

# 000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 LEARNING A ZEROTH-ORDER OPTIMIZER FOR FINE-TUNING LLMs

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## ABSTRACT

Zeroth-order optimizers have recently emerged as a practical approach for fine-tuning large language models (LLMs), significantly reducing GPU memory consumption compared to traditional first-order methods. Yet, existing zeroth-order methods rely on hand-crafted, static sampling strategies that are not adaptable to model-specific structures. To address this, we propose ZO Fine-tuner, a learning-based zeroth-order optimizer for LLMs that automatically learns efficient perturbation strategies through a compact and memory-efficient design. Crucially, our approach is motivated by the observation that only a small number of foundation models and their derivatives are widely adopted in practice. Therefore, learning the optimizer once for a given LLM and reusing it across diverse downstream tasks is both feasible and highly desirable. Accordingly, ZO Fine-tuner is designed to scale learning to learn (L2L) to the foundation-model era by supporting one-time training per LLM with minimal overhead. Experiments on 4 LLMs and 7 datasets show that ZO Fine-tuner outperforms prior zeroth-order baselines in 82.1% of task-model combinations, thereby demonstrating strong performance and scalability for efficient LLM fine-tuning.

## 1 INTRODUCTION

Nowadays fine-tuning pre-trained foundation models on downstream tasks has become a standard paradigm. However, as model sizes grow, traditional first-order optimizers such as Adam become increasingly expensive. In particular, these methods impose significant memory overhead, up to 12 times (Malladi et al., 2023) more than inference. Even with parameter-efficient fine-tuning (PEFT) methods such as LoRA (Hu et al., 2022) and Prefix-Tuning (Li & Liang, 2021), the overall memory requirement during training remains substantial.

To address these challenges, memory-efficient zeroth-order (MeZO) optimizer (Malladi et al., 2023) has been proposed. This approach only requires two forward passes per step and achieves competitive performance to first-order methods while maintaining memory usage comparable to inference. Many subsequent methods, such as HIZOO (Zhao et al., 2025), LOZO (Chen et al., 2024), MeZO-SVRG (Gautam et al., 2024), ZO-AdamU (Jiang et al., 2023), and ZO-DAP (Ma & Huang, 2025) attempt to improve upon MeZO by manually designing more sophisticated parameter-updating rules. However, these designs are often based on intuition or mathematical approximations, and still typically require extensive hyperparameter tuning beyond learning rates to perform well in practice.

We argue that prior works have largely overlooked the potential of learning to learn (L2L) techniques (Andrychowicz et al., 2016) in this context. Unlike hand-designed optimizers, L2L provides a data-driven approach to automatically learn effective optimization strategies. Rather than manually tuning update rules and hyperparameters, L2L leverages auxiliary neural networks that adaptively guide the optimization process. These learned optimizers typically rely on the same information accessible to conventional optimizers, such as gradient signals or their approximations. By leveraging such inputs, they often outperform their manually designed counterparts in both convergence speed and final performance, as they are able to explore the loss landscape more effectively during optimization (Wichrowska et al., 2017a). For example, learned optimizers have been shown to surpass SGD and even Adam across a variety of models and tasks (Lv et al., 2017a). Similar improvements have also been observed in zeroth-order optimization settings on small-scale models (Ruan et al., 2020).

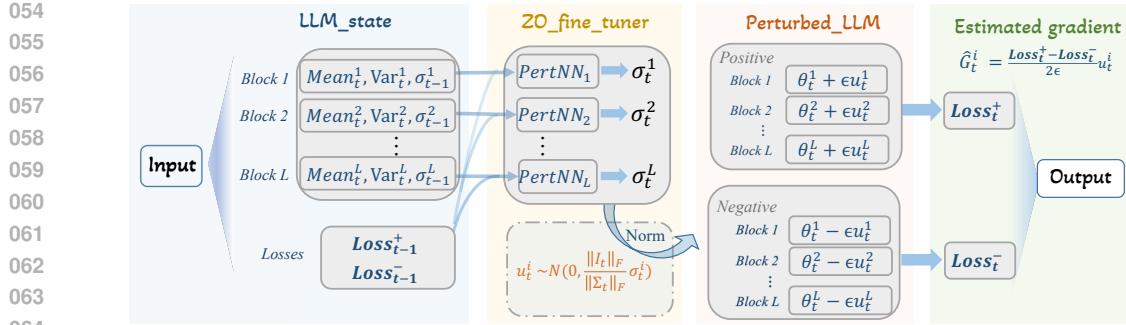


Figure 1: Fine-tune the LLM using trained ZO Fine-tuner. Each block of the LLM is equipped with a lightweight neural network that predicts its perturbation variance. For LLM parameter  $\theta_t^i$  in block  $i$  at step  $t$ , PertNN $_i$  takes in compact summarizing statistics containing the Mean $_t^i$ , Var $_t^i$  of the  $\theta_t^i$ . Additionally, it takes in the last perturbation variance  $\sigma_{t-1}^i$ , and the two losses recorded at the last step. It outputs the updated perturbation variance  $\sigma_t^i$  and then applies normalization. By learning non-uniform, layer-specific perturbation scales and plugging them into standard zeroth-order updates, the fine-tuner enables efficient, high-performance gradient-free optimization of LLM.

While L2L methods have shown promise on small-scale models (Chen et al., 2021), we believe their potential is even greater in the era of foundation models. In the small-model regime, different tasks typically require different models, and L2L optimizers often exhibit limited transferability across model architectures (Wichrowska et al., 2017a). As a result, a separate optimizer must be trained for each model-task pair, leading to substantial additional costs. In contrast, a recent LLM supply chain study shows that while there are many specialized checkpoints on platforms like Huggingface, most are derivatives of a handful of core base models like Llama and Qwen (Shahedur Rahman et al., 2025). Moreover, for a given LLM, the structure or properties leveraged by certain optimizers are often consistent across tasks (Guo et al., 2024). This provides a great opportunity for L2L methods, where **a learned optimizer trained once for a base LLM can be potentially reused across diverse derivative models and tasks**. Toward practical adoption, if model creators were to ship a pretrained learned finetuner alongside each base model, it would unlock a memory-efficient fine-tuning path with competitive performance for downstream users.

In the context of zeroth-order optimization for LLMs, learning a perturbation distribution with non-uniform and adaptive variance scales, rather than relying on a standard normal distribution, could be beneficial (Ye et al., 2018; Gao & Sener, 2022; Zhao et al., 2025). However, the sheer number of parameters of LLMs introduces new challenges when applying L2L as it requires differentiating through the optimization process itself, which demands storing a substantial number of activations for backpropagation. Moreover, naively applying coordinate-wise auxiliary networks at the LLM scale can result in prohibitive memory overhead. To address this, we draw inspiration from a careful theoretical analysis, which suggests that LLMs' approximately block-diagonal Hessian implies that sharing a single variance per block can already yield strong performance gains. We thus propose ZO Fine-tuner, which consists of highly compact and memory-efficient per-parameter-block auxiliary networks that learns shared effective perturbation variances. As a result, the **memory cost is minimal**: for OPT-30B, the total storage required for all auxiliary networks is less than 2MB under FP16 precision, which is negligible compared to the 60GB model itself. In the meantime, through extensive experiments, we demonstrate that our ZO Fine-tuner **trained on a single dataset is highly generalizable across model derivatives and datasets**, which strongly underscores the potential of the “train once, reuse widely” goal. Our contributions are summarized as follows:

- We extend L2L framework to LLMs and show that a single learned optimizer trained on a base model can generalize across downstream tasks and derivative checkpoints.
- Motivated by block-diagonal Hessian structure, we learn a shared perturbation variance per parameter block via compact per-block auxiliary networks. This dramatically reduced memory overhead compared to coordinate-wise or fully connected designs, which made L2L practical at LLM scale.
- Across four models and seven datasets (28 task-model pairs), ZO Fine-tuner outperforms the strongest baseline (lower training loss) in 82.1% of the combinations, achieving an average of 2.5% improvement in accuracy with tiny memory and time overhead.

108 **2 RELATED WORK**  
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110 **Zer0th-order optimization.** Zer0th-order optimization appears in a wide range of applications  
 111 where either the objective function is implicit or its gradient is impossible or too expensive to  
 112 compute. For example, methods (Tang et al., 2021; Hajinezhad & Zavlanos, 2018) consider derivative-  
 113 free distributed algorithms for non-convex multi-agent optimization. ZO-BCD (Cai et al., 2021),  
 114 ZOO (Chen et al., 2017), ZO-signSGD (Liu et al., 2019) and ZO-HessAware (Ye et al., 2019) utilize  
 115 zer0th-order stochastic optimization to generate black-box adversarial example in deep learning.  
 116 Beyond that, MeZO (Malladi et al., 2023) firstly adapted the classical ZO-SGD method to fine-  
 117 tune LLMs, while achieving comparable performance with extremely great memory reduction.  
 118 Subsequently, ZO-AdaMU (Jiang et al., 2023) improved ZO-SGD by incorporating momentum into  
 119 its stochastic approximation process. HIZOO (Zhao et al., 2025) leverages Hessian information to  
 120 enhance performance in a memory-efficient manner. Other works explore structural properties of  
 121 the gradient to improve MeZO, such as utilizing low-rank approximations (Chen et al., 2024) or  
 122 exploiting gradient sparsity (Guo et al., 2024; Liu et al., 2024).

123 **Learning to learn.** Previous studies have investigated using neural networks to improve optimization  
 124 update rules, replacing manually crafted algorithms such as Adam (Kingma & Ba, 2015). (Cotter  
 125 & Conwell, 1990) tried to use recurrent neural networks (RNNs) to model the optimization process  
 126 to learn adaptively. After that, (Baxter, 1998) gave an overview of the idea and techniques of  
 127 learning to learn; for example, they proposed to train RNNs to optimize basic convex functions. Then  
 128 (Andrychowicz et al., 2016; Wichrowska et al., 2017b; Metz et al., 2019; 2022; Lv et al., 2017b)  
 129 introduced a variety of sophisticated strategies to enhance the performance of optimizers in deep  
 130 learning. Additionally, (Li & Malik, 2016) and (Li & Malik, 2017) adopted reinforcement learning  
 131 (RL) policy search techniques into the L2L framework. In the context of zer0th-order optimization,  
 (Ruan et al., 2020) applied L2L techniques to enhance performance on small-scale models.

132 **3 METHOD**  
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134 **3.1 MOTIVATION**  
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136 The foundational work in zer0th-order optimization for LLM fine-tuning, MeZO (Malladi et al.,  
 137 2023), simply estimates the directional derivative as the step size on a certain sampled direction by  
 138 evaluating the model at two perturbed parameter points. This approach only requires two forward  
 139 passes and avoids backpropagation, making it attractive for memory-constrained training. Given a  
 140 model with parameters  $\theta \in \mathbb{R}^d$  and loss function  $\mathcal{L}$ , MeZO estimates the gradient on a mini-batch  $\mathcal{B}_t$   
 141 as:

$$142 \hat{g}(\theta_t; \mathcal{B}_t) = \frac{\mathcal{L}(\theta_t + \epsilon u_t; \mathcal{B}_t) - \mathcal{L}(\theta_t - \epsilon u_t; \mathcal{B}_t)}{2\epsilon} u_t, \quad u_t \sim \mathcal{N}(0, I_d), \quad (1)$$

144 and the model parameter is updated via  $\theta_{t+1} = \theta_t - \eta \hat{g}(\theta_t; \mathcal{B}_t)$ .

145 We argue that a fixed sampling rule from  $\mathcal{N}(0, I)$  is suboptimal: the quality of zer0th-order gradient  
 146 estimates depends on local properties of the landscape at each step (Ye et al., 2018; Gao & Sener,  
 147 2022; Zhao et al., 2025). Therefore, through learning, L2L approach has the potential to generate  
 148 perturbations  $u_t$  that are informed by such local signals and thus allocate perturbation effort more  
 149 effectively. However, naive implementations of this idea can incur prohibitive memory overhead.  
 150 For example, learning a separate perturbation for each individual parameter using a fully connected  
 151 auxiliary network would require at least  $O(d^2)$  parameters for a model with  $d$  parameters.

152 We thus turn to exploit the geometric structure exhibited by LLM. Empirical evidence suggests that  
 153 the Hessian of LLM is approximately block-diagonal, with blocks aligned to natural parameter groups  
 154 (e.g., embeddings, attention Q,K,V matrices, projections, etc.) (Zhang et al., 2024b). This structure  
 155 motivates a coarse control of the perturbation: rather than learning coordinate-wise perturbations,  
 156 we target per-block adaptation. We now formalize this idea in theory, demonstrating that a simple  
 157 adaptive variance change across parameter groups could lead to potential improvements over MeZO.

158 **Theorem 1** (Informal Version). *Define the expected change in loss after performing a one step update  
 159 in parameter  $\theta_t$  as  $d(\theta_t) := \mathbb{E} [\mathcal{L}(\theta_{t+1}) \mid \theta_t] - \mathcal{L}(\theta_t)$ . Suppose now the Hessian matrix  $H(\theta_t)$  is block-  
 160 diagonal  $H(\theta_t) := \text{diag}(H_1(\theta_t), \dots, H_b(\theta_t))$ , then by varying the  $\sigma_i$ 's in  $\Sigma := \text{diag}(\sigma_1 I, \dots, \sigma_b I)$   
 161 the same gradient estimation equation 3.1 but with  $u_t \sim \mathcal{N}(0, \Sigma)$  can yield tighter upper bound on  
 162  $d(\theta_t)$  compared to MeZO.*

The formal version and proof of this theorem can be found in appendix E. At a high level, this theorem states that if the Hessian exhibits a block wise structure, then learning an adaptive per-block shared variance can improve convergence over MeZO. Crucially, the per-block parameterization yields this improvement *without* incurring prohibitive memory cost as the number of parameter blocks is far less than the number of parameters. For instance, in LLaMA-8B, the model contains only 291 parameter blocks, despite having over 8 billion individual parameters. This result thus motivates and justifies our design of ZO Fine-tuner, a per-block variance learner. Below, we first discuss the architecture of our ZO Fine-tuner and how to finetune downstream LLMs using a given ZO Fine-tuner. Then we introduce the training scheme for ZO Fine-tuner to enable generalizations.

### 3.2 ZO FINE-TUNER

**Architecture.** As we discussed in the motivation section, we design ZO Fine-tuner to dynamically generate a block diagonal variance matrix  $\Sigma_t$  corresponding to each parameter group at each optimization step via lightweight neural networks named PertNN. To incorporate all the dynamic information, PertNN takes in model parameters  $\theta_t$ , previously used perturbation variances  $\Sigma_{t-1}$ , and their observed loss as inputs  $\ell_{t-1}$ . Intuitively, these inputs encourage PertNN to consider the effectiveness of past updates, where perturbations that lead to sharper loss changes might indicate more informative directions. However, we notice that the model parameters are still too memory-intensive as an input feature. Therefore, we further compress the memory usage by only feeding the summarizing statistics of the model parameters  $\theta_t$  into PertNN, such as  $\text{Mean}(\theta_t)$  and  $\text{Var}(\theta_t)$ .

Formally, the perturbation variance at each step  $t$  is generated as follows. For parameter block  $i$ ,  $\sigma_{t-1}^{(i)}$  is the previous perturbation variance,  $d_i$  is the number of parameters in this block, and  $\text{Mean}_t^{(i)}$  and  $\text{Var}_t^{(i)}$  represent the current mean and variance of the block's parameter values.  $\omega^{(i)}$  denotes the learnable parameters of the auxiliary neural network assigned to block  $i$ .

$$\begin{aligned}\sigma_t^{(i)} &= \text{PertNN}^{(i)} \left( \ell_{t-1}, \sigma_{t-1}^{(i)}, \text{Mean}_t^{(i)}, \text{Var}_t^{(i)}; \omega^{(i)} \right), \\ \Sigma_t &= \text{diag}(\sigma_t^{(1)} I_{d_1}, \sigma_t^{(2)} I_{d_2}, \dots, \sigma_t^{(n)} I_{d_n}).\end{aligned}\tag{2}$$

With this variance, ZO Fine-tuner then updates model parameters with

$$\begin{aligned}\hat{g}(\theta_t; \mathcal{B}_t; \omega) &:= \frac{\mathcal{L}(\theta_t + \epsilon u_t; \mathcal{B}_t) - \mathcal{L}(\theta_t - \epsilon u_t; \mathcal{B}_t)}{2\epsilon} u_t, \quad u_t \sim \mathcal{N}(0, \Sigma_t) \\ \theta_{t+1} &:= \theta_t - \eta \hat{g}(\theta_t; \mathcal{B}_t)\end{aligned}\tag{3}$$

Importantly, we should note that  $\hat{g}$  is inherently a function of  $u_t$ , which is a function of  $\Sigma_t$ , and thus a function of the parameter of PertNN  $\omega$ . To enable gradient-based training of PertNN within the L2L framework, we adopt the reparameterization trick: instead of sampling  $u_t$  directly from  $\mathcal{N}(0, \Sigma_t)$ , we sample  $z_t \sim \mathcal{N}(0, I_d)$  and compute  $u_t = \Sigma_t^{1/2} z_t$ . This makes the entire perturbation process differentiable, allowing gradients to flow back through the perturbation generation module.

**Normalization.** Although effective, this non-uniform variance introduced a new challenge when using ZO Fine-tuner as an optimizer. From the two-point ZO estimator, we see that

$$\mathbb{E}[\hat{g}(\theta_t; \mathcal{B}_t)] = \mathbb{E}\left[\frac{\mathcal{L}(\theta_t + \epsilon u_t; \mathcal{B}_t) - \mathcal{L}(\theta_t - \epsilon u_t; \mathcal{B}_t)}{2\epsilon}\right] \approx \mathbb{E}[u_t u_t^\top] \nabla \mathcal{L}(\theta_t; \mathcal{B}_t).\tag{4}$$

Therefore, when fine-tuning downstream tasks, we note that the effective learning rate became  $\eta \cdot \frac{\|u_t\|^2}{d}$  on average. This makes controlling the effective learning rate difficult, and the learned variance  $\Sigma_t$  became a confounding variable in the update size. In reality, we wish  $\Sigma_t$  to only carry information about relative block-wise variance, and we could still use a single learning rate to control the overall step size to ensure stable training. Therefore, we introduce the following normalization, which ensures the decoupling of the variance and the learning rate. We note that if  $u_t = \Sigma_t^{1/2} z_t$  with  $z_t \sim \mathcal{N}(0, I)$ , then  $\mathbb{E}\|u_t\|^2 = \text{tr}(\Sigma_t \Sigma_t^\top) = \|\Sigma_t\|^2$ . We then normalize by fixing the total variance budget and let  $\|\Sigma_t\|_F^2 = \|I_d\|_F^2 = d$ . Thus, only the relative block-wise variances are learned. In practice, this keeps  $\|u_t\|$  approximately constant (by concentration in high dimensions). For example, with our generated  $\Sigma_t$ , if  $u_t \sim \mathcal{N}(0, \Sigma_t)$ ,  $\|u_t\|$  concentrates around  $\|\Sigma_t\|_F$  and we achieve the desired control over the effective learning rate.

216 **Complete Optimization Algorithm.** As summarized in Algorithm 1 and Figure 1, ZO Fine-tuner  
 217 first compute the block-wise non-uniform perturbation variance  $\Sigma_t$  using the learned neural network  
 218 PertNN. Then it applies normalization to control the overall magnitude of the perturbation. Finally,  
 219 it uses the normalized perturbation to update the LLM following equation 3. We notice this incurs  
 220 minimal overhead compared to MeZO, in terms of both memory and speed. In particular, the only  
 221 memory overhead compared to MeZO is the light-weight per-block PertNN, whereas the only speed  
 222 overhead is the query to PertNN.

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**Algorithm 1** Finetuning a LLM with ZO Fine-tuner
 

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225 **Require:** LLM parameters  $\theta$ , PertNN parameters  $\omega$ , training step  $T$ , learning rate  $\eta$   
 226 1: Initialize variance  $\Sigma_0$  as  $I_d$ , LLM parameter as  $\theta_0$ .  
 227 2: **for**  $t = 1, \dots, T$  **do**  
 228 3:   Sample a batch  $\mathcal{B}_t$  from  $\mathcal{T}$   
 229 4:    $\Sigma_t \leftarrow \text{PertNN}(\theta_t, \Sigma_{t-1}, \ell_{t-1}; \omega)$   
 230 5:   Sample  $u_t \sim N(0, \frac{\|I_d\|_F}{\|\Sigma_t\|_F} \Sigma_t)$  ▷ Sample after normalization  
 231 6:   Compute LLM loss with perturbed parameter to obtain  $\ell_t$   
 232 7:    $\hat{g}_t = \frac{\ell_t^+ - \ell_t^-}{2\epsilon} u_t$   
 233 8:    $\theta_{t+1}^t = \theta_t - \eta \hat{g}_t$   
 234 9: **end for**

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 236 3.3 TRAINING ZO FINE-TUNER
 

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238 We now turn to training ZO Fine-tuner in a L2L fashion. The key idea is to treat the model’s  
 239 own finetuning trajectory as supervision. After a single update by ZO Fine-tuner, we evaluate the  
 240 post-update loss and adjust PertNN so as to reduce this quantity across tasks. We next formalize this  
 241 meta-objective and outline several practical choices that make training stable.

242 **Data Source and Objective Function.** First, we need a source of training data for our ZO Fine-tuner.  
 243 In our setting, this data corresponds to different model states with various losses. A key insight of us  
 244 is to notice that the fine-tuning process of LLMs under a first-order optimizer naturally produces a  
 245 trajectory of intermediate model states, and we can directly leverage this trajectory to optimize the  
 246 perturbation variance generator.

247 Along the first order optimization trajectory with loss function  $\mathcal{L}$ , we obtain a set of model parameters  
 248  $\{\theta_0^k\}_k$ . We then attempt to perform a one-step zeroth-order update using our ZO Fine-tuner with  
 249 update rule 3 to get  $\theta_1^k$  and use the resulting loss as a feedback signal to assess and optimize the  
 250 effectiveness of the current perturbation strategy. Specifically, at each step we aim to minimize the  
 251 post-update loss  $\mathcal{L}(\theta_1^k)$ . As we discussed in section 3.2, the estimated gradient  $\hat{g}$  is implicitly a  
 252 differentiable function of the parameters  $\omega$  of PertNN per the reparametrization trick. Therefore, we  
 253 can use a gradient-based method to update ZO Fine-tuner. Formally, the objective for training ZO  
 254 Fine-tuner is therefore:

$$\min_{\omega} \mathcal{L}_{\text{ZO}}(\theta_0^k; \omega) := \min_{\omega} \mathcal{L}(\theta_1^k) = \min_{\omega} \mathcal{L}(\theta_0^k - \eta \hat{g}(\theta_0^k, \omega)) \quad (5)$$

255 After the update, we move the parameters  $\theta_0^k$  along the first-order trajectory to get  $\theta_0^{k+1}$  and continue  
 256 learning. As the inputs to ZO Fine-tuner are task and model-agnostic state summaries, rather than  
 257 task-specific features, the learned decisions are largely invariant to differences across datasets or  
 258 nearby checkpoints. As we will demonstrate in experiments, our ZO Fine-tuner trained on one single  
 259 dataset can be transferred to efficiently finetune other datasets and model derivatives.

260 **Periodic Reset of Model Parameters.** During the training of our ZO Fine-tuner, a lot of data needs to  
 261 be generated. However, since the optimizer is trained along the fine-tuning trajectory of a model using  
 262 a first-order optimizer, the auxiliary network tends to receive inputs that are chronologically ordered.  
 263 In particular, it will get more data from the low-loss region. As a result, it may lead to overfitting to  
 264 the low-loss region of the parameters while learning the crucial high-loss region insufficiently.

265 To address this issue, we introduce a periodic re-initialization mechanism. After each complete  
 266 optimization cycle or when the loss has sufficiently decreased, we reset the model parameters to their  
 267 original pre-finetuning state and restart the fine-tuning process. This approach introduces diversity  
 268 into the input distribution by exposing the optimizer to model states from multiple phases of training.

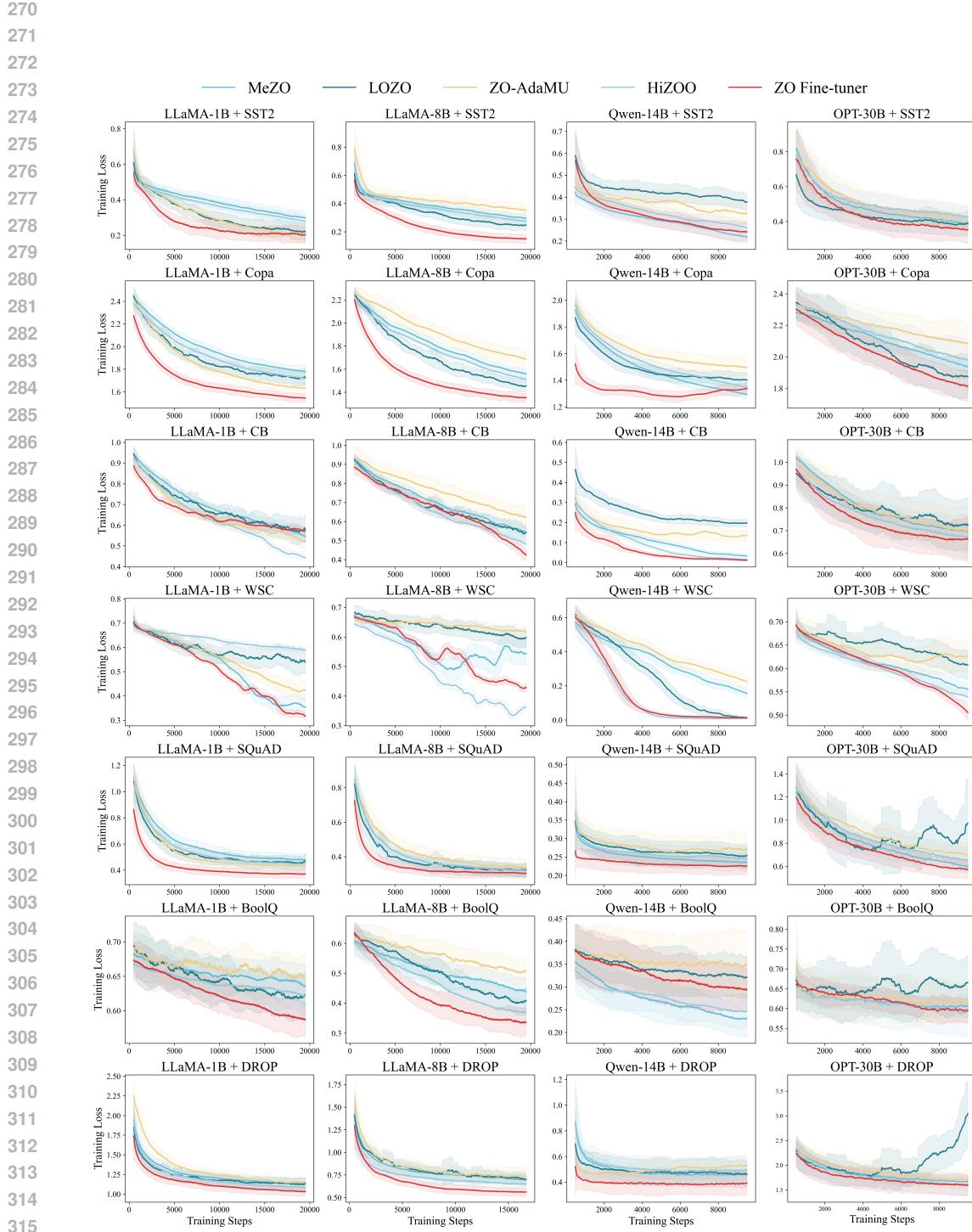


Figure 2: Loss comparison across different methods on various datasets and LLMs. Models (columns) are LLaMA-3.2-1B, LLaMA-3.1-8B, Qwen2.5-14B and OPT-30B, while datasets (rows) cover COPA, SST-2, CB, SQuAD, WSC, BoolQ and DROP. All curves use the best hyperparameters found for each method. The shaded region around each curve shows the standard deviation of the smoothed loss—the wider the shade, the larger the fluctuation. ZO Fine-tuner shows advantages in both convergence speed and final loss value across most settings.

## Algorithm 2 Learning to Learn Framework

**Complete Learning to Learn Framework for ZO Fine-tuner Training.** Algorithm 2 illustrates the complete L2L framework for training ZO Fine-tuner. At each training step, we sample a training dataset and a batch from this dataset to perform a one-step update to ZO Fine-tuner as described above. Moreover, we periodically reset the model parameters to mitigate the bias discussed previously. Despite the complexity of this training algorithm, we would like to emphasize that it is a *one-time* cost: once ZO Fine-tuner is learned, deployment reduces to Algorithm 1.

## 4 EXPERIMENT

Following MeZO Malladi et al. (2023), we evaluate ZO Fine-tuner with four LLMs: LLaMA-3.2-1B (Grattafiori et al., 2024), LLAMA-3.1-8B (Grattafiori et al., 2024), Qwen2.5-14B (Bai et al., 2023), and OPT-30B (Zhang et al., 2022) using seven diverse benchmark datasets including SST-2 (Socher et al., 2013), CB (De Marneffe et al., 2019), COPA (Roemmele et al., 2011), BoolQ (Clark et al., 2019), WSC (Levesque et al., 2012), SQuAD (Rajpurkar et al., 2016), and DROP (Dua et al., 2019).

We compare our approach against four representative zeroth-order optimization baselines for LLM fine-tuning: HIZOO (Zhao et al., 2025), LÖZO (Chen et al., 2024), MeZO and MeZO-Adam (Malladi et al., 2023). Due to computational resource constraints, we replace the expensive MeZO-Adam with a more efficient variant MeZO-AdamU (Jiang et al., 2023) for models larger than LLaMA-3.2-1B. To ensure a fair comparison, we perform the same grid search over learning rates for each method and pick the best learning rate when reporting. More details can be found in appendix C.2.

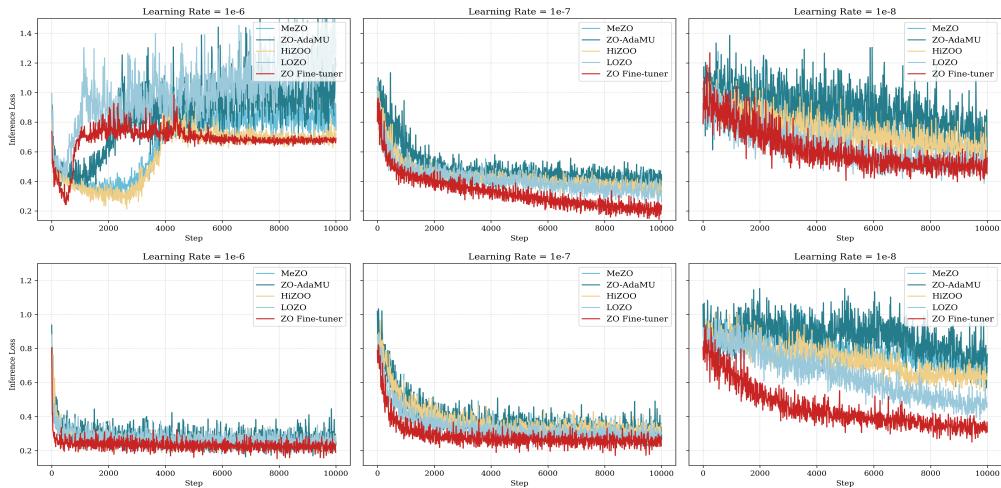
For our ZO Fine-tuner, we train it once using algorithm 2 on the COPA dataset. This choice is mainly due to COPA’s consistently smooth loss decrease during standard fine-tuning. Its small size and also yield fast cycles. **Unless otherwise noted, the ZO Fine-tuner trained on COPA is reused as is across all other tasks and models.** In appendix D.5, we also discussed more about multi-dataset training. Other hyperparameters and training details can be found in section C.3.

## 4.1 MAIN RESULTS

**Generalization Across Datasets.** Figure 2 compares convergence across all 28 dataset-model pairs using each method’s best learning rate. We observe that ZO Fine-tuner (red) consistently reaches lower loss faster. The effect is especially clear on SST-2, CB, COPA, SQuAD, and DROP, where curves descend more steeply early on and settle at a better plateau. In addition, we report the final loss and accuracy values for all 28 combinations in Table 1. On average, ZO Fine-tuner achieves an average accuracy improvement of 2.5% over MeZO. Overall, Our method outperforms the baselines in 75.0% of the task-model combinations in accuracy and 82.1% in the converged loss. These results indicate strong generalization capability of ZO Fine-tuner, as training ZO Fine-tuner on a single COPA dataset already yields consistent gains across datasets.

378  
 379 Table 1: Average training loss in the final epoch and accuracy on seven datasets for each method and  
 380 model combination under the best hyperparameter. We report both loss (↓) and accuracy / F1 (↑)  
 381 across tasks of diverse formats to evaluate the overall performance.

382 Model	383 Method	COPA		SST-2		CB		SQuAD		WSC		BoolQ		DROP	
		384 Loss	385 Acc	386 Loss	387 Acc	388 Loss	389 Acc	390 Loss	391 F1	392 Loss	393 Acc	394 Loss	395 Acc	396 Loss	397 Acc
388 LLaMA-3.2-1B	389 MeZO	1.77	0.75	0.29	0.90	0.55	0.70	0.48	0.75	0.35	<b>0.62</b>	0.63	0.63	1.16	0.29
	390 MeZO-Adam	1.62	0.79	0.20	0.92	0.53	0.66	0.41	<b>0.78</b>	0.42	0.61	0.66	0.62	1.14	0.29
	391 HIZOO	1.71	0.78	0.27	0.90	<b>0.44</b>	<b>0.71</b>	0.43	0.75	0.55	0.54	0.62	0.61	1.09	0.29
	392 LOZO	1.72	0.74	0.20	0.92	0.58	0.64	0.47	<b>0.78</b>	0.51	0.61	0.62	0.64	1.15	0.32
	393 ZO Fine-tuner	<b>1.54</b>	<b>0.80</b>	<b>0.14</b>	<b>0.93</b>	0.57	0.67	<b>0.37</b>	<b>0.78</b>	<b>0.31</b>	0.56	<b>0.58</b>	<b>0.66</b>	<b>1.03</b>	<b>0.35</b>
394 LLaMA-3.1-8B	395 MeZO	1.54	0.92	0.29	0.92	0.54	0.71	0.32	0.89	0.55	0.63	0.42	0.78	0.69	0.64
	396 MeZO-AdamU	1.67	0.89	0.36	0.92	0.61	0.70	0.35	0.86	0.61	<b>0.64</b>	0.50	0.75	0.73	0.59
	397 HIZOO	1.50	<b>0.93</b>	0.27	0.92	0.47	0.71	0.32	0.88	<b>0.36</b>	0.62	0.36	0.79	0.64	0.60
	398 LOZO	1.46	0.89	0.25	<b>0.94</b>	0.54	0.70	0.33	<b>0.90</b>	0.61	0.63	0.41	0.83	0.74	0.65
	399 ZO Fine-tuner	<b>1.35</b>	0.91	<b>0.18</b>	<b>0.94</b>	<b>0.26</b>	<b>0.76</b>	<b>0.31</b>	<b>0.90</b>	0.44	0.62	<b>0.34</b>	<b>0.87</b>	<b>0.54</b>	<b>0.66</b>
400 Qwen2.5-14B	401 MeZO	1.28	0.86	<b>0.21</b>	0.88	0.05	<b>0.93</b>	0.24	0.88	0.18	0.76	<b>0.23</b>	0.84	0.45	0.66
	402 MeZO-AdamU	1.43	0.85	0.35	0.89	0.13	0.91	0.28	0.90	0.25	0.75	0.35	0.84	0.50	0.64
	403 HIZOO	<b>1.34</b>	0.87	0.26	0.93	<b>0.03</b>	0.89	0.24	0.89	<b>0.02</b>	<b>0.79</b>	0.25	0.86	0.49	0.68
	404 LOZO	1.40	0.91	0.38	0.93	0.19	0.91	0.26	0.90	0.04	<b>0.79</b>	0.32	0.86	0.46	0.67
	405 ZO Fine-tuner	<b>1.34</b>	<b>0.92</b>	0.24	<b>0.94</b>	<b>0.03</b>	<b>0.93</b>	<b>0.22</b>	<b>0.91</b>	<b>0.02</b>	0.76	0.29	<b>0.89</b>	<b>0.40</b>	<b>0.70</b>
406 OPT-30B	407 MeZO	1.93	0.83	0.38	0.89	0.69	0.64	0.59	0.74	0.55	<b>0.63</b>	<b>0.60</b>	0.66	1.66	<b>0.31</b>
	408 MeZO-AdamU	2.07	0.80	0.43	0.84	0.70	0.66	0.67	0.73	0.62	<b>0.63</b>	0.62	0.66	1.70	0.30
	409 HIZOO	1.97	0.81	0.43	0.86	0.67	0.66	0.65	0.75	0.53	0.61	0.62	0.65	1.61	0.30
	410 LOZO	1.86	0.82	0.40	<b>0.90</b>	0.73	0.64	0.96	0.75	0.58	0.62	0.70	0.66	2.83	0.27
	411 ZO Fine-tuner	<b>1.81</b>	<b>0.85</b>	<b>0.35</b>	0.87	<b>0.66</b>	<b>0.70</b>	<b>0.56</b>	<b>0.77</b>	<b>0.51</b>	0.60	0.61	<b>0.67</b>	<b>1.59</b>	<b>0.31</b>



416 Figure 3: Loss curves under varying learning rates for different optimizers on (top) SST2 with  
 417 LLaMA-3.1-8B, and (bottom) SQuAD with Qwen2.5-14B.

420 **Generalization Across Model Derivatives.** We further investigate the generalization capability of  
 421 ZO Fine-tuner to derived models. We take the ZO Fine-tuner trained with LLaMA-3.1-8B and use  
 422 it to finetune Llama-3.1-8B-Instruct. As can be seen in table 3, ZO Fine-tuner can also generalize  
 423 to effectively finetune derived models across a single model family. On both datasets evaluated,  
 424 ZO Fine-tuner beats MeZO both in terms of both average loss and final accuracy. Practically, this  
 425 supports the *train-once, reuse-across-derivatives* paradigm we have mentioned. If model developers  
 426 could release a pretrained finetuner with each base model, then model users can then efficiently  
 427 finetune the model further on derivative checkpoints with near-inference memory.

## 4.2 ABLATION STUDIES

431 **Learning Rate.** Figure 3 further demonstrates the sensitivity of different methods to the choice of  
 432 learning rate. Notably, ZO Fine-tuner often achieves comparable loss at a learning rate of  $1 \times 10^{-8}$  to

432  
 433 Table 2: Ablation results on Normalization and Periodic Reset. We report the final loss and the  
 434 final accuracy. Consistently, both techniques individually improve performance across models and  
 435 datasets, and combining them achieves the best results.

436 <b>Setting</b>	437 <b>LLaMA-8B + SQuAD</b>	438 <b>LLaMA-8B + SST2</b>	439 <b>Qwen-14B + SQuAD</b>	440 <b>Qwen-14B + SST2</b>
437 Base	438 0.3950 / 0.840	439 0.3976 / 0.874	440 0.3582 / 0.844	441 0.4086 / 0.800
437 Reset alone	438 0.3682 / 0.856	439 0.3891 / 0.881	440 0.3551 / 0.851	441 0.4039 / 0.810
437 Normalization alone	438 0.3071 / 0.899	439 0.3061 / 0.920	440 0.2380 / 0.904	441 0.3885 / 0.844
437 Reset+Normalization	438 <b>0.3065 / 0.905</b>	439 <b>0.1789 / 0.941</b>	440 <b>0.2246 / 0.911</b>	441 <b>0.2403 / 0.935</b>

441 Table 3: We demonstrate that the ZO  
 442 Fine-tuner trained from LLaMA-3.1-  
 443 8B generalizes well to LLaMA-3.1-8B-  
 444 Instruct. Across datasets it outperforms  
 445 MeZO in final loss and accuracy.

446 <b>Method</b>	447 <b>Dataset</b>	448 <b>Loss / Acc</b>
448 SST2	MeZO	0.276 / 0.92
	ZO Fine-tuner	<b>0.164 / 0.95</b>
449 SQuAD	MeZO	0.291 / 0.90
	ZO Fine-tuner	<b>0.287 / 0.92</b>

446  
 447 Table 4: Ablation results on parameter sharing strategy.  
 448 We compare our block-wise scheme to a simpler layer-  
 449 wise baseline. As shown below, block-wise sharing con-  
 450 stently achieves lower final loss and higher accuracy.

451 <b>Model</b>	452 <b>Sharing</b>	453 <b>SST2 Loss / Acc</b>	454 <b>SQuAD Loss / Acc</b>
452 LLaMA-8B	layer wise	0.23 / 0.92	0.32 / 0.88
	block wise	<b>0.18 / 0.94</b>	<b>0.31 / 0.90</b>
453 Qwen-14B	layer wise	0.27 / 0.91	0.25 / 0.88
	block wise	<b>0.24 / 0.94</b>	<b>0.22 / 0.91</b>

455 that of baseline methods operating at  $1 \times 10^{-7}$ . When the learning rate further increases to  $1 \times 10^{-6}$ ,  
 456 many baseline methods suffer from instability and fail to converge, falling short of the performance  
 457 that ZO Fine-tuner achieves at  $1 \times 10^{-7}$ . More results can be found in D.1.

458 **Normalization & Periodic Reset.** We also conduct experiments to evaluate the effectiveness of our  
 459 design choices including normalization introduced in section 3.2 and periodic reset in section 3.3.  
 460 From table 2, it is clear that both normalization and periodic reset helps ZO Fine-tuner for achieving  
 461 better performance.

462 **Parameter Sharing Strategy.** Finally, we evaluate the granularity of sharing in ZO Fine-tuner,  
 463 comparing our *block-wise* scheme to a simpler *layer-wise* sharing baseline. As shown in Table 4,  
 464 block-wise sharing consistently achieves lower final loss and higher accuracy. Importantly, this choice  
 465 is theory-driven: when the Hessian is (approximately) block-diagonal, theorem 1 indicates that the  
 466 natural unit for variance sharing is the Hessian block itself.

#### 467 4.3 MEMORY USAGE AND TIME EFFICIENCY ANALYSIS

468 **Memory Usage.** The memory overhead of ZO Fine-tuner when using to finetune LLMs mainly  
 469 comes from the additional memory taken by PertNN. However, the parameter number of our ZO  
 470 Fine-tuner is extremely small, even negligible compared to the LLMs. Consequently, the memory  
 471 footprint of our method remains essentially identical to that of MeZO. Under equivalent experimental  
 472 settings, it requires only 1/4 of the memory overhead incurred by Adam. For example, MeZO and ZO  
 473 Fine-tuner peak at 61GB and 62GB of GPU memory when fine-tuning OPT-30B, whereas first-order  
 Adam reaches 312GB with FP16. More details can be found in D.3.

474 **Time Efficiency.** Similarly, the time overhead comes directly from the query to PertNN. However,  
 475 this overhead is typically minimal. For example, when fine-tuning on DROP using LLaMA-3.2-1B  
 476 on an L40S GPU with a batch size of 16, the generation of perturbation takes only 0.025 seconds,  
 477 while all other operations take approximately 0.70 seconds. This means our method introduces  
 478 less than 3.4% additional overhead, demonstrating strong time efficiency. This overhead becomes  
 479 even less significant with larger models. For instance, under the same setting with LLaMA-3.1-8B,  
 480 perturbation generation takes only 0.052 seconds compared to a total runtime of 3.14 seconds. More  
 481 details can be found in D.2.

## 482 5 CONCLUSION

483 We introduced ZO Fine-tuner, a learning-to-learn zeroth-order optimizer that uses adaptive, per-block  
 484 perturbation variances. The finetuner trained once on a single dataset is demonstrated to transfer  
 485 across tasks and to finetuned derivatives, supporting a practical “train once, reuse widely” path.

## 486 ETHICS STATEMENT

487  
488 This work does not raise any known ethical concerns, as far as the authors concern.

## 489 REPRODUCIBILITY STATEMENT

490  
491 Our code repository is available at [https://anonymous.4open.science/r/ZO\\_Fine\\_](https://anonymous.4open.science/r/ZO_Fine_tuner_ICLR-F69A)  
492 tuner\_ICLR-F69A. We have also more detailedly discuss the hyperparameters and experiment  
493 setting in appendix C.2 and C.3.

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648 **A LLM USAGE**649  
650 We used LLM only to polish writing and retrieve related works for this work.651 **B DISCUSSIONS**653 **B.1 LIMITATIONS AND FUTURE WORK**655 The coordinate-wise structure has already been shown to be effective and reasonable in the era of L2L  
656 for zeroth-order optimizer on small-scale models (Ruan et al., 2020). While our current design uses a  
657 diagonal variance matrix  $\Sigma_t$  for its memory efficiency and strong empirical performance, exploring  
658 non-diagonal structures is a potential improvement, though it may require additional techniques to  
659 mitigate the associated memory overhead.660 Moreover, more properties of LLM gradient and Hessian could be potentially exploited. For example,  
661 Chen et al. (2024); Sun et al. (2025) explicitly exploits the low-rank structure of LLM gradients. A  
662 potential future direction is to leverage these properties to generate more informed perturbations or  
663 cut the memory usage even more.665 **C IMPLEMENTATION DETAILS**666 **C.1 DATASETS AND MODELS**668 We evaluate all optimizers on seven NLP tasks spanning multiple formats, including natural language  
669 inference, question answering, and commonsense reasoning. SST-2 (Socher et al., 2013) is a binary  
670 sentiment classification benchmark from the GLUE suite. CB (De Marneffe et al., 2019) and  
671 COPA (Roemmele et al., 2011) are low-resource natural language inference tasks from SuperGLUE,  
672 requiring models to recognize textual entailment or choose causal relationships. BoolQ (Clark et al.,  
673 2019) involves answering yes/no questions given short passages. WSC (Levesque et al., 2012)  
674 tests pronoun resolution in challenging coreference contexts. SQuAD (Rajpurkar et al., 2016) and  
675 DROP (Dua et al., 2019) are span-based question answering datasets that require locating answer  
676 spans in context paragraphs. For most classification tasks, we report accuracy as the evaluation metric.  
677 For SQuAD and DROP, we follow standard practice and report F1 score to better capture partial  
678 match quality.679 We evaluate our optimizers on four representative large language models with diverse architectures  
680 and scales: LLaMA-3.2-1B (Grattafiori et al., 2024), LLaMA-3.1-8B (Grattafiori et al., 2024),  
681 Qwen2.5-14B (Bai et al., 2023), and OPT-30B (Zhang et al., 2022).683 **C.2 HYPERPARAMETERS**685 We use a two-layer MLP with 64 hidden units and a tanh activation function as the auxiliary neural  
686 network for each parameter block. Table 5 presents the hyperparameter search grids used in our  
687 experiments to facilitate reproducibility. We primarily perform a grid search over three learning rate  
688 values:  $10^{-4}$ ,  $10^{-5}$ , and  $10^{-6}$  for MeZO-Adam, and  $10^{-6}$ ,  $10^{-7}$ , and  $10^{-8}$  for all other methods.  
689 For the WSC task, we additionally include  $3 \times 10^{-7}$ , as most methods exhibit slow loss decay at  
690  $10^{-7}$  and become unstable when using  $10^{-6}$ . We run 20,000 optimization steps on LLaMA-1B  
691 and LLaMA-8B, and 10,000 steps on Qwen-14B and OPT-30B due to resource limitation. A batch  
692 size of 16 is used for all models by default, except for OPT-30B, where we reduce it to 4 due to  
693 GPU memory constraints. Also due to computational resource constraints, we replace the expensive  
694 MeZO-Adam with its more efficient variant MeZO-AdamU (Jiang et al., 2023) for models larger  
695 than LLaMA-1B. As shown in the hyperparameter table, our method, together with MeZO, requires  
696 the smallest number of tunable hyperparameters among all baselines.697 **C.3 LEARNING TO LEARN DETAILS**698  
699 In Section 3.3, we introduced our learning to learn framework. Here, we elaborate on additional  
700 implementation details. We use a two-layer MLP with 64 hidden units and a tanh activation function  
701 as the auxiliary neural network for each parameter block. Empirically, we set  $\epsilon = 10^{-3}$ ,  $\eta_1 = 10^{-6}$ ,  
and  $\eta_2 = 10^{-2}$  in Algorithm 2. When the task list  $\mathcal{T}_{\text{list}}$  contains only a single task, the framework

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704 Table 5: Hyperparameter configurations for ZO Fine-tuner and all baseline methods.  
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Method	Hyperparameters	Values
MeZO	Batch size Learning rate $\epsilon$	16 for LLaMA-1B/8B/Qwen-14B; 4 for OPT-30B $\{10^{-6}, 10^{-7}, 10^{-8}\}$ (plus $3 \times 10^{-7}$ for WSC only) $10^{-3}$
MeZO-Adam	Batch size Learning rate $\epsilon$ $\epsilon_{\text{Adam}}$	16 $\{10^{-4}, 10^{-5}, 10^{-6}\}$ (plus $3 \times 10^{-6}$ for WSC only) $10^{-3}$ $\{10^{-6}, 10^{-7}, 10^{-8}\}$
ZO-AdamU	Batch Size Learning Rate $\epsilon$ $\alpha$ $\beta^{(1)}$ $\beta^{(2)}$	16 for LLaMA-1B/8B/Qwen-14B; 4 for OPT-30B $\{10^{-6}, 10^{-7}, 10^{-8}\}$ (plus $3 \times 10^{-7}$ for WSC only) $10^{-3}$ $\{0.2, 0.5, 0.7\}$ $\{0.9, 0.8, 0.7\}$ $\{0.01, 0.05, 0.1\}$
HIZOO	Batch Size Learning Rate $\epsilon$ Smooth Constant	16 for LLaMA-1B/8B/Qwen-14B; 4 for OPT-30B $\{10^{-6}, 10^{-7}, 10^{-8}\}$ (plus $3 \times 10^{-7}$ for WSC only) $10^{-3}$ $\{10^{-7}, 10^{-8}\}$
LOZO	Batch Size Learning Rate $\epsilon$ Rank ( $r$ ) Interval ( $\nu$ )	16 for LLaMA-1B/8B/Qwen-14B; 4 for OPT-30B $\{10^{-6}, 10^{-7}, 10^{-8}\}$ (plus $3 \times 10^{-7}$ for WSC only) $10^{-3}$ $\{2, 4\}$ $\{50, 100\}$
ZO Fine-Tuner	Batch Size Learning Rate $\epsilon$	16 for LLaMA-1B/8B/Qwen-14B; 4 for OPT-30B $\{10^{-6}, 10^{-7}, 10^{-8}\}$ (plus $3 \times 10^{-7}$ for WSC only) $10^{-3}$

730  
731 reduces to single-dataset training as a special case. We find that training on a single dataset can yield  
732 competitive performance with reduced cost. In our experiments, the optimizer is trained on COPA. A  
733 comparison between single-dataset and multi-dataset training results is provided in Section D.5. We  
734 also block certain gradient flows to reduce memory consumption during learning-to-learn. Specifically,  
735 recall that  
736

$$737 \hat{g}(\theta_t; \omega) = \frac{\mathcal{L}(\theta_t + \epsilon u_t) - \mathcal{L}(\theta_t - \epsilon u_t)}{2\epsilon} u_t \\ 738 u_t = \text{PertNN}(\theta_t, \Sigma_{t-1}, \ell_{t-1}; \omega) z_t, \quad z_t \sim \mathcal{N}(0, I_d).$$

739 The gradient of the ZO loss, defined as  $\mathcal{L}_{\text{ZO}}(\theta; \omega) := \mathcal{L}(\theta - \eta \hat{g}(\theta; \omega))$ , propagates first to  $\hat{g}(\theta_t; \omega)$ ,  
740 and then further through both components used to construct it: the perturbation direction  $u_t$  and  
741 the finite-difference estimator  $\frac{\mathcal{L}(\theta_t + \epsilon u_t) - \mathcal{L}(\theta_t - \epsilon u_t)}{2\epsilon}$ . To save memory, we cut off the gradient flow  
742 through the finite-difference term, which eliminates the need to back-propagate through the inner loss  
743 evaluations and store their activations. Despite this approximation, we still observe strong empirical  
744 performance.  
745

## 746 D ADDITIONAL EXPERIMENTAL RESULTS

### 747 D.1 ADDITIONAL RESULTS ON LEARNING RATE SENSITIVITY

748 We previously presented the sensitivity of different optimization methods to the learning rate in  
749 Section 4.2. Due to space constraints, only a subset of the results was shown. Here, we provide the  
750 complete loss curves across the three benchmarks SQuAD, SST-2, and COPA using LLaMA-1B,  
751 LLaMA-8B, Qwen-14B, and OPT-30B, as shown in Figure 4 and Figure 5.  
752

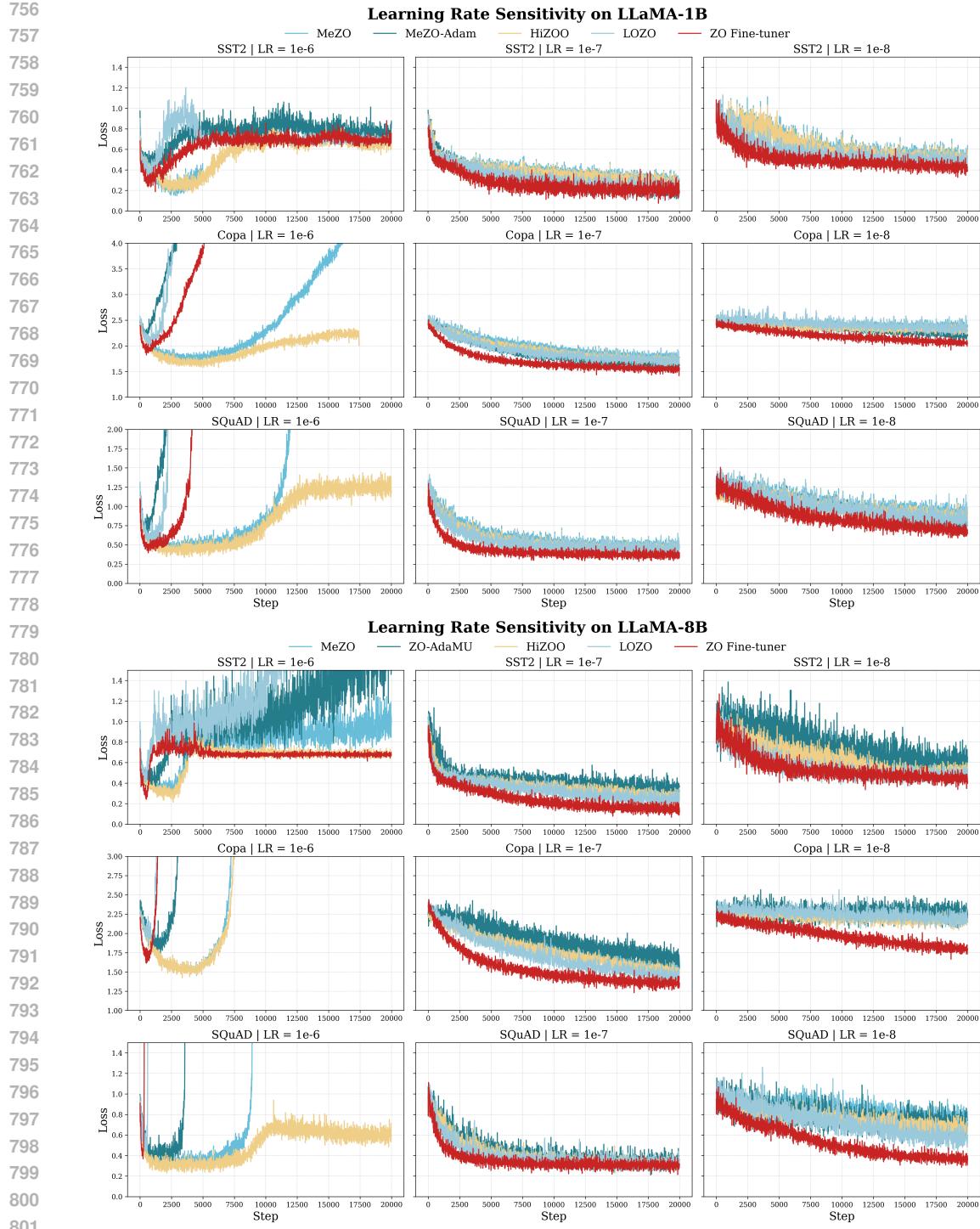


Figure 4: Loss curves under varying learning rates for different optimizers with LLaMA-1B (top) and Qwen-14B (bottom). We report results on SST2, Copa, and SQuAD. For MeZO-Adam, note that the actual learning rates used were  $10^{-4}$ ,  $10^{-5}$ , and  $10^{-6}$ , corresponding to the plotted values of  $10^{-6}$ ,  $10^{-7}$ , and  $10^{-8}$ , respectively.

Across the grid search over learning rates  $10^{-6}$ ,  $10^{-7}$ , and  $10^{-8}$ , the ZO Fine-tuner consistently achieves superior results compared to all baselines when comparing their best-performing settings. On LLaMA-1B, LLaMA-8B, Qwen-14B and OPT-30B, our method exhibits faster convergence and

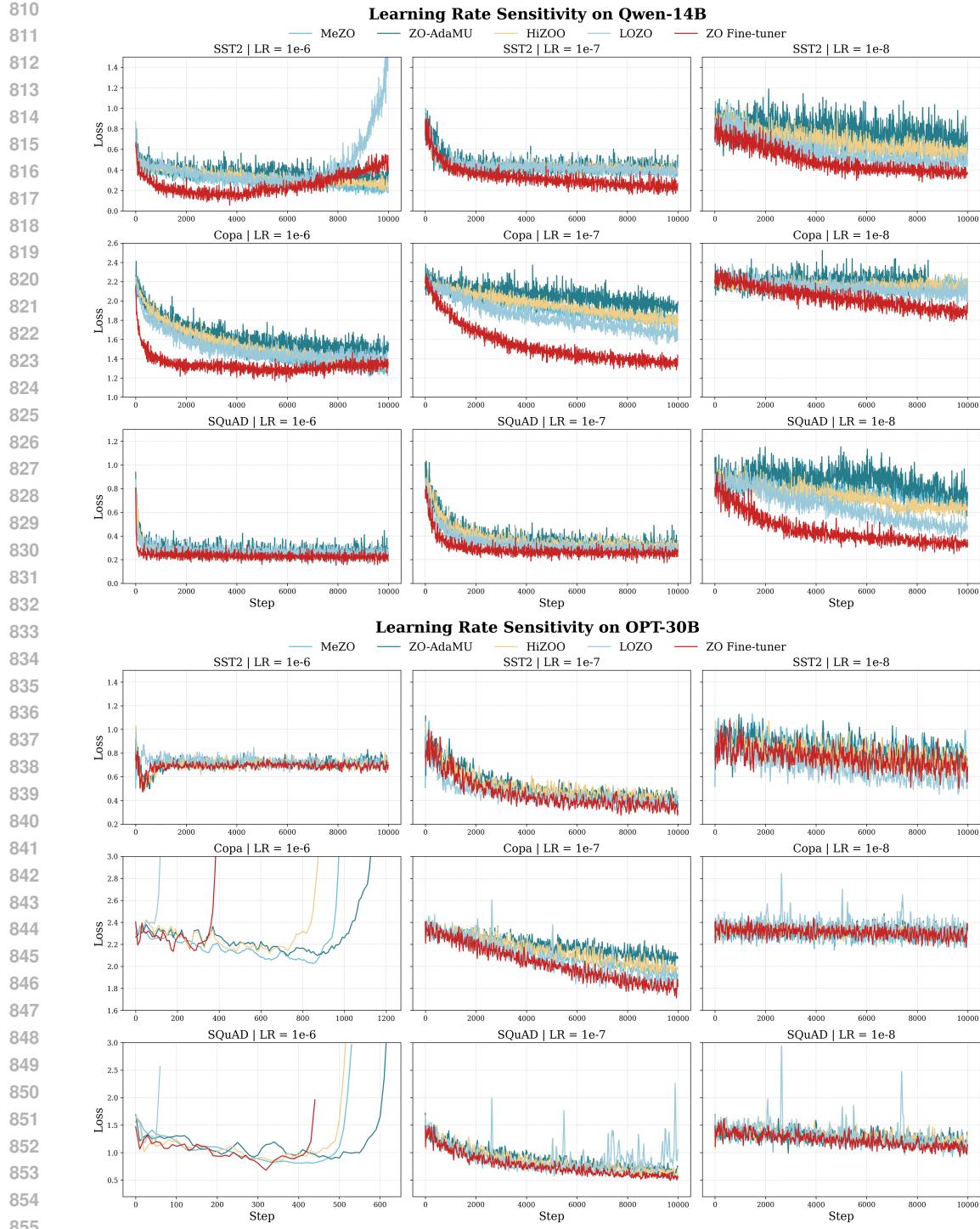


Figure 5: Loss curves under varying learning rates for different optimizers with Qwen-14B (top) and OPT-30B (bottom). We report results on SST2, Copa, and SQuAD.

achieves lower final loss, particularly under the two learning rates  $10^{-7}$  and  $10^{-8}$ . Notably, ZO Fine-tuner often matches or exceeds the best performance of other methods at  $10^{-7}$ , even when operating at  $10^{-8}$  on LLaMA-1B, LLaMA-8B and Qwen-14B.

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867 Table 6: Component-wise runtime breakdown (in seconds and percentage of total time) for different  
868 models. All results are tested on DROP and L40S GPU with a batch size of 16 using FP16.  
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Model	Generate Var	Perturb Param	Update Param	Compute Loss	Total Time
LLaMA-1B	0.025s (3.39%)	0.052s (7.07%)	0.021s (2.86%)	0.631s (86.65%)	0.729s
LLaMA-8B	0.052s (1.66%)	0.460s (14.67%)	0.192s (6.11%)	2.433s (77.55%)	3.137s
Qwen-14B	0.119s (2.06%)	0.395s (6.82%)	0.164s (2.84%)	5.106s (88.27%)	5.785s
OPT-30B	0.142s (1.64%)	0.214s (2.48%)	0.090s (1.04%)	8.183s (94.82%)	8.630s

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873  
874 When increasing the learning rate from  $10^{-8}$  to  $10^{-7}$ , ZO Fine-Tuner continues to improve. In  
875 contrast, baseline methods tend to suffer from instability at higher learning rates like  $10^{-6}$  when  
876 increasing from  $10^{-7}$ , especially on LLaMA-1B and LLaMA-8B. At the low end ( $10^{-8}$ ), many  
877 baselines exhibit stagnation, which means their loss decreases slowly or plateaus. This suggests  
878 limited adaptivity in low-gradient regimes. These observations underscore the robustness of ZO  
879 Fine-Tuner across a wide range of learning rates and tasks, highlighting its strong default behavior  
880 even without fine-tuned hyperparameters.  
881

## 882 D.2 TIME ANALYSIS

883  
884 We further break down the runtime of each component involved in the optimizer and summarize  
885 the results in Table 6. Among these, the variance generation step is extremely lightweight. It only  
886 accounts for 3.39% of total runtime on LLaMA-1B, and less than 2.06% on larger models such as  
887 LLaMA-8B, Qwen-14B, and OPT-30B. This highlights the efficiency of our design: although we  
888 introduce an additional learned component to control perturbation variance, it imposes almost no  
889 computational overhead.  
890

891 This can be easily explained as we only employ a lightweight neural network for each parameter  
892 block. More specifically, a two-layer MLP with just 32 hidden units. In addition, both the input and  
893 output of these networks are compressed, further reducing the computational cost.  
894

895 In contrast, the dominant cost comes from loss computation, which includes forward passes for both  
896 positive and negative perturbations. This accounts for over 77%–95% of total runtime and is intrinsic  
897 to all zeroth-order optimization frameworks. Overall, our method introduces minimal additional cost  
898 while achieving adaptive and effective optimization.  
899

## 900 D.3 MEMORY ANALYSIS

901 Table 7 reports the peak GPU memory usage of various optimization methods across different model  
902 sizes on the SST-2 dataset. We observe that all zeroth-order (ZO) methods, including MeZO, LOZO,  
903 and HiZOO, exhibit similar memory footprints. The only notable exception is ZO-AdaMU, which  
904 incurs higher memory usage due to its additional momentum tracking. Compared to the first-order  
905 method like Adam, all ZO methods consume significantly less memory, highlighting the efficiency of  
906 ZO-based optimization. Notably, our ZO Fine-Tuner achieves comparable memory usage to other ZO  
907 baselines, indicating that it introduces no additional memory overhead beyond standard ZO designs.  
908

909 Table 7: Peak GPU memory usage (GB) of different optimization methods across models on the  
910 SST-2 dataset, using batch size = 1 and FP16 precision.  
911

Method	LLaMA-1B	LLaMA-8B	Qwen-14B	OPT-30B
MeZO	5	20	35	61
LOZO	5	20	35	61
HiZOO	6	23	40	65
ZO-AdaMU	9	39	69	122
ZO Fine-Tuner	5	21	36	62
FO-SGD	9	40	74	126
FO-Adam	13	84	163	316

918  
 919 Table 8: Time and memory cost of meta-training the ZO Fine-Tuner in our L2L framework. GPU  
 920 memory usage and GPU time (in minutes) are reported for different foundation models. This cost is  
 921 incurred only once per base model, and the trained fine-tuner can be reused across downstream tasks.  
 922

Model	GPU Memory (GB)	Meta-training GPU Time (min)
LLaMA-1B	13	3
LLaMA-8B	83	15
Qwen-14B	150	25
OPT-30B	332	51

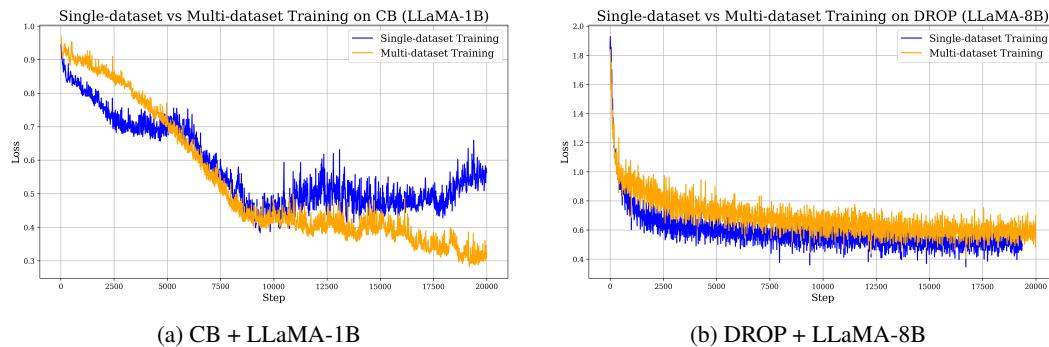
#### 926 927 D.4 COST OF LEARNING TO LEARN

928  
 929 We also assess the time and memory overhead incurred during the training of the ZO Fine-Tuner in  
 930 table 8. In general, L2L takes approximately  $2.4\times$  the time of standard first-order fine-tuning. This  
 931 is expected, as the L2L process inherently includes a full fine-tuning phase using SGD. However,  
 932 importantly, this cost is incurred only **once**, as a single ZO Fine-Tuner trained for a given model can  
 933 be reused across diverse downstream tasks, effectively amortizing the training cost.  
 934

#### 935 D.5 COMPARISON BETWEEN SINGLE-DATASET AND MULTI-DATASET TRAINING

936  
 937 We also compare the performance of ZO Fine-tuner under single-dataset and multi-dataset training  
 938 settings. In the multi-dataset setting, we construct a diverse training set by selecting one representative  
 939 dataset from each task type: SST-2 for sentiment analysis, CB and COPA for natural language  
 940 inference, and SQuAD for question answering. For the single-dataset setting, the optimizer is trained  
 941 solely on COPA.

942  
 943 The multi-dataset setting could lead to better performance. However, as shown in Figure 6, in some  
 944 cases, the ZO Fine-tuner trained on a single dataset can outperform its multi-dataset counterpart.  
 945 Overall, the two settings yield comparable performance. Single-dataset training is also simpler to  
 946 implement and tune, while still achieving competitive results. And that's why we choose to use it  
 947 throughout the main experiments.



948  
 949 Figure 6: Comparison of inference loss between ZO Fine-tuners trained with single-dataset and  
 950 multi-dataset settings. Results are reported on CB task using LLaMA-1B (left) and on DROP task  
 951 LLaMA-8B (right). The single-dataset variant is trained solely on COPA, while the multi-dataset  
 952 variant is jointly trained on COPA, SST-2, and SQuAD.  
 953

#### 954 E THEORETICAL ANALYSIS

955  
 956 In this section, we formally discuss our theoretical results and derive theorem 1. First, we set up the  
 957 notations and definition we need and formally present the theorem.  
 958

959 **Definition 1** (Expected Loss Change). *The expected change in loss after performing a one-step  
 960 update from parameter  $\theta_t$  is defined as*

$$961 \quad d(\theta_t) := \mathbb{E}[\mathcal{L}(\theta_{t+1}) \mid \theta_t] - \mathcal{L}(\theta_t).$$

972 **Assumption 1** (Local  $r$ -effective rank). Let  $G(\theta_t) = \max_{(x,y) \in D} \|\nabla \mathcal{L}(\theta_t; \{(x,y)\})\|$ . There exists  
 973 a matrix  $H(\theta_t) \leq \ell \cdot I_d$  such that:  
 974

- 975 1. For all  $\theta$  such that  $\|\theta - \theta_t\| \leq \eta d G(\theta_t)$ , we have  $\nabla^2 \mathcal{L}(\theta) \preceq H(\theta_t)$ .
- 976 2. The effective rank of  $H(\theta_t)$ , i.e.  $\text{tr}(H(\theta_t)) / \|H(\theta_t)\|_{\text{op}}$ , is at most  $r$ .

977 **Theorem 2.** Under Assumption 1, the expected loss change after one-step update of MeZO has upper  
 978 bound as follows, where  $\Sigma_{MB} = \text{Cov}(\nabla \mathcal{L}(\theta_t; \{(x_i, y_i)\}))$ :

$$979 d_{\text{MeZO}}(\theta_t) = \mathbb{E}[\mathcal{L}(\theta_{t+1}) | \theta_t] - \mathcal{L}(\theta_t) \\ 980 \leq -\eta \|\nabla \mathcal{L}(\theta_t)\|^2 + \frac{\eta^2 \ell}{2} \cdot \left( \frac{dr + d - 2}{d + 2} + 1 \right) \cdot \left( \|\nabla \mathcal{L}(\theta_t)\|^2 + \frac{1}{B} \text{tr}(\Sigma_{MB}(\theta_t)) \right) \\ 981 982$$

983 **Assumption 2** (Local Block-wise  $r_i$ -Effective Rank). The Hessian matrix  $H(\theta_t)$  in Assumption 1 sat-  
 984 isfies the following property:  $H(\theta_t) = \text{diag}(H_1(\theta_t), \dots, H_m(\theta_t))$  and  $r_i := \text{tr}(H_i(\theta_t)) / \|H_i(\theta_t)\|_{\text{op}}$   
 985 have different upper bounds  $r_i$ .

986 **Theorem 3.** Under Assumption 1 and Assumption 2 and ideal situation, assigning distinct perturba-  
 987 tion variances across parameter blocks can yield a tighter upper bound than that of  $d_{\text{MeZO}}(\theta_t)$ .

988 Assumption 1 and Theorem 1 are directly from the theoretical analysis of MeZO Malladi et al. (2023).  
 989 MeZO states the Lipschitz condition alone does not guarantee convergence in high-dimensional  
 990 settings and it is necessary to **leverage the low-rank structure of the Hessian matrix**. Assumption  
 991 2 is consistent with the actual situation, which has been deeply researched and checked by work like  
 992 Adam-mini (Zhang et al., 2024b;a).

993 In this section, we consider a setting where, at each iteration, we sample a perturbation block-by-block:  
 994 for the  $i$ -th parameter block, we draw noise from the distribution  $N(0, \sigma_i I_{d_i})$ , apply the perturbation  
 995 to the  $i$ -th block of parameters, and perform a zeroth-order update accordingly. Let the total number  
 996 of blocks be  $b$ , and denote by  $\mathcal{L}(\theta_{t,j})$  the loss after perturbing the  $j$ -th block at iteration  $t$ . We further  
 997 denote the full loss after perturbing all  $b$  blocks as  $\mathcal{L}(\theta_{t+1})$ , and let  $\nabla_j \mathcal{L}(\theta_{t,j})$  denote the  $j$ -th block  
 998 of the gradient evaluated at  $\theta_{t,j}$ . Here, we adopt the sphere (normalized-Gaussian) perturbation used  
 999 in the original MeZO analysis for its built-in step-size control. An analogous convergence form also  
 1000 holds for Gaussian perturbations, as shown in prior work (Malladi et al., 2023), when the probability  
 1001 of large updates  $\|\theta_{t+1} - \theta_t\|$  is kept small, which ensures the required local assumptions hold with  
 1002 high probability.

1003 *Proof.* As shown in Theorem 2, the expected loss decrease under MeZO is bounded by

$$1004 d_{\text{MeZO}}(\theta_t) = \mathbb{E}[\mathcal{L}(\theta_{t+1}) | \theta_t] - \mathcal{L}(\theta_t) \\ 1005 \leq -\eta \|\nabla \mathcal{L}(\theta_t)\|^2 + \frac{\eta^2 \ell}{2} \cdot \left( \frac{dr + d - 2}{d + 2} + 1 \right) \cdot \left( \|\nabla \mathcal{L}(\theta_t)\|^2 + \frac{1}{B} \text{tr}(\Sigma_{MB}(\theta_t)) \right) \\ 1006 1007$$

1008 Since each block-wise gradient estimate is still an unbiased estimator of the true gradient restricted to  
 1009 the corresponding block, we can get:

$$1011 \mathbb{E}[\mathcal{L}(\theta_{t,j+1}) | \theta_{t,j}] - \mathcal{L}(\theta_{t,j}) \leq \\ 1012 -\eta \sigma_j^2 \|\nabla_j \mathcal{L}(\theta_{t,j})\|^2 + \frac{\eta^2 \sigma_j^4 \ell}{2} \cdot \left( \frac{dr_j + d - 2}{d + 2} + 1 \right) \cdot \left( \|\nabla_j \mathcal{L}(\theta_{t,j})\|^2 + \frac{1}{B} \text{tr}(\Sigma_{MB,j}(\theta_{t,j})) \right) \\ 1013 1014$$

1015 By summing both sides over  $j = 1$  to  $b$  and taking expectation, we eliminate the dependence on  $\theta_{t,j}$ :  
 1016 the right-hand side becomes a function of  $\theta_t$  only, while the left-hand side depends only on  $\theta_{t+1}$  and  
 1017  $\theta_t$ . This yields:

$$1018 \mathbb{E}[\mathcal{L}(\theta_{t+1}) | \theta_t] - \mathcal{L}(\theta_t) \leq -\eta \sum_{j=1}^b \sigma_j^2 \mathbb{E}[\|\nabla_j \mathcal{L}(\theta_{t,j})\|^2 | \theta_t] \\ 1019 + \sum_{j=1}^b \frac{\eta^2 \sigma_j^4 \ell}{2} \cdot \left( \frac{dr_j + d - 2}{d + 2} + 1 \right) \cdot \left( \mathbb{E}[\|\nabla_j \mathcal{L}(\theta_{t,j})\|^2 | \theta_t] + \frac{1}{B} \text{tr}(\mathbb{E}[\Sigma_{MB,j}(\theta_{t,j}) | \theta_t]) \right) \\ 1020 \\ 1021 = \sum_{j=1}^b \left[ -\eta \sigma_j^2 \|\nabla_j \mathcal{L}(\theta_t)\|^2 + \frac{\eta^2 \sigma_j^4 \ell}{2} \cdot \left( \frac{dr_j + d - 2}{d + 2} + 1 \right) \cdot \left( \|\nabla_j \mathcal{L}(\theta_t)\|^2 + \frac{1}{B} \text{tr}(\Sigma_{MB}(\theta_t)) \right) \right] \\ 1022 1023$$

1026 The equality in the last line follows from the condition that the Hessian matrix is block-diagonal  
 1027 according to Assumption 2. Specifically, when updating block  $j_1$ , the change in the gradient of block  
 1028  $j_2$  ( $j_2 \neq j_1$ ) can be expressed as:  
 1029

$$1030 \quad \nabla_{j_2} \mathcal{L}(\theta_{t,j_1}) - \nabla_{j_2} \mathcal{L}(\theta_t) = \int_0^1 H_{j_2 j_1}(\theta_t + s P_{j_1} \delta) \delta \, ds,$$

$$1031$$

1032 where  $P_{j_1}$  denotes the projection onto block  $j_1$ , and  $H_{j_2,j_1}(\cdot)$  is the  $(j_2, j_1)$  block of the Hessian.  
 1033 The perturbation direction  $\delta$  is sampled from the standard multivariate normal distribution and  
 1034 scaled by the corresponding block-wise variance, i.e.,  $\delta \sim \mathcal{N}(0, \sigma_{j_1}^2 I_{d_{j_1}})$  for block  $j_1$ . Under the  
 1035 block-diagonal assumption,  $H_{j_2,j_1}(\cdot) = 0$  for all  $j_2 \neq j_1$ , and thus cross-block gradient changes  
 1036 vanish.

1037 Even if Assumption 2 does not hold exactly, the effect of cross-block interactions can still be  
 1038 controlled by bounding the operator norm of the off-diagonal blocks of the Hessian. As long as these  
 1039 terms remain small, the overall error introduced in the bound remains negligible.

1040 Note that if we set  $\sigma_j = 1$  for all  $j$ , our upper bound reduces to the standard MeZO bound:

$$1042 \quad \mathbb{E}[\mathcal{L}(\theta_{t+1})|\theta_t] - \mathcal{L}(\theta_t)$$

$$1043$$

$$1044 \leq \sum_{j=1}^b \left[ -\eta \|\nabla_j \mathcal{L}(\theta_t)\|^2 + \frac{\eta^2 \ell}{2} \cdot \left( \frac{dr_j + d - 2}{d + 2} + 1 \right) \cdot \left( \|\nabla_j \mathcal{L}(\theta_t)\|^2 + \frac{1}{B} \text{tr}(\Sigma_{MB}(\theta_t)) \right) \right]$$

$$1045$$

$$1046$$

$$1047 \leq -\eta \|\nabla \mathcal{L}(\theta_t)\|^2 + \frac{\eta^2 \ell}{2} \cdot \left( \frac{dr + d - 2}{d + 2} + 1 \right) \cdot \left( \|\nabla \mathcal{L}(\theta_t)\|^2 + \frac{1}{B} \text{tr}(\Sigma_{MB}(\theta_t)) \right)$$

$$1048$$

1049 where  $r$  is the (uniform) effective rank used in MeZO and  $r \geq r_j$  for any  $j$ . Therefore, by optimizing  
 1050  $\sigma_j$  for each block according to its local structure (e.g.,  $r_j$ ), we can obtain a strictly tighter upper  
 1051 bound than  $d_{\text{MeZO}}(\theta_t)$ .  $\square$

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