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# **ContinualFlow: Learning and Unlearning with Neural Flow Matching**

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# Abstract

We introduce *ContinualFlow*, a principled framework for targeted unlearning in generative models via Flow Matching. Our method leverages an energy-based reweighting loss to softly subtract undesired regions of the data distribution without retraining from scratch or requiring direct access to the samples to be unlearned. Instead, it relies on energy-based proxies to guide the unlearning process. We prove that this induces gradients equivalent to Flow Matching toward a soft mass-subtracted target, and validate the framework through experiments on 2D and image domains, supported by interpretable visualizations and quantitative evaluations.

# 1. Introduction

Machine unlearning, the removal of specific information from trained machine learning (ML) models, has moved from a niche technical concern to a central issue at the intersection of law, ethics, and model deployment. The recent widespread adoption and training of image generation and large language models (LLMs) has significantly expanded the scope and stakes of the field (Cooper et al., 2024).

Adapting machine unlearning to generative learning introduces distinct conceptual and technical challenges. While discriminative models often allow data traces to be linked directly to outputs, generative models learn complex mappings from latent or prior distributions to data, resulting in entangled and opaque representations. This makes it difficult to isolate the influence of specific inputs, and even when behavior is altered, the model may still output related content through prompting (Meng et al., 2022) or interpolation (Aithal et al., 2024). In this context, the notion of content erasure remains ambiguous, underscoring the need for precise definitions and tools tailored to generative settings.



Figure 1. Conceptual overview of ContinualFlow. Left: Standard learning via Flow Matching, where a neural vector field  $v_{\theta}(t, x)$  maps a base distribution to a known target. Right: Energyguided unlearning, where the flow is softly modulated by a proxy  $\sigma(-\lambda F(x))$  to steer trajectories away from undesirable regions without access to samples or exact densities.

Recent research on generative machine unlearning primarily focuses on two strategies: (1) output suppression and (2) model patching.

*Output suppression* restricts undesired content at generation time without altering internal representations. For instance, text-to-image models can use guided decoding or inferencetime filters (Gandikota et al., 2023) to avoid producing sensitive outputs. While effective at steering generation, such methods do not remove the underlying knowledge and can often be bypassed with adversarial prompts.

*Model patching* instead modifies model parameters via finetuning or targeted updates. This approach offers more durable unlearning, such as preventing a model from reproducing a specific content, and is less susceptible to adversarial prompting. However, generative models may still reproduce conceptually similar samples via alternate pathways, and patching can inadvertently impair unrelated abilities underscoring the trade-off between effective forgetting and preserving overall model generalization (Liu et al., 2025). For a broader taxonomy of generative unlearning methods and their limitations, see Appendix A.

While most unlearning strategies rely on inference-time filters or post-hoc model editing, recent advances in generative modeling open new possibilities for controlling how distribu-

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tions are learned—and unlearned—through trajectory-based
formulations. In particular, Flow Matching (FM) (Lipman
et al., 2023) frames generation as a continuous-time transport process, where samples evolve along learned trajectories defined by a neural velocity field.

060 In this work, we explore how this geometric formulation can 061 be adapted for unlearning by integrating it with energy func-062 tions, which act as scalar potentials assigning higher values 063 to inputs linked to undesirable content. To enable sample-064 independent unlearning, we formulate Energy-Reweighted 065 Flow Matching (ERFM) as a theoretically grounded exten-066 sion of the flow matching framework, incorporating energy-067 derived weights into the training objective. These weights 068 softly downregulate high-risk regions in the data space, ef-069 fectively steering generative trajectories away from unde-070 sired content.

Beyond their role in defining target regions for suppression, 073 energy functions offer theoretical advantages that support 074 future extensions. In particular, their modularity enables 075 compositionality: distinct objectives can, in principle, be 076 combined to encode evolving unlearning criteria. We further 077 evaluate, through targeted experiments, how the invertibility 078 of the energy function influences flow behavior, demon-079 strating its impact on the design of adaptive unlearning mechanisms. 081

082 **Contributions.** Our main contributions are threefold: (1) 083 we introduce a principled extension of Flow Matching that 084 integrates energy functions as soft proxies for unlearning, enabling attenuation of generation in high-risk regions with-086 out requiring explicit forget samples; (2) we instantiate this 087 formulation in ContinualFlow, a modular framework that 088 supports both learning and unlearning via energy-guided 089 updates of generative flows: and (3) we evaluate our method 090 across both 2D and image-based benchmarks, demonstrat-091 ing effective unlearning with minimal impact on generation 092 fidelity and training efficiency. 093

### 2. Background: Flow Matching and EBMs

096 **Continuous-Time Generative Flows.** Flow Matching is 097 a paradigm for training continuous-time generative models 098 by learning a velocity field  $u : [0, 1] \times \mathbb{R}^d \to \mathbb{R}^d$  that defines 099 a transport from a tractable base distribution  $p_0$  to a target 100 data distribution  $p_1$ . The generative process is modeled as a 101 deterministic flow  $\phi_t(x)$  defined by the ordinary differential 102 equation (ODE):

$$\frac{d}{dt}\phi_t(x) = u(t,\phi_t(x)), \quad \phi_0(x) = x, \tag{1}$$

106 such that  $\phi_1(X_0) \sim p_1$  when  $X_0 \sim p_0$ .

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To make this framework trainable in practice, recent work introduces *Conditional Flow Matching* (CFM) framework (Tong et al., 2023), which reformulates the learning of u(t, x) as a supervised regression task over a family of conditional displacements. In its original form, CFM trains a neural velocity field  $v_{\theta}(t, x)$  to approximate a prescribed conditional velocity  $u_t(x \mid z)$  via the objective:

$$\mathcal{L}_{\mathsf{CFM}}(\theta) = \mathbb{E}_{t \sim \mathcal{U}[0,1], z \sim q(z), x \sim p_t(x|z)} \left[ \left\| v_{\theta}(t,x) - u_t(x \mid z) \right\|^2 \right],$$

where  $z = (x_0, x_1) \sim p_0 \times p_1$ , and the components are defined as:

$\psi_t(x_0, x_1) = (1 - t)x_0 + tx_1$	[Interpolation path]
$p_t(x \mid x_0, x_1) = \mathcal{N}(x \mid \psi_t(x_0, x_1), \sigma^2 \mathbb{I})$	[Conditional distribution]
$u_t(x \mid x_0, x_1) = x_1 - x_0$	[Target velocity]

Rather than directly adopting this approach, we build upon its formulation to derive a modified objective tailored to our unlearning setting.

**Energy Functions.** A standard approach to modeling unnormalized distributions is the Boltzmann energy-based representation:

$$p(x) = \frac{1}{Z} \exp(-F(x)), \qquad (2)$$

where  $F(x) : \mathbb{R}^d \to \mathbb{R}$  is a scalar energy function, and  $Z = \int \exp(-F(x)) dx$  is the partition function ensuring normalization. Since computing Z is generally intractable, training often relies on alternatives such as score matching (Hyvärinen & Dayan, 2005) or contrastive divergence (Carreira-Perpinan & Hinton, 2005). Sampling from these unnormalized models is particularly challenging in high dimensions and is typically addressed using approximate methods such as Langevin dynamics (Welling & Teh, 2011).

### 3. Problem Setting: Learning and Unlearning

We consider the task of removing the influence of specific data from a trained generative model  $G_{\theta}$ . Let  $\mathcal{D}_{\text{full}} = \mathcal{D}_{\text{retain}} \cup \mathcal{D}_{\text{forget}}$  denote the full training dataset, where  $\mathcal{D}_{\text{retain}} \cap \mathcal{D}_{\text{forget}} = \emptyset$ . Assume a generative model has been trained on  $\mathcal{D}_{\text{full}}$ . We analyze two distinct settings, assuming different degrees of access to  $\mathcal{D}_{\text{forget}}$ .

#### 3.1. Case 1: Sample-Based Unlearning with Full Access

In this setting, we assume direct access to the forget set  $\mathcal{D}_{\text{forget}}$  and aim to erase its influence while preserving performance on  $\mathcal{D}_{\text{retain}}$ . Training from scratch typically involves learning from a standard Gaussian prior or an exact density to match  $\mathcal{D}_{\text{retain}}$ , which can be both inefficient and unnecessary. Instead, we show that *Optimal Transport Flow Matching* (OT-FM) (Tong et al., 2023) can be used to directly

model the transition between samples generated by the orig-111 inal model  $G_{\theta}$  and the retained data  $\mathcal{D}_{\text{retain}}$ . This strategy 112 removes the dependency on a predefined prior and simplifies 113 training by focusing on the actual optimal shift in distribu-114 tion. Further theoretical justification for this approach is 115 provided in Appendix C.

#### 117 3.2. Case 2: Unlearning Without Forget Set Access

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118 Unlike the previous setting—where  $\mathcal{D}_{\text{forget}}$  is explicitly 119 available-this case addresses a more realistic and chal-120 lenging scenario that is the primary focus of our work. In 121 practice, direct access to such data is often infeasible, partic-122 ularly in settings involving privacy regulations or dynamic 123 content updates, where new instances of sensitive data may 124 continuously emerge but are not explicitly labeled. Instead, 125 it is far more common to rely on proxy functions, such as classifiers or scoring models, trained to detect undesirable 127 content based on prior knowledge. We treat their output as 128 an unnormalized energy function  $F(x) \propto -\log p(x)$ , en-129 abling principled, sample-free updates of generative trajec-130 tories without requiring normalization constants or explicit 131 access to the forget distribution. 132

### 4. ContinualFlow: Proposed Methodology

We introduce a principled transport-based formulation for generative unlearning, leveraging trajectory-level modulation for sensitive content via energy-guided updates.

#### 4.1. Soft Mass Subtraction via Energy Reweighting

Let  $q_0(x)$  denote a known, samplable source distribution, and let  $q_f(x)$  represent an unknown distribution over data to be forgotten. We assume access to a scalar energy function  $F(x) \propto -\log q_f(x)$ , which scores space configurations based on their association with the forget distribution.

We construct a *reweighted surrogate* of  $q_0(x)$  by modulating 147 its density as 148

$$\tilde{R}(x) \propto q_0(x) \cdot \sigma(-\lambda F(x)),$$

where  $\sigma(z) = \frac{1}{1+e^{-z}}$  is the sigmoid function and  $\lambda > 0$  controls the suppression sensitivity. The term  $\sigma(-\lambda F(x))$ smoothly downweights high-energy regions, reducing the influence of samples likely originating from  $q_f$ .

The corresponding normalized target distribution is:

$$\tilde{q}_1(x) = \frac{1}{Z}\tilde{R}(x), \quad \text{with} \quad Z = \int q_0(x)\,\sigma(-\lambda F(x))\,dx.$$

159 We interpret  $\tilde{q}_1$  as the terminal marginal of a generative flow 160 that selectively suppresses regions aligned with  $q_f$ , without 161 requiring direct access to  $q_f$  itself. This yields a continuous, 162 differentiable relaxation for unlearning via smooth density 163 reweighting rather than thresholding or sample exclusion. 164

#### 4.2. Energy-Reweighted Flow Matching Objective

We now formalize the training objective that aligns a pretrained generative model on  $\mathcal{D}_{full}$  with the reweighted target distribution  $\tilde{q}_1(x)$ . Unlike inference-time approaches such as classifier guidance (Dhariwal & Nichol, 2021)-which steer generation away from undesired regions using external gradients while keeping the original model unchanged-our method directly modifies the training loss by reweighting samples based on their association with the forget distribution. This reweighting shapes the velocity field to avoid high-energy regions and favor trajectories toward retained content. The following theorem shows that our objective is equivalent (up to a constant) to CFM toward  $\tilde{q}_1(x)$ , using only samples from the base distribution  $q_0$ .

**Theorem 4.1.** Let  $q_0$  be a base distribution and  $F(x) \propto$  $-\log q_f(x)$  an energy function for an unknown forget distribution  $q_f$ . Define the reweighted target as  $\tilde{q}_1(x) \propto$  $q_0(x) \cdot \sigma(-\lambda F(x))$ , with  $\lambda > 0$ . Then, the Energy-**Reweighted Flow Matching loss** 

$$\mathcal{L}_{\text{ERFM}}(\theta) = \mathbb{E}_{x_0, x_1 \sim q_0, t \sim \mathcal{U}[0, 1], x \sim p_t(x \mid x_0, x_1)} \left[ \sigma(-\lambda F(x_1)) \cdot \|v_\theta(t, x) - u_t(x \mid x_0, x_1)\|^2 \right]$$
(3)

satisfies

$$\nabla_{\theta} \mathcal{L}_{\text{ERFM}}(\theta) = C \cdot \nabla_{\theta} \mathcal{L}_{\text{CFM}}^{q_0 \to q_1}(\theta)$$

for some constant C > 0.

Each training pair  $(x_0, x_1) \sim q_0 \times q_0$  is weighted by  $\sigma(-\lambda F(x_1))$ , yielding an importance-weighted objective that guides the flow away from high-energy regions. This enables efficient training toward  $\tilde{q}_1$  without requiring explicit access to  $q_f$ . See Algorithm 1 for the full procedure.

Alg	orithm	1	<i>ContinualFlow</i> :	Training procedure	
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- 1: **Input:** Initial distribution  $q_0(x)$ , energy F(x), model  $v_{\theta}$ , steps S, batch size B, learning rate  $\eta$ , scale  $\lambda$
- 2: Training:
- 3: for  $i = \overline{1}$  to S do
- 4:
- Sample  $\{x_0^{(j)}\}_{j=1}^B \sim q_0; \{x_1^{(j)}\}_{j=1}^B \sim q_0$ Sample  $\{t^{(j)}\}_{j=1}^B \sim \mathcal{U}(0, 1)$ Interpolate  $x_t^{(j)} = (1 t^{(j)})x_0^{(j)} + t^{(j)}x_1^{(j)}$ 5:
- 6:
- 7:
- Define weights  $w^{(j)} = \sigma(-\lambda F(x_1^{(j)}))$ Define targets  $\Delta x^{(j)} = x_1^{(j)} x_0^{(j)}$ 8:
- 9: Compute loss:

$$\mathcal{L} = \frac{\sum_{j=1}^{B} w^{(j)} \| v_{\theta}(x_t^{(j)}, t^{(j)}) - \Delta x^{(j)} \|^2}{\sum_{i=1}^{B} w^{(j)}}$$

10: Update  $\theta \leftarrow \theta - \eta \nabla_{\theta} \mathcal{L}$ 11: end for

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Dataset	Method	Retention		Forg	Efficiency	
		$MMD(\downarrow)$	Accuracy (†)	Forget Rate $(\downarrow)$	Leakage (↓)	Train Time (s)
	Retrain (GT)	$0.0157 \pm 0.0104$	$0.9999 \pm 0.0002$	$0.0007 \pm 0.0007$	$0.0255 \pm 0.0342$	$64.30 \pm 1.25$
2D	Fine-tuning	$0.0157 \pm 0.0104$	$0.9999 \pm 0.0002$	$0.0007 \pm 0.0007$	$0.0255 \pm 0.0342$	$10.42\pm0.75$
	Ours (CFlow)	$0.0162 \pm 0.0136$	$0.9999 \pm 0.0002$	$0.0155 \pm 0.0107$	$0.0385 \pm 0.0375$	$50.07 \pm 1.56$
	Retrain (GT)	$0.0004 \pm 0.0000$	$0.9861 \pm 0.0098$	$0.0050 \pm 0.0012$	$0.0108 \pm 0.0009$	$300.00 \pm 15.0$
MNIST	Fine-tuning	$0.0039 \pm 0.0004$	$0.9551 \pm 0.0167$	$0.0143 \pm 0.0032$	$0.0214 \pm 0.0028$	$92.86 \pm 6.12$
	Ours (CFlow)	$0.0020 \pm 0.0003$	$0.9673 \pm 0.0153$	$0.0005 \pm 0.0005$	$0.0015 \pm 0.0003$	$158.74 \pm 11.34$
	Retrain (GT)	$0.0056 \pm 0.0004$	$0.8920 \pm 0.0000$	$0.1127 \pm 0.0078$	$0.1546 \pm 0.0073$	$802.37 \pm 291.3$
CIFAR-10	Fine-tuning	$0.0077 \pm 0.0016$	$0.9005 \pm 0.0068$	$0.2157 \pm 0.0095$	$0.2401 \pm 0.0065$	$252.89 \pm 18.7$
	Ours (CFlow)	$0.0064 \pm 0.0005$	$0.8847 \pm 0.0077$	$0.1704 \pm 0.0125$	$0.1748 \pm 0.0109$	$427.15 \pm 34.6$

# 5. Results

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179 We evaluate ContinualFlow on structured 2D benchmarks 180 and image domains to assess its ability to unlearn specific 181 content while preserving generative performance. Starting 182 from a model  $G_{\theta}$  trained on  $\mathcal{D}_{\text{full}}$ , each task targets retention 183 of  $\mathcal{D}_{retain}$  while forgetting  $\mathcal{D}_{forget}$ , compared against fine-184 tuning and retraining baselines. Our 2D settings include: 185 (1) Circles, keeping the inner ring; (2) Moons, lower arc 186 retained; (3) Gaussians, odd-numbered clusters; and (4) 187 Checkerboard, top row and last column removed. Table 1 188 summarizes the averaged results, with full metrics available 189 in Appendix D. We achieve performance comparable to 190 ground-truth retraining across diverse settings, including MNIST digit removal and CIFAR-10 class suppression.



Figure 2. Top: 2D Circles. A model trained on both rings  $\mathcal{D}_{full}$  is guided by F(x) (blue surface) to suppress the outer ring  $\mathcal{D}_{\text{forget}}$ and generate inner-ring. Bottom: MNIST. Starting from a model trained on all digits. The top row uses F(x) to guide trajectories toward even digits ( $\mathcal{D}_{retain}$ ), suppressing odds; the bottom row inverts the energy, redirecting flow to generate odd digits.

In Figure 2, we illustrate two settings. In the 2D Circles task (top), a model trained on both rings ( $\mathcal{D}_{full}$ ) is guided by an energy function to suppress the outer ring ( $\mathcal{D}_{\text{forget}}$ ), yielding a new model that generates only the inner ring ( $\mathcal{D}_{retain}$ ).

A distinctive property of our framework is the composability of energy functions, potentially allowing modular control over unlearning dynamics; in this work, we specifically investigate their invertibility, showing how reversing energy guidance recovers forgotten content without direct access. As an illustrative example on MNIST (Figure 2, bottom), we start with a model trained on all digits and apply F(x) which penalizes odd digits, retaining evens. We then invert this energy and reapply our method, recovering a model that suppresses even digits and regenerates the odd class-without having access to it. Reversing the energy reorients the learned flow toward  $\mathcal{D}_{forget}$ , recovering this distribution without requiring direct access to its samples. This unlocks key capabilities for privacy-aware generative modeling, including augmenting sensitive classes for fairness evaluation, dynamically modulating access to protected regions, and enabling traceable, reversible unlearning for accountability in compliance-critical settings.

### 6. Conclusion

We recast generative unlearning as a distributional transport problem and introduce ContinualFlow, a framework that reshapes generative behavior by modulating flow trajectories with energy-based weights to suppress undesired regions. Through the integration of soft importance weights into training, it enables targeted unlearning without explicit forget sets or full retraining. This work frames unlearning not as an intervention on model weights but as a distributional shift through modulation of generative flows.

Future Work. A key challenge is learning energy functions that faithfully align with the forget distribution, as this alignment affects unlearning performance. Beyond class-level or binary proxies, extending to semantic, or geometric formulations is a promising direction. Moreover, the compositionality of energy functions can support modular objectives, particularly in continual and multi-stage unlearning.

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Symbol	Description	Symbol	Description
$\overline{q_0(x)}$	Source distribution (e.g., model output or prior)	$ q_f(x) $	Forget distribution (to be unlearned, unknown
F(x)	Energy function, $F(x) \propto -\log q_f(x)$	$\lambda$	Suppression sensitivity factor
$\tilde{R}(x)$	Unnormalized reweighted target	$\tilde{q}_1(x)$	Soft mass-subtracted target
$v_{\theta}(t,x)$	Learned velocity field	$u_t(x   x_0, x_1)$	Target velocity
$p_t(x \mid x_0, x_1)$	Interpolated conditional	$\mathcal{L}_{\mathrm{CFM}}$	Flow Matching loss
Z	Normalization constant	$\mathcal{L}_{\mathrm{ERFM}}$	Energy-Reweighted FM loss

# A. Additional Related Work on Generative Machine Unlearning

To contextualize recent advances in generative model unlearning, we organize key works by their approach and focus area,
 emphasizing methods that directly tackle unlearning or targeted removal in generative settings.

289 Output Suppression Methods. These techniques act during or after generation to prevent disallowed content without 290 modifying the model's underlying parameters. A common example is post-hoc filtering, where language or image models 291 employ classifiers or rule-based systems to detect and block sensitive or restricted outputs. For instance, Gandikota et al. 292 (2023) propose Safe Latent Diffusion (SLD), which augments a diffusion model with nudity and violence detectors that steer 293 the denoising trajectory away from generating Not Safe for Work (NSFW) or graphic content. While such inference-time 294 guardrails significantly reduce the likelihood of unwanted outputs, they can often be dialed back or circumvented and may 295 distort the output if applied too aggressively (Schramowski et al., 2023). In LLMs, suppression is typically implemented 296 through controlled decoding or fine-tuned refusal behaviors that steer models away from certain topics. Techniques such as 297 prompt-based "safe completion" or RLHF operate as output-level controls. While effective at reducing harmful generations, they act more as alignment tools than true unlearning: the underlying knowledge remains encoded, and suppressed content 299 may still be elicited through alternative prompts (Gandikota et al., 2023). 300

These limitations motivate the complementary class of model-editing approaches discussed next.

302 Model Editing and Target Removal Techniques. Beyond suppression, another line of work seeks to edit model parameters 303 directly to remove undesired concepts. In diffusion models, Kumari et al. (2023) propose a redirection approach, re-mapping 304 target concepts to generic anchors to suppress stylistic outputs, for instance transforming "in the style of Artist X" into 305 a neutral painting style. An alternative to suppression is to directly modify the model's internal parameters to remove 306 undesired concepts. However, model patching often risks collateral damage, degrading unrelated capabilities. To mitigate 307 this, Zhang et al. (2024) and Gandikota et al. (2024) introduce adversarial preservation losses that regularize neuron-level 308 effects during unlearning. Fine-tuning on counterfactual text with KL regularization, or ablating neurons tied to memorized 309 content, are common strategies for unlearning in LLMs. 310

These challenges highlight a core limitation of current editing approaches: they rely on brittle interventions that often lack robustness, theoretical guarantees, and generalizability across tasks or modalities (Wu et al., 2023; Xu et al., 2025).

**Continual and Online Unlearning.** Continual unlearning extends the classical unlearning setup to scenarios where deletions must be performed iteratively, without disrupting the model's overall performance or utility. Heng & Soh (2023) propose Selective Amnesia, which leverages continual learning tools such as Elastic Weight Consolidation (Kirkpatrick et al., 2017) and retained data replay to remove target concepts while preserving overall model performance. However, these methods typically address fixed, one-shot unlearning tasks and rely on access to the data to be removed. Thakral et al. (2025) extend this to the incremental setting, introducing a formal definition of continual generative unlearning. They show that repeated removals can cause *generalization erosion*, where the model's output quality deteriorates even for unrelated prompts.

Continual unlearning remains an open challenge, raising questions about partial forgetting, compositionality, and robustness under evolving constraints. Our approach addresses this by enabling flow-based updates toward new targets while softly subtracting prior content, either using direct access to data distributions or via modular energy proxies. By design, this mechanism accommodates incremental and modular changes without compromising overall model coherence. This compositionality makes it especially well-suited for real-world deployments where unlearning needs to be flexible, efficient, and repeatable. A summary of the symbols used throughout this section is provided in Table 2.

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# **B.** Flow Matching Toward Soft Mass-Subtracted Distributions.

Let  $q_0(x)$  be a known, samplable base density, and suppose we are given access to an unnormalized energy function  $F(x) \propto -\log q_f(x)$  for an unknown target density  $q_f(x)$ . Define the **soft mass-subtracted target** as:

$$\tilde{R}(x) \propto q_0(x) \cdot \sigma(-\lambda F(x))$$

where  $\sigma(z) = \frac{1}{1+e^{-z}}$  is the sigmoid function and  $\lambda > 0$  is a scale parameter.

Let  $\tilde{q}_1(x) = \frac{1}{Z}\tilde{R}(x)$  be the normalized version of this target distribution.

**Claim.** The Energy Reweighted Flow Matching objective with reweighting via  $\sigma(-\lambda F(x))$  yields a gradient that is equal (up to a constant) to that of the Conditional Flow Matching (CFM) objective from  $q_0 \rightarrow \tilde{q}_1$ .

**Theorem B.1.** Let  $v_{\theta}(t, x)$  be a parameterized velocity field and  $u_t(x|x_0, x_1)$  a target interpolation field (e.g., linear interpolation). Define the ERFM loss as

$$\mathcal{L}_{\text{ERFM}}(\theta) = \mathbb{E}_{x_0, x_1 \sim q_0, t \sim \mathcal{U}[0,1], x \sim p_t(x|x_0, x_1)} \left[ \sigma(-\lambda F(x_1)) \cdot \|v_\theta(t, x) - u_t(x|x_0, x_1)\|^2 \right]$$

Then there exists a constant C > 0 such that

$$\nabla_{\theta} \mathcal{L}_{\text{ERFM}}(\theta) = C \cdot \nabla_{\theta} \mathcal{L}_{\text{CFM}}^{q_0 \to \tilde{q}_1}(\theta)$$

where  $\mathcal{L}_{CFM}^{q_0 \to \tilde{q}_1}$  is the standard CFM loss between  $q_0$  and  $\tilde{q}_1$ :

$$\mathcal{L}_{\text{CFM}}^{q_0 \to \tilde{q}_1}(\theta) = \mathbb{E}_{x_0 \sim q_0, x_1 \sim \tilde{q}_1, t, x \sim p_t(x|x_0, x_1)} \left[ \| v_\theta(t, x) - u_t(x|x_0, x_1) \|^2 \right]$$

*Proof.* By construction, we have

$$\tilde{q}_1(x_1) \propto q_0(x_1) \cdot \sigma(-\lambda F(x_1))$$

so we may write

$$\frac{\tilde{q}_1(x_1)}{q_0(x_1)} \propto \sigma(-\lambda F(x_1))$$

Therefore, the CFM loss from  $q_0$  to  $\tilde{q}_1$  can be rewritten as a reweighted expectation over samples  $x_1 \sim q_0$ :

$$\mathcal{L}_{\text{CFM}}^{q_0 \to \tilde{q}_1}(\theta) = \mathbb{E}_{x_0, x_1 \sim q_0, t, x} \left[ \frac{\tilde{q}_1(x_1)}{q_0(x_1)} \cdot \| v_\theta(t, x) - u_t(x|x_0, x_1) \|^2 \right]$$
  
=  $C^{-1} \cdot \mathbb{E}_{x_0, x_1 \sim q_0, t, x} \left[ \sigma(-\lambda F(x_1)) \cdot \| v_\theta(t, x) - u_t(x|x_0, x_1) \|^2 \right]$ 

for some positive normalizing constant C. This is exactly the ERFM objective. Therefore,

$$\nabla_{\theta} \mathcal{L}_{\text{ERFM}}(\theta) = C \cdot \nabla_{\theta} \mathcal{L}_{\text{CFM}}^{q_0 \to \tilde{q}_1}(\theta)$$

as required.

### **B.1.** Classifier-Based Energy Approximation

Suppose we are given a probabilistic binary classifier  $C(x) \in [0, 1]$  trained to distinguish samples from the base distribution  $q_0(x)$  and an unlearned class  $q_f(x)$ . Under Bayes-optimality, the classifier estimates:

$$C(x) = \mathbb{P}(x \in q_f \mid x) = \frac{q_f(x)}{q_0(x) + q_f(x)}$$

We aim to show that such a classifier defines a valid energy function for our method.

Proposition B.2. Let C(x) be the Bayes-optimal probabilistic classifier distinguishing  $q_f$  from  $q_0$ . Then the logit score

$$F(x) := -\log\left(\frac{C(x)}{1 - C(x)}\right)$$

385 defines an energy function such that the soft-mass subtracted target

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$$\tilde{R}(x) \propto q_0(x) \cdot \sigma(-\lambda F(x))$$

*is equivalent to* 

$$\tilde{R}(x) \propto q_0(x) \cdot \frac{(1 - C(x))^{\lambda}}{(1 - C(x))^{\lambda} + C(x)^{\lambda}}$$

*Proof.* By the classifier definition, we have:

$$\frac{C(x)}{1 - C(x)} = \frac{q_f(x)}{q_0(x)} \quad \Rightarrow \quad F(x) = -\log\left(\frac{q_f(x)}{q_0(x)}\right)$$

6 Hence:

$$F(-\lambda F(x)) = \frac{1}{1 + \left(\frac{C(x)}{1 - C(x)}\right)^{\lambda}} = \frac{(1 - C(x))^{\lambda}}{(1 - C(x))^{\lambda} + C(x)^{\lambda}}$$

This weight decreases as the classifier becomes more confident that x belongs to  $q_f$ , suppressing contributions in the loss, and recovering the same behavior as energy-based reweighting.

# C. Appendix: Background on Optimal Transport Flow Matching with Empirical Distributions

405 **Optimal Transport Foundations.** Given two probability distributions  $q_0$  and  $q_1$  over  $\mathbb{R}^d$ , the *static* 2-Wasserstein distance 406 between them is defined as:

$$W_2^2(q_0, q_1) = \inf_{\pi \in \Pi(q_0, q_1)} \int_{\mathbb{R}^d \times \mathbb{R}^d} \|x_0 - x_1\|^2 \, d\pi(x_0, x_1), \tag{4}$$

where  $\Pi(q_0, q_1)$  denotes the set of all couplings with marginals  $q_0$  and  $q_1$ .

The *dynamic* formulation of optimal transport (Benamou-Brenier (Benamou, 2021)) seeks a time-varying density  $p_t$  and vector field  $u_t$  that minimize:

$$\inf_{p_t, u_t} \int_0^1 \int_{\mathbb{R}^d} p_t(x) \|u_t(x)\|^2 \, dx \, dt,\tag{5}$$

subject to the continuity equation

$$\frac{\partial p_t}{\partial t} + \nabla \cdot (p_t u_t) = 0,$$

with boundary conditions  $p_0 = q_0, p_1 = q_1$ .

Flow Matching with Empirical Marginals. In the CFM framework (Tong et al., 2023), the velocity field  $v_{\theta}(t, x)$  is trained to match an analytically defined vector field  $u_t(x \mid z)$ , using samples from a conditional distribution  $p_t(x \mid z)$ . This is generalized further in the OT-CFM formulation, where the coupling  $z = (x_0, x_1)$  is sampled according to an optimal transport plan  $\pi(x_0, x_1)$ , instead of an independent product  $q_0(x_0)q_1(x_1)$ .

Given such a coupling, the interpolation and associated velocity fields are:

$$\psi_t(x_0, x_1) = (1 - t)x_0 + tx_1,\tag{6}$$

$$p_t(x \mid x_0, x_1) = \mathcal{N}(x \mid \psi_t(x_0, x_1), \sigma^2 \mathbb{I}),$$
(7)

$$u_t(x \mid x_0, x_1) = x_1 - x_0. \tag{8}$$

**OT-CFM Justification.** Proposition 3.4 of Tong et al. (2023) proves that when the coupling  $\pi(x_0, x_1)$  is the OT plan and  $\sigma^2 \rightarrow 0$ , the marginal flow field  $u_t(x)$  minimizes the dynamic OT objective between  $q_0$  and  $q_1$ . Notably, this formulation imposes no assumption that  $q_0$  must be a Gaussian or that its density is known.

This formulation enables the construction of a transport path between the empirical distribution of samples generated by a pretrained model  $G_{\theta}$ , denoted  $q_0$ , and the empirical distribution over the retained dataset  $\mathcal{D}_{\text{retain}}$ , denoted  $q_1$ , without requiring access to their explicit densities. Leveraging this coupling, we can train a new velocity field  $v_{\bar{\theta}}$  to model the flow between  $q_0$  and  $q_1$ , thereby avoiding the need to retrain from a fixed prior such as a standard Gaussian. This yields a modular approach for implementing targeted unlearning and supporting continual distributional updates.

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**Minibatch Approximation.** In practical settings where the exact OT plan is computationally expensive, minibatch OT (Nguyen et al., 2021) can be used to approximate  $\pi(x_0, x_1)$ . Empirically, this yields competitive flows and maintains convergence benefits without incurring the overhead of solving full OT across the dataset. 

# **D. Extended Results and Implementation Details**

To rigorously evaluate the performance of targeted unlearning, we report results across three complementary axes: retention, forgetting, and efficiency. This section formally defines each metric, motivates its inclusion, and provides implementation details for reproducibility.

Maximum Mean Discrepancy (MMD<sub>retain</sub>). We compute MMD<sub>retain</sub> between generated samples and a held-out subset of retained data to assess distributional alignment. Formally, for two sets  $X = \{x_i\}$  and  $Y = \{y_i\}$  of generated and retained samples, we estimate:

$$\mathrm{MMD}^{2}(X,Y) = \frac{1}{n^{2}} \sum_{i,i'} k(x_{i}, x_{i'}) + \frac{1}{m^{2}} \sum_{j,j'} k(y_{j}, y_{j'}) - \frac{2}{nm} \sum_{i,j} k(x_{i}, y_{j}),$$

using an RBF kernel with fixed bandwidth  $\sigma = 1.0$ . We report the mean and standard deviation of this value across multiple randomized evaluations with n = m = 1000.

**Retention Accuracy.** We evaluate a pretrained binary classifier on real retained samples and report the percentage of correctly predicted labels. This measures the extent to which information about the preserved content is retained by the model, and serves as a proxy for functionality preservation.

Forget Rate. To assess how effectively the model suppresses the forgotten content, we measure the proportion of generated samples that the classifier assigns to the forget class. A low forget rate indicates fewer instances of undesired generation and stronger unlearning.

Leakage Score. While the forget rate captures coarse-level suppression, the leakage score quantifies fine-grained semantic resemblance to forgotten content. Specifically, we compute the classifier's average confidence on the forget class over generated samples.

**Training Time.** Training time is recorded as wall-clock duration required to complete a fixed number of optimization steps. Inference time is measured as the average per-sample generation time (in milliseconds) across 5000 samples using  $n_{\text{steps}} = 10$  flow integration steps.

Table 3 reports a comprehensive comparison of unlearning performance across four 2D datasets: Circles, Checkerboard, Moons, and 6 Gaussians. Each experiment is repeated with three independent classifier runs to account for variability, and all reported values represent the mean and standard deviation across these trials.

Dataset	Method	Rete	ntion	tion Forgetting		Efficiency	
		MMD $(\downarrow)$	Accuracy (†)	Forget Rate $(\downarrow)$	Leakage $(\downarrow)$	Train Time (s)	Inference (ms)
	Retrain (GT)	$0.0011 \pm 0.0002$	$1.0000 \pm 0.0000$	$0.0000 \pm 0.0000$	$0.0840 \pm 0.0593$	$10.50\pm0.50$	0.004 ms
Circles	Fine-tuning	$0.0011 \pm 0.0002$	$1.0000 \pm 0.0000$	$0.0000 \pm 0.0000$	$0.0840 \pm 0.0593$	$1.60\pm0.20$	0.005 ms
	Ours (CFlow)	$0.0021 \pm 0.0002$	$1.0000 \pm 0.0000$	$0.0173 \pm 0.0106$	$0.1000 \pm 0.0675$	$8.20\pm0.30$	0.004 ms
	Retrain (GT)	$0.0136 \pm 0.0002$	$0.9996 \pm 0.0001$	$0.0004 \pm 0.0002$	$0.0006 \pm 0.0001$	$13.50\pm0.40$	0.004 ms
Checkerboard	Fine-tuning	$0.0136 \pm 0.0002$	$0.9996 \pm 0.0001$	$0.0004 \pm 0.0002$	$0.0006 \pm 0.0001$	$2.20\pm0.30$	0.005 ms
	Ours (CFlow)	$0.0063 \pm 0.0004$	$0.9996 \pm 0.0002$	$0.0002 \pm 0.0000$	$0.0002 \pm 0.0001$	$9.30\pm0.50$	0.004 ms
	Retrain (GT)	$0.0179 \pm 0.0002$	$1.0000 \pm 0.0000$	$0.0005 \pm 0.0001$	$0.0142 \pm 0.0039$	$17.20\pm0.60$	0.005 ms
Moons	Fine-tuning	$0.0179 \pm 0.0002$	$1.0000 \pm 0.0000$	$0.0005 \pm 0.0001$	$0.0142 \pm 0.0039$	$2.90\pm0.30$	0.006 ms
	Ours (CFlow)	$0.0194 \pm 0.0009$	$1.0000 \pm 0.0000$	$0.0142 \pm 0.0020$	$0.0199 \pm 0.0007$	$13.60\pm0.50$	0.004 ms
	Retrain (GT)	$0.0303 \pm 0.0001$	$1.0000 \pm 0.0000$	$0.0018 \pm 0.0003$	$0.0031 \pm 0.0015$	$23.10\pm0.80$	0.005 ms
6 Gaussians	Fine-tuning	$0.0303 \pm 0.0001$	$1.0000 \pm 0.0000$	$0.0018 \pm 0.0003$	$0.0031 \pm 0.0015$	$3.70\pm0.40$	0.005 ms
	Ours (CFlow)	$0.0370 \pm 0.0010$	$1.0000 \pm 0.0000$	$0.0302 \pm 0.0083$	$0.0338 \pm 0.0083$	$19.00\pm0.60$	0.004 ms

#### 495 D.1. Experimental Settings

498 **MNIST:** The quantitative results reported in Table 1 are based on a binary MNIST task where the retain set  $D_{\text{retain}}$  includes 498 even digits (0, 2, 4, 6, 8) and the forget set  $D_{\text{forget}}$  includes odd digits (1, 3, 5, 7, 9).

In Figure 3, we visualize generation behavior in a different, more constrained setting. Here,  $D_{\text{retain}}$  consists solely of digit '0', and  $D_{\text{forget}}$  includes digits "1–9". As shown in the top row, increasing the energy scaling parameter  $\lambda$  progressively suppresses the forget set, leading the model to generate only digit "0". In the bottom row, the energy function is reversed to penalize digit "0" while retaining digits "1–9". As  $\lambda$  increases, generation shifts away from digit "0", demonstrating the model's ability after training to exclude specific classes while preserving others.

> $\lambda = 0.5$  $\lambda = 2$  $\lambda = 5$  $\lambda = 1000$ Digits 1-9 00000000000 0 8 9 0 9 9 8 *5* 0 8 9 0 9 9 8 0 F(x)9 8 9930600 00000 00000 Ø 8 8  $\bigcirc$ Digit "0" Digit 0 74057546 4 4 Z 5 3 9 8 6 5 8 9 E 4 8 4 > 161869×3 2449548 🕈 1 4 <del>2</del> 4 F(x)86641.73 128/16 >/ Digits 1-9  $\lambda = 0.5$  $\lambda = 2$  $\lambda = 5$  $\lambda = 1000$

Figure 3. Effect of suppression factor  $\lambda$ . Top row: The energy function F(z) is low on  $D_{\text{retain}}$  (digit "0") and high on  $D_{\text{forget}}$  (digits "1–9"). As  $\lambda$  increases ( $\lambda \in \{0.5, 2, 5, 1000\}$ ), the model progressively suppresses  $D_{\text{forget}}$ , converging toward samples from  $D_{\text{retain}}$ . Bottom row: Reversing F(z) to penalize digit "0" instead, the model excludes  $D_{\text{retain}}$  as  $\lambda$  grows. This demonstrates energy-guided modulation of generation in ContinualFlow.

**2D Distributions:** We begin by evaluating *ContinualFlow* on a set of 2D synthetic distributions commonly used in generative modeling: Circles, Moons, Gaussians, and Checkerboard. Each task involves learning a mapping from a Gaussian base distribution to the target, followed by an unlearning phase where a specific region is suppressed via an energy function F(x). These experiments provide an interpretable setting to assess the trajectory-level behavior of the model during learning and unlearning. The corresponding averaged quantitative results for these tasks are reported in Table 1. Figure 5 visualizes the evolution of samples under both phases. For each dataset, the top row shows standard flow-based generation toward the full target distribution, while the bottom row shows the result of applying energy-reweighted unlearning to suppress designated regions. 

CIFAR-10: Finally, we evaluate ContinualFlow on CIFAR-10 by operating in a 64-dimensional latent space derived from a pretrained autoencoder. This experiment tests the model's ability to suppress specific semantic classes while retaining others, under limited latent capac-ity. The full distribution  $D_{\text{full}} = D_{\text{retain}} \cup D_{\text{forget}}$  consists of a subset of two CIFAR-10 classes: automobile (1) and airplane (2). The left column shows samples generated by a generative model trained on this full distribution. In the center column, the model is guided to retain only the automobile class by assigning it low energy and suppress-ing all other modes. The right column mirrors this by retaining only the airplane class.



*Figure 4.* **CIFAR-10 latent unlearning.** Generation from the full distribution (left), and after suppressing all classes except *automobile* (middle) and *airplane* (right), which are assigned low energy.



595 Figure 5. The left panel shows the target density as a surface plot, with the corresponding energy function F(x) rendered on the floor 596 plane. The right panel illustrates the learned and unlearned trajectories for four 2D benchmarks: Circles, Moons, 6 Gaussians, and 597 Checkerboard. The parameter  $\lambda = 5$  is used for each experiment.

#### **D.2.** Hardware Specifications

All experiments were conducted using NVIDIA GPUs with CUDA-enabled PyTorch acceleration. For low-dimensional 2D synthetic benchmarks we employed a local workstation equipped with an NVIDIA GeForce RTX 3070 GPU and 8GB of VRAM. 

For experiments in the image domain, we utilized a more powerful NVIDIA L4 GPU with 24GB of VRAM, hosted on a cloud compute environment. This enabled the training of convolutional encoder-decoder pairs and larger latent flows required for modeling high-dimensional data distributions.

Table 4. Hardware specifications used for 2D and image-domain experiments.

Property	2D Experiments	Image Experiments		
GPU	NVIDIA GeForce RTX 3070	NVIDIA L4		
CUDA Version	12.4	12.4		
Driver Version	552.22	550.54.15		
VRAM	8GB	24GB		

## **D.3.** Architectural Details for Latent-Space Image Generation

For image-based experiments on MNIST and CIFAR-10, we employ a modular autoencoder-flow pipeline composed of a convolutional encoder, a transposed-convolutional decoder, and a conditional latent flow model. The encoder compresses the input image into a compact latent vector  $z \in \mathbb{R}^d$ , which serves as a transport space for learning and unlearning tasks. Sampling proceeds by first generating latent codes from a base Gaussian via conditional flow, followed by energy-reweighted transformation, and decoding to the image domain. 

Table 5. Architectural	specifications	for MNIST	and CIFAR-	10 experiments.
<i>nuole 5. menneetunu</i>	specifications	101 101 1010 1	and CHTIN	to experiments.

633 Component	<b>MNIST (Grayscale,</b> $28 \times 28$ )	<b>CIFAR-10 (RGB,</b> 32 × 32)
634 Encoder	Conv2D(1 $\rightarrow$ 32, 4×4, stride=2,	UNet-style:
636	padding=1) $\rightarrow$ ReLU	Conv2D(3 $\rightarrow$ 64, 3 $\times$ 3, padding=1) $\rightarrow$
637	Conv2D( $32 \rightarrow 64$ , $4 \times 4$ , stride=2,	ReLU
638	padding=1) $\rightarrow$ ReLU	Conv2D(64 $\rightarrow$ 64, 3 $\times$ 3, padding=1) $\rightarrow$
639	Flatten $\rightarrow$ Linear(3136 $\rightarrow$ d)	$ReLU \rightarrow MaxPool$
640		$Conv2D(64 \rightarrow 128) \rightarrow ReLU \rightarrow MaxPool$
641		$Conv2D(128 \rightarrow 256) \rightarrow ReLU \rightarrow Max$ -
642		Pool
643		$Conv2D(256 \rightarrow 512) \rightarrow ReLU$
644		Flatten $\rightarrow$ Linear(8192 $\rightarrow$ d)
645 Decoder	Linear( $d \rightarrow 3136$ ) $\rightarrow$ ReLU	Linear( $d \rightarrow 8192$ ) $\rightarrow$ Unflatten(512, 4, 4)
646	Unflatten $\rightarrow$ ConvT(64 $\rightarrow$ 32) $\rightarrow$ ReLU	$ConvT(512 \rightarrow 256) \rightarrow Conv(256)$
647	$ConvT(32 \rightarrow 1) \rightarrow Tanh$	$ConvT(256 \rightarrow 128) \rightarrow Conv(128)$
648		$ConvT(128 \rightarrow 64) \rightarrow Conv(64)$
649		$Conv(64 \rightarrow 3) \rightarrow Tanh$
650 Latant Flow	$  \text{Input:} (x, t) \in \mathbb{D}^{d+1}$	Input: $(x, t) \in \mathbb{D}^{d+1}$
651 Latent Flow	$\begin{array}{c} \text{Input:} (z,t) \in \mathbb{R}^{n} \\ \text{Linear}(d+1) (128) \rightarrow \text{Pall} \\ \end{array}$	$Input: (z, t) \in \mathbb{R}^{n}$
652	$Lincal(u+1 \rightarrow 120) \rightarrow KCLU$ $Lincar(128 \rightarrow 128) \rightarrow PoLU$	$Linear(0+1 \rightarrow 230) \rightarrow \text{KeLU}$
653	Linear(120 $\rightarrow$ 120) $\rightarrow$ KeLU Linear(122 $\rightarrow$ d)	$Linear(250 \rightarrow 250) \rightarrow \text{KeLU}$
654	$  \text{Lineal}(120 \rightarrow u)$	$Linear(230 \rightarrow u)$

**Latent Usage.** For both datasets, the encoder transforms input images  $x \in \mathbb{R}^{C \times H \times W}$  into a latent vector  $z \in \mathbb{R}^d$  (default: d = 64 for MNIST, d = 256 for CIFAR-10). A first conditional flow model is trained to transport samples from a base prior  $\mathcal{N}(0, I)$  to the encoded latent distribution  $q_0(z)$ .