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Inferring missing edges from Cartesian products of graphs.

Keywords: Cartesian Products, Edge Inference, Network Algorithms

Extended Abstract

In large, designed networks, it is normal to have multiple sets of networks components which all follow a nearly identical design [1]. This use of repetition makes network operations vastly simpler and has been observed in real-world networks [1]. Because of this, the Cartesian products of graphs are a useful tool for studying the topology of designed networks. We can leverage this Cartesian product-based model to help us infer the existence of missing edges in networks when we are unable to fully observe them.

Our work presents an empirical study of edge reconstruction methods for noisy measurements of Cartesian products. In the literature, this problem has often been tackled from a graph theoretic perspective, typically focussing on cases where only a single edge is missing and the original graph can be reconstructed exactly [2]. In our study we look at the problem of inferring multiple missing from a Cartesian product graph. We show that when multiple edges are removed, it is not always possible to reconstruct the original graph.

In our study, we analyse several algorithms designed to infer missing edges in Cartesian products [2–5]. We will be comparing these algorithms in terms of reliability, and efficiency as well as a broader study into the problem of edge reconstruction of Cartesian product graphs. We intend to infer the probability with which a Cartesian product graph can be uniquely reconstructed. How many edges are required for the graph to no longer be recoverable? How does this probability change conditioned on the relative sizes of factors?

Our research also presents a comparison of the accuracy and efficiency of other algorithms designed for edge reconstruction of Cartesian products. There is currently very little in the literature about how feasible it is to infer missing edges in large Cartesian products. Preliminary investigations have shown that there is a combinatorial explosion present making some methods of edge reconstruction infeasibly slow. Our research presents an empirical analysis of the problem's complexity using different algorithms. It is well-known that the problem of inferring missing edges in a Cartesian product is a special case of a known NP-hard problem [6], we investigate the potential for heuristics and improvements to existing algorithms that can increase the speed of finding missing edges.

In the future we intend to study extend our research in consider the zig-zag product which is used frequently in the design of data centers.

This research used publicly available data, and no ethical considerations beyond adherence to data use policies were needed.

References

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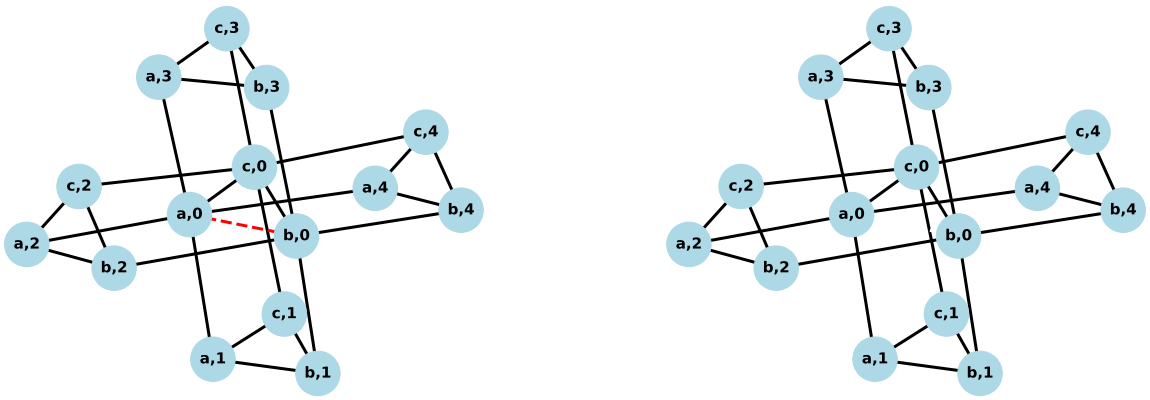

Figure 1: An example of a Cartesian product graph G with an edge e highlighted (a), and the same graph G with e removed.