

000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 TOWARD BIT-EFFICIENT DATASET CONDENSATION: A GENERAL FRAMEWORK

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ABSTRACT

Dataset condensation aims to distill a large-scale dataset into a compact set of synthetic samples for efficient training. Existing methods primarily focus on reducing the number of samples but generally assume full-precision representations. While effective, this assumption limits their applicability in resource-constrained scenarios due to several major drawbacks: (1) *Transmission bottlenecks*—full-precision datasets consume excessive bandwidth and introduce latency during network transfer, especially in cloud–edge collaborative learning; (2) *Memory overhead*—storing and processing full-precision data rapidly exhausts GPU memory or RAM, restricting batch sizes; and (3) *Hardware underutilization*—modern accelerators are optimized for low-precision operations, yet full-precision data prevents full efficiency gains in training and inference. To address these challenges, we propose a novel approach that fine-tunes distilled full-precision datasets into compact low-bit representations, substantially reducing memory usage with minimal computational overhead. Central to our method is a differentiable bit-conscious optimization framework. This framework allows more synthetic samples to be stored within the same memory budget, thereby improving downstream performance. Beyond the algorithmic contribution, we provide theoretical analysis that characterizes (1) the trade-off between compression error and generalization error under memory constraints, and (2) the extent to which Fisher information is preserved under bit compression. Extensive experiments compared to state-of-the-art baselines validate both the effectiveness and efficiency of our method.

1 INTRODUCTION

Dataset condensation is an emerging technique that synthesizes a compact dataset from a large-scale one, enabling more efficient training of machine learning models. Unlike traditional data selection or compression methods, condensation generates artificial samples that are explicitly optimized to preserve the key learning properties of the original data. As a result, models trained on these synthetic datasets can achieve comparable performance to those trained on the full dataset. This paradigm is particularly appealing in resource-constrained scenarios, such as deployment on edge devices, distributed learning environments, or continual learning settings where storage and communication overheads are critical bottlenecks.

Despite advances in dataset condensation (Zhao & Bilen, 2021; Zhou et al., 2022), most methods assume that synthetic samples are stored and used in full precision. While effective, this design limits their practicality in low-resource settings. Using low-bit representations addresses several major challenges: (1) *Transmission*: Large high-precision datasets can saturate networks, especially wireless links, causing bandwidth and latency issues. In contrast, low-bit data requires less bandwidth, enabling efficient transfer between cloud servers and edge devices in distributed or federated learning. (2) *Memory*: Data storage in GPU memory or RAM is often a bottleneck. Low-bit datasets reduce the memory footprint, allowing larger batches of data to fit in memory and improving training efficiency—crucial for devices with limited RAM. (3) *Hardware efficiency*: Many modern accelerators are optimized for low-precision operations. Storing distilled datasets in low-bit formats allows full use of these hardware capabilities, yielding faster training and inference.

In contrast, the benefits of bit compression—widely studied in the context of model compression and efficient inference—have not yet been explored for representing the synthetic datasets themselves.

Moreover, bit compression introduces non-differentiable operations, making it challenging to integrate into the gradient-based dataset condensation optimization frameworks that underpin most dataset condensation methods. To bridge this gap, we introduce a novel and efficient dataset condensation framework that *fine-tunes* full-precision distilled datasets to generate highly compact low-bit data representations with only minimal computational burden, significantly enhancing storage efficiency. Specifically, we introduce a differentiable condensation optimization algorithm tailored for reducing the number of bits to represent synthetic data. Our approach allows the condensation process to jointly optimize data content and its low-bit representation, enabling the generation of a substantially larger number of samples under the same memory budget. This flexibility is crucial in maximizing the utility of condensed data in memory-constrained applications. We name our approach as *Bit-Conscious Dataset Condensation* (BCDC).

In addition to our algorithmic contributions, we provide extensive theoretical analysis that characterizes: (1) the fundamental trade-off between the quantization error and generalization error under fixed memory constraints; and (2) Fisher information preservation under bit compression. These analyses offer deeper insights into how quantization affects the effectiveness of condensed datasets and guide practical design choices.

Comprehensive experiments across multiple benchmarks and state-of-the-art baselines show that: (1) our method delivers strong performance while significantly reducing memory usage when using the same number of distilled images; and (2) under a fixed memory budget, it accommodates more distilled images, resulting in markedly improved performance and advancing the practicality and scalability of dataset condensation.

Our main contributions in this paper are summarized as follows:

- We propose a novel and general dataset condensation framework through low-bit compression, enabling memory-efficient learning.
- We develop an efficient dataset condensation optimization algorithm, facilitating more effective dataset distillation.
- The theoretical analysis is derived to characterize: (1) the trade-off between quantization error and generalization error under memory constraints; and (2) Fisher information preservation under bit compression.
- Extensive experiments are conducted on multiple benchmarks and state-of-the-art baselines to validate the effectiveness and efficiency of the proposed method.

2 RELATED WORK

2.1 CORESET SELECTION AND DATA CONDENSATION

Coreset Selection Coreset selection (Har-Peled & Mazumdar, 2004) aims to identify a small, representative subset of the original dataset such that training a model on this subset yields performance comparable to training on the full dataset. Importantly, the selected subset consists of actual data samples rather than synthetic ones. This idea has been extensively explored in domains such as active learning (Settles, 2009) and continual learning (Lopez-Paz & Ranzato, 2017), where the goal is to select the most informative examples for training (Yang et al., 2023b; Welling, 2009; Chen et al., 2010; Rebuffi et al., 2017; Aljundi et al., 2019). However, coreset selection inherently relies on choosing a portion of the original dataset and thus may fail to capture useful information contained in the remaining samples.

Data Condensation Dataset condensation (DC) (Wang et al., 2018) synthesizes compact data that retains the essential information of the original dataset, enabling efficient model training with fewer samples. Unlike coreset selection, which chooses real data points, condensation generates synthetic ones informed by the full dataset. Approaches include: (I) *Bi-level Optimization* (e.g., DD (Wang et al., 2018), Qin et al. (Qin et al., 2024), EDC (Shao et al., 2024)); (II) *Analytical Methods* such as KRR (Nguyen et al., 2021); (III) *Surrogate Matching*, including gradient (DC (Zhao et al., 2021), DSA (Zhao & Bilen, 2021)), trajectory (MTT (Cazenavette et al., 2022)), and loss/feature matching (LCMat (Shin et al., 2023), CAFE (Wang et al., 2022a), DM (Zhao & Bilen, 2023)); (IV) *Parameter-Efficient Methods* using data partitioning (IDC (Kim et al., 2022), IDM (Zhao et al., 2023), DQ (Zhou

108 et al., 2023)), basis factorization (HaBa (Liu et al., 2022), RememberThePast (Deng & Russakovsky, 109 2022)), or low-rank techniques (LoDC (Yang et al., 2023a)); (V) *Regularization* (DWA (Du et al., 110 2024), CMI (Zhong et al., 2025)); (VI) *Diffusion Models* (D³HR (Zhao et al., 2025), D⁴M (Su et al., 111 2024)); and (VII) *Optimization-Free Methods* like RDED (Sun et al., 2024).

112 Despite these advances, most approaches rely on full-precision representations, limiting efficiency
113 in resource-constrained settings. In contrast, our method targets low-bit synthetic data generation,
114 significantly reducing memory usage while maintaining competitive model performance.
115

116 2.2 CONTINUAL LEARNING

117 Continual learning (CL) seeks to enable models to learn from non-stationary data distributions without
118 forgetting knowledge acquired from previously encountered tasks. Most existing CL approaches rely
119 on storing and replaying raw data samples (Kirkpatrick et al., 2017; Schwarz et al., 2018; Zenke
120 et al., 2017; Rebuffi et al., 2017; Chaudhry et al., 2018; Lopez-Paz & Ranzato, 2017; Riemer et al.,
121 2019; Chaudhry et al., 2019; Buzzega et al., 2020; Prabhu et al., 2020; Pham et al., 2021; Verwimp
122 et al., 2021; Arani et al., 2022; Caccia et al., 2022; Wang et al., 2022c) or on using synthetic data
123 generated at full precision (Yang et al., 2023a). However, data efficiency and privacy remain critical
124 challenges in CL, as raw data from previous tasks may be unavailable or sensitive during the training
125 of new tasks. In this paper, we integrate our BCDC into CL, enabling the training of CL models that
126 simultaneously improve data efficiency, enhance privacy, and maintain strong performance.
127

128 2.3 LOW BITS QUANTIZATION

129 To our best knowledge, BCDC is the first to explore bit-efficient dataset distillation. Unlike network
130 quantization (Wang et al., 2022b; Gong et al., 2019; Yao et al., 2021), which targets model parameters,
131 our method focuses on reducing the memory footprint of the distilled dataset itself. Moreover, our
132 approach is seamlessly compatible with existing dataset distillation pipelines.

133 There is no prior work that integrates quantization into dataset condensation. Existing DC methods
134 always operate with full-precision samples, and quantization research focuses exclusively on model
135 parameters or activations rather than synthetic data. This gap directly motivates BCDC, which is the
136 first framework to introduce quantization-aware dataset condensation.
137

139 3 METHOD

141 3.1 PROBLEM DEFINITION

143 **Traditional Dataset Distillation** The goal of dataset distillation is to distill a large-scale dataset of
144 $\mathcal{T} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{i=N}$ into a small-scale dataset $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{i=m}$, where $m \ll N$. The objective is to
145 ensure that the network trained on the compressed dataset, with parameters $\boldsymbol{\theta}^{\mathcal{S}}$, achieves performance
146 comparable to that of the network trained on the original dataset \mathcal{T} , with parameters $\boldsymbol{\theta}^{\mathcal{T}}$, where:

$$\boldsymbol{\theta}^{\mathcal{T}} = \arg \min_{\boldsymbol{\theta}^{\mathcal{T}}} [\mathcal{L}(\boldsymbol{\theta}^{\mathcal{T}}, \mathcal{T})] = \frac{1}{|\mathcal{T}|} \sum_{(\mathbf{x}, y) \sim \mathcal{T}} \mathcal{L}(\mathbf{x}, y, \boldsymbol{\theta}^{\mathcal{T}}) \quad (1)$$

$$\boldsymbol{\theta}^{\mathcal{S}} = \arg \min_{\boldsymbol{\theta}^{\mathcal{S}}} [\mathcal{L}(\boldsymbol{\theta}^{\mathcal{S}}, \mathcal{S})] = \frac{1}{|\mathcal{S}|} \sum_{(\mathbf{x}, y) \sim \mathcal{S}} \mathcal{L}(\mathbf{x}, y, \boldsymbol{\theta}^{\mathcal{S}}) \quad (2)$$

153 The dataset distillation can be formulated as a bi-level optimization problem:

$$\mathcal{S} = \arg \min_{\mathcal{S}} \mathcal{L}(\boldsymbol{\theta}^{\mathcal{S}}, \mathcal{T}) \quad \text{satisfy} \quad \boldsymbol{\theta}^{\mathcal{S}} = \arg \min_{\boldsymbol{\theta}^{\mathcal{S}}} \mathcal{L}(\boldsymbol{\theta}^{\mathcal{S}}, \mathcal{S}) \quad (3)$$

156 where the inner loop optimizes a network to train on the synthetic dataset \mathcal{S} and the outer loop
157 optimization optimizes on the original dataset to learn the synthetic dataset.
158

159 **Low-Bits Dataset Quantization** Given an original dataset $\mathcal{T} = \{(\mathbf{x}_i, y_i)\}_{i=1}^N$, we first compress it
160 into a small-scale full-precision dataset $\mathcal{S} = \{(\mathbf{x}_i, y_i)\}_{i=1}^m$, where $m \ll N$. Our goal is to further
161 *fine-tune* a condensed low-bit dataset $\mathcal{S}_{\text{quant}} = \{(\tilde{\mathbf{x}}_i, y_i)\}_{i=1}^m$, where each $\tilde{\mathbf{x}}_i$ is a quantized version of
162 \mathbf{x}_i represented using b -bit precision.

162 3.2 PROPOSED METHOD
163164 We adopt a uniform quantization method:
165

166
$$x_{\text{low}} = \min \mathbf{x}, \quad x_{\text{high}} = \max \mathbf{x}, \quad \Delta = \frac{x_{\text{high}} - x_{\text{low}}}{2^b - 1} \quad (4)$$

167

168 where b denotes the number of bits used to encode each dimension of a data sample. Δ denotes the
169 length of the quantization interval. The uniform quantization-dequantization can be defined as:
170

171
$$Q_b(\mathbf{x}) = \text{round}\left(\frac{\mathbf{x} - x_{\text{low}}}{\Delta}\right) \Delta \quad (5)$$

172

173 **Differentiable Data Quantization (DDQ)** The near-zero gradients of the uniform quantization
174 function (Eq. 5) at most input values hinder effective training on the quantized data, resulting in
175 unstable learning dynamics. To address this issue, we introduce a differentiable asymptotic function
176 that approximates a uniform data quantizer. Specifically, DDQ replaces hard quantization with a
177 smooth, continuous approximation.
178

179
$$\phi(\mathbf{x}) = s \cdot \tanh(k(\mathbf{x} - \mathbf{m}_i)), \quad \text{if } \mathbf{x} \in P_i = [x_{\text{low}} + i\Delta, x_{\text{low}} + (i+1)\Delta]$$

180

181
$$\text{where } \mathbf{m}_i = x_{\text{low}} + (i + 0.5)\Delta, \quad s = \frac{1}{\tanh(0.5k\Delta)}. \quad (6)$$

182

183 Where in Eq. 6, s ensures that the outputs of $\phi(\mathbf{x})$ are normalized to -1 and $+1$ at the boundaries of
184 quantization intervals. \mathbf{m}_i denotes the midpoint of each quantization interval. We then define a soft
185 quantization function (Eq. 7) to provide a smooth and differentiable approximation to Eq. (5):
186

187
$$Q_V(\mathbf{x}) = x_{\text{low}} + \left(i + \frac{\phi(\mathbf{x}) + 1}{2}\right)\Delta \quad (7)$$

188

189 **Bi-Level Optimization for Low-Bit Condensation** We optimize \mathcal{S} in full precision but penalize
190 deviations from quantized values:
191

192
$$\min_{\mathcal{S}} \underbrace{\mathbb{E}_{(\mathbf{x}, y) \sim \mathcal{T}_{\text{val}}} [\mathcal{L}(f_{\theta^*}(\mathbf{x}), y)]}_{\text{Validation loss}} + \lambda \underbrace{\|\mathcal{S} - Q_V(\mathcal{S})\|_2^2}_{\text{Quantization loss}}$$

193
194
$$\text{s.t. } \theta^* = \arg \min_{\theta} \mathbb{E}_{(\tilde{\mathbf{x}}, y) \sim Q_V(\mathcal{S})} [\mathcal{L}(f_{\theta}(\tilde{\mathbf{x}}), y)]$$

195

196
197 *Inner Loop (Model Training):* Update θ on $Q_V(\mathcal{S})$ via SGD:
198

199
$$\theta \leftarrow \theta - \eta_1 \nabla_{\theta} \mathcal{L}(f_{\theta}(Q_V(\mathcal{S})), \mathbf{y})$$

200

201 *Outer Loop (Dataset Update):* Compute $\nabla_{\mathcal{S}} \mathcal{L}_{\text{val}}$ via chain rule:
202

203
$$\nabla_{\mathcal{S}} \mathcal{L}_{\text{val}} = \frac{\partial \mathcal{L}_{\text{val}}}{\partial \mathcal{S}} + \frac{\partial \mathcal{L}_{\text{val}}}{\partial \theta^*} \cdot \frac{\partial \theta^*}{\partial Q_V(\mathcal{S})} \cdot \frac{\partial Q_V(\mathcal{S})}{\partial \mathcal{S}} \quad (8)$$

204

205 where

206
$$\frac{\partial \theta^*}{\partial Q_V(\mathcal{S})} = - \left(\frac{\partial^2 \mathcal{L}_{\text{train}}}{\partial \theta^2} \right)^{-1} \frac{\partial^2 \mathcal{L}_{\text{train}}}{\partial Q_V(\mathcal{S}) \partial \theta} \quad (9)$$

207
208

209 Derivations of Eq. 9 are presented in Appendix A. Then, we update \mathcal{S} with the following gradients:
210

211
$$\mathcal{S} \leftarrow \mathcal{S} - \eta_2 (\nabla_{\mathcal{S}} \mathcal{L}_{\text{val}} + \lambda \nabla_{\mathcal{S}} \|\mathcal{S} - Q_V(\mathcal{S})\|_2^2)$$

212

213 **Final Quantization** After convergence, we apply the following quantization to each image:
214

215
$$\tilde{x}_i = \text{round}\left(\frac{\text{clip}(\mathbf{x}_i, x_{\text{low}}, x_{\text{high}}) - x_{\text{low}}}{\Delta}\right)$$

216 3.3 INTEGRATE BCDC WITH EXISTING APPROACHES
217

218 **Bi-level Dataset Condensation with Quantized Data Representations** Our proposed BCDC
219 serves as a general, versatile framework that integrates seamlessly with existing approaches. The
220 proposed algorithm integrated with bi-level dataset condensation loss function (Wang et al., 2018) is
221 presented in Algorithm 1. Integrating BCDC with surrogate loss function, e.g., DM (Zhao & Bilen,
222 2023) is presented in Algorithm 2 in Appendix.

223 **Algorithm 1** Bi-level Dataset Condensation with Quantized Data Representations

224 **Input:** Training set \mathcal{T} , distilled dataset learning rates γ
225 **Initialize:** Initialize distilled dataset and labels \mathcal{S} .
226 **while** not converged **do**
227 Sample a training batch from the training set: $\{\mathbf{x}, y\} \sim \mathcal{T}$
228 Perform K optimization steps on inner objective to obtain θ_i^*
229 Compute synthetic data gradient $\nabla_{\mathcal{S}} \mathcal{L}_{\text{val}}$ by Eq. (8)
230 Update the distilled dataset: $\mathcal{S} \leftarrow \mathcal{S} - \gamma \nabla_{\mathcal{S}} \mathcal{L}_{\text{val}}$
231 Train the model θ_i on the current distilled dataset \mathcal{S} for one step
232 **end while**

233
234 4 THEORETICAL ANALYSIS
235

236 We use \mathcal{D} to represent the *ground truth* distribution governing data generation, $R_{\mathcal{D}}(f_{\theta}) =$
237 $\mathbb{E}_{(\mathbf{x}, y) \sim \mathcal{D}} \mathcal{L}(f_{\theta}(\mathbf{x}), y)$, which is the expected loss (or risk) over the true data distribution and also
238 known as generalization error. $\hat{R}_{Q_b(\mathcal{S})}(f_{\theta}) = \mathbb{E}_{(\mathbf{x}, y) \sim Q_b(\mathcal{S})} \mathcal{L}(f_{\theta}(\mathbf{x}), y)$ denotes the empirical risk on
239 the quantized data.

240 **Assumption 4.1.** A hypothesis function $f_{\theta} : \mathbb{R}^d \rightarrow \mathbb{R}$ is called Lipschitz continuous with constant
241 $L > 0$ if

$$243 \quad \|f_{\theta}(\mathbf{x}_1) - f_{\theta}(\mathbf{x}_2)\| \leq L \|\mathbf{x}_1 - \mathbf{x}_2\|, \quad \forall \mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^d.$$

244 If h is differentiable, this implies a bound on its gradient: $\|\nabla f_{\theta}(\mathbf{x})\| \leq L, \forall \mathbf{x} \in \mathbb{R}^d$.

245 **Assumption 4.2.** The empirical risk function $R_{\mathcal{D}}(f_{\theta}) : \mathbb{R}^d \rightarrow \mathbb{R}$ is β -smooth if

$$246 \quad R_{\mathcal{S}}(f_{\theta}) \leq R_{\mathcal{D}}(f_{\theta}) + \langle \nabla R_{\mathcal{D}}(f_{\theta}), f_{\theta}(\mathcal{S}) - f_{\theta}(\mathcal{D}) \rangle + \frac{\beta}{2} \|f_{\theta}(\mathcal{S}) - f_{\theta}(\mathcal{D})\|^2,$$

248 **Assumption 4.3.** Q_b be a b -bit quantizer with $\mathbb{E}[\|Q_b(\mathbf{x}) - \mathbf{x}\|] \leq C2^{-b}$ (following (Gray & Neuhoff,
249 2002)) where C is a constant.

250 **Assumption 4.4.** \mathcal{H} be a hypothesis class with Rademacher complexity (Bartlett & Mendelson, 2002)
251 $\mathfrak{R}_n(\mathcal{H}) \leq \kappa/\sqrt{n}$, where κ is a constant and n denotes the number of training samples.

253 Let M denote the total memory budget in bits and d be the data dimension. $m(b) = \lfloor M/(bd) \rfloor$
254 denotes the number of stored samples under b -bit quantization.

255 **Theorem 4.5** (Memory-Constrained Quantization-Generalization Trade-off). *With probability \geq
256 $1 - \delta$:*

$$257 \quad R_{\mathcal{D}}(f_{\theta}) \leq \underbrace{\hat{R}_{Q_b(\mathcal{S})}(f_{\theta})}_{\text{Empirical Risk}} + \underbrace{\frac{2\kappa}{\sqrt{m(b)}}}_{\text{Generalization Error}} + \underbrace{\sqrt{\frac{\log(2/\delta)}{2m(b)}} + LC2^{-b} + \beta C^2 2^{-2b}}_{\text{Quantization Error}} \quad (10)$$

261 **Implications:** Each term in the above generalization bound can be interpreted as the following:

262 (1) The term $\frac{2\kappa}{\sqrt{m(b)}}$ reflects the reduced model complexity from more samples; (2) $\sqrt{\frac{\log(2/\delta)}{2m(b)}}$ is
263 the classical Hoeffding concentration term; (3) $LC2^{-b}$ shows the first-order quantization error; (4)
264 $\beta C^2 2^{-2b}$ captures the second-order quantization effects.

266 The generalization bound in equation 10 exhibits a trade-off between quantization error and generalization
267 error influenced by the choice of b , as shown in Table 1: selecting a smaller b tightens
268 the generalization bound but at the cost of increased quantization error; selecting a larger b reduces
269 quantization error but may lead to looser generalization bounds due to fewer samples. Balancing
these factors is crucial for optimizing model performance under memory constraints.

270
271 Table 1: Trade-off between generalization error and quantization error.
272
273

$m(b)$	$\frac{2\kappa}{\sqrt{m(b)}} + \sqrt{\frac{\log(2/\delta)}{2m(b)}}$	$LC2^{-b} + \beta C^2 2^{-2b}$
b decreases (\downarrow)	\uparrow	\downarrow
b increases (\uparrow)	\downarrow	\uparrow

274
275
276 **Theorem 4.6** (Fisher Information Retention in Bit-Conscious Condensation). *Let $I(\mathcal{T}; \theta)$ and*
277 *$I(Q_b(\mathcal{S}); \theta)$ denote the Fisher information of the original and quantized condensed datasets, respec-*
278 *tively. For a b -bit quantizer:*

$$279 \quad I(\mathcal{T}; \theta) - I(Q_b(\mathcal{S}); \theta) \leq \frac{L^2 \Delta^2}{8} \text{tr}(\mathbb{E}[\nabla_{\theta} \log p(\theta|\mathcal{S}) \nabla_{\theta} \log p(\theta|\mathcal{S})^{\top}])$$

$$280$$

$$281$$

282 where $\Delta = 2^{-b+1}(\max(\mathcal{S}) - \min(\mathcal{S}))$. tr denotes the trace of the Fisher information matrix
283 $(\mathbb{E}[\nabla_{\theta} \log p(\theta|\mathcal{S}) \nabla_{\theta} \log p(\theta|\mathcal{S})^{\top}])$.

284
285 **Implications:** As b increases, Δ_1 decreases and the right-hand side bound becomes tighter, since
286 more bits allow finer-grained image details to be preserved. Conversely, as b decreases, Δ_1 increases
287 and the bound becomes looser, due to the loss of fine-grained detail with fewer bits. Due to space
288 limitations, we provide detailed theorem proof in Appendix B.

290 5 EXPERIMENT

$$291$$

292 5.1 DATASET CONDENSATION FOR DEEP LEARNING

$$293$$

294 **Datasets** We assess the effectiveness of BCDC on the following benchmark datasets: MNIST (LeCun
295 et al., 1998), CIFAR10 (Krizhevsky et al., 2009), CIFAR100 (Krizhevsky et al., 2009), TinyImageNet
296 (Le & Yang, 2015) and ImageNet-1K (Deng et al., 2009).

297 **Baselines** We compare to both coresets selection and dataset distillation methods.

298 (I) *Coreset Selection*: Selecting a representative subset of real data: (1) *Random*: Selects images
299 randomly from the dataset; (2) *Herding*: Selects samples heuristically, aiming for those closest to the
300 class center (Welling, 2009; Belouadah & Popescu, 2020); (3) *Forgetting*: Selects samples that are
301 most likely to be forgotten during model training (Toneva et al., 2019).

302 (II) *Dataset Distillation*: *DD* (Wang et al., 2018), *LD* (Bohdal et al., 2020), *DC* (Zhao et al., 2021),
303 *DSA* (Zhao & Bilen, 2021), *MTT* (Cazenavette et al., 2022), *IDC* (Kim et al., 2022), *HaBa* (Liu et al.,
304 2022), *RememberThePast* (Deng & Russakovsky, 2022) *DM* (Zhao & Bilen, 2023), *DataDAM* (Sajedi
305 et al., 2023), *TESLA* (Cui et al., 2023), *SRe²L* (Yin et al., 2023), *DWA* (Du et al., 2024), *RDED* (Sun
306 et al., 2024), *D⁴M* (Su et al., 2024), *CMI* (Zhong et al., 2025), *D³HR* (Zhao et al., 2025).

307 **Implementation Details** Following the experimental protocol established in (Kim et al., 2022), we
308 ensure that all methods operate under an equal memory budget. For each trial, we either select
309 a coresset (Random, Herding, or Forgetting) or optimize a synthetic dataset (DD, LD, DC, DSA,
310 DM, etc), and then use it to train 20 independently initialized ConvNet models (Rocco et al., 2017).
311 All other hyperparameters are aligned with those used in prior work (Zhao et al., 2021; Zhao &
312 Bilen, 2021; 2023). Each experiment setup is repeated five times, and we report the average test
313 accuracy across runs. In addition, as detailed in Sec. 5.2, we assess the generalization capability of
314 the synthetic datasets across architectures by evaluating them on five commonly used deep networks:
315 ConvNet (Rocco et al., 2017), LeNet (LeCun et al., 1998), AlexNet (Krizhevsky et al., 2017),
316 VGG11 (Simonyan & Zisserman, 2015), and ResNet18 (He et al., 2016). For this set of experiments,
317 we use 2, 4, 4 and 4-bit representations for MNIST, CIFAR10, CIFAR100 and TinyImageNet
318 respectively. The number of bits for each dataset is selected from {2, 4, 8}, based on the configuration
319 that yields the highest validation performance. $\lambda = 0.2$. All experiments are conducted on a single
320 NVIDIA A6000 GPU.

321 **Results and Analysis** Table 2 and 4 compares dataset condensation with coresets selection approaches,
322 showing that dataset condensation generally outperforms coresets selection. In Table 3, we compare
323 our BCDC with conventional dataset condensation methods (DC, DSA, DM) under two scenarios:
using the same number of images (SI) and using the same memory budget (SM). The results highlight

Table 2: Comparison with coresets selection methods and dataset condensation methods.

DataSet	Img/Clss	Coreset Selection Methods			Dataset Condensation Methods					
		Random	Herding	Forgetting	DD	LD	DC	DSA	DM	DM+BCDC (Ours)
MNIST	1	64.9 \pm 3.5	89.2 \pm 1.6	35.5 \pm 5.6	-	60.9 \pm 3.2	91.7\pm0.5	88.7 \pm 0.6	89.7 \pm 0.6	91.5\pm0.3
	10	95.1 \pm 0.9	93.7 \pm 0.3	68.1 \pm 3.3	79.5 \pm 8.1	87.3 \pm 0.7	97.4\pm0.2	97.1 \pm 0.1	96.5 \pm 0.2	97.8\pm0.3
	50	97.9 \pm 0.2	94.8 \pm 0.2	88.2 \pm 1.2	-	93.3 \pm 0.3	98.8\pm0.2	99.2\pm0.1	97.5 \pm 0.5	98.4 \pm 0.2
CIFAR10	1	14.4 \pm 2.0	21.5 \pm 1.2	13.5 \pm 1.2	-	25.7 \pm 0.7	28.3 \pm 0.5	28.8 \pm 0.7	26.0 \pm 0.8	45.1\pm0.8
	10	26.0 \pm 1.2	31.6 \pm 0.7	23.3 \pm 1.0	36.8 \pm 1.2	38.3 \pm 0.4	44.9 \pm 0.5	51.1 \pm 0.5	48.9 \pm 0.6	60.6\pm0.5
	50	43.4 \pm 1.0	40.4 \pm 0.6	23.3 \pm 1.1	-	42.5 \pm 0.4	53.9 \pm 0.5	60.6 \pm 0.5	63.0 \pm 0.4	65.9\pm0.2
CIFAR100	1	4.2 \pm 0.3	8.4 \pm 0.3	4.5 \pm 0.2	-	11.5 \pm 0.4	12.8 \pm 0.3	13.9 \pm 0.3	11.4 \pm 0.3	26.3\pm0.5
	10	14.6 \pm 0.5	17.3 \pm 0.3	15.1 \pm 0.3	-	-	25.2 \pm 0.3	32.3 \pm 0.3	29.7 \pm 0.3	38.3\pm0.7
TinyImageNet	1	1.4 \pm 0.1	2.8 \pm 0.2	1.6 \pm 0.1	-	-	4.61 \pm 0.2	4.79 \pm 0.2	3.9 \pm 0.2	10.6\pm0.4
	10	5.0 \pm 0.2	6.3 \pm 0.2	5.1 \pm 0.2	-	-	11.6 \pm 0.3	14.7 \pm 0.2	12.9 \pm 0.4	18.9\pm0.5

Table 3: Comparison with dataset distillation methods on the same number of image (SI) or same memory (SM).

DataSet	Img/Clrs	DC	+BCDC (SI)	+BCDC (SM)	DSA	+BCDC (SI)	+BCDC (SM)	DM	+BCDC (SI)	+BCDC (SM)
MNIST	1	91.7±0.5	-	93.5±0.4	88.7±0.6	-	90.3±0.7	89.7±0.6	88.1±0.8	91.5±0.3
	10	97.4±0.2	96.0±0.2	97.7±0.4	97.1±0.1	96.2±0.3	97.8±0.3	96.5±0.2	95.7±0.6	97.8±0.2
	50	98.6±0.3	97.8±0.3	98.9±0.5	98.0±0.2	97.1±0.4	98.6±0.4	97.2±0.3	96.6±0.5	98.9±0.3
CIFAR10	1	28.3±0.5	28.1±0.4	36.1±0.6	28.8±0.7	28.6±0.7	42.2±0.3	26.0±0.8	25.0±0.4	45.1±0.8
	10	44.9±0.5	43.1±0.5	51.2±0.4	51.1±0.5	49.7±0.6	57.3±0.4	48.9±0.6	47.1±0.8	60.6±0.5
	50	53.9±0.5	52.8±0.6	60.5±0.5	60.6±0.5	58.6±0.5	68.2±0.3	63.0±0.4	58.2±0.7	65.9±0.2
CIFAR100	1	12.8±0.3	12.5±0.4	19.1±0.2	13.9±0.3	13.7±0.3	23.7±0.4	11.4±0.3	10.5±0.6	26.3±0.5
	10	25.2±0.3	24.6±0.3	27.6±0.5	32.3±0.3	30.9±0.2	34.2±0.6	29.7±0.3	28.6±0.4	38.3±0.7
TinyImageNet	1	4.61±0.2	4.32±0.3	7.10±0.2	4.79±0.2	4.52±0.4	10.80±0.5	3.9±0.2	3.7±0.3	10.6±0.4
	10	11.6±0.3	10.2±0.3	18.31±0.3	14.7±0.2	13.61±0.5	20.70±0.6	12.9±0.4	11.6±0.4	18.9±0.5

the following observations: (i) With an equal number of images, our method achieves more reductions in storage requirements—while maintaining performance close to that of the traditional DC approach. (ii) When the memory budget is the same, our method is able to store more samples within the same budget, thereby retaining richer information from the original dataset. As a result, it significantly outperforms existing condensation methods. For example, on CIFAR10 with 1 image per class, BCDC achieves improvements of 13.4% and 19.1% over DSA and DM, respectively. In Table 5, we compare our method against recent approaches—MTT (Cazenavette et al., 2022), DataDAM (Sajedi et al., 2023), TESLA (Cui et al., 2023), SRe²L (Yin et al., 2023) and DWA (Du et al., 2024)—on both Tiny-ImageNet and ImageNet-1K benchmarks. The results show that our BCDC can enhance dataset distillation performance under the same memory budget.

5.2 ABLATION STUDY

Cross-Architecture Transferability Analysis To assess how well BCDC generalizes across different model architectures, we perform a cross-architecture evaluation. Specifically, we generate the condensed dataset using a single architecture (such as AlexNet or ConvNet) and then use it to train five different network architectures from scratch. We evaluate the resulting models on the CIFAR-10 test set. As shown in Table 7, BCDC consistently delivers strong performance across these diverse architectures, highlighting its effectiveness in supporting cross-architecture knowledge transfer.

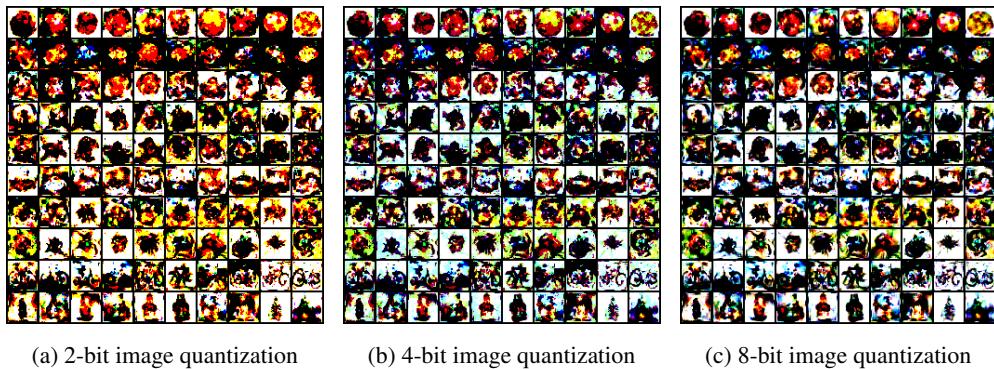


Figure 1: Visualization of different bits image quantization by integrating BCDC with DC.

Computation Efficiency Evaluation To evaluate the training efficiency of our proposed BCDC compared to traditional dataset condensation methods without quantization, we report the training

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Table 4: Compare with other advanced dataset condensation methods.

CIFAR10 (Img/Clss=1)	MTT 46.3%	IDC-I 36.7%	IDC 50.6%	HaBa 48.3%	RememberThePast 66.4%
	MTT+BCDC 59.0%	IDC-I+BCDC 50.1%	IDC+BCDC 56.7%	HaBa+BCDC 65.3%	RememberThePast+BCDC 68.4%
	MTT 24.3%	IDC-I 16.6%	IDC 24.9%	HaBa 33.4%	RememberThePast -
CIFAR100 (Img/Clss=1)	MTT+BCDC 31.2%	IDC-I+BCDC 27.6%	IDC+BCDC 33.4%	HaBa+BCDC 36.5%	RememberThePast+BCDC -

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Table 5: Evaluation against state-of-the-art dataset distillation methods on Tiny-ImageNet and ImageNet-1K. Unless stated otherwise, we adopt the same model architecture during both the distillation and evaluation phases. Consistent with the configurations reported in their respective works, MTT (Cazenavette et al., 2022) and TESLA (Cui et al., 2023) employ ConvNet-128. In contrast, SRe²L (Yin et al., 2023) generates synthetic data using ResNet-18 and assesses performance across ResNet-18, ResNet-50, and ResNet-101. The symbol † denotes that MTT is applied to a 10-class subset of the complete ImageNet-1K dataset.

Dataset	ipc	ConvNet			ResNet-18			ResNet-50			ResNet-101		
		MTT	DataDAM	TESLA	SRe ² L	DWA	+BCDC	SRe ² L	DWA	+BCDC	SRe ² L	DWA	+BCDC
Tiny-ImageNet	50	28.0±0.3	28.7±0.3	-	41.1±0.4	52.8±0.2	55.1±0.3	42.2±0.5	53.7±0.2	55.6±0.3	42.5±0.2	54.7±0.3	57.8±0.4
	100	-	-	-	49.7±0.3	56.0±0.2	59.6±0.3	51.2±0.4	56.9±0.4	59.1±0.5	51.5±0.3	57.4±0.3	59.6±0.4
ImageNet-1K	10	64.0±1.3†	6.3±0.0	17.8±1.3	21.3±0.6	37.9±0.2	39.6±0.3	28.4±0.1	43.0±0.5	45.5±0.6	30.9±0.1	46.9±0.4	49.2±0.3
	50	-	-	27.9±1.2	46.8±0.2	55.2±0.2	57.5±0.3	55.6±0.3	62.3±0.1	65.6±0.2	60.8±0.5	63.3±0.7	64.5±0.8
	100	-	-	-	52.8±0.3	59.2±0.3	61.7±0.4	61.0±0.4	65.7±0.4	67.8±0.3	62.8±0.2	66.7±0.2	68.0±0.4

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cost comparison in Table 9 in Appendix D. Although BCDC introduces additional quantization-aware fine-tuning cost, it results in only a modest increase in training cost—ranging from 17% to 20%—while offering improved performance under quantized settings.

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Table 8: Comparison of testing performance using naive quantization versus our BCDC.

Method	Without Quantization	+ Naive Quantization (Naive-Q)	+ BCDC (Ours)
DC	28.3±0.5	31.8±0.5	36.1±0.6
DSA	28.8±0.7	35.6±0.7	42.2±0.3
DM	26.0±0.8	34.3±0.2	45.1±0.8

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Effect of Bit-Width b : Table 10 (in Appendix D) illustrates the effect of bit-width on dataset distillation performance. Lower bit-widths reduce the memory required per image, enabling the storage of a larger number of samples within a fixed memory budget. However, this reduction in precision also degrades image quality, which can hinder model performance. Conversely, higher bit-widths preserve more visual detail and improve image fidelity but increase the memory footprint per image, limiting the number of samples that can be stored. Empirically, we observe that performance improves as the bit-width increases from 2 bits, reaching its peak at 4 bits. Beyond this point, however, performance begins to decline despite the improved image quality, primarily due to the reduced number of stored samples. This highlights a trade-off between image quality and sample quantity, emphasizing the need to select an optimal bit-width that balances memory constraints and model performance in practical applications.

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Comparison Between Our BCDC and Direct Quantization: To assess the effectiveness of our quantization-aware fine-tuning strategy, we compare the performance of our BCDC against a baseline approach that applies direct quantization to the distilled dataset without any additional fine-tuning. This comparison is presented in Table 8. The results clearly demonstrate that BCDC achieves substantially better performance, highlighting the importance of adapting the distilled representations to the quantized setting. By incorporating quantization into the fine-tuning process, BCDC effectively mitigates the performance degradation typically caused by direct quantization, thereby preserving the utility of the condensed data under limited bit precision.

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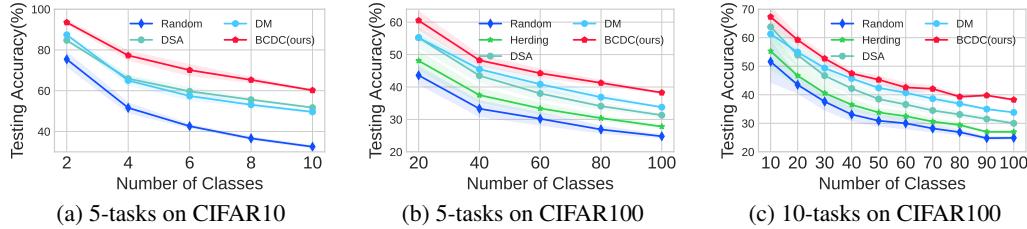


Figure 2: Test accuracy on the class-incremental learning task.

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434 Table 6: Comparison of dataset condensation methods on Tiny-ImageNet and ImageNet-1K. Results
are reported as test accuracy (%) on condensed dataset.

Dataset	Img/Cl	D ¹ M	RDED	CMI	DWA	+BCDC	D ³ HR	+BCDC
Tiny-ImageNet	50	46.2	58.2 ± 0.1	53.7 ± 0.3	52.8 ± 0.2	55.1 ± 0.3	56.9 ± 0.2	59.3 ± 0.4
Tiny-ImageNet	100	51.4	—	56.9 ± 0.3	56.0 ± 0.2	59.6 ± 0.3	59.3 ± 0.1	61.8 ± 0.2
ImageNet-1K	10	27.9	42.0 ± 0.1	38.5 ± 0.3	37.9 ± 0.2	39.6 ± 0.3	44.3 ± 0.3	46.9 ± 0.5
ImageNet-1K	50	55.2	56.5 ± 0.1	55.6 ± 0.3	55.2 ± 0.2	57.5 ± 0.3	59.4 ± 0.1	62.6 ± 0.3
ImageNet-1K	100	59.3	—	59.8 ± 0.4	59.2 ± 0.3	61.7 ± 0.4	62.5 ± 0.0	64.7 ± 0.2

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442 Table 7: Cross-architecture evaluation on CIFAR-10 using 10 images per class. *Train* denotes the
443 architecture used to condense the dataset, while *Transfer* refers to the architecture trained on the
444 condensed data.

Method	Train \ Transfer	ConvNet	LeNet	AlexNet	VGG11	ResNet18
DSA	AlexNet	30.4±0.7	24.2±0.4	28.3±0.4	27.2±1.0	27.8±1.1
	ConvNet	31.4±1.1	21.7±1.6	25.9±0.8	27.6±0.8	27.6±1.4
DM	AlexNet	41.4±0.8	31.4±0.2	37.5±0.9	36.8±0.5	34.9±1.1
	ConvNet	42.2±0.5	33.4±0.6	38.8±1.3	36.2±1.0	34.6±0.5
DM+BCDC (ours)	AlexNet	57.6±0.5	33.8±0.7	51.2±0.6	52.3±0.7	52.1±0.8
	ConvNet	58.0±0.5	46.9±0.8	54.6±0.9	52.0±0.7	51.8±0.9

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Comparison with standard image compression techniques (JPEG, WebP.) We compare our approach with standard image compression methods, such as JPEG and WebP, in Table 11 in Appendix. Notably, our method can be seamlessly combined with these compression techniques and yields substantial additional improvements in dataset distillation performance.

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Dataset Condensation Visualization We present visualizations of the condensed images under different quantization levels integrating with DC in Figure 1 and integrating with DM in Figure 3 in Appendix. As the number of bits used to represent each image increases, more fine-grained visual details are preserved, resulting in higher image quality. However, this comes at the cost of increased storage requirements per image. Conversely, using fewer bits reduces the fidelity of the image, leading to some loss of detail, but significantly lowers the memory footprint. This trade-off enables the storage of a larger number of images within the same memory budget, which is particularly beneficial in resource-constrained scenarios.

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Hyperparameter Sensitivity We present hyperparameter sensitivity, e.g., λ , in Appendix D.

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5.3 DATASET CONDENSATION FOR CONTINUAL LEARNING

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In this section, we evaluate the effectiveness of BCDC in the context of continual learning. We adopt a class-incremental learning setting under tight memory constraints—specifically, 10 images per class for CIFAR10 and 20 images per class for CIFAR100. We integrate BCDC with the GDumb framework (Prabhu et al., 2020) using either coreset selection strategies (Random, Herding) or dataset distillation techniques (DSA, DM). Experiments are conducted on two standard benchmarks: CIFAR10, split into 5 sequential tasks, and CIFAR100, evaluated under both 5-task and 10-task settings. As shown in Fig. 2, GDumb combined with BCDC consistently outperforms all other variants. This demonstrates that our low-bit condensed data remains highly informative and effective in CL scenarios.

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6 CONCLUSION
In this paper, we proposed a low-bit data representation quantization method to compress datasets into small-scale condensed versions, significantly reducing memory storage costs. Through extensive experiments across multiple datasets and settings, we demonstrated the effectiveness of our approach in maintaining data utility while achieving substantial compression. The results highlight the potential of our method to facilitate efficient data storage and processing in resource-constrained environments. Future work could explore adaptive quantization strategies to further enhance performance.

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Appendix705
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A DERIVATIONS FOR DATA-GRADIENTS708
709
710 When adapting parameters θ to a task via fine-tuning, the optimal post-adaptation parameters $Q(\mathcal{S})$
711 depend implicitly on θ . We compute $\frac{\partial Q(\mathcal{S})}{\partial \theta}$ using chain rule on the optimality condition:712
713
714 **Fine-Tuning (Inner Loop)** At convergence, the gradient of the fine-tuning loss L_{train} w.r.t. $Q(\mathcal{S})$ is
715 zero:

716
717
$$\frac{\partial L_{\text{train}}(\theta)}{\partial \theta} = 0 \quad (\text{Optimality condition})$$

718
719 Differentiate the optimality condition w.r.t. θ :

720
721
$$\frac{\partial}{\partial Q(\mathcal{S})} \left(\frac{\partial L_{\text{train}}(\theta)}{\partial \theta} \right) = 0$$

722
723
$$\frac{\partial^2 L_{\text{train}}}{\partial Q(\mathcal{S}) \partial \theta} + \frac{\partial^2 L_{\text{train}}}{\partial \theta^2} \frac{\partial \theta}{\partial Q(\mathcal{S})} = 0$$

724
725
726 **Solve for $\frac{\partial \theta}{\partial Q(\mathcal{S})}$**

727
728
$$\frac{\partial \theta}{\partial Q(\mathcal{S})} \approx - \left(\frac{\partial^2 L_{\text{train}}}{\partial \theta^2} \right)^{-1} \frac{\partial^2 L_{\text{train}}}{\partial Q(\mathcal{S}) \partial \theta}$$

729 where $\frac{\partial^2 L_{\text{train}}}{\partial Q(\mathcal{S})^2}$: Hessian of the fine-tuning loss; $\frac{\partial^2 L_{\text{train}}}{\partial Q(\mathcal{S}) \partial \theta}$: Mixed partial derivative; The inverse
730 Hessian adjusts the meta-gradient for inner-loop dynamics.731
732
B THEOREM PROOF733
734 B.1 PROOF FOR THEOREM 4.5735
736 *Proof.* We decompose the true risk $R_{\mathcal{D}}(h)$ as:

737
738
$$R_{\mathcal{D}}(h) = \underbrace{\hat{R}_{Q_b(\mathcal{S})}(h)}_{\text{Empirical Risk}} + \underbrace{R_{\mathcal{D}}(h) - R_{\mathcal{S}}(h)}_{\text{Condensation Error}} + \underbrace{R_{\mathcal{S}}(h) - \hat{R}_{Q_b(\mathcal{S})}(h)}_{\text{Quantization Error}} \quad (11)$$

739
740 Using Rademacher complexity and Hoeffding's inequality:

741
742
$$R_{\mathcal{D}}(h) - R_{\mathcal{S}}(h) \leq 2\mathfrak{R}_{m(b)}(\mathcal{H}) + \sqrt{\frac{\log(2/\delta)}{2m(b)}} \quad (12)$$

743
744
745
$$\leq \frac{2\kappa}{\sqrt{m(b)}} + \sqrt{\frac{\log(2/\delta)}{2m(b)}} \quad (13)$$

746
747
748 **Rademacher Complexity Foundation** For any hypothesis class \mathcal{H} , the generalization gap can be
749 bounded via Rademacher complexity:

750
751
752
$$R_{\mathcal{D}}(h) - R_{\mathcal{S}}(h) \leq 2\mathfrak{R}_m(\mathcal{H}) + \sup_{h \in \mathcal{H}} |R_{\mathcal{D}}(h) - R_{\mathcal{S}}(h)| \quad (14)$$

753
754
755 Where $\mathfrak{R}_m(\mathcal{H}) = \mathbb{E}_{\mathcal{S}} \mathbb{E}_{\sigma} [\sup_{h \in \mathcal{H}} \frac{1}{m} \sum_{i=1}^m \sigma_i \ell(h(x_i^s), y_i^s)]$ with $\sigma_i \in \{\pm 1\}$ being Rademacher
variables.

756 **Finite-Sample Concentration** Applying Hoeffding's inequality to the second term, for any fixed
 757 h :

$$759 \quad 760 \quad 761 \quad \mathbb{P}(|R_{\mathcal{D}}(h) - R_{\mathcal{S}}(h)| \geq \epsilon) \leq 2 \exp\left(-\frac{2m\epsilon^2}{L^2}\right) \quad (15)$$

762 Taking a union bound over \mathcal{H} with finite VC-dimension d :

$$764 \quad 765 \quad 766 \quad \mathbb{P}\left(\sup_{h \in \mathcal{H}} |R_{\mathcal{D}}(h) - R_{\mathcal{S}}(h)| \geq \epsilon\right) \leq 2\mathcal{N}(\mathcal{H}, \epsilon) \exp\left(-\frac{2m\epsilon^2}{L^2}\right) \quad (16)$$

767 Where $\mathcal{N}(\mathcal{H}, \epsilon)$ is the covering number. For parametric models, $\log \mathcal{N}(\mathcal{H}, \epsilon) \asymp d \log(1/\epsilon)$.

769 **Solving for High Probability Bound** Set the RHS to $\delta/2$ and solve for ϵ :

$$772 \quad 773 \quad 774 \quad 2 \exp\left(d \log(1/\epsilon) - \frac{2m\epsilon^2}{L^2}\right) = \delta/2 \quad (17)$$

$$775 \quad 776 \quad \Rightarrow \epsilon \asymp \sqrt{\frac{d \log(m/d) + \log(2/\delta)}{m}} \quad (18)$$

778 This gives the standard generalization bound:

$$780 \quad 781 \quad 782 \quad R_{\mathcal{D}}(h) - R_{\mathcal{S}}(h) \leq \frac{2\kappa}{\sqrt{m}} + \sqrt{\frac{\log(2/\delta)}{2m}} \quad (19)$$

783 Using Lipschitz and β -smoothness properties:

$$785 \quad |R_{\mathcal{S}}(h) - \hat{R}_{Q_b(\mathcal{S})}(h)| \leq L\mathbb{E}[\|h(\mathbf{x}) - h(Q_b(\mathbf{x}))\|] + \frac{\beta}{2}\mathbb{E}[\|h(\mathbf{x}) - h(Q_b(\mathbf{x}))\|^2] \quad (20)$$

$$787 \quad \leq LC2^{-b} + \beta C^2 2^{-2b} \quad (21)$$

789 Applying the union bound and rescaling δ :

$$791 \quad 792 \quad 793 \quad R_{\mathcal{D}}(h) \leq \hat{R}_{Q_b(\mathcal{S})}(h) + \frac{2\kappa}{\sqrt{m(b)}} + \sqrt{\frac{\log(2/\delta)}{2m(b)}} + LC2^{-b} + \beta C^2 2^{-2b} \quad (22)$$

794 The following is a detailed derivation of how Rademacher complexity and Hoeffding's inequality are
 795 applied in the proof with probabilistic bounds and their interaction with quantization:

797 **Incorporating Quantization Effects** When replacing \mathcal{S} with $Q_b(\mathcal{S})$, the key modification appears
 798 in the Rademacher term:

$$801 \quad 802 \quad 803 \quad \mathfrak{R}_m(\mathcal{H}, Q_b) = \mathbb{E} \left[\sup_h \frac{1}{m} \sum_i \sigma_i \ell(h(Q_b(x_i^s)), y_i^s) \right] \quad (23)$$

$$804 \quad 805 \quad 806 \quad \leq \mathfrak{R}_m(\mathcal{H}) + L\mathbb{E} \left[\sup_h \frac{1}{m} \sum_i \sigma_i (h(Q_b(x_i^s)) - h(x_i^s)) \right] \quad (24)$$

$$807 \quad 808 \quad 809 \quad \leq \mathfrak{R}_m(\mathcal{H}) + LC2^{-b} \sqrt{\frac{d}{m}} \quad (25)$$

The second inequality uses the Lipschitz property and the quantization error bound.

810 **Final Composition** Combining all terms while accounting for the memory constraint $m(b) =$
 811 $M/(bd)$:

$$813 \quad R_{\mathcal{D}}(h) \leq \hat{R}_{Q_b(\mathcal{S})}(h) + \frac{2\kappa}{\sqrt{m(b)}} + \sqrt{\frac{\log(2/\delta)}{2m(b)}} + LC2^{-b} + \beta C^2 2^{-2b} \quad (26)$$

816 \square

819 **C DERIVATION OF KL DIVERGENCE BOUND**

821 The Fisher Information Matrix (FIM) is defined as the covariance of the score function (gradient of
 822 log-posterior):

$$824 \quad \mathbf{F} = I(\mathcal{S}; \boldsymbol{\theta}) = \mathbb{E}_{p(\boldsymbol{\theta}|\mathcal{S})} [\nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta}|\mathcal{S}) \nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta}|\mathcal{S})^\top] \quad (27)$$

827 This matrix captures the **local curvature** of the log-posterior, measuring how sensitive the distribution
 828 is to small changes in $\boldsymbol{\theta}$.

829 **Lemma C.1.**

$$830 \quad D_{KL} \leq \frac{L^2 \Delta^2}{8} \text{tr}(\mathbf{F}) \quad (28)$$

833 *Proof.* Using the Largest Eigenvalue (λ_{\max})

$$834 \quad \Delta^\top \mathbf{F} \Delta \leq \lambda_{\max}(\mathbf{F}) \|\Delta\|^2 \quad (29)$$

836 This gives:

$$837 \quad D_{KL} \leq \frac{1}{2} \lambda_{\max}(\mathbf{F}) \|\Delta\|^2 \quad (30)$$

839 Since $\text{tr}(\mathbf{F}) = \sum_i \lambda_i \geq \lambda_{\max}(\mathbf{F})$, we can write:

$$841 \quad \Delta^\top \mathbf{F} \Delta \leq \text{tr}(\mathbf{F}) \|\Delta\|^2 \quad (31)$$

843 Thus:

$$844 \quad D_{KL} \leq \frac{1}{2} \text{tr}(\mathbf{F}) \|\Delta\|^2 \quad (32)$$

846 If $\|\nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta}|\mathcal{S})\| \leq L$, then:

848 The perturbation Δ may be scaled by L (i.e., $\|\Delta\| \leq \frac{L\Delta}{2}$). This leads to:

$$850 \quad D_{KL} \leq \frac{L^2 \Delta^2}{8} \text{tr}(\mathbf{F}) \quad (33)$$

852 Combining these results:

$$854 \quad D_{KL} \leq \frac{L^2 \Delta^2}{8} \text{tr}(I(\mathcal{S}; \boldsymbol{\theta})) \quad (34)$$

856 where $\text{tr}(I(\mathcal{S}; \boldsymbol{\theta}))$ measures total sensitivity, L is the Lipschitz constant, Δ is the perturbation
 857 magnitude

858 \square

860 **C.1 THEOREM PROOF FOR THEOREM 4.6**

862 We aim to prove:

$$863 \quad I(\mathcal{T}; \boldsymbol{\theta}) - I(Q_b(\mathcal{S}); \boldsymbol{\theta}) \leq \mathbb{E}_{\mathcal{D}} [D_{KL}(p(\boldsymbol{\theta}|\mathcal{T}) \| p(\boldsymbol{\theta}|Q_b(\mathcal{S})))],$$

864 EXPRESS MUTUAL INFORMATION AS KL DIVERGENCE
865

866 Mutual information can be written as:

867
$$I(\mathcal{T}; \boldsymbol{\theta}) = \mathbb{E}_D [D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta}))],$$

868
$$I(Q_b(\mathcal{S}); \boldsymbol{\theta}) = \mathbb{E}_{Q_b(\mathcal{S})} [D_{\text{KL}}(p(\boldsymbol{\theta}|Q_b(\mathcal{S}))\|p(\boldsymbol{\theta}))].$$

869

870 Since $Q_b(\mathcal{S})$ is a function of D , we rewrite:

871
$$I(Q_b(\mathcal{S}); \boldsymbol{\theta}) = \mathbb{E}_D [D_{\text{KL}}(p(\boldsymbol{\theta}|Q_b(\mathcal{S}))\|p(\boldsymbol{\theta}))].$$

872

873
$$I(\mathcal{T}; \boldsymbol{\theta}) - I(Q_b(\mathcal{S}); \boldsymbol{\theta}) = \mathbb{E}_D [D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta})) - D_{\text{KL}}(p(\boldsymbol{\theta}|Q_b(\mathcal{S}))\|p(\boldsymbol{\theta}))].$$

874

875 EXPAND KL DIVERGENCE
876877 Using $D_{\text{KL}}(p\|q) = \mathbb{E}_p \left[\log \frac{p}{q} \right]$:

879
$$D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta})) = \mathbb{E}_{\boldsymbol{\theta}|\mathcal{T}} \left[\log \frac{p(\boldsymbol{\theta}|\mathcal{T})}{p(\boldsymbol{\theta})} \right],$$

880
881
$$D_{\text{KL}}(p(\boldsymbol{\theta}|Q_b(\mathcal{S}))\|p(\boldsymbol{\theta})) = \mathbb{E}_{\boldsymbol{\theta}|Q_b(\mathcal{S})} \left[\log \frac{p(\boldsymbol{\theta}|Q_b(\mathcal{S}))}{p(\boldsymbol{\theta})} \right].$$

882
883

884 REWRITE THE DIFFERENCE INSIDE EXPECTATION
885

886
$$D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta})) - D_{\text{KL}}(p(\boldsymbol{\theta}|Q_b(\mathcal{S}))\|p(\boldsymbol{\theta}))$$

887
$$= \mathbb{E}_{\boldsymbol{\theta}|\mathcal{T}} \left[\log \frac{p(\boldsymbol{\theta}|\mathcal{T})}{p(\boldsymbol{\theta})} - \log \frac{p(\boldsymbol{\theta}|Q_b(\mathcal{S}))}{p(\boldsymbol{\theta})} \right]$$

888
$$= \mathbb{E}_{\boldsymbol{\theta}|\mathcal{T}} \left[\log \frac{p(\boldsymbol{\theta}|\mathcal{T})}{p(\boldsymbol{\theta}|Q_b(\mathcal{S}))} \right]$$

889
$$= D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta}|Q_b(\mathcal{S}))).$$

890
891

892 Take Expectation Over D
893

894
$$I(\mathcal{T}; \boldsymbol{\theta}) - I(Q_b(\mathcal{S}); \boldsymbol{\theta}) = \mathbb{E}_D [D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta}|Q_b(\mathcal{S})))].$$

895

896 INEQUALITY FOR STOCHASTIC $Q_b(\mathcal{S})$
897898 For stochastic $Q_b(\mathcal{S})$ (e.g., variational approximations), we have:

899
$$I(\mathcal{T}; \boldsymbol{\theta} | Q_b(\mathcal{S})) \leq \mathbb{E}_D [D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta}|Q_b(\mathcal{S})))],$$

900

901 where $I(\mathcal{T}; \boldsymbol{\theta} | Q_b(\mathcal{S})) = I(\mathcal{T}; \boldsymbol{\theta}) - I(Q_b(\mathcal{S}); \boldsymbol{\theta})$. This holds because equality is achieved when
902 $Q_b(\mathcal{S})$ is deterministic in D . For stochastic $Q_b(\mathcal{S})$, the KL divergence overcounts discrepancies,
903 making it an upper bound.
904

905
$$I(\mathcal{T}; \boldsymbol{\theta}) - I(Q_b(\mathcal{S}); \boldsymbol{\theta}) \leq \mathbb{E}_D [D_{\text{KL}}(p(\boldsymbol{\theta}|\mathcal{T})\|p(\boldsymbol{\theta}|Q_b(\mathcal{S})))]$$

906

907 The second-order Taylor expansion of $\log p(\boldsymbol{\theta} | Q_b(\mathcal{S}))$ around \mathcal{S} is:
908

909
$$\log p(\boldsymbol{\theta} | Q_b(\mathcal{S})) \approx \log p(\boldsymbol{\theta} | \mathcal{S}) + \nabla_{\mathcal{S}} \log p(\boldsymbol{\theta} | \mathcal{S})^{\top} (Q_b(\mathcal{S}) - \mathcal{S}) + \frac{1}{2} (Q_b(\mathcal{S}) - \mathcal{S})^{\top} \nabla_{\mathcal{S}}^2 \log p(\boldsymbol{\theta} | \mathcal{S}) (Q_b(\mathcal{S}) - \mathcal{S}).$$

910
911

912 The KL divergence between $p(\boldsymbol{\theta} | \mathcal{S})$ and $p(\boldsymbol{\theta} | Q_b(\mathcal{S}))$ is:
913

914
$$D_{KL}(p(\boldsymbol{\theta} | \mathcal{S}) \| p(\boldsymbol{\theta} | Q_b(\mathcal{S}))) = \mathbb{E}_{p(\boldsymbol{\theta} | \mathcal{S})} [\log p(\boldsymbol{\theta} | \mathcal{S}) - \log p(\boldsymbol{\theta} | Q_b(\mathcal{S}))]. \quad (36)$$

915

916 Substituting the Taylor expansion:
917

918
$$D_{KL} \approx \mathbb{E}_{p(\boldsymbol{\theta} | \mathcal{S})} \left[-\nabla_{\mathcal{S}} \log p(\boldsymbol{\theta} | \mathcal{S})^{\top} (Q_b(\mathcal{S}) - \mathcal{S}) - \frac{1}{2} (Q_b(\mathcal{S}) - \mathcal{S})^{\top} \nabla_{\mathcal{S}}^2 \log p(\boldsymbol{\theta} | \mathcal{S}) (Q_b(\mathcal{S}) - \mathcal{S}) \right].$$

919
920

918 • The score function $\nabla_{\mathcal{S}} \log p(\boldsymbol{\theta} \mid \mathcal{S})$ has zero expectation under $p(\boldsymbol{\theta} \mid \mathcal{S})$:

919
$$\mathbb{E}_{p(\boldsymbol{\theta} \mid \mathcal{S})} [\nabla_{\mathcal{S}} \log p(\boldsymbol{\theta} \mid \mathcal{S})] = 0. \quad (38)$$

920 • The Fisher information matrix is defined as:

921
$$\mathbf{F} = -\mathbb{E}_{p(\boldsymbol{\theta} \mid \mathcal{S})} [\nabla_{\mathcal{S}}^2 \log p(\boldsymbol{\theta} \mid \mathcal{S})]. \quad (39)$$

922 Thus, the KL divergence simplifies to:

923
$$D_{KL} \approx \frac{1}{2} (Q_b(\mathcal{S}) - \mathcal{S})^\top \mathbf{F} (Q_b(\mathcal{S}) - \mathcal{S}). \quad (40)$$

924 We define $\epsilon = Q_b(\mathcal{S}) - \mathcal{S}$

925 The KL divergence becomes:

926
$$D_{KL} \approx \frac{1}{2} \mathbb{E} [\epsilon^\top (-\nabla_{\mathcal{S}}^2 \log p(\boldsymbol{\theta} \mid \mathcal{S})) \epsilon]$$

927 **Bounding the Terms:**

928
$$\|\nabla_{\mathcal{S}}^2 \log p(\boldsymbol{\theta} \mid \mathcal{S})\|_{\text{op}} \leq L^2 \Rightarrow -\nabla_{\mathcal{S}}^2 \log p(\boldsymbol{\theta} \mid \mathcal{S}) \preceq L^2 I$$

929
$$\|\epsilon\|_2 = \|\mathcal{S} - Q_b(\mathcal{S})\|_2 \leq \sqrt{d} \Delta / 2$$

930
$$D_{KL} \leq \frac{\Delta^2}{8} \text{tr} (\mathbb{E} [\nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta} \mid \mathcal{S}) \nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta} \mid \mathcal{S})^\top]) = \frac{L^2 \Delta^2}{8} \text{tr}(I(\mathcal{S}; \boldsymbol{\theta}))$$

931 The Fisher information loss due to b -bit quantization is bounded by:

932
$$I(\mathcal{T}; \boldsymbol{\theta}) - I(Q_b(\mathcal{S}); \boldsymbol{\theta}) \leq \frac{L^2 \Delta^2}{8} \text{tr} (\mathbb{E} [\nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta} \mid \mathcal{S}) \nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta} \mid \mathcal{S})^\top])$$

933 where $\Delta = 2^{-b+1} (\max(\mathcal{S}) - \min(\mathcal{S}))$.

934 **D MORE EXPERIMENTAL RESULTS**

935 **Table 9: Training Efficiency Comparison of Dataset Distillation Methods**

Method	DC	DC+BCDC	DM	DM+BCDC
Running Time (hours)	10.97	12.83	7.15	8.62

936 **Table 10: Effect of Bandwidth on Dataset Distillation Performance on CIFAR10 Under the Same**
937 **Memory Budget**

Bandwidth (bits/sample)	2-bit	4-bit	8-bit
DC+BCDC (SM)	25.2 ± 0.9	36.1 ± 0.6	27.3 ± 0.7
DSA+BCDC (SM)	28.3 ± 0.6	42.2 ± 0.3	29.4 ± 0.5
DM+BCDC (SM)	31.7 ± 0.3	45.1 ± 0.8	30.5 ± 0.6

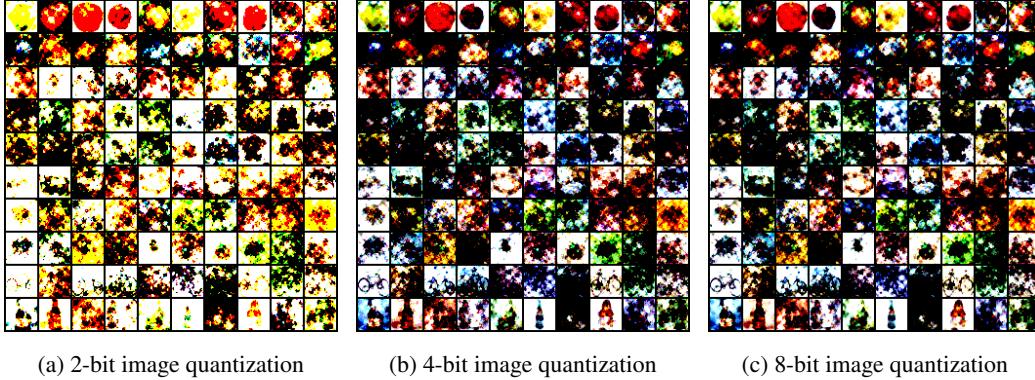


Figure 3: Visualization of different bits image quantization by integrating BCDC with DM.

987 D.1 COMPARISON WITH STANDARD IMAGE COMPRESSION TECHNIQUES (JPEG, WEBP.)

989 We compare our approach with standard image compression methods, such as JPEG and WebP, in
 990 Table 11. Notably, our method can be seamlessly combined with these compression techniques and
 991 yields substantial additional improvements in dataset distillation performance.

Dataset	Img/Clss	DM	DM+BCDC (Ours)	BCDC+JPEG	BCDC+WebP
CIFAR100	10	29.7 ± 0.3	38.3 ± 0.7	41.2 ± 0.6	43.6 ± 0.8

Table 11: Comparison with standard image compression techniques (JPEG, WebP, etc.) on CIFAR100.

998 **Dataset Condensation with Distribution Matching By Quantized Data Representations** Integrating BCDC with surrogate loss function, e.g., DM (Zhao & Bilen, 2023) is presented in Algorithm
 999 2.

Algorithm 2 Dataset Condensation with Distribution Matching By Quantized Data Representations

1: **Require:** Training set \mathcal{T} , randomly initialized synthetic samples \mathcal{S} for J classes, deep neural
 2: network ψ_{θ} parameterized by θ , parameter distribution P_{θ} , differentiable augmentation \mathcal{A}_{ω}
 3: parameterized by ω , augmentation distribution Ω , training iterations K , learning rate η
 4: **for** $k = 0$ to $K - 1$ **do**
 5: Sample $\theta \sim P_{\theta}$
 6: **for** each class $j = 0$ to $J - 1$ **do**
 7: Sample mini-batch pairs $B_c^{\mathcal{T}} \sim \mathcal{T}$, $B_c^{\mathcal{S}} \sim \mathcal{S}$, and $\omega_c \sim \Omega$
 8: **end for**
 9: Compute the loss:
 10:
$$\mathcal{L} = \sum_{c=0}^{C-1} \left\| \frac{1}{|B_c^{\mathcal{T}}|} \sum_{(\mathbf{x}, y) \in B_c^{\mathcal{T}}} \psi_{\theta}(\mathcal{A}_{\omega_c}(\mathbf{x})) - \frac{1}{|B_c^{\mathcal{S}}|} \sum_{(\mathbf{s}, y) \in B_c^{\mathcal{S}}} \psi_{\theta}(\mathcal{A}_{\omega_c}(Q_V(\mathbf{s}))) \right\|^2$$

 11: Update $\mathcal{S} \leftarrow \mathcal{S} - \eta \nabla_{\mathcal{S}} \mathcal{L}$
 12: **end for**
 13: 10: **Output:** \mathcal{S}
