Hierarchical structure of redundancy and synergy in brain networks across goal-directed learning

Keywords: Brain networks, Temporal networks, Whole-brain modelling, Information theory, Hierarchical network structure

Extended Abstract

Redundancy and synergy are complementary principles of information processing in complex systems. Within the Partial Information Decomposition (PID) framework, redundancy quantifies the information about a target duplicated across sources, ensuring robustness and reliable read-out, while synergy captures information available only from the joint state of multiple sources, supporting integration and flexible representations [1]. Applied to brain activity, redundancy has been linked to stable transmission of signals, whereas synergy is associated with cognitive flexibility and the emergence of novel information [2,3].

We analyse a magnetoencephalographic (MEG) dataset previously employed in studies of goal-directed learning [4]. Participants performed an arbitrary visuomotor task, discovering by trial and error the mapping between three visual stimuli and five finger movements. This design induces reproducible exploration–exploitation phases and allows estimation of information gain (IG), the reduction of uncertainty about action–outcome contingencies. MEG data were source-reconstructed in the high-gamma band (60–120 Hz) across cortical parcels, and time-resolved functional networks were constructed by quantifying redundancy and synergy of cortical interactions with respect to IG, using the HOI Python toolbox [5].

To characterise their hierarchical organisation, we employed two graph-theoretic methods. The k-core decomposition identifies the maximal subgraph where each node has degree $\geq$ k, providing a nested hierarchy of cores and peripheries. The span-core decomposition extends this to temporal networks, detecting nodes that maintain degree $\geq$ k over contiguous intervals, thus capturing the persistence of structural cores [6]. Redundant networks exhibit a marked core–periphery structure with dense, central backbones, whereas synergistic networks show a flatter organisation with more uniform participation across shells (Fig. A). Span-core analysis reveals phase-dependent reconfiguration: early short-lived cores in visual regions, mid-trial involvement of temporal and executive areas, and late, long-lived cores in ventromedial prefrontal and orbitofrontal cortices, consistent with IG convergence. Redundant span-cores are longer-lived and higher-k, while synergistic span-cores are shorter and more evenly distributed, indicating transient but flexible integration (Fig. B).

To assess the origin of these patterns, we complement empirical analyses with whole-brain simulations. Starting from an empirical structural connectome, we generate synthetic time series using a network of nonlinear oscillators in the normal form of a supercritical Hopf bifurcation (Stuart–Landau model) [7]. This model couples anatomical connectivity with amplitude–phase dynamics and noise, and accounts for collective oscillatory behaviour in different brain states. Injecting task-related signals into this dynamical system yields synthetic redundancy- and synergy-based networks that reproduce the empirical hierarchical structures. This provides a mechanistic explanation for the emergence of specific hierarchical patterns and highlights general principles by which redundancy and synergy shape cortical information dynamics.

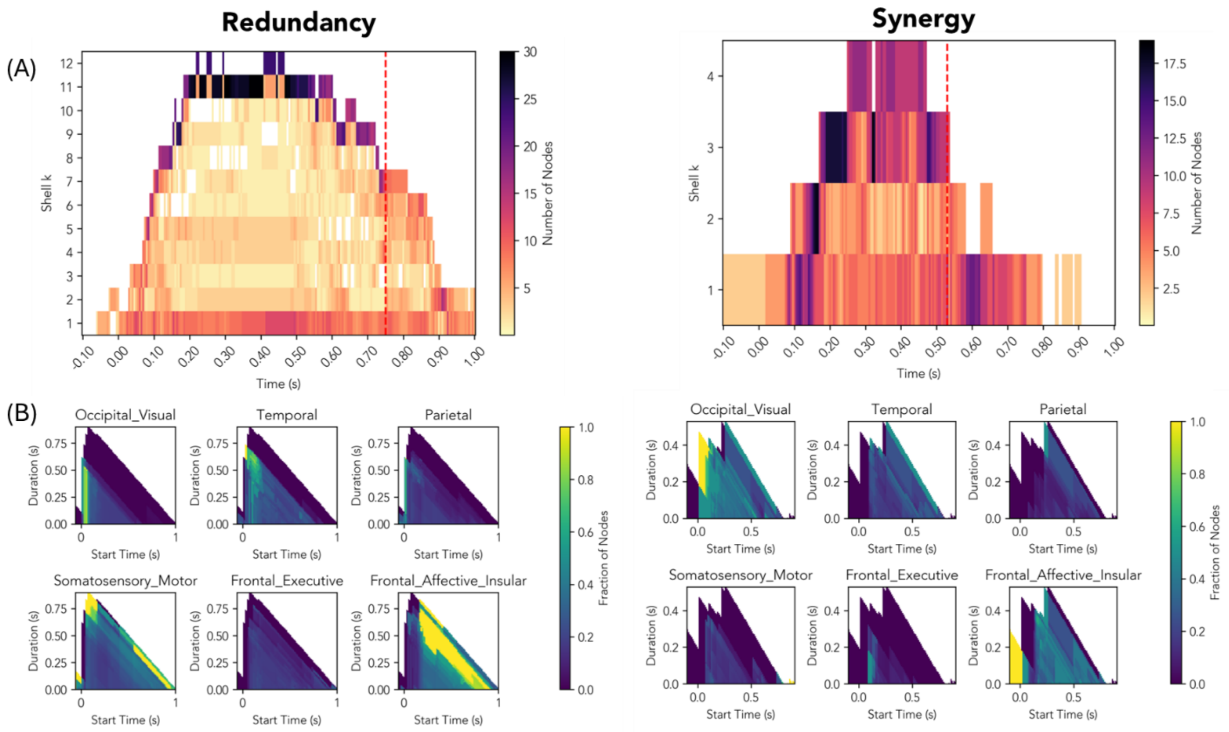

Figure 1. (A) Time-resolved k-core decomposition of redundancy (left) and synergy (right) networks. The x-axis denotes time (s), the y-axis the k-shell index, and the color scale the number of nodes in each shell. (B) Span-core distributions across cortical macro-areas for redundancy (left) and synergy (right). The x-axis denotes the start time of each span-core, the y-axis its duration (s), and the color scale the fraction of nodes in each macro-area.

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