnew in nonconvex settings 175 has not been investigated either. Our goal is to go beyond the convex setting and simplistic logistic 176 regression experiments and to study compression in Scaffnew in realistic nonconvex settings with 177 large datasets such as Federated CIFAR and MNIST. Our integration of compression in Scaffnew is 178 heuristic but backed by the findings and theoretical guarantees of CompressedScaffnew in the con-179 vex setting, which shows a twofold acceleration with respect to the conditioning κ and the dimension d, thanks to LT and compression, respectively. 180

181 182

183 184

185

187

188

189 190

193

194

195 196 197

198

205

208

209

3 PROPOSED ALGORITHM FedComLoc

3.1 SPARSITY AND QUANTIZATION

Let us define the sparsifying $\text{Top}K(\cdot)$ and quantization $Q_r(\cdot)$ operators. 186

Definition 3.1. Let $x \in \mathbb{R}^d$ and $K \in \{1, 2, \dots, d\}$. We define the sparsifying compressor Top K: $\mathbb{R}^d \to \mathbb{R}^d$ as:

$$\operatorname{Top} K(x) \coloneqq \arg \min_{y \in \mathbb{R}^d} \left\{ \|y - x\| \mid \|y\|_0 \le K \right\},\$$

where $||y||_0 := |\{i : y_i \neq 0\}|$ denotes the number of nonzero elements in the vector $y = (y_1, \dots, y_d)^{\mathsf{T}} \in \mathbb{R}^d$. In case of multiple minimizers, TopK is chosen arbitrarily. 191 192

Definition 3.2. For any vector $x \in \mathbb{R}^d$, with $x \neq 0$ and a number of bits r > 0, its binary quantization $Q_r(x)$ is defined componentwise as

$$Q_{r}(x) = (\|x\|_{2} \cdot \text{sgn}(x_{i}) \cdot \xi_{i}(x, 2^{r}))_{1 \le i \le d},$$

where $\xi_i(x, 2^r)$ are independent random variables. Let $y_i := \frac{|x_i|}{||x||_2}$. Then their probability distribution is given by

$$\xi_i(x, 2^r) = \begin{cases} \left\lceil 2^r y_i \right\rceil / 2^r & \text{with proba. } 2^r y_i - \left\lfloor 2^r y_i \right\rfloor; \\ \left\lfloor 2^r y_i \right\rfloor / 2^r & \text{otherwise.} \end{cases}$$

If x = 0, we define $Q_r(x) = 0$. 203

204 The distributions of the $\xi_i(x,r)$ minimize variance over distributions with support $\{0, 1/r, \ldots, 1\}$, ensuring unbiasedness, i.e. $\mathbb{E}[\xi_i(x,r)] = |x_i| / ||x||_2$. This definition is based on an equivalent one 206 in Alistarh et al. (2017). 207

3.2 INTRODUCTION OF THE ALGORITHMS

210 FedComLoc (Algorithm 1) is an adaptation of Scaffnew, with modifications for compression. 211 $Top K(\cdot)$ is used as the default compression technique for simplicity, although quantization is 212 equally applicable. We examine three distinct variants in our ablations:

213

• FedComLoc-Com: This variant addresses the communication bottleneck. It focuses on 214 compressing the uplink network weights transmitted from each client to the server. This 215 setup is adopted as our default setting.





Figure 1: Performance outcomes for various Top-K ratios.

Figure 2: Training loss and test accuracy for a density ratio of K = 10%.

10%.

• FedComLoc-Local: Here, the local model is compressed during each training step. This addresses limited computational power and resources available to each client.

100%.

 FedComLoc-Global: Here, the server model is compressed before sending it to the clients. This variant is tailored for FL situations where downloading the model from the server is costly e.g. due to network bandwidth constraints.

EXPERIMENTS

Baselines. Our evaluation comprises three distinct aspects. Firstly, we conduct experiments to assess the impact of compression on communication costs. FedComLoc is assessed for varying sparsity and quantization ratios. Secondly, we compare FedComLoc-Com with FedComLoc-Local and FedComLoc-Global. Thirdly, we explore the efficacy of FedComLoc against non-accelerated local training methods, including FedAvg (McMahan et al., 2016), its Top-K sparsified counterpart sparseFedAvg, and Scaffold (Karimireddy et al., 2020).

Datasets. Our experiments are conducted on FedMNIST (LeCun, 1998) and FedCI-FAR10 (Krizhevsky et al., 2009) with the data processing framework FedLab (Zeng et al., 2023). For FedMNIST, we employ MLPs with three fully-connected layers, each coupled with a ReLU activation function. For FedCIFAR10, we utilize CNNs with two convolutional layers and three fully-connected layers. Comprehensive statistics for each dataset, details on network architecture and training specifics can be found in Appendix A.

Heterogeneous Setting. We explore different heterogeneous settings. Similar to (Zhang et al., 2023; Yi et al., 2024), we create heterogeneity in data by using a Dirichlet distribution, which assigns each client a vector indicating class preferences. This vector guides the unique selection of labels and images for each client until all data is distributed. The Dirichlet parameter α indicates the level of non-identical distribution. We also include a visualization of this distribution for the CIFAR10 dataset in Appendix B.1.1.



Tuned Stepsize.

Fixed Stepsize of 0.01.

Figure 3: CNN Performance on the FedCIFAR10 Dataset. For the left most columns, the step size was optimized for each density ratio K. For the two rightmost columns, a fixed stepsize of 0.01 is used. This is the maximum feasible step size which ensures convergence across all configurations.

288 Default Configuration. In the absence of specific clarifications, we adopt the Dirichlet factor 289 $\alpha = 0.7$. To balance both communication and local computation costs, we use p = 0.1, result-290 ing in an average of 10 local iterations per communication round. The learning rate is chosen by 291 conducting a grid search over the set $\{0.005, 0.01, 0.05, 0.1, 0.5\}$. With communication costs being 292 of most interest, our study employs FedComLoc-Com as the default strategy. The experiments are 293 run for 2500 communication rounds for the CNN on FedCIFAR10 and 500 rounds for the MLP on 294 FedMNIST. Furthermore, the dataset is distributed across 100 clients from which 10 are uniformly 295 chosen to participate in each global round.

Furthermore, in our Definition 3.1 of Top*K*, *K* is the number of nonzero parameters. However, we will rather specify the enforced density ratio, i.e. the ratio of nonzero parameters. For instance, specifying K = 30% means retaining 30% of parameters.

299 300 301

281

283

284

285

286 287

4.1 TOP-K SPARSITY RATIOS

This section investigates the effects of different sparsity rations by investigating Top*K* ratios on FedMNIST. The outcomes can be found in Table 1. Notably, K = 10% in Top*K* yields an accuracy of 0.9374, merely 3.94% lower than the 0.9758 unsparsified baseline. Remarkably, a 70% sparsity level (K = 30%) attains commendable performance, with only a 1.07% accuracy reduction, alongside a 70% reduction in communication costs. Furthermore, from the communication bits depicted in Figure 1 it is evident that sparsity yields faster convergence, the more so with increased sparsity (smaller *K*).

309 310

4.2 DATA HETEROGENEITY/DIRICHLET FACTORS

This subsection aims to assess the impact of varying degrees of data heterogeneity on FedMNIST. Hence, an analysis of the Dirichlet distribution factor α is presented, exploring the range of values $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9, 1.0\}$. Remember that a lower α means increased heterogeneity. Alongside, we examine the influence of different Top K factors, specifically 10%, 50% and 100%. The results are shown in Table 2. Figure 2 reports training loss and test accuracy for a sparsity ratio of 90% (K = 10%). Additionally, round-wise visualizations for K = 50% and K = 100% (nonsparse) are presented in Figure 11 in the Appendix.

319 Key observations from our study include:

a) When examining the effects of the heterogeneity degree α (as seen in each column of Table 2), we observe that sparsity performance is influenced by heterogeneity degrees. For instance, $\alpha = 0.1$ results in a relative performance drop of 9.79% from an unsparsified to a sparsified model with K = 10%. In contrast, for $\alpha = 0.3$, this drop is 5.80%, and for $\alpha = 1.0$, it is 3.63%. Interestingly, for commonly used heterogeneity ratios in literature ($\alpha = 0.3, 0.5, 0.7$), the performance drop does

324





Figure 5: FedComLoc employing $Q_r(\cdot)$. The number of quantization bits r is set to $r \in \{4, 8, 16, 32\}$.

Figure 6: Data heterogeneity ablations for Fed-ComLoc utilizing $Q_r(\cdot)$ with number of bits r either 8 or 16. The same results plotted over the number of communication rounds can be found in Figure 13 in the Appendix.

not decrease substantially when moving from $\alpha = 0.3$ to $\alpha = 0.5$, or from $\alpha = 0.5$ to $\alpha = 0.7$, unlike the shift from $\alpha = 0.1$ to $\alpha = 0.3$.

b) Focusing on the rows of Table 2, we find that lower sparsity ratios are more sensitive to heterogeneous distributions. In particular, observe that with K = 10%, the absolute performance improvement from $\alpha = 0.1$ to $\alpha = 1$ is 7.01%. However, for K = 50%, this improvement is only 1.22%.

c) It should be noted that each method was run with a fixed learning rate without scheduling, and
the maximum communication round was set to 1000. Previous studies suggest that higher sparsity ratios require more communication rounds in centralized settings (Kuznedelev et al., 2023).
This phenomenon was also observed in our FL experiments. Therefore, there is the potential for
performance enhancement through sufficient model rounds and adaptive learning rate adjustments,
especially for methods with higher sparsity.

361 4.3 CNNs on FedCIFAR10

362 This section repeats the experiments for CIFAR10 and a Convolutional Neural Network (CNN). We 363 explored a range of stepsizes ($\gamma \in \{0.005, 0.01, 0.05, 0.1|\}$). Further information is provided in Ap-364 pendix A. The CIFAR10 results, which involve optimizing a Convolutional Neural Network (CNN), 365 are presented in Figure 3 for both tuned and a fixed step size. Observe the accelerated convergence 366 of sparsified models in terms of communicated bits when the step size is tuned. Interestingly, a 367 sparsity of 90% (K = 10%) shows faster convergence in terms of communication rounds (as shown 368 in the first column), suggesting the potential for enhanced training efficiency in sparsified models. 369 For a fixed step size (the two rightmost columns of Figure 3) and K = 10%, one can observe slower 370 convergence compared to other configurations. This indicates that sparsity training requires more data and benefits from either increased communication rounds or a larger initial stepsize. This aligns 371 with recent similar findings in the centralized setting (Kuznedelev et al., 2023). 372

373

360

374 4.4 QUANTIZATION

375

This section explores using quantization Q_r for compression with the number of bits, r, set to $r \in \{4, 8, 16, 32\}$ on FedMNIST. This approach aligns with the methodologies outlined in Alistarh et al. (2017). The results after 1000 communication rounds are illustrated in Figure 5. Our



Figure 7: Loss and test accuracy over communication rounds and total costs. Total costs are a combined measurement of both communication costs and local computation cost. A communication round has unit cost while a local training round has cost τ . In a realistic FL system, τ is typically much less than 1, as the primary bottleneck is often communication and hence we set $\tau = 0.01$.

393

394

395

396

397

398

399

415

416 417 418

419

420

421

422

423 424 425



Figure 8: Comparison among FedAvg, Scaffold, FedDyn, FedComLoc-Local, FedComLoc-Com, and FedComLoc-Global. First column: K = 50%; second column: K = 100% (no sparsity).



Figure 9: Sparsity ablation studies of FedComLoc-Local, FedComLoc-Com, and FedComLoc-Global on FedCIFAR10 and tuned stepsizes.

analysis reveals that quantization offers superior performance compared to TopK-style sparsity. For instance, with 16-bit quantization corresponding to a 50% reduction in communication cost, the performance decrease is a mere 0.14%, Furthermore, Figure 6 shows outcomes for different degrees of data heterogeneity. These findings demonstrate that quantization reduces communication at minor performance tradeoff while also exhibiting only minor sensitivity to data heterogeneity. The Appendix gives further results for both FedMNIST (section B.2.1) and FedCIFAR10 (section B.2.2).

4.5 NUMBER OF LOCAL ITERATIONS

This section explores the performance impact of varying the expected number of local iterations on FedMNIST. The expected number of local iterations is 1/p where p is the communication probability. Hence, we investigate the influence of p ranging from $p \in \{0.05, 0.1, 0.2, 0.3, 0.5\}$. Furthermore, K = 30% is used. The results are presented in Figure 7. A key finding is that more local training rounds (i.e. smaller p) not only accelerate convergence but can also improve the final performance.

432 4.6 FEDCOMLOC VARIANTS

434 In this section, we compare FedComLoc-Local, FedComLoc-Com, and FedComLoc-Global on Fed-CIFAR10. The findings are illustrated in Figure 9. Observe that at high levels of sparsity (indicated 435 by a small K in TopK), FedComLoc-Com underperforms the other algorithms. This could be at-436 tributed to the heterogeneous setting of our experiment: each client's model output is inherently 437 skewed towards its local dataset. When this is coupled with extreme Top K sparsification, more bias 438 is introduced, which adversely affects performance. Conversely, at low sparsity (e.g. K = 90%), 439 FedComLoc-Com surpasses FedComLoc-Global. In addition, we observe that sparsity during local 440 training (i.e. FedComLoc-Local) tends to yield better results. One possible explanation is that due 441 to the local data bias the communication bandwidth between client and server might be crucial. Re-442 member that FedComLoc-Local had no communication compression while both FedComLoc-Com 443 and FedComLoc-Global do. Further FedMNIST results are shown in the Appendix. 444

4.7 FEDAVG AND SCAFFOLD

447 In this section, the performance of FedComLoc is compared with baselines in form of FedAvg (McMahan et al., 2016) and Scaffold (Karimireddy et al., 2020) on FedCIFAR10. Further-448 more, a sparsified version of FedAvg is employed, termed as sparseFedAvg. For sparseFedAvg a 449 learning rate of 0.1 is used, whereas for FedComLoc, a lower rate of 0.05 is utilized. The outcomes 450 of this analysis are depicted in Figure 8. The left part illustrates the performance of compressed 451 models. We observe notably faster convergence for FedComLoc-type methods in comparison with 452 sparseFedAvg despite the lower learning rate. The right part of the figure compares FedAvg with 453 Scaffold, devoid of sparsity, using an identical learning rate of 0.005. This uniform rate ensures 454 that each method achieves satisfactory convergence over a sufficient number of epochs. Here again, 455 faster convergence is demonstrated with FedComLoc.

456 457 458

445

446

5 CONCLUSION AND FUTURE WORK

This paper advances the field of FL by tackling one of its main challenges, namely its high communication cost. Building on the accelerated Scaffnew algorithm (Mishchenko et al., 2022), we introduced FedComLoc. This novel approach blends the practical compression techniques of model sparsity and quantization into the efficient local training framework. Our extensive experimental validation shows that FedComLoc preserves computational integrity while notably cutting communication costs.

Future research could explore the reduction of internal variance in stochastic gradient estimation, akin to the approach described in Malinovsky et al. (2022). The FedComLoc-Global algorithm we propose offers potential for obtaining a sparsified model suitable for deployment. Additionally, investigating the development of an efficiently sparsified deployed model extensively presents an intriguing avenue for further study.

471 472 REFERENCES

- 473 Dan Alistarh, Demjan Grubic, Jerry Li, Ryota Tomioka, and Milan Vojnovic. QSGD:
 474 Communication-efficient SGD via gradient quantization and encoding. In *Advances in Neural*475 *Information Processing Systems*, volume 30. Curran Associates, Inc., 2017.
- 476
 477
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
 478
- Yash Bhalgat, Jinwon Lee, Markus Nagel, Tijmen Blankevoort, and Nojun Kwak. Lsq+: Improving
 low-bit quantization through learnable offsets and better initialization. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops*, pp. 696–697, 2020.
- Brian Chmiel, Ron Banner, Gil Shomron, Yury Nahshan, Alex Bronstein, Uri Weiser, et al. Robust quantization: One model to rule them all. *Advances in neural information processing systems*, 33:5308–5317, 2020.

499

500

501

523

524

525

526

527

528 529

530

531

532

533

534

- L. Condat and P. Richtárik. RandProx: Primal-dual optimization algorithms with randomized proximal updates. In *Proc. of Int. Conf. Learning Representations (ICLR)*, 2023.
- L. Condat, I. Agarský, and P. Richtárik. Provably doubly accelerated federated learning: The first theoretically successful combination of local training and compressed communication. *preprint arXiv:2210.13277*, 2022.
- 492
 493
 493
 493
 494
 494
 495
 495
 495
 496
 497
 498
 498
 498
 499
 499
 499
 490
 490
 491
 491
 492
 493
 494
 495
 494
 495
 494
 495
 494
 495
 494
 495
 495
 495
 494
 495
 495
 495
 494
 495
 495
 495
 495
 495
 496
 497
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
 498
- Tim Dettmers and Luke Zettlemoyer. Sparse networks from scratch: Faster training without losing
 performance. *preprint arXiv:1907.04840*, 2019.
 - Utku Evci, Trevor Gale, Jacob Menick, Pablo Samuel Castro, and Erich Elsen. Rigging the lottery: Making all tickets winners. In *International Conference on Machine Learning*, pp. 2943–2952. PMLR, 2020.
- Trevor Gale, Erich Elsen, and Sara Hooker. The state of sparsity in deep neural networks. *preprint arXiv:1902.09574*, 2019.
- E. Gorbunov, F. Hanzely, and P. Richtárik. Local SGD: Unified theory and new efficient methods. In *Proc. of Conf. Neural Information Processing Systems (NeurIPS)*, 2020.
- M. Grudzień, G. Malinovsky, and P. Richtárik. Can 5th Generation Local Training Methods Support Client Sampling? Yes! In *Proc. of Int. Conf. Artificial Intelligence and Statistics (AISTATS)*, April 2023.
- Kartik Gupta, Marios Fournarakis, Matthias Reisser, Christos Louizos, and Markus Nagel. Quantization robust federated learning for efficient inference on heterogeneous devices. *preprint arXiv:2206.10844*, 2022.
- F. Haddadpour and M. Mahdavi. On the convergence of local descent methods in federated learning.
 preprint arXiv:1910.14425, 2019.
- Farzin Haddadpour, Mohammad Mahdi Kamani, Aryan Mokhtari, and Mehrdad Mahdavi. Federated learning with compression: Unified analysis and sharp guarantees. In *International Conference on Artificial Intelligence and Statistics*, pp. 2350–2358. PMLR, 2021.
- Tiantian Han, Dong Li, Ji Liu, Lu Tian, and Yi Shan. Improving low-precision network quantization
 via bin regularization. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pp. 5261–5270, 2021.
 - Hong Huang, Lan Zhang, Chaoyue Sun, Ruogu Fang, Xiaoyong Yuan, and Dapeng Wu. Fedtiny: Pruned federated learning towards specialized tiny models. *preprint arXiv:2212.01977*, 2022.
 - Yuang Jiang, Shiqiang Wang, Victor Valls, Bong Jun Ko, Wei-Han Lee, Kin K Leung, and Leandros Tassiulas. Model pruning enables efficient federated learning on edge devices. *IEEE Transactions on Neural Networks and Learning Systems*, 2022.
 - P. Kairouz et al. Advances and open problems in federated learning. *Foundations and Trends in Machine Learning*, 14(1–2):1–210, 2019.
 - S. Karimireddy, S. Kale, M. Mohri, S. Reddi, S. Stich, and A. Suresh. SCAFFOLD: Stochastic controlled averaging for on-device federated learning. In *Proc. of Int. Conf. Machine Learning* (*ICML*), 2020.
- A. Khaled, K. Mishchenko, and P. Richtárik. First analysis of local GD on heterogeneous data. paper arXiv:1909.04715, presented at NeurIPS Workshop on Federated Learning for Data Privacy and Confidentiality, 2019.
- 539 A. Khaled, K. Mishchenko, and P. Richtárik. Tighter theory for local SGD on identical and heterogeneous data. In *Proc. of 23rd Int. Conf. Artificial Intelligence and Statistics (AISTATS)*, 2020.

541 2009. 542 Denis Kuznedelev, Eldar Kurtic, Eugenia Iofinova, Elias Frantar, Alexandra Peste, and Dan Al-543 istarh. Accurate neural network pruning requires rethinking sparse optimization. preprint 544 arXiv:2308.02060, 2023. 546 Yann LeCun. The MNIST database of handwritten digits. http://yann. lecun. com/exdb/mnist/, 1998. 547 X. Li, K. Huang, W. Yang, S. Wang, and Z. Zhang. On the convergence of FedAvg on non-IID data. 548 In Proc. of Int. Conf. Learning Representations (ICLR), 2020. 549 G. Malinovsky, D. Kovalev, E. Gasanov, L. Condat, and P. Richtárik. From local SGD to local fixed 550 point methods for federated learning. In Proc. of 37th Int. Conf. Machine Learning (ICML), 2020. 551 552 G. Malinovsky, K. Yi, and P. Richtárik. Variance reduced Proxskip: Algorithm, theory and applica-553 tion to federated learning. In Proc. of Conf. Neural Information Processing Systems (NeurIPS), 554 2022. 555 A. Maranjyan, M. Safaryan, and P. Richtárik. GradSkip: Communication-accelerated local gradient 556 methods with better computational complexity. preprint arXiv:2210.16402, 2022. B. McMahan, E. Moore, D. Ramage, and B. Agüera y Arcas. Federated learning of deep networks 558 using model averaging. preprint arXiv:1602.05629, 2016. 559 H Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Agüera y Arcas. 561 Communication-efficient learning of deep networks from decentralized data. In Proceedings of 562 the 20th International Conference on Artificial Intelligence and Statistics (AISTATS), 2017. 563 K. Mishchenko, G. Malinovsky, S. Stich, and P. Richtárik. ProxSkip: Yes! Local gradient steps provably lead to communication acceleration! Finally! In Proc. of 39th Int. Conf. Machine 565 Learning (ICML), 2022. 566 A. Mitra, R. Jaafar, G. Pappas, and H. Hassani. Linear convergence in federated learning: Tack-567 ling client heterogeneity and sparse gradients. In Proc. of Conf. Neural Information Processing 568 Systems (NeurIPS), 2021. 569 570 P. Moritz, R. Nishihara, I. Stoica, and M. I. Jordan. SparkNet: Training deep networks in Spark. In 571 Proc. of Int. Conf. Learning Representations (ICLR), 2016. 572 D. Povey, X. Zhang, and S. Khudanpur. Parallel training of DNNs with natural gradient and param-573 eter averaging. preprint arXiv:1410.7455, 2014. 574 Shai Shalev-Shwartz and Shai Ben-David. Understanding machine learning: from theory to algo-575 rithms. Cambridge University Press, 2014. 576 577 Juncheol Shin, Junhyuk So, Sein Park, Seungyeop Kang, Sungjoo Yoo, and Eunhyeok Park. Nipq: 578 Noise proxy-based integrated pseudo-quantization. In Proceedings of the IEEE/CVF Conference 579 on Computer Vision and Pattern Recognition, pp. 3852–3861, 2023. 580 J. Wang et al. A field guide to federated optimization. preprint arXiv:2107.06917, 2021. 581 582 Kai Yi, Laurent Condat, and Peter Richtárik. Explicit personalization and local training: Double communication acceleration in federated learning. preprint arXiv:2305.13170, 2023. 583 584 Kai Yi, Nidham Gazagnadou, Peter Richtárik, and Lingjuan Lyu. Fedp3: Federated personalized 585 and privacy-friendly network pruning under model heterogeneity. International Conference on 586 Learning Representations (ICLR), 2024. Dun Zeng, Siqi Liang, Xiangjing Hu, Hui Wang, and Zenglin Xu. FedLab: A flexible federated 588 learning framework. Journal of Machine Learning Research, 24(100):1-7, 2023. URL http: 589 //jmlr.org/papers/v24/22-[]0440.html. 590

Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images.

Hao Zhang, Chenglin Li, Wenrui Dai, Junni Zou, and Hongkai Xiong. Fedcr: Personalized feder ated learning based on across-client common representation with conditional mutual information
 regularization. In *International Conference on Machine Learning*, pp. 41314–41330. PMLR, 2023.

594 595	Contents			
596 597	1	Intr	oduction	1
598 599	2	Rela	ted Work	2
600		2.1	Local Training	2
601 602		2.2	Model Compression in Federated Learning	3
603				
604	3	Prop	osed Algorithm FedComLoc	4
605		3.1	Sparsity and Quantization	4
607		3.2	Introduction of the Algorithms	4
608 609	4	Exp	eriments	5
610		4.1	Top-K Sparsity Ratios	6
611 612		4.2	Data Heterogeneity/Dirichlet Factors	6
613		43	CNNs on FedCIFAR10	7
614		A A	Quantization	, 7
615 616		т.т 4 5		, 0
617		4.5		0
618		4.0	FedComLoc variants	9
619 620		4.7	FedAvg and Scattold	9
621 622	5	Con	clusion and Future Work	9
623 624	A	Exp	erimental Details	13
625		A.1	Datasets and Models	13
626 627		A.2	Training Details	13
628	B	Con	plementary Experiments	13
630		B .1	Exploring Heterogeneity	13
631			B.1.1 Visualization of Heterogeneity	13
632 633			B.1.2 Influence of Heterogeneity with Non-Compressed Models	14
634		R 2	Complementary Quantization Results	14
635		D .2	P 2.1 Additional Quantization Results on FedMNIST	14
636			D.2.1 Additional Quantization Results on Federations 1	14
638			B.2.2 Quantization on FedCIFAR10	14
639		В.З	Double Compression by Sparsity and Quantization	16
640				
641 642				
643				

EXPERIMENTAL DETAILS А 649

A.1 DATASETS AND MODELS

650

651

654

657

659

661

662 663

664

692 693 694

695 696

697

Our research primarily focuses on evaluating the effectiveness of our proposed methods and var-652 ious baselines on widely recognized FL datasets. These include Federated MNIST (FedMNIST) 653 and Federated CIFAR10 (FedCIFAR10), which are benchmarks in the field. The use of the terms FedMNIST and FedCIFAR10 is intentional to distinguish our federated training approach from the 655 centralized training methods typically used with MNIST and CIFAR10. The MNIST dataset consists 656 of 60,000 samples distributed across 100 clients using a Dirichlet distribution. For this dataset, we employ a three-layer Multi-Layer Perceptron (MLP) as our default model. CIFAR10, also compris-658 ing 60,000 samples, is utilized in our experiments to conduct various ablation studies. The default setting for our FedCIFAR10 experiments is set with 10 clients. The model chosen for CIFAR10 is 660 a Convolutional Neural Network (CNN) consisting of 2 convolutional layers and 3 fully connected layers (FCs). The network architecture is chosen in alignment with (Zeng et al., 2023).

A.2 TRAINING DETAILS

Our experimental setup involved the use of NVIDIA A100 or V100 GPUs, allocated based on their 665 availability within our computing cluster. We developed our framework using PyTorch version 666 1.4.0 and torchvision version 0.5.0, operating within a Python 3.8 environment. The FedLab frame-667 work (Zeng et al., 2023) was employed for the implementation of our code. For the FedMNIST 668 dataset, we established the default number of global iterations at 500, whereas for the FedCIFAR10 669 dataset, this number was set at 2500. We conducted a comprehensive grid search for the optimal 670 learning rate, exploring values within the range of [0.005, 0.01, 0.05, 0.1]. Our intention is to make 671 the code publicly available upon the acceptance of our work.



Figure 10: Data distribution with different Dirichlet factors on CIFAR10 distributed over 100 clients

В **COMPLEMENTARY EXPERIMENTS**

EXPLORING HETEROGENEITY **B**.1

698 B.1.1 VISUALIZATION OF HETEROGENEITY 699

The Dirichlet non-iid model serves as our primary means to simulate realistic FL scenarios. 700 Throughout this paper, we extensively explore the effects of varying the Dirichlet factor α and ex-701 amine how our algorithms perform under different degrees of data heterogeneity. In Figure 10, we

702 present a visualization of the class distribution in the FedCIFAR10 dataset. We visualize the first 10 703 clients. This illustration clearly demonstrates that a smaller α results in greater data heterogeneity, 704 with $\alpha = 1000$ approaching near-homogeneity. To further our investigation, we conduct thorough 705 ablation studies using values of α in the range of [0.1, 0.3, 0.5, 0.7, 0.9, 1.0]. It is important to note 706 that an α value of 1.0, while on the higher end of our test spectrum, still represents a heterogeneous data distribution.

B.1.2 INFLUENCE OF HETEROGENEITY WITH NON-COMPRESSED MODELS

In our previous analyses, the impact of sparsified models with a sparsity factor K = 10% was 712 illustrated in Figure 2, and the effects of quantized models were depicted in Figure 6. Extending this line of inquiry, we now present additional experimental results that explore the influence of data 714 heterogeneity on models with K = 50% and those without compression, as shown in Figure 11. Our findings indicate that while model compression can result in slower convergence rates, it also 716 potentially reduces the total communication cost, thereby enhancing overall efficiency. Notably, a Dirichlet factor of $\alpha = 0.1$ creates a highly heterogeneous setting, impacting both the speed of convergence and the final accuracy, with results being considerably inferior compared to other degrees of heterogeneity.



708 709

710 711

713

715

717



Figure 11: Exploration of variations in loss and accuracy across diverse sparsity ratios, communica-736 tion rounds, and communicated bits is depicted through our figures. The first set of four figures on 737 the left showcases results obtained with a sparsity ratio of K = 50%. In contrast, the corresponding 738 set on the right, consisting of another four figures, represents scenarios where K = 100%, indicative 739 of scenarios without model compression.

740 741 742

744

746

747

B.2 **COMPLEMENTARY QUANTIZATION RESULTS** 743

B.2.1 ADDITIONAL QUANTIZATION RESULTS ON FEDMNIST 745

In Figure 6, we presented the quantization results in terms of communicated bits. For completeness, we also display the results with respect to communication rounds in Figure 13.

748 749 750

751

B.2.2 QUANTIZATION ON FEDCIFAR10

752 Previously, in Figure 5, we detailed the outcomes of applying quantization to the FedMNIST dataset. 753 This section includes an additional series of experiments conducted on the FedCIFAR10 dataset. The results of these experiments are depicted in Figure 14. Consistent with our earlier findings, we 754 observe that quantization considerably reduces communication costs with only a marginal decline 755 in performance.



Figure 13: FedComLoc utilizing $Q_r(\cdot)$ with a fixed r value of 8 (as shown in the left figure) and 16 (in the right figure) with respected to communication rounds. We conduct ablations across various α .



Figure 14: FedComLoc with $Q_r(\cdot)$ on CIFAR10.

