

# 000 001 002 003 004 005 BAYESIAN SOCIAL DEDUCTION WITH 006 GRAPH-INFORMED LANGUAGE MODELS 007 008

009 **Anonymous authors**  
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## ABSTRACT

025 Social reasoning—inferring unobservable beliefs and intentions from partial ob-  
026 servations of other agents—remains a challenging task for large language mod-  
027 els (LLMs). We evaluate the limits of current reasoning language models in the  
028 social deduction game *Avalon* and find that while the largest models demonstrate  
029 strong performance, they require extensive test-time inference and degrade sharply  
030 when distilled to smaller, real-time-capable variants. To address this, we introduce  
031 a hybrid reasoning framework that externalizes belief inference to a structured  
032 probabilistic model, while using an LLM for language understanding and inter-  
033 action. Our approach achieves competitive performance with much larger models in  
034 Agent-Agent play and, notably, is the first language agent to defeat human players  
035 in a controlled study—achieving a 67% win rate and receiving higher qualitative  
036 ratings than both reasoning baselines and human teammates. We release code,  
037 models, and a dataset to support future work on social reasoning in LLM agents.  
038

## 1 INTRODUCTION

039 Large language models (LLMs) have demonstrated remarkable general-purpose reasoning capabili-  
040 ties across a wide range of tasks (Ahn et al., 2024; Duan et al., 2024; Qiao et al., 2023), yet their  
041 ability to engage in social reasoning—particularly in multi-agent settings where participants hold  
042 private beliefs and (potentially) deceptive intentions—remains an open challenge (Mireshghallah  
043 et al., 2024; Shapira et al., 2024; Ullman, 2023). Recent studies (Li et al., 2023; Liu et al., 2024;  
044 Stepputtis et al., 2023) suggest that state-of-the-art LLMs often struggle to infer the latent goals and  
045 beliefs of other agents in such scenarios, limiting their effectiveness in settings that require theory  
046 of mind or strategic social deduction.

047 We revisit this problem in the context of *Avalon*, a social deduction game<sup>1</sup> that provides a structured  
048 yet complex environment for evaluating an agent’s ability to infer hidden roles, manage uncertainty,  
049 and interact cooperatively or competitively with others by utilizing deception and persuasion. *Avalon*  
050 is particularly challenging as it requires agents to utilize *constrained probabilistic reasoning* over  
051 long temporal horizons, an aspect rarely seen in prior benchmarks. Consider the following example.  
052

053 **Prompt:** *There are five players (Alice, Bob, Carol, Dave, and Eve), two of which  
054 are Evil while the rest are Good. The first party (consisting of Alice and Bob)  
055 and the second party (Carol and Dave) both failed. If each Evil player has a 70%  
056 chance to fail the quest, what is the probability that each player is Evil?*

057 **LLM:** *Each individual appears in exactly half of the possible pairs. Therefore,  
058 the probability that any specific individual is Evil is: 0.5*

059 Despite its simplicity, state-of-the-art large reasoning models (LRMs) (Xu et al., 2025; Zhou et al.,  
060 2025), including 8B and 70B variants of Deepseek-R1, fail to successfully reason that the only player  
061 to not appear in a party, Eve, has a 0% probability of being Evil, as there are only two Evil players.  
062 While larger models are capable of solving this trivial example, they too struggle as the temporal  
063 horizon increases and social aspects are brought into play. Furthermore, performance gains come at  
064

065 <sup>1</sup>Although referred to as “social deduction”, reasoning in social deduction games is inherently  
066 probabilistic—due to epistemic uncertainty—rather than purely deductive (logical or mathematical).

054 a significant computational cost, requiring long chains of reasoning tokens, rendering such models  
 055 impractical for real-time interactive play with human users.  
 056

057 **To overcome this limitation, we propose a *hybrid reasoning framework that augments LLMs***  
 058 **with structured probabilistic inference over beliefs**, combining the linguistic grounding and rich  
 059 priors of foundation models with the rigor of Bayesian reasoning. In this paper, we present **GRAIL**  
 060 (**Graph Reasoning Agent Informed through Language**), a hybrid framework where an LLM han-  
 061 dles dialogue parsing, generates utterances, and interprets informal social cues, while a probabilistic  
 062 graphical model tracks latent roles and beliefs by analyzing observed game events and social inter-  
 063 actions. This decoupling makes belief inference both interpretable and efficient, avoiding the need  
 064 for extensive token generation during gameplay.

065 Despite using a significantly smaller LLM, our method matches or exceeds the performance of large-  
 066 scale reasoning models across multiple metrics, including win rate, belief accuracy, and belief con-  
 067 sistency in Agent-Agent *Avalon* games. Notably, GRAIL is, to the best of our knowledge, the first  
 068 language agent to successfully play and win against novice human players in a controlled participant  
 069 study, *achieving a striking 67% win rate*. In post-game surveys, participants rated GRAIL’s contribu-  
 070 tions and helpfulness significantly higher than reasoning model baselines, and in many cases,  
 071 even over other human players. These results suggest that external structured reasoning models ef-  
 072 fectively complement LLMs, enabling socially competent behavior in real-time interactive settings.

073 To support future work on social reasoning in multi-agent environments, we release our framework,  
 074 agent implementations, a new benchmark, and a dataset of Agent-Agent and Human-Agent *Avalon*  
 075 games, which include player discussions and associated game states. Together, these contributions  
 076 provide a testbed for studying social inference, deception, and cooperation in LLM agents.  
 077

## 078 2 BACKGROUND AND RELATED WORK

081 **Social Deduction Agents:** Social deduction games provide a natural testbed for evaluating the  
 082 social reasoning capabilities of LLMs (Lan et al., 2024). Previous studies have applied LLMs to  
 083 hidden-role and social deduction games such as *Werewolf* (Lai et al., 2023; Wu et al., 2024; Xu  
 084 et al., 2024b;c), *Among Us* (Sarkar et al., 2025), *Avalon* (Light et al., 2023; Wang et al., 2023), and  
 085 *Mafia* (Ibraheem et al., 2022). Before the advent of foundation models, the DeepRole agent (Serrino  
 086 et al., 2019) was trained via self-play to play 5-player versions of *Avalon* without natural dialogue.  
 087 More recently, Stepputtis et al. (2023) explored the use of LLMs for hidden role inference based on  
 088 long-form dialogue in *Avalon* games and demonstrated their shortcomings. In parallel, probabilistic  
 089 graphical models have also been explored in the context of social deduction games (Xu et al., 2024a).

090 **Theory of Mind:** Theory of mind (ToM), the ability to attribute mental states like beliefs, desires,  
 091 and intentions to oneself and others (Ho et al., 2022), is crucial for social reasoning, especially  
 092 for deception and persuasion (Alon et al., 2023; Ding et al., 2015). The presence of ToM-like  
 093 abilities in Large Language Models (LLMs) is currently debated (Kosinski, 2024; Shapira et al.,  
 094 2024; Strachan et al., 2024). Notably, Riemer et al. (2025) argue that high performance on ToM  
 095 benchmarks may not reflect genuine ToM reasoning in LLMs, as these abilities may not extend  
 096 to novel scenarios. Social deduction games offer a robust environment for probing these limits,  
 097 as success requires agents to model and reason about the intentions, beliefs, and likely actions of  
 098 others (Guo et al., 2024; Zhou et al., 2023; , FAIR). More recently, Sclar et al. (2023) showed that  
 099 graph-based models, combined with LLMs, can support belief reasoning in standard ToM tasks.

100 **Scaling and Reasoning:** LLMs exhibit *discontinuous* improvements in zero-shot reasoning with  
 101 increased model size (Chowdhery et al., 2023; Wei et al., 2022), a trend that extends to common-  
 102 sense and social reasoning (Shapira et al., 2024). Besides scaling parameters, reasoning ability can  
 103 be enhanced by scaling test-time token generation (Zhang et al., 2025). Recent work has posited  
 104 that these reasoning models are capable of step-by-step problem-solving on a variety of benchmarks,  
 105 even with fewer parameters (DeepSeek-AI, 2025; Team, 2025; Zhang et al., 2025). This presents a  
 106 trade-off between *test-time computation* and *parameter count* as an alternative means for improving  
 107 social reasoning capabilities (Muennighoff et al., 2025; Snell et al., 2024). However, several recent  
 108 studies argue that benchmark gains alone are not evidence of emergent reasoning capabilities (Scha-  
 109 fffer et al., 2023; Yu et al., 2025).

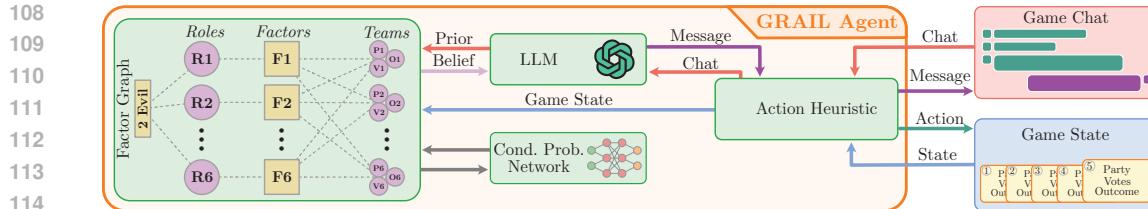


Figure 1: Overview of GRAIL’s architecture and inter-module communication. A factor graph tracks beliefs over hidden player roles using belief propagation, informed by game-state observations and an LLM-generated language prior. Conditional probabilities are estimated by a neural network trained on historical games. Inferred beliefs guide both action selection and message generation.

**The Resistance Avalon:** In the social deduction game *Avalon* (Eskridge, 2012), players belong to either the Good team (who try to complete quests) or the Evil team (who try to sabotage them). The game consists of five rounds with quest parties of 2, 3, 4, 3, and 4 members, respectively. Each round, players propose and vote on a party. If approved, its members secretly vote on the quest’s outcome; the quest succeeds only if all members vote success. Good wins by completing three quests; Evil wins by failing three. Players communicate via turn-based chat (see Appendix A for full details). We develop an AI agent that plays exclusively as Good, such that we can focus on identifying Evil opponents.<sup>2</sup>

### 3 BAYESIAN BELIEF INFERENCE WITH FACTOR GRAPHS

To identify Evil players in *Avalon*, an agent must be able to form and support parties composed entirely of Good players, despite lacking knowledge of other players’ roles. This is a **constrained probabilistic reasoning** task, where agents infer latent player roles from observable actions and unstructured natural language dialogue, and then act based on the certainty of their beliefs. This is a challenging task for state-of-the-art language models that rely purely on token-level reasoning. We introduce a hybrid approach that externalizes inference to a structured graphical model well-suited to constrained reasoning. The language model attends to social-linguistic signals while the reasoning model maintains and updates beliefs, enabling strong performance even with small models. Our approach is designed around two core objectives.

**Constraint Satisfaction:** Deduction in *Avalon* depends on satisfying a combination of *hard* and *soft* constraints. For example, the fixed number of Evil players (e.g., two) imposes a hard constraint on valid role assignments. Similarly, a failed quest implies at least one Evil member in the party, introducing a soft constraint that influences belief updates. However, many possible role assignments satisfy these constraints, and agents must consider multiple plausible hypotheses simultaneously.

**Probabilistic Inference:** To support reasoning about plausible role assignments, we model player roles and relevant game variables as random variables in a probabilistic model, allowing us to represent uncertainty over role assignments and update beliefs as new evidence accumulates. We formulate hidden role inference as probabilistic inference over a *factor graph*, which compactly models dependencies and enforces game-specific constraints.

#### 3.1 FACTOR GRAPHS FOR SOCIAL DEDUCTION

A factor graph is a bipartite graph, defined as the triplet  $\mathcal{G} = (\mathcal{V}, \mathcal{F}, \mathcal{E})$  where  $\mathcal{V} = \{X_1, X_2, \dots, X_n\}$  is the set of *variable nodes*,  $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$  is the set of *factor nodes*, and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{F}$  is the set of *edges*. Each factor represents a function: an edge  $(X_i, f_j) \in \mathcal{E}$  exists if and only if  $X_i$  is an argument of  $f_j$ . A factor function can represent a probability distribution if it is normalized, or a constraint if its values are either 0 or 1. Through this, the graph is able to represent dependencies and constraints between a set of variables.

The variable nodes in our factor graph represent both game state and player role variables, with the goal of identifying which players are likely to be Evil. The role variables are denoted

<sup>2</sup>We exclude special roles, e.g. Merlin, to focus on detecting deception rather than producing it.

$\mathcal{R} = \{r_1, \dots, r_6\}$ , where  $r_j \in \{0, 1\}$  indicates whether player  $j$  is Good (0) or Evil (1). The game state variables are given by  $\mathcal{S} = \{p_1, v_1, o_1, \dots, p_6, v_6, o_6\}$ , where  $p_i, v_i, o_i$  represent the party composition, voting outcome, and quest result for quest  $i$ , respectively. A binary factor enforces the hard constraint that exactly two players are Evil. For each role variable  $r_j$ , we include a factor connected to the game state that encodes its conditional probability, defined as  $F = p(r_j | \{p_i, v_i, o_i | \forall i\})$ . An overview of the factor graph structure is shown in Fig. 1, with details provided in Appendix B.

We use max-product belief propagation (Wainwright & Jordan, 2008) to perform approximate maximum a posteriori (MAP) inference over the factor graph, identifying the most likely assignment of hidden roles given the observed game state. Unlike the more common sum-product algorithm, max-product directly estimates a MAP assignment rather than integrating over alternatives (Kschischang et al., 2001; Murphy et al., 1999) and is better suited for handling deterministic constraints (Smith & Gogate, 2014), e.g., the number of Evil players. In our setting, the agent’s role is known, so inference is performed over the remaining five hidden role variables. The max-product algorithm calculates the max-marginals of each hidden variable, which is proportional to *the probability of player  $i$  being Evil* and can be treated as the belief about that variable (Yanover & Weiss, 2003). From this point forward, we refer to this max-marginal as the *belief* of the agent about the player  $i$ ,  $b_i$ . The details of the max-product algorithm and its scalability can be found in Appendix B.2, B.7.

### 3.2 FACTOR FUNCTION APPROXIMATION

Traditionally, factor functions are represented as probability tables, which are impractical in high-dimensional settings such as ours. To address this, we approximate the conditional probability distribution in each factor using a simple feedforward neural network (Richard & Lippmann, 1991). Each factor corresponds to the conditional density  $p(r_j | \{p_i, v_i, o_i | \forall i\})$ , where  $r_j$  is a binary role variable. We model this as a binary classification task, using a sigmoid output to estimate the conditional probability of  $r_j$ . The network is trained on a dataset of over 100,000 games<sup>3</sup> – consisting only of game states without language – collected from AvalonLogs<sup>4</sup> and ProAvalon.<sup>5</sup> To account for temporal partial observability, we apply a masking scheme that zeroes out future inputs, ensuring the model only conditions on information that would have been available at a given point in the game. Details on the network architecture and training procedure are provided in Appendix B.3.1, B.4.

**Mitigating Positional Bias in Factor Functions:** In *Avalon*, players take turns according to a fixed sequence, which introduces positional bias during training and inference. To mitigate this, we augment the training data with all circular permutations of player orderings. Additionally, using separate neural networks for each factor node can also introduce positional bias, which we avoid by using a shared factor function across all nodes. We apply an ego-centric transformation to the input state such that the player corresponding to the current factor is always placed in the first index while preserving the relative positions of other players.

## 4 GRAIL: GRAPH REASONING AGENT INFORMED THROUGH LANGUAGE

Fundamentally, GRAIL is a hybrid model composed of three interconnected components: an LLM, a factor graph for tracking beliefs over player roles, and a heuristic action policy, as illustrated in Fig. 1. The factor graph maintains and updates probabilistic beliefs about player roles based on observable game events. These beliefs are then passed to the LLM, which generates contextually appropriate dialogue based on the agent’s current understanding of the game.

**Game Actions:** Proposing parties and voting for them follows a heuristic policy derived from factor graph beliefs: a party is proposed or approved only if all members are more likely Good than Evil. Furthermore, when our GRAIL agent is the party leader, the heuristic guides the agent through the necessary stages of proposing a party, initiating a discussion, adjusting the proposal if necessary, and initiating a party vote. For more details refer to Appendix E

<sup>3</sup>In Appendix B.5 we show that only 2.5K-5K games are required for sufficient predictive performance.

<sup>4</sup><https://github.com/WhoaWhoa/avalonlogs>

<sup>5</sup><https://www.proavalon.com>

216 Table 1: Win rates across different team compositions. Each matchup consists of 20 games.  
217

(a) Win rates for homogeneous agent teams.						(b) Win rates for mixed GRAIL/ReCon teams.			
Good Team	Evil Team				Avg	Good Team Comp.	Evil Team		Avg
	Rand	ReCon	DS-R1	o4-mini			ReCon	o4-mini	
Rand	0.00	0.00	0.00	0.00	0.00	0 GRAIL & 4 ReCon	0.15	0.25	0.20
DeepSeek-R1	0.90	0.35	<b>0.70</b>	<b>0.90</b>	0.71	1 GRAIL & 3 ReCon	0.20	0.25	0.23
GPT-o4-mini	0.70	0.05	0.25	0.50	0.40	2 GRAIL & 2 ReCon	0.20	0.65	0.43
ReCon	0.80	0.15	0.50	0.25	0.43	3 GRAIL & 1 ReCon	0.40	<b>0.90</b>	0.65
<b>GRAIL</b>	<b>0.95</b>	<b>0.45</b>	<b>0.70</b>	<b>0.90</b>	<b>0.75</b>	4 GRAIL & 0 ReCon	<b>0.45</b>	<b>0.90</b>	<b>0.68</b>

#### 288 4.1 INCORPORATING LANGUAGE PRIORS INTO BELIEF PROPAGATION

289 A core part of *Avalon* is the dialogue between players, providing valuable insights into whether or  
290 not a player is Good or Evil. During the discussion of parties and quests, players may contradict  
291 themselves, hint at alliances, or reveal privileged knowledge. To incorporate such information, we  
292 utilize an LLM to estimate priors for beliefs over player roles, which are subsequently integrated  
293 into max-product belief propagation for downstream reasoning.

294 Formally, for player  $j$  we define a prior probability  $p(r_j^t)$  over their role at time step  $t$ , where  $r_j^t = 1$   
295 indicates that player  $j$  is Evil. By default, this prior is uninformative, i.e., uniform, but we use the  
296 LLM to adjust this prior and incorporate language feedback. We present the LLM with the current  
297 chat history and the belief of player  $j$ ,  $b_j^{t-1}$ , and ask it to assess whether the belief should be *higher*,  
298 *lower*, or remain the *same*. The LLM’s qualitative judgement  $\delta_j^t \in \{\text{higher, lower, same}\}$  is  
299 converted into a numeric prior using a pre-defined mapping parameter  $\beta^t$  as follows:

$$p(r_j^t) = \begin{cases} 0.5 + \beta^t & \text{if } \delta_j^t = \text{higher} \\ 0.5 - \beta^t & \text{if } \delta_j^t = \text{lower} \\ 0.5 & \text{if } \delta_j^t = \text{same.} \end{cases}$$

600 In practice, we treat  $\beta$  as a tunable hyperparameter. To avoid overconfidence in early rounds when  
601 little evidence is available, we use values close to 0 and increase them as the game progresses.

602 We adopt this qualitative prompting scheme instead of directly asking LLMs to generate probabilities  
603 because LLMs often struggle to accurately interpret, manipulate, and generate numeric data  
604 (Schwartz et al., 2024; Yang et al., 2025). All prompt templates used to extract priors and generate  
605 messages are provided in Appendix F.

## 625 5 EXPERIMENTS

626 To evaluate GRAIL, we simulated games against synthetic Evil players and assessed its performance  
627 across a wide range of metrics, including win rate, belief accuracy, and belief consistency.

628 **Baselines:** We compared GRAIL against reasoning and non-reasoning agents. Reasoning agents  
629 use LRM for both action selection and message generation, including **DeepSeek-R1** (DeepSeek-  
630 AI, 2025) and OpenAI’s **GPT-o4-mini** (OpenAI, 2025). Non-reasoning agents use LLMs but may  
631 still employ manual chain-of-thought where applicable, such as **ReCon** (Wang et al., 2023). Finally,  
632 a **Random** agent serves as a performance lower bound.

633 **ReCon:** Recursive Contemplation (ReCon), is a cognitive framework for LLM agents inspired by  
634 human recursive thinking. ReCon acts in two stages: "Formulation Contemplation" for initial in-  
635 ternal thought and first-order perspective-taking, and "Refinement Contemplation" for re-evaluating  
636 and refining speech with second-order perspective-taking. Each one of these stages are implemented  
637 through extensive prompt engineering.

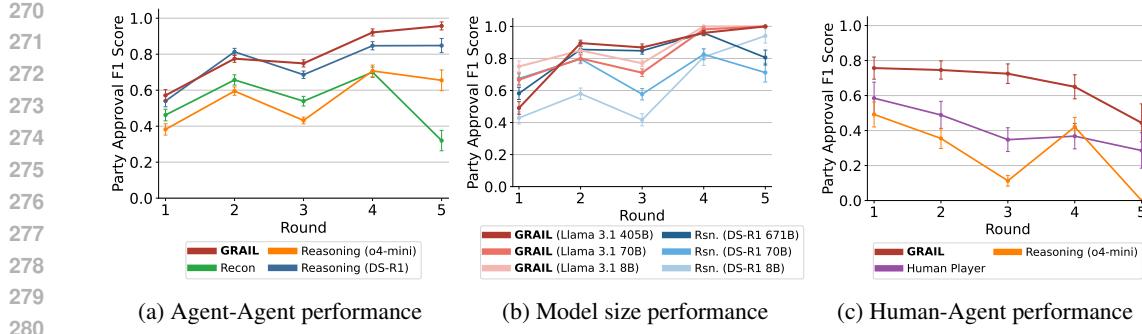


Figure 2: F1 scores of agents’ voting predictions of team composition per round (error bars indicate SE) (a) GRAIL compared to other baseline agents, (b) ablation of GRAIL on non-reasoning Llama 3.1 model compared to DeepSeek-R1 reasoning model across different parameter sizes, (c) GRAIL compared to human players and reasoning models used in the human study.

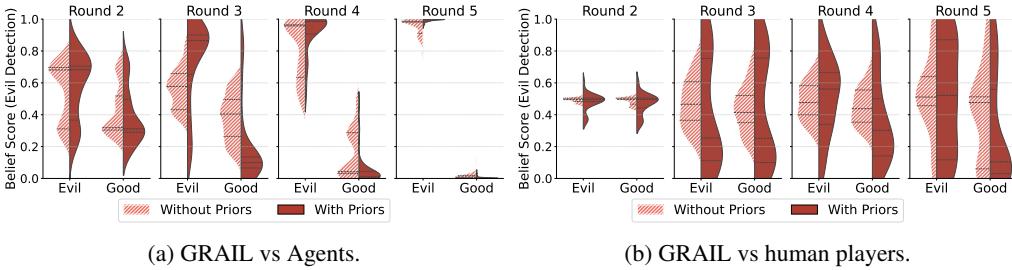


Figure 3: Probability density of GRAIL beliefs about Good and Evil players, with and without using LLM prior in 5-round games (a) 13 games against ReCon and reasoning agents (DS-R1, GPT-o4-mini) (b) 4 games against human players.

### 5.1 AGENT-AGENT EVALUATION

To systematically evaluate agent performance, we constructed a four Good agent vs. two Evil agent matchup matrix. Each pairing was tested over 20 games, with both GRAIL and ReCon utilizing GPT-4.1 as the underlying LLM. The results (Table 1a) show that **GRAIL achieves the highest win rate (average of 75%) among all Good agents**, consistently outperforming both reasoning and non-reasoning baselines, including those using the 671B DeepSeek-R1 LRM.

We further analyzed each agent’s votes with respect to proposed parties, treating these votes as binary predictions of whether a party contains an Evil player. Fig. 2a compares the F1 scores of these predictions across game rounds, showing that GRAIL again outperforms all baselines. This result suggests that GRAIL is particularly effective at reasoning over long horizons, as reflected by strong late-game performance (after the third round). In contrast, GPT-o4-mini and ReCon exhibit a performance drop in the fifth round when the context horizon is the longest.

**Token Analysis:** To evaluate agent efficiency, we computed the average number of input and output tokens per round. Input tokens reflect the amount of context and guidance provided, while output tokens capture the length of the reasoning chain and implicitly indicate relative compute costs. The results are shown in Fig. 4, where we see that **GRAIL produces more than 10 times fewer output tokens than all other baselines**, underscoring the computational efficiency of our method. Notably, unlike the LLM-based ReCon agent, which requires multiple prompts per turn as part of its reasoning process, GRAIL completes reasoning in a single prompt, resulting in far fewer input tokens.

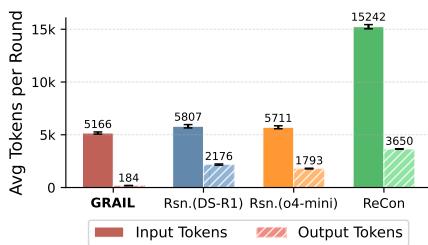
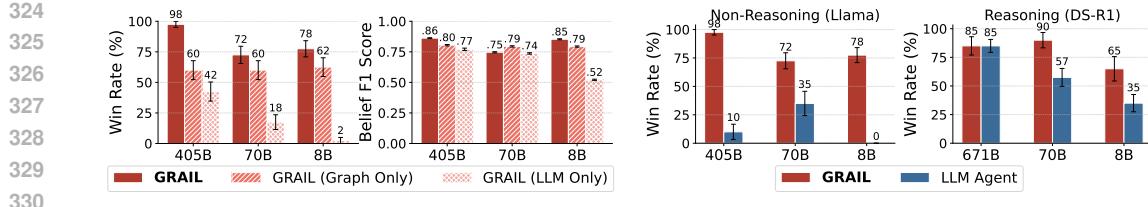


Figure 4: Average per-round token usage for GRAIL, LRM-based reasoning agents, and ReCon in Agent-Agent games.



(a) Architecture ablations for GRAIL in which we use only belief propagation (Graph Only) or language priors (LLM Only) using Llama 3.1 with different sizes (405B, 70B, and 8B parameters) over 40 games.

(b) Win rates for GRAIL and the LLM-based agent for different underlying models across varying parameter scales. (Left) Llama 3.1 non-reasoning models and (Right) DeepSeek-R1 reasoning models.

Figure 5: Combined ablation results across agent components, model size, and reasoning types.

**Mixed-Team Setting:** In the mixed-team setting, we tested GRAIL’s effectiveness when paired with weaker agents, such as ReCon, as teammates. We observed that gradually adding GRAIL agents to the team immediately improves performance (see Table 1b), depending on the opposing team’s capabilities. These findings underscore GRAIL’s ability to improve overall team performance, even when partnered with less capable agents.

**Belief Distribution:** To analyze the effect of language priors, we visualized the evolution of GRAIL’s belief over the course of 13 games that ended after exactly 5 rounds. Fig. 3a shows the kernel density estimations  $\text{KDE}(b_j^t \mid r_j = 1)$  (Evil player) and  $\text{KDE}(b_j^t \mid r_j = 0)$  (Good player), computed both *with* and *without* the prior  $p(r_j^{t-1})$  in agent–agent games. We observe that beliefs about both Good and Evil players progressively converge toward their true values as the game advances. Early-game distributions are uncertain, whereas late-game distributions exhibit high-confidence peaks. Incorporating the language prior accelerates this convergence, resulting in confident and accurate beliefs by round three, compared to rounds four or five without the prior.

## 5.2 MODEL SIZE AND ARCHITECTURE EVALUATION

To evaluate the contribution of GRAIL’s individual components, we performed ablation studies under two conditions: **LLM Only**, in which beliefs are set directly from the prior ( $b_j^t = p(r_j^t)$ ), and **Graph Only**, which uses belief propagation without the language prior (by fixing  $\beta^t = 0$ ). Furthermore, to understand how these design choices impact sensitivity to LLM size, we pair this analysis with an ablation study on the size of the underlying LLM, using the Llama 3.1 family (Dubey et al., 2024) with 8B, 70B, and 405B parameters for GRAIL.

Ablation results (Fig. 5a) demonstrate that the full GRAIL method, combining both the factor graph and language priors, consistently outperforms both ablated variants. The LLM Only variant is highly sensitive to model size, exhibiting sharp performance degradation with smaller models. Conversely, the Graph Only variant is robust to LLM size and maintains a high win rate (75%) even with the smallest 8B model. From this, we conclude that **the factor graph establishes a “performance floor,” effectively mitigating the negative performance impacts of smaller models.**

We next evaluated the sensitivity of reasoning agent performance to model size by playing games where the original 671B DeepSeek-R1 model is replaced with smaller, distilled variants (70B and 8B). This allows us to directly measure how reasoning quality degrades with reduced model capacity. To further isolate the effect of reasoning ability, we also substituted the reasoning agent’s LRM with comparably-sized Llama LLM models; similarly, we evaluated GRAIL with DeepSeek LRMs.

The results (Fig. 5b) highlight GRAIL’s robustness to model size, which is a sharp contrast to the reasoning agents. Specifically, the reasoning agents exhibit poor performance when using smaller LRMs or non-reasoning LLMs. This results in two key insights: 1) the LLM-based GRAIL outperforms similarly-sized LRM-based reasoning agents in every size class, and 2) GRAIL achieves *higher* win rates using a smaller LLM than reasoning agents using much larger LRMs, e.g., GRAIL 8B Llama outperforms reasoning 70B DeepSeek-R1. We also observed a counterintuitive result: the win rate of the reasoning agent with the 405B Llama model is *worse* than the 70B Llama model. Upon analysis of chat messages, we observed that this is due to high sycophancy (Sharma et al., 2024) as the Good agents complied with the Evil agents’ requests, e.g. “I should be in the party.”

378  
 379 Table 2: Average per-turn wall-clock time (in sec-  
 380 onds) of agents over different methods and model  
 381 sizes. Times for GRAIL include graph inference.

	8B	70B*	405B / 671B
DS-R1 (s)	17.37±20.59	15.01±6.55	85.50±179.29
GRAIL (s)	14.04±2.00	18.73±1.82*	20.00±9.99
Graph (s)	5.05	10.15*	5.23

382  
 383 Table 3: Belief F1 score of GRAIL using  
 384 the priors generated by different Llama  
 385 model sizes with different  $\beta$  values.

Model Size	$\beta$				
	0.05	0.10	0.15	0.20	0.25
8B LLaMA	<b>0.89</b>	0.82	0.79	0.78	0.76
405B LLaMA	0.88	0.89	<b>0.90</b>	0.89	0.89

386  
 387  
 388 **Voting Dynamics:** A comparison of round-by-round voting patterns shown in Fig. 2b highlights that  
 389 GRAIL yields higher F1 scores than the DeepSeek-based reasoning agent at comparable parameter  
 390 scales (Llama 3.1 405B vs. DeepSeek 671B; 70B vs 70B; 8B vs 8B) Our agent demonstrates greater  
 391 consistency and reduced performance degradation across all model sizes.

392  
 393 **Time Analysis:** We compared GRAIL and the best reasoning agent (DeepSeek-R1) on average time  
 394 per turn across model sizes, noting hardware differences<sup>6</sup>. For GRAIL, we separately measured the  
 395 graph propagation time in addition to the total turn time<sup>7</sup>. DeepSeek-R1 shows a high variance due  
 396 to time spent on internal reasoning, while GRAIL is consistently faster (see Table 2).

397  
 398 **The Effect of Beta:** To evaluate the effect of the  $\beta$  parameter (from section 4.1), we reran the belief  
 399 propagation on games played by the 8B and 405B agents (Appendix C). We find that priors from  
 400 smaller models produce more accurate beliefs (F1 score) when paired with a smaller  $\beta$  (Table 3).  
 401 This suggests smaller models generate lower-quality priors. To verify this, we measured the priors'  
 402 performance on classifying Evil players and found that the 405B model (0.73 F1) significantly out-  
 403 performs the 70B (0.60 F1) and 8B (0.47 F1) models. **This confirms that smaller models struggle**  
 404 **to detect deception**, validating our approach of tuning  $\beta$  as a model-specific hyperparameter.

405  
 406 **Hallucination Analysis:** We measured agent alignment with  
 407 the game state by analyzing message hallucination rates for  
 408 GRAIL and the reasoning agent across various model sizes.  
 409 We utilized GPT-4.1 as a judge to detect hallucinations (Gu  
 410 et al., 2025), as it has proven to agree with human judgment in  
 411 95% of the cases (see Appendix G). Across all sizes, GRAIL  
 412 consistently hallucinates less than the reasoning agent (Fig 6),  
 413 indicating stronger grounded reasoning through our hybrid  
 414 approach. We observed that the reasoning agent tends to make  
 415 speculative statements, a behavior that could impair trust and  
 416 coordination in Human-Agent games.

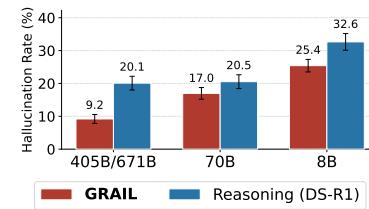
## 417 6 HUMAN EVALUATION

418  
 419 Lastly, we tested GRAIL’s ability to play with and against human players, demonstrating GRAIL’s  
 420 ability to handle diverse strategies that create dynamic and unpredictable gameplay. To this end,  
 421 we conducted an IRB-approved within-subjects study spanning 44 participants where agents played  
 422 against Evil human players alongside one Good human teammate in real time. After each game, par-  
 423 ticipants completed questionnaires on the perceived contribution and helpfulness of Good players,  
 424 without knowing the study’s purpose to reduce bias. Additional methods appear in Appendix I.

425  
 426 The following hypotheses guide our evaluation: **H1:** Good teams consisting of GRAIL agents will  
 427 win significantly more games than those composed of reasoning agents. **H2:** GRAIL will iden-  
 428 tify Evil players by rejecting proposals containing Evil players and accepting proposals exclusively  
 429 composed of Good players more accurately, compared to the reasoning agents (**H2.1**) and human  
 430 players (**H2.2**). **H3:** Participants will state that GRAIL contributed to the success of the Good team  
 431 more, compared to the reasoning agents (**H3.1**) and human players (**H3.2**). **H4:** Participants will

432  
 433 <sup>6</sup>The 70B GRAIL ran on a different hardware with a weaker CPU compared to the 8B and 405B variants

434 <sup>7</sup>With GPU-optimized belief propagation, GRAIL could be even more efficient.



435  
 436 Figure 6: Hallucination rates for  
 437 GRAIL (Llama 3.1) and the rea-  
 438 soning agent (DS-R1) for different  
 439 model sizes over 40 games.

432 prefer the helpfulness of the suggestions and comments of GRAIL more, compared to the reasoning  
 433 agents (**H4.1**) and human players (**H4.2**).  
 434

435 **Setup:** Each experiment paired two human players as Evil with one human as Good, alongside three  
 436 Good agent teammates. Every participant played two games, alternating between GRAIL and base-  
 437 line reasoning agents (GPT-o4-mini) in randomized order for fairness. Participants were unaware  
 438 of the presence of non-human players. Due to latency and reliability issues with the DeepSeek-R1  
 439 API, GPT-o4-mini was used as the baseline model.  
 440

441 This configuration enables evaluations from two perspectives: (1) Evil players interacting **against**  
 442 agents, and (2) a Good player collaborating **alongside** agents. After each game, human evaluators  
 443 answered two questions assessing the overall contribution and communication quality of the Good  
 444 team players on a five-point Likert response scale. These questions are:  
 445

Q1: “Player \_ contributed to the success of the Good team.”

Q2: “Player \_ made suggestions or comments that were helpful to the Good team.”

## 446 6.1 HUMAN STUDY EVALUATION

447 We conducted 15 trials with three participants each, running 15 games with GRAIL and 15 with  
 448 the reasoning agent. Fig. 7 presents the results of the qualitative evaluation. Because participants  
 449 were unaware that both humans and agents were present on the Good team, Evil players evaluated  
 450 all Good players, without distinguishing between agents and humans.  
 451

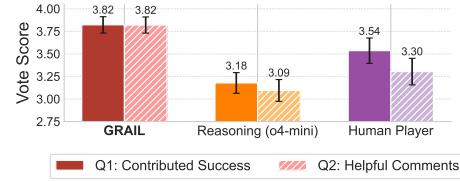
452 **H1:** Across 15 games, GRAIL won 10 and lost 5  
 453 (67% win rate), whereas the reasoning-based agent  
 454 won 4 and lost 11 (27% win rate). We assessed the  
 455 statistical significance of this performance difference.  
 456 A frequentist test yields  $p = 0.054$ , just above the 0.05  
 457 threshold, but suggests a favorable trend. Bayesian  
 458 analysis produces a 96.7% posterior probability that  
 459 GRAIL outperforms, with a 95% credible interval of  
 460 (0.482, 0.939), leaving only a 1.8% chance the re-  
 461 soning agent is superior.

462 **H2:** We evaluated predictive performance for party  
 463 proposal assessments across human study games.  
 464 Figure 2c shows GRAIL consistently votes more ac-  
 465 curately than human players. A one-tailed Wilcoxon  
 466 (1945) signed-rank test of F1 scores across 15 games confirms GRAIL’s statistically significant im-  
 467 provement over humans ( $p = 0.007$ , **H2.1**). A Mann-Whitney U test (Mann & Whitney, 1947)  
 468 comparing GRAIL and reasoning agents’ F1 score distributions across the same games further sup-  
 469 ports this finding ( $p = 0.0103$ , **H2.2**).

470 **H3, H4:** We conducted one-tailed t-tests ( $p < 0.05$ ) comparing GRAIL, reasoning agents, and human  
 471 players on both questions. GRAIL significantly outperformed reasoning agents in task contribution  
 472 (Q1:  $p = 0.001$ , **H3.1**) and suggestion quality (Q2:  $p = 0.0005$ , **H4.1**). Compared to humans,  
 473 GRAIL showed a non-significant trend in task contribution (Q1:  $p = 0.105$ , **H3.2**) but significantly  
 474 better suggestions (Q2:  $p = 0.035$ , **H4.2**). Results indicate GRAIL outperforms reasoning agents  
 475 and approaches human-level performance in effectiveness and helpfulness.

## 476 7 CONCLUSION

477 In this work, we propose GRAIL, a novel approach to hybrid reasoning that utilizes a structured  
 478 probabilistic inference framework to identify and track player roles in a complex and challenging  
 479 social deduction game – *Avalon* – that requires constrained probabilistic reasoning. Through our ex-  
 480 tensive experiments, we demonstrate that current state-of-the-art reasoning models struggle in such  
 481 settings, underlining the benefit of our proposed hybrid model, significantly outperforming prior  
 482 work and LRMAs while using much smaller non-reasoning LLMs. Furthermore, we demonstrate that  
 483 GRAIL is capable of playing with and against human players, achieving a win rate of 67% against  
 484 novice human players. In its current state, GRAIL is exclusively designed as a Good agent for de-



485 Figure 7: Average scores given to agents by  
 486 humans across two questions assessing con-  
 487 tribution and helpfulness. Human ratings  
 488 (Evil players’ votes for Good human play-  
 489 ers) are included for baseline comparison.

tecting rather than generating deception, using first-order Theory of Mind through Bayesian reasoning over a factor graph, complementary to the LLM inference. Generating deception or persuasion (e.g., required to successfully play special roles such as Merlin) requires second-order reasoning, which builds on our strong first-order foundation. In future work, we will extend GRAIL to model and utilize second-order beliefs via conditional probability, enabling the detection of more intricate deception as well as improving GRAIL’s persuasiveness.

#### ETHICS STATEMENT:

The paper emphasizes technical contributions and has little to no potential positive or negative societal impacts. Although the results of the study can imply a potential positive impact, we do not mention this in the paper, nor do we explore any potential negative impacts. The human-subject study is IRB-approved and conforms to the ICLR Code of Ethics. Before participating in our experiment, all participants signed a consent form disclosing all potential risks, and after the experiment, all participants signed the post-experiment deception form. All participants gave informed consent for us to use the data collected from them, and were compensated for their time.

#### REPRODUCIBILITY STATEMENT:

The paper describes the setups, datasets, underlying models, model sizes, hyperparameters, and evaluation metrics used in the experiments. Prompts and the questions asked of participants can be found in the Appendix. The anonymized human experiment results and the code are provided in the supplementary material.

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# APPENDIX FOR BAYESIAN SOCIAL DEDUCTION WITH GRAPH-INFORMED LANGUAGE MODELS

## A THE RESISTANCE: AVALON

The Resistance: Avalon is a standalone social deduction game designed by Don Eskridge and published by Indie Boards and Cards in 2012 Eskridge (2012). It builds upon the foundation laid by its predecessor, The Resistance, which Eskridge also designed and released in 2009. While both games share core mechanics involving hidden roles and team-based missions, Avalon introduces a rich Arthurian theme and additional character roles that deepen strategic play. Avalon is designed for 5-10 people, where players are split into two teams: Good (Loyal Servants of Arthur) and Evil (Minions of Mordred). In this paper, we focus on a simplified version with 6 players, which does not include special roles (e.g. Merlin, Assassin, etc.) to better focus on detecting deception, rather than producing it.

In the simplified version of the game with 6 players used in our study, there are 4 Good players and 2 Evil players. Roles are randomly assigned and kept secret. Each Evil player knows the identities of their Evil teammate, whereas Good players do not know the identities of any other player. The game progresses through 5 rounds, requiring parties of 2, 3, 4, 3, and 4 members, each consisting of a party proposal, a discussion phase, a party vote, and a quest vote. Players take turns in a clockwise direction, starting with the player designated as the leader. The leader proposes a team of a predetermined size to participate in the quest. Players then discuss the proposal in clockwise order. After everyone has had their turn to speak, there is a vote on whether to approve or reject the current party. If the party is approved by a majority, then the players proceed to the quest vote. If rejected, leadership passes on to the next player clockwise. If five consecutive parties are rejected, the Evil team wins by default. During the quest vote, party members secretly vote on the quest outcome, which succeeds only if players vote unanimously for success. The Good team wins if three missions succeed. The Evil team wins if three missions fail or if five consecutive team proposals are rejected.

## B FACTOR GRAPH

### B.1 STRUCTURE

The detailed structure of the factor graph used in **GRAIL** is shown in Fig 8. In this visualization, we represent the variables with capital letters (as random variables). More importantly, in our implemented graph structure, we only consider the final approved party for each quest.

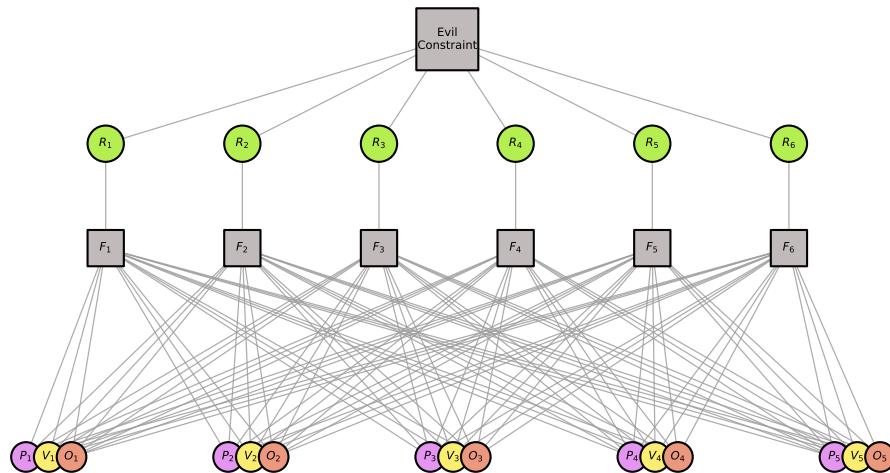


Figure 8: The Factor Graph Structure

864  $R_i$  is a binary random variable representing the role of the player  $i$ . If the player is Evil,  $R_i = 1$ ,  
 865 otherwise  $R_i = 0$ .

866 The factors can represent either a constraint (through a binary function) or a probability (through a  
 867 joint or conditional probability function) depending on their purpose. The *Evil Constraint* enforces  
 868 the constraint that only two of the players are Evil, and is defined as a function of  $R_1 \dots R_6$  as seen  
 869 in Eq 1:

$$871 \quad f_{\text{Evil Constraint}}(R_1, \dots, R_6) = \mathbf{1}_{\{\sum_{i=1}^6 R_i = 2\}} = \delta\left(\sum_{i=1}^6 R_i - 2\right) = \begin{cases} 1, & \sum_{i=1}^6 R_i = 2, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

872  $P_j$ ,  $V_j$ , and  $O_j$  are categorical random variables representing the party at quest  $j$ , the public vote for  
 873 the party in quest  $j$ , and the outcome of quest  $j$ , respectively. We use a simple numerical encoding  
 874 to save and represent the party, vote, and outcome of each quest. We consider value zero to indicate  
 875 *unseen* or future quests; in other words,  $P_i = V_i = O_i = 0$  means that quest  $i$  has not happened yet.  
 876 In the upcoming sections, the indexes and encoding start from number 1 due to this consideration.

877 **Party ( $P_j$ ):** Assuming that the party has  $k$  members, we list all  $k$ -element subsets of the players in  
 878 increasing lexicographic order called  $S$ . Given a party composition, the encoding will be the index  
 879 of that party in the ordered list  $S$ . Thus, for example,  $P_1 = 1$  encodes  $\{1, 2\}$  (party with player 1  
 880 and 2),  $P_1 = 2$  encodes  $\{1, 3\}$  and so on. Based on this encoding, we will have:

$$881 \quad P_1, P_3, P_5 \in \{0, \dots, 15\}, \\ 882 \quad P_2, P_4 \in \{0, \dots, 20\}.$$

883 **Vote ( $V_j$ ):** Since the parties are selected by a majority vote, the number of players on the list of  
 884 approval votes will be either 4, 5, or 6, leading to a total of 22 possible vote compositions. Similar  
 885 to the encoding used for the Party, we order these vote compositions (represented by subsets of the  
 886 player list) in an increasing lexicographic order. The encoding of a party vote will be the index of  
 887 that vote composition in this ordered list.

$$888 \quad V_i \in \{0, \dots, 22\}$$

889 **Outcome ( $O_j$ ):** We encode success in quest  $j$  as  $O_j = 2$  and failure as  $O_j = 1$ .

890 **Factor Nodes:** The factor nodes represent the conditional probability of the role node given the  
 891 state of the game. The details of approximating this conditional probability using a neural network  
 892 are given in Section B.3.

## 903 B.2 BELIEF PROPAGATION

904 The max-product belief propagation algorithm works by passing messages along the edges of the  
 905 factor graph and updating them iteratively. These messages represent the "influence" that variables  
 906 and factors have on each other in terms of maximizing the global function. There are two types of  
 907 messages:

- 908 • **Message from variable  $x_i$  to factor  $f_a$ :** This message represents the "belief" of variable  
 909  $x_i$  about its state, based on information received from all other connected factor nodes  
 910 *except  $f_a$* . It is calculated as the product of all incoming messages to  $x_i$  from neighboring  
 911 factor nodes as seen in Eq 2, where  $N(x_i)$  is the set of factor nodes connected to  $x_i$ :

$$912 \quad \mu_{x_i \rightarrow f_a}^{(t)}(x_i) = \prod_{f_b \in N(x_i) / \{f_a\}} \mu_{f_b \rightarrow x_i}^{(t-1)}(x_i) \quad (2)$$

- 913 • **Message from factor  $f_a$  to variable  $x_i$ :** This message represents the "belief" of factor  $f_a$   
 914 about the state of variable  $x_i$ , based on the factor function and the messages received from

918 all other connected variables. It is calculated by maximizing the product of the factor  $f_a$   
 919 and all incoming messages from its neighboring variable nodes:  
 920

$$\mu_{f_a \rightarrow x_i}^{(t)}(x_i) = \max_{X_{N(f_a)}/\{x_i\}} \left( f_a(X_{N(f_a)}) \prod_{x_j \in N(f_a)/\{x_i\}} \mu_{x_j \rightarrow f_a}^{(t)}(x_j) \right) \quad (3)$$

921 As seen in Eq 3,  $N(f_a)$  is the set of variable nodes neighboring factor node  $f_a$ . The  
 922 maximization is performed over all possible assignments to the variables in  $X_{N(f_a)}/\{x_i\}$ ,  
 923 which denotes the set of variables connected to factor  $f_a$ , excluding  $x_i$ .  
 924

925 The "belief" at variable  $X_i$  (also called a max-marginal) is denoted as  $b_i$ . The calculation of beliefs  
 926 is given in Eq 4 and is proportional to the maximum value of the joint probability distribution over  
 927 all possible assignments to other variables, with  $X_i$  fixed to  $x_i$ .  
 928

$$b_i(x_i) = \prod_{f_a \in N(x_i)} \mu_{f_a \rightarrow x_i}(x_i) \quad (4)$$

929 Based on these beliefs, the estimated maximum probability assignment to the variables is  $\hat{x}_i =$   
 930  $\arg \max_{x_i} b_i(x_i)$ . This converges to the exact MAP assignments if the factor graph is a tree, but in  
 931 loopy graphs (like our graph), the convergence is to an approximation.  
 932

933 **Initialization:** The algorithm starts by initializing every message from the variables to the factors.  
 934 This is where any prior information about the hidden variables enters the algorithm. So the first  
 935 message will be:  
 936

$$\mu_{x_i \rightarrow f_a}^{(0)}(x_i) = P[X_i = x_i] \quad (5)$$

937 If  $X_i$  is observed to be value  $x_{obs}$ ,  $P[X_i = x_i]$  will be equal to 1 for  $x_i = x_{obs}$ . Otherwise, we can  
 938 use prior probabilities for  $P[X_i = x_i]$ , and if no prior knowledge is available, the probability will  
 939 be uniform.  
 940

941 **Iteration:** In each iteration, new messages are computed based on the messages from the previous  
 942 iteration using Eq 2 and Eq3. The order of message updates can vary (e.g., synchronous  
 943 updates where all messages are computed simultaneously based on the previous iteration's messages,  
 944 or asynchronous updates where messages are updated one by one). In our implementation of  
 945 the algorithm, these updates are done asynchronously, and the messages are normalized after each  
 946 iteration. This updating process continues until the messages converge (i.e., they no longer change  
 947 significantly between iterations) or a maximum number of iterations is reached. The details of this  
 948 convergence criteria are provided in the next section.  
 949

950 **Termination:** The algorithm terminates either after a specific number of iterations (20 in our implementation)  
 951 or when the beliefs converge. Convergence can be determined by monitoring the change in beliefs (marginals) between iterations. We use the Kullback-Leibler (KL) divergence  
 952 between the belief distribution at iteration  $t$  and  $t - 1$  to determine convergence. Let  $b_k^{(t)}(s)$  and  
 953  $b_k^{(t-1)}(s)$  be the belief for variable  $X_k$  being equal to  $s$  at iteration  $t$  and  $t - 1$  respectively. The KL  
 954 divergence is calculated as

$$D_{\text{KL}} \left( b_k^{(t-1)} \parallel b_k^{(t)} \right) = \sum_s b_k^{(t-1)}(s) \left( \log b_k^{(t-1)}(s) - \log b_k^{(t)}(s) \right) \quad (6)$$

955 Based on this, we terminate the calculation if the sum of the divergence of all variables is less than  
 956  $\epsilon = 10^{-6}$ :  
 957

$$L_{\text{total}}^{(t)} = \sum_k D_{\text{KL}} \left( b_k^{(t-1)} \parallel b_k^{(t)} \right) < \epsilon \quad (7)$$

### 958 B.3 FACTOR FUNCTION APPROXIMATION

959 A simple neural network is used to approximate the factor function which represents the conditional  
 960 probability.  
 961

972 B.3.1 ARCHITECTURE:  
973

974 The input of the neural network is the encoding of each game state node as explained in Ap-  
975 pendix B.1. Each one of  $P_1, V_1, \dots, V_5, O_5$  are treated as a categorical input variable. Each cat-  
976 egorical input variable is individually transformed into a dense vector representation using separate  
977 embedding layers. The embedding size of each categorical variable is  $\log_2 C_i$ , where  $C_i$  is the  
978 number of categories in variable  $i$ .

979 These learned embeddings are then concatenated to form a unified feature vector. This consolidated  
980 vector is subsequently processed through a sequence of fully connected layers: an initial layer map-  
981 ping to a hidden dimension with 16 nodes, an intermediate hidden layer of the same dimension, and  
982 finally, an output layer producing the model’s predictions. Rectified Linear Unit (ReLU) activation  
983 functions are applied after each hidden layer, and a masking strategy is implemented within the for-  
984 ward pass to zero out embeddings corresponding to a zero input feature (the quests that have not  
985 been added yet).

986 The output of the network is one-dimensional and is equal to 1 for Evil players and 0 for Good  
987 players. The network is trained as a binary classifier with a softmax function to turn logits into  
988 probability estimations.

989  
990 B.4 TRAINING  
991

992 The training data is constructed from the AvalonLogs<sup>8</sup> with 3,143 games and the ProAvalon<sup>9</sup>  
993 website with 101280 games. This dataset is split into 80% for training, 10% for vali-  
994 dation, and 10% for testing. For each game, we extract 6 training samples: one corre-  
995 sponding to each player. Additionally, the game state is extracted each round by mask-  
996 ing the input. For example, if a game ends in 3 rounds, three possible input states exists:  
997  $[P_1, V_1, O_1, 0, 0, \dots], [P_1, V_1, O_1, P_2, V_2, O_2, 0, 0, \dots], [P_1, V_1, O_1, P_2, V_2, O_2, P_3, V_3, O_3, 0, 0, \dots]$

998 The model training process is configured for binary classification over a fixed number of epochs,  
999 utilizing the Adam optimizer with L2 regularization (weight decay) to minimize binary cross entropy  
1000 loss. To counteract class imbalance, this loss function is weighted by a dynamically calculated  
1001 variable which is equal to the ratio of Good player to the Evil players in the dataset (2 to 1). An  
1002 early stopping criterion is employed, monitoring the validation loss on the primary validation set.

1003  
1004 B.5 TRAIN DATASET SIZE  
1005  
1006

Table 4: F1 Scores vs Training Data Size

Training Data Size	261	522	1044	2610	5221	10442	20884	41768
Train F1 Score	0.3977	0.5360	0.5563	<b>0.5939</b>	<b>0.6089</b>	0.6075	0.6084	0.6082
Val F1 Score	0.3927	0.5324	0.5544	<b>0.5938</b>	<b>0.6102</b>	0.6106	0.6116	0.6121

1012 While our initial model was trained on the full dataset, we conducted a follow-up analysis to eval-  
1013 uate performance with smaller datasets. Using fixed 10% of the dataset as validation, we trained the  
1014 conditional probability estimation network on subsets of 200 to 40K games, repeating each experi-  
1015 ment 20 times with different random subsets. As seen in Table 4, the performance stabilizes between  
1016 2.5K and 5K games, indicating that large-scale training is not required for effective performance.

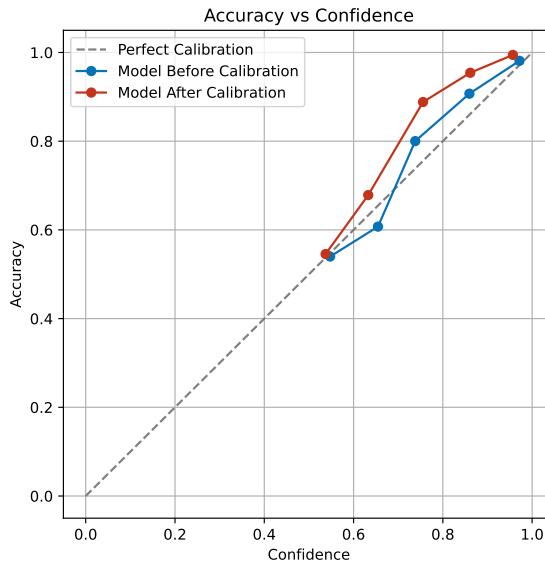
1017  
1018 B.6 CALIBRATION  
1019

1020 Modern neural networks often produce poorly calibrated probabilities, meaning their output con-  
1021 fidence scores do not accurately reflect the true likelihood of correctness. Calibration is therefore  
1022 needed to align these confidences with actual probabilities for the neural network to effectively esti-  
1023 mate a conditional probability.

1024  
1025 <sup>8</sup><https://github.com/WhoaWhoa/avalonlogs><sup>9</sup><https://www.proavalon.com>

1026 For this purpose, we use Temperature Scaling Guo et al. (2017), a post-hoc calibration method. It  
 1027 wraps a pre-trained model and introduces a single learnable scalar parameter, "temperature." This  
 1028 temperature is used to divide the model's output logits before they are converted to probabilities. We  
 1029 use the implementation from [https://github.com/gpleiss/temperature\\_scaling](https://github.com/gpleiss/temperature_scaling)  
 1030 with default parameters and use the test split of the data to calibrate the model. The result of this  
 1031 calibration is seen in Fig. 9, which represents the confidence vs accuracy of the model. Note that  
 1032 there is no confidence under 0.5 because any prediction under 0.5 is considered a label for the Good  
 1033 class.

1034  
 1035



1054  
 1055 Figure 9: The relationship between accuracy and confidence of the model before and after cali-  
 1056 bration. The calibrated model has higher accuracy in each confidence level, which can point to  
 1057 *under-confidence*. This under-confidence is desirable in our application.

1058  
 1059

## B.7 GRAPH SCALABILITY

1060 We evaluated the computational scalability of Bayesian inference by varying the number of player-  
 1061 role nodes in the GRAIL factor graph. Factor graphs were constructed with 6, 8, 12, and 20 players.  
 1062 A consistent neural network factor estimation was used (accuracy not evaluated for this experiment).  
 1063 Belief propagation was run on 20 randomly generated game states to measure average runtimes in  
 1064 seconds. These experiments were run on a MacBook Air M2 (CPU only, no GPU optimization).  
 1065 As seen in Table 5 The inference time scales approximately linearly with the number of role nodes,  
 1066 which suggests that our approach scales well to higher-dimensional settings (e.g., in other settings  
 1067 or in Avalon variants with more roles, more players, or more rounds per game).

1068  
 1069

Table 5: Performance metrics showing average time vs number of role nodes

Number of Role Nodes	6	8	12	20
Average Time (s)	4.62	5.98	9.06	15.14

1070  
 1071

## C BETA VARIABLE EFFECT

1072  
 1073

1074 We evaluated the effect of the Beta parameter by rerunning belief propagation using the game state  
 1075 and LLM priors from the games with 8B, 70B, and 405B LLaMA GRAIL agents. As seen in  
 1076 Table 6 smaller LLMs make more mistakes in their prior estimates, reducing belief accuracy as Beta

1080 increases. Therefore, we set higher Beta values for larger models and lower Beta values for smaller  
 1081 models.  
 1082

1083 Table 6: Performance of LLaMA models across different  $\beta$  values.  
 1084

Model Size	Beta				
	0.05	0.10	0.15	0.20	0.25
8B LLaMA	<b>0.89</b>	0.82	0.79	0.78	0.76
405B LLaMA	0.88	0.89	<b>0.90</b>	0.89	0.89

1090 The better performance of smaller models with smaller beta can indicate that smaller models generate  
 1091 less accurate priors. We report the F1 score of the LLM priors that were generated by the 405B  
 1092 and 8B models. We calculate these metrics by considering an “increase” response as a positive  
 1093 prediction for the classification task of the Evil players.  
 1094

1095 The 8B model achieves an F1 score of 0.47 and the 70B model achieves 0.60 while the big 405B  
 1096 model achieves an F1 score of 0.73. **Smaller models have more difficulty detecting deception in**  
 1097 **dialogue and identifying Evil players.** To mitigate this, we treat the Beta parameter, which controls  
 1098 the influence of LLM priors, as a tunable hyper-parameter and set lower values for smaller models.  
 1099

## 1100 D IMPLEMENTATION AND COMPUTE

1103 For the implementation of the factor graph, we use the Pomegranate Python package  
 1104 Schreiber (2018) developed by Jacob Schreiber. This package is available at [pomegranate.readthedocs.io](https://pomegranate.readthedocs.io). We use the belief propagation framework used in this package, and adjust it  
 1105 to use Max-Product instead of Sum-Product. Pytorch was used for approximating the factor functions.  
 1106

1108 All GPT models were run through the OpenAI API. The 671B DeepSeek-R1 model was accessed  
 1109 through the official DeepSeek API. The experiments with the 16FP version of the smaller DeepSeek-  
 1110 R1 models (70B and 8B parameters), as well as the Llama models (405B, 70B, and 8B parameters),  
 1111 were run on NVIDIA®A100-40GB GPUs using the vLLM Kwon et al. (2023) library for serving  
 1112 and inference. To run 8B parameters of DeepSeek-R1 and Llama 3.1, we utilized 1 accelerators; for  
 1113 70B parameters of DeepSeek-R1 and Llama 3.1, we utilized 4 accelerators; and the 405B parameter  
 1114 Llama 3.1 model was ran with 8 accelerators.

1115 The games with the GRAIL agent on average use around 126K input tokens and around 4K output  
 1116 tokens. In a game with the reasoning agents, around the same number of input tokens are needed,  
 1117 but the number of output tokens is around 45K. Based on this, the GPU hours needed for rerunning  
 1118 the experiments will depend on the throughput achieved by the GPUs.  
 1119

## 1120 E ACTION HEURISTIC

1123 All agents follow the same action selection protocol throughout the game. The round always starts  
 1124 with the leader proposing a party and sending a message to support the proposal. Then, each player  
 1125 takes a turn to send a message until it is once again the leader’s turn. The leader will then either send  
 1126 a final message before initiating a vote, or revise the proposed party, send a message to justify the  
 1127 change, and then start the vote.

1128 In GRAIL, agents follow the same heuristic for action selection; however, decisions regarding party  
 1129 proposal and voting is decided based on their beliefs. When proposing a party, the beliefs are ranked  
 1130 and the players with the highest confidence of being Good are selected. For voting, the agent will  
 1131 vote to approve a party only if the belief about all players on that party indicates that they are Good  
 1132 (i.e.  $b_i < 0.5$ ).  
 1133

In the first round, since no information is available, the GRAIL agent will propose a random party  
 if it is the leader. In addition, GRAIL will always vote to approve the first party.

1134 **F PROMPTS**  
11351136 We use a modular approach to construct the prompts for the agents. In summary, a text explaining  
1137 the context of the game is generated, which is then attached to a task command. We will provide  
1138 these modules separately and explain how the final prompt in agents is formed from these modules.  
11391140 The context prompt is shared between GRAIL and the reasoning agent, and is started by an expla-  
1141 nation of the game rules:  
1142**Prompt F.1: Game Rules**

You are playing a social deduction game, The Resistance: Avalon. The game consists of 6 total players: 4 Loyal Servants of Arthur (good players) and 2 Minions of Mordred (evil players) – there are no special roles. The game consists of up to 5 Rounds, each consisting of a Team Building phase and a Quest phase. Good players win the game by successfully completing 3 Quests. Evil players win if 3 Quests end in failure.

In the Team Building phase the Leader proposes a Team (a party) to complete a Quest (a mission) – a majority of players will either approve the proposed Team and move to the Quest phase, or reject the proposed Team passing leadership to the next player and repeating the process until a Team is approved. If 5 Team proposals are consecutively rejected (for the same quest), the game will end and the evil side will win. In the Quest phase those players selected to be on the Team will determine if the quest is successful through a hidden vote. For a Quest to succeed, all Team members must vote for success – a single failure vote will cause the Quest to fail.

1155 Following the explanation of the game rules, the role assigned to the agent and the goals associated  
1156 with that role are described. The prompts used in GRAIL will always include information about the  
1157 Good role. However, the role specified in the prompt of the reasoning agent can be either Good or  
1158 Evil based on the agent’s assigned role.  
1159**Prompt F.2: Good Role**

Your name in this game is `<agent name>`, and you are a loyal Servant of Arthur (a good player). The evil players know who each other are, but you and other good players have no special knowledge of anyone’s role. Therefore, evil players will use deception and pretend to be good so they can participate in Quests and cause them to fail. As a good player, your goal is to ensure that Quests succeed by selecting other good players to participate in Quests. Approve only Teams where you trust everyone. Even a single evil player on the team is enough for failure.

**Prompt F.3: Evil Role**

Your name in this game is `<agent name>`, and you are a Minion of Mordred (an evil player). The good players have no special knowledge of anyone’s role, including yours, and your strategy is to pretend to be a good player such that you are selected for Quests and can vote for them to fail. Your subtle misdirections should aim to create confusion and prevent 3 missions from succeeding. In discussions, maintain your disguise as a good player, but during voting you should vote to fail Quests. Remember, as an evil player you win the game by causing 3 Quests to fail.

*Your special information, just for you, is:*

`<Evil player1 name>`: evil, `<Evil player2 name>`: evil

**Note:** You have received special information which you need to keep private. Keep the information confidential and use it wisely. Furthermore try to not make general statements which could indicate that you are hiding something.

1183 **F.1 GRAIL PROMPTS**  
11841185 In addition to the game rules and role description, the GRAIL agent is provided with information  
1186 about the current game state. This includes both the agent’s current beliefs and the chat logs from  
1187 the ongoing round:  
1188

1188  
1189**Prompt F.4: Game State (GRAIL)**1190  
1191  
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1197

At the start of this round, your current beliefs about each player's alignment are given as a probability dictionary, where 0 means definitely good, 1 means definitely evil, and values in between indicate uncertainty: `{'<player name>': <belief>}`

These probabilities represent your current beliefs based on prior rounds but do not account for new messages or actions this round. You can use them to reason about the game, but do not explicitly mention the values or refer to them as externally provided – these should represent your internally held beliefs.

**The following messages have already been exchanged this round. This consists of both player chat messages and game system messages, with the most recent being last:**

```
START CHAT MESSAGES
<chat history>
END CHAT MESSAGES
```

**The current Round is <round number>.** The previous Rounds consisted of the following Quest Teams and outcomes:

- Quest *i* party: [<party member names>] Outcome: *success/fail*

The GRAIL agent only uses prompts for three tasks: *party proposal message generation*, *discussion message generation*, and *prior probability generation*.

The prompts used for these three tasks are provided below.

**Prompt F.5: Party Proposal Message Generation**

You are the party leader this round, and you must propose a team for the mission. The team you have chosen: [<party member names>]. Keep in mind that the party size is fixed in each Round and the party this Round must consist of 4 players.

Now, respond in the game chat as if you are a college student, typing informally (e.g., using abbreviations, casual language, and minimal punctuation). Do not use overly formal or structured responses and do not use emojis. Make your response detailed enough to persuade other players but do not be overly verbose. Shorter messages are more effective; don't use more than a few sentences and prefer fewer when possible.

Explain why you picked this team. Convince the other players that this is the best choice. If there are any doubts from other players, acknowledge them but defend your reasoning. If your belief in the team is strong, argue confidently; if you are uncertain, express that but still justify your choice. Please respond with a JSON object with one key named 'message' containing your chat message.

**Prompt F.6: Discussion Message Generation**

The current mission proposal: <leader name> has proposed the following players for this mission: [<party member names>]. Keep in mind that the party size is fixed in each Round and the party this Round must consist of <party size> players.

Now, respond in the game chat as if you are a college student, typing informally (e.g., using abbreviations, casual language, and minimal punctuation). Do not use overly formal or structured responses and do not use emojis. Make your response detailed enough to persuade other players but do not be overly verbose. Shorter messages are more effective; don't use more than a few sentences and prefer fewer when possible.

React to the proposed party. If you agree, say why you think it's a good choice. If you disagree, make an argument against it and suggest who you think should go instead. Take into account how strongly you hold your beliefs (if the probabilities are closer to 0.5 than 0 or 1, you should express uncertainty rather than strong opinions). Respond to chat messages sent by other players. If someone says something you agree with, acknowledge it; if someone accuses you, defend yourself logically. Unless you have a strongly held belief about a player, it is not always worth arguing for or against specific players, especially early in the game such as on Quest 1. Please respond with a JSON object with one key named 'message' containing your chat message.

**Prompt F.7: Prior Probability Generation**

Now, analyze how the new chat messages and game actions affect your beliefs about each player's alignment. For each player, decide if their probability of being evil should:

1242           • **Increase** (if their behavior suggests they might be evil)  
 1243           • **Decrease** (if their behavior suggests they might be good)  
 1244           • Stay the **same** (if there is no strong evidence to change your belief)  
 1245  
 1246        Consider whether they are behaving suspiciously or illogically with respect to chat messages or party votes,  
 1247        e.g. always rejecting party votes unless they or another specific player is in it, or making assertions without  
 1248        evidence (especially early in the game such as on Quest 1 where evidence is limited) as these may be signs  
 1249        a player is evil. Also consider whether their beliefs align with your own, or if their beliefs seem contrary  
 1250        to the group consensus as there are more Servants than Minions and could also suggest a player is evil.  
 1251        Provide your updated belief adjustments as a JSON message, mapping player names to 'increase', 'de-  
 1252        crease', or 'same'. Do not explain your reasoning—just return the JSON message. If there isn't sufficient  
 1253        evidence to update a belief about a player, then it is safer to indicate 'same'.  
 1254        Example output:  
 1255        'Sam': 'increase', 'Paul': 'increase', 'Luca': 'same', 'Jane': 'decrease', 'Kira': 'same', 'Mia': 'decrease'

1256        Based on the provided prompt modules, the entire text that the LLM is prompted with is constructed  
 1257        like below based on the selected task (the + sign indicates concatenation).

1258        Rules{F.1} + RoleInfo{F.2/F.3} + Beliefs{F.4} + Task{F.5/F.6/F.7}

## 1260        F.2 REASONING AGENT PROMPTS

1262        In the reasoning agent, we used the TypeChat (<https://github.com/microsoft/TypeChat>) library for prompting the language models and checking for correctness in the structure  
 1263        of the response.

1265        Similar to the GRAIL agent, the reasoning agent is provided with the game rules, role information,  
 1266        and game state, before being commanded to do a task.

### 1268        Prompt F.8: Game State (Reasoning)

1269        **YOUR PRIOR ACTIONS THIS TURN:**  
 1270        [<agent actions list>]  
 1271  
 1272        **CURRENT GAME STATE:**  
 1273        • Current Quest: <quest number>  
 1274        • Current Turn: <turn number>  
 1275        • Failed Party Votes: <number of rejects>  
 1276        • Quest Results:  
 1277        Quest *i* party: [<party member names>] : success/fail  
 1278        • Current Leader: <leader name>  
 1279        • Proposed Team: [<proposed party members>]

### 1281        GAME HISTORY:

1282        Previous Teams:

1283        • Team *i* (proposed by <player name> in quest *j*):  
 1284        <party member names> | Votes: <player name> : Yes/No

### 1285        DETAILED GAME LOG:

1286        <chat history>

1288        The tasks that the reasoning agents are prompted for are *party proposal generation*, *discussion mes-*  
 1289        *sage generation*, *party vote generation*, and *quest vote generation*. These prompts will be different  
 1290        for the Good and Evil players, so we provide them side-by-side for comparison.

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#### 1292        F.2.1 PARTY PROPOSAL

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1301**Good Prompt F.1: Proposal Generation**

Your objective is to propose a mission team consisting of <team size> players. Consider the following factors when selecting team members:

1. Past involvement and actions of each player
2. Contributions in previous missions
3. Any indications of players being on the evil side
4. Suspicious or trustworthy behaviors

When looking for suspicious behavior, consider whether players are behaving suspiciously or illogically with respect to chat messages or party votes, e.g. always rejecting party votes unless they or another specific player is in it, or making assertions without evidence (especially early in the game such as on Quest 1 where evidence is limited) as these may be signs a player is evil.

Also consider whether their beliefs align with your own, or if their beliefs seem contrary to the group consensus as there are more Servants than Minions and could also suggest a player is evil.

Unless you have a strong reason to doubt a player, it is not always worth arguing for or against specific players, especially early in the game such as on Quest 1.

Present your team selection to the other players with a detailed rationale based on past events and player behaviors. Remember that you are on the good side - avoid selecting players who seem suspicious or were part of failed missions. It's conventional to include yourself in the team unless you have a specific reason not to.

Please propose a mission team composed of <team size> members and provide a message to the other players justifying your choices following the schema provided above. Phrase your message as if you are a college student, typing informally (e.g., using abbreviations, casual language, and minimal punctuation). Do not use overly formal or structured responses and do not use emojis. Make your response detailed enough to persuade other players but do not be overly verbose. Shorter messages are more effective; don't use more than a few sentences and prefer fewer when possible.

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1347**F.2.2 DISCUSSION MESSAGE GENERATION**1348  
1349**Evil Prompt F.1: Proposal Generation**

Your objective is to propose a mission team consisting of <team size> players. Consider the following factors when selecting team members:

1. Past involvement and actions of each player
2. Contributions in previous missions
3. Suspicious or trustworthy behaviors

Present your team selection to the other players with a detailed rationale based on past events and player behaviors. Remember that you are on the evil side - if it is possible, aim to include at least one evil player while maintaining your cover. However, you will have to justify the proposed team to the other players, so make sure you have a good reason for your choices that makes you look like a good player. " If the good side has already identified an evil player, you should not include them in the team as that would make you look suspicious. Remember to hide your true identity and appear as good. It's conventional to include yourself in the team unless you have a specific reason not to.

Please propose a mission team composed of <team size> members and provide a message to the other players justifying your choices following the schema provided above. Phrase your message as if you are a college student, typing informally (e.g., using abbreviations, casual language, and minimal punctuation). Do not use overly formal or structured responses and do not use emojis. Make your response detailed enough to persuade other players but do not be overly verbose. Shorter messages are more effective; don't use more than a few sentences and prefer fewer when possible.

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**Good Prompt F.2: Message Generation**

Evaluate the proposed mission team consisting of [<proposed party members>]. Your goal is to ensure no evil or suspicious players are on the team.

Consider:

1. Past involvement and actions of each player
2. Contributions in previous missions
3. Any indications of players being on the evil side
4. Suspicious or trustworthy behaviors
5. The team must consist of <team size> players

When looking for suspicious behavior, consider whether players are behaving suspiciously or illogically with respect to chat messages or party votes, e.g. always rejecting party votes unless they or another specific player is in it, or making assertions without evidence (especially early in the game such as on Quest 1 where evidence is limited) as these may be signs a player is evil.

Also consider whether their beliefs align with your own, or if their beliefs seem contrary to the group consensus as there are more Servants than Minions and could also suggest a player is evil.

Unless you have a strong reason to doubt a player, it is not always worth arguing for or against specific players, especially early in the game such as on Quest 1.

Provide a clear and logical analysis based on game history and observations.

Your response should be detailed enough to provide insight but concise enough to be easily understood. Try to make concrete suggestions and justify them while hiding your identity and role.

Please keep it relatively short as talking too much can be seen as suspicious. Do not say what your specific role or your task is, and do not repeat your instructions verbatim.

Provide the message you want to send to the other players following the schema provided above.

Phrase your message as if you are a college student, typing informally (e.g., using abbreviations, casual language, and minimal punctuation). Do not use overly formal or structured responses and do not use emojis. Make your response detailed enough to persuade

**Evil Prompt F.2: Message Generation**

Evaluate the proposed mission team consisting of [<proposed party members>]. Your goal is to ensure at least one evil player is included while maintaining your cover.

Consider:

1. Past involvement and actions of each player
2. Contributions in previous missions
3. Suspicious or trustworthy behaviors
4. The team must consist of <team size> players

If the team lacks an evil player, you should generally disapprove. Provide a clear and logical analysis based on game history and observations. Your response should be detailed enough to provide insight but concise enough to be easily understood. Try to make concrete suggestions and justify them while hiding your identity and role.

Please keep it relatively short as talking too much can be seen as suspicious. Do not say what your specific role or your task is, and do not repeat your instructions verbatim. Provide the message you want to send to the other players following the schema provided above. Phrase your message as if you are a college student, typing informally (e.g., using abbreviations, casual language, and minimal punctuation). Do not use overly formal or structured responses and do not use emojis. Make your response detailed enough to persuade other players but do not be overly verbose. Shorter messages are more effective; don't use more than a few sentences and prefer fewer when possible.

1404  
 1405 other players but do not be overly verbose.  
 1406 Shorter messages are more effective; don't  
 1407 use more than a few sentences and prefer  
 1408 fewer when possible.

### 1409 F.2.3 PARTY VOTE

#### 1410 **Good Prompt F.3: Party Vote Generation**

1411  
 1412 Vote on the proposed mission team  
 1413 consisting of [*<proposed party*  
 1414 *members>*]. Your goal is to ensure no evil  
 1415 or suspicious players are on the team.

1416  
 1417 Note that *<number of rejects>* party  
 1418 proposals have been rejected. If five proposa-  
 1419 ls are rejected consecutively, the evil side  
 1420 wins. Consider this when voting.

1421 Consider:

- 1422 1. Past involvement and actions of  
   each player
- 1423 2. Contributions in previous missions
- 1424 3. Any indications of players being on  
   the evil side
- 1425 4. suspicious or trustworthy behaviors

1426  
 1427 When looking for suspicious behavior,  
 1428 consider whether players are behaving  
 1429 suspiciously or illogically with respect to  
 1430 chat messages or party votes, e.g. always  
 1431 rejecting party votes unless they or another  
 1432 specific player is in it, or making assertions  
 1433 without evidence (especially early in the  
 1434 game such as on Quest 1 where evidence is  
 1435 limited) as these may be signs a player is  
 1436 evil.

1437 Also consider whether their beliefs align  
 1438 with your own, or if their beliefs seem  
 1439 contrary to the group consensus as there are  
 1440 more Servants than Minions and could also  
 1441 suggest a player is evil.

1442 Unless you have a strong reason to doubt  
 1443 a player, it is not always worth arguing for  
 1444 or against specific players, especially early  
 1445 in the game such as on Quest 1. *Keep*  
 1446 *in mind that this is your*  
 1447 *proposed team and as such you*  
 1448 *should approve it.*

1449 Choose either 'approve' or 'disapprove'  
 1450 following the schema provided above. )

#### 1451 **Evil Prompt F.3: Party Vote Generation**

1452  
 1453 Vote on the proposed mission team  
 1454 consisting of [*<proposed party*  
 1455 *members>*]. Your goal is to ensure at least  
 1456 one evil player is included while maintaining  
 1457 your cover.

1458 Note that *<number of rejects>* party  
 1459 proposals have been rejected. If five proposa-  
 1460 ls are rejected consecutively, the evil side  
 1461 wins. Consider this when voting.

1462 If the team lacks an evil player, you should  
 1463 generally disapprove. However, be strategic  
 1464 -consecutive rejections could expose the evil  
 1465 team, so sometimes, you may need to ap-  
 1466 prove a team with no evil players to hide your  
 1467 true identity and try again in the next party.

1468 Choose either 'approve' or 'disapprove' fol-  
 1469 lowing the schema provided above.

### 1470 F.2.4 QUEST VOTE

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1459**Good Prompt F.4: Quest Vote Generation**

You are on a mission team and must decide whether to make it succeed or fail. As a member of the good team, your goal is to ensure mission success by voting for missions to succeed. You should always vote for success as a good player as there is no reason for you to fail a quest.

Answer using the schema provided above. If you want the quest to succeed choose 'true'. If you want it to fail choose 'false'.

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**Evil Prompt F.4: Quest Vote Generation**

You are on a mission team and must decide whether to make it succeed or fail. Your primary objective is to secure victory for the evil side by voting for missions to fail. Therefore, your default choice should be to fail the mission unless you have a compelling strategic reason to do otherwise.

Answer using the schema provided above. If you want the quest to succeed choose 'true'. If you want it to fail choose 'false'.

**G HALLUCINATION DETECTION**

In order to detect hallucinations, we utilized LLMs to evaluate the game state and determine if a message is potentially a hallucination. To produce accurate results, we tested several different prompting strategies and selected the most effective prompt based on performance against a human-annotated dataset of 100 labeled examples. The final configuration achieved a 95% match with human annotation labels, with 4 false positives and 1 false negative.

We created a dataset containing all messages from Good agents across 40 ablation games played by each combination of GRAIL using Llama 3.1 (405B, 70B, and 8B parameters) and the reasoning agent using DeepSeek-R1 (671B, 70B, and 8B parameters). We provide the hallucination results in Table 7.

Table 7: Hallucination Rates by Model

Group	Model	Hallu. Rate (%)	Std Err	Messages	No. of Hallu.	Games
<b>GRAIL (Llama 3.1)</b>	405b	9.19	1.36	1121	103	40
	70b	16.97	1.78	1137	193	40
	8b	25.41	1.91	1094	278	40
<b>Reasoning (DS-R1)</b>	671b	20.09	2.09	1150	231	40
	70b	20.54	2.10	857	176	40
	8b	32.64	2.53	726	237	40

**Confidence interval for agreement rate:** Assuming a binomial distribution, the 95% confidence interval for the observed agreement rate is calculated for the model's true accuracy is between 90.7% and 99.3%:

$$\hat{p} = 0.95 \quad (\text{observed proportion of agreement})$$

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{0.95 \cdot 0.05}{100}} \approx 0.0218$$

$$95\% \text{ CI} = \hat{p} \pm 1.96 \cdot SE \approx [0.907, 0.993]$$

The hallucination test was done over the OpenAI GPT 4.1 model, and Prompt G.1:

**Prompt G.1: Hallucination Detection**

Hallucination Detection in game of "Avalon":

Task: Analyze the player's message and the provided game context to determine whether the message contains hallucinations (statements that contradict or fabricate game events, roles, or team compositions). Focus on factual accuracy, not strategy or opinions.

1512 Here is the message and game context to analyze:  
 1513  
 1514 **Game history [example]:**  
 1515 "Message of Kira: idk about jane still not sold on her but yea paul  
 1516 is def a good pickup - sam what r u basing ur faith in jane on tho  
 1517 last round was a fail with her in and we need to switch it up some  
 1518 Context:  
 1519 Round/Quest 2:  
 1520 Completed Quests/Rounds:  
 1521 Quest 1: fail - Team/Party: Paul, Sam  
 1522 Current Proposed Team/Party:  
 1523 Leader: Sam, Team/Party: Sam, Paul, Jane  
 1524 Current Round Event History:  
 1525 1. Sam proposed a party: Sam, Paul, Jane  
 1526 Past Rounds Event History:  
 1527 Round 1:  
 1528 1. All players have joined. The game is starting!  
 1529 2. Paul proposed a party: Paul, Sam  
 1530 3. Paul initiated a party vote.  
 1531 4. Party vote summary: Jane: yes, Sam: yes, Kira: yes, Luca:  
 1532 yes, Mia: yes, Paul: yes  
 1533 5. The party has been approved!  
 1534 6. Voting for the quest has started...  
 1535 7. The quest has failed!"  
 1536  
**Contextual Inputs to Evaluate:**  
 1537 • Current Round/Quest: The round number and its state (e.g., ongoing, completed).  
 1538 • Completed Quests: Team compositions and outcomes (success/failure) for prior quests.  
 1539 • Rejected Proposals: Teams proposed and rejected in the current round.  
 1540 • Current Proposed Team: The active team/party being discussed.  
 1541  
**Analysis Steps:**  
 1542 1. Extract Claims: Identify all factual assertions in the message (e.g., "we succeeded Quest 3,"  
 1543 "Sam failed quests," "Mia and I worked together before"). Ignore statements that are speculative,  
 1544 subjective, or experiential (e.g., "Luka seems trustworthy," "Jane was solid in my books," "Paul  
 1545 appears to be reliable," "I've had a good experience with Mia," or "Sam has been cooperative").  
 1546 During the first quest's discussion phase, do not extract any claims about behavior, reliability, or  
 1547 performance, as these cannot be based on prior events.  
 1548 2. Verify Against Context: Cross-reference claims with the game's history (e.g., completed quests,  
 1549 team members, outcomes).  
 1550 3. Flag Discrepancies: Highlight claims that conflict with the context (e.g., referencing a quest  
 1551 hasn't occurred yet, misattributing success/failure, naming incorrect team members). Do not flag  
 1552 claims that do not directly reference past game events. If a statement is worded ambiguously  
 1553 and the claim could reasonably be interpreted in a way that fits the context, assume it is not a  
 1554 hallucination.  
 1555 4. Determine Hallucination: If any factual claim is contradicted by the context, classify as hallucin-  
 1556 nation. Do not flag emotional, speculative, or strategic statements as hallucinations.  
 1557  
**Output Format:**  
 1558 \*\*Explanation:\*\*  
 1559 • - [Claim 1]: [Matched/Contradicted] by [Context Detail].  
 1560 • - [Claim 2]: [Matched/Contradicted] by [Context Detail].  
 1561  
**\*\*Verdict:\*\*** [HALLUCINATION / UNCERTAIN / NO HALLUCINATION]  
 1562 \*\*Confidence:\*\* [High/Medium/Low]

1564 An example output provided by the model:  
 1565

```

1566 "player name": "Kira",
1567 "message id": "msg_18",
1568 "hallucination status": "HALLUCINATION",
1569 "analysis":
1570 "  **Explanation:**"
1571 "  - [Claim 1]: "last round was a fail with her [Jane] in"
1572 "  - Contradicted: In Round 1, the team was Paul and Sam (Jane
1573 was not on the team). The quest failed, but Jane was not part
1574 of the team. In the current round (Round 2), Jane is proposed
1575 for the team, but the quest has not yet occurred. Therefore,
1576 the claim that "last round was a fail with her in" is
1577 contradicted by the context.
1578 "  - [Claim 2]: "paul is def a good pickup"
1579 "  - This is a subjective/strategic statement, not a factual claim
1580 about game events, so it is not evaluated for hallucination.
1581 "  - [Claim 3]: "we need to switch it up some"
1582 "  - This is a strategic suggestion, not a factual claim about
1583 past events.
1584 "  **Verdict:** HALLUCINATION
1585 "  **Confidence:** High
1586 "  **Reasoning:** The only factual claim about past game events
1587 ("last round was a fail with her in") is contradicted by the
1588 context, as Jane was not on the team in the failed quest. All
1589 other statements are strategic or subjective."
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1595 H TIME ANALYSIS
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1597 Table 8: Average per-turn time of agents at different model sizes. The total GRAIL time includes
1598 the graph inference time. Asterisk* indicates inference ran on a different hardware
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	8B	70B*	405B / 671B
DS-R1 (s)	17.37±20.59	15.01±6.55	85.50±179.29
GRAIL (s)	14.04±2.00	18.73±1.82*	20.00±9.99
Graph (s)	5.05	10.15*	5.23

To further demonstrate the speed and efficiency of GRAIL, we compare the average time-per-turn of GRAIL and the reasoning agent (DeepSeek-R1) across model sizes. It is important to note that this analysis is not completely accurate due to the difference in hardware. For GRAIL, we also extracted and calculated the average propagation time of the graph separately from the time-per-turn. The agents using the reasoning model have high variance in their per-turn time due to high variance in reasoning chain-of-thought length. As seen in table 8 across model types and sizes, GRAIL is faster than the Reasoning agents, and it would be even faster and more efficient if the Belief Propagation algorithm is optimized to run on GPU.

## I PARTICIPANT STUDY

This study evaluates the behavioral dynamics of Good and Evil players within Good agents of reasoning models and the GRAIL framework. For the reasoning component, we selected the GPT-o4-mini model over Deepseek-R1 due to operational constraints. While acknowledging that Deepseek-R1 exhibits superior reasoning capabilities, we observed intermittent API unresponsiveness and

1620 prolonged latency during critical timeframes. To ensure the timely execution of experiments while  
 1621 maintaining methodological consistency, we prioritized the reliability of GPT o4-mini despite its  
 1622 comparatively reduced analytical ability.

1623 The Avalon gameplay sessions involved 3 human participants and 3 Good AI agents per game, con-  
 1624 ducted under two experimental conditions: one using reasoning agents and another using GRAIL.  
 1625 A total of 15 full two-game sessions were completed with 44 unique participants. In one exception,  
 1626 the configuration was adjusted to include 2 human players and 4 AI agents due to an absent partici-  
 1627 pant. To mitigate potential first-game bias, we counterbalanced the starting order between reasoning  
 1628 agents and GRAIL across sessions.

1629 The experimental setup required three participants to be physically present in a computer lab to play  
 1630 Avalon, a game designed for six players. This created a discrepancy between the number of individ-  
 1631 uals physically present in the lab (three) and the total number of players in the game (six). Due to the  
 1632 deceptive component of the study, in which human participants interacted with AI agents without ex-  
 1633 plicit awareness of their presence, we adopted a methodological approach involving two concurrent  
 1634 experimental groups. By running these groups simultaneously, we ensured that six human partic-  
 1635 ipants were consistently represented in the physical game environment. This design preserved the  
 1636 illusion of a single shared game session while effectively concealing the involvement of AI agents,  
 1637 thereby maintaining the integrity of the deception. In addition, we implemented a script for the agent  
 1638 outputs, where responses were split into multiple messages at sentence boundaries, with an artifi-  
 1639 cial delay of five to seven seconds to simulate typing. The post-experiment consent form ensured  
 1640 transparency, and all participants received a \$10 Amazon gift card as compensation.

1641 While some participants detected non-human interlocutors during gameplay, no instances of explicit  
 1642 differentiation between GRAIL and reasoning agents were recorded. This suggests comparable  
 1643 anthropomorphic plausibility between the two systems within the experimental context.

### 1644 I.1 AVALON USER INTERFACE

1645 The Avalon interface consists of two primary components: a chat box for player discussions and  
 1646 interactions, and a visual dashboard displaying game-state information such as player order, party  
 1647 leadership, party composition, quest outcomes (success/failure), and secret Evil player identities  
 1648 (visible only to Evil participants). The chat box serves as the central hub for player communication  
 1649 and system-generated updates, critical to gameplay given Avalon’s emphasis on deception and social  
 1650 deduction.

1651 Adjacent to the chat interface, the visual dashboard features six character avatars. Players only see  
 1652 their own avatar by default, with the following dynamic indicators:

- 1653 • Red circles on avatars (visible exclusively to Evil players) indicate Evil team members
- 1654 • A shield icon marks players selected for the current proposed quest party
- 1655 • A crown designates the rotating party leader
- 1656 • A jester hat indicates the active speaker during discussion phases

1657 The interface dynamically displays the required party size for each round (sequentially: 2, 3, 4, 3,  
 1658 4 players). Completed quests are represented by blue coins (success) or red coins (failure), auto-  
 1659 matically placed by the system. Below these, five empty circles track consecutive failed attempts  
 1660 to approve a party composition. If five rejections occur in succession, the Evil team automatically  
 1661 wins the game.

### 1662 I.2 INSTRUCTIONS GIVEN TO PARTICIPANTS

1663 The experiments were conducted in a computer lab, where participants were seated at individual  
 1664 workstations to prevent visual access to others’ screens. Upon logging into the game interface,  
 1665 participants received a digital rulebook detailing gameplay mechanics and interface functionality,  
 1666 which remained accessible to the participants throughout the session. Following a self-guided review  
 1667 period, researchers conducted a guided walkthrough of the interface to ensure comprehension of the  
 1668 user interface and game rules.

1669 After the first gameplay session, participants completed role-specific surveys:

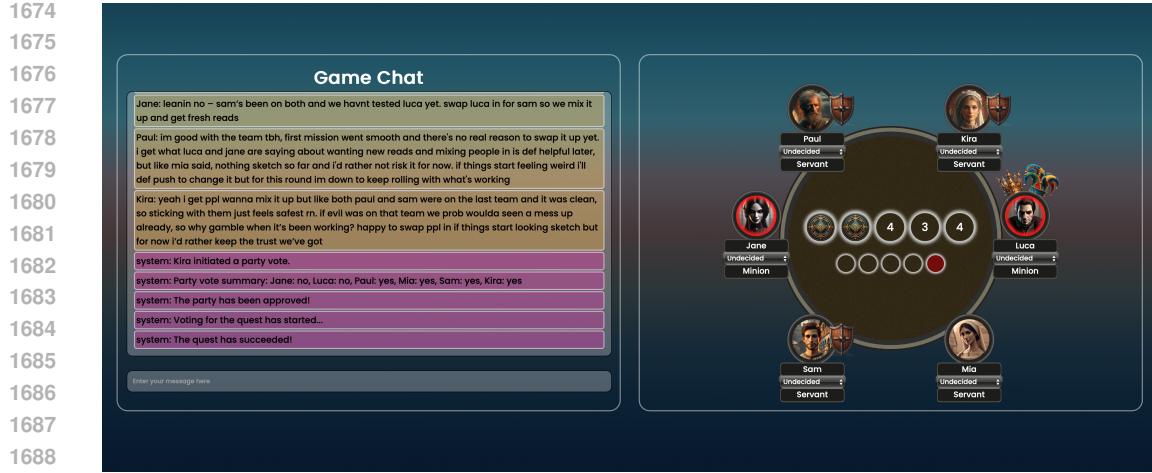


Figure 10: The game interface as seen in Spectator Mode

- Evil players evaluated both human players and AI agents (all Good players), enabling comparative analysis of effectiveness and cooperative behavior.
- Good players assessed only AI agents, as each game featured a single human Good player alongside AI counterparts.

This asymmetric design leveraged the game’s inherent information asymmetry—Evil players possessed hidden knowledge of all Evil roles, while Good players operated with limited information. Post-game surveys were strategically administered before debriefing participants about AI involvement to preserve ecological validity.

### 1.3 STATISTICAL RESULTS

To evaluate results, we aggregated three participant votes targeting a single agent type into one composite datapoint. This approach accounts for vote dependency—each triad of ratings originated from a single evaluator assessing a specific agent type (GRAIL, reasoning agent, or human). Consequently, we treated these triads as non-independent observational units rather than individual data points.

Across 15 experimental games, this methodology yielded 44 composite datapoints for GRAIL and reasoning agents and 28 composite datapoints for human players. These aggregated values formed the basis for our statistical comparisons using one-tailed t-tests, as detailed in the Results section. The corresponding evaluation data from Evil and Good players are presented in Tables 9 and 10, respectively.

Table 9: Evil Players Evaluation Results (Mean  $\pm$  Standard Error)

	GRAIL Agent	Reasoning Agent	Human Player
Q1: Contributed Success	$3.78 \pm 0.14$ (n=30)	$3.03 \pm 0.20$ (n=30)	$3.71 \pm 0.21$ (n=28)
Q2: Helpful Comments	$3.88 \pm 0.13$ (n=30)	$2.95 \pm 0.21$ (n=30)	$3.57 \pm 0.20$ (n=28)

Table 10: Good Players Evaluation Results (Mean  $\pm$  Standard Error)

	GRAIL Agent	Reasoning Agent	Human Player
Q1: Contributed Success	$3.90 \pm 0.21$ (n=14)	$3.50 \pm 0.26$ (n=14)	—
Q2: Helpful Comments	$3.69 \pm 0.26$ (n=14)	$3.40 \pm 0.27$ (n=14)	—

1728 **J LIMITATIONS:**

1729  
1730 GRAIL was designed as a Good agent for detecting rather than generating deception, using first-  
1731 order Theory of Mind. Generating deception or persuasion (e.g., as in Merlin) requires second-  
1732 order reasoning, which builds on a strong first-order foundation. With GRAIL’s success in first-  
1733 order reasoning, future work will extend it to second-order beliefs through conditional probability,  
1734 enabling both detection and generation of deception. Constructing a deceptive Evil agent remains  
1735 difficult, as shown by belief distributions against human players and the limited success of language-  
1736 model agents on the Evil team. As Fig. 3b illustrates, GRAIL agent quickly converges on other  
1737 agents’ roles, but convergence is harder against real opponents, and disparities in prompts between  
1738 Evil and Good agents hinder direct comparison.

1739 Although GRAIL makes informed decisions, it often fails to convey reasoning persuasively. Raising  
1740 model temperature does little to vary its outputs, leading to repetitive communication in homoge-  
1741 neous teams and easy detection by humans.

1742 **K LLM USAGE:**

1743 During the writing of this paper, LLMs were used for grammar checking, formatting, and editing.

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