Robust Trajectory Optimization for Safe Locomotion over Uncertain Terrain

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***Abstract*—Safe and autonomous locomotion for legged robots in real-world environments requires generating motion strategies that are robust to uncertainty in the terrain. Current trajectory optimization methods rely on specifying the geometry and friction properties of the terrain; however, errors in the terrain model can lead to failure through slipping and falling. Here we develop a trajectory optimization approach that explicitly incorporates parametric uncertainty in the terrain model. We demonstrate that our method produces a spectrum of robust trajectories: the method produces robust trajectories when uncertainty is large and the nominal optimal trajectories when uncertainty is small. Our study represents a step towards generating safe locomotion behaviors which are robust against uncertainty in the terrain.**

I. Introduction

Designing safe and autonomous locomotion behaviors for bipedal robotics poses a challenge for deploying these systems in real-world environments. Safely traversing a terrain to a goal requires avoiding slipping and falling on the terrain, in addition to avoiding obstacles. Current contact-implicit trajectory optimization methods typically require precisely specifying terrain geometry and friction characteristics which in real-world scenarios are prone to uncertainty [5, 4]. Previous works have addressed model uncertainty by perturbing individual model parameters, resulting in an ensemble of trajectories [3].

Our work focuses on explicitly reasoning about uncertainty in the environment. In particular, we aim to encode uncertainty about the terrain geometry and the friction characteristics into robust objectives for trajectory optimization. The goal of our study is to demonstrate that the robust objectives produce robust trajectories when the terrain uncertainty is large and non-robust optimal trajectories when the uncertainty is small.

II. Robust Trajectory Optimization

Contact-implicit trajectory optimization has been used in recent years to compute the contact forces while the trajectory is optimized [5], and traditionally takes the form:

$$\min_{z=(q,\dot{q},u,\lambda)} \int_0^T L(z)dt \tag{1a}$$

$$\text{s.t.} \begin{cases} M(q)\ddot{q} + C(\dot{q},q) = B(q)u + J_c^\top(q)\lambda & \text{(1b)} \\ 0 \leq \lambda_N \perp \phi(q) \geq 0 & \text{(1c)} \\ 0 \leq \lambda_T \perp \gamma + J_T\dot{q} \geq 0 & \text{(1d)} \\ 0 \leq \gamma \perp \mu\lambda_N - e^\top\lambda_T \geq 0 & \text{(1e)} \end{cases}$$

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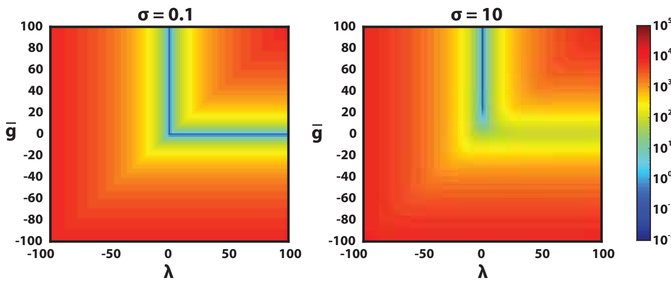

Fig. 1. Comparison of the ERM cost landscape for low uncertainty (left) and high uncertainty (right). Axes represent the forces λ and the expected value $\bar{g}$ of the uncertain constraint. When uncertainty σ is small, the ERM cost landscape resembles the solution set for the complementarity constraint.

where z is the set of decision variables, L is the running cost, (1b) represents the dynamics, $\phi(q)$ is the normal distance to the terrain, γ is a slack variable related to sliding velocity, μ is the coefficient of friction and $e^\top = [1, ..., 1]$. The complementarity conditions (1c)-(1e) can be expressed as $0 \leq \lambda \perp g(z) \geq 0$, where g represents an exact model of the terrain.

In general, solutions to (1a) are sensitive to the contact model encoded in the constraints. Our study encodes uncertainty in the terrain by employing stochastic complementarity constraints in place of the deterministic constraints. Specifically, we assume Gaussian uncertainty in the friction coefficient and in the contact distance. Then, following previous developments [6], we derived an expected residual minimization (ERM) cost to represent the uncertain contact:

$$\begin{aligned}\mathbb{E}[\|\psi(\lambda_i, g(z_i))\|^2] = \lambda_i^2 &- \sigma(\lambda_i + \bar{g})p(\lambda_i) \\ &+ (\sigma^2 + \bar{g}^2 - \lambda_i^2)P(\lambda_i)\end{aligned} \tag{2}$$

where ψ is the min function, $g(z_i, \omega) \sim \mathcal{N}(\bar{g}, \sigma^2)$ is the constraint with Gaussian uncertainty, and $p(\lambda_i)$ and $P(\lambda_i)$ are the Gaussian probability and cumulative distribution functions. The ERM costs are then added to the running cost:

$$\min_{q,\dot{q},u,\lambda} \sum_{i=0}^{N-1} \Big(L(z_i) + \beta\mathbb{E}[\|\psi(\lambda_i, g(z_i))\|^2] \Big) \tag{3}$$

Previous developments in the theory of stochastic complementarity problems have shown that solutions to the ERM problem are robust to variations in the uncertain parameters [2, 1]. The present study explores the robustness of ERM solutions in the context of trajectory optimization problems, and also explores the behavior of the solutions under different amounts of uncertainty. Moreover, our work shows that, as the uncertainty decreases, the ERM problem converges to the deterministic complementarity problem (Figure 1).

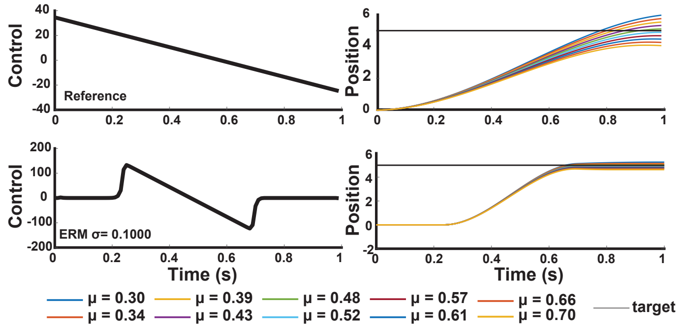

Fig. 2. Comparison of trajectories generated by the reference, non-ERM method and the ERM method for the sliding block example. (Left) Optimized controls from the reference trajectory optimization problem (top) and the ERM problem (bottom). (Right) Position trajectories simulated using the optimized controls under different friction coefficients on the terrain.

III. Simulation Experiments

We explored robustness to uncertainty in friction coefficients in a sliding block simulation. We assumed the friction coefficient was normally distributed and derived the ERM cost for the friction cone. Compared to the non-ERM reference control, the control generated with the ERM was nonzero for a shorter period of time and its amplitude was larger (Figure 2). The position trajectories generated by simulating the controls forward showed smaller final position variation with respect to changing terrain friction coefficients for the ERM controls compared to the non-ERM controls.

We tested robustness to uncertainty in the contact distance in a simulation experiment where a cart must pedal across a frictionless rail via a two-link pendulum that can contact the ground (Figure 3(b)). We assumed the terrain normal vector was known but that the contact distance was normally distributed and we derived an ERM cost on the normal distance. Compared to the non-ERM reference trajectory, trajectories generated with the ERM cost and contact distance uncertainties $\sigma \geq 0.1$ showed increased foot clearance height (Figure 3b,c). For uncertainties $\sigma < 0.01$, the ERM trajectories converged to the reference trajectory with little error.

IV. Discussion

In contrast to previous work [6], which presented ERM as a smoothing method for contact, we presented ERM as a method for robust optimization in the face of parametric uncertainty in the terrain model. Specifically, our work demonstrated that the ERM produces shorter sliding durations under friction uncertainty and greater foot clearances under contact location uncertainty. The ERM controls in the sliding example also produced less terminal position variation compared to non-ERM controls in simulation, demonstrating that the controls can inherit robustness from the ERM solution.

One advantage of the ERM method is that the robustness is adjustable through the uncertainty σ. As we demonstrated, increasing the uncertainty leads to more robust solutions, while decreasing the uncertainty leads to more optimal solutions. However, robustness does not always increase with uncertainty; if the uncertainty is too large, the ERM cost ceases to represent the discontinuous contact conditions and the resulting solution could be physically infeasible. In future work, the uncertainty could be made variable across the terrain, biasing the system away from regions where terrain information is poor and towards regions where terrain information is known.

The ERM method we presented here represents a step towards contact-implicit trajectory optimization that is robust against uncertainty in the terrain model. More work is required to generalize our results to uncertainty in the geometry of the terrain and to demonstrate that the results hold in systems with more complex dynamics. Given the probabilistic framework underlying the ERM method, future studies could fuse the ERM method with data collected during run-time locomotion experiments to update the uncertainty and robustness online.

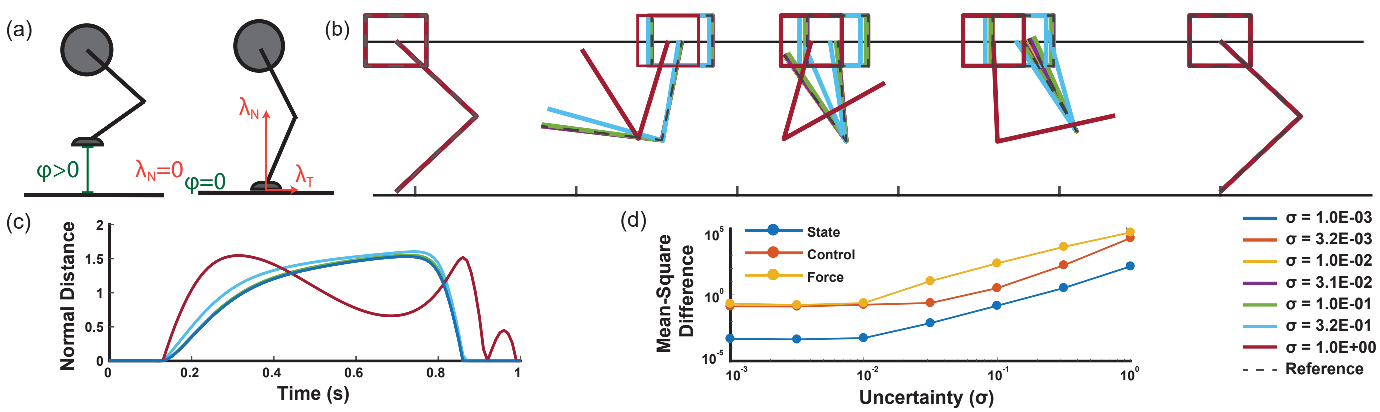

Fig. 3. Illustration of the relationship between contact uncertainty and locomotion foot clearance height. (a) Illustration of the complementarity relationship between normal distance ϕ and normal force λ_N. In this example, the normal distance ϕ is uncertain. (b) Selected configurations of the contact-driven cart under different values of uncertainty, where the cart is constrained along a horizontal track. For $\sigma < 0.1$, the configurations are indistinguishable from one another and from the non-ERM reference trajectory. (c) The normal distance between the endpoint of the contact-driven cart and the terrain over the entire trajectory. As uncertainty increases, the distance increases until the second link flips over, decreasing the distance again. (d) Mean-squared difference between the ERM solutions and the non-ERM reference. As uncertainty decreases, the ERM trajectories converge to the reference with little error. Colors in (b) and (c) represent solutions at different levels of uncertainty and a legend is given in the lower right corner.

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