

# I&I ADAPTIVE FORCE-MOTION CONTROL FOR ARM MANIPULATION

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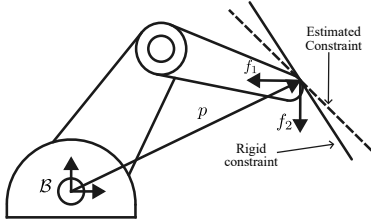
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## Introduction

We propose an adaptive force-motion controller for 2-DoF robot arm which is constrained to move in a line in  $\mathbb{R}^2$  of unknown orientation. Our proposed design is based on adaptive immersion and invariance (I&I) control theory. In particular, we adapt to the unknown geometry of the motion constraint. The design is verified through numerical simulation and compared with a traditional hybrid force-motion controller.

## Methods

Consider the 2-DoF arm with base fixed to the origin of the task frame  $\mathcal{B}$  (Fig. 1). The end-effector of the arm stays in contact with a frictionless linear constraint.



**Figure 1:** Constrained 2-DoF Arm Manipulator.

The task space dynamics with normalized mass, and the linear constraint are

$$\ddot{p} = u + f \quad (1)$$

$$\phi(p) = p_1 + Ap_2 + b = 0, \quad A > 0 \quad (2)$$

where  $p \in \mathbb{R}^2$  is the end-effector position,  $A$  is the constraint parameter,  $f \in \mathbb{R}^2$  is the force due to the constraint,  $b$  is a constant, and  $u$  is the force that the robot exerts on the end-effector. The coordinate transformation  $x = [\phi(p) \ p_2]^T$  decouples motion and force control of the end-effector, allowing us to independently track position and force trajectories  $x_{2d}, f_{1d}$ . We have

$$-A\ddot{x}_2 = u_1 + f_1 \quad (3a)$$

$$(A^2 + 1)\ddot{x}_2 = -Au_1 + u_2 \quad (3b)$$

With an auxiliary input  $s = \ddot{x}_{2d} + k_v\dot{e}_2 + k_pe_2 + k_ie_{2i}$ , where  $e_2 = x_{2d} - x_2$ ,  $e_{2i} = \int_0^t e_2 d\sigma$ ,  $k_p, k_v, k_i > 0$  are constant, we define a traditional hybrid force-motion control (HMFC)

$$u = [-As - f_{1d} \quad s - Af_{1d}]^T \quad (4)$$

Substituting (4) into (3) leads to a stable closed-loop dynamics

$$A(\ddot{e}_2 + k_v\dot{e}_2 + k_pe_2 + k_ie_{2i}) = -e_{f1} \quad (5)$$

$$(A^2 + 1)(\ddot{e}_2 + k_v\dot{e}_2 + k_pe_2 + k_ie_{2i}) = 0 \quad (6)$$

When using an inaccurate parameter  $\bar{A}$ , an error in this parameter can lead to an unstable closed-loop:

$$A(\ddot{e}_2 + k_v\dot{e}_2 + k_pe_2 + k_ie_{2i}) = -e_{f1} - sz$$

$$(A^2 + 1)(\ddot{e}_2 + k_v\dot{e}_2 + k_pe_2 + k_ie_{2i}) = -(As - f_{1d})z$$

where  $e_{f1} = f_{1d} - f_1$ ,  $z$  is the parameter error. With an function  $\beta$  to be determined and  $\hat{A}$  to estimate, define

$$z = \bar{A} - A = \hat{A} - A + \beta(e_{2i}, e_2, \dot{e}_2) \quad (7)$$

In I&I theory, if we can stabilize  $z$ -subsystem about 0 by designing  $\hat{A}$  and  $\beta$ , then the  $(z, x)$  system is rendered stable. Using this idea, a rigorous Lyapunov function design leads to the adaptive state feedback

$$\dot{\hat{A}} = -k_p\dot{x}_2^2 + s(s - \ddot{x}_{2d}) + f_{1d}e_{f1} \quad (8a)$$

$$\beta = \dot{e}_2\ddot{x}_{2d} + k_pe_2\dot{e}_2 + k_v/2\dot{e}_2^2 + k_ie_{2i}\dot{e}_2 \quad (8b)$$

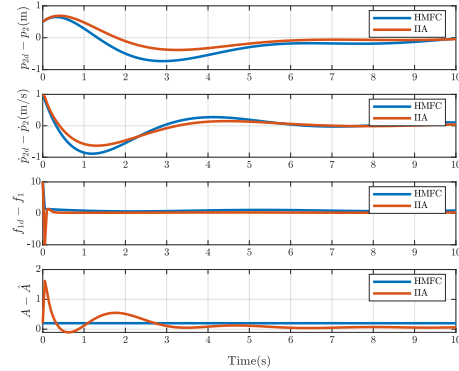
$$u_1 = -(\hat{A} + \beta)s - f_{1d} \quad (8c)$$

$$u_2 = s - (\hat{A} + \beta)f_{1d} \quad (8d)$$

This leads to the closed-loop (1), (8) has a global exponential stable equilibrium  $(x_2, \hat{A}) = (0, A)$ ,  $\lim_{t \rightarrow \infty} e_{f1}(t) = 0$ , and  $\lim_{t \rightarrow \infty} e_2(t) = 0$ .

## Results and Discussion

Figure 2 shows a comparison between the traditional HMFC (4) and our approach under noisy force measurement. Both controllers are initialized with an incorrect value of  $A$ . Our approach achieves exponential convergence of  $e_2$  and drives the force tracking error to a smaller neighborhood of 0 than the traditional HMFC.



**Figure 2:** Setpoint tracking errors of ordinary and I&I adaptive force motion controller.

## Conclusion

A novel I&I adaptive hybrid force-motion controller is developed and verified for arm manipulation. The method identifies the unknown linear constraint parameter under a noisy force measurement. Numerical simulation verified the controller stability and shows a better force tracking performance for our approach over its non-adaptive counterpart.

## References

- [1] N.H. McClamroch et al. IEEE Trans. Automat. Contr. 33(5):419–426, 1988.
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