Learn, Unlearn and Relearn: An Online Learning Paradigm for Deep Neural Networks

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Abstract

Deep neural networks (DNNs) are often trained with the premise that the complete training data set is provided ahead of time. However, in real-world scenarios, data often arrive in chunks over time. This leads to important considerations about the optimal strategy for training DNNs, such as whether to fine-tune them with each chunk of incoming data (warmstart) or to retrain them from scratch with the entire corpus of data whenever a new chunk is available. While employing the latter for training can be computationally inefficient, recent work has pointed out the lack of generalization in warm-start models. Therefore, to strike a balance between efficiency and generalization, we introduce Learn, Unlearn, and Relearn (LURE) an online learning paradigm for DNNs. LURE interchanges between the unlearning phase, which selectively forgets the undesirable information in the model through weight reinitialization in a data-dependent manner, and the relearning phase, which emphasizes learning on generalizable features. We show that our training paradigm provides consistent performance gains across datasets in both classification and few-shot settings. We further show that it leads to more robust and well-calibrated models.¹

1 Introduction

"A little learning is a dangerous thing." -Alexander Pope

In recent years, supervised learning has achieved human-level performance in many computer vision tasks in which the learner is trained in an offline learning environment with a fixed set of training data. However, DNNs deployed in the real world are expected to work in an environment where the data arrive in a sequence of large chunks (mega-batches). Several learning paradigms have been proposed to learn from a stream of data, including, but not limited to, continual learning (Van de Ven & Tolias, 2019; Thrun, 1995), active online learning (Settles, 2009), and anytime learning (Grefenstette & Ramsey, 1992; Caccia et al., 2021). Although previous efforts established a solid theoretical foundation, certain subtle issues make it inapplicable to the practical environment.

For an online learning system, it is important that the learner produces high accuracy and generalizes well at any point in time while using limited computational resources (Caccia et al., 2021). Recent research in online learning, however, has shown that training from a previously trained model (warm-start rather than fresh initialization) hinders its ability to adapt to new input (Achille et al., 2018), thus incapacitating the generalization of DNNs (Ash & Adams, 2020; Caccia et al., 2021). These implications of warm-starting have also been observed in online active learning (Huang, 2021; Sener & Savarese, 2017; Ash et al., 2019), where they mitigate it by retraining from scratch after every selection. However, training the DNNs from scratch each time the new data arrive is computationally inefficient, and the lack of generalization with warm-starting undermines the benefits of training with learned features (Ash & Adams, 2020). Thus, the failure of current online learning systems to generalize across data streams without bartering computation efficiency presents a striking lacuna for large-scale deployment of machine learning systems.

Humans, on the other hand, learn in succession over the lifespan and readily generalize by applying prior knowledge to novel situations and stimuli without the need to learn from scratch. This in the brain is

 $^{^{1}\}mathrm{The}$ code will be made publically available upon acceptance.

facilitated by a complex set of neurophysiological processes (Goyal & Bengio, 2020). One such glaring aspect of the brain that allows humans to generalize better is the inherent process of active forgetting (Hardt et al., 2013; Davis & Zhong, 2017). It plays an active role in the regulation of the learning process for better generalizability in the real world. The growing evidence in neuroscience and cognitive psychology (Gravitz, 2019; Izawa et al., 2019) suggests that the brain actively forgets through selective extinction of neurons, which shapes the learning-memory process and, therefore, prevents humans from overfitting to experiences (Shuai et al., 2010). Thus, emulating this aspect of selective forgetting might hold the key to improving generalization in DNNs.

Therefore, we propose a general learning paradigm, which we refer to as Learn, Unlearn, and RElearn (LURE), to address the problem of generalization of parameterized networks on sequential data. For simplicity, we focus mainly on an online learning scenario in which models achieve good performance at any point in time, termed Anytime Learning (Caccia et al., 2021). We consciously simulate the process of selective forgetting (unlearning) in the DNNs by re-randomizing a subset of weights before training on the new samples. With extensive experiments on multiple datasets, we show that our proposed training paradigm boosts the performance and generalization of the models to a greater extent. Compared to standard online training, LURE significantly improves the robustness of DNNs in tackling more challenging real-world scenarios such as learning with noisy labels, natural corruption, and adversarial attacks. Finally, our training paradigm leads to well-calibrated models that are less susceptible to changes in hyperparameters. Our main contributions are as follows.

- Learn, Unlearn, and Relearn (LURE), a bio-inspired online training paradigm to improve the performance and generalization of DNNs through the lens of selective forgetting.
- We demonstrate the efficacy of LURE in multiple architectures across different datasets in online learning and a few-shot classification scenario.
- Our method exhibits robustness in solving more common challenges in real-world problems, including learning with noisy labels, natural corruption, and adversarial attacks.
- Our proposed training paradigm is robust to changes in hyper parameters and leads to well-calibrated models.

2 Related Work

Computational systems operating in the real world are exposed to a continuous stream of data that often arrives in chunks over time. Lifelong learning (Thrun, 1995), online learning (Ash & Adams, 2020), anytime learning (Caccia et al., 2021), have gained increasing attention from the deep learning community due to its relevance in practical settings. Caccia et al. (2021) describes an online learning system as a learner that produces high accuracy and generalizes well at any point in time while using limited computational resources. Recent research on online learning, (Caccia et al., 2021; Ash & Adams, 2020), has noticed a lack of generalization in DNNs when trained in online settings. Ash & Adams (2020) points out that if a model is finetuned from a pre-trained model (a "warm-start"), the resulting new model performs worse than a model trained from scratch (a "cold-start") even though the new data is sampled from the same distribution as the previously trained data. Thus, the lack of generalization in DNN renders them inapplicable to real-world scenarios.

Recently, several weight reinitialization methods (Taha et al., 2021; Li et al., 2020; Alabdulmohsin et al., 2021; Ash & Adams, 2020; Zhou et al., 2022) have been proposed to improve the generalization performance of DNNs by partially or fully refining the learned solution. Zhou et al. (2022) propose a forget and relearn hypothesis to unify disparate existing iterative algorithms under the lens of forgetting. Their approach is based on the consideration that early layers learn generalized representation, whereas later layers memorize. Therefore, they reinitialize and retrain the later layers of the model repeatedly, thereby erasing the information pertaining to the memorized difficult examples. Similarly, Ash & Adams (2020) propose a method to improve generalization by shrinking the magnitude of the weights and perturbing it by injecting a small noise. However, these weight reinitialization methods have architecture-specific assumptions independent of

the data and are handled based on the assumed properties that are inherent to the model and its learning. These methods lack a priori knowledge of where and what features, layers, etc. should be reinitialized in the general case. Motivated by the biological phenomenon of active forgetting (Gravitz, 2019; Shuai et al., 2010; Davis & Zhong, 2017) that selectively forgets extraneous information, we propose a bio-inspired training paradigm for learning from sequential data, LURE, to improve the generalization of DNNs through the lens of active forgetting.

3 Method

We propose *Learn*, *Unlearn*, and *Relearn* (*LURE*), a training paradigm for learning from sequence of data, which interchanges the unlearning (selective forgetting) and relearning steps alternatively. Our proposed online training paradigm consists of three steps: a) learn, b) unlearn, and c) relearn. Our proposed approach is illustrated in Figure 1 and is detailed in the Appendix (Algorithm 1).

Learn We define the Anytime Learning at Macroscale (ALMA) learning environment as envisioned in Caccia et al. (2021) where the authors focus on real-world settings. The data is provided to the learner in the form of a stream SB consisting of t consecutive batches of samples. Therefore, we also focus on the general classification problem, where data are sampled from an underlying data distribution $\mathcal{D}_{x,y}$ with input $x \in \mathbb{R}^{\mathcal{D}}$ and label $y \in \{1, ..., C\}$.

Let \mathcal{M}_i be a collection of $N \gg 0$ in-distribution samples randomly selected from $\mathcal{D}_{x,y}$, for $i \in \{1,...,t\}$. The stream is then defined as the ordered sequence $S_B = \{\mathcal{M}_1,...,\mathcal{M}_t\}$. We refer to each dataset \mathcal{M}_i as a mega-batch, as it is composed of a large number of samples. Consider a model $f_\theta : \mathbb{R}^{\mathcal{D}} \to \{1,...,C\}$ updates its parameters by processing a mini-batch of $n \ll N$ examples at the time of each mega-batch \mathcal{M}_i in such a way as to minimize its objective function. Since the data are passed as a stream, the model does not have access to the future mega-batches and is limited to one pass through the entire stream. However, the model might make several passes over the current and some previous mega-batches depending on the available computational budget. In ALMA, it is assumed that the rate at which mega-batches arrive is slower than the training time of the model on each mega-batch, and therefore the model can iterate over the mega-batches at its disposal based on its discretion to maximize performance, resulting in an overall data distribution that is not i.i.d. by the end of the stream. This implies a trade-off between effectively generalizing and learning from the current data at each mega-batch. Therefore, in such settings, we train the randomly initialized network $f_{\theta_{Reinit}}$ on a mega-batch \mathcal{M}_t belonging to the data stream $\mathcal{D}_{x,y}$ for e epochs until convergence. The loss function employed for learning is defined as follows:

$$L_{\mathcal{T}} = \sum_{i=1}^{t} \underset{(x,y) \sim \mathcal{M}_{i}}{\mathbb{E}} \left[\mathcal{L}_{ce} \left(\sigma \left(f_{\theta} \left(x \right) \right), y \right) \right], \tag{1}$$

where \mathcal{L}_{ce} is a cross-entropy loss, t is the number of mega-batch sequences, and σ is the softmax function.

Unlearn

The human brain, on the other hand, has the remarkable capacity to acquire, store and recall information that allows humans to learn continuously and generalize better (Hardt et al., 2013). Davis & Zhong (2017) partially attributes the ability of humans to generalize new experiences to the phenomenon of active forgetting. Forgetting plays a key role in selectively regulating and rebalancing the learning memory process to prevent humans from overfitting to the experiences (Gravitz, 2019). These neuroscience findings provide substantial evidence for the existence of a symbiotic link between generalization and active forgetting in biological neural networks that are lacking in DNNs. Therefore, we emulate this aspect of selective forgetting to improve the generalization of DNNs in online settings.

We introduce an unlearning step where the network selectively forgets the connections that are less relevant for the current mega-batch and retains those that are specific for the current mega-batch. We quantify the sensitivity (importance) of each connection in the network in a data-dependent manner to identify taskspecific connections. We employ SNIP (Lee et al., 2018), which harnesses the sensitivity of the connection

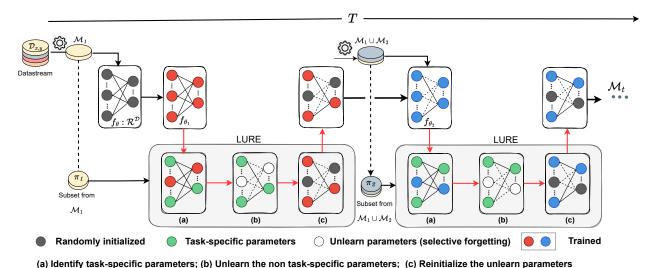


Figure 1: Schematics of the proposed LURE framework. LURE alternates between the unlearning phase, which selectively forgets the undesirable information in the model through weight reinitialization, and the relearning phase, which emphasizes learning generalizable features.

by decoupling the weight from the loss function, to find relevant connections. To determine the connection sensitivity, we sample a small subset of data from the current mega-batch ($\pi_i = 0.2 \times \mathcal{M}_i$). We define a connection sensitivity mask $\mathbf{M} \in \{0,1\}^{|\theta|}$ that is proportional to the number of parameters (m) in the network. Then we apply a sparsity constraint k that specifies the percentage of parameters that must be retained. We compute the connection sensitivity as follows:

$$g_{j}(\theta; \pi) = \lim_{\delta \to 0} \frac{\mathcal{L}_{ce}(\mathbf{M} \odot \theta; \pi) - \mathcal{L}_{ce}((\mathbf{M} - \delta \mathbf{e}_{j}) \odot \theta; \pi)}{\delta} \bigg|_{\mathbf{M} = 1}$$
(2)

where j corresponds to the parameter index and e_j is the mask vector of the index j where the magnitude of the derivatives is then used to calculate the saliency criteria (s_j) :

$$s_j = \frac{|g_j(\theta; \pi)|}{\sum_{k=1}^m |g_k(\theta; \pi)|}.$$

Following the saliency computation, the connection sensitivity mask is set to only retain the top-k task-specific connections based on the sparsity constraint k which is given as follows:

$$\mathbf{M}_{j} = \mathbb{1}\left[s_{j} - \tilde{s}_{\kappa} \ge 0\right], \quad \forall j \in \{1 \dots m\},$$

where \tilde{s}_k is the k^{th} largest element in the saliency vector s and $\mathbb{1}[.]$ is the indicator function.

Then, based on the saliencies pertaining to the connection sensitivity, we retain the top-k important connection and we unlearn the parameters that are unimportant for the current data. Thus, we induce active forgetting through reinitialization of the connections that are less desirable for the current mega-batch. Finally, the network parameters pertaining to unlearned connections are reinitialized to random values:

$$\theta_{new}^{j} = \begin{cases} \theta^{j} & \text{if } \mathbf{M}_{j} = 1\\ \theta_{Reinit} & \text{Otherwise} \end{cases}$$
 (3)

where θ are the weight parameters for the previous mega-batch and θ_{Reinit} corresponds to the random initialized value sampled from a uniform distribution. The network with new parameters θ_{new} is then trained on new consecutive mega-batch.

Relearn

In this stage, the network with the new initialization $(f_{\theta_{new}})$ is updated with the new incoming data \mathcal{M}_{i+1} (in case of no-replay), or with the joint of all the seen data $\mathcal{M}_{i+1} \cup \mathcal{M}_i$ (in case of full-replay) for the e epochs, where e is kept the same for each iteration. The network is trained with the loss function shown in Equation 1. The unlearn and relearn phases are alternatively repeated after the completion of each megabatch training. Thus, by alternating between unlearning and relearning, we favor the preservation of the task-specific connections that can guide the network towards those desirable traits that efficiently improve performance and generalization.

4 Experimental set-up

We provide details on the datasets, implementation details (see Appendix Section A.2), and metrics used to evaluate our LURE framework.

Baselines. To evaluate and benchmark our proposed method, we compare LURE with (1) Baseline (BL) method which is continuously trained in ALMA settings without reinitialization as proposed by Caccia et al. (2021). (2) Shrink and Perturb method (S&P) (Ash & Adams, 2020) which is proposed for online learning. (3) Later-Layer forgetting (LLF) which is proposed for improving generalization in the small data regime (Zhou et al., 2022).

Metrics. For a thorough evaluation, we use CER along with the test accuracy and the generalization gap.

• Cumulative Error Rate (CER): This can be defined as follows:

$$CER = \sum_{i=1}^{\mathcal{M}_t} \sum_{j=1}^{|\mathcal{T}_{x,y}|} \mathbb{1}\left(f_{\theta}\left(x_j\right) \neq y_j\right),\tag{4}$$

where $\mathcal{T}_{x,y}$ represents the held-out test set, f_{θ}^{i} is trained on \mathcal{M}_{i} , y is the ground truth label. A model must have a lower CER at each mega-batch of training with a data stream in order to be an effective anytime learner.

• **Generalization gap:** We use the standard generalization gap as a measure to understand whether the model is overfitting or underfitting at anytime learning which is given by the difference between the training and the validation accuracy.

5 Results

5.1 Analysis of Short Sequence

Table 1 shows the results of the ResNet18 model training on multiple datasets with and without different forms of reinitialization. All experiments on CIFAR10 and CIFAR100 were carried out using full replay $(S_B = \bigcup_{i=1}^8 \mathcal{M}_i)$ for a total of 8 mega-batches with each mega-batch containing 6250 samples, while the experiments on Restricted ImageNet are performed for $|S_B| = 3$ mega-batches. Our observations from Table 1 are as follows: (1) Reinitialization-based training for online learning improves test accuracy, CER, and generalization to a greater extent consistently on all three datasets compared to standard training (BL). (2) LURE outperforms the baseline by 4.8%, 6.64% and 4.13% on CIFAR10, CIFAR100, and Restricted ImageNet respectively, and shows the strongest performance on all the datasets when compared to the other reinitialization methods. (3) Online training using LURE results in the lowest CER and generalization gap compared to other methods, improving the model's anytime learning capabilities. Thus, selectively forgetting extraneous information and relearning it through weight reinitialization brings discernible benefits to the model in online settings.

Table 1: Evaluation of the model (ResNet18) trained with various reinitialization methods in ALMA settings. CIFAR10 and CIFAR100 were trained in a sequence of $|S_B| = 8$, while restricted ImageNet was trained with $|S_B| = 3$ mega-batches.

Datasets	Methods	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
	BL	89.47 ±0.51	11760	8.98
CIEA D10	$_{ m LLF}$	$91.43{\scriptstyle~\pm0.70}$	10103	7.22
CIFAR10	S&P	$91.76{\scriptstyle~\pm0.26}$	10206	7.72
	LURE	$93.32 \scriptstyle{\pm 0.58}$	$\boldsymbol{9622}$	$\boldsymbol{6.60}$
	BL	$62.96{\scriptstyle~ \pm 0.53}$	39010	31.54
CIEA D100	$_{ m LLF}$	$67.04{\scriptstyle~\pm0.68}$	34575	21.23
CIFAR100	S&P	$64.48 \scriptstyle~ \pm 0.11$	36303	28.58
	LURE	69.60 ± 0.82	33037	22.37
	BL	81.39 ±0.48	2967	4.90
Restricted ImageNet	LLF	$82.10{\scriptstyle~\pm0.85}$	2854	4.88
	S&P	$80.80{\scriptstyle~\pm0.39}$	2996	5.10
	LURE	85.52 ± 0.22	2699	4.84

Table 2: Evaluation of the model (ResNet18) trained with buffered replay (buffer size=186) in ALMA settings. CIFAR10 and CIFAR100 were trained in a sequence of $|S_B| = 8$ mega-batch, while restricted ImageNet was trained for $|S_B| = 3$.

Datasets	Methods	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
	BL	88.01±0.93	15784	13.01
	$_{ m LLF}$	$87.51{\scriptstyle\pm0.82}$	15045	13.22
CIFAR10	S&P	$84.62 \scriptstyle{\pm 0.74}$	15549	14.72
	LURE	$89.8 \scriptstyle{\pm 0.58}$	$\boldsymbol{13629}$	11.92
	BL	$62.68{\scriptstyle\pm0.74}$	39178	31.33
	LLF	$67.29{\scriptstyle\pm0.58}$	34416	24.86
CIFAR100	S&P	$65.07 \scriptstyle{\pm 0.60}$	36040	27.79
	LURE	$72.04_{\pm0.35}$	33037	26.59
	BL	$75.97{\scriptstyle\pm0.55}$	3284	8.72
	LLF	$76.40{\scriptstyle\pm0.65}$	3245	9.07
Restricted ImageNet	S&P	$74.75{\scriptstyle\pm0.38}$	3857	8.20
	LURE	$78.58_{\pm0.49}$	3376	7.94

5.2 Analysis of Short Sequence with Buffered/No Replay

We investigate a real-world scenario in which access to the entire dataset used to train the model is restricted due to data privacy or memory restrictions. Tables 2 and 3 show the results of training the ResNet18 model on multiple datasets with buffered replay (buffer size =187) and without replay, respectively. All experiments on CIFAR10 and CIFAR100 were carried out for a total of 8 mega-batches with each mega-batch containing 6250 samples, while the experiments on Restricted ImageNet are performed for $|S_B| = 3$ mega-batches. LURE with and without buffered replay consistently outperforms baselines and other methods across all datasets. For example, LURE with buffered replay improves performance by 1.7%, 9.4%, and 2.5% over standard training (BL) on CIFAR10, CIFAR100, and R-ImageNet, respectively. In the challenging case of no-replay settings, our proposed method has a profound impact on standard training compared to other reinitialization methods. This demonstrates that the observed benefits of LURE are not limited to replay-based methods alone. Thus, unlearning and relearning at each mega-batch of training help to learn more generalizable representation in different replay scenarios.

Table 3: Evaluation of the model (ResNet18) trained without replay in ALMA settings. CIFAR10 and CIFAR100 were trained in a sequence of $|S_B| = 8$ mega-batch, while restricted ImageNet was trained for $|S_B| = 3$.

Datasets	Methods	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
	BL	80.86±1.01	16789	16.85
CIEA D10	S&P	$81.92 \scriptstyle{\pm 0.62}$	16133	16.36
CIFAR10	LLF	$86.11{\scriptstyle\pm0.59}$	15294	14.66
	LURE	$88.96 \scriptstyle{\pm 0.24}$	13953	12.14
	BL	$48.79 \scriptstyle{\pm 0.75}$	44128	49.95
CIEA D100	LLF	$50.38 \scriptstyle{\pm 0.54}$	44816	50.80
CIFAR100	S&P	$49.26 \scriptstyle{\pm 0.59}$	44581	51.45
	LURE	$55.37 \scriptstyle{\pm 0.63}$	43348	46.69
	BL	$73.36 \scriptstyle{\pm 0.88}$	3450	9.68
Restricted ImageNet	LLF	$76.40{\scriptstyle\pm0.65}$	3515	8.15
	S&P	$73.36{\scriptstyle\pm0.78}$	3464	8.13
	LURE	$77.97_{\pm 0.43}$	3367	7.90

5.3 Analysis of Moderate and Long Sequence ($|S_B| = 25, 50, 100$)

Computational systems deployed in the real world are often exposed to longer mega-batch sequences of data and need to be updated frequently. Therefore, it is quintessential for the online model to perform well under longer mega-batch sequences. Table 4 shows the results of the ResNet18 model training on the CIFAR10 dataset. All experiments were carried out using full replay for a longer sequence of mega-batches 25, 50, and 100. We observe that LURE consistently outperforms the baseline and other methods across varying sequences of mega-batches. As the sequence of mega-batches increases, the number of samples available per mega-batch reduces drastically. Similar to Caccia et al. (2021), we observe that regularly updating the model on fewer samples significantly exacerbates the CER resulting in poor anytime learning performance. Therefore, long-sequence online learning with reinitialization, especially LURE, reduces CER and the generalization gap to a greater extent compared to standard training, thus enriching the predictive capabilities of the model at any given time.

5.4 Analysis of Few-Shot Experiments on Restricted ImageNet

For many real-world classification problems, machine learning models deployed often need to be updated on labeled data that are scarce and may not be initially available for training. It is possible for new sets of labeled data to become available gradually as they are labeled. Therefore, it is important for the system to function properly in such online few-shot settings. Table 5 shows the results of the methods in a few-shot settings where we limit the number of samples to 270 per class and vary the sequence of mega-batches. We observe that reinitialization-based training improves performance and generalization over the baseline, even in challenging few-shot classification. For a mega-batch sequence of 70, LURE outperforms baseline, LLF, and S&P by a relative improvement of 4.11%, 2.01%, and 3.15%, respectively, while for a sequence of 30 it is on par with LLF in terms of accuracy. In both settings, LURE achieves the lowest CER and the generalization gap, demonstrating the superiority of our proposed approach.

5.5 Analysis on Various Architectures

Here, we examine the versatility of our proposed LURE framework for multiple architectures on the CIFAR10 dataset. We consider ResNet18 (He et al., 2016), ResNet50 (He et al., 2016), wider-Resnet50-2 (Zagoruyko & Komodakis, 2016), VGG16 (Simonyan & Zisserman, 2014). We chose these models explicitly because of their widespread popularity in common computer vision tasks and the breadth of research done on them for different learning paradigms. Table 6 shows the performance and generalization gap of the model trained in

	Table 4: Evaluation of the model	(ResNet18) o	on CIFAR10 for longer sequences of mega-ba	tches.
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# Mega-batches	Methods	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
	BL	89.94 ±0.54	43029	6.80
25	$_{ m LLF}$	$89.80{\scriptstyle~\pm0.62}$	42961	6.52
20	S&P	$88.37{\scriptstyle~\pm0.26}$	43578	5.94
	LURE	90.55 ± 0.34	42790	6.79
	BL	89.12 ±0.61	87843	6.02
50	$_{ m LLF}$	$90.26{\scriptstyle~\pm0.82}$	87826	5.75
	S&P	$88.32{\scriptstyle~\pm 0.35}$	85798	6.11
	LURE	90.97 ± 0.67	$\boldsymbol{85487}$	5.18
	BL	89.64 ±0.69	176954	6.46
100	$_{ m LLF}$	$89.48{\scriptstyle~ \pm 0.77}$	173505	7.21
	S&P	$88.01{\scriptstyle~\pm0.44}$	182294	5.56
	LURE	91.95 ± 0.53	170178	5.66

Table 5: Few-shot experiments using ResNet50 on Restricted ImageNet.

# Mega-batches	Methods	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
	BL	$45.82{\scriptstyle~\pm0.55}$	73514	47.93
20	$_{ m LLF}$	47.60 ± 0.61	72868	43.50
30	S&P	$45.92{\scriptstyle~\pm0.38}$	72954	45.61
	LURE	$46.97{\scriptstyle~\pm0.79}$	71542	$\boldsymbol{42.56}$
	BL	$53.43{\scriptstyle~\pm 0.49}$	156332	41.17
70	$_{ m LLF}$	$54.53{\scriptstyle~\pm0.68}$	149493	32.57
	S&P	$53.90{\scriptstyle~\pm0.24}$	150332	40.87
	LURE	55.65 ± 0.52	148478	28.35

different architectures. The experiments are performed with full replay for $|S_B| = 4$. The results demonstrate that LURE significantly outperforms standard training across multiple architectures while the LLF and S&P fail to improve. Therefore, reinitialization of the weight parameter in a data-dependent manner using connection sensitivity by LURE is more effective to improve generalization in different architectures than reinitialization based on assumed model properties and learning (as done by LLF and S&P).

6 Robustness Analyses

6.1 Robustness to Natural Corruptions

In practice, DNNs are often deployed in real-world scenarios where they are exposed to constantly changing environments, often influenced by changes in lighting and weather. Therefore, the robustness of the DNNs to data distributions that are subjected to natural corruption is pertinent. Here, we evaluate the benefit of LURE on robustness to common corruption using CIFAR10-C (Hendrycks & Dietterich, 2019). The models are trained on clean images and tested on CIFAR10-C. Following Hendrycks & Dietterich (2019), we use the mean Corruption Accuracy (mCA) to measure performance under natural corruption. Figure 2 shows the accuracy of the models on 19 different corruptions averaged on five severity levels. Compared to baseline (68%), LLF (72%), and S&P (71%), LURE (74%) delivers a higher mCA in all types of corruption. Evidently, unlearning and relearning at each mega-batch of training bring discernible benefits in terms of robustness to natural corruptions.

Architechture	Methods	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
	Baseline	89.31 ±0.61	5657	7.90
ResNet18	$_{ m LLF}$	$91.80{\scriptstyle~\pm0.43}$	5466	7.68
nesnetto	S&P	$90.50{\scriptstyle~\pm0.51}$	5667	7.80
	LURE	93.73 ± 0.35	4409	7.48
	Baseline	$89.53_{\pm0.58}$	6854	7.83
ResNet50 Wide-ResNet50-2	$_{ m LLF}$	$89.49{\scriptstyle~\pm0.45}$	6782	6.78
	S&P	$89.15{\scriptstyle~\pm0.61}$	6940	6.98
	LURE	92.75 ± 0.73	$\boldsymbol{6682}$	$\boldsymbol{6.62}$
	Baseline	90.10 ±0.69	5978	6.84
	$_{ m LLF}$	$89.72{\scriptstyle~\pm0.36}$	6000	5.70
	S&P	$89.38{\scriptstyle~\pm0.61}$	6292	5.96
	LURE	$93.78 \scriptstyle{\pm 0.54}$	5557	5.81
VCC16 DN	Baseline	89.32 ±0.74	5650	9.62
	$_{ m LLF}$	$87.85 \scriptstyle~\pm 0.58$	6124	$\boldsymbol{6.17}$
VGG16-BN	S&P	$88.25{\scriptstyle~\pm0.86}$	5720	8.11
	LURE	$92.67 \scriptstyle{\pm 0.47}$	4439	8.55

Table 6: Evaluation of methods using different architectures on CIFAR10 dataset ($|S_B| = 4$).

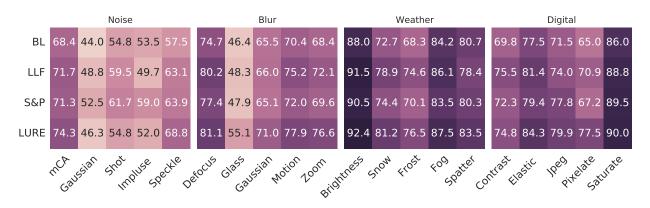


Figure 2: Robustness to natural corruptions on CIFAR10-C (Hendrycks & Dietterich, 2019). LURE is more robust against majority of corruptions compared to other reinitialization methods.

6.2 Robustness to Adversarial Attacks

DNNs have been shown to be vulnerable to adversarial attacks in which imperceptible perturbations are added to inputs during inference. The adversarial images are designed to fool the network to make false predictions (Szegedy et al., 2013). We perform a PGD-10 attack (Madry et al., 2017) on the models trained on the CIFAR10 dataset with varying attack strengths. As observed in Figure 3(Left), LURE exhibits greater resistance to these attacks of varying strengths. Thus, compared to standard training, training a model in the LURE framework facilitates online learners to learn high-level abstractions that are not sensitive to small perturbations in the data.

6.3 Robustness to Noisy Labels

The success of supervised learning often depends on the availability of large amounts of high-quality annotations. However, the availability of high-quality annotated datasets can be extremely expensive and time-consuming to collect. Therefore, it is paramount to have robust training on noisy labels, as studies

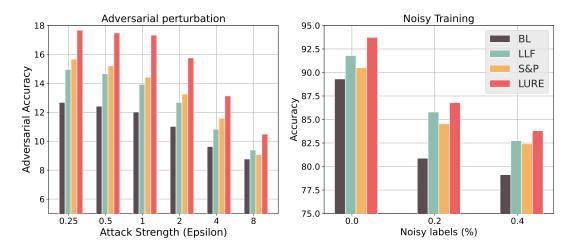


Figure 3: (Left) Robustness to adversarial attacks; (Right) Robustness to training under noisy labels. In both robustness analyses, LURE shows a significant performance improvement compared to the baselines considered.

have shown that DNNs can easily memorize the samples and are susceptible to noisy labels (Arpit et al., 2017). we train ResNet18 on sequential CIFAR10 with noisy labels for a mega-batch sequence of $|S_B| = 4$ and evaluate the performance on a clean test set. We corrupt every ground-truth label with a specified probability (noise rate) by randomly sampling from a uniform distribution over a large number of classes. Table 9 in the Appendix shows the detailed results under noisy label training, including the generalization gap and CER. Figure 3(Right) presents the test accuracy of the reinitialization methods under different percentages of noisy labels. The results show that the reinitialization-based training paradigm is robust to the presence of noisy labels during training compared to the warm-started baseline. Furthermore, LURE consistently outperforms LLF and S&P baseline comfortably across different noisy label rates. Thus, reinitializing based on selective forgetting helps to learn a generic representation that is less sensitive to noise in the dataset.

7 Characteristic Analyses

7.1 Model Calibration

DNNs are often deployed in safety-critical applications where it is essential to have a model that has a sufficient sense of uncertainty about its predictions. Therefore, we evaluate the calibration of models trained with different reinitialization methods in ALMA settings. The common metric for identifying miscalibration in classification is the Expected Calibration Error (ECE) (Naeini et al., 2015). The ECE measures the discrepancy between absolute accuracy and average confidence as a weighted average. The lower the ECE, the better calibrated the model is. Figure 4 shows the ECE values along with a reliability diagram on CIFAR10 using the calibration library by Kuppers et al. (2020). The result shows that BL, LLF, and S&P are highly miscalibrated and far more overconfident than the proposed LURE framework. Thus, in addition to improving performance and generalization, online learning using selective forgetting can effectively improve calibration, thus improving reliability in contexts where safety is of absolute importance.

7.2 Convergence to Flatter Minima

DNNs that converge to flatter minima in a loss landscape have greater adaptability to new tasks without straying too far from the optimal parameters for previous tasks. Furthermore, solutions that reside in flatter minima are more robust because the predictions do not change significantly with minor perturbations. We apply independent Gaussian noise to all parameters of the CIFAR-10 trained model, as described in (Alabdulmohsin et al., 2021). Figure 5(left) shows that the solution reached by LURE, LLF, and S&P is

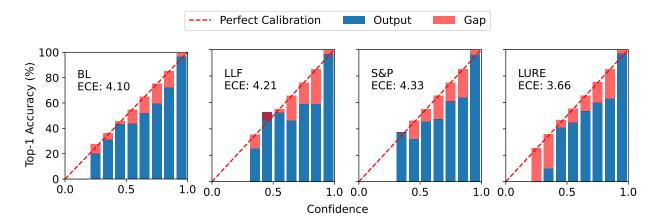


Figure 4: Confidence estimates and the corresponding Expected Calibration Error (ECE) of the CIFAR-10 ALMA trained models. Lower ECE is better. Our method is well calibrated, with confidence estimates closer to perfect calibration compared to BL, LLF, and S&P.

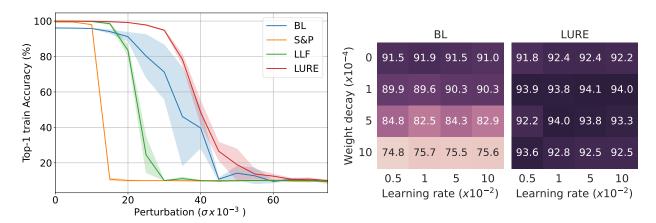


Figure 5: (Left) Robustness of the model perturbed by varying degrees of Gaussian noise. Our method is considerably robust to Gaussian perturbations as the decline in performance is gradual, suggesting convergence to flatter minima. (Right) Sensitivity to hyperparameters. LURE is more robust to the changes in weight decay and learning rate to standard online training.

more robust to model perturbation than standard training. Our method is significantly less sensitive to perturbations than the other methods, and the performance decreases gradually. More specifically, for every amount of noise introduced into the model parameters θ , the change in the training accuracy for LURE is smaller than in standard training, implying that the solution provided by LURE appears to reside in flatter local minima. We argue that training the model by alternating between learning and unlearning stages leads to a larger valley, which could better explain our model's ability to consolidate generalizable features.

7.3 Sensitivity to Hyperparameters

Machine learning systems are often deployed in the real world, where explicitly running a hyperparameter search for each update can be computationally exhaustive. Therefore, similar to Zaidi et al. (2022), we explore the sensitivity of our method to the choice of weight decay and learning rate. Figure 5(right) demonstrates the test accuracy achieved by changing the learning rate and the weight decay values intended for CIFAR-10 training in ALMA scenarios for the mega-batch sequence of $|S_B| = 4$. Compared to baseline training without reinitialization, the performance of LURE is less sensitive to the choice of hyperparameters.

Detailed comparison with other methods is provided in Appendix, Figure 6. For example, the performance of normal training decreases to 75% with a learning rate of 0.005 and a weight decay of 0.001, while the performance of LURE remains above 94% throughout. Therefore, LURE can improve generalization in regimes where it is infeasible to perform exhaustive hyperparameter tuning.

We underline that LURE's effectiveness far exceeds that of other reinitialization techniques and that it should be viewed as a general-purpose online training paradigm, as it is more resilient to typical problems found in real world datasets than the conventional online training method.

8 Conclusion

We introduce Learn, Unlearn, and RElearn (LURE), a bio-inspired online training paradigm to improve the performance and generalization of DNNs through the lens of selective forgetting. LURE alternates between the unlearning phase, which selectively forgets undesirable information in the model, and the relearning phase, which emphasizes learning generalizable features. Empirical results show that the proposed framework improves performance and generalization across a wide range of architectures and datasets, both online and in challenging few-shot classification. Our framework is robust to learning with noisy labels and adversarial attacks and increases generalization in many real-world scenarios. Additionally, training with LURE results in well-calibrated models that are less susceptible to change in hyperparameters. Finally, our model training by unlearning and relearning leads to flatter minima, which results in better generalization. In the future, it will be interesting to study the dynamics of reinitialization in other lifelong scenarios, such as continuous learning, where domain shifts and catastrophic forgetting are more prevalent.

8.1 Broader Impact and Societal Relevance

We believe that our findings can potentially be harnessed to enhance the test accuracy and robustness of any machine learning system deployed in the real world where the aspect of generalization is crucial, as they are continuously trained on sequential data. For example, consider a large-scale social media website in which users continually upload images and content. To recommend material, filter out inappropriate media, and choose adverts, the organization requires up-to-date prediction models. Every day, millions of fresh data points may arrive, which must be quickly integrated into operational ML pipelines. In this scenario, it is logical to envision having a single model that is regularly updated with the most recent data. Everyday additional training on the model with the updated and larger dataset might be undertaken. In these scenarios, the proposed framework (LURE) can improve the generalization and performance of the model to a greater extent as opposed to new training from the parameters of yesterday's model without reinitialization.

Furthermore, in applications such as autonomous driving and industrial robotics, where the deployed model needs to be frequently updated in order to stay in sync with the surroundings. Using LURE as a training paradigm to update the model can boost performance and generalization in a computationally efficient way as it provides a better initialization for continuous training compared to warm-starting or updating the model from scratch.

In addition to the above scenarios, our proposed framework can be conceivably harnessed in applications of deep active learning where the goal is to find the most informative data to label with an oracle and incorporate into the training set. However, current active learning frameworks retrain models from scratch after each querying step, which is computationally expensive and partially responsible for deleterious environmental ramifications. The LURE framework allows models to be efficiently updated without sacrificing generalization and performance, thus having a positive impact on society.

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A Appendix

A.1 Ablation studies

To examine the influence of the individual components of our LURE network in ALMA settings, we perform the following ablation study.

Effect of Ratio of Reinitialized Parameters. Table 7 shows the effect of varying the number of initialized parameters on the performance and generalization of the model in CIFAR10. We train the model in ALMA settings using the LURE framework by varying different percentages of reinitialized parameters (5%, 10%, 20%, 30%, and 40%). The experiments were carried out using full replay with ResNet18 for a $|S_B| = 8$. The results show that the unlearning of a 5% percentage of parameters has no impact on performance, while the unlearning of more than 30% has less impact on the test accuracy. We find that reinitialization 20% of the parameters results in the best performance.

Table 7: Evaluation varying the percentage of reinitialized parameters during training on CIFAR10 dataset using ResNet18.

Method	Reinitialized Params (%)	Test Acc (↑)	CER (\downarrow)	Generalization Gap (\downarrow)
LURE	5	89.84	11955	6.83
	10	91.81	11571	7.22
	20	$\boldsymbol{94.32}$	$\boldsymbol{9622}$	6.60
	30	90.73	11806	7.91
	40	90.79	11559	6.57

Effect of Importance Estimation Method. We investigate the effect of various importance estimation methods on our proposed training paradigm. For this, we consider Fisher Importance (FIM), weight magnitude, random, and SNIP. Table 8 demonstrates the performance and generalization of the model trained with the LURE framework with different selections to estimate important parameters on CIFAR10 using ResNet18 for $|S_B| = 4$. Unlearning and relearning with SNIP, Fisher information, and weight magnitude results in better performance compared to baseline. This shows that our training paradigm is not only limited to SNIP, but any importance estimation criterion can be used to identify the dataset-specific connections.

A.2 Datasets and Implementation Details

Datasets. We empirically evaluate our proposed framework on three different data sets: (a) CIFAR-10 (Krizhevsky et al., 2009) (b) CIFAR-100 (Krizhevsky et al., 2009) and (c) Restricted Imagenet (balanced) (Ilyas et al., 2019; Tsipras et al., 2018). CIFAR10 and CIFAR100 consist of 50,000 training images and 10,000

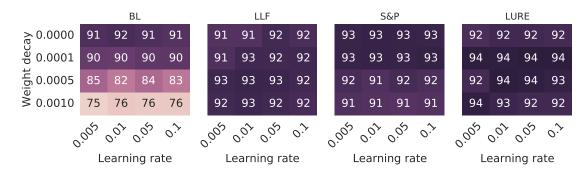


Figure 6: The sensitivity of different reinitialization methods to hyperparameters. The initialization-based training paradigm is more robust against the change in weight decay and learning rate compared to standard online training.

Table 8: Evaluation with different importance estimation on CIFAR10 dataset.

Method	Importance criteria	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
LURE	BL	89.31	5657	7.904
	FIM	92.73	4346	8.39
	Weight Magnitude	92.18	4547	8.53
	SNIP	93.73	4409	7.48

Algorithm 1 Training LURE in ALMA settings

```
input: Data stream S_B = \{M_1, ..., M_t\}, Model f_{\theta}^{i=0}, replay, Sparsity \alpha
 1: i \leftarrow 1
 2: while i \leq |S_B| do
 3:
           if replay then
                \mathcal{M}_i \leftarrow \bigcup_{i=1}^t \mathcal{M}_i
 4:
 5:
           \mathcal{M}_{i} \leftarrow \mathcal{M}_{i}
f_{\theta}^{i} \leftarrow f_{\theta}^{i-1}.train(\mathcal{M}_{i})
\pi_{i} \leftarrow 0.2\mathcal{M}_{i}
 6:
 7:
                                                                                                                                   ▶ Training or learning step
 8:
           M \leftarrow \text{Importance Estimation}(f_{\theta}^{i}, \pi_{i}, \alpha)
 9:
           Retain the task specific weights based on M
10:
                                                                                                                                            ▷ Selective forgetting
           Randomly reinitialize the task irrelevant parameters in f_{\theta}^{i}
11:
           Model with this new initialization for next \mathcal{M}_{i+1} training
12:
```

test images, each of size 32×32 , divided into 10 and 100 classes, respectively. Restricted ImageNet (balanced) is a subset of the original ImageNet data set (Russakovsky et al., 2015) consisting of 89517 training images and 3450 test images, each of 224×224 size divided into 14 classes consisting of five subclasses each. For ease of computation, we resize the images to 32×32 for our experimental settings.

Implementation Details. The efficacy of our framework is demonstrated in ALMA settings. ResNet18 (He et al., 2016) is used as the backbone for most of the experiments on CIFAR10 and CIFAR100, while ResNet50 (He et al., 2016) is used for restricted ImageNet experiments. We initialize the networks randomly and use stochastic gradient descent (SGD) with momentum 0.9 and weight decay 1e-4 to optimize it. We followed the same procedure as that followed by Caccia et al. (2021); Misra et al. (2022). Networks are trained iteratively for t mega-batches with a batch size b = 64 for 50 epochs per mega-batch of training without early stopping. A step learning rate scheduler with an initial learning rate of 0.1 decayed at steps 20 and 40 is employed during mega-batch training of our method. The standard data augmentation technique, i.e., flipping and random cropping, is used. All training settings (lr, b, e) are kept constant throughout the mega-batch training. We randomly divide the data set into mega-batches with an equal number of samples in each mega-batch. For each mega-batch \mathcal{M}_t , we divide it into a train set with 90% of the samples and a validation set with the remaining 10% samples in it. We then randomly sampled 20% of the training data from each mega-batch to build the set π used to identify task-specific parameters through SNIP after each mega-batch of training. We use the default parameters for the SNIP algorithm specified above. Finally, we maintain a separate held-out test set, which is used to evaluate the model's performance after training on each mega-batch. Unless specified, we keep the number of mega-batches to 8 for all the experiments. For training S&P, a shrink coefficient of 0.4 and a noise of 0.001 are applied for the weights of the entire network before training on the new mega-batch of data. Similarly, for LLF, we reinitialize blocks 3 and 4 of ResNet (He et al., 2016) before the start of each mega-batch of training.

Table 9: Evaluation of the model on CIFAR10 with different percentages of noisy labels.

Noisy labels (%)	Methods	Test Accuracy (\uparrow)	CER (\downarrow)	Generalization Gap (\downarrow)
	Baseline	89.31	5657	7.90
	$_{ m LLF}$	91.80	5466	7.68
0.0	S&P	90.50	5667	7.80
	LURE	93.73	4409	7.48
	Baseline	80.88	9460	18.44
	$_{ m LLF}$	85.80	7827	9.90
0.2	S&P	84.54	8081	12.41
	LURE	86.79	7551	8.70
	Baseline	79.13	10854	20.76
	$_{ m LLF}$	82.74	9411	8.40
0.4	S&P	82.41	9839	10.48
	LURE	83.80	$\boldsymbol{9291}$	7.58