

FURINA: FREE FROM UNMERGEABLE ROUTER VIA LINEAR AGGREGATION OF MIXED EXPERTS

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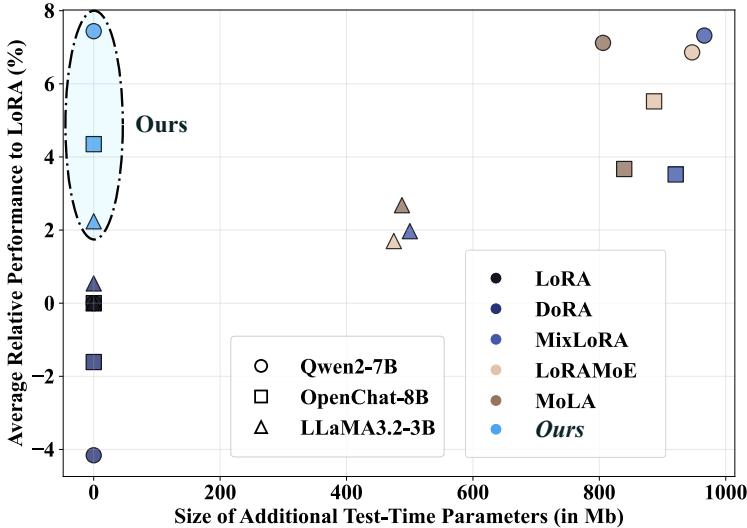


Figure 1: Relationship between additional test-time parameters and relative performance gain compared to standard LoRA. FURINA achieves comparable or superior performance gains to standard MoE-LoRA approaches while sharing the same architecture with standard LoRA during inference, resulting in up to $1.5\times$ speedup.

To overcome these limitations while preserving the performance benefits of MoE, we propose FURINA (Free from Unmergeable Router via LINear Aggregation), a novel fully mergeable MoE-enhanced LoRA architecture which matches the performance of the standard MoE-LoRA methods while overcoming the deployment complexity and overhead. The core of our approach is a Mergeable Self-Routing mechanism, which replaces the traditional router with three key components: (1) Decoupled Learning of Direction and Magnitude: decouples the LoRA adapters by applying column-wise normalization to the weight matrices, isolating their directional components. (2) Shared Learnable Magnitude Vector: introduces a single, shared magnitude vector that scales the outputs of all experts uniformly, ensuring the norm of an expert’s output directly reflects its activation strength. (3) Expert Selection Loss: employs a loss function that encourages sparse, divergent expert activation by maximizing the contribution of the most relevant experts. Specifically, the input is first projected by the normalized LoRA matrices to produce normalized logits, then scaled by the shared magnitude vector. This design allows the norm of each expert’s output to naturally represent its relevance to the input, enabling dynamic, router-free routing. We also introduce a Shared Expert (SE) within the MoE-LoRA block. This expert provides foundational knowledge across the data corpus without introducing non-linearities that would prevent merging. By integrating these modules, FURINA seamlessly transitions between two phases: during training, it operates as a full, capacity-enhanced MoE architecture; during inference, the experts and the shared expert are linearly aggregated and can be merged into a single LoRA adapter or directly into the backbone model, introducing zero overhead. A comparative summary of FURINA against full fine-tuning and other PEFT methods is provided in Tab. 1.

Table 1: Comparison of the proposed FURINA with different fine-tuning strategies

Method	PEFT	MoE During Training	Zero Extra Test Time Cost
Full-SFT	✗	✗	✓
LoRA (Hu et al. (2022))			
DoRA (Liu et al. (2024b))	✓	✗	✓
MixLoRA (Li et al. (2024))			
LoRAMoE (Dou et al. (2024))			
MoLA (Gao et al. (2025))	✓	✓	✗
SLIM (Han et al. (2025))			
FURINA (Ours)	✓	✓	✓

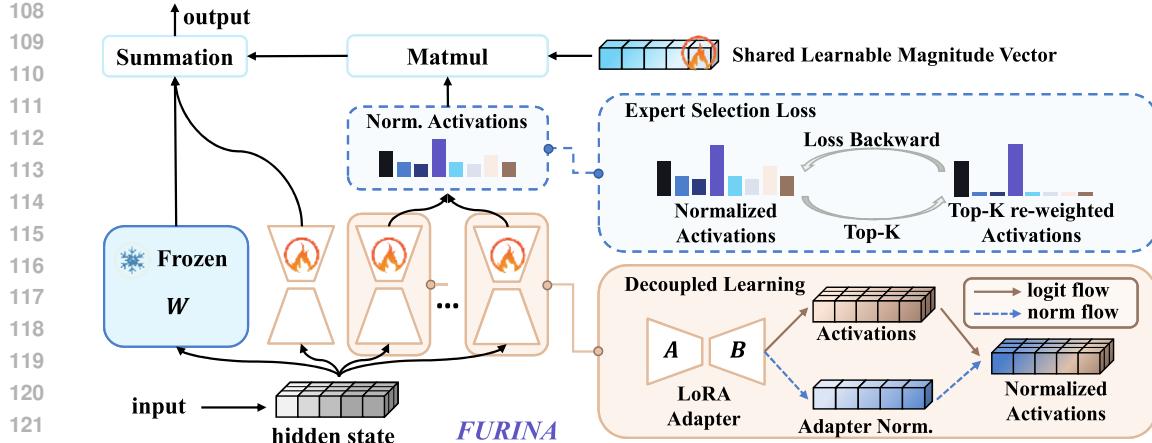


Figure 2: The overall framework of FURINA during training. LoRA weights are normalized, multiplied by the input hidden state, and scaled by a shared magnitude vector before aggregation with the backbone and shared expert outputs, while an expert selection loss sharpens activation sparsity.

Our contributions are threefold:

1. We propose FURINA, the first, to our knowledge, fully mergeable MoE–LoRA architecture. Unlike existing MoE–LoRA methods, FURINA can be seamlessly re-parameterized into a single LoRA adapter or directly into the backbone LLM after training. This ensures full compatibility with high-performance inference frameworks like vLLM and introduces **zero additional latency or complexity** during deployment.
2. We introduce a novel Self-Routing mechanism that eliminates the need for a discrete router via three key innovations: (a) decoupling the learning of direction and magnitude in adapters, (b) a shared magnitude vector for uniform activation scaling, and (c) an expert selection loss that promotes specialization. We also propose a shared expert to mitigate the diminished output norm, preserving model capacity.
3. We conduct extensive experiments on multiple LLMs and benchmarks. The results demonstrate that FURINA significantly enhances the performance of standard LoRA and achieves competitive or superior results compared to state-of-the-art non-mergeable MoE–LoRA methods, while eliminating the corresponding inference-time costs.

2 PRELIMINARIES

Low-Rank Adaption LoRA introduces trainable low-rank matrices to adapt large-scale pre-trained models efficiently. Typically applied to the multi-layer perceptron (MLP) layers, given a frozen weight matrix $W \in \mathbb{R}^{c_2 \times c_1}$ and an input hidden state $x \in \mathbb{R}^{c_1}$, the output of a LoRA-adapted layer is formulated as:

$$y = Wx + BAx, \quad (1)$$

where $A \in \mathbb{R}^{r \times c_1}$ and $B \in \mathbb{R}^{c_2 \times r}$ are learnable low-rank matrices with rank $r \ll \min(c_1, c_2)$.

MoE–LoRA The MoE paradigm has been integrated with LoRA to enhance model capacity while maintaining parameter efficiency. This approach initializes N distinct LoRA adapters $(B_i, A_i)_{i=1}^N$ and employs a router function ϕ to select a sparse combination of K experts for each input. The router is typically implemented as a trainable gating network. For input x , the routing weights are computed as

$$\phi(x)_i = \begin{cases} \frac{1}{Z} r(x)_i, & i \in \arg \max_i r(x)_i, \\ 0, & \text{otherwise,} \end{cases} \quad r(x) = \text{Softmax}(W_G x), \quad (2)$$

162 where $W_G \in \mathbb{R}^{N \times c_1}$ is a learnable projection matrix and Z is a normalization constant. The final
163 output becomes

$$164 \quad 165 \quad y = Wx + \sum_{i \in U}^N \phi(x)_i B_i A_i x, \quad U = \arg \max_j \text{topK}(r(x)_j), \quad (3)$$

166 while MoE-LoRA enhances model capacity, it introduces a critical limitation: the discrete routing
167 function $\phi(x)$ prevents the merging of adapters into the backbone model. The adapted weight
168 $W' = W + \sum_i \phi(x)_i B_i A_i$ varies with each input x , necessitating separate execution of router
169 and experts during inference. This significantly increases implementation complexity and computa-
170 tional overhead compared to standard LoRA. The proposed FURINA framework addresses this fun-
171 damental limitation while preserving the capacity benefits of MoE-LoRA, enabling both enhanced
172 expressivity and efficient deployment.

174 3 METHOD

175 3.1 OVERVIEW OF THE PROPOSED FURINA

176 As detailed in the previous section, the MoE-enhanced LoRA architecture employs a router to dy-
177 namically activate specific LoRA adapters. To emulate this behavior, the proposed FURINA in-
178 corporates three key components: (1) Decoupled learning of direction and magnitude for LoRA
179 adapters, (2) Shared learnable magnitude vector for uniform activation scaling, and (3) Expert se-
180 lection loss that encourages divergent activation like MoE. As depicted in Fig. 2, for each LoRA
181 adapter, we simultaneously calculate the activation of the input hidden state and the norm of the
182 adapter. The activations are multiplied by the reciprocal of the adapter norm for normalization. To
183 avoid the uniform activation, the expert selection loss maximizes the activation share of the top-K
184 LoRA experts. We also introduce shared experts to capture the shared knowledge across FURINA
185 experts. Without a router, the shared experts could be combined with the backbone.
186

187 3.2 MERGEABLE SELF-ROUTING MECHANISM

188 Mergeable Self-routing is the most important mechanism of FURINA, enabling the merging of
189 LoRA experts. Since W is frozen, we focus mainly on the adapted part of the output hidden state.
190 For simplicity, we denote the adapted part of the MLP layer for MoE-LoRA as follows:

$$191 \quad 192 \quad \Delta y = y - Wx = \sum_i \phi(x)_i B_i A_i x. \quad (4)$$

193 The simplest way to eliminate the routers is to assign the same weight to all experts:

$$194 \quad 195 \quad \Delta y = \sum_i^N B_i A_i x. \quad (5)$$

196 However, this formulation will degrade the MoE of LoRA to the naive LoRA, where $B' =$
197 (B_1, B_2, \dots, B_N) and $A' = (A_1^T, A_2^T, \dots, A_N^T)^T$.

198 **Decoupled Learning** To solve the aforementioned issue, we need to review the original MoE of
199 LoRA. Denote the full-size weight matrix of the i th LoRA adapter as $W_i = B_i A_i$, and its column-
200 wise norm could be calculated as follows:

$$201 \quad 202 \quad d_i = W_i e, \quad e = \underbrace{[1, 1, \dots, 1]^T}_{\times c_1}. \quad (6)$$

203 Denote $\hat{W}_i = \text{diag}(d_i + \epsilon)^{-1} W_i$, in which ϵ is a small positive number to prevent division of 0,
204 and diag expands a vector to the corresponding diagonal matrix. The formulation of MoE-LoRA in
205 Eq. 4 could be reformulated as follows:

$$206 \quad 207 \quad \Delta y = \sum_i^N \underbrace{\text{diag}(d_i + \epsilon)}_{\text{magnitude}} \cdot \underbrace{\phi(x)_i \hat{W}_i \hat{x}}_{\text{reweighted similarity}} \cdot \|x\|, \quad (7)$$

208 in which $\hat{x} = x / \|x\|$. Similarly, after eliminate the router, Eq. 5 could be reformulated as follows:

$$209 \quad 210 \quad \Delta y = \sum_i^N \underbrace{\text{diag}(d_i + \epsilon)}_{\text{magnitude}} \cdot \underbrace{\hat{W}_i \hat{x}}_{\text{similarity}} \cdot \|x\|. \quad (8)$$

Different from Eq. 7, eliminating the router creates a problematic coupling between the magnitude and similarity terms. The similarity calculation embeds the inverse of the magnitude without any independent operations. Consequently, when the similarity term is multiplied by the magnitude term, they cancel each other out due to the inverse relationship. This cancellation nullifies the intended effect entirely and disables measuring input-to-adapter similarity via the output norm. To address this, we decouple the magnitude and the similarity by replacing the magnitude with a learnable vector $v_i \in \mathcal{R}^{c_2}$, and approximate the MoE of LoRA adapters as follows:

$$\Delta y = \sum_i^N \text{diag}(v_i)(\hat{W}_i \hat{x} \|x\|) = \sum_i^N \text{diag}(v_i)(\hat{W}_i x). \quad (9)$$

Shared Magnitude Vector Although decoupled learning addresses the issue of magnitude-similarity cancellation, the following issue still exists: the output norm of an expert could be large even if the similarity term is small (indicating the expert should not be activated), when the magnitude v_i is large. To address this, we introduce a shared magnitude vector v for all experts. It is worth noting that we do not need to calculate the full-size \hat{W} and $\text{diag}(v)$ during training. The output could be further reformulated as follows:

$$\Delta y = \sum_i^N \text{diag}(v)(\hat{W}_i x) = \sum_{i=1}^N \left(v \otimes \frac{1}{d_i + \epsilon} \right) \otimes B_i A_i x, \quad (10)$$

in which \otimes represents element-wise multiplication. Note that \tilde{B}_i is only calculated once per batch.

Shared Experts Unlike standard MoE-LoRA methods that normalize routing weights to enforce exactly K active experts, cross-expert re-weighting is not applicable in our approach. In extreme cases, the output of the MoE may approach zero, thereby limiting its learning capacity. To mitigate this issue, we introduce a shared expert component to FURINA. Specifically, the incremental output Δy with shared expert is calculated as follows:

$$\Delta y = \sum_{i=1}^{N_{SE}} B_i A_i x + \sum_{j=N_{SE}+1}^N \text{diag}(v) \hat{W}_i x. \quad (11)$$

3.3 TRAINING OBJECTIVES

The training objective consists of two parts: (1) the supervised fine-tuning (SFT) loss, and (2) the expert selection loss. The SFT loss, similar to the prior approaches, is defined as follows:

$$\mathcal{L}_{\text{SFT}} = \frac{1}{|y|} \sum_t \text{CE}(f(y_t | x, y_{<t})), \quad (12)$$

in which CE represents the cross-entropy loss.

Expert Selection Loss We propose the expert selection loss to encourage the self-routing of LoRA adapters to approximate the function of routers. Specifically, denote the activations of the i_{th} LoRA adapter (apart from the shared experts, if any) as \mathbf{a}_i . To encourage divergent activation per token, we introduce the divergence loss as follows:

$$\mathcal{L}_{\text{div}} = -\log \left(\frac{\sum_{i \in \mathcal{S}} |\text{sum}(\mathbf{a}_i)|}{\sum_j |\text{sum}(\mathbf{a}_j)|} \right), \mathcal{S} = \arg \text{top}K \left(\left| \sum_j \mathbf{a}_{k,j} \right| \right), \mathbf{a}_i = \hat{W}_i x, \quad (13)$$

in which \mathcal{S} demonstrates the indices of the selected experts. Moreover, we also introduce the balance loss of expert selection. Specifically, given a batch of logits $\mathcal{X} \in \mathbb{R}^{B, T, N}$ in which B, T, N demonstrate the batch size, number of tokens per sample, and the total number of experts. First, we calculate the activation frequency of each expert:

$$\mathcal{F}_i = \left(\sum_u \sum_v \mathbb{I}(i \in \mathcal{S}_{u,v}) \right) / T, \mathcal{P}_i = \sum_u \sum_v |\mathcal{X}_{u,v,i}| / T, \quad (14)$$

in which $\mathcal{S}_{u,v}$ represents the selected expert of token $\mathcal{X}_{u,v,:}$, $\mathbb{I}(\cdot) = 1$ if the input condition is “True”, otherwise it equals to 0. Then the balance loss could be calculated as follows:

$$\mathcal{L}_{\text{bal}} = N \times \sum_i \mathcal{F}_i \mathcal{P}_i. \quad (15)$$

Denote it as \mathcal{L}_{bal} , given a certain batch of input, the expert selection loss \mathcal{L}_{sel} is defined as their summation. The overall training objective is defined as follows:

$$\mathcal{L} = \mathcal{L}_{\text{SFT}} + \alpha \mathcal{L}_{\text{sel}}, \quad \mathcal{L}_{\text{sel}} = \mathcal{L}_{\text{div}} + \mathcal{L}_{\text{bal}}, \quad (16)$$

in which α represents the loss coefficient, which is set to 0.01 in our work.

270 3.4 MERGING OF EXPERTS
271

272 During inference, unlike the standard MoE-LoRA approaches, the multiple LoRA adapters of the
273 proposed FURINA could be merged without loss of information. Without loss of generality, we
274 start from the full FURINA to merge all the experts into one LoRA adapter, and furthermore, into
275 the backbone network. First, we re-write Eq. 11 as follows:

$$\begin{aligned}
 \Delta y &= \sum_{i=1}^{N_{SE}} B_i A_i x + \sum_{j=N_{SE}+1}^N \text{diag}(v) \hat{W}_j x \\
 &= \sum_{i=1}^{N_{SE}} B_i A_i x + \sum_{j=N_{SE}+1}^N \text{diag}(v) \otimes \frac{1}{d_j + \epsilon} (B_j A_j x) \\
 &= \sum_{i=1}^{N_{SE}} B_i A_i x + \sum_{j=N_{SE}+1}^N \text{diag}\left(v \otimes \frac{1}{d_j + \epsilon}\right) B_j A_j x \\
 &= \sum_{i=1}^{N_{SE}} B_i A_i x + \sum_{j=N_{SE}+1}^N \tilde{B}_j A_j x.
 \end{aligned} \tag{17}$$

289 Then we could merge all these LoRA adapters and the backbone as follows:

$$\begin{cases} \mathbf{B} = (B_1, B_2, \dots, B_{N_{SE}}, \tilde{B}_{N_{SE}+1}, \dots, \tilde{B}_N), \\ \mathbf{A} = (A_1^T, A_2^T, \dots, A_N^T)^T. \end{cases} \tag{18}$$

293 Then the output could be formulated as:

$$y + \Delta y = Wx + \mathbf{B}\mathbf{A}x = (W + \mathbf{B}\mathbf{A})x. \tag{19}$$

296 For FURINA without shared experts, $N_{SE} = 0$, \mathbf{B} could be reformulated as follows:

$$\mathbf{B} = (\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_N). \tag{20}$$

300 4 EXPERIMENTS
301302 4.1 IMPLEMENTATION DETAILS
303

304 For each model, we leverage 7 benchmarks to evaluate the capacity of the PEFT methods, including
305 CSQA (Talmor et al. (2019)), HellaSwag (Zellers et al. (2019)), Winogrande (Sakaguchi et al.
306 (2021)), ARC-c and ARC-e (Clark et al. (2018)), OBQA (Mihaylov et al. (2018)), and BoolQ (Clark
307 et al. (2019)). Details of the benchmarks are provided in the Appendix. Three different LLMs are
308 included in our experiment: Qwen2-7B (Yang et al. (2024)), OpenChat-8B (Wang et al. (2024)),
309 and LLaMA3.2-3B, covering different architectures and model scales. Following the setting in
310 MixLoRA (Li et al. (2024)), the learning rate, the rank of each LoRA adapter r , the number of
311 experts N , and the number of activated experts K are set to 2×10^{-4} , 16, 8, and 2, respectively.
312 For our proposed FURINA method, we include one shared expert alongside the self-routed experts
313 (maintaining eight experts in total). All experiments are conducted on Nvidia GPUs.

314 4.2 COMPARISON WITH SOTA APPROACHES
315

316 We compare FURINA against state-of-the-art PEFT methods, including mergeable baselines
317 (LoRA, DoRA) and non-mergeable MoE-LoRA approaches (MixLoRA, LoRAMoE, MoLA). We
318 measure Time to First Token (TTFT) and latency using Llama3.2-3B. Mergeable methods (LoRA,
319 DoRA, FURINA) are evaluated using vLLM+EvalScope, while non-mergeable methods use MoE-
320 PEFT. For a fair comparison under equivalent parameter budgets, we set LoRA and DoRA rank to
321 128, matching the total rank of MoE methods. As shown in Tab. 2, FURINA significantly improves
322 upon standard LoRA (+4.8% average gain) and achieves competitive performance with MoE-LoRA
323 methods. Meanwhile, FURINA maintains inference latency equivalent to standard LoRA, with de-
tailed settings and results in Appendix A.3.

324 Table 2: Comparison with LoRA-styled PEFT approaches on downstream tasks. We employ **blue**
 325 and **bold** to indicate the best and second-best results for each model and the average performance.
 326 \dagger represents FURINA without shared experts. \ddagger represents that the method is not applicable to
 327 vLLM, thus evaluated on the MoE-PEFT framework. FURINA achieves competitive performance
 328 with standard MoE-LoRA methods with much less computational overhead.

Method	AVG	OpenChat-8B	Llama3.2-3B	Qwen2-7B	TTFT (ms, \downarrow)	Latency (ms, \downarrow)
Single Adapter LoRA						
LoRA	78.8	80.6	77.4	78.5	≈ 10	≈ 800
DoRA	77.1	79.0	77.9	74.3		
Standard MoE of LoRA						
MixLoRA	83.1	84.1	79.3	85.8	$\approx 550^\ddagger$	$\approx 9000^\ddagger$
LoRAMoE	83.5	86.1	79.1	85.3	$\approx 450^\ddagger$	$\approx 6000^\ddagger$
MoLA	83.4	84.3	80.3	85.6	$\approx 700^\ddagger$	$\approx 14500^\ddagger$
Fully Mergeable MoE of LoRA						
FURINA† (Ours)	81.1	86.1	73.9	83.3	≈ 10	≈ 800
FURINA (Ours)	83.6	85.3	79.7	85.9		

4.3 COMPATIBILITY WITH LORA VARIATIONS

We also evaluate the compatibility of FURINA with existing LoRA variations, including LoRA+ (Hayou et al. (2024)) and rsLoRA (Kalajdzievski (2023)). For LoRA+, we maintain a learning rate ratio of 5.0 between matrices B and A . As shown in Tab. 3, FURINA is consistently compatible with these variants, achieving superior performance compared to standard MoE-LoRA approaches.

Table 3: Compatibility of FURINA with LoRA variations

Method	OpenChat-8B	Llama3.2-3B	Qwen2-7B	AVG
LoRA	80.6	77.4	78.5	78.8
LoRAMoE	86.1	79.1	85.3	83.5
SLIM	87.4	79.2	85.9	84.2
FURINA	85.3	79.7	85.9	83.6
FURINA w/ LoRA+	87.2	81.3	86.9	85.1
FURINA w/ rsLoRA	87.6	81.2	86.9	85.2

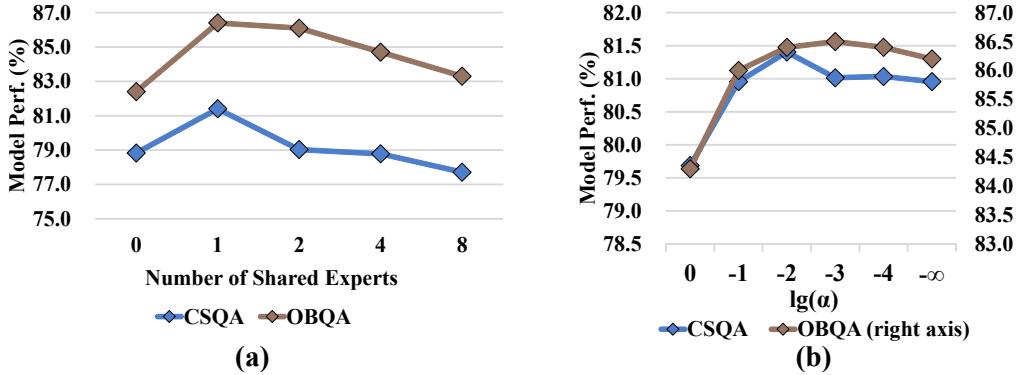
4.4 ABLATION STUDY

Ablations on the Main Modules We conduct the ablation study on the main modules proposed in our work. We include Qwen2-7B and LLaMA3.2-3B with all benchmarks, apart from the HellaSwag (because of its size), in the main ablation study. “Decoupled Learning” demonstrates the column normalization of the LoRA adapters, and “Shared Magnitude Vector” represents the introduction of the shared magnitude vector v after normalization. The result shown in Tab. 4 demonstrates that normalizing the LoRA adapters significantly improves the model performance by 3.4%, representing the importance of balanced effectiveness of each expert. The introduction of shared experts and shared magnitude vector also boosts the model performance by 1.4%, demonstrating the benefit of learning mutual foundation knowledge. Further mimicking the activation pattern of standard MoE by \mathcal{L}_{sel} boosts the model by 0.3%, indicating the importance of emulating the top-K pattern of MoE.

Effect of Number of Shared Experts We conduct an experiment on the OBQA and CSQA datasets to validate the influence of the number of shared experts. We average the performance of

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379
400 Table 4: Ablation study of the main modules. The performance (Perf.) is the average of Qwen2-7B
401 and LLaMA-3.2-3B.
402

Decoupled Learning	Shared Experts	Shared Magnitude Vector	\mathcal{L}_{sel}	Perf (%)
✗	✗	✗	✗	75.9
✓	✗	✗	✗	79.3
✓	✓	✗	✗	80.2
✓	✓	✓	✗	80.7
✓	✓	✓	✓	81.0

401
402 Figure 3: The effect of the number of shared experts (a) and the scale of the loss coefficient α (b)
403404
405 LLaMA-3.2-3B and Qwen2-7B. The result is demonstrated in Fig. 3(a). Validation across different
406 LLMs demonstrates that setting the number of shared experts to 1 is sufficient. Further increasing it
407 may result in model performance degradation and convergence to the original LoRA approach.
408409
410 **Effect of Loss Coefficient** We also conduct an experiment to validate the influence of the loss
411 coefficient α . The result is demonstrated in Fig. 3(b). A large α results in over-focus on the imitation
412 of the expert selection patterns of MoE models and limits the learning capacity of FURINA. On the
413 contrary, a minor α cannot guide the LoRA adapters of the MoE to mimic the selective expert
414 activation pattern of standard MoE architectures. Experiments demonstrate that setting α to the
415 range of $[1e - 2, 1e - 3]$ could achieve an optimal trade-off for these two factors.
416417
418 **Comparison of Shared Magnitude Vector with Decoupling LoRA Adapters** To validate the
419 utilization of the shared magnitude vector instead of simply decouple the direction and magni-
420 tude of the LoRA adapters, we conduct an experiment on the OBQA and CSQA datasets and the
421 LLaMA3.2-3B model. The results are shown in Tab. 5. Utilizing the shared magnitude vector
422 achieves higher performance with fewer trainable parameters. This may be because, although uti-
423 lizing different magnitude vectors for the adapters increases the number of trainable parameters, it
424 results in an imbalance in the activation of the adapters.
425426 Table 5: Comparison of shared magnitude vector with decoupling of LoRA direction and magnitude
427

Method	OBQA	CSQA	AVG
Decoupling	82.4	78.5	80.4
Shared Mag. Vec. (Ours)	84.0	78.5	81.2

428
429 **Evaluation on GSM8K and HumanEval Datasets** To validate the model capacity on reasoning
430 tasks, we also conduct an experiment on the GSM8K (Cobbe et al. (2021)) and HumanEval (Chen
431 et al. (2021)) datasets. We keep the hyperparameters as in the main experiments, and all models
432

432 are fine-tuned for 1k steps. For the HumanEval dataset, the model is trained on the CodeAlpaca
 433 (Chaudhary (2023)) dataset, and the metric is set to Pass@1. The Evalscope framework is adopted
 434 for evaluation. The results in Tab. 6 demonstrate that, compared to LoRA, the proposed FURINA
 435 could significantly boost the model performance, indicating its generalizability.
 436

437 Table 6: Comparison of LoRA and FURINA on GSM8K and HumanEval datasets

Method	OpenChat-8B	Llama-3.2-3B	Qwen2-7B
LoRA	61.1	47.8	62.5
FURINA (Ours)	65.1	53.4	65.7

444 5 RELATED WORKS

445 **LoRA-style PEFT** LoRA (Hu et al. (2022)) has emerged as a prominent method for PEFT of
 446 large language models. Unlike prompt tuning or adapter-based approaches, LoRA’s key advantage
 447 lies in its mergeability: after training, the low-rank adapters can be consolidated into the original pre-
 448 trained weights, introducing zero additional inference latency. DoRA (Liu et al. (2024b)) decouples
 449 weight updates into magnitude and direction components, while rsLoRA (Kalajdzievski (2023))
 450 introduces a scaling mechanism to improve training stability. Hayou et al. (2024) identifies an
 451 imbalance in learning dynamics between the A and B matrices and proposes differentiated learning
 452 rates to address it. Recently, several works have integrated mixture-of-experts (MoE) architectures
 453 with LoRA-style PEFT. MixLoRA (Li et al. (2024)) incorporates the sparse MoE structure from
 454 Mixtral into LoRA, and MoLA (Gao et al. (2025)) advocates for layer-wise expert configurations
 455 with dynamic expert counts. SLIM (Han et al. (2025)) further enhances this approach by blending
 456 identity mappings with LoRA adapters to better preserve pre-trained knowledge. A fundamental
 457 limitation of these MoE-LoRA methods, however, is their inability to merge experts into the base
 458 model post-training, resulting in persistent inference overhead and increased deployment complexity
 459 compared to standard LoRA.
 460

461 **Mixture of Experts Architecture** MoE has gained significant traction for scaling large language
 462 models efficiently. Mixtral 8×7B (Jiang et al. (2024)) stands as the first widely adopted open-source
 463 MoE LLM, demonstrating that sparse expert activation can substantially improve parameter effi-
 464 ciency without compromising performance. DeepSeek MoE (Dai et al. (2024)) introduces shared
 465 experts that remain active across all inputs to capture common knowledge, thereby enhancing the
 466 model’s representational capacity. DeepSeek-V3 (Liu et al. (2024a)) further increases the number
 467 of shared experts and incorporates a loss-free balancing mechanism to improve training stability.
 468 Recent innovations in MoE architectures also explore MoE frameworks for improved computational
 469 efficiency. Jin et al. (2025) proposes incorporating non-computational experts—such as identity
 470 or constant-output layers—into the MoE framework, reducing inference costs while maintaining
 471 model performance. Lv et al. (2025) proposes to remove the routers from MoE, but still retains the
 472 non-linear cross-expert operations that prevent merging of experts.
 473

474 6 CONCLUSION

475 In this work, we propose FURINA, a novel fully mergeable MoE-LoRA framework that overcomes
 476 the deployment complexity and overhead of the standard MoE-LoRA while matching and even
 477 surpassing their learning capacity. FURINA introduces a Mergeable Self-Routing mechanism with
 478 three key components: (1) Decoupled learning of adapter direction and magnitude, (2) Shared mag-
 479 nitude vector for uniform scaling, and (3) Expert selection loss that promotes sparse expert acti-
 480 vation. These elements collectively ensure that each expert’s output norm reflects its relevance to
 481 the input, enabling mergeable routing. We also incorporate shared experts within the MoE-LoRA
 482 block that provides essential foundational knowledge so that the other experts can focus on spe-
 483 cific tasks. Comprehensive experiments on 9 datasets and 3 LLMs demonstrate that FURINA sig-
 484 nificantly enhances standard LoRA performance while achieving competitive results compared to
 485 SOTA MoE-LoRA methods, while overcoming their complexity and overhead during deployment,
 486 and plug-and-play with mainstream deployment frameworks.
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REPRODUCIBILITY STATEMENT

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We have included the necessary information to reproduce the results reported in our manuscript. The code base is attached to this manuscript in the supplementary material, and we plan to open-source the code and checkpoints upon publication.

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608 A APPENDIX

609 A.1 DETAILS OF THE INVOLVED DATASETS

610 611 612 613 Table 7: Summary of datasets used in the experiments.

614 615 Dataset	616 Task Type	617 Train	618 Dev	619 Test
620 OpenBookQA (OBQA)	Multiple-choice QA	4,957	500	500
621 BoolQ	Binary QA	9,400	3,200	3,200
622 HellaSwag	Sentence completion	40,000	—	10,000
623 WinoGrande (Debiased)	Commonsense reasoning	9,248	1,267	1,767
624 CommonsenseQA (CSQA)	Commonsense QA	9,798	1,224	1,225
625 ARC-c	Commonsense reasoning	1,418	—	1,172
626 ARC-e	Commonsense reasoning	2,821	—	2,376
627 GSM8K	Mathematics	7,200	—	1,300
628 HumanEval	Code generation	—	—	164
629 Code Alpaca	Code generation	20,000	—	—

630 All the mentioned datasets are open-sourced and allow academic use. We report results for the test
 631 set when the ground truth is available. Otherwise, we use the dev set.

632 A.2 COMPARISON ON HOLD OUT DATASETS

633 We compare the catastrophic forgetting resulting from LoRA and FURINA. N/A represents the
 634 original model. We fine-tune the Llama3.2-3B model with different PEFT approaches on the OBQA
 635 dataset, and evaluate on the hold-out datasets, GSM8K and MMLU. The result demonstrates that,
 compared with LoRA, FURINA could significantly mitigate catastrophic forgetting. Note that the
 “Rel. Perf. Drop” is calculated as follows:

$$636 \text{Rel. Perf. Drop} = \frac{1}{N} \sum_{i=1}^N \frac{\text{ACC}_i^0 - \text{ACC}_i}{\text{ACC}_i^0}, \quad (21)$$

637 in which ACC_i^0 and ACC_i represent the performance of the original and the fine-tuned model on
 638 task i , respectively.

642 A.3 INFERENCE TIME OF PEFT APPROACHES

644 To evaluate the inference latency of different PEFT methods, we conduct a controlled comparison
 645 using a fixed sequence length of 100 input and 100 output tokens. Time to First Token (TTFT)
 646 measures the latency until the first output token is generated, while total latency refers to the time
 647 cost for generating the entire output sequence. The evaluation is performed across two distinct
 frameworks to assess both unmerged and merged deployment scenarios:

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Table 8: Comparison on out-of-domain dataset. The model is trained on the OBQA dataset.

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Method	GSM8K(%)	MMLU(%)	Rel. Perf. Drop (%↓)
N/A	74.7	55.8	N/A
LoRA	60.4 (14.3↓)	49.7 (6.1↓)	10.1
FURINA	74.5 (0.2↓)	52.6 (3.1↓)	2.0
FURINA w/ rsLoRA	73.5 (1.2↓)	50.6 (5.2↓)	3.7
FURINA w/ LoRA+	73.5 (1.2↓)	51.5 (4.3↓)	3.1

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1. **MoE-PEFT Framework:** This framework is used to evaluate standard MoE-LoRA approaches directly. For a fair comparison, single-adapter LoRA methods are also evaluated in this framework without merging the adapters into the backbone model. Our proposed FURINA method is merged into the standard LoRA architecture under this framework.
2. **vLLM Framework:** This setup represents a production-ready deployment environment. In this scenario, adapters are merged into the backbone model prior to inference, which is only feasible for mergeable PEFT approaches, such as vanilla LoRA and our FURINA. The router-based MoE-LoRA baselines cannot be evaluated in this setting as they are fundamentally unmergeable due to their routing mechanisms.

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All prompts and hyperparameters are aligned across the compared approaches. To account for potential variance in time measurements, each test is repeated 5 times per model, with reported results representing the averaged values. Our results in Tab. 9 demonstrate that FURINA achieves comparable inference latency to standard LoRA while significantly outperforming conventional MoE-LoRA methods. This confirms that FURINA maintains the efficiency benefits of mergeable PEFT methods while delivering the performance advantages of MoE approaches.

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677Table 9: Inference time of different PEFT approaches on different implementation frameworks. [†] represents FURINA without shared experts.

Method	MoE-PEFT		vLLM	
	TTFT(ms)	Latency(ms)	TTFT(ms)	Latency(ms)
Single Adapter LoRA				
LoRA	≈350	≈3500	≈10	≈800
DoRA	≈400	≈9000	≈10	≈800
Router-based MoE of LoRA				
MixLoRA	≈550	≈9000	N/A	N/A
LoRAMoE	≈450	≈6000	N/A	N/A
MoLA	≈700	≈14500	N/A	N/A
SLIM	≈650	≈17000	N/A	N/A
Router-free MoE of LoRA				
FURINA [†] (Ours)	≈350	≈3500	≈10	≈800
FURINA (Ours)	≈350	≈3500	≈10	≈800

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A.4 COMPARISON WITH DIRECTLY ELIMINATING THE ROUTERS OF STANDARD MOE-LORA

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To evaluate the effectiveness of the proposed FURINA, we also compare FURINA with directly eliminating the routers during inference. We utilize SLIM as the baseline and merge the adapters as:

$$\mathbf{B} = \frac{1}{N}(B_1, B_2, \dots, B_N), \mathbf{A} = (A_1^T, A_2^T, \dots, A_N^T)^T. \quad (22)$$

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We utilize $\frac{1}{N}$ to synthesize the re-weighting operation of the routers. The experiment is conducted on Llama3.2-3B. The results are demonstrated in Tab. 10. Directly eliminating the routers from the MoE-LoRA could significantly decrease its performance, even worse than the original model.

702 Table 10: Effect of directly eliminating routers from MoE-LoRA approaches during inference
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Method	Original Model	SLIM	SLIM w/ merge	FURINA
Perf. (%)	67.5	79.2	37.4 (41.8 ↓)	79.7

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708 **A.5 ACTIVATION PATTERN OF LORA AND FURINA**
709710 In Fig. 4, we also compare the activation pattern of FURINA and LoRA. Note that we compare the
711 normalized activation of both approaches on OBQA dataset, with the Llama-3.2-3B model. The
712 result demonstrates that, compared with LoRA, FURINA achieves significantly larger normalized
713 activations, indicating that the proposed approach could effectively increase the angular similarity
714 of input and LoRA weights.715
716 **A.6 HYPER-PARAMETERS OF EVALSCOPE.**717 To validate the fine-tuned models on GSM8K and HumanEval datasets with the EvalScope frame-
718 work in the main paper, the hyper-parameters are set as in Tab. 11.
719720 Table 11: Hyper-parameters for evaluating GSM8K and HumanEval
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#max tokens	temperature	# shots
2048	0.0	0

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723 **A.7 USE OF LLM**
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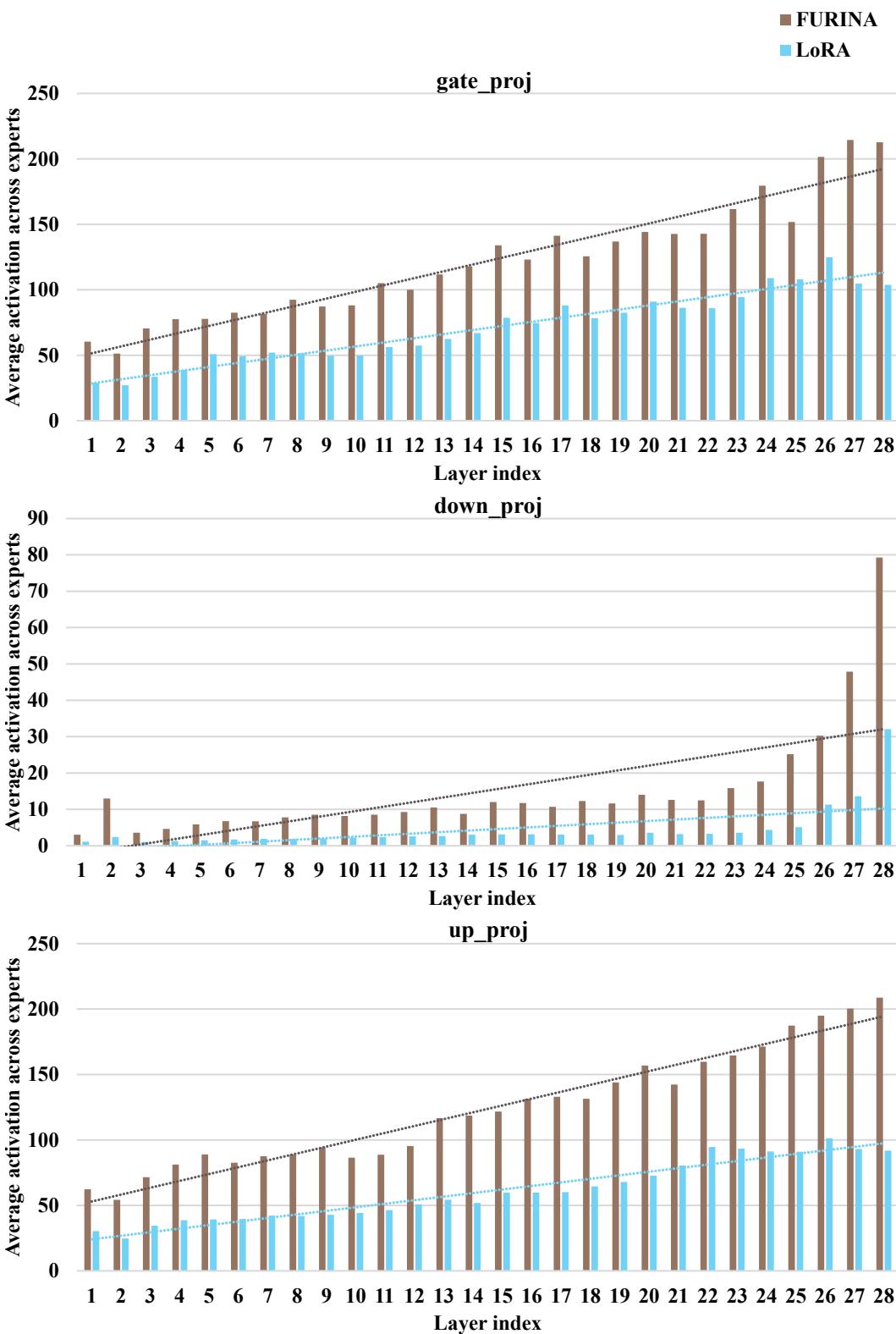


Figure 4: Comparison of activation pattern of FURINA and LoRA across layers