ESTIMATION OF ACOUSTIC FIELD IN 1-D DUCT USING ARTIFICIAL NEURAL NETWORKS

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ABSTRACT

Estimation of sound field in one-dimension finds extensive applications in many areas such as speech, automotive, aerospace and biomedical industries. Traditionally, it is obtained by solving Helmholtz equation using analytical and numerical methods (finite difference, finite element, etc.). This paper discusses a neural network methodology to solve 1-D Helmholtz equation subjected to some constraints. Unlike other governing equations, Helmholtz equation poses a biasing problem with the loss functions at higher frequencies. In the current work, an automatic weight update algorithm is proposed to bypass this difficulty. The results obtained from the proposed methodology are compared with those of the analytical method. A good correlation has been observed between the two methods. The robustness of the methodology with respect to the frequency is also verified.

1 INTRODUCTION

In many engineering applications, sound propagation is dominant in one-dimension(Munjal, 2014). For instance, in the modelling and analysis of speech, automotive intake/exhaust sounds, human respiratory sounds, etc., the acoustic field variation is primarily significant in the direction of largest dimension as compared to the other directions. In those cases, 1-D Helmholtz equation with appropriate boundary condition will provide an acoustic field distribution required to perform the intended study. Traditionally, this task is performed by means of analytical(Veerababu & Venkatesham, 2020; 2021) and numerical methods(Ihlenburg & Babuška, 1995; Shen & Liu, 2007; Lourier et al., 2012).

Recently, data-driven methods using machine learning techniques have emerged as a new alternative in providing reliable solutions(Megalmani et al., 2021; Rao et al., 2017). However, there is always a possibility of contamination of measured data which affects the accuracy of the solution. Hence, there is need to develop methods that are driven by physics apart from the data(Raissi et al., 2019). Towards this direction, a neural network solution for the 1-D Helmholtz equation is presented in the current article.

According to universal approximation theorem, the solution of a well-posed problem can be approximated to a desired accuracy with a feedforward neural network of infinite number of neurons(Hornik et al., 1989). Using this fact, a feedforward neural network is constructed to approximate the 1-D acoustic field. Unlike other governing equations, Helmholtz equation encounters biasing problem between the loss functions at higher frequencies(Wang et al., 2021). This problem can be bypassed by introducing unequal weights to the loss functionsMaddu et al. (2022); Basir & Senocak (2022). In this current article, an algorithm that automatically adjust the weights based on the calculated loss functions is proposed. Section 2 describes the neural network formulation for the 1-D Helmholtz equation. Problems associated with the higher frequencies, algorithm implementation to address this issue, and correlation with the true solution are presented in detail in Section 3. The article is concluded with final remarks in Section 4.

2 NEURAL NETWORK FORMULATION FOR THE 1-D HELMHOLTZ EQUATION

In many engineering applications such as human speech synthesis, automotive intake/exhaust systems, HVAC ducts, etc., the acoustic field variation is primarily dominated in the direction of longest dimension. The acoustic field in that direction can be obtained by solving 1-D Helmholtz equation given below(Munjal, 2014)

$$\left(\frac{d^2}{dx^2} + k^2\right)\psi(x) = 0, \qquad x \in [0, L] \tag{1}$$

subjected to the boundary conditions $\psi(0) = \psi_0$ and $\psi(L) = \psi_L$. Here, $\psi(x)$ is the acoustic field, $k = 2\pi f/c$ is the wavenumber, f is the frequency and c is the speed of sound in air (340 m/s).

According to the universal approximation theorem(Hornik et al., 1989), the acoustic field $\psi(x)$ can be approximated with a feedforward neural network $\hat{\psi}(x;\theta)$ and can be estimated by minimizing the following loss functional $\mathcal{L}$ with respect the parameters θ(Raissi et al., 2019).

$$\mathcal{L}(\theta) = \mathcal{L}_F(\theta) + \mathcal{L}_B(\theta), \tag{2}$$

where $\mathcal{L}_F(\theta)$ is the loss functions associated with the Helmholtz equation

$$\mathcal{L}_F(\theta) = \frac{1}{N}\sum_{i=1}^{N}\left\| \frac{d^2}{dx^2}\hat{\psi}(x^{(i)};\theta) + k^2\hat{\psi}(x^{(i)};\theta)\right\|_2^2, \tag{3}$$

and $\mathcal{L}_B(\theta)$ is the loss functions associated with the boundary conditions

$$\mathcal{L}_B(\theta) = \frac{1}{2}\left(|\hat{\psi}(0;\theta) - \psi_0|^2 + |\hat{\psi}(L;\theta) - \psi_L|^2\right). \tag{4}$$

Here, N is the number of internal collocation points in $[0, L]$.

3 Results and Discussion

A feedforward neural network with 4 hidden layers and 90 neurons in each layer is constructed to predict the acoustic field ψ. The loss functional $\mathcal{L}$ is optimised using *Adam* algorithm with *tanh* activation function. The domain is divided into 10000 random collocation points and 10000 full-batch iterations were performed with a learning rate of 10^{-3}.

Figure 1 shows the acoustic field predicted by the neural network against the true solution obtained from the analytical method (refer Appendix A) for different frequencies. It can be observed that the neural network is able to learn the underlying physics from the governing equations successfully at lower frequencies (100 Hz). At higher frequencies (above 200 Hz), there is a drastic deviation from the true solution. The reason for this behaviour can best be understood by observing the variation of individual loss functions with respect to iterations.

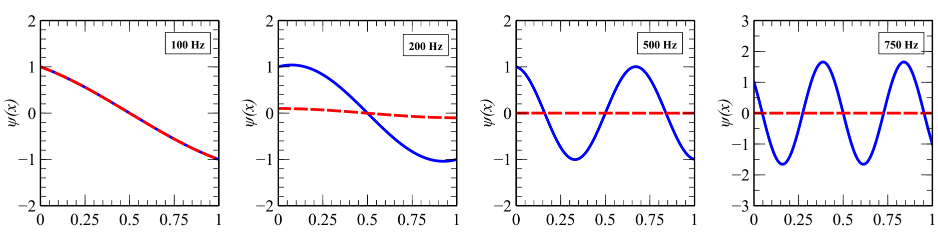

Figure 1: Correlation between true (———) and predicted solution (- - - - -) for different frequencies.

Figure 2 shows the variation of the loss functions $\mathcal{L}_F$, $\mathcal{L}_B$ and $\mathcal{L}$ with respect to iteration for different frequencies. It can be observed that both the loss functions decreases with respect to iterations at lower frequencies (100 Hz). However, at higher frequencies (200 Hz and above), the loss function associated with the Helmholtz equation $\mathcal{L}_F$ decreases, whereas the loss function associated with the boundary conditions $\mathcal{L}_B$ does not decrease and becomes stable at particular value.

In other words, a biasing behaviour is introduced during the training process at higher frequencies, which enforces the optimization process to favour $\mathcal{L}_F$ alone and minimises it, leaving $\mathcal{L}_B$. This problem can be bypassed by assigning weights to the loss functions as given below

$$\mathcal{L} = \lambda_F\mathcal{L}_F + \lambda_B\mathcal{L}_B, \tag{5}$$

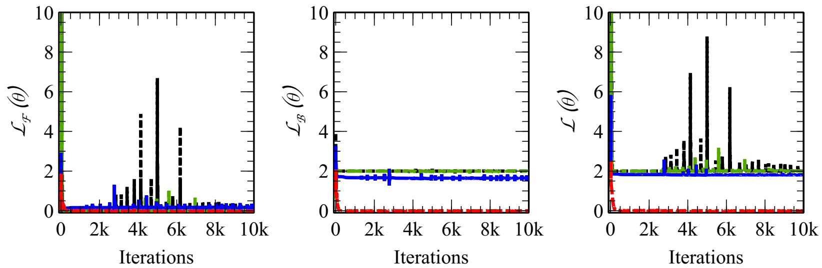

Figure 2: Variation of the loss functions with respect to the iterations: 100 Hz, 200 Hz, 500 Hz, 750 Hz.

where λ_F and λ_B are the weights associated with the loss functions $\mathcal{L}_F$ and $\mathcal{L}_B$, respectively.

One of the simplest method to find λ_F and λ_B is to choose them on the trial-and-error basis method. Even though this method is relatively easy, finding appropriate weights for each frequency makes it cumbersome. In addition, any change in the boundary conditions will alter the problem and the procedure needs to be repeated. Therefore, there is a need to develop an automatic weight update procedure that guarantees the correlation for the chosen frequency range and the boundary conditions. In the current work, an automatic weight update algorithm that works on the values of the loss functions is proposed. In this algorithm λ_F is chosen as unity and λ_B is updated with each iteration as follows

Algorithm: Training algorithm to update weights

Input: E — // Input number of iterations
$\lambda_F, \lambda_B \leftarrow 1$ — // Initialize weights
$\beta \leftarrow 10^{-3}$ — // Assign hyperparameter
$\epsilon \leftarrow 10^{-3}$ — // Assign tolerance for the weight update

for *iter 1* **to** *E* **do**
  if $\mathcal{L}_F/\lambda_B \leq \epsilon$ **then**
    $\hat{\lambda}_B \leftarrow 1$
  else
    $\hat{\lambda}_B \leftarrow \mathcal{L}_B/\mathcal{L}_F/\epsilon$ — // Update intermediate weight
  end
  $\lambda_B \leftarrow (1-\beta)\lambda_B + \beta\hat{\lambda}_B$ — // Update weight associated with boundary loss
end

Figure 3 shows the comparison of results obtained with the weight update algorithm and the true solution. It can be observed that a good correlation has been achieved by updating the weights based on the loss functions. In other words, the algorithm is able to avoid the biasing behaviour of the training process and is helping to optimise both the loss functions simultaneously. This can be confirmed from the loss function plots draw in Fig. 4 with the automatic weight update algorithm.

Figure 4 shows the loss functions with respect to iterations for various frequencies. It can be observed that unlike with the case of equal weights, $\mathcal{L}_B$ with the weight update algorithm starts reducing with respect to iterations at higher frequencies. This ensures the neural network to learn physics properly from the Helmholtz equation as well as the boundary conditions.

Table 1 shows the percentage of relative error between the prediction solution and the true solution calculated as per Eq. (6) at different frequencies using the two methods

$$\%\,Error = \frac{\left\|\hat{\psi}_t(x;\theta) - \psi(x)\right\|_2}{\left\|\psi(x)\right\|_2} \times 100. \tag{6}$$

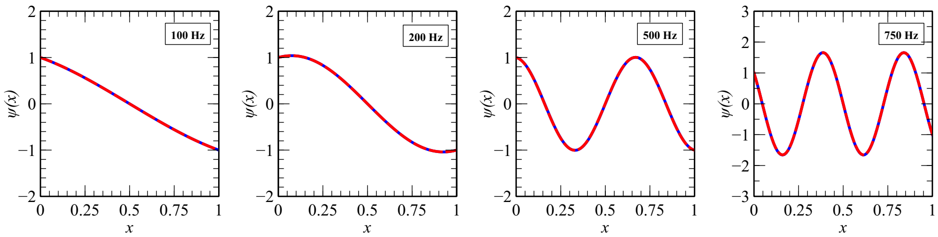

Figure 3: Correlation between true (———) and predicted solution (- - - - -) for different frequencies.

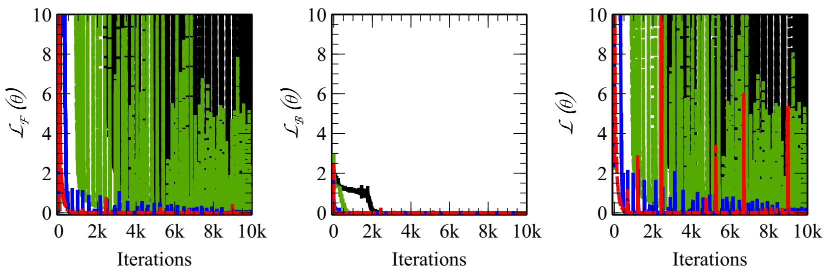

Figure 4: Variation of the loss functions with respect to the iterations: - - - - - 100 Hz, ——— 200 Hz, -·-·-·- 500 Hz, ········ 750 Hz.

Table 1: Comparison of relative error (in percentage) at different frequencies.

Frequency (Hz)	Equal weights	Weight update
100	0.03	0.02
200	91.08	0.01
500	99.99	0.58
750	100	0.31

The weight update algorithm significantly reduces the error at higher frequencies and ensures proper correlation with the true solution.

4 CONCLUSION

The acoustic field in a 1-D duct is estimated using neural network by posing the problem of governing differential equations as an unconstrained optimization problem with two loss functions. One associated with the Helmholtz equation and the other associated with the boundary conditions. The results reveal that at higher frequencies, the training process exhibits biasing behaviour. The optimization process favours the loss function associated with the Helmholtz equation, and leaves that associated with the boundary conditions. This problem is bypassed by introducing weights which adjust their values based on the individual loss functions. The proposed automatic weight update algorithm ensures the boundary loss to converge with respect to the iterations and helps the network to learn the underlying physics properly. Quantitatively, the proposed algorithm keeps the relative error to a maximum value of 0.58% within the frequency range considered for the analysis. The method is having a potential to predict the acoustic field governed by the higher dimensional Helmholtz equation as well as complicated boundary conditions and is need to be explored.

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A Analytical solution

The analytical solution for the 1-D Helmholtz equation

$$\left(\frac{d^2}{dx^2} + k^2\right)\psi(x) = 0, \qquad x \in [0, L] \tag{7}$$

subjected to the boundary conditions

$$\psi(0) = \psi_0; \qquad \psi(L) = \psi_L \tag{8}$$

can be obtained by assuming a solution of the form

$$\psi(x) = Ce^{\gamma x}, \tag{9}$$

where C and γ are the constants.

Upon substituting Eq. (9) in Eq. (7) yields the general solution

$$\psi(x) = C_1 \cos(kx) + C_2 \sin(kx). \tag{10}$$

By substituting the boundary conditions, the constants C_1 and C_2 can be evaluated and the actual solution can be written as

$$\psi(x) = \psi_0 \cos(kx) + \left[\frac{\psi_L - \psi_0 \cos(kL)}{\sin(kL)}\right] \sin(kx). \tag{11}$$