# Date Fragments: A Hidden Bottleneck of Tokenization for Temporal Reasoning

Anonymous ACL submission

#### Abstract

Modern BPE tokenizers often split calendar dates into meaningless fragments, e.g., "20250312" → "202", "503", "12", inflating token counts and obscuring the inherent structure needed for robust temporal reasoning. In this work, we (1) introduce a simple yet interpretable metric, termed date fragmentation ratio, that measures how faithfully a tokenizer preserves multi-digit date components; (2) release DATEAUGBENCH, a suite of 6500 examples spanning three temporal reasoning tasks: context-based date resolution, formatinvariance puzzles, and date arithmetic across historical, contemporary, and future regimes; and (3) through layer-wise probing and causal attention-hop analyses, uncover an emergent date-abstraction mechanism whereby large language models sequentially assemble the fragments of month, day, and year components into a unified "date" concept. Our experiments show that excessive fragmentation correlates with accuracy drops of up to 10 points on uncommon dates like historical and futuristic dates. Further, we find that the larger the model, the more quickly the emergent date abstraction that heals date fragments is accomplished. Lastly, we observe a reasoning path that LLMs follow to interpret dates, relying on subword fragments that statistically represent year, month and day, and stitch these fragments in a flexible order that is subject to date formats.

## 1 Introduction

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Understanding and manipulating dates is a deceptively complex challenge for modern large language models (LLMs). Unlike ordinary words, dates combine numeric and lexical elements in rigidly defined patterns—ranging from compact eight-digit strings such as 20250314 to more verbose forms like "March 14, 2025" or locale-specific variants such as "14/03/2025." Yet despite their structured nature, these date expressions often fall prey to subword tokenizers that fragment them into semantically meaningless pieces. A tokenizer that



Figure 1: Internal processing of dates for temporal reasoning. Here F=0.4 shows the date fragmentation ratio.

splits "2025-03-14" into "20", "25", "-0", "3", "-1", "4" not only inflates the token count but also severs the natural boundaries of year, month, and day. This fragmentation obscures temporal cues and introduces a hidden bottleneck: even state-of-the-art LLMs struggle to resolve, compare, or compute dates accurately when their internal representations have been so badly fragmented. This issue is critical for real-world applications:

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Mis-tokenized dates can undermine scheduling and planning workflows, leading to erroneous calendar invites or appointments (Vasileiou and Yeoh, 2024). They can skew forecasting models in domains ranging from time-series analysis (Tan et al., 2024; Chang et al., 2023) to temporal knowledge graph reasoning (Wang et al., 2024). In digital humanities and historical scholarship, incorrect splitting of date expressions may corrupt timelines and misguide interpretative analyses (Zeng, 2024). As LLMs are increasingly deployed in cross-temporal applications, such as climate projection(Wang and Karimi, 2024), economic forecasting (Carriero et al., 2024; Bhatia et al., 2024), and automated curriculum scheduling (Vasileiou and Yeoh, 2024), the brittleness introduced by subword fragmentation poses a risk of propagating temporal biases and inaccuracies into downstream scientific discoveries and decision-making systems (Tan et al., 2024).

In this work, we provide a pioneer outlook on the impact of date tokenization on downstream tem-

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higher compression fidelity predicts better down-

flexibility for future models.

**Related Works** 

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Tokenisation as an information bottleneck.

cent scholarship interrogates four complementary

facets of sub-word segmentation: (i) tokenisation

fidelity, i.e. how closely a tokenizer preserves se-

mantic units: Large empirical studies show that

stream accuracy in symbol-heavy domains such as code, maths and dates (Goldman et al., 2024;

poral reasoning. Figure 1 illustrates how dates are

processed internally for temporal reasoning. Our

(i) We introduce DATEAUGBENCH, a benchmark

dataset comprising 6,500 examples with 21

date formats. It is leveraged to evaluate a

diverse array of LLMs from 8 model families

that measures how fragmented the tokeniza-

tion outcome is compared to the actual year,

month, and day components. We find that

the fragmentation ratio generally correlates

with temporal reasoning performance, namely

that the more fragmented the tokenization, the

how LLMs "heal" fragmented date embed-

dings in their layer stack—an emergent ability

that we term *date abstraction*. We find that

larger models quickly stitch fragmented date

inputs into a unified "date" concept for tem-

LLMs understand dates. Our results show that

LLMs follow a reasoning path that is typically

not aligned with human interpretation (year  $\rightarrow$  month  $\rightarrow$  day), but relies on subword frag-

ments that statistically represent year, month,

and day, and stitch them in a flexible order.

Our work fills the gap between tokenisation re-

search (Goldman et al., 2024; Schmidt et al., 2024)

and temporal reasoning (Su et al., 2024; Fatemi

et al., 2024), and motivates the design of date-aware

vocabularies and adaptive tokenizers that preserve

temporal coherence without sacrificing numeric

(iv) We leverage causal analysis to interpret how

(iii) We analyse internal representations by tracing

(ii) We present date fragmentation ratio, a metric

contributions are summarized as follows:

in three temporal reasoning tasks.

worse the reasoning performance.

poral reasoning at early layers.

Schmidt et al., 2024); (ii) numeric segmenta-

tion strategies that decide between digit-level or

multi-digit units: Previous work demonstrates that

the choice of radix—single digits versus 1-3-digit

chunks-induces stereotyped arithmetic errors and

can even alter the complexity class of the computations LLMs can realise (Singh and Strouse, 2024; Zhou et al., 2024); (iii) probabilistic or learnable tokenisers whose segmentations are optimised jointly with the model: Theory frames tokenisation as a stochastic map whose invertibility controls whether maximum-likelihood estimators over tokens are consistent with the underlying word distribution (Gastaldi et al., 2024; Rajaraman et al., 2024) and (iv) pre-/post-tokenisation adaptations that retrofit a model with a new vocabulary: Zheng et al. (2024) introduce an adaptive tokenizer that co-evolves with the language model, while Liu et al. (2025) push beyond the "sub-word" dogma with *SuperBPE*, a curriculum that first learns subwords and then merges them into cross-whitespace "superwords", cutting average sequence length by 27 %. Complementary studies expose and correct systematic biases introduced by segmentation (Phan et al., 2024) and propose trans-tokenization to transfer vocabularies across languages without re-training the model from scratch (Remy et al., 2024). Our work builds on these insights but zooms in on calendar dates—a hybrid of digits and lexical delimiters whose multi-digit fields are routinely shredded by standard BPE, obscuring cross-field regularities

crucial for temporal reasoning.

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Temporal reasoning in large language models. Despite rapid progress on chain-of-thought and process-supervised reasoning, temporal cognition remains a conspicuous weakness of current LLMs. Benchmarks such as TIMEBENCH (Chu et al., 2024), TEMPREASON (Tan et al., 2023), TEST-OF-TIME (Fatemi et al., 2024), MENATQA (Wei et al., 2023) and TIMEQA (Chen et al., 2021) reveal large gaps between model and human performance across ordering, arithmetic and co-temporal inference. Recent modelling efforts attack the problem from multiple angles: temporal-graph abstractions (Xiong et al., 2024), instruction-tuned specialists such as TIMO (Su et al., 2024), pseudoinstruction augmentation for multi-hop QA (Tan et al., 2023), and alignment techniques that reground pretrained models to specific calendar years (Zhao et al., 2024). Yet these approaches assume a faithful internal representation of the input dates themselves. By introducing the notion of date fragmentation and demonstrating that heavier fragmentation predicts up to ten-point accuracy drops on DATEAUGBENCH, we uncover a failure mode that is orthogonal to reasoning algorithms or supervision: errors arise before the first transformer layer,

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at the level of subword segmentation. Addressing this front-end bottleneck complements, rather than competes with, existing efforts to improve temporal reasoning in LLMs.

## 3 DateAugBench

We introduce DATEAUGBENCH, benchmark designed to isolate the impact of date tokenisation on temporal reasoning in LLMs. DATEAUGBENCH comprises 6,500 augmented examples drawn from two established sources, TIMEQA (Chen et al., 2021) and TIMEBENCH (Chu et al., 2024), distributed across three tasks splits (see Table 1). Across all the splits, our chosen date formats cover a spectrum of common regional conventions (numeric with slashes, dashes, or dots; concatenated strings; two-digit versus four-digit years) and deliberately introduce fragmentation for atypical historical (e.g. "1799") and future (e.g. "2121") dates. This design enables controlled measurement of how tokenization compression ratios and subsequent embedding recovery influence temporal reasoning performance.

**Context-based task.** In the *Context-based* split, we sample 500 question-context pairs from TIMEQA, each requiring resolution of a date mentioned in the passage (e.g. Which team did Omid Namazi play for in 06/10/1990?). Every date expression is systematically rendered in six canonical serialisations—including variants such as MM/DD/YYYY, DD-MM-YYYY, YYYY.MM.DD and concatenations without delimiters—yielding 3,000 examples that jointly probe tokenisation fragmentation and contextual grounding.

212 Simple Format Switching task. The Simple Format Switching set comprises 150 unique date pairs 213 drawn from TIMEBENCH, posed as binary same-214 day recognition questions (e.g. "Are 20251403 and 14th March 2025 referring to the same date?"). 216 Each pair is presented in ten different representa-217 tions, spanning slash-, dash-, and dot-delimited for-218 mats, both zero-padded and minimally notated, to 219 stress-test format invariance under maximal tokeni-220 sation drift. This produces 1,500 targeted examples 222 of pure format robustness. We also have examples where the dates are not equivalent, complicating 223 the task.

Date Arithmetic task. The *Date Arithmetic* split
uses 400 arithmetic instances from TIMEBENCH
(e.g. What date is 10,000 days before 5/4/2025?).
With the base date serialised in five distinct ways—

from month-day-year and year-month-day with various delimiters to compact eight-digit forms. This results in 2,000 examples that examine the model's ability to perform addition and subtraction of days, weeks, and months under various token fragmentation.

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## 4 Experiment Design

## 4.1 Date Tokenization

**Tokenizers.** For tokenization analysis, we compare a deterministic, rule-based *baseline tokenizer* against model-specific tokenizers. The baseline splits each date into its semantic components—year, month, day or Julian day—while preserving original delimiters. For neural models, we invoke either the OpenAI tiktoken encodings (for gpt-4, gpt-3.5-turbo, gpt-4o, text-davinci-003) or Hugging Face tokenizers for open-source checkpoints. Every date string is processed to record the resulting sub-tokens, token count, and reconstructed substrings.

**Distance metric.** To capture divergence from the ideal, we define a distance metric  $\theta$  between a model's token distribution and the baseline's:

$$\theta(\mathbf{t}, \mathbf{b}) = 1 - \frac{\mathbf{t} \cdot \mathbf{b}}{|\mathbf{t}|, |\mathbf{b}|},$$
 (1)

where t and b are vectors of sub-token counts for the model and baseline, respectively. A larger  $\theta$ indicates greater sub-token divergence.

**Date fragmentation ratio.** Building on  $\theta$ , we introduce the date fragmentation ratio F, which quantifies how fragmented a tokenizer's output is relative to the baseline. We initialise F = 0.0 for a perfectly aligned segmentation and apply downward adjustments according to observed discrepancies: a 0.10 penalty if the actual year/month/day components are fragmented (i.e.,  $\mathbf{1}_{split} = 1$ ), a 0.10 penalty if original delimiters are lost (i.e.,  $\mathbf{1}_{\text{delimiter}} = 1$ ), a 0.05 penalty multiplied by the token count difference  $(N - N_b)$  between a tokenizer and the baseline, and a  $0.30 \times \theta$  penalty for distributional divergence. The resulting  $F \in [0, 1]$ provides an interpretable score: values close to 0 denote minimal fragmentation, and values near 1 indicate severe fragmentation.

$$F = 0.10 * \mathbf{1}_{\text{split}} + 0.10 * \mathbf{1}_{\text{delimiter}} + 0.05 * (N - N_b) + 0.30 * \theta$$
(2) 272

Dataset and Task	# Formats	# Raw	Size	Evaluation		
		Example		Example	GT	
Context based	6	500	3000	Which team did Omid Namazi play for in 06/10/1990?	Maryland Bays	
Date Format Switching	10	150	1500	Are 20251403 and March 14th 2025 referring to the same date?	Yes	
Date Arithmetic	5	400	2000	What date is 10,000 days be- fore 5/4/2025?	18 November 1997; 17 Decem- ber 1997	
Total	21	1500	6500			

Table 1: Overview and examples of task splits in DATEAUGBENCH.

This date fragmentation ratio is pivotal because tokenisation inconsistencies directly impair a model's ability to represent and reason over temporal inputs. When date strings are split nonintuitively, models face inflated token sequences and fragmented semantic cues, potentially leading to errors in tasks such as chronological comparison, date arithmetic, and context-based resolution. By quantifying fragmentation explicitly through F, we reveal hidden limitations in existing tokenizers, inform selections of robust architectures for time-sensitive applications.

## 4.2 Temporal Reasoning Evaluation

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Models. We evaluate a spectrum of model ranging from 0.5 B to 14 B parameters: five opensource Qwen 2.5 models (0.5 B, 1.5 B, 3 B, 7 B, 14 B) (Yang et al., 2024), two Llama 3 models (3 B, 8 B) (Touvron et al., 2023b), and two OLMo (Groeneveld et al., 2024) models (1 B, 7 B). For comparison with state-of-the-art closed models, we also query the proprietary GPT-40 and GPT-40-mini endpoints via the OpenAI API (OpenAI et al., 2024).

LLM-as-a-judge. To measure how date tokenization affects downstream reasoning, we employ an 297 LLM-as-judge framework using GPT-40. For each test instance in DATEFRAGBENCH, we construct a JSONL record that includes the question text, the model's predicted answer, and a set of acceptable 301 gold targets to capture all semantically equivalent 302 date variants (e.g., both "03/04/2025" and "April 3, 2025" can appear in the gold label set). This record is submitted to GPT-40 via the OpenAI API with a system prompt instructing it to classify the prediction as CORRECT, INCORRECT, or NOT ATTEMPTED. 307 A prediction is deemed CORRECT if it fully contains any one of the gold target variants without contradiction; INCORRECT if it contains factual errors 310



Figure 2: Illustration of how LLMs with various model sizes process dates. TCP means Tokenization Compensation Point, defined as the earliest layer at which LLMs achieve above-chance accuracy (see more details in Section 6).

relative to all gold variants; and NOT ATTEMPTED if it omits the required information. We validate GPT-4o's reliability by randomly sampling 50 judged instances across all splits and obtaining independent annotations from four human reviewers. GPT-4o's classifications agree with the human consensus on 97% of cases, yielding a Cohen's  $\kappa$  of 0.89, which affirms the reliability of our automated evaluation. 311

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#### 4.3 Internal Representations

Layerwise probing. We use four Qwen2.5 (Yang et al., 2024) model checkpoints (0.5B, 1.5B, 3B, and 7B parameters) to trace how temporal information is processed internally across different layers. During inference, each question is prefixed with a fixed system prompt and a chain-of-thought cue, then passed through the model in evaluation mode. At each layer *i*, we extract the hidden-state vector corresponding to the final token position, yielding an embedding  $h_i \in \mathbb{R}^d$  for that layer. Repeating over all examples produces a collection of layerwise representations for positive and negative cases. We then quantify the emergence of temporal reasoning by training lightweight linear probes on these embeddings. For layer *i*, the probe is trained to distinguish "same-date" vs "different-date" examples. To explain when the model's date understanding is achieved, we define the *tokenization compensation point* as the layer at which the model's representation correctly represents the date in the given prompt. We experiment with this idea across various model sizes, aiming to test our hypothesis: larger models would recover calendar-level semantics from fragmented tokens at earlier stages, i.e., tokenization compensation is accomplished at early layers, as illustrated in Figure 2.

Causal attention-hop analysis. To reveal the mechanisms by which LLMs parse and resolve date strings, we conduct a two-stage causal analysis (Lindsey et al., 2025) that combines activation tracing with targeted interventions. First, we instrument the model's residual stream across all layers to capture when and where temporal information emerges. Given an input prompt requiring a date resolution (e.g., "Is 12/05/2020 the same date as 354 12th of May 2020?"), we identify two sets of tokens: (1) concept tokens corresponding to year, month, and day fragments, and (2) decision tokens corresponding to the final "yes" or "no" output. For each layer-wise token, we project its hidden state through the output embedding, producing an activation map whose peaks locate the layer and position that best encode each fragment or verdict. Attention peaks indicate the layer and position where temporal fragments and the judgment receive the most attention from input tokens. In the second stage, we perform causal interventions at the final transformer layer to quantify each token's influence on the model's decision. For each concept token that shows a strong activation peak, we generate a corrupted prompt by replacing that fragment with a contrasting value (e.g., swapping "12" for "31"). We then re-evaluate the model on the corrupted prompt and measure the change in the logit differ-373 ence between "yes" and "no." The magnitude of this change reflects the causal strength of the original token's contribution to the final judgment. To 376 build a sparse importance map, we multiply each 377 token's normalised peak height by the absolute size of its causal effect. This causal framework not only 379 pinpoints where and when temporal concepts are represented, but also how they sequentially combine to drive the model's final decision.

#### **5** Experiment Results

#### 5.1 Date fragmentation

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**Cross-temporal performance.** Table 2 reports the mean date fragmentation ratio across four

Model	Past	Near Past	Present	Future	Avg
Baseline	0.00	0.00	0.00	0.00	0.00
OLMo	0.15	0.14	0.07	0.25	0.15
GPT-3	0.17	0.14	0.06	0.25	0.16
Llama 3	0.29	0.28	0.27	0.30	0.29
GPT-40	0.32	0.31	0.22	0.30	0.29
GPT-3.5	0.47	0.22	0.26	0.36	0.33
GPT-4	0.36	0.26	0.29	0.39	0.33
Qwen	0.58	0.55	0.49	0.58	0.55
Gemma	0.58	0.55	0.49	0.58	0.55
DeepSeek	0.58	0.55	0.49	0.58	0.55
LlaMa	0.63	0.63	0.63	0.63	0.63
Phi	0.63	0.63	0.63	0.63	0.63

Table 2: Date fragmentation ratio across models and data splits over time.

Context Rlt	Fmt Switch	Date Arth.	Avg.
53.20	95.66	56.67	68.51
32.13	97.24	64.72	64.70
47.56	94.56	51.35	64.49
39.56	91.24	40.56	57.12
25.45	90.10	39.45	51.67
26.20	90.22	34.50	50.31
21.32	89.65	32.34	47.77
10.23	88.95	31.32	43.50
9.26	90.09	25.90	41.75
9.51	88.45	23.66	40.54
	53.20 32.13 47.56 39.56 25.45 26.20 21.32 10.23 9.26	53.20         95.66           32.13         97.24           47.56         94.56           39.56         91.24           25.45         90.10           26.20         90.22           21.32         89.65           10.23         88.95           9.26         90.09	53.20         95.66         56.67           32.13         97.24         64.72           47.56         94.56         51.35           39.56         91.24         40.56           25.45         90.10         39.45           26.20         90.22         34.50           21.32         89.65         32.34           10.23         88.95         31.32           9.26         90.09         25.90

Table 3: Average accuracies per task. Context Rlt stands for context based resolution task, Fmt Switch refers to the format switching task, and Date Arth. refers to the date arithmetic task.

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temporal regimes—Past (pre-2000), Near Past (2000-2009), Present (2010-2025), and Future (post-2025)-for each evaluated model. A ratio of 0.00 signifies perfect alignment with our rule-based baseline tokenizer, whereas higher values indicate progressively greater fragmentation. The rule-based Baseline unsurprisingly attains the maximal ratio of 0.00 in all periods, serving as a lower bound. Among neural architectures, OLMo (Groeneveld et al., 2024) demonstrates the highest robustness, with an average fragmentation ratio of 0.15, closely followed by GPT-3 at 0.16. Both maintain strong fidelity across temporal splits, although performance dips modestly in the Future category (0.25), reflecting novel token sequences not seen during pre-training.

**Impact of subtoken granularity.** A closer look, from Table 4, at sub-token granularity further explains these trends. Llama 3 (Touvron et al., 2023b) and the GPT (OpenAI et al., 2023) families typically segment each date component into three-digit sub-tokens (e.g., "202", "504", "03"), thus preserving the semantic unit of "MMDDYYYY" as compact pieces. OLMo (Groeneveld et al., 2024)

Model	Tokenized output	Frag-ratio	
Baseline	10 27 1606	0.00	
OLMo	10 27 16 06	0.34	
Llama 3	102 716 06	0.40	
GPT-3	1027 16 06	0.40	
GPT-40	102 716 06	0.40	
Gemma	1 0 2 7 1 6 0 6	0.55	
DeepSeek	1 0 2 7 1 6 0 6	0.55	
Cohere	1 0 2 7 1 6 0 6	0.55	
Qwen	1 0 2 7 1 6 0 6	0.55	
Phi 3.5	_ 1 0 2 7 1 6 0 6	0.60	
Llama 2	_ 1 0 2 7 1 6 0 6	0.60	

Table 4: Tokenisation of the MMDDYYYY string "10271606" across models.



Figure 3: Date fragmentation ratio versus date resolution accuracy, stratified by temporal regime and six LLMs: OLMo, Llama 3, GPT-4o, Qwen, Gemma, Phi.

splits the date tokens into two digit tokens (e.g., "20", "25"). By contrast, Qwen (Yang et al., 2024) and Gemma (Team et al., 2024) models break dates into single-digit tokens (e.g., "2", "5"), whereas Phi (Abdin et al., 2024) and LLama (Touvron et al., 2023a) divide it into single-digit tokens with an initial token (e.g. "\_", "2", "0", "2", "5"), inflating the token count. Although single-digit tokenisation can enhance models' ability to perform arbitrary numeric manipulations (by treating each digit as an independent unit), it comes at the expense of temporal abstraction: the tight coupling between day, month, and year is lost, inflating the compression penalty and increasing the  $\theta$  divergence from the baseline.

## 5.2 DATEFRAGBENCH Evaluation

427 Performance on temporal reasoning tasks. We
428 compare model accuracies in three tasks: Context429 based Resolution, Format Switching, and Date
430 Arithmetic (see the results in Table 3). All mod431 els effectively solve Format Switching (e.g. 97.2%)



Figure 4: Date fragmentation ratio versus date resolution accuracy, stratified by six formats and six LLMs.

for OLMo-2-7B, 95.7% for GPT-4o-mini, 94.6% for Qwen2.5-14B, 90.2% for Llama3.1-8B). By contrast, Context Resolution and Arithmetic remain challenging: GPT-4o-mini scores 53.2% and 56.7%, Qwen2.5-14B 47.6% and 51.4%, Llama3.1-8B 26.2% and 34.5%, and OLMo-2-7B 32.1% and 64.7%, respectively. The fact that arithmetic performance consistently exceeds resolution suggests that, given a correctly tokenized date, performing addition or subtraction is somewhat easier than resolving the date within free text—which requires encyclopedic knowledge.

Correlating date fragmentation with model accuracy over time. Figure 3 plots date fragmentation ratio against resolution accuracy, with 24 data points across six models and four temporal splits. Accuracy rises as we move from Past (1600-2000) to the Near Past (2000-2009) and peaks in the Present (2010–2025), mirroring the negative correlation between fragmentation and accuracy (dashed line, Pearson correlation of -0.61). We note that the correlation is not particularly strong. This is because (i) for some models, their date fragmentation ratios remain unchanged across temporal data splits and (ii) models differ greatly by their sizes: a larger model often outperforms a substantially smaller model in resolution accuracy, even if the former has a much higher fragmentation ratio.

As seen from Table 5, GPT-4o-mini climbs from 61.7 % in the Past to 67.9 % in the Near Past, peaks at 70.5 % for Present, and falls to 58.2 % on Future dates. Qwen-2.5-14B and Llama-3.1-8B trace the same contour at lower absolute levels. OLMo-2-7B

shows the steepest Near-Past jump (49.5  $\rightarrow$  62.4 465 %) and achieves the highest Present accuracy (73.6 466 %), consistent with its finer-grained tokenisation of 467 "20XX" patterns. These results indicate that while 468 finer date tokenisation (i.e., lower fragmentation 469 ratios) boosts performance up to contemporary ref-470 erences, today's models still generalise poorly to 471 genuinely novel (post-2025) dates, highlighting an 472 open challenge for robust temporal reasoning. 473

Correlating date fragmentation with model ac-474 curacy over formats. Figure 4 plots model ac-475 476 curacy against date fragmentation ratio across six date formats and six LLMs. A moderate nega-477 tive trend emerges (dashed line, Pearson corre-478 lation of -0.42): formats that contain explicit 479 separators (DD-MM-YYYY, DD/MM/YYYY, 480 481 YYYY/MM/DD) are tokenised into more pieces and, in turn, resolved more accurately than com-482 pact, separator-free strings (DDMMYYYY, MMD-483 DYYYY, YYYYMMDD). As shown in Table 484 6, GPT-40-mini tops every format and receives 485 a moderate performance drop from 71.2 % on 486 DD/MM/YYYY to 61.2 % on DDMMYYYY, with 487 the highest overall average (66.3 %). OLMo-2-7B 488 and Qwen-2.5-14B both exceed 70 % on the highly 489 fragmented YYY/MM/DD form, but slip into 490 the low 50s on MMDDYYYY and YYYYMMDD. 491 Lower date fragmentation ratio models, such as 492 Llama-3.1-8B and Phi-3.5, lag behind; their accu-493 494 racy plunges below 40 %. Even so, all models score much better on separator-rich formats compared 495 to the date formats without separators. In sum-496 mary, model accuracy is correlated to how cleanly 497 a model can tokenize the string into interpretable 498 tokens: more visual structure (slashes or dashes) means lower fragmentation, which suggests more straightforward reasoning, and in turn, leads to better performance.

## 6 When do LLMs understand dates?

Layerwise linear probing. To pinpoint in which layer a model learns to recognize two equiva-505 lent dates, we define the tokenization compensa-506 tion point (TCP) as the earliest layer at which 507 a lightweight linear probe on the hidden state achieves above-chance accuracy, which is defined 510 as 80%, on the date equivalence task. Figure 5a reports TCPs for the DATES\_PAST benchmark 511 (1600-2010): Qwen2.5-0.5B reaches TCP at layer 512 12 (50% depth), Qwen2.5-1.5B at layer 15 (53.6%), 513 Qwen2.5-3B at layer 8 (22.2%), and Qwen2.5-514 7B at layer 4 (14.3%). The leftward shift of 515

the 3B and 7B curves suggests how larger models recover calendar-level semantics from fragmented tokens more rapidly. Figure 5b shows the DATES PRESENT benchmark (2010–2025), where only the 1.5B, 3B, and 7B models surpass TCP-at layers 16 (57.1%), 21 (58.3%), and 17 (60.7%), respectively—while the 0.5B model never does. The deeper TCPs here reflect extra layers needed to recombine the two-digit "20" prefix, which is fragmented unevenly by the tokenizer. In Figure 10, we evaluate DATES\_FUTURE (2025–2599), where novel four-digit sequences exacerbate fragmentation. Remarkably, TCPs mirror the Past regime: layers 12, 15, 8, and 4 for the 0.5B, 1.5B, 3B, and 7B models, respectively. This parallelism indicates that model scale dictates how quickly fragmented inputs are stitched into a unified "date" concept for temporal reasoning, even when dates are novel.

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**Tokenization compensation point.** Overall, we observe a sharp decline in TCP as model size increases: small models defer date reconstruction to middle layers, whereas the largest model does so within the first quarter of layers. Across all the three temporal benchmarks, TCP shifts steadily toward the first layers as model size grows, and its absolute position is mainly independent of date ranges being tested.

## 7 *How* do LLMs understand dates?

**Causal path tracing.** To investigate how LLMs like Llama 3 (Touvron et al., 2023b) internally understand a date string, we traced the dominant attention "hops" from the model answer token back through the input digits. Figure 6 plots model layers on the y axis against prompt tokens (e.g., Is 03122025 a valid date?) on the x axis. Green arrows mark the attention path with the highest weight that is responsible for generating the answer "yes". Activation peaks at the final layers sequentially highlight the different fragments "25", "220", "031", the abstract "date" concept, and finally the "yes" output. The model tokenizes the input into "220", "031", and "25" as tokens. The LLM understands these tokens differently from the input. This chain of token-level jumps reveals that the LLM performs a kind of discrete, step-by-step pattern aggregation, stitching together substrings of the input until a binary valid/invalid verdict emerges.

**Date understanding and explainability.** In contrast, human readers parse dates by immediately mapping each component to a coherent temporal schema: "03" is March, "12" is day of month,



Figure 5: Layer-wise accuracies in the two periods: Past and Present.



Prompt: is 03122025 a valid date? Answer: Reasoning Path:  $25 \rightarrow 220 \rightarrow 031 \rightarrow date \rightarrow yes$ 

Figure 6: Causal-tracing of the "03122025 is a valid date" judgment.

"2025" is year, and then checking whether the 567 day falls within the calendar bounds of that month. 568 Humans bring rich world knowledge of calendars and leap-year rules to bear in parallel. However, LLMs exhibit no explicit calendar "module"; instead, they rely on learned statistical associations 572 between digit-patterns and the training-time supervisory signal for "valid date." The causal-tracing path in Figure 6 thus illustrates a fundamentally different mechanism of date comprehension in LLMs, based on token-level attention re-routing rather than holistic semantic interpretation. We repeated 578 causal tracing on 100 date strings in 6 different date formats to test whether this attention trajectory is 581 consistent across date formats. In most of cases, we observe that the model's attention hops (i.e., reasoning paths) are not aligned with human interpretation 583 (year  $\rightarrow$  month  $\rightarrow$  day), rather *sub-word fragments* that statistically represent year, month, and day in 585 a flexible order that is subject to date formats (see examples in Figures 7-8). However, such date understanding becomes tricky when a date is greatly 588 fragmented: given the date abstraction is learned from frequency rather than hard-coded rules, the abstraction is biased toward standard Western for-591

mats and contemporary years. As a result, a model often addresses popular dates with higher model accuracy and similar date reasoning paths. However, the reasoning path becomes obscure on rare, historical, or locale-specific strings outside the distribution of pre-training data (see Figure 9).

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#### 8 Conclusion

In this paper, we identified date tokenization as a critical yet overlooked bottleneck in temporal reasoning with LLMs. We demonstrated a correlation between date fragmentation and task performance in temporal reasoning, i.e., the more fragmented the tokenization, the worse the reasoning performance. Our layerwise and causal analyses in LLMs further revealed an emergent "date abstraction" mechanism that explains when and how LLMs understand and interpret dates. Our results showed that larger models can compensate for date fragmentation by stitching fragments into a unified "date" concept, while the stitching process appears to be accomplished via a reasoning path that connects date fragments in a flexible order, differing from human interpretation from year to month to day.

## 615 Limitations

While our work demonstrates the impact of date tokenization on LLMs for temporal reasoning, there 617 are several limitations. First, DATEAUGBENCH fo-618 cuses on a finite set of canonical date serialisations 619 and does not capture the full diversity of naturallanguage expressions (e.g., "the first Monday of 621 May 2025") or noisy real-world inputs like OCR 622 outputs. Second, our experiments evaluate a representative but limited pool of tokenizers and model checkpoints (up to 14B parameters); therefore, the generalizability of date fragmentation ratio and our probing and causal analyses to very large models 627 628 with 15B+ parameters remains unknown. Finally, while the fragmentation ratio measures front-end segmentation fidelity, it does not account for deeper world-knowledge factors such as leap-year rules, timezone conversions, and culturally grounded calendar systems, all of which would influence temporal interpretation. Future work should extend to more diverse date expressions, broader model and tokeniser families, and equipping tokenisers with external calendar-wise knowledge to further improve robust temporal reasoning. 638

### Ethical Considerations

DATEAUGBENCH is derived solely from the public, research-licensed TIMEQA and TIMEBENCH corpora that do not contain sensitive data; our augmentation pipeline rewrites only date strings. However, our dataset focuses on 21 Anglo-centric Gregorian formats. Therefore, our data potentially reinforce a Western default and overlook calendars or numeral systems used in many other cultures, and our date fragmentation metric may over-penalise tokenisers optimised for non-Latin digits.

## References

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- Marah Abdin, Jyoti Aneja, Hany Awadalla, Ahmed Awadallah, Ammar Ahmad Awan, Nguyen Bach, Amit Bahree, Arash Bakhtiari, Jianmin Bao, Harkirat Behl, Alon Benhaim, Misha Bilenko, Johan Bjorck, Sébastien Bubeck, Martin Cai, Qin Cai, Vishrav Chaudhary, Dong Chen, Dongdong Chen, and 110 others. 2024. Phi-3 technical report: A highly capable language model locally on your phone.
  - Gagan Bhatia, El Moatez Billah Nagoudi, Hasan Cavusoglu, and Muhammad Abdul-Mageed. 2024. Fintral: A family of gpt-4 level multimodal financial large language models. *Preprint*, arXiv:2402.10986.
- Andrea Carriero, Davide Pettenuzzo, and Shubhranshu Shekhar. 2024. Macroeconomic forecast-

ing with large language models. *arXiv preprint arXiv:2407.00890*.

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- Ching Chang, Wei-Yao Wang, Wen-Chih Peng, and Tien-Fu Chen. 2023. Llm4ts: Aligning pre-trained llms as data-efficient time-series forecasters. *arXiv preprint arXiv:2308.08469*.
- Wenhu Chen, Xinyi Wang, and William Yang Wang. 2021. A dataset for answering time-sensitive questions. *Preprint*, arXiv:2108.06314.
- Zheng Chu, Jingchang Chen, Qianglong Chen, Weijiang Yu, Haotian Wang, Ming Liu, and Bing Qin. 2024. Timebench: A comprehensive evaluation of temporal reasoning abilities in large language models. *Preprint*, arXiv:2311.17667.
- Bahare Fatemi, Mehran Kazemi, Anton Tsitsulin, Karishma Malkan, Jinyeong Yim, John Palowitch, Sungyong Seo, Jonathan Halcrow, and Bryan Perozzi. 2024. Test of time: A benchmark for evaluating llms on temporal reasoning.
- Juan Luis Gastaldi, John Terilla, Luca Malagutti, Brian DuSell, Tim Vieira, and Ryan Cotterell. 2024. The foundations of tokenization: Statistical and computational concerns.
- Omer Goldman, Avi Caciularu, Matan Eyal, Kris Cao, Idan Szpektor, and Reut Tsarfaty. 2024. Unpacking tokenization: Evaluating text compression and its correlation with model performance.
- Dirk Groeneveld, Iz Beltagy, Pete Walsh, Akshita Bhagia, Rodney Kinney, Oyvind Tafjord, Ananya Harsh Jha, Hamish Ivison, Ian Magnusson, Yizhong Wang, Shane Arora, David Atkinson, Russell Authur, Khyathi Raghavi Chandu, Arman Cohan, Jennifer Dumas, Yanai Elazar, Yuling Gu, Jack Hessel, and 24 others. 2024. Olmo: Accelerating the science of language models.
- Jack Lindsey, Wes Gurnee, Emmanuel Ameisen, Brian Chen, Adam Pearce, Nicholas L. Turner, Craig Citro, David Abrahams, Shan Carter, Basil Hosmer, Jonathan Marcus, Michael Sklar, Adly Templeton, Trenton Bricken, Callum McDougall, Hoagy Cunningham, Thomas Henighan, Adam Jermyn, Andy Jones, and 8 others. 2025. On the biology of a large language model. *Transformer Circuits Thread*.
- Alisa Liu, Jonathan Hayase, Valentin Hofmann, Sewoong Oh, Noah A. Smith, and Yejin Choi. 2025. Superbpe: Space travel for language models. *arXiv preprint arXiv:2503.13423*.
- OpenAI, :, Aaron Hurst, Adam Lerer, Adam P. Goucher, Adam Perelman, Aditya Ramesh, Aidan Clark, AJ Ostrow, Akila Welihinda, Alan Hayes, Alec Radford, Aleksander Mądry, Alex Baker-Whitcomb, Alex Beutel, Alex Borzunov, Alex Carney, Alex Chow, Alex Kirillov, and 401 others. 2024. Gpt-40 system card.

774

OpenAI, Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman, Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, Red Avila, Igor Babuschkin, Suchir Balaji, Valerie Balcom, Paul Baltescu, Haiming Bao, Mohammad Bavarian, Jeff Belgum, and 262 others. 2023. Gpt-4 technical report.

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- Buu Phan, Marton Havasi, Matthew Muckley, and Karen Ullrich. 2024. Understanding and mitigating tokenization bias in language models. *arXiv preprint arXiv:2406.16829*.
- Nived Rajaraman, Jiantao Jiao, and Kannan Ramchandran. 2024. Toward a theory of tokenization in llms.
  - François Remy, Pieter Delobelle, Hayastan Avetisyan, Alfiya Khabibullina, Miryam de Lhoneux, and Thomas Demeester. 2024. Trans-tokenization and cross-lingual vocabulary transfers: Language adaptation of llms for low-resource nlp. *arXiv preprint arXiv:2408.04303*.
  - Craig W. Schmidt, Varshini Reddy, Haoran Zhang, Alec Alameddine, Omri Uzan, Yuval Pinter, and Chris Tanner. 2024. Tokenization is more than compression.
  - Aaditya K. Singh and DJ Strouse. 2024. Tokenization counts: the impact of tokenization on arithmetic in frontier llms.
  - Zhaochen Su, Jun Zhang, Tong Zhu, Xiaoye Qu, Juntao Li, Min Zhang, and Yu Cheng. 2024. Timo: Towards better temporal reasoning for language models.
  - Mingtian Tan, Mike A. Merrill, Vinayak Gupta, Tim Althoff, and Thomas Hartvigsen. 2024. Are language models actually useful for time series forecasting? In Advances in Neural Information Processing Systems.
  - Qingyu Tan, Hwee Tou Ng, and Lidong Bing. 2023. Towards robust temporal reasoning of large language models via a multi-hop qa dataset and pseudoinstruction tuning.
- Gemma Team, Morgane Riviere, Shreya Pathak, Pier Giuseppe Sessa, Cassidy Hardin, Surya Bhupatiraju, Léonard Hussenot, Thomas Mesnard, Bobak Shahriari, Alexandre Ramé, Johan Ferret, Peter Liu, Pouya Tafti, Abe Friesen, Michelle Casbon, Sabela Ramos, Ravin Kumar, Charline Le Lan, Sammy Jerome, and 179 others. 2024. Gemma 2: Improving open language models at a practical size. *Preprint*, arXiv:2408.00118.
- Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timothée Lacroix, Baptiste Rozière, Naman Goyal, Eric Hambro, Faisal Azhar, Aurelien Rodriguez, Armand Joulin, Edouard Grave, and Guillaume Lample. 2023a. Llama: Open and efficient foundation language models.
- Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaei, Nikolay Bashlykov, Soumya Batra, Prajjwal Bhargava, Shruti Bhosale, Dan Bikel, Lukas Blecher, Cristian Canton

Ferrer, Moya Chen, Guillem Cucurull, David Esiobu, Jude Fernandes, Jeremy Fu, Wenyin Fu, and 49 others. 2023b. Llama 2: Open foundation and fine-tuned chat models.

- Stylianos Loukas Vasileiou and William Yeoh. 2024. Trace-cs: A synergistic approach to explainable course scheduling using llms and logic. *arXiv preprint arXiv:2409.03671*.
- Jiapu Wang, Kai Sun, Linhao Luo, Wei Wei, Yongli Hu, Alan Wee-Chung Liew, Shirui Pan, and Baocai Yin. 2024. Large language models-guided dynamic adaptation for temporal knowledge graph reasoning. *arXiv preprint arXiv:2405.14170*.
- Yang Wang and Hassan A Karimi. 2024. Exploring large language models for climate forecasting. *arXiv* preprint arXiv:2411.13724.
- Yifan Wei, Yisong Su, Huanhuan Ma, Xiaoyan Yu, Fangyu Lei, Yuanzhe Zhang, Jun Zhao, and Kang Liu. 2023. Menatqa: A new dataset for testing the temporal comprehension and reasoning abilities of large language models.
- Siheng Xiong, Ali Payani, Ramana Kompella, and Faramarz Fekri. 2024. Large language models can learn temporal reasoning.
- An Yang, Baosong Yang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang Zhou, Chengpeng Li, Chengyuan Li, Dayiheng Liu, Fei Huang, Guanting Dong, Haoran Wei, Huan Lin, Jialong Tang, Jialin Wang, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Ma, and 43 others. 2024. Qwen2 technical report.
- Yifan Zeng. 2024. Histolens: An llm-powered framework for multi-layered analysis of historical texts – a case application of yantie lun. arXiv preprint arXiv:2411.09978.
- Bowen Zhao, Zander Brumbaugh, Yizhong Wang, Hannaneh Hajishirzi, and Noah A. Smith. 2024. Set the clock: Temporal alignment of pretrained language models.
- Mengyu Zheng, Hanting Chen, Tianyu Guo, Chong Zhu, Binfan Zheng, Chang Xu, and Yunhe Wang. 2024. Enhancing large language models through adaptive tokenizers. In *Proc. NeurIPS*.
- Zhejian Zhou, Jiayu Wang, Dahua Lin, and Kai Chen. 2024. Scaling behavior for large language models regarding numeral systems: An example using pythia.

## A Appendix

**Implementation details of evaluation.** The evaluation pipeline is implemented in Python and supports asynchronous API requests with retry logic, as well as multiprocessing to handle thousands of examples efficiently. After collecting GPT-4o's label for each instance, we map CORRECT/INCORRECT NOT ATTEMPTED to categorical scores A, B, and

- C. We then compute three core metrics: overall
  accuracy (proportion of A scores), given-attempted
  accuracy (A over A+B), and the F1 score, defined as the harmonic mean of overall and givenattempted accuracy. Results are reported both globally and stratified by task split (Context-based, Format Switching, Date Arithmetic) and by temporal
  category (Past, Near Past, Present, Future).
- **Date ambiguities.** We explicitly enumerate all valid variants in the gold label set for each example to handle multiple correct answers arising from date-format ambiguities. This ensures that any prediction matching one of these variants is marked correct, avoiding penalisation for format differences.

Synthetic benchmark construction for linear probing. We construct a suite of synthetic true-false benchmarks to isolate temporal reasoning across different reference frames. For the DATES\_PAST, DATES\_PRESENT, and 846 DATES FUTURE datasets, we sample 1,000 847 848 date-date pairs each, drawing calendar dates uniformly from the appropriate range and rendering 849 them in two randomly chosen, distinct formatting patterns (Ymd vs d/m/Y). Exactly half of each set are "YES" examples (identical dates under differ-852 ent formats), which are our positive examples, and half are "NO" (different dates), which are our neg-855 ative examples. All three datasets are balanced, shuffled, and split into equal positive and negative 856 subsets to ensure fair probing. 857

Models	Past	Near Past	Present	Future
GPT-40-mini	61.66	67.93	70.51	58.23
OLMo-2-7B	49.45	62.35	73.56	43.45
Qwen2.5 14B	58.97	64.80	67.22	55.69
Qwen2.5 7B	51.41	55.98	57.98	48.55
Qwen2.5 3B	46.50	50.25	51.98	43.91
LLama3.1 8B	45.28	48.82	50.48	42.76
Qwen2.5 1.5B	42.99	46.16	47.69	40.60
Qwen2.5 0.5B	39.15	41.68	43.00	36.98
OLMo-2-1B	36.07	38.09	40.49	34.07
LLama3.2 3B	36.48	38.57	39.74	34.46

Table 5: Model accuracy on context-based questionsacross four data splits over time.



Prompt: Is 03/12/2025 a valid date? Reasoning Path: 03  $\rightarrow$  12  $\rightarrow$  202  $\rightarrow$  5  $\rightarrow$  date  $\rightarrow$  yes

Figure 7: Causal-tracing of the "03/12/2025 is a valid date" judgment.



Figure 8: Causal-tracing of the "03-12-2025 is a valid date" judgment.



Prompt: is 03121325 a valid date? Answer: Reasoning Path:  $25 \rightarrow 213 \rightarrow 031 \rightarrow date \rightarrow no$ 

Figure 9: Causal-tracing of the "03121325 is a valid date" (Date in past) judgment.



Figure 10: Layer-wise accuracies in the Future period

Model	DD-MM-YYYY	DD/MM/YYYY	YYYY/MM/DD	DDMMYYYY	MMDDYYYY	YYYYMMDD	Avg.
OLMo	64.70	64.56	65.35	52.35	54.56	50.41	58.65
Llama 3	50.31	50.89	53.45	38.45	40.24	34.56	44.65
GPT-40	68.51	71.23	69.24	61.23	62.34	64.98	66.25
Qwen	64.49	62.35	73.56	46.50	50.25	51.98	58.19
Gemma	58.90	58.97	64.80	47.22	46.50	50.25	54.44
Phi	47.23	46.07	48.09	39.15	41.68	43.00	44.20

Table 6: Model accuracy on context-based questions across date formats.