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# What Matters in Branch Specialization? Using a Toy Task to Make Predictions

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Anonymous Author(s)

Affiliation

Address

email

## Abstract

1       What motivates the brain to allocate tasks to different regions and what distin-  
2       guishes multiple-demand brain regions and the tasks they perform from ones in  
3       highly specialized areas? Here we explore these neuroscientific questions using a  
4       purely computational framework and theoretical insights. In particular, we focus  
5       on how branches of a neural network learn representations contingent on their  
6       architecture and optimization task. We train branched neural networks on families  
7       of Gabor filters as the input training distribution and optimize them to perform  
8       combinations of angle, average color, and size approximation tasks. We find that  
9       networks predictably allocate tasks to the branches with appropriate inductive  
10      biases. However, this task-to-branch matching is not required for branch spe-  
11      cialization, as even identical branches in a network tend to specialize. Finally,  
12      we show that branch specialization can be controlled by a curriculum in which  
13      tasks are alternated instead of jointly trained. Longer training between alternation  
14      corresponds to more even task distribution among branches, providing a possible  
15      model for multiple-demand regions in the brain.

## 16   1 Introduction

17   Brain specialization has been an active topic in neuroscience for decades, and has helped us discover  
18   many brain regions dedicated to particular tasks across humans despite low-level differences in  
19   plasticity (*eg* the Fusiform Face Area (Kanwisher et al., 1997) that serves a pivotal role for face  
20   recognition; though also see Gauthier et al. (1999); Arcaro et al. (2017, 2020); Hesse & Tsao (2020)).  
21   However, we have yet to build a full explanation of why some "multiple-demand" brain systems  
22   (Fedorenko et al., 2013) are involved in a variety of tasks while others are extremely narrow in scope.  
23   We do not currently know all the factors that distinguish general and specialized brain regions, nor  
24   what causes them to emerge or develop; in addition, they are challenging to study *in vivo* because  
25   biological network architectures cannot be easily modified and paired with the proper controls.  
26   Consequently, we turn to deep learning, where computational cognitive neuroscientists may now  
27   test their ideas on computer models rather than living organisms and can also play the role of a  
28   "virtual neurophysiologist" by inspecting artificial neural network activations (Zeiler & Fergus, 2014;  
29   Kriegeskorte, 2015; Olah et al., 2020; Hamblin & Alvarez, 2021).

30   Indeed, the field of machine learning has shown interest in neural network specialization: Voss et al.  
31   (2021) at OpenAI showed that branching still occurs implicitly in neural networks even when the  
32   network architecture does not explicitly bifurcate. On the neuroscience side, examples of previous  
33   works on branching in audition include Kell et al. (2018) where a branched neural network jointly  
34   trained to learn speech and music learned to correlate well with human auditive behaviour. And in  
35   vision, Dobs et al. (2021) found that jointly training networks on a face and object classification task  
36   resulted in specialized branching along the hierarchy of the CNNs.

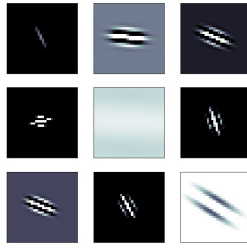


Figure 1: Sample images generated for the Gabor dataset. See Appendix A for details.

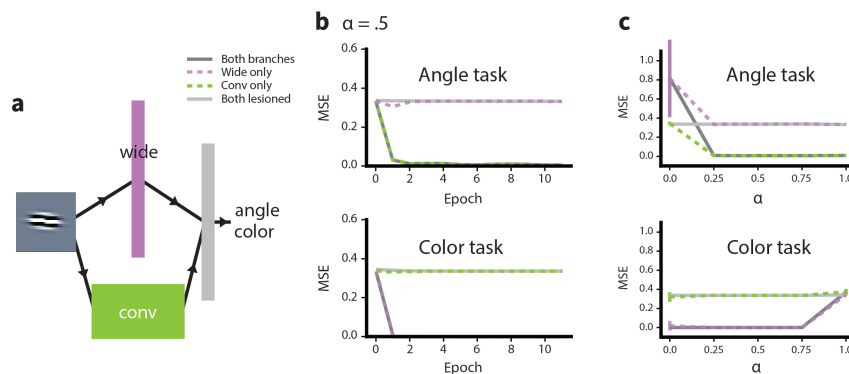


Figure 2: **a**. A branched network with one fully connected wide branch and a convolutional branch. Outputs are the angle and color of the  $32 \times 32$  Gabor filter image. **b**. Mean squared errors (MSEs) of branched network during training in four conditions: intact, with the wide branch only, with the convolutional branch only, and with both branches lesioned. Branches specialize early on in training and do not appear to change afterward. **c**. Statistics of MSEs for ten random seeds for the experiment in **b**, for different values of  $\alpha$  (see text). Standard deviations shown in error bars. Task weights as defined by  $\alpha$  do not affect branch specialization.

37 Here we hope to complement observations from the previous studies, and begin identifying the  
 38 factors involved in branch specialization. In particular we focus on specialization in a precise  
 39 visual task where the stimuli are Gabor patches (Fig. 1) rather than natural images such as those of  
 40 ImageNet (Russakovsky et al., 2015), VGGFace2 (Cao et al., 2018), THINGS (Hebart et al., 2019)  
 41 or Places (Zhou et al., 2017). The benefit of Gabors is that we can design tasks that need not be  
 42 compositional or hierarchically-local such as object recognition, but can be global (*e.g.* average  
 43 color/luminance) and/or order-1 hierarchically local (*e.g.* orientation) (Deza et al., 2020). And  
 44 in contrast to most prior work, we change our tasks, architectures, and training protocols to try to  
 45 unravel the *causal factors* for branches to specialize rather than only observing this phenomena after  
 46 training.

47 The rest of the paper is organized as follows. In section 2, we show that branch specialization is robust  
 48 and happens with diverse or identical branches. In the case of branches with different architectures  
 49 trained on two simultaneous tasks where each is better suited for only one of the branches, a network  
 50 specializes in predictable ways that align with the branches' inductive biases. In section 3, we show  
 51 that branch specialization can be controlled by a curriculum learning scheme that alternates task  
 52 training, and that the faster the alternation rate, the more likely specialization is to occur. In the  
 53 discussion, we note some neuroscientific implications of our results and describe planned future work  
 54 on both the mechanisms of branch specialization and its consequences.

## 55 2 Task allocation in branches can be predicted by inductive biases

56 This section asks how consistently branch specialization occurs, and how much architectural biases  
57 affect it. In Figure 2, we used a Gabor filter dataset and asked our networks to simultaneously  
58 output the images’ angle and average color. Our network architecture consisted of two branches (see  
59 Figure 2a). One was a convolutional network with two convolutional layers followed by two fully  
60 connected layers (see Appendix for details). The other branch was a fully connected network with  
61 two fully connected layers. The outputs of both branches are then fed into a linear output layer, from  
62 which the two output values for the dual task (angle and color) are read. See the associated code  
63 for implementation.<sup>1</sup> We expected the angle task to be better suited to the inductive biases of the  
64 convolutional branch, perhaps requiring edge detection or similar computations that convolutions  
65 can more easily learn. Conversely, mean color estimation is a simple average that is better suited to  
66 the fully connected branch. In this way, we have designed each branch of the network to have an  
67 inductive bias that matches only one of the tasks. In our experiments, however, the fully connected  
68 branch has almost three times fewer parameters than the convolutional branch (see Appendix B.2.1  
69 for details), so in one sense, it may be surprising if the fully connected branch learns a task at all.

70 Results are shown in Figure 2b and c. We train the entire network on the dual task but then evaluate  
71 branches individually by zeroing out (*i.e.* lesioning) the final outputs of the other branch before  
72 it is fed into the linear output layer. We compare those results to ones where both branches were  
73 lesioned and where the network is intact. Figure 2b shows that training converges quickly and each  
74 task is entirely localized to one branch. Over ten random seeds for the training in Figure 2b, an intact  
75 network had an average MSE of .0074(.0044) on the angle task (standard deviation in parentheses).  
76 The fully connected branch alone had an average MSE of .3367(.0045) on the angle task, whereas the  
77 convolutional branch alone had an average MSE of .0074(.0044). With both branches lesioned, the  
78 average MSE was .3367(.0045). We can conclude, then, that the convolutional branch is responsible  
79 for the angle task. These data are shown in Figure 2c. Figure 2c also tells a similar story for the  
80 color estimation task, except that the fully connected network is now responsible for it instead.  
81 In ten random seeds, the angle task was always localized to the convolutional branch while color  
82 estimation was always localized to the fully connected branch. These allocations align precisely with  
83 the inductive biases we initially described.

84 We then wanted to see if branch specialization was robust to the two tasks’ relative contributions  
85 to loss, so we scaled the losses for both tasks with a convex-combination parameter  $\alpha$  to define a  
86 new loss  $\mathcal{L} = \alpha\mathcal{L}_{angle} + (1 - \alpha)\mathcal{L}_{color}$ . We tried a range of  $\alpha$  values from 0 to 1 with the same  
87 network architectures in Figure 2a and the same dual task. One might expect that if one task’s relative  
88 importance were to increase, a network may allocate more resources to it rather than continuing to  
89 split resources evenly. However, we did not see any gradual change in resource allocation. Rather,  
90 we saw the same branch specializations as before regardless of task importance, even for losses that  
91 were heavily biased toward one task. Furthermore, even when the branched network was trained on  
92 only the color or only the angle task (corresponding to  $\alpha = 0$  and  $\alpha = 1$  respectively), one branch  
93 was left unused while other shouldered the entire task burden. Thus, relative contributions of each  
94 task to the loss did not affect task allocation to branches.

95 Next, we wanted to know what would happen with two identical branches and two more similar tasks.  
96 If branches didn’t have asymmetrical biases for tasks, would they still exhibit branch specialization?  
97 We used the architecture described in Figure 3a with two convolutional network branches this time,  
98 with all else the same as the network in Figure 2a. For training we used another simultaneous dual  
99 task setup where the input was a Gabor filter and the output was two values: angle and size, where  
100 size was set by the parameter  $\omega$  in the Gabor filter generation function (see Appendix).

101 We plot training progress in Figure 3b. As before, branches quickly specialize to one task. Because  
102 the branches were identical, we used their effect on the size task to label them "conv 1" or "conv 2" in  
103 all of Figure 3 and ordered the branches by their performance on the size task to maintain functional  
104 identity. Tasks are consistently allocated to different branches over five random initializations,  
105 although in the angle task, both branches were occasionally involved. However, one branch was  
106 always more important: in an intact network with  $\alpha = .5$ , the average MSE was .0079(.0036) for  
107 the angle task and with "conv 1" it was .0498(.0832). "Conv 2" had a significantly larger MSE  
108 of .2724(.1229) while lesioning both branches only increased that number to .3381(.0022). We

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<sup>1</sup>Code is available at <https://github.com/ccli3896/branches-svrhm>.

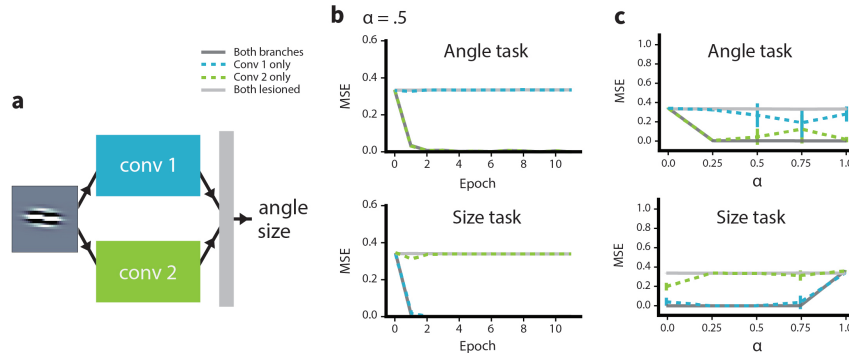


Figure 3: **a.** Branched network architecture with two identical convolutional branches. In this figure branches are distinguished by functionality after training. “Conv 1” is more important for performance in the size task than “conv 2.” Outputs are now angle and size of Gabor filters. **b.** Training progress on the simultaneous angle/size task. Plots show performance during training of MSEs for the intact network, “conv 1 only”, “conv 2” only, or both branches lesioned. Again, specialization happens early. **c.** MSE statistics over five random seeds for different values of  $\alpha$  (see text), with standard deviations in error bars. Complete task separation does not always happen as in Figure 2, but there is clear specialization particularly for the size task.

109 tried different relative task weightings as in Figure 2c, again showing that our results were not too  
 110 sensitive to the loss function. Overall, branch specialization happened even with more similar tasks  
 111 and identical branch architectures, so although specialization can be predicted by matching inductive  
 112 biases to tasks, it is not dependent on task-branch asymmetry.

### 113 3 Branch specialization can be controlled by curriculum learning

114 In this section we ask whether branch specialization can be controlled. Looking at individual networks’  
 115 branch allocations from the experiments for Figure 3c (not shown), we noticed that branched networks  
 116 trained on one task were more likely to use both branches for it. We hypothesized that alternating  
 117 between tasks could lead to task sharing between branches rather than specialization.<sup>2</sup> To test the  
 118 hypothesis, we trained the branched architecture in Figure 4a on the same Gabor angle and size task  
 119 as in Figure 3. This time, losses came entirely from one task for  $n$  epochs and switched to the other  
 120 task for the next  $n$  epochs, alternating for 500 epochs. We tested 1, 5, 10, and 20 for values of  $n$ .

121 For small values of  $n$ , training is more like the simultaneous dual task and we expect branch  
 122 specialization to happen. Our results for  $n = 1$ , or task alternation between every epoch, are shown  
 123 in Figure 4a. Aside from some epochs where EWC (see footnote) fails to maintain task performance  
 124 as seen in the spikes in Figure 4a-bottom, we observe consistent branch specialization for the angle  
 125 and size tasks. The MSEs over five random seeds for the angle task, for instance, were .0065(.0017)  
 126 for the intact network and .0065(.0017) when we lesioned the branch with worse performance on  
 127 the angle task. When the branch with better performance was lesioned, MSEs were .3392(.0038),  
 128 compared to both branches lesioned at .3392(.0037). Data for the size task are in Figure 4c.

129 When  $n$  is increased to 10, we see more distribution of both tasks over both branches. Figure 4b  
 130 displays the errors from trained networks with all combinations of branch lesions. Now, MSEs for  
 131 five random seeds for the angle task were .0835(.0189) for the intact network and .1247(.0332) with  
 132 the worse branch lesioned. With the better branch lesioned MSEs were .2453(.0330), compared  
 133 to both lesioned with an average MSE of .3397(.0034). On both tasks, both branches have non-  
 134 negligible contribution to task performance. Results for all tested values of  $n$ , the epochs between  
 135 task alternation, are in Figure 4c. Faster alternation (corresponding to smaller values of  $n$ ) more

<sup>2</sup>To prevent catastrophic forgetting, we used Elastic Weight Consolidation (Kirkpatrick et al., 2017), or EWC during training. EWC is inspired by synaptic weight consolidation in the brain, where synapses that are involved in long-term memories are strengthened and stay strong for days, weeks, or years (Clopath, 2012). In a similar way, when a network is trained on a new task after having already learned another, EWC prevents weights that are important to previous tasks from changing as much as unimportant weights.

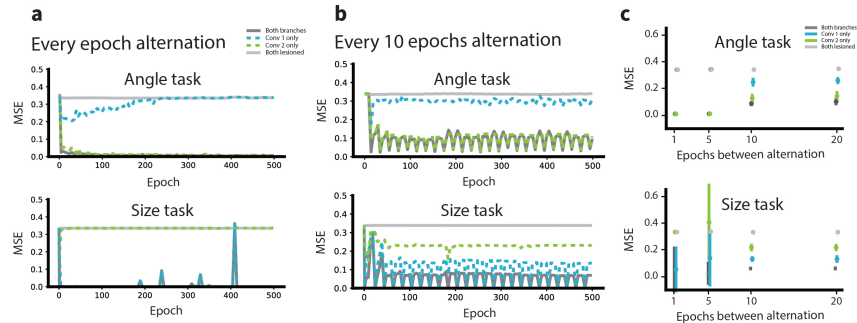


Figure 4: The effects of task alternation on branch specialization. **a.** and **b.** show training progress with intact networks, one branch only, or both branches lesioned. **a** is data from alternating between angle and size tasks every epoch and **b** alternates them every ten epochs. Complete branch specialization happens as usual in **a** but not in **b**, which distributes tasks more between the branches. **c.** shows MSEs for intact networks, one branch, and both branches lesioned over five seeds at the end of training, with standard deviations in error bars. More epochs between task switching decreases specialization, but further decreasing alternation frequency does not further reduce specialization in these experiments.

136 often led to branches that were entirely responsible for one of the two tasks. For slower alternation,  
 137 both branches affect both tasks. However, one branch tended to be more important in each task, and  
 138 this did not change as alternation times grew. Nonetheless, we could partially control the degree of  
 139 branch specialization by changing the learning curriculum.

#### 140 4 Discussion

141 Understanding how tasks distribute themselves within a neural network has relevance to machine  
 142 learning research, perhaps to aid in architecture design, and to neuroscientists, to understand why  
 143 parts of the brain can be highly specialized or be involved in a broad set of functions. We use small  
 144 networks and a simple task set to try to gain intuitions for larger models and more complex tasks. As  
 145 a consequence, our conclusions may serve best as guides for larger experiments in the future.

146 Using a toy Gabor filter dataset and dual task experiments, we show that branch specialization is a  
 147 robust phenomenon with either identical or different branches. In the case of branches with different  
 148 architectures that are better suited to one of a set of tasks, task allocation to branches can be predicted  
 149 based on branches' inductive biases. We also demonstrate that specialization is sensitive to training  
 150 curricula and if multiple tasks alternate during training, branch specialization can be reduced.

151 These results suggest some neuroscientific hypotheses about how and when branch specialization  
 152 occurs. One can predict that brain regions that are consistently allocated to the same tasks across  
 153 individuals have architectural inductive biases for those tasks, as mentioned by previous studies  
 154 such as Dobs et al. (2021). Another prediction is that that the difference between multiple-demand  
 155 and specialized brain regions may be dependent on the statistics of task presentation throughout an  
 156 animal's life—that is, perhaps more functions in multiple-demand regions are blocked and infrequent  
 157 compared to a task like visual perception, which the highly specialized human visual cortex must  
 158 perform almost every waking hour.

159 For future work, we would like to better understand both the mechanisms leading to branch special-  
 160 ization and the consequences of it. In terms of mechanisms, training dynamics would be interesting  
 161 to study, since based on how reliably we see branch specialization in small models and large vision  
 162 models (Voss et al., 2021) alike, we expect the loss surfaces of branches within one architecture  
 163 to interact with each other in a feedback loop that pushes branch functions away from each other  
 164 during training. With regards to the consequences of branch specialization, we want to know when  
 165 specialization is or is not useful. If both very specific and very broadly-used regions exist in the brain,  
 166 does the distribution of neural substrate that these tasks are computed on affect performance? In the  
 167 future we would like to understand this task localization trade-off, and its broader implications to  
 168 generalization in human and machine visual intelligence.

169 **Checklist**

- 170 1. For all authors...
- 171 (a) Do the main claims made in the abstract and introduction accurately reflect the paper’s  
172 contributions and scope? [Yes] We note that larger experiments are warranted and can  
173 be guided by our results in the Discussion.
- 174 (b) Did you describe the limitations of your work? [Yes] See Discussion.
- 175 (c) Did you discuss any potential negative societal impacts of your work? [N/A]
- 176 (d) Have you read the ethics review guidelines and ensured that your paper conforms to  
177 them? [Yes]
- 178 2. If you are including theoretical results...
- 179 (a) Did you state the full set of assumptions of all theoretical results? [N/A]
- 180 (b) Did you include complete proofs of all theoretical results? [N/A]
- 181 3. If you ran experiments...
- 182 (a) Did you include the code, data, and instructions needed to reproduce the main experi-  
183 mental results (either in the supplemental material or as a URL)? [Yes] See Section  
184 2.
- 185 (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they  
186 were chosen)? [Yes] See Appendix B.
- 187 (c) Did you report error bars (e.g., with respect to the random seed after running experi-  
188 ments multiple times)? [Yes] See Figure 2, 3, 4.
- 189 (d) Did you include the total amount of compute and the type of resources used (e.g., type  
190 of GPUs, internal cluster, or cloud provider)? [Yes] See Appendix A.
- 191 4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
- 192 (a) If your work uses existing assets, did you cite the creators? [N/A]
- 193 (b) Did you mention the license of the assets? [N/A]
- 194 (c) Did you include any new assets either in the supplemental material or as a URL? [N/A]
- 195
- 196 (d) Did you discuss whether and how consent was obtained from people whose data you’re  
197 using/curating? [N/A]
- 198 (e) Did you discuss whether the data you are using/curating contains personally identifiable  
199 information or offensive content? [N/A]
- 200 5. If you used crowdsourcing or conducted research with human subjects...
- 201 (a) Did you include the full text of instructions given to participants and screenshots, if  
202 applicable? [N/A]
- 203 (b) Did you describe any potential participant risks, with links to Institutional Review  
204 Board (IRB) approvals, if applicable? [N/A]
- 205 (c) Did you include the estimated hourly wage paid to participants and the total amount  
206 spent on participant compensation? [N/A]

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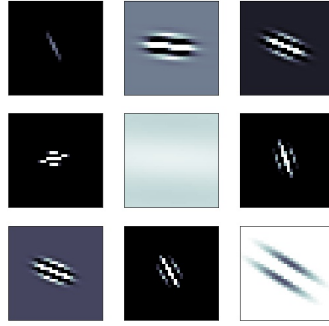


Figure 5: Sample images generated for the Gabor dataset.

266 Bolei Zhou, Agata Lapedriza, Aditya Khosla, Aude Oliva, and Antonio Torralba. Places: A 10  
 267 million image database for scene recognition. *IEEE transactions on pattern analysis and machine*  
 268 *intelligence*, 40(6):1452–1464, 2017.

## 269 A Creation of Gabor Dataset

270 Gabor filters are defined by the following sets of equations.  $\theta$  is the angle parameter mentioned in  
 271 Figures 2, 3, and 4.  $\omega$  is the size parameter in Figures 3 and 4.

$$g(x', y', \omega) = \frac{\omega^2}{4\pi^3} \exp\left(\frac{-\omega^2}{8\pi^2} * (4x'^2 + y'^2)\right) \exp(\pi^2/2) * \cos(\omega x')$$

$$x' = x \cos \theta + y \sin \theta$$

$$y' = -x \sin \theta + y \cos \theta$$

272 We then added a random number to the image from a uniform distribution  $\in [-1, 1]$  to vary the  
 273 image colors. For all experiments,  $\theta \in [0, \pi]$  and  $\omega \in [.1, 3]$ , both drawn randomly from uniform  
 274 distributions. Image sizes were  $32 \times 32$ . The training set was 20k images and the test set was 10k  
 275 images.

## 276 B Training networks

277 All code is available online at <https://github.com/ccli3896/branches-svrhm>.

### 278 B.1 Hardware

279 All experiments use neural networks written in Pytorch (Paszke et al., 2019). Experiments were run  
 280 on shared GPUs in Google Colab (with GPU models allocated from NVIDIA K80, T4, P4, and P100)  
 281 and the FASRC cluster, supported by the FAS Division of Science Research Computing Group at  
 282 Harvard University (containing automatically allocated GPU models from among NVIDIA K20m,  
 283 K40m, K80, M40, 1080, TITAN X, TITAN V, P100, V100, and RTX2080TI).



## 284 **B.2 Network and training parameters**

### 285 **B.2.1 Network sizes**

286 All branched networks had the following traits: they took  $32 \times 32$  image inputs fed immediately  
287 into two branches. The two branches' outputs were concatenated and fed into a linear readout layer,  
288 which always output two values.

289 Convolutional branches had two convolutional layers, max pooling, and two fully connected layers.  
290 The first convolutional layer took in one channel and output 32 channels with a kernel size of 3 and  
291 padding of 1. The second layer was the same except that the input was 32 channels as well. The max  
292 pooling layer had a kernel size of  $2 \times 2$ . It was followed by two dense layers. The first took an input  
293 size of 2048 (the flattened output of the max pool layer). Its output size was 120. The second dense  
294 layer took an input size of 120 and output of 84. All layers used relu as an activation function. In  
295 total the convolutional branch had 265612 trainable parameters.

296 The fully connected branch used in Figure 2 was a two-layer fully connected network with an input  
297 size of  $32 \times 32$  (the flattened image). The first layer had an output size of 86; the second layer took  
298 an input of size 86 and output 10. Both layers used relu for an activation function. In total there were  
299 89020 trainable parameters.

300 Thus, the networks with fully connected and convolutional branches had  $265612 \text{ conv branch} +$   
301  $89020 \text{ dense branch} + (84 \text{ conv outputs} + 10 \text{ dense outputs}) * 2 \text{ outputs} + 2 \text{ biases for outputs} =$   
302  $354822 \text{ parameters}$ . Networks with two convolutional branches had  $265612 * 2 + (84 * 2) * 2 + 2 =$   
303  $531562 \text{ parameters}$ .

### 304 **B.2.2 Training**

305 For training all models, we used Adam optimizer with a learning rate of .001. For experiments in  
306 Figure 4, we used dropout with a rate of .5 and elastic weight consolidation with an importance (see  
307 Kirkpatrick et al. (2017))  $1e4$  for the size task and  $1e7$  for the angle task, both chosen after a period  
308 of tuning.