# Network spreading from network dimension

*spreading dynamics, network correlation dimension, basic reproduction number, susceptible-infected-recovered processes, self-similarity*

## Extended Abstract

Spreading on networks encompasses numerous influential processes, including rumour propagation on social networks and contagion on contact networks [1]. Decision makers may wish to promote a beneficial quantity or contain a harmful one, but in either case they benefit from accurate network spreading models. However, many of these models are limited by their reliance on local features, such as degree [2] or adjacency [3], which can only represent each node's immediate neighbourhood and reflect just one or two of the immediately succeeding transmission steps. They achieve excellent performance on synthetic networks satisfying the appropriate assumptions, but can be challenged by alternative structures [4].

Towards addressing these limitations of popular models, we consider a network property ideally suited to capturing spreading. This is the network correlation dimension $D$ [5], which characterises how the number of entities within network distance $s$ of a source scales with network distance via the power-law relationship

$$c(s) \propto s^{D-1}, \qquad 1 \leq s \leq s_{\max}, \tag{1}$$

where $s_{\max}$ is the upper cutoff of the considered dimension-based structural model and the correlation $c(s)$ is the fraction of distinct nodes at network distance $s$ (see Fig. 1). Leveraging this property leads to a simple but accurate model. On a wide range of synthetic and empirical networks the proposed method yields better predictions of early susceptible-infected-recovered spreading dynamics than established techniques of substantially higher parametric complexity (see Fig. 2). The proposed model also leads to a basic reproduction number–one of the most important terms in studies of epidemics and spreading–providing additional information about final system state (see Fig. 3). The insights from our proposal highlight the importance of incorporating global properties such as dimension into spreading models, and suggest a promising direction for more accurate characterisations of complex dynamics.

## References

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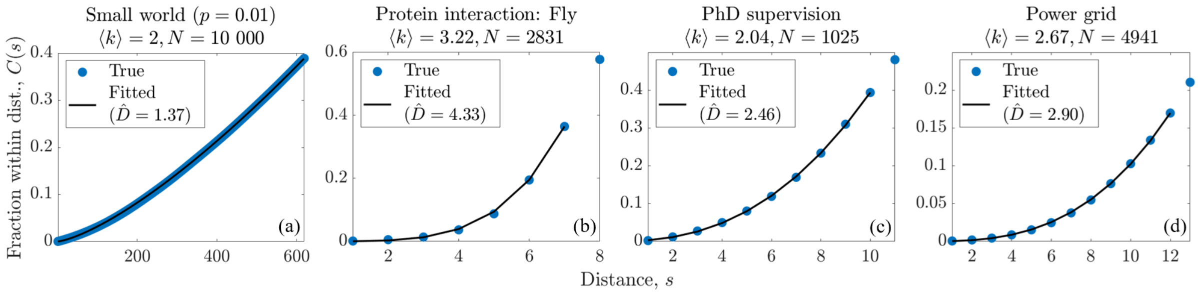


Figure 1: Network correlation dimension governs scaling of small network distances in a wide range of networks. Fraction $C(s) = \sum_{r=0}^{s} c(r)$ of pairs of distinct nodes within network distance $s$ for observed networks (filled blue circles), together with estimates based on maximum likelihood fits to Eq. (1) (black lines). (a) Synthetic network. (b)-(d) Empirical networks.

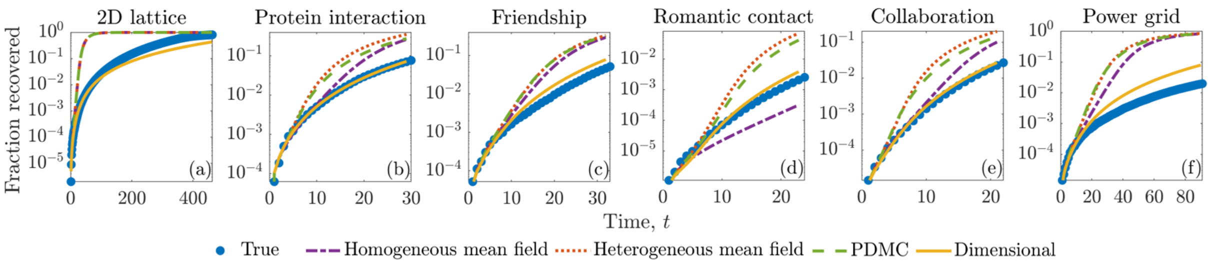


Figure 2: The proposed dimensional spreading model captures the early stages of spreading more accurately than established models of higher complexity. (a) Synthetic network. (b)-(f) Empirical networks. Curves represent true mean state (filled blue circles) and estimates from homogeneous mean field (purple dot-dashed), heterogeneous mean field [2] (red dotted), probabilistic discrete-time Markov chain [3] (green dashed), and proposed dimensional spreading (yellow solid) model.

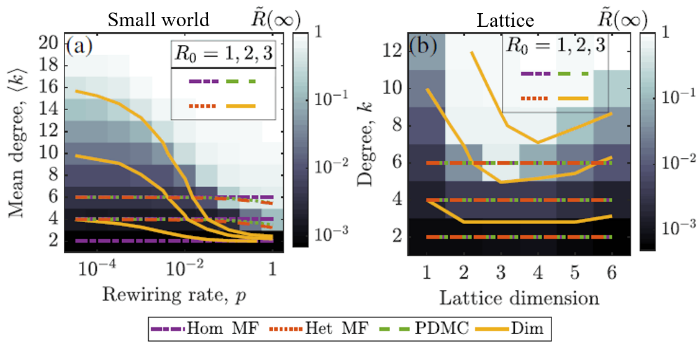


Figure 3: The dimensional spreading model provides information about ultimate system state not available from other models. Variation of final affected ratio $\tilde{R}(\infty)$ (the fraction of nodes who eventually enter the recovered state) with: (a) rewiring rate $p$ and mean degree $\langle k \rangle$ on a small world network; and (b) lattice dimension and degree $k$ on a regular lattice. Curves show level sets of basic reproduction number $R_0$ inferred from the homogeneous mean field (purple dot-dashed), heterogeneous mean field (red dotted), probabilistic discrete-time Markov chain (green dashed), and proposed dimensional spreading (yellow solid) model.