POD-KAN-NO: a physically interpretable neural operator

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Abstract

POD-KAN-NO is a novel neural operator framework that combines the interpretability of modal decomposition with the expressive power of modern neural networks. By integrating Proper Orthogonal Decomposition (POD) with Kolmogorov–Arnold Networks (KAN), our method facilitates transparent and physically interpretable spatial reconstruction, while preserving strong nonlinear representation capabilities. Compared to traditional empirical models and black-box neural operators, POD-KAN-NO offers a promising balance between interpretability and accuracy. Preliminary results show promising performance in tasks such as spatial reconstruction, highlighting the framework's capacity to integrate interpretability with nonlinear modeling flexibility.

1. Introduction

Neural operators have transformed data-driven modeling of physical systems by enabling flexible and accurate learning of complex mappings between function spaces. However, most existing architectures—such as the Fourier Neural Operator (FNO) (Li et al., 2021)—remain fundamentally opaque to human interpretation. This black-box nature limits their reliability and practical adoption, particularly in safety-critical or scientific applications.

In contrast, traditional empirical models and modal decomposition techniques like Proper Orthogonal Decomposition (POD) offer explicit mathematical structures and physically interpretable modes, yet they struggle to capture nonlinear phenomena. This dichotomy raises a key question: Can we design a neural operator that preserves the interpretability of classical methods while leveraging the representational power of deep learning?

To address this challenge, we propose POD-KAN-NO, a hybrid neural operator that integrates the modal transparency of POD with the expressive nonlinearity of Kolmogorov–Arnold Networks (KAN). POD-KAN-NO bridges the gap between physically grounded understanding and the powerful learning capabilities of modern neural operators.

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2. Methodology: POD-KAN-NO Architecture

Classical neural operators are composed of multiple functional layers, as illustrated in Figure 1. While architectures such as the Fourier Neural Operator (FNO) and its variants enable powerful end-to-end learning of solution operators, they often do so at the expense of physical interpretability.

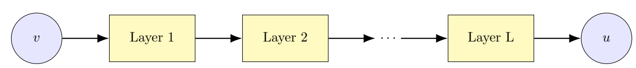

Figure 1. neural operator layer

Modal decomposition methods, including POD, have been widely used to extract dominant physical patterns and achieve dimensionality reduction in complex systems (Zhou et al., 2021). In a typical workflow, given data from a physical system, POD is applied to obtain a set of orthonormal basis functions $\phi_k(\mathbf{x})_{k=1}^{r}$, where r denotes the truncation rank. The approximate solution $\hat{u}(\mathbf{x}, t)$ is then represented as:

$$\hat{u}(\mathbf{x}, t) = \sum_{k=1}^{r} a_k(t)\phi_k(\mathbf{x}),$$

While modal decomposition methods provide interpretable basis functions and enable efficient inference, their inherent linearity significantly limits their ability to model nonlinear mappings or generalize across varying physical conditions.

Recent advances, such as KAN(Liu et al., 2025), offer a promising alternative to MLPs by enabling structured and efficient modeling of highly nonlinear mappings. This architecture complements POD by enhancing the nonlinear representational power within interpretable modal frameworks. The formulation of KAN is given by:

$$y_i = \sum_j c_{i,j} B_j(x_i)$$

Here, $c_{i,j}$ denotes the weight, $B_j(x_i)$ denotes the j-th basis function (e.g., B-spline) applied to input dimension x_i.

Building on the above theoretical foundations, POD-KAN-NO bridges the gap between interpretability and nonlinear expressiveness by coupling a truncated POD expansion with a KAN-based residual learner. The POD basis captures dominant linear modes, while KAN corrects the approximation by modeling nonlinear discrepancies.

This separation ensures that spatial structures remain physically interpretable through POD modes, while nonlinear corrections are captured by the KAN component. The overall architecture is illustrated in Figure 2. The ”Linear POD” block is precomputed from a representative dataset, producing interpretable modal bases. The ”Nonlinear KAN” block learns the residual component through explicit functional decomposition. The final field is reconstructed by combining the POD approximation and the learned residual, achieving a balance between physical insight and modeling flexibility.

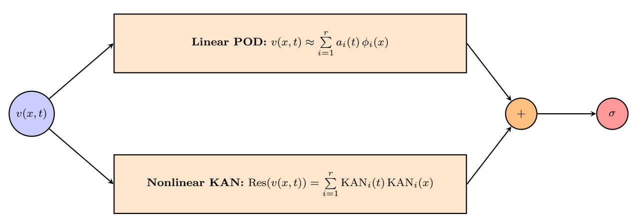

Figure 2. POD-KAN-NO layer

The ”Nonlinear KAN” component shown in Figure 2 adopts the KAN-aϕ strategy. Alternative formulations of the nonlinear modeling strategy are summarized in Table 1, highlighting the flexibility of the POD-KAN-NO framework.

Table 1. Different Nonlinear KAN modeling strategies.

Model Name	**Mathematical Expression**	**Description**
KAN-a	$\sum_i \mathrm{KAN}_i(t)\,\phi_i(x)$	Learn temporal coefficients
KAN-ϕ	$\sum_i a_i(t)\,\mathrm{KAN}_i(x)$	Learn spatial modes
KAN-aϕ	$\sum_i \mathrm{KAN}_{ai}(t)\,\mathrm{KAN}_{\phi i}(x)$	Learn both temporal and spatial components
KAN-Direct	$\mathrm{KAN}(x,t)$	Fully end-to-end regression model

3. Results

POD-KAN-NO demonstrates strong potential across a range of tasks, including time-series prediction and spatial reconstruction. Figure 3 presents the results for flow field reconstruction from sparse measurements, where the KAN-Direct modeling strategy is employed. Compared to linear POD, which tends to smooth out high-gradient regions and underestimate localized structures, the proposed method effectively restores fine-scale details. Notably, the POD-KAN reconstruction recovers sharper wake regions. These results suggest that POD-KAN-NO enhances linear reconstructions with data-driven nonlinear corrections, effectively bridging the gap between physical interpretability and expressive modeling capacity.

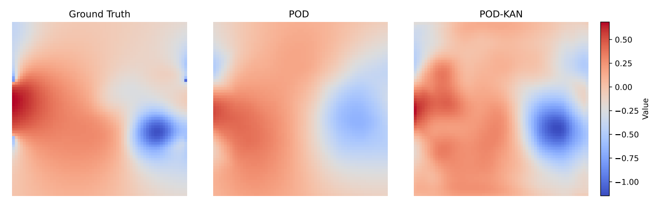

Figure 3. Flow field reconstruction using POD-KAN-NO

POD-KAN-NO maintains interpretability through physically meaningful modal components, while the KAN module captures complex nonlinear relationships. Leveraging physical structure, the framework also generalizes well with limited data.

4. Conclusion and Future Work

POD-KAN-NO advances the frontier of operator learning by integrating interpretable modal decomposition with expressive nonlinear modeling. We believe this paradigm offers a promising step toward interpretable AI for scientific applications. Future work will involve comprehensive empirical evaluations and ablation studies to further understand the impact of architectural choices.

Impact Statement

This work contributes to building transparent and reliable neural operators for scientific applications. It may benefit domains such as fluid mechanics, where interpretability and data efficiency are crucial. No foreseeable negative societal impacts are associated with this research.

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