
Adversarially Perturbed Batch Normalization: A Simple Way to Improve Image Recognition

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Abstract

1 Recently, it has been shown that adversarial training (AT) by injecting adversarial
2 samples can improve the quality of recognition. However, the existing AT methods
3 suffer from the performance degradation on the benign samples, leading to a
4 gap between robustness and generalization. We argue that this gap is caused
5 by the inaccurate estimation of the Batch Normalization (BN) layer, due to the
6 distributional discrepancy between the training and test set. To bridge this gap, this
7 paper identifies the adversarial robustness against the indispensable noise in BN
8 statistics. In particular, we proposed a novel strategy that adversarially perturbs the
9 BN layer, termed ARAPT. The ARAPT leverages the gradients to shift BN statistics
10 and helps models resist the shifted statistics to enhance robustness to noise. Then,
11 we introduce ARAPT into a new paradigm of AT called model-based AT, which
12 strengthens models' tolerance to noise in BN. Experiments indicate that the APART
13 can improve model generalization, leading to significant improvements in accuracy
14 on benchmarks like CIFAR-10, CIFAR-100, Tiny-ImageNet, and ImageNet.

15 1 Introduction

16 Recent works [1, 2, 3, 4] show that deep neural networks are sensitive to adversarial perturbations,
17 which gives rise to the rapid development of adversarial training (AT) methods [5, 6, 7, 8]. These AT
18 methods enhance models' robustness against adversarial samples by solving a min-max optimization
19 problem [5]. However, many efforts [6, 9, 10] have corroborated that there is a trade-off between
20 standard and robust accuracy, in which AT usually degrades models' performance on benign samples,
21 though they enjoy the accuracy gains on adversarial samples.

22 Xie *et al.* [11] challenges the widely accepted idea that AT hurts models' generalization. They
23 proposed adversarial propagation (AdvProp) to exploit the adversarial features via auxiliary Batch
24 Normalization (BN) [12] layers. However, the huge computational overhead discourages more efforts
25 in its applications. Thus, the further work proposed fast AdvProp [13] to reduce the computations by
26 leveraging the acceleration of AT [14]. Nevertheless, leveraging adversarial samples to perform AT
27 in *non-safety* tasks leads to the following question:

28 *What does the model defend against?*

29 Indeed, adversarially trained models appear closely tied to the robustness against adversarial attacks
30 for safety concerns. However, in non-safety situations, there is an open question of what these
31 models defend against. This question is essentially related to the gap between models' generalization
32 and adversarial robustness. Prior efforts [15, 16, 17] reposition the adversarial robustness as the

33 robustness against the worst-case unseen domains, in attempts to bridge the generalization-robustness
34 gap. They usually enhance models’ robustness to adversarially generated domains to improve the
35 generalization. Nonetheless, there is inevitably a mismatch between such generated domains and the
36 actual domains. The mismatch hinders their further applications.

37 In this paper, we answer this question by identifying the robustness against the noise of BN statistics
38 that are the estimated mean and variance. The statistics noise is indispensable due to the distributional
39 discrepancy between training and test domains [18, 19, 20, 21, 22, 23]. Moreover, insufficient batch
40 size will cause the severer noise in some computation-demanding tasks [18, 19, 20]. In this study, we
41 cast the statistics noise as a numerical problem to avoid the issue of how to match the adversarially
42 generated domains with the actual ones. Since the noise degrades BN’s performance, numerically
43 strengthening models’ tolerance to such noise will boost the generalization of BN-based models.

44 In this work, we train models by **Adversarially Perturbed bAtch noRmalizaTion** (APART) that
45 perturbs BN statistics and updates the model parameters to resist the perturbation on the fly. More
46 concretely, APART performs backward passes twice over each batch of benign samples. The
47 first backward pass produces two gradient computations: one is normal gradient that helps update
48 parameters of model *w.r.t.* samples’ patterns, and the other one is statistics gradient that is used to
49 perturb the statistics parameters in BN. Then, the second pass is performed to generate the defensive
50 gradient that helps the model resist the adversarial statistics perturbation. The normal and defensive
51 gradients are combined to improve both generalization and robustness of the model. All gradients are
52 computed by the regular gradient descent algorithm. Note that APART combines the normal gradient
53 with the defensive one without changing the update strategy and without crafting the adversarial
54 samples. This process follows a paradigm of AT performing attacks and defense within models
55 instead of on samples, hence the name model-based AT. Besides, as suggested by AdvProp [11], the
56 BN statistics computed over the adversarial passes are dropped to avoid the corruption.

57 Experimentally, APART makes models less brittle to noisy BN statistics. As a consequence, the mod-
58 els enjoy significant accuracy gains on CIFAR [24], Tiny-ImageNet [25] and ImageNet [26] datasets.
59 Moreover, the improvement brought by APART only depends on BN, allowing the combination with
60 other training methods, *e.g.* data augmentation [27] and sharpness-aware minimization [28].

61 **Summary of contributions:**

- 62 • We identify the adversarial robustness against the noise in BN statistics to bridge the gap between
63 models’ generalization and robustness. Enhancing such robustness by AT improves models’
64 generalization on benign samples.
- 65 • We proposed APART to achieve the robustness against the statistics noise. APART follows a
66 new paradigm of AT utilizing the gradients efficiently. By strengthening BN-based models’
67 tolerance to BN statistics noise, APART significantly improves the models’ performance.
- 68 • With its plug-and-play nature, APART allows the combination with other training methods and
69 enjoys the further accuracy gains.

70 **2 Related Work**

71 **2.1 Adversarial Training**

72 Adversarial training (AT) [1, 2, 5, 29] is empirically demonstrated to be one of the most effective
73 defense methods for models’ safety concerns. Instead, many non-AT methods [30, 31, 32, 33] fail
74 to defend against adaptive attacks [4]. However, AT sacrifices the standard accuracy on benign
75 samples to increase models’ robustness [9]. Thus, there is a trade-off between the robustness and
76 generalization [6]. Furthermore, many efforts [34, 9, 35, 36, 37] theoretically and experimentally
77 corroborate the difficulty of achieving adversarial robustness over limited data. Besides adversarial
78 robustness, other works [38, 14] focus on the efficiency of AT due to the high computational overhead
79 of vanilla AT methods [5, 6]. The proposed fast AT method [14] accelerates the training in a

80 simple way, but suffers from catastrophic overfitting [39, 40]. This problem gives rise to more
81 efforts [39, 40, 41, 42].

82 Besides performing AT over samples, Adversarial Weight Perturbation [8] additionally perturbs
83 parameters to enhance the generalization from a perspective of loss landscape. In non-safety tasks,
84 some efforts [28, 43, 44, 45] have been devoted to such parameter-based AT and show promise
85 in improving models’ generalization. In this study, the proposed method follows a more generic
86 paradigm of AT that allows the attacks on each component of models even including the non-parameter
87 BN statistics.

88 2.2 Adversarial Robustness Beyond Safety

89 Though the disadvantage of AT in models’ generalization discourages the efforts of its non-safety
90 applications, Xie *et al.* [11] proposed AdvProp to challenge this issue. AdvProp utilizes auxiliary
91 BN layers to avoid corrupting the BN statistics estimated over benign samples. In this manner,
92 AdvProp improves models’ generalization and inspires the further studies [46, 16, 47, 15, 48, 49]
93 of the adversarial robustness beyond safety. Indeed, AT provides the framework of crafting and
94 countering the worse-case unseen domains [15, 16, 17], and enhances adversarial robustness varying
95 in different contexts. Besides, Mei *et al.* [13] utilize the acceleration of fast AT [14] to significantly
96 reduce the computational overhead of AdvProp [11].

97 In this work, the proposed AT method, termed APART, increases models’ robustness against the
98 noise of BN statistics. Though the perturbation formula of the statistics is somewhat similar to that
99 of Adversarial Batch Normalization (AdvBN) [15], there are three major differences between them
100 in implementations: 1) APART perturbs the entire network by slightly changing each BN layer,
101 instead of perturbing the features generated from a specific non-BN layer [15]; 2) in each iteration,
102 APART performs backward passes only twice to carry out the attack and defense efficiently, instead
103 of performing multiple backward passes inefficiently [15]; 3) APART trains each model from the
104 scratch, instead of fine-tuning a pre-trained model [15], which leads to incomparability between
105 APART and AdvBN.

106 2.3 Normalization

107 Batch Normalization (BN) [12] has successfully boosted a broad range of deep neural networks by
108 accelerating the training. However, the noisy statistics of BN degrade its performance experimen-
109 tally [20] and theoretically [50]. Many efforts have been devoted to more accurate estimators of the
110 statistics [18, 19, 20, 21, 22, 23]. Some estimators perform the normalization along different axes,
111 *e.g.* Layer Normalization [18], Instance Normalization [19] and Group Normalization [20]. They
112 reduce the noise in the case of tiny batch but suffer from performance degradation under large batch
113 as the alternative to BN. More efforts [21, 51, 52, 16] exploit the combination of these normalization
114 methods. They selectively use the axis-specific statistics to perform normalization in response to
115 different domains. Additionally, the on-the-fly estimation of BN statistics over adversarial samples is
116 experimentally found to have negative impacts on the standard accuracy [11, 53]. This finding leads
117 to more exploration in BN under AT [11, 13, 15, 46, 49].

118 The noisy statistics result from a mismatch between the seen and unseen domains and are therefore
119 indispensable without the domain-specific knowledge. Furthermore, tiny batch size caused by the
120 computation-demanding tasks results in the severer noise. From an opposite perspective of these
121 methods denoising the statistics, our method hardens BN-based models’ robustness against the noise.

122 3 Method

123 In this section, we firstly introduce a new paradigm of AT that allows us to perform attacks and
124 defense within models rather than on samples. Then, we propose APART to implement this paradigm
125 in a simple way. Finally, we discuss the enhancement of APART, which is derived from the potential
126 link between APART and the other training method.

127 3.1 Model-Based Adversarial Training

128 The vanilla AT formulates a min-max game [5] by adversarially crafting and defensively countering
 129 the imperceptible perturbations to samples. Specifically, given the ground truth \mathbf{y} and sample \mathbf{x} 's
 130 allowed neighborhood $\mathcal{S}(\mathbf{x})$, we minimize the expectation of a θ -parameterized loss $\mathcal{L}(\mathbf{x}^*, \mathbf{y}; \theta)$
 131 with $\mathbf{x}^* \in \mathcal{S}(\mathbf{x})$ maximizing $\mathcal{L}(\cdot, \mathbf{y}; \theta)$, *i.e.*,

$$\min_{\theta} \mathbb{E}_{\mathbf{x}, \mathbf{y}} \mathcal{L}(\mathbf{x}^*, \mathbf{y}; \theta), \quad \text{where } \mathbf{x}^* := \operatorname{argmax}_{\mathbf{x}' \in \mathcal{S}(\mathbf{x})} \mathcal{L}(\mathbf{x}', \mathbf{y}; \theta). \quad (1)$$

132 Empirically, the maximization of Eq. (1) is achieved by a gradient ascent method for each sample.
 133 The gradient $\nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \mathbf{y}; \theta)$ is iteratively computed by full forward and backward passes on the model.
 134 This process merely requires the inputs' gradients $\nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \mathbf{y}; \theta)$ and drops all the internal gradients
 135 $\nabla_{\theta} \mathcal{L}(\mathbf{x}, \mathbf{y}; \theta)$ without their further utilization after finishing the backward pass. Thus, such vanilla
 136 AT suffers from low efficiency of utilizing the gradients. Meanwhile, AT's potency is limited by such
 137 sample-based attacks and defense. Therefore, we propose a paradigm of *model-based* AT to leverage
 138 internal gradients efficiently and allow the attacks and defense within models.

139 To perform such AT, each component of a model is categorized into two types: one for the attacker, and
 140 one for the defender. Denoting by θ, ϕ the parameters of the adversarial and defensive components
 141 respectively, we formulate the model-based AT as follows

$$\min_{\phi} \mathbb{E}_{(\mathbf{x}, \mathbf{y})} \left[\mathcal{R}(\mathbf{x}, \mathbf{y}; \phi) + \max_{\theta \in \Theta} \mathcal{L}(\mathbf{x}, \mathbf{y}; \theta, \phi) \right], \quad (2)$$

142 where Θ is a parameter space that can be bounded to avoid trivial results; $\mathcal{R}(\mathbf{x}, \mathbf{y}; \phi)$ is a task-
 143 specific loss allowing models to learn the normal patterns in samples, and $\mathcal{L}(\mathbf{x}, \mathbf{y}; \theta, \phi)$ can share
 144 a similar formulation of \mathcal{R} to enable more pattern exploration in an adversarial manner. Overall,
 145 Eq. (2) provides a generic formulation of model-based AT. For example, Generative Adversarial
 146 Networks (GANs) [54] can be repositioned as a special case of such AT, in which the generator and
 147 discriminator are regarded as the attacker and defender respectively, and the discriminative losses are
 148 cast as proper \mathcal{R} and \mathcal{L} in Eq. (2). Next, we introduce APART to implement this model-based AT in
 149 a simple way.

150 3.2 Adversarially Perturbed Batch Normalization

151 Model-based AT helps models harden their robustness against a specific problem. We shift our
 152 attention to the noisy BN statistics [20], and apply the proposed AT to address this problem.

153 Firstly, we embed two temporary parameters $\delta_{\mu}, \delta_{\sigma}$ into each BN layer as the adversarial parameters
 154 θ in Eq. (2), which will be dropped after the training. Inspired by AdvBN [15], with $\delta_{\mu}, \delta_{\sigma} \leftarrow \mathbf{0}$, we
 155 reformulate the BN mapping as

$$\mathbf{BN}(\mathbf{x}; \delta_{\mu}, \delta_{\sigma}) = \gamma(1 + \delta_{\sigma}) \cdot \frac{\mathbf{x} - (1 + \delta_{\mu})\hat{\boldsymbol{\mu}}}{\hat{\boldsymbol{\sigma}}} + \beta, \quad (3)$$

156 where each operator is element-wise; $\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\sigma}}$ are the mean and standard deviation estimated over a batch
 157 of \mathbf{x} respectively; γ, β are the parameters of BN's affine mapping. We bound the d -dimensional
 158 $\delta_{\mu}, \delta_{\sigma}$ such that $\delta_{\mu}, \delta_{\sigma} \in [-\epsilon, \epsilon]^d$ for a small sufficient perturbation radius $\epsilon > 0$. The bound avoids
 159 trivial results, *e.g.* $\mathbf{BN}(\mathbf{x}; \delta_{\mu}, \delta_{\sigma})|_{\delta_{\sigma}=-1} \equiv \mathbf{0}$. Now all the other trainable parameters within the
 160 entire model including γ, β are naturally the defensive parameters ϕ . The losses \mathcal{R}, \mathcal{L} in Eq. (2) are
 161 both the same task-specific loss, *i.e.*, the cross entropy in recognition. Therefore, Eq. (3) implies (θ
 162 indicates all BN layers' $\delta_{\mu}, \delta_{\sigma}$)

$$\mathcal{L}(\mathbf{x}, \mathbf{y}; \theta, \phi)|_{\theta=\mathbf{0}} = \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi) \quad \nabla_{\phi} \mathcal{L}(\mathbf{x}, \mathbf{y}; \theta, \phi)|_{\theta=\mathbf{0}} = \nabla_{\phi} \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi), \quad (4)$$

163 by which we can get both $\nabla_{\theta} \mathcal{L}(\mathbf{x}, \mathbf{y}; \theta, \phi)|_{\theta=\mathbf{0}}$ and $\nabla_{\phi} \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi)$ in a single backward pass with
 164 $\mathcal{L}(\mathbf{x}, \mathbf{y}; \theta, \phi)|_{\theta=\mathbf{0}}$ as the loss.

165 Secondly, we propose APART that follows a gradient accumulation strategy, instead of the alternative
 166 update strategy used by GANs [54]. Specifically, in each iteration of a normal gradient descent
 167 algorithm over a batch of samples $\mathcal{D} := \{(\mathbf{x}_i, \mathbf{y}_i), 1 \leq i \leq M\}$,

Algorithm 1: Pseudo code of APART getting the gradient for a batch of samples, given some perturbation radius ϵ , number of samples in the second pass N , and group number n

Data: A batch of samples $\mathcal{D} := \{(\mathbf{x}_i, \mathbf{y}_i), 1 \leq i \leq M\}$

Result: The gradient \mathbf{g} for this batch of samples

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1  $\boldsymbol{\theta} \leftarrow \mathbf{0}$ 
2 Perform forward and backward passes once over  $\mathcal{D}$ , generating  $\mathbf{g}_\theta$  and  $\mathbf{g}_\phi$  simultaneously
3  $\mathbf{g}_\theta \leftarrow \mathbb{E}_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{D}} \nabla_{\boldsymbol{\theta}} \mathcal{L}(\mathbf{x}_i, \mathbf{y}_i; \boldsymbol{\theta}, \phi)|_{\boldsymbol{\theta}=\mathbf{0}}$ 
4  $\mathbf{g}_\phi \leftarrow \mathbb{E}_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{D}} \nabla_{\phi} \mathcal{R}(\mathbf{x}_i, \mathbf{y}_i; \phi)$ 
5  $\boldsymbol{\theta} \leftarrow \epsilon \text{sign}(\mathbf{g}_\theta)$ 
6 Randomly draw  $N$  samples  $\mathcal{S} \subseteq \mathcal{D}$ 
7 Group  $\mathcal{S}$  into  $n$  equally sized subsets  $\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n$ 
8  $\mathbf{h}_\phi \leftarrow \mathbf{0}$ 
9 for  $j \leftarrow 1$  to  $n$  do
10 |  $\mathbf{h}_\phi \leftarrow \mathbf{h}_\phi + \frac{1}{n} \mathbb{E}_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{S}_j} \nabla_{\phi} \mathcal{L}(\mathbf{x}_i, \mathbf{y}_i; \boldsymbol{\theta}, \phi)|_{\boldsymbol{\theta}=\epsilon \text{sign}(\mathbf{g}_\theta)}$ 
11 end
12  $\mathbf{g} \leftarrow \frac{M}{M+N} \mathbf{g}_\theta + \frac{N}{M+N} \mathbf{h}_\phi$ 

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168 • **Step 1:** With $\boldsymbol{\theta} \leftarrow \mathbf{0}$, APART performs the forward and backward passes once over this
169 batch of samples to generate the gradients *w.r.t.* the adversarial and defensive parameters, *i.e.*,
170 $\mathbf{g}_\theta := \mathbb{E}_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{D}} \nabla_{\boldsymbol{\theta}} \mathcal{L}(\mathbf{x}_i, \mathbf{y}_i; \boldsymbol{\theta}, \phi)|_{\boldsymbol{\theta}=\mathbf{0}}$ and $\mathbf{g}_\phi := \mathbb{E}_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{D}} \nabla_{\phi} \mathcal{R}(\mathbf{x}_i, \mathbf{y}_i; \phi)$ according
171 to Eq. (4). Like Fast Gradient Sign Method [2], APART uses $\epsilon \text{sign}(\mathbf{g}_\theta)$ to assign $\boldsymbol{\theta}$, which
172 empirically performs the inner maximization of Eq. (2) and generates the adversarially perturbed
173 statistics in each BN layer.

174 • **Step 2:** With the adversarial BN statistics, APART performs the forward and backward passes
175 again, over a full/incomplete batch of the same samples $\mathcal{S} \subseteq \mathcal{D}$. This backward pass yields
176 the gradient resisting the attack, *i.e.*, $\mathbf{h}_\phi := \mathbb{E}_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{S}} \nabla_{\phi} \mathcal{L}(\mathbf{x}_i, \mathbf{y}_i; \boldsymbol{\theta}, \phi)|_{\boldsymbol{\theta}=\epsilon \text{sign}(\mathbf{g}_\theta)}$. The
177 weighted gradient $\mathbf{g} := (1 - r)\mathbf{g}_\theta + r\mathbf{h}_\phi$ is finally used in the outer minimization of Eq. (2) for
178 this batch of samples, where $r \in [0, 0.5]$ re-balances the gradients.

179 Apparently, using a full batch of the samples in the second pass leads to the best performance,
180 but results in more computational overhead. Instead, using the incomplete batch of these samples
181 allows the less computation but suffers from insufficient defense against the attack. Thus, the ratio
182 $r = N/(M + N)$ is introduced to re-balance the gradients with $N = |\mathcal{S}|$ the number of the samples
183 used in the second pass. Additionally, the on-the-fly BN statistics estimated in the second pass are
184 completely dropped to avoid corrupting the statistics at inference, like the auxiliary BN layers [11].

185 Note that stronger attacks in AT indirectly enhance the adversarial robustness [5]. Thus, we slightly
186 modify the process of the second pass to strengthen the attack. The modification increases the noise
187 in the adversarial BN statistics without additional computation. In details, we group the samples into
188 equally sized sets and stop their group-to-group communications in BN layers during the second
189 forward pass. This is inspired by the fact that smaller batch size results in larger noise in the statistics.
190 In this manner, the BN statistics are estimated over less samples without reducing the entire batch
191 size, giving rise to the less adversarial accuracy.

192 Overall, APART only changes the way of getting gradients in each iteration, without involving in
193 data augmentation or network modification. Therefore, APART has plug-and-play nature that allows
194 the combination with a broad range of training methods. We summarize APART in Algorithm 1, and
195 then introduce the enhancement of APART.

196 3.3 Enhancement by Combination with Sharpness-Aware Minimization

197 Besides APART, Sharpness-Aware Minimization (SAM) [28] also belongs to and use the proposed
198 model-based AT paradigm. SAM improves network training from a perspective of loss landscape
199 relating to models' generalization. Given the empirical loss function $\mathcal{L}_{\mathcal{D}}$ estimated over a dataset \mathcal{D} ,

200 SAM minimizes $\mathcal{L}_{\mathcal{D}}$ with a sharpness measure:

$$\min_{\mathbf{w}} \left[\max_{\|\delta\|_2 \leq \rho} \mathcal{L}_{\mathcal{D}}(\mathbf{w} + \delta) - \mathcal{L}_{\mathcal{D}}(\mathbf{w}) \right] + \mathcal{L}_{\mathcal{D}}(\mathbf{w}) + \frac{\lambda}{2} \|\mathbf{w}\|_2^2, \quad (5)$$

201 where \mathbf{w} is the model’s trainable parameters; $\rho > 0$ is small sufficient to restrict the perturbation
 202 δ ; $\lambda > 0$ is used to control the regularizer $\frac{\lambda}{2} \|\mathbf{w}\|_2^2$; the term in the square bracket measures $\mathcal{L}_{\mathcal{D}}$ ’s
 203 sharpness. Obviously, Eq. (5) is equivalent to

$$\min_{\mathbf{w}} \max_{\|\delta\|_2 \leq \rho} \mathcal{L}_{\mathcal{D}}(\mathbf{w} + \delta) + \frac{\lambda}{2} \|\mathbf{w}\|_2^2. \quad (6)$$

204 Actually, Eq.(6) performs the model-based AT formulated by Eq. (2), in which we treat δ, \mathbf{w}
 205 as the adversarial and defensive parameters θ, ϕ respectively, and let $\mathcal{L}(\mathbf{x}, \mathbf{y}; \theta, \phi) = \mathcal{L}_{\mathcal{D}}(\mathbf{w} +$
 206 $\delta), \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi) \equiv 0$ with the regularizer $\frac{\lambda}{2} \|\mathbf{w}\|_2^2$ included in the empirical optimization. Therefore,
 207 SAM and APART essentially share the same training paradigm. Surprisingly, these two methods
 208 implement this paradigm in a complementary way: SAM focuses on the trainable parameters
 209 optimized by gradient descent, while APART concentrates on the non-trainable BN statistics requiring
 210 estimation instead of optimization. The training paradigm enables them to enhance models’ robustness
 211 in different contexts, which inspires a combination of them.

212 Note that the inner maximization in Eq. (6) is approximately achieved by one-step gradient ascent.
 213 Then, the outer minimization is performed by estimating the gradient *w.r.t.* the adversarially shifted
 214 parameters $\mathbf{w} \leftarrow \mathbf{w} + \delta$ [28]. Thus, SAM shares a similar two-step strategy of APART. Such similarity
 215 allows us to perform APART and SAM simultaneously by a slight modification of Algorithm 1,
 216 termed APART-SAM. Specifically, we adopt the weights’ perturbations, *i.e.*, Eq. (2) therein [28],
 217 reformulated as

$$\hat{\delta}(\phi) = \rho \text{sign}(\nabla_{\phi} \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi)) |\nabla_{\phi} \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi)|^{q-1} / \left(\|\nabla_{\phi} \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi)\|_q^q \right)^{1/p}, \quad (7)$$

218 where $1/p + 1/q = 1$ and experimentally let $p = 2$ as suggested by [28]. Then, we modify
 219 APART’s first step by additionally perturbing the defensive parameters $\phi \leftarrow \phi + \hat{\delta}(\phi)$ to enhance the
 220 attacks with the second step unchanged. The additional perturbation $\hat{\delta}(\phi)$ just employs the gradient
 221 $\nabla_{\phi} \mathcal{R}(\mathbf{x}, \mathbf{y}; \phi)$ previously computed by APART’s first step, and normalizes them with ignorable extra
 222 computations. Therefore, APART-SAM is computation-friendly enhancement of APART.

223 4 Experiments

224 4.1 Experimental Setup

225 **Datasets and Models.** We evaluate APART on CIFAR-10, CIFAR-100 [24], Tiny-ImageNet [25] and
 226 ImageNet [26]. On CIFAR datasets, we employ a WideResNet-40-2 [55] (as implemented in [56]),
 227 PreAct-ResNet-18 [57] (as implemented in [58]). We use a PreAct-ResNet-18 on Tiny-ImageNet,
 228 and ResNet-18 [59] (as implemented in *torchvision* library [60]) on ImageNet.

229 **Implementation Details.** On CIFAR-10 and CIFAR-100, we run all experiments by five different
 230 random seeds and report the mean and standard derivation of test accuracy. We employ SGD
 231 with initial learning rate 0.1, momentum 0.9 and weight decay 0.0005. We train models for 200
 232 epochs and reduce the learning rate by 0.1 at the 100-th and 150-th epoch with batch size of
 233 128. We use only the standard augmentations (*i.e.*, random flipping and translation) in the basic
 234 experiments, and additionally leverage mixup [27] for further comparison. The hyperparameter α
 235 of mixup is set to 1 in the baseline as suggested by [27] and is properly chosen for APART. For
 236 comparison, we report the empirical results of SAM-trained WideResNet-40-2 and PreAct-ResNet-18
 237 under standard augmentation, where we set $\rho = 0.05$ and $\rho = 0.1$ on CIFAR-10 and CIFAR-100
 238 respectively, as suggested by [28]. On Tiny-ImageNet, we use batch size of 256 and set other
 239 hyperparameters in the same way of the CIFAR experiments; under mixup, we set $\alpha = 0.2$ for
 240 both the standard method and APART. On ImageNet, We employ SGD with initial learning rate
 241 0.1, momentum 0.9 and weight decay 0.0001. We train models with batch size of 256 for 105
 242 epochs, where the learning rate is reduced by 0.1 at the 30-th, 60-th, 90-th and 100-th epoch. We
 243 randomly resize and crop images to 224×224 resolution with random flipping to perform the

244 standard augmentation. For the hyperparameters of APART and APART-SAM, we evaluate a few
 245 combinations and show the best performance in the main results. More results of these combinations
 246 are reported in the ablation studies and appendix A.1. Considering the $2\times$ training budget of
 247 APART, we also conduct the experiments of the standard training with $2\times$ total and decay epochs
 248 to show APART’s non-trivial performance. Our implementations use Pytorch [61], and all models
 249 are trained on a server with three NVIDIA RTX 3090 GPUs. Please see appendix B and the code at
 250 <https://github.com/unknown9567/apart.git> for more details.

251 **4.2 Main Results**

Table 1: Results on CIFAR-10 and CIFAR-100.

Method (Augmentation)	Budget	CIFAR-10	CIFAR-100
WideResNet-40-2			
Standard (Standard)	1×	94.67 \pm 0.10(+0.00)	76.10 \pm 0.24(+0.00)
Standard (Standard)	2×	94.99 \pm 0.11(+0.32)	76.73 \pm 0.27(+0.63)
SAM (Standard)	2×	95.39 \pm 0.14(+0.72)	77.47 \pm 0.09(+1.37)
APART (Standard)	2×	95.69 \pm 0.13(+1.02)	79.05 \pm 0.25(+2.95)
APART-SAM (Standard)	2×	95.81\pm0.27(+1.14)	79.21\pm0.23(+3.11)
Standard (Mixup)	1×	95.43 \pm 0.11(+0.76)	76.63 \pm 0.34(+0.53)
Standard (Mixup)	2×	96.03\pm0.11(+1.36)	77.96 \pm 0.43(+1.86)
APART (Mixup)	2×	95.86 \pm 0.05(+1.19)	79.22\pm0.22(+3.12)
APART-SAM (Mixup)	2×	95.78 \pm 0.08(+1.11)	79.00 \pm 0.09(+2.90)
PreAct-ResNet-18			
Standard (Standard)	1×	94.60 \pm 0.17(+0.00)	76.30 \pm 0.11(+0.00)
Standard (Standard)	2×	94.76 \pm 0.12(+0.16)	75.34 \pm 0.21(−0.96)
SAM (Standard)	2×	95.56 \pm 0.16(+0.96)	78.57 \pm 0.17(+2.27)
APART (Standard)	2×	95.84 \pm 0.16(+1.24)	79.48 \pm 0.15(+3.18)
APART-SAM (Standard)	2×	96.12\pm0.06(+1.52)	80.07\pm0.18(+3.77)
Standard (Mixup)	1×	95.76 \pm 0.11(+1.16)	77.30 \pm 0.50(+1.00)
Standard (Mixup)	2×	96.19 \pm 0.12(+1.59)	78.81 \pm 0.45(+2.51)
APART (Mixup)	2×	96.28\pm0.09(+1.68)	80.07 \pm 0.17(+3.77)
APART-SAM (Mixup)	2×	96.08 \pm 0.18(+1.48)	80.19\pm0.15(+3.89)

252 **Evaluation on CIFAR-10 and CIFAR-100.**

253 As is shown in Table 1, APART helps mod-
 254 els significantly outperform their counter-
 255 parts trained by the standard method. Under
 256 standard augmentation, without considering
 257 the training budget, APART improves the
 258 accuracy by over 1.02% on CIFAR-10 and
 259 2.95% on CIFAR-100 for each model; con-
 260 sidering the training budget leads to the accu-
 261 racy gains of over 0.70% on CIFAR-10 and
 262 2.32% on CIFAR-100; enhanced by SAM,
 263 APART-SAM further improves the accuracy
 264 of APART-trained models by over 0.12%
 265 on CIFAR-10 and 0.16% on CIFAR-100.
 266 In addition, APART and APART-SAM out-
 267 perform SAM under this experimental set-
 268 ting. Under mixup [27], the improvements
 269 achieved by APART are generally consistent,
 270 though the APART-trained WideResNet-40-2 is somewhat inferior to the standard counterpart with
 271 $2\times$ budget on CIFAR-10; on the other hand, APART-SAM slightly degenerates due to the potential
 272 conflicts between SAM and mixup on CIFAR datasets. Besides, mixup with sufficient training
 273 budgets boosts the standard models more significantly, reducing the accuracy gap between them and
 274 the APART-trained counterparts.

Table 2: Results on Tiny-ImageNet and ImageNet.

Method (Augmentation)	Budget	Accuracy (%)
Tiny-ImageNet		
Standard (Standard)	1×	63.52 (+0.00)
Standard (Mixup)	1×	64.34 (+0.82)
Standard (Standard)	2×	63.94 (+0.42)
Standard (Mixup)	2×	64.54 (+1.02)
APART (Standard)	2×	67.00 (+3.48)
APART (Mixup)	2×	67.26 (+3.74)
APART-SAM (Standard)	2×	67.53 (+4.01)
APART-SAM (Mixup)	2×	68.66 (+5.14)
ImageNet		
Standard (Standard)	1×	70.24 (+0.00)
Standard (Standard)	2×	71.25 (+1.01)
Standard (Standard)	4×	71.45 (+1.21)
APART (Standard)	2×	70.86 (+0.62)
APART (Standard)	4×	72.14 (+1.90)
APART-SAM (Standard)	2×	70.82 (+0.58)

275 **Evaluation on Tiny-ImageNet and ImageNet.** As is shown in Table 2, APART and APART-SAM
 276 consistently improve the accuracy on ImageNet and its variant. On Tiny-ImageNet, the accuracy
 277 gains are significant, *e.g.* the models trained by APART-SAM outperforms the standard counterparts
 278 by over 4% and 3.5% for $1\times$ and $2\times$ training budgets respectively under standard augmentation.
 279 Furthermore, APART-SAM enjoys the combination with mixup and improves the accuracy by over
 280 5%. On ImageNet, APART with $2\times$ budget outperforms the baseline with $1\times$ budget, but is inferior
 281 to the standard training with $2\times$ budget. However, scaling the training budgets leads to a different
 282 result: APART with $4\times$ budget outperforms the standard method with both $2\times$ and $4\times$ training
 283 budgets. It seems that APART employed on the large-scale dataset requires more steps to show
 284 its promise. Besides, APART-SAM slightly degenerates due to the insufficient tuning of its more
 285 hyperparameters.

Table 3: Ablation studies of APART’s hyperparameters.

286 4.3 Ablation Study

287 Table 3 shows the performance of the
 288 WideResNet-40-2 trained by APART with
 289 different hyperparameters in Algorithm 1
 290 on CIFAR-100. Overall, APART-trained
 291 models outperform all standard models de-
 292 spite training budgets and hyperparameters.
 293 For example, even the model trained by
 294 APART with $1.19\times$ budget performs better
 295 than the standard model with $2.00\times$ budget.
 296 On the other hand, APART’s hyperparam-
 297 eters have impacts at different levels on its
 298 performance.

299 **Impact of N .** The number of samples used
 300 in APART’s second step has a significant
 301 impact on its performance. Indeed, models
 302 with APART’s adversarial BN statistics implicitly generate adversarial features within the models
 303 in the second pass. Therefore, more samples in this pass lead to more diversity required by models’
 304 robustness against the noisy BN statistics and improve the performance more significantly.

305 **Impact of ϵ .** Large perturbation radii (*e.g.* $\epsilon = 0.4$) degenerate models’ performance, since strong
 306 attacks caused by such radii force the models to sacrifice their generalization for more robustness. In
 307 contrast, smaller radii reduce both the robustness and accuracy, which illustrates the link between the
 308 generalization and robustness of APART-trained models.

309 **Impact of n .** The group number has a relatively slight impact on the accuracy, since it implicitly
 310 enhances the attack. A properly chosen n can help APART achieve the best performance.

311 4.4 Evaluation on APART’s Attacks

312 **Experimental Setup.** We evaluate
 313 APART’s attacks to provide a basic insight
 314 of its effectiveness. We use a WideResNet-
 315 40-2 [55] pretrained on CIFAR-100. We
 316 perform APART’s first step to adversarially
 317 shift its BN statistics without changing the
 318 other parameters. We use only a batch of
 319 training samples for the attack, but evalu-
 320 ate the accuracy over the entire training
 321 dataset. For comparison, we provides the
 322 accuracy in the cases of random perturba-
 323 tions, *i.e.*, $\delta_\mu, \delta_\sigma \sim \text{Uniform}[-\epsilon, \epsilon]^d$ or

Budget	N	ϵ	n	Accuracy (%)
Standard Training				
1.00×				76.10±0.24(+0.00)
1.20×	NA	NA	NA	76.24±0.30(+0.14)
2.00×				76.73±0.27(+0.63)
APART				
1.19×	24	0.1	1	77.58±0.17(+1.48)
		0.1	2	77.50±0.17(+1.40)
		0.1	8	77.35±0.36(+1.25)
2.00×	128	0.05	8	78.45±0.12(+2.35)
		0.1	8	78.80±0.23(+2.70)
		0.2	8	77.95±0.30(+1.85)
		0.4	8	71.86±0.12(−4.24)
		0.1	1	78.36±0.22(+2.26)
		0.1	2	78.54±0.39(+2.44)
		0.1	16	79.05±0.25(+2.95)
		0.1	32	78.69±0.19(+2.59)

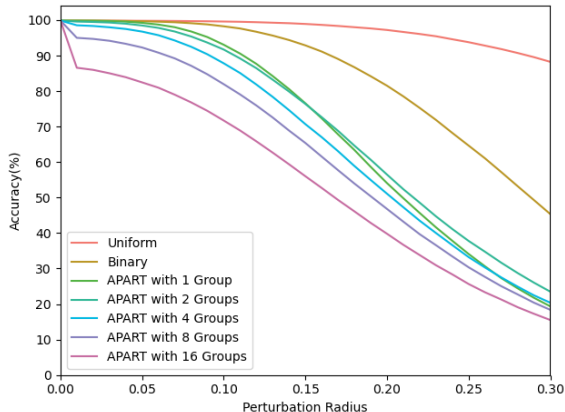


Figure 1: Evaluation on APART’s Attacks.

324 randomly drawing $\delta_\mu, \delta_\sigma$ from $\{-\epsilon, \epsilon\}^d$ formed by the binary values. Besides, we test different
 325 group numbers of APART to substantiate our insight of this trick.

326 **Results.** As is shown in Figure 1, the uniform random perturbations result in almost no reduction in
 327 the accuracy despite the radii, while the binary random perturbations require sufficient large radii for
 328 the attack. In contrast, APART uses only a batch of samples to generate the effective perturbations
 329 that reduce the accuracy even under a small radius. Additionally, the larger group numbers of APART
 330 provide more significant accuracy reduction especially when the radii are more limited, demonstrating
 331 our insight.

332 4.5 Robustness against Perturbed BN Statistics

333 **Experimental Setup.** We evaluate the robustness of the APART-trained models against perturbed BN
 334 statistics to provide the insight of APART’s effectiveness. We employ the WideResNet-40-2 trained
 335 by the standard method with $1\times$ and $2\times$ training budgets and APART with different perturbation
 336 radii and group numbers on CIFAR-100. First, we randomly draw a direction \mathbf{v} from $\{-1, 1\}^d$ for
 337 each BN statistics with the same initial random seed shared across each experiment. Second, we
 338 scale \mathbf{v} by different perturbation radii ϵ to perturb the estimated BN statistics, *i.e.*, $\hat{\boldsymbol{\mu}} \leftarrow (1 + \epsilon\mathbf{v})\hat{\boldsymbol{\mu}}$ or
 339 $\hat{\boldsymbol{\sigma}} \leftarrow (1 + \epsilon\mathbf{v})\hat{\boldsymbol{\sigma}}$. Then, each model with the perturbed statistics is evaluated over the test samples.

340 **Results.** As is shown in Figure 2, mod-
 341 els’ generalization is measured by the non-
 342 perturbed accuracy, and their robustness
 343 is illustrated by the accuracy reduction result-
 344 ing from the perturbations. APART-
 345 trained models generally outperform the
 346 standard models for both the generaliza-
 347 tion and robustness. Specifically, the stan-
 348 dard models (dashed lines) yield lower non-
 349 perturbed accuracy and suffers from more
 350 accuracy reduction as the perturbations in-
 351 crease. Meanwhile, more training epochs
 352 (dashed orange line) slightly improve the
 353 performance of the standard methods. On
 354 the other hand, APART performs better but
 355 requires a trade-off between the generaliza-
 356 tion and robustness. Increasing APART’s radii improves both the robustness and generalization to
 357 some extent. However, a large radius ($\epsilon = 0.4$) results in the severe degeneration (solid pink line)
 358 in the generalization. Additionally, different group numbers of APART lead to improvement of
 359 the generalization and robustness to varying degrees but have no clear trend. In summary, APART
 360 consolidates models’ robustness against noisy BN statistics to boost models’ performance but requires
 361 a further generalization-robustness trade-off achieved by tuning the hyperparameters.

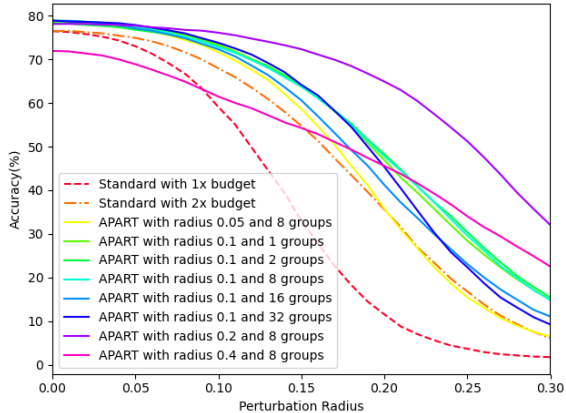


Figure 2: Robustness of WideResNet-40-2 against perturbed BN statistics on CIFAR-100.

362 5 Conclusion and Discussion

363 In this paper, we identify the robustness against the noise in BN statistics to bridge the generalization-
 364 robustness gap. Then, we proposed APART that implements a new AT paradigm, termed model-
 365 based AT, to achieve such robustness. APART performs attacks and defense within models by two
 366 backward passes over each batch of benign samples, utilizing gradients efficiently. The empirical
 367 results demonstrate APART’s effectiveness in improving the robustness, which further boosts model
 368 generalization on benign samples.

369 **Limitations.** Though APART improves models by solving a BN-specific problem, it and its variant
 370 suffer from the potential degeneration in case of the combination with other training methods
 371 implicitly involving BN, which results in more demand for fine-tuning the hyperparameters.

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577 Checklist

- 578 1. For all authors...
- 579 (a) Do the main claims made in the abstract and introduction accurately reflect the paper’s
580 contributions and scope? [Yes]
- 581 (b) Did you describe the limitations of your work? [Yes]
- 582 (c) Did you discuss any potential negative societal impacts of your work? [No]
- 583 (d) Have you read the ethics review guidelines and ensured that your paper conforms to
584 them? [Yes]
- 585 2. If you are including theoretical results...
- 586 (a) Did you state the full set of assumptions of all theoretical results? [N/A]
- 587 (b) Did you include complete proofs of all theoretical results? [N/A]
- 588 3. If you ran experiments...
- 589 (a) Did you include the code, data, and instructions needed to reproduce the main ex-
590 perimental results (either in the supplemental material or as a URL)? [Yes] See the
591 supplemental materials
- 592 (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they
593 were chosen)? [Yes] See the experimental setup
- 594 (c) Did you report error bars (e.g., with respect to the random seed after running experi-
595 ments multiple times)? [Yes]

- 596 (d) Did you include the total amount of compute and the type of resources used (e.g., type
597 of GPUs, internal cluster, or cloud provider)? [Yes]
- 598 4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
- 599 (a) If your work uses existing assets, did you cite the creators? [Yes]
- 600 (b) Did you mention the license of the assets? [N/A]
- 601 (c) Did you include any new assets either in the supplemental material or as a URL? [No]
602 Only existing datasets are used.
- 603 (d) Did you discuss whether and how consent was obtained from people whose data you're
604 using/curating? [N/A]
- 605 (e) Did you discuss whether the data you are using/curating contains personally identifiable
606 information or offensive content? [N/A]
- 607 5. If you used crowdsourcing or conducted research with human subjects...
- 608 (a) Did you include the full text of instructions given to participants and screenshots, if
609 applicable? [N/A]
- 610 (b) Did you describe any potential participant risks, with links to Institutional Review
611 Board (IRB) approvals, if applicable? [N/A]
- 612 (c) Did you include the estimated hourly wage paid to participants and the total amount
613 spent on participant compensation? [N/A]