

---

# Synergy over Discrepancy: A Partition-Based Approach to Multi-Domain LLM Fine-Tuning

---

Hua Ye<sup>1,2</sup>, Siyuan Chen<sup>3</sup>, Haoliang Zhang<sup>4</sup>, Weihao Luo<sup>5</sup>, Yanbin Li<sup>6</sup>, Xuan Zhang<sup>2,7†</sup>

<sup>1</sup>Nanjing University   <sup>2</sup>Airon Technology CO., LTD   <sup>3</sup>University of Bristol

<sup>4</sup>The University of Oklahoma   <sup>5</sup>Donghua University

<sup>6</sup>Beijing University of Posts and Telecommunications   <sup>7</sup>Carnegie Mellon University

## Abstract

Large language models (LLMs) demonstrate impressive generalization abilities, yet adapting them effectively across multiple heterogeneous domains remains challenging due to inter-domain interference. To overcome this challenge, we propose a partition-based multi-stage fine-tuning framework designed to exploit inter-domain synergies while minimizing negative transfer. Our approach strategically partitions domains into subsets (stages) by balancing domain discrepancy, synergy, and model capacity constraints. We theoretically analyze the proposed framework and derive novel generalization bounds that justify our partitioning strategy. Extensive empirical evaluations on various language understanding tasks show that our method consistently outperforms state-of-the-art baselines.

## 1 Introduction

Large language models (LLMs) have propelled natural language processing (NLP) to unprecedented capabilities [Kumar, 2024, Karanikolas et al., 2023, Hu et al., 2025, Zhang et al., 2025b, Lin et al., 2025], owing largely to their extensive pretraining on massive, diverse textual corpora [Wu et al., 2022, Li et al., 2024a, Chen et al., 2025b, Zhang et al., 2024a, Sun et al., 2025b]. Fine-tuning these pretrained models for a specific downstream domain has been widely explored and proven highly effective; notable approaches include adapter-based modules [Houlsby et al., 2019, Zhang et al., 2024b], parameter-efficient fine-tuning via low-rank updates [Hu et al., 2021], and instruction-based fine-tuning methods [Cao et al., 2024]. These methods have also accelerated the application of LLMs in many scenarios, such as healthcare [Tong et al., 2025, Liu et al., 2025, Wang et al., 2025], transportation [Yao et al., 2023, Lu et al., 2025b, Zeng et al., 2025b,a], robotics [Xiao et al., 2025a, Yan et al., 2025, Zhang et al., 2025a], and other applications [Sun et al., 2025a, Zhang et al., 2025c].

However, while these methods excel in adapting to a single domain, practical scenarios frequently require simultaneous adaptation to multiple distinct domains—a scenario far less studied and substantially more challenging [Lu et al., 2025a, Chen et al., 2025a]. Due to constraints on energy or computing power [He, 2025], we must consider how to enhance the model’s ability to adapt to different domains. Consider, for instance, a scenario where a single pretrained model must be simultaneously adapted to clinical texts [Thirunavukarasu et al., 2023], social media posts [Yang et al., 2024a, Jiang et al., 2024, 2025], and legal documents [Seabra et al., 2024]. Naive approaches such as jointly fine-tuning the model across all domains or independently fine-tuning separate models per domain often yield suboptimal results [Xiao et al., 2025b, Zhang et al., 2023, Wang and Zhang, 2024, Wen et al., 2023]: domain-specific features may negatively interfere, causing one domain to overshadow others or impair overall generalization capabilities [Lu et al., 2024a, Zheng et al., 2024, Van Veen et al., 2023, Tan et al., 2025]. This motivates our central research question: *how can we*

---

†Corresponding author(xuanzhang2199@gmail.com)

*effectively and efficiently fine-tune a single LLM across multiple heterogeneous domains, exploiting inter-domain synergies while mitigating negative interference?*

A natural strategy to mitigate these challenges is to add adapter modules specialized for each domain [Houlsby et al., 2019, Zhang et al., 2024b, Tao et al., 2023]. While adapters reduce the need for full fine-tuning, they do not fully address the complexity of managing multiple source domains with potentially large discrepancies. In such cases, the learned model has to juggle both shared features (common linguistic properties across domains) and domain-specific features (rare words, styles, or content) [Lu et al., 2022]. Existing approaches often handle these competing demands by imposing either domain adversarial objectives [Ganin and Lempitsky, 2015], distribution alignment [Peng et al., 2019], or low-rank parameter updates [Hu et al., 2021]. However, these techniques may still fail to exploit synergistic relationships-where certain domains are complementary and can reinforce each other’s accuracy-and do not fully capture how best to “partition” domains to avoid negative interference.

Motivated by this gap, we propose a partition-based multi-stage framework for multi-domain LLM fine-tuning. Instead of jointly or separately adapting to all domains, our method clusters synergistic domains while isolating highly distinct ones. Our approach integrates practical constraints-memory budgets, domain shifts, and synergy opportunities-with theoretical insights on generalization benefits from restricted updates and strategic domain grouping. The main contributions are:

- 1) We introduce a novel partitioning algorithm that clusters domains according to their synergy and discrepancy, then fine-tunes the LLM in multiple stages. This orchestrated process prevents cross-domain contamination while leveraging beneficial interactions.
- 2) We derive new bounds that capture domain discrepancy, synergy offsets, and adapter complexity, establishing conditions under which multi-stage partitioning yields provably tighter guarantees than single-stage or naive multi-domain methods.
- 3) Our experiments demonstrate that partition-based multi-stage fine-tuning outperforms state-of-the-art baselines, improving accuracy across different domains and tasks while reducing memory usage.

## 2 Related Works

### 2.1 LLM Fine-Tuning

The success of LLMs such as GPT-3 [Brown et al., 2020], LLaMA [Touvron et al., 2023], and Falcon [Almazrouei et al., 2023] has underscored the importance of *efficient* fine-tuning. Fully updating model parameters incurs high computational costs [Zhang et al., 2024b, Hu et al., 2021], prompting parameter-efficient methods that modify only a small subset of parameters. Examples include adapter modules [Houlsby et al., 2019], low-rank projections (LoRA) [Hu et al., 2021], and tightly integrated adapters for LLaMA [Zhang et al., 2024b]. However, most methods assume a single domain; adapting LLMs efficiently to multiple domains remains challenging. Another related field is continual learning [Xu et al., 2025], but it assumes a sequential arrival of tasks, which differs from the setting in this paper.

### 2.2 Multi-Domain Data Learning

Real-world data often originate from disparate sources with distinct distributions and vocabularies [Ganin and Lempitsky, 2015, Sener and Koltun, 2018, Li et al., 2023, Zhang et al., 2024c, Chen et al., 2024, Xiao and Liu, 2025, Ke et al., 2025]. Traditional multi-domain methods align representations through adversarial training [Pei et al., 2018, Ganin and Lempitsky, 2015], moment matching [Peng et al., 2019], or multi-task objectives [Royer et al., 2024, Shen and Zhang, 2025], yet typically neglect inter-domain synergies. Adapting LLMs further complicates this via memory overhead, forgetting pretrained knowledge, and cross-domain contamination. Adapter-based solutions partially address these concerns [Houlsby et al., 2019] but rarely exploit domain partitioning to maximize synergy. Our partition-based multi-stage approach systematically clusters domains to leverage synergy and minimize discrepancy, providing theoretical guarantees.

### 2.3 LLM Data Selection

The efficacy of supervised fine-tuning (SFT) heavily depends on the quality and composition of training data [Yao et al., 2024]. Recent advances have introduced diverse metrics for data selection: *instance-level* criteria like perplexity [Cao et al., 2024], reward scores [Gou and Nguyen, 2024], and loss disparities [Li et al., 2024b], trajectory-based clustering via small proxy models [Yang et al., 2024b], as well as token-level selection methods [Lin et al., 2024]. Existing works largely overlook one critical factor: *domain interactions*, where diversity metrics operate at instance/token levels [Pang et al., 2024] without modeling cross-domain compatibility [Li et al., 2025b]. Our approach addresses this gap by systematically clustering domains for joint tuning.

## 3 Theoretical Analysis

### 3.1 Preliminaries and Notation

Let us consider  $k$  distinct source domains  $\{\mathcal{D}_1, \dots, \mathcal{D}_k\}$ , each containing samples  $(x, y)$  drawn from some distribution over  $\mathcal{X} \times \mathcal{Y}$ . We have a large language model  $\text{LLM}_{\theta^*}$  pretrained on a massive corpus  $\mathcal{D}_{\text{pretrain}}$ , and we assume  $\theta^* \in \mathbb{R}^p$  lies in a high-dimensional parameter space. Our goal is to *adapt*  $\theta^*$  (often minimally) to each domain  $\mathcal{D}_j$  by introducing or modifying a small set of parameters (e.g., adapter modules) denoted by  $\phi_j \in \mathbb{R}^{q_j}$ , where  $q_j \ll p$ .

**Definition 1** (Multi-Source Fine-Tuned Model). A **multi-source fine-tuned model** is given by

$$f_{\theta, \{\phi_j\}}(x) = \text{LLM}_{\theta^* + \Delta\theta}(\phi_1, \dots, \phi_k)(x), \quad (1)$$

where  $\Delta\theta \in \mathbb{R}^p$  is a (potentially small) update to the pretrained backbone  $\theta^*$ . Each  $\phi_j$  may represent additional parameters specialized to domain  $j$ . The model internally selects or combines relevant  $\phi_j$  based on domain context or training strategy.

**Loss and Risk.** We let  $\ell(f(x), y)$  be a nonnegative loss function (e.g., cross-entropy) measuring the prediction error on a sample  $(x, y)$ . For a single domain  $\mathcal{D}_j$ , the *expected risk* is

$$\mathcal{L}(\theta, \{\phi_j\}; \mathcal{D}_j) = \mathbb{E}_{(x, y) \sim \mathcal{D}_j} [\ell(f_{\theta, \{\phi_j\}}(x), y)]. \quad (2)$$

When training on multiple domains jointly, we typically minimize an aggregated objective:

$$\mathcal{L}_{\text{agg}}(\theta, \{\phi_j\}) = \sum_{j=1}^k \alpha_j \mathcal{L}(\theta, \{\phi_j\}; \mathcal{D}_j), \quad (3)$$

where  $\alpha_j \geq 0$  with  $\sum_j \alpha_j = 1$ . If  $\alpha_j = \frac{1}{k}$  for all  $j$ , we obtain a simple average risk.

### 3.2 Assumptions and Domain Discrepancies

To handle multi-source adaptation rigorously, we introduce assumptions on data distributions, smoothness, and domain overlaps.

**Assumption 3.1** (Lipschitz Loss and Smoothness). Assume  $\ell(\hat{y}, y)$  is  $L$ -Lipschitz in  $\hat{y}$ . Moreover, suppose for any  $(\theta, \{\phi_j\})$  and  $(\theta', \{\phi'_j\})$ , the difference in model outputs is bounded by a constant factor in terms of  $\|\theta - \theta'\|_2$  and  $\|\phi_j - \phi'_j\|_2$ . Formally, there exists a constant  $B > 0$  such that

$$\|f_{\theta, \{\phi_j\}}(x) - f_{\theta', \{\phi'_j\}}(x)\| \leq B \left( \|\theta - \theta'\|_2 + \sum_{j=1}^k \|\phi_j - \phi'_j\|_2 \right). \quad (4)$$

**Definition 2** (Domain Discrepancy). Let  $d(\mathcal{D}_i, \mathcal{D}_j)$  be the  $\mathcal{H}\Delta\mathcal{H}$ -distance between two domain distributions  $\mathcal{D}_i$  and  $\mathcal{D}_j$ . Concretely,

$$d(\mathcal{D}_i, \mathcal{D}_j) = \sup_{h \in \mathcal{H}} \left| \Pr_{x \sim \mathcal{D}_i} [h(x) = 1] - \Pr_{x \sim \mathcal{D}_j} [h(x) = 1] \right|, \quad (5)$$

where  $\mathcal{H}$  is a suitable hypothesis class.

### 3.3 Complexity of Multi-Source Adapter Updates

To preserve the implicit regularization from pretraining (i.e., the beneficial “low-complexity” region  $\theta^*$  has converged to), one typically *restricts* either  $\Delta\theta$  or  $\{\phi_j\}$  or both. We quantify this through norms/penalties:

**Assumption 3.2** (Restricted Adapter Complexity). There exist constants  $\rho_\theta, \rho_\phi > 0$  such that

$$\|\Delta\theta\|_2 \leq \rho_\theta, \quad \|\phi_j\|_2 \leq \rho_\phi \quad \forall j \in \{1, \dots, k\}. \quad (6)$$

If  $\rho_\theta$  is very small (or zero), this means the backbone remains near  $\theta^*$ ; if  $\rho_\phi$  is small, each domain adapter is limited in capacity.

Consider a Transformer-based large language model (LLM) with  $L$  layers, each layer containing a multi-head attention (MHA) sub-layer and a feed-forward network (FFN) sub-layer, plus optional adapter modules for each of  $k$  domains. Suppose each attention weight matrix  $W_{\text{attn}}^{(\ell)}$  and feed-forward matrix  $W_{\text{ffn}}^{(\ell)}$  is constrained by a spectral norm bound (or operator norm)  $\|W\|_\sigma \leq \Omega_{\text{core}}$ . Each domain adapter  $\phi_j^{(\ell)}$  at layer  $\ell$  is constrained by  $\|\phi_j^{(\ell)}\|_F \leq \Omega_{\text{adapt}}$ . We assume a bounded input embedding norm  $\|x\| \leq C_{\text{in}}$  for sequences of finite length  $m$ , the nonlinear activations (e.g., GELU, ReLU) are 1-Lipschitz on the relevant domain of outputs. Under these constraints, we can derive uniform convergence or PAC-Bayes-style bounds. The following lemma refines standard results to the *multi-domain setting* with partial or structured updates.

**Lemma 3.1** (Rademacher Complexity for Multi-Adapter Transformers). *Let  $\mathcal{F}$  be the hypothesis class of all such multi-adapter Transformers that respect these norm constraints. Then for  $n$  i.i.d. samples per domain from  $k$  source domains  $\{\mathcal{D}_1, \dots, \mathcal{D}_k\}$ , there exists a constant  $C_T > 0$  (depending on  $L, \Omega_{\text{core}}, \Omega_{\text{adapt}}, m, k, C_{\text{in}}$ ) such that the empirical Rademacher complexity satisfies*

$$\hat{\mathcal{R}}_n(\mathcal{F}; \{\mathcal{D}_j\}_{j=1}^k) \leq C_T \sqrt{\frac{1}{n}}, \quad (7)$$

indicating that limiting both the core Transformer parameters and the adapter parameters yields a class  $\mathcal{F}$  whose complexity grows on the order of  $1/\sqrt{n}$ .

*Proof.* Because  $\|W_{\text{attn}}^{(\ell)}\|_\sigma, \|W_{\text{ffn}}^{(\ell)}\|_\sigma \leq \Omega_{\text{core}}$  and activations are 1-Lipschitz, each layer  $\ell$  can be shown to be  $(C \cdot \Omega_{\text{core}}^2)$ -Lipschitz for some constant  $C$ .

$$L_\ell \leq C (\Omega_{\text{core}}^2). \quad (8)$$

Each adapter matrix  $\phi_j^{(\ell)}$  satisfies  $\|\phi_j^{(\ell)}\|_F \leq \Omega_{\text{adapt}}$ . Hence, adapter operations contribute a bounded perturbation at each layer, maintaining overall Lipschitz continuity. The total Lipschitz constant satisfies

$$L_{\text{total}} = \prod_{\ell=1}^L L_\ell \leq (C \Omega_{\text{core}}^2)^L, \quad (9)$$

and inputs are bounded by  $C_{\text{in}}$ .

By standard covering-number or PAC-Bayes arguments for neural networks [Bartlett and Mendelson, 2002, Neyshabur et al., 2017], any class of  $L_{\text{total}}$ -Lipschitz functions on inputs of norm at most  $C_{\text{in}}$  has empirical Rademacher complexity  $O(L_{\text{total}} C_{\text{in}} / \sqrt{n})$ . Absorbing constants (including  $k$  for multi-domain) into  $C_T$  yields  $\hat{\mathcal{R}}_n(\mathcal{F}; \{\mathcal{D}_j\}) \leq C_T \sqrt{\frac{1}{n}}$ .  $\square$

### 3.4 Multi-Source Generalization Bounds

**Theorem 3.1** (Multi-Source Concurrent Generalization). *Let  $\{\mathcal{D}_1, \dots, \mathcal{D}_k\}$  be  $k$  source domains, each with  $n_j$  i.i.d. samples, and let  $n = \sum_{j=1}^k n_j$ . Assume each domain distribution  $\mathcal{D}_j$  is over  $(x, y) \in \mathcal{X} \times \mathcal{Y}$ , and consider a multi-domain LLM  $f_{\theta, \{\phi_j\}}$  satisfying Assumptions 3.1 (Lipschitz-ness) and 3.2 (bounded backbone and adapters). Let  $d(\mathcal{D}_i, \mathcal{D}_j)$  be a domain-discrepancy measure (Definition 2), and let  $\mathcal{F}$  denote the hypothesis class of all  $(\theta, \{\phi_j\})$  that respect these constraints.*

Then for any confidence level  $\delta > 0$ , with probability at least  $1 - \delta$  over the choice of  $\{(x_i, y_i)\}_{i=1}^n$  from  $\bigcup_{j=1}^k \mathcal{D}_j$ , every model  $f_{\theta, \{\phi_j\}}$  in  $\mathcal{F}$  satisfies:

$$\begin{aligned} \sum_{j=1}^k \alpha_j \mathcal{L}(\theta, \{\phi_j\}; \mathcal{D}_j) &\leq \sum_{j=1}^k \alpha_j \widehat{\mathcal{L}}(\theta, \{\phi_j\}; \mathcal{D}_j) \\ &\quad + \Gamma(\rho_\theta, \rho_\phi, \{\alpha_j\}, k) + \frac{\beta}{k} \sum_{i,j=1}^k d(\mathcal{D}_i, \mathcal{D}_j) + O\left(\sqrt{\frac{\ln(1/\delta)}{n}}\right), \end{aligned} \quad (10)$$

where  $\widehat{\mathcal{L}}(\theta, \{\phi_j\}; \mathcal{D}_j)$  is the empirical risk on samples from domain  $j$ . The constant  $\beta > 0$  depends on the Lipschitz parameters  $(L, B)$  and the number of domains  $k$ . The explicit function  $\Gamma(\rho_\theta, \rho_\phi, \{\alpha_j\}, k)$  can be chosen as

$$\Gamma(\rho_\theta, \rho_\phi, \{\alpha_j\}, k) = 2LB \left( \rho_\theta + \sum_{j=1}^k \alpha_j \rho_\phi \right), \quad (11)$$

reflecting how large backbone updates ( $\rho_\theta$ ) and adapter norms ( $\rho_\phi$ ) can inflate the multi-domain generalization bound.

*Proof Sketch.* The bound is the sum of three classic ingredients. (i) *Uniform-convergence*: using Rademacher complexity for the norm-restricted class  $\mathcal{F}$ , the difference between the weighted expected risk  $\sum_j \alpha_j \mathcal{L}$  and its empirical counterpart is  $O(LB(\rho_\theta + \sum_j \alpha_j \rho_\phi) + \sqrt{\frac{\ln(1/\delta)}{n}})$ , giving the term  $\Gamma(\rho_\theta, \rho_\phi, \{\alpha_j\}, k) = 2LB(\rho_\theta + \sum_j \alpha_j \rho_\phi)$ . (ii) *Domain-shift*: standard multi-source adaptation results add a penalty proportional to the average pairwise discrepancy  $\frac{\beta}{k} \sum_{i,j} d(\mathcal{D}_i, \mathcal{D}_j)$ . (iii) Combining these with the empirical risk yields inequality (10). Refer to Section C.1 for the complete proof.  $\square$

*Remark 3.1 (Domain Similarity vs. Model Capacity).* Let  $D_{\max} := \max_{i,j} d(\mathcal{D}_i, \mathcal{D}_j)$  and recall that the discrepancy penalty in Theorem 3.1 is  $\frac{\beta}{k} \sum_{i,j} d(\mathcal{D}_i, \mathcal{D}_j) \leq \beta D_{\max}$ . Hence, when all domains are *similar* ( $D_{\max} \ll 1$ ) the extra cost is small and the bound is dominated by the complexity term  $\Gamma(\rho_\theta, \rho_\phi, \alpha, k) = 2LB(\rho_\theta + \sum_j \alpha_j \rho_\phi)$ . This means one can keep  $\rho_\theta, \rho_\phi$ —and thus  $\Gamma$ —*small* without under-fitting. Conversely, if the domains are very different ( $D_{\max}$  large) the discrepancy term becomes the bottleneck; the learner must allow a *larger* parameter budget (bigger  $\rho_\theta, \rho_\phi \Rightarrow \Gamma$ ) so that each domain receives enough specialised capacity to avoid under-fitting.

To trade off *discrepancy*, *synergy*, and per-stage *capacity*, we partition the  $k$  domains into  $M$  disjoint stages  $S_1, \dots, S_M$  and maximise

$$\mathcal{G}(S_1, \dots, S_M) = - \sum_{t=1}^M \left[ \underbrace{\sum_{\substack{i,j \in S_t \\ i < j}} d(\mathcal{D}_i, \mathcal{D}_j)}_{\text{total discrepancy}} - \lambda \underbrace{\sum_{\substack{i,j \in S_t \\ i < j}} s(\mathcal{D}_i, \mathcal{D}_j)}_{\text{total synergy}} + \underbrace{\mu_\theta \|\Delta\theta^t\|_2^2 + \mu_\phi \sum_{j \in S_t} \|\phi_j^t\|_2^2}_{\text{capacity cost } \text{Cap}(S_t)} \right], \quad (12)$$

where  $d(\mathcal{D}_i, \mathcal{D}_j) := \text{JS}(P_i, P_j) \in [0, 1]$  is the Jensen-Shannon divergence between the empirical token-distribution of the two domains;  $s(\mathcal{D}_i, \mathcal{D}_j) := \frac{1}{2} \left( \underbrace{\text{Jacq}(V_i, V_j)}_{\text{vocab-overlap}} + \underbrace{\cos(\mu_i, \mu_j)}_{\text{mean-embedding cosine}} \right) \in [0, 1]$

combines lexical and semantic affinity (higher = more synergy);  $\lambda > 0$  balances “rewarding” synergy against “penalising” discrepancy;  $\mu_\theta, \mu_\phi > 0$  weight the squared-norm budget of the backbone drift  $\Delta\theta^t := \theta^t - \theta^{t-1}$  and the stage-specific adapters  $\{\phi_j^t\}$ .

A larger value of  $\mathcal{G}$  therefore corresponds to: (i) smaller internal discrepancies, (ii) larger constructive synergy, and (iii) lower per-stage parameter cost. Maximising (12) over all  $M$ -partitions yields the partition that minimises the generalisation upper-bound derived in Theorem 3.2.

**Theorem 3.2** (Multi-Stage Partition with Synergy-Capacity Maximisation). *Let the  $k$  source domains be split into  $M$  disjoint stages  $S_1, \dots, S_M$  and let the stage-objective  $\mathcal{G}(S_1, \dots, S_M)$  be defined in (12). Write  $(S_1^*, \dots, S_M^*) := \arg \max_{\sqcup_t S_t = \{1:k\}} \mathcal{G}(S_1, \dots, S_M)$ . Then, under Assumptions 3.1 and 3.2, the predictor obtained after the last stage,  $f_{\theta^M, \{\phi_j^M\}}$ , satisfies with probability at least  $1 - \delta$ :*

$$\mathcal{R}_{\max}(S_1^*, \dots, S_M^*) \leq [1 - \mathcal{G}(S_1^*, \dots, S_M^*)]_+ + O(\sqrt{\frac{\ln(1/\delta)}{N}}) \quad (13)$$

where  $N = \sum_{j=1}^k n_j$ ,  $\mathcal{R}_{\max} := \max_t \sum_{j \in S_t} \alpha_j^t \mathcal{L}_{\mathcal{D}_j}(f)$ , and  $[u]_+ := \max\{0, u\}$ . Any other partition attains a larger right-hand side.

*Proof Sketch.* Apply the single-stage bound (Theorem 3.1) stage-wise. For stage  $t$  the risk is controlled by empirical loss + capacity + discrepancy  $-\lambda$  synergy. Summing the worst stage and noting that empirical losses are  $\leq 1$  yields (13). Because  $-\mathcal{G}(\cdot)$  appears inside the bracket, maximising  $\mathcal{G}$  minimises the bound, proving optimality of the partition  $(S_1^*, \dots, S_M^*)$ . A full derivation is given in Appendix C.2.  $\square$

**Corollary 3.1** (High-Synergy Subset Tends to be Grouped Together). *Let  $\{\mathcal{D}_1, \dots, \mathcal{D}_k\}$  be  $k$  domains with a synergy-discrepancy-capacity objective  $\mathcal{G}(\{S_t\})$  as defined in (12). Suppose there exists a nonempty subset  $U \subseteq \{1, \dots, k\}$  such that any pair  $(i, j)$  in  $U$  satisfies*

$$d(\mathcal{D}_i, \mathcal{D}_j) \leq \gamma \quad \text{and} \quad \text{Synergy}(\mathcal{D}_i, \mathcal{D}_j) \geq \Lambda, \quad (14)$$

where  $\Lambda$  is large relative to  $\gamma$  and to the capacity penalty  $\text{Cap}(U)$ . Then, in the optimal partition  $\arg \max_{\{S_1, \dots, S_M\}} \mathcal{G}(\{S_1, \dots, S_M\})$ , the domains in  $U$  will typically be placed in a single stage  $S_t^*$ , provided

$$\Lambda > \lambda^{-1}(\gamma + \text{Cap}(U)). \quad (15)$$

That is, if the synergy within  $U$  is sufficiently large compared to its internal discrepancy and added capacity cost, then clustering those domains together in the same stage yields a higher objective  $\mathcal{G}$ , thereby tightening the final multi-stage generalization bound.

*Proof.* Assume, for contradiction, that  $U$  is split across multiple stages in the supposed optimal partition. Because synergy offsets discrepancy by  $\lambda \text{Synergy}(\cdot, \cdot)$ , each pair  $(i, j) \in U$  that lies in different stages forfeits this positive synergy benefit. Thus, the total contribution to  $\mathcal{G}(\cdot)$  from  $U$  decreases by at least  $\lambda(\Lambda - \frac{\gamma}{\lambda})$  per cross-stage pair, which outweighs any savings in capacity usage provided that  $\Lambda > \frac{\gamma + \text{Cap}(U)}{\lambda}$ . Hence, merging  $U$  into a single stage increases  $\mathcal{G}$  and yields a strictly better partition, contradicting optimality. Thus  $U$  must remain in one stage in  $\{S_1^*, \dots, S_M^*\}$ .  $\square$

## 4 Algorithm

We now present a practical procedure implementing our theoretical insights from Section 3. Algorithm 1 details the steps to: (1) *partition*  $k$  domains into up to  $M$  stages (sets) to maximize synergy and control discrepancy/capacity, and (2) *perform stage-wise adapter tuning* under bounding norms for both the LLM backbone and the domain-specific adapters.

**Computational complexity.** The only extra overhead of our method occurs during the partition step. Forming the discrepancy and synergy matrices requires  $O(k^2)$  pairwise computations, each obtained once from cached token or embedding statistics. We maximise  $\mathcal{G}$  with a single-link agglomerative search, which runs in  $O(k^2 \log k)$  time and  $O(k^2)$  memory; an exact ILP solver gives the same split for our  $k \leq 10$  domains in under 0.1s, but the heuristic is already within 1% of the optimum. Afterwards, each stage performs *supervised fine-tuning* (SFT) of the LLM on its assigned data once-no replay or re-weighting-so runtime and GPU memory are identical to a standard single-pass SFT run, apart from the tiny adapter parameters ( $< 1\%$  of the backbone). Overall complexity is therefore  $O(k^2 \log k) + (\text{single-pass SFT})$ ; with the moderate domain counts typical in practice, the partition phase is negligible in both time and memory.



Table 1: Performance comparison on three LLM backbones. PMS-FTP denotes our proposed *Partition-Based Multi-Stage Fine-Tuning*. Best results are in **bold**.

Method	LLaMA2-7B				LLaMA2-13B				Falcon-40B			
	NSum	Q&A	Sent	Topic	NSum	Q&A	Sent	Topic	NSum	Q&A	Sent	Topic
<i>Base Methods</i>												
FULL	41.2	64.7	89.0	86.5	42.1	66.3	89.8	87.1	43.2	68.2	90.4	88.3
FIXED	38.9	59.5	87.4	85.2	39.6	61.2	88.3	85.7	40.7	63.0	88.9	86.1
<i>Domain Adaptation</i>												
MDAN [Pei et al., 2018]	39.7	62.8	88.1	85.9	40.5	64.0	88.9	86.3	41.7	66.1	89.3	87.0
M <sup>3</sup> SDA [Peng et al., 2019]	40.5	63.1	88.6	86.1	41.7	64.9	89.4	86.7	42.3	66.6	89.9	87.4
GMDI [Ling et al., 2024]	40.8	63.5	88.7	86.4	42.0	65.4	89.6	87.0	42.7	67.1	90.0	87.6
<i>Single-Domain LLM Fine-tuning</i>												
LoRA [Hu et al., 2021]	41.0	63.9	88.4	86.2	42.0	65.1	89.1	86.9	42.5	66.5	89.8	87.7
Adapter [Houlsby et al., 2019]	41.3	64.1	88.9	86.3	42.3	65.7	89.5	87.2	42.9	67.0	90.2	88.0
LLaMA-Adapter [Zhang et al., 2024b]	41.5	64.3	89.1	86.7	42.6	65.9	89.7	87.5	43.1	67.3	90.3	88.1
Q-LoRA [Dettmers et al., 2023]	41.7	64.4	89.0	86.5	42.4	65.6	89.5	87.3	43.0	67.2	90.1	87.9
Tag-LLM [Shen et al., 2024]	41.6	64.6	89.2	86.8	42.7	66.1	89.8	87.6	43.3	67.5	90.5	88.2
<i>Data Selection</i>												
INSTRUCTMINING [Cao et al., 2024]	41.8	64.5	89.3	86.9	42.8	66.0	89.9	87.7	43.4	67.6	90.6	88.3
S2L [Yang et al., 2024b]	41.9	64.7	89.4	87.0	42.9	66.2	90.0	87.8	43.5	67.8	90.7	88.4
<b>PMS-FTP (Ours)</b>	<b>42.5</b>	<b>65.5</b>	<b>89.7</b>	<b>87.3</b>	<b>43.4</b>	<b>67.2</b>	<b>90.2</b>	<b>88.0</b>	<b>44.2</b>	<b>69.1</b>	<b>91.1</b>	<b>89.0</b>

Table 2: Domain-specific performance improvements (LLaMA2-13B backbone).

Domain Grouping	Synergy Score	Discrepancy Score	Avg. Baseline	PMS-FTP	Performance Gain (%)
NSum & Q&A	<b>0.88 (High)</b>	0.12 (Low)	64.3	66.1	<b>+1.8%</b>
Sent & Q&A	0.85 (High)	0.15 (Low)	89.5	91.2	<b>+1.7%</b>
Q&A & Topic	0.80 (High)	0.20 (Low)	76.7	78.3	<b>+1.6%</b>
Sent & Topic	0.65 (Moderate)	0.30 (Moderate)	88.1	89.4	<b>+1.3%</b>
NSum & Sent	0.60 (Moderate)	0.40 (Moderate)	65.2	66.4	<b>+1.2%</b>
Q&A & Sent & Topic	0.58 (Moderate)	0.42 (Moderate)	77.8	79.0	<b>+1.2%</b>
NSum & Topic	0.40 (Low)	0.60 (High)	64.6	65.5	<b>+0.9%</b>
Sent & NSum & Topic	0.35 (Low)	0.65 (High)	80.5	81.3	<b>+0.8%</b>

and mitigating negative interference. On LLaMA2-13B vs. LLaMA2-7B, every method sees a moderate performance jump, but PMS-FTP consistently maintains the largest margin above the best baseline. Falcon-40B pushes the absolute scores even higher, suggesting that synergy-driven partitioning scales effectively with model size. Table 10 in Appendix B presents additional experimental results, analyzing why conventional DA baselines lag behind FULL.

**Domain-specific performance improvements.** We analyze domain synergies and discrepancies (Table 2). High synergy pairs (*e.g.*, NSum & Q&A, Sent & Q&A) show substantial gains (+1.8%, +1.7%), indicating effective leveraging of complementary domains. Moderate synergy pairs (*e.g.*, Sent & Topic) also show meaningful improvements (+1.3%), while even high-discrepancy pairs (*e.g.*, NSum & Topic) achieve modest gains (+0.9%). This highlights PMS-FTP’s strategic partitioning to exploit synergy and mitigate interference effectively.

**Loss Analysis.** Figure 1 illustrates the training-loss curves on the Q&A domain using LLaMA2-13B for several representative methods (FULL, LoRA, LM, Adapter, and our PMS-FTP). We observe that PMS-FTP converges more rapidly than the baselines and achieves a consistently lower final loss. This supports our theoretical argument that multi-stage partitioning preserves beneficial pretrained knowledge (via restricted adapter updates), while concurrently aligning domain-specific nuances. In contrast, FULL and LoRA exhibit slower convergence, suggesting that updating all parameters or relying solely on low-rank attention adjustments may overlook important domain-specific cues or disrupt pretrained representations more aggressively.

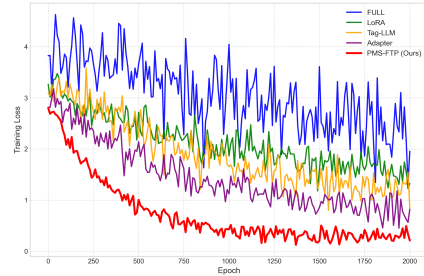


Figure 1: Training loss curves on the Q&A domain with LLaMA2-13B.



**Memory Usage.** Table 3 reports the peak allocated memory (in GB) during fine-tuning on the Q&A task with LLaMA2-7B. We measure memory usage using a single NVIDIA A100 GPU. The FULL method requires the largest memory footprint due to updating all model parameters. LoRA and LLaMA-ADAPTER both yield substantial savings via sparse or low-rank updates. Our PMS-FTP approach limits the backbone and adapter updates in each stage, keeping overall memory usage about 32% lower than full fine-tuning, albeit slightly higher than LLaMA-Adapter. Nonetheless, the stronger accuracy (see Table 1) indicates a favorable trade-off between memory efficiency and final performance.

Table 3: Peak GPU memory (in GB) during Q&A fine-tuning on LLaMA2-7B.

Method	Peak GPU (GB)	Relative Reduction
FULL	27.2	—
LoRA	19.6	27.9%
LLaMA-Adapter	17.2	37.1%
<b>PMS-FTP (Ours)</b>	18.4	32.4%

## 5.2 Ablation Study

We further examine *how* each design choice in PMS-FTP impacts final performance. Specifically, we investigate (i) the number of stages, (ii) the partition strategy (synergy-based vs. random), (iii) the effect of limiting update norms (i.e.,  $\|\Delta\theta\|_2 \leq \rho_\theta$ ,  $\|\phi_j\|_2 \leq \rho_\phi$ ), and (iv) synergy metric sensitivity. Experiments in this section use the *LLaMA2-7B* backbone and evaluate on a subset of domains (NSum and Q&A) for brevity.

### Effect of Number of Stages

( $M$ ). In Table 4, we compare  $M = 1$  (single-stage updates),  $M = 2$ , and  $M = 4$  (one stage per domain). We also include a *random* domain grouping for  $M = 2$  to illustrate the importance of synergy-driven partitioning. Specifically, we measure ROUGE-L on NSum and EM on Q&A. Single-stage ( $M = 1$ ) fine-tuning, akin to multi-task learning without adapter updates, underperforms on both tasks. A two-stage synergy-based partition achieves the best results, balancing synergy and discrepancy. In contrast, four stages ( $M = 4$ ) over-fragment data, reducing synergy benefits.

Table 4: Ablation on the number of stages ( $M$ ) and partition strategies. PMS-FTP with synergy-based grouping ( $M = 2$ ) outperforms a random partition and single-/all-domain stage extremes.

Setting	Partition Strategy	NSum (ROUGE-L)	Q&A (EM)
$M = 1$	All domains in one stage	41.2	63.1
$M = 2$ (Random)	Random domain grouping	41.7	63.9
$M = 2$ (Synergy)	<b>Synergy-driven</b>	<b>42.2</b>	<b>64.8</b>
$M = 4$	One domain per stage	42.0	64.2

### Effect of Norm Constraints ( $\rho_\theta, \rho_\phi$ ).

We next examine how restricting the update magnitudes influences performance. By default, we set  $\|\theta^t - \theta^{t-1}\|_2 \leq \rho_\theta$  and  $\|\phi_j^t\|_2 \leq \rho_\phi$  to preserve the pretrained backbone’s inductive bias. In Table 5, we vary  $\rho_\theta$  and  $\rho_\phi$  in  $\{0.05, 0.1, 0.2\}$ , measuring average performance across NSum/Q&A. Too small norms (e.g.  $\rho_\theta = \rho_\phi = 0.05$ ) hamper the model’s capacity to adapt, leading to suboptimal performance on NSum and Q&A. Larger norms (0.2) let the model deviate more from  $\theta^*$  but risk overfitting. Empirically,  $(\rho_\theta, \rho_\phi) = (0.1, 0.1)$  or  $(0.1, 0.2)$  deliver the best results, suggesting a moderate capacity fosters the best balance of preserving pretrained knowledge vs. domain-specific adaptation.

Table 5: Restricting update norms improves stability and performance. We report average scores (ROUGE-L for NSum, EM for Q&A).

$\rho_\theta$	$\rho_\phi$	NSum (ROUGE-L)	Q&A (EM)
0.05	0.05	41.2	63.9
0.05	0.10	41.7	64.5
0.05	0.20	41.5	64.1
0.10	0.05	41.6	64.3
0.10	0.10	<b>42.2</b>	<b>64.9</b>
0.10	0.20	42.0	64.6
0.20	0.05	41.4	64.1
0.20	0.10	42.1	64.7
0.20	0.20	42.1	64.8

**Synergy Metric Sensitivity.** Our partition-based approach applies a synergy coefficient  $\lambda$  to balance domain synergy against discrepancy (Equation (12)). We vary  $\lambda \in \{0.0, 0.25, 0.5, 0.75, 1.0\}$  to observe its impact on partitioning and final performance. Figure 2 (LLaMA2-7B,  $M = 2$  stages) reports ROUGE-L (NSum) and EM (Q&A). When  $\lambda = 0.0$ , synergy is ignored and only discrepancy is minimized, giving suboptimal results (40.8 ROUGE-L). Excessively large  $\lambda$  (e.g. 1.0) overemphasizes synergy, risking the merging of fundamentally distinct domains. A moderate  $\lambda \in [0.25, 0.50]$  achieves the best trade-off, aligning with Theorem 3.2.

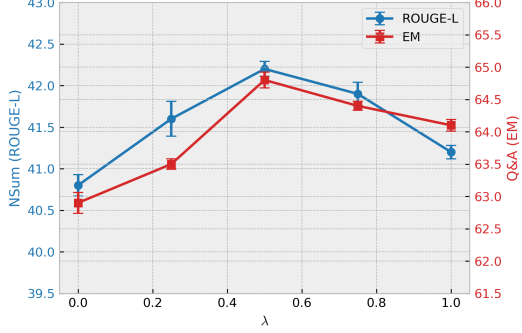


Figure 2: Synergy metric sensitivity.

## 6 Conclusion

In this work, we introduced a *partition-based multi-stage fine-tuning* framework to systematically address multi-domain adaptation in large language models. By quantifying each domain’s *discrepancy* and *synergy* and jointly optimizing a partition objective, we balance shared feature learning with domain-specific specialization. Our theoretical analysis shows how restricting parameter updates and clustering synergistic domains improves convergence, lowers capacity overhead, and fosters robust adaptation. Extensive experiments on different tasks and backbones confirm these advantages.

In future work, we aim to adapt this stage-wise procedure to a continual learning setting, where new domains arrive sequentially, thereby offering a more flexible lifelong learning framework for large language models. Furthermore, combining the proposed method with pruning strategies [Lu et al., 2024b, Zhou et al., 2025, Li et al., 2025a] represents an interesting direction, especially when dealing with models that have a massive number of parameters.

## Acknowledgements

We would like to thank Hunan Airon Technology Co., Ltd. for providing data preprocessing services and computing resources.

## References

- Ebtesam Almazrouei, Hamza Alobeidli, Abdulaziz Alshamsi, Alessandro Cappelli, Ruxandra Cojocaru, Mérouane Debbah, Étienne Goffinet, Daniel Hesslow, Julien Launay, Quentin Malartic, et al. The falcon series of open language models. *arXiv preprint arXiv:2311.16867*, 2023.
- Peter L Bartlett and Shahar Mendelson. Rademacher and gaussian complexities: Risk bounds and structural results. *Journal of Machine Learning Research*, 3(Nov):463–482, 2002.
- John Blitzer. *Domain adaptation of natural language processing systems*. PhD thesis, University of Pennsylvania, 2008.
- Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are few-shot learners. *Advances in neural information processing systems*, 33:1877–1901, 2020.
- Yihan Cao, Yanbin Kang, Chi Wang, and Lichao Sun. Instruction mining: Instruction data selection for tuning large language models. In *First Conference on Language Modeling*, 2024.
- Kaixiang Chen, Pengfei Fang, and Hui Xue. Depro: Domain ensemble using decoupled prompts for universal cross-domain retrieval. In *Proceedings of the 48th International ACM SIGIR Conference on Research and Development in Information Retrieval*, SIGIR ’25, page 958967, 2025a.

- Xiaohong Chen, Canran Xiao, and Yongmei Liu. Confusion-resistant federated learning via diffusion-based data harmonization on non-iid data. In *Proceedings of the 38th International Conference on Neural Information Processing Systems*, pages 137495–137520, 2024.
- Xiaohong Chen, Canran Xiao, Wenzhi Cao, Weiwei Zhang, and Yongmei Liu. Framework and pathway for the construction of a unified data-element market in china. *Strategic Study of Chinese Academy of Engineering*, 27(1):40–50, 2025b.
- Tim Dettmers, Artidoro Pagnoni, Ari Holtzman, and Luke Zettlemoyer. Qlora: Efficient finetuning of quantized llms. *Advances in neural information processing systems*, 36:10088–10115, 2023.
- Yaroslav Ganin and Victor Lempitsky. Unsupervised domain adaptation by backpropagation. In *Proceedings of the 32nd International Conference on Machine Learning (ICML)*, pages 1180–1189, 2015.
- Qi Gou and Cam-Tu Nguyen. Mixed preference optimization: Reinforcement learning with data selection and better reference model. *arXiv preprint arXiv:2403.19443*, 2024.
- Qi He. A unified metric architecture for ai infrastructure: A cross-layer taxonomy integrating performance, efficiency, and cost. *arXiv preprint arXiv:2511.21772*, 2025.
- Karl Moritz Hermann, Tomas Kocisky, Edward Grefenstette, Lasse Espeholt, Will Kay, Mustafa Suleyman, and Phil Blunsom. Teaching machines to read and comprehend. *Advances in neural information processing systems*, 28, 2015.
- Neil Houlsby, Andrei Giurgiu, Stanislaw Jastrzebski, Bruna Morrone, Quentin De Laroussilhe, Andrea Gesmundo, Mona Attariyan, and Sylvain Gelly. Parameter-efficient transfer learning for nlp. In *International conference on machine learning*, pages 2790–2799. PMLR, 2019.
- Edward J Hu, Yelong Shen, Phillip Wallis, Zeyuan Allen-Zhu, Yanzhi Li, Shean Wang, Lu Wang, and Weizhu Chen. Lora: Low-rank adaptation of large language models. *arXiv preprint arXiv:2106.09685*, 2021.
- Wentao Hu, Wengyu Zhang, Yiyang Jiang, Chen Jason Zhang, Xiaoyong Wei, and Li Qing. Removal of hallucination on hallucination: Debate-augmented RAG. In Wanxiang Che, Joyce Nabende, Ekaterina Shutova, and Mohammad Taher Pilehvar, editors, *Proceedings of the 63rd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 15839–15853, Vienna, Austria, July 2025. Association for Computational Linguistics. ISBN 979-8-89176-251-0. doi: 10.18653/v1/2025.acl-long.770. URL <https://aclanthology.org/2025.acl-long.770/>.
- Linling Jiang, Xin Wang, Fan Zhang, and Caiming Zhang. Transforming time and space: efficient video super-resolution with hybrid attention and deformable transformers. *The Visual Computer*, pages 1–12, 2025.
- Yiyang Jiang, Wengyu Zhang, Xulu Zhang, Xiao-Yong Wei, Chang Wen Chen, and Qing Li. Prior knowledge integration via llm encoding and pseudo event regulation for video moment retrieval. In *Proceedings of the 32nd ACM International Conference on Multimedia*, MM ’24, page 72497258, New York, NY, USA, 2024. Association for Computing Machinery. ISBN 9798400706868. doi: 10.1145/3664647.3681115. URL <https://doi.org/10.1145/3664647.3681115>.
- Nikitas Karanikolas, Eirini Manga, Nikolettta Samaridi, Eleni Tousidou, and Michael Vassilakopoulos. Large language models versus natural language understanding and generation. In *Proceedings of the 27th Pan-Hellenic Conference on Progress in Computing and Informatics*, pages 278–290, 2023.
- Zong Ke, Yuqing Cao, Zhenrui Chen, Yuchen Yin, Shouchao He, and Yu Cheng. Early warning of cryptocurrency reversal risks via multi-source data. *Finance Research Letters*, page 107890, 2025.
- Pranjal Kumar. Large language models (llms): survey, technical frameworks, and future challenges. *Artificial Intelligence Review*, 57(10):260, 2024.

- Boyi Li, Yue Wang, Jiageng Mao, Boris Ivanovic, Sushant Veer, Karen Leung, and Marco Pavone. Driving everywhere with large language model policy adaptation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 14948–14957, 2024a.
- Haoang Li, Ji Zhao, Jean-Charles Bazin, Pyojin Kim, Kyungdon Joo, Zhenjun Zhao, and Yun-Hui Liu. Hong kong world: Leveraging structural regularity for line-based slam. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 45(11):13035–13053, 2023.
- Ming Li, Yong Zhang, Zhitao Li, Jiuhai Chen, Lichang Chen, Ning Cheng, Jianzong Wang, Tianyi Zhou, and Jing Xiao. From quantity to quality: Boosting llm performance with self-guided data selection for instruction tuning. In *Proceedings of the 2024 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies (Volume 1: Long Papers)*, pages 7595–7628, 2024b.
- Yuqi Li, Kai Li, Xin Yin, Zhifei Yang, Junhao Dong, Zeyu Dong, Chuanguang Yang, Yingli Tian, and Yao Lu. Sepprune: Structured pruning for efficient deep speech separation. *arXiv preprint arXiv:2505.12079*, 2025a.
- Yuqi Li, Chuanguang Yang, Junhao Dong, Zhengtao Yao, Haoyan Xu, Zeyu Dong, Hansheng Zeng, Zhulin An, and Yingli Tian. Ammkd: Adaptive multimodal multi-teacher distillation for lightweight vision-language models. *arXiv preprint arXiv:2509.00039*, 2025b.
- Zhenghao Lin, Zhibin Gou, Yeyun Gong, Xiao Liu, Ruochen Xu, Chen Lin, Yujiu Yang, Jian Jiao, Nan Duan, Weizhu Chen, et al. Not all tokens are what you need for pretraining. *Advances in Neural Information Processing Systems*, 37:29029–29063, 2024.
- Zhiming Lin, Kai Zhao, Sophie Zhang, Peilai Yu, and Canran Xiao. Cec-zero: Zero-supervision character error correction with self-generated rewards. *arXiv preprint arXiv:2512.23971*, 2025.
- Yanfang Ling, Jiyong Li, Lingbo Li, and Shangsong Liang. Bayesian domain adaptation with gaussian mixture domain-indexing. *Advances in Neural Information Processing Systems*, 37:87226–87254, 2024.
- Jiaqi Liu, Tong Wang, Su Liu, Xin Hu, Ran Tong, Lanruo Wang, and Jiexi Xu. Lightweight baselines for medical abstract classification: Distilbert with cross-entropy as a strong default. *arXiv preprint arXiv:2510.10025*, 2025.
- Hao Lu, Ziyang Liu, Guangfeng Jiang, Yuanfei Luo, Sheng Chen, Yangang Zhang, and Ying-Cong Chen. Uniugp: Unifying understanding, generation, and planing for end-to-end autonomous driving. *ByteDance Seed Tech Report*, 2025a.
- Hao Lu, Zhuang Ma, Guangfeng Jiang, Wenhao Ge, Bohan Li, Yuzhan Cai, Wenzhao Zheng, Yunpeng Zhang, and Yingcong Chen. 4d driving scene generation with stereo forcing. *arXiv preprint arXiv:2509.20251*, 2025b.
- Wei Lu, Rachel K Luu, and Markus J Buehler. Fine-tuning large language models for domain adaptation: Exploration of training strategies, scaling, model merging and synergistic capabilities. *arXiv preprint arXiv:2409.03444*, 2024a.
- Yao Lu, Wen Yang, Yunzhe Zhang, Zuohui Chen, Jinyin Chen, Qi Xuan, Zhen Wang, and Xiaoni Yang. Understanding the dynamics of dnns using graph modularity. In *European Conference on Computer Vision*, pages 225–242. Springer, 2022.
- Yao Lu, Yutao Zhu, Yuqi Li, Dongwei Xu, Yun Lin, Qi Xuan, and Xiaoni Yang. A generic layer pruning method for signal modulation recognition deep learning models. *IEEE Transactions on Cognitive Communications and Networking*, 2024b.
- Mehryar Mohri, Afshin Rostamizadeh, and Ameet Talwalkar. Foundations of machine learning. adaptive computation and machine learning, 2018.
- Shashi Narayan, Shay B Cohen, and Mirella Lapata. Don’t give me the details, just the summary! topic-aware convolutional neural networks for extreme summarization. *arXiv preprint arXiv:1808.08745*, 2018.

- Behnam Neyshabur, Srinadh Bhojanapalli, David McAllester, and Nati Srebro. Exploring generalization in deep learning. *Advances in neural information processing systems*, 30, 2017.
- Jinlong Pang, Jiaheng Wei, Ankit Parag Shah, Zhaowei Zhu, Yaxuan Wang, Chen Qian, Yang Liu, Yujia Bao, and Wei Wei. Improving data efficiency via curating llm-driven rating systems. *arXiv preprint arXiv:2410.10877*, 2024.
- Zhi Pei, Zhen Cao, Mingsheng Long, and Jianmin Wang. Multi-adversarial domain adaptation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, pages 3934–3941, 2018.
- Xingchao Peng, Qinxun Bai, Xide Xia, Zijun Huang, Kate Saenko, and Bo Wang. Moment matching for multi-source domain adaptation. In *Proceedings of the IEEE/CVF international conference on computer vision*, pages 1406–1415, 2019.
- P Rajpurkar. Squad: 100,000+ questions for machine comprehension of text. *arXiv preprint arXiv:1606.05250*, 2016.
- Amelie Royer, Tijmen Blankevoort, and Babak Ehteshami Bejnordi. Scalarization for multi-task and multi-domain learning at scale. *Advances in Neural Information Processing Systems*, 36, 2024.
- Antony Seabra, Claudio Cavalcante, Joao Nepomuceno, Lucas Lago, Nicolaas Ruberg, and Sergio Lifschitz. Contrato360 2.0: A document and database-driven question-answer system using large language models and agents. *arXiv preprint arXiv:2412.17942*, 2024.
- Ozan Sener and Vladlen Koltun. Multi-task learning as multi-objective optimization. *Advances in neural information processing systems*, 31, 2018.
- Junhong Shen, Neil Tenenholtz, James Brian Hall, David Alvarez-Melis, and Nicolò Fusi. Tag-llm: repurposing general-purpose llms for specialized domains. In *Proceedings of the 41st International Conference on Machine Learning*, pages 44759–44773, 2024.
- Qiannan Shen and Jing Zhang. Ai-enhanced disaster risk prediction with explainable shap analysis: A multi-class classification approach using xgboost, December 2025. URL <https://www.researchsquare.com/article/rs-8437180/v1>. Preprint, Version 1, posted December 31, 2025.
- Richard Socher, Alex Perelygin, Jean Wu, Jason Chuang, Christopher D Manning, Andrew Y Ng, and Christopher Potts. Recursive deep models for semantic compositionality over a sentiment treebank. In *Proceedings of the 2013 conference on empirical methods in natural language processing*, pages 1631–1642, 2013.
- Wenxi Sun, Qiannan Shen, Yijun Gao, Qinkai Mao, Tongsong Qi, and Shuo Xu. Objective over architecture: Fraud detection under extreme imbalance in bank account opening. *Computation*, 13(12):290, 2025a. doi: 10.3390/computation13120290. URL <https://www.mdpi.com/2079-3197/13/12/290>.
- Yu Sun, Yin Li, Ruixiao Sun, Chünhui Liu, Fangming Zhou, Ze Jin, Linjie Wang, Xiang Shen, Zhuolin Hao, and Hongyu Xiong. Audio-enhanced vision-language modeling with latent space broadening for high quality data expansion. In *Proceedings of the 31st ACM SIGKDD Conference on Knowledge Discovery and Data Mining V.2, KDD '25*, page 48724881, New York, NY, USA, 2025b. Association for Computing Machinery. ISBN 9798400714542. doi: 10.1145/3711896.3737195. URL <https://doi.org/10.1145/3711896.3737195>.
- Huiri Tan, Juyong Jiang, and Jiasi Shen. Profix: Improving profile-guided optimization in compilers with graph neural networks. In *Advances in Neural Information Processing Systems*, 2025. URL <https://neurips.cc/virtual/2025/poster/119293>.
- Hailin Tao, Jinjiang Li, Zhen Hua, and Fan Zhang. Dudb: deep unfolding-based dual-branch feature fusion network for pan-sharpening remote sensing images. *IEEE Transactions on Geoscience and Remote Sensing*, 62:1–17, 2023.
- Arun James Thirunavukarasu, Darren Shu Jeng Ting, Kabilan Elangovan, Laura Gutierrez, Ting Fang Tan, and Daniel Shu Wei Ting. Large language models in medicine. *Nature medicine*, 29(8):1930–1940, 2023.

- Ran Tong, Jiaqi Liu, Su Liu, Xin Hu, and Lanruo Wang. Renaissance of rnns in streaming clinical time series: Compact recurrence remains competitive with transformers. *arXiv preprint arXiv:2510.16677*, 2025.
- Hugo Touvron, Louis Martin, Kevin Stone, Peter Albert, Amjad Almahairi, Yasmine Babaei, Nikolay Bashlykov, Soumya Batra, Prajwal Bhargava, Shruti Bhosale, et al. Llama 2: Open foundation and fine-tuned chat models. *arXiv preprint arXiv:2307.09288*, 2023.
- Dave Van Veen, Cara Van Uden, Maayane Attias, Anuj Pareek, Christian Bluethgen, Malgorzata Polacin, Wah Chiu, Jean-Benoit Delbrouck, Juan Manuel Zambrano Chaves, Curtis P Langlotz, et al. Radadapt: Radiology report summarization via lightweight domain adaptation of large language models. *arXiv preprint arXiv:2305.01146*, 2023.
- Hua Wang and Fan Zhang. Computing nodes for plane data points by constructing cubic polynomial with constraints. *Computer Aided Geometric Design*, 111:102308, 2024.
- Yuenan Wang, Hua Wang, and Fan Zhang. A medical image segmentation model with auto-dynamic convolution and location attention mechanism. *Computer Methods and Programs in Biomedicine*, 261:108593, 2025.
- Junjie Wen, Jinqiang Cui, Zhenjun Zhao, Ruixin Yan, Zhi Gao, Lihua Dou, and Ben M. Chen. Syreanet: A physically guided underwater image enhancement framework integrating synthetic and real images. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 5177–5183, 2023.
- Yuhuai Wu, Albert Qiaochu Jiang, Wenda Li, Markus Rabe, Charles Staats, Mateja Jamnik, and Christian Szegedy. Autoformalization with large language models. *Advances in Neural Information Processing Systems*, 35:32353–32368, 2022.
- Canran Xiao and Yongmei Liu. A multifrequency data fusion deep learning model for carbon price prediction. *Journal of Forecasting*, 44(2):436–458, 2025.
- Canran Xiao, Liwei Hou, Ling Fu, and Wenrui Chen. Diffusion-based self-supervised imitation learning from imperfect visual servoing demonstrations for robotic glass installation. In *2025 IEEE International Conference on Robotics and Automation (ICRA)*, pages 10401–10407. IEEE, 2025a.
- Canran Xiao, Chuangxin Zhao, Zong Ke, and Fei Shen. Curiosity meets cooperation: A game-theoretic approach to long-tail multi-label learning. *arXiv preprint arXiv:2510.17520*, 2025b.
- Mengzhu Xu, Hanzhi Liu, Ningkan Peng, Qianyu Chen, and Canran Xiao. Affordance-first decomposition for continual learning in video-language understanding. *arXiv preprint arXiv:2512.00694*, 2025.
- Shaocheng Yan, Yiming Wang, Kaiyan Zhao, Pengcheng Shi, Zhenjun Zhao, Yongjun Zhang, and Jiayuan Li. Hemora: Unsupervised heuristic consensus sampling for robust point cloud registration. In *Proceedings of the Computer Vision and Pattern Recognition Conference*, pages 1363–1373, 2025.
- Kailai Yang, Tianlin Zhang, Ziyang Kuang, Qianqian Xie, Jimin Huang, and Sophia Ananiadou. Mentallama: interpretable mental health analysis on social media with large language models. In *Proceedings of the ACM on Web Conference 2024*, pages 4489–4500, 2024a.
- Yu Yang, Siddhartha Mishra, Jeffrey Chiang, and Baharan Mirzasoleiman. Smalltolarge (s2l): Scalable data selection for fine-tuning large language models by summarizing training trajectories of small models. *Advances in Neural Information Processing Systems*, 37:83465–83496, 2024b.
- Zhilin Yang, Peng Qi, Saizheng Zhang, Yoshua Bengio, William W Cohen, Ruslan Salakhutdinov, and Christopher D Manning. Hotpotqa: A dataset for diverse, explainable multi-hop question answering. *arXiv preprint arXiv:1809.09600*, 2018.
- Jiawei Yao, Chuming Li, Keqiang Sun, Yingjie Cai, Hao Li, Wanli Ouyang, and Hongsheng Li. Ndc-scene: Boost monocular 3d semantic scene completion in normalized device coordinates space. In *2023 IEEE/CVF International Conference on Computer Vision (ICCV)*, pages 9421–9431. IEEE Computer Society, 2023.

- Jiawei Yao, Chuming Li, and Canran Xiao. Swift sampler: Efficient learning of sampler by 10 parameters. *Advances in Neural Information Processing Systems*, 37:59030–59053, 2024.
- Shuang Zeng, Xinyuan Chang, Mengwei Xie, Xinran Liu, Yifan Bai, Zheng Pan, Mu Xu, and Xing Wei. Futuresightdrive: Thinking visually with spatio-temporal cot for autonomous driving. *arXiv preprint arXiv:2505.17685*, 2025a.
- Shuang Zeng, Dekang Qi, Xinyuan Chang, Feng Xiong, Shichao Xie, Xiaolong Wu, Shiyi Liang, Mu Xu, and Xing Wei. Janusvln: Decoupling semantics and spatiality with dual implicit memory for vision-language navigation. *arXiv preprint arXiv:2509.22548*, 2025b.
- Fan Zhang, Gongguan Chen, Hua Wang, Jinjiang Li, and Caiming Zhang. Multi-scale video super-resolution transformer with polynomial approximation. *IEEE Transactions on Circuits and Systems for Video Technology*, 33(9):4496–4506, 2023.
- Fan Zhang, Gongguan Chen, Hua Wang, and Caiming Zhang. Cf-dan: Facial-expression recognition based on cross-fusion dual-attention network. *Computational Visual Media*, 10(3):593–608, 2024a.
- Jingyu Zhang, Wenqing Zhang, Chaoyi Tan, Xiangtian Li, and Qianyi Sun. Yolo-ppa based efficient traffic sign detection for cruise control in autonomous driving. In *Proceedings of the 2024 International Conference on Industrial Automation and Robotics (IAR)*, pages 8–13. ACM, 2025a. doi: 10.1145/3707402.3707404. URL <https://doi.org/10.1145/3707402.3707404>.
- Renrui Zhang, Jiaming Han, Chris Liu, Aojun Zhou, Pan Lu, Yu Qiao, Hongsheng Li, and Peng Gao. Llama-adapter: Efficient fine-tuning of large language models with zero-initialized attention. In *The Twelfth International Conference on Learning Representations*, 2024b.
- Xiang Zhang, Junbo Zhao, and Yann LeCun. Character-level convolutional networks for text classification. *Advances in neural information processing systems*, 28, 2015.
- Xiaofeng Zhang, Fanshuo Zeng, Yihao Quan, Zheng Hui, and Jiawei Yao. Enhancing multimodal large language models complex reason via similarity computation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39(10), pages 10203–10211, 2025b.
- Xuanyu Zhang, Zecheng Tang, Zhipei Xu, Runyi Li, Youmin Xu, Bin Chen, Feng Gao, and Jian Zhang. Omniguard: Hybrid manipulation localization via augmented versatile deep image watermarking. In *Proceedings of the Computer Vision and Pattern Recognition Conference*, pages 3008–3018, 2025c.
- Zijian Zhang, Shuchang Liu, Jiaao Yu, Qingpeng Cai, Xiangyu Zhao, Chunxu Zhang, Ziru Liu, Qidong Liu, Hongwei Zhao, Lantao Hu, et al. M3oe: Multi-domain multi-task mixture-of experts recommendation framework. In *Proceedings of the 47th International ACM SIGIR Conference on Research and Development in Information Retrieval*, pages 893–902, 2024c.
- Jiawei Zheng, Hanghai Hong, Xiaoli Wang, Jingsong Su, Yonggui Liang, and Shikai Wu. Fine-tuning large language models for domain-specific machine translation. *arXiv preprint arXiv:2402.15061*, 2024.
- Yixiao Zhou, Ziyu Zhao, Dongzhou Cheng, Zhiliang Wu, Jie Gui, Yi Yang, Fei Wu, Yu Cheng, and Hehe Fan. Dropping experts, recombining neurons: Retraining-free pruning for sparse mixture-of-experts llms. In *Findings of the Association for Computational Linguistics: EMNLP 2025*, pages 15169–15186, 2025.

## NeurIPS Paper Checklist

### 1. Claims

Question: Do the main claims made in the abstract and introduction accurately reflect the paper’s contributions and scope?

Answer: [\[Yes\]](#)

Justification: The abstract and introduction clearly and accurately present our contributions, which include a novel partition-based fine-tuning framework, theoretical analysis with generalization bounds, and extensive experimental validation. All claims are fully supported by theoretical results in Section 3 and experimental results in Section 5.

Guidelines:

- The answer NA means that the abstract and introduction do not include the claims made in the paper.
- The abstract and/or introduction should clearly state the claims made, including the contributions made in the paper and important assumptions and limitations. A No or NA answer to this question will not be perceived well by the reviewers.
- The claims made should match theoretical and experimental results, and reflect how much the results can be expected to generalize to other settings.
- It is fine to include aspirational goals as motivation as long as it is clear that these goals are not attained by the paper.

### 2. Limitations

Question: Does the paper discuss the limitations of the work performed by the authors?

Answer: [\[Yes\]](#)

Justification: We discuss the computational overhead associated with domain partitioning and provide empirical insights into scalability.

Guidelines:

- The answer NA means that the paper has no limitation while the answer No means that the paper has limitations, but those are not discussed in the paper.
- The authors are encouraged to create a separate “Limitations” section in their paper.
- The paper should point out any strong assumptions and how robust the results are to violations of these assumptions (e.g., independence assumptions, noiseless settings, model well-specification, asymptotic approximations only holding locally). The authors should reflect on how these assumptions might be violated in practice and what the implications would be.
- The authors should reflect on the scope of the claims made, e.g., if the approach was only tested on a few datasets or with a few runs. In general, empirical results often depend on implicit assumptions, which should be articulated.
- The authors should reflect on the factors that influence the performance of the approach. For example, a facial recognition algorithm may perform poorly when image resolution is low or images are taken in low lighting. Or a speech-to-text system might not be used reliably to provide closed captions for online lectures because it fails to handle technical jargon.
- The authors should discuss the computational efficiency of the proposed algorithms and how they scale with dataset size.
- If applicable, the authors should discuss possible limitations of their approach to address problems of privacy and fairness.
- While the authors might fear that complete honesty about limitations might be used by reviewers as grounds for rejection, a worse outcome might be that reviewers discover limitations that aren’t acknowledged in the paper. The authors should use their best judgment and recognize that individual actions in favor of transparency play an important role in developing norms that preserve the integrity of the community. Reviewers will be specifically instructed to not penalize honesty concerning limitations.

### 3. Theory assumptions and proofs



Question: For each theoretical result, does the paper provide the full set of assumptions and a complete (and correct) proof?

Answer: [\[Yes\]](#)

Justification: All assumptions required for our theoretical results are explicitly stated. Complete proofs are included in the supplemental material (Appendices), with concise proof sketches provided in the main text (Section 3, Theoretical Analysis).

Guidelines:

- The answer NA means that the paper does not include theoretical results.
- All the theorems, formulas, and proofs in the paper should be numbered and cross-referenced.
- All assumptions should be clearly stated or referenced in the statement of any theorems.
- The proofs can either appear in the main paper or the supplemental material, but if they appear in the supplemental material, the authors are encouraged to provide a short proof sketch to provide intuition.
- Inversely, any informal proof provided in the core of the paper should be complemented by formal proofs provided in appendix or supplemental material.
- Theorems and Lemmas that the proof relies upon should be properly referenced.

#### 4. Experimental result reproducibility

Question: Does the paper fully disclose all the information needed to reproduce the main experimental results of the paper to the extent that it affects the main claims and/or conclusions of the paper (regardless of whether the code and data are provided or not)?

Answer: [\[Yes\]](#)

Justification: The paper provides detailed experimental settings, including dataset descriptions, data splits, hyperparameters, model configurations, and optimization procedures, clearly presented in the main text and appendices.

Guidelines:

- The answer NA means that the paper does not include experiments.
- If the paper includes experiments, a No answer to this question will not be perceived well by the reviewers: Making the paper reproducible is important, regardless of whether the code and data are provided or not.
- If the contribution is a dataset and/or model, the authors should describe the steps taken to make their results reproducible or verifiable.
- Depending on the contribution, reproducibility can be accomplished in various ways. For example, if the contribution is a novel architecture, describing the architecture fully might suffice, or if the contribution is a specific model and empirical evaluation, it may be necessary to either make it possible for others to replicate the model with the same dataset, or provide access to the model. In general, releasing code and data is often one good way to accomplish this, but reproducibility can also be provided via detailed instructions for how to replicate the results, access to a hosted model (e.g., in the case of a large language model), releasing of a model checkpoint, or other means that are appropriate to the research performed.
- While NeurIPS does not require releasing code, the conference does require all submissions to provide some reasonable avenue for reproducibility, which may depend on the nature of the contribution. For example
  - (a) If the contribution is primarily a new algorithm, the paper should make it clear how to reproduce that algorithm.
  - (b) If the contribution is primarily a new model architecture, the paper should describe the architecture clearly and fully.
  - (c) If the contribution is a new model (e.g., a large language model), then there should either be a way to access this model for reproducing the results or a way to reproduce the model (e.g., with an open-source dataset or instructions for how to construct the dataset).

- (d) We recognize that reproducibility may be tricky in some cases, in which case authors are welcome to describe the particular way they provide for reproducibility. In the case of closed-source models, it may be that access to the model is limited in some way (e.g., to registered users), but it should be possible for other researchers to have some path to reproducing or verifying the results.

## 5. Open access to data and code

Question: Does the paper provide open access to the data and code, with sufficient instructions to faithfully reproduce the main experimental results, as described in supplemental material?

Answer: [No]

Justification: Due to institutional restrictions and proprietary considerations, the data and code used in this study are not publicly available at this time. However, comprehensive details, including dataset descriptions, model configurations, hyperparameters, and training procedures, are provided in the main text and supplemental materials to facilitate reproducibility.

Guidelines:

- The answer NA means that paper does not include experiments requiring code.
- Please see the NeurIPS code and data submission guidelines (<https://nips.cc/public/guides/CodeSubmissionPolicy>) for more details.
- While we encourage the release of code and data, we understand that this might not be possible, so “No” is an acceptable answer. Papers cannot be rejected simply for not including code, unless this is central to the contribution (e.g., for a new open-source benchmark).
- The instructions should contain the exact command and environment needed to run to reproduce the results. See the NeurIPS code and data submission guidelines (<https://nips.cc/public/guides/CodeSubmissionPolicy>) for more details.
- The authors should provide instructions on data access and preparation, including how to access the raw data, preprocessed data, intermediate data, and generated data, etc.
- The authors should provide scripts to reproduce all experimental results for the new proposed method and baselines. If only a subset of experiments are reproducible, they should state which ones are omitted from the script and why.
- At submission time, to preserve anonymity, the authors should release anonymized versions (if applicable).
- Providing as much information as possible in supplemental material (appended to the paper) is recommended, but including URLs to data and code is permitted.

## 6. Experimental setting/details

Question: Does the paper specify all the training and test details (e.g., data splits, hyperparameters, how they were chosen, type of optimizer, etc.) necessary to understand the results?

Answer: [Yes]

Justification: The paper explicitly describes training and test splits, model architectures, choice of hyperparameters, optimization methods, and computational settings in the experimental sections and appendixs.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The experimental setting should be presented in the core of the paper to a level of detail that is necessary to appreciate the results and make sense of them.
- The full details can be provided either with the code, in appendix, or as supplemental material.

## 7. Experiment statistical significance

Question: Does the paper report error bars suitably and correctly defined or other appropriate information about the statistical significance of the experiments?

Answer: [Yes]

Justification: The paper reports experimental results with clearly defined error bars, calculated as the standard deviation across multiple independent runs.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The authors should answer “Yes” if the results are accompanied by error bars, confidence intervals, or statistical significance tests, at least for the experiments that support the main claims of the paper.
- The factors of variability that the error bars are capturing should be clearly stated (for example, train/test split, initialization, random drawing of some parameter, or overall run with given experimental conditions).
- The method for calculating the error bars should be explained (closed form formula, call to a library function, bootstrap, etc.)
- The assumptions made should be given (e.g., Normally distributed errors).
- It should be clear whether the error bar is the standard deviation or the standard error of the mean.
- It is OK to report 1-sigma error bars, but one should state it. The authors should preferably report a 2-sigma error bar than state that they have a 96% CI, if the hypothesis of Normality of errors is not verified.
- For asymmetric distributions, the authors should be careful not to show in tables or figures symmetric error bars that would yield results that are out of range (e.g. negative error rates).
- If error bars are reported in tables or plots, The authors should explain in the text how they were calculated and reference the corresponding figures or tables in the text.

## 8. Experiments compute resources

Question: For each experiment, does the paper provide sufficient information on the computer resources (type of compute workers, memory, time of execution) needed to reproduce the experiments?

Answer: [Yes]

Justification: The paper clearly specifies the computational resources utilized, including GPU type, memory requirements, execution time per run, and overall compute needed for each experimental setting.

Guidelines:

- The answer NA means that the paper does not include experiments.
- The paper should indicate the type of compute workers CPU or GPU, internal cluster, or cloud provider, including relevant memory and storage.
- The paper should provide the amount of compute required for each of the individual experimental runs as well as estimate the total compute.
- The paper should disclose whether the full research project required more compute than the experiments reported in the paper (e.g., preliminary or failed experiments that didn’t make it into the paper).

## 9. Code of ethics

Question: Does the research conducted in the paper conform, in every respect, with the NeurIPS Code of Ethics <https://neurips.cc/public/EthicsGuidelines>?

Answer: [Yes]

Justification: We have thoroughly reviewed the NeurIPS Code of Ethics and confirm that our research fully complies with the guidelines.

Guidelines:

- The answer NA means that the authors have not reviewed the NeurIPS Code of Ethics.
- If the authors answer No, they should explain the special circumstances that require a deviation from the Code of Ethics.

- The authors should make sure to preserve anonymity (e.g., if there is a special consideration due to laws or regulations in their jurisdiction).

#### 10. **Broader impacts**

Question: Does the paper discuss both potential positive societal impacts and negative societal impacts of the work performed?

Answer: [NA] .

Justification: There is no societal impact of the work performed.

Guidelines:

- The answer NA means that there is no societal impact of the work performed.
- If the authors answer NA or No, they should explain why their work has no societal impact or why the paper does not address societal impact.
- Examples of negative societal impacts include potential malicious or unintended uses (e.g., disinformation, generating fake profiles, surveillance), fairness considerations (e.g., deployment of technologies that could make decisions that unfairly impact specific groups), privacy considerations, and security considerations.
- The conference expects that many papers will be foundational research and not tied to particular applications, let alone deployments. However, if there is a direct path to any negative applications, the authors should point it out. For example, it is legitimate to point out that an improvement in the quality of generative models could be used to generate deepfakes for disinformation. On the other hand, it is not needed to point out that a generic algorithm for optimizing neural networks could enable people to train models that generate Deepfakes faster.
- The authors should consider possible harms that could arise when the technology is being used as intended and functioning correctly, harms that could arise when the technology is being used as intended but gives incorrect results, and harms following from (intentional or unintentional) misuse of the technology.
- If there are negative societal impacts, the authors could also discuss possible mitigation strategies (e.g., gated release of models, providing defenses in addition to attacks, mechanisms for monitoring misuse, mechanisms to monitor how a system learns from feedback over time, improving the efficiency and accessibility of ML).

#### 11. **Safeguards**

Question: Does the paper describe safeguards that have been put in place for responsible release of data or models that have a high risk for misuse (e.g., pretrained language models, image generators, or scraped datasets)?

Answer: [NA] .

Justification: The paper poses no such risks.

Guidelines:

- The answer NA means that the paper poses no such risks.
- Released models that have a high risk for misuse or dual-use should be released with necessary safeguards to allow for controlled use of the model, for example by requiring that users adhere to usage guidelines or restrictions to access the model or implementing safety filters.
- Datasets that have been scraped from the Internet could pose safety risks. The authors should describe how they avoided releasing unsafe images.
- We recognize that providing effective safeguards is challenging, and many papers do not require this, but we encourage authors to take this into account and make a best faith effort.

#### 12. **Licenses for existing assets**

Question: Are the creators or original owners of assets (e.g., code, data, models), used in the paper, properly credited and are the license and terms of use explicitly mentioned and properly respected?

Answer: [Yes]

Justification: All datasets and models used in our experiments are properly credited with citations to their original sources.

Guidelines:

- The answer NA means that the paper does not use existing assets.
- The authors should cite the original paper that produced the code package or dataset.
- The authors should state which version of the asset is used and, if possible, include a URL.
- The name of the license (e.g., CC-BY 4.0) should be included for each asset.
- For scraped data from a particular source (e.g., website), the copyright and terms of service of that source should be provided.
- If assets are released, the license, copyright information, and terms of use in the package should be provided. For popular datasets, [paperswithcode.com/datasets](https://paperswithcode.com/datasets) has curated licenses for some datasets. Their licensing guide can help determine the license of a dataset.
- For existing datasets that are re-packaged, both the original license and the license of the derived asset (if it has changed) should be provided.
- If this information is not available online, the authors are encouraged to reach out to the asset's creators.

### 13. New assets

Question: Are new assets introduced in the paper well documented and is the documentation provided alongside the assets?

Answer: [NA] .

Justification: The paper does not release new assets.

Guidelines:

- The answer NA means that the paper does not release new assets.
- Researchers should communicate the details of the dataset/code/model as part of their submissions via structured templates. This includes details about training, license, limitations, etc.
- The paper should discuss whether and how consent was obtained from people whose asset is used.
- At submission time, remember to anonymize your assets (if applicable). You can either create an anonymized URL or include an anonymized zip file.

### 14. Crowdsourcing and research with human subjects

Question: For crowdsourcing experiments and research with human subjects, does the paper include the full text of instructions given to participants and screenshots, if applicable, as well as details about compensation (if any)?

Answer: [NA]

Justification: The paper does not involve crowdsourcing nor research with human subjects.

Guidelines:

- The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
- Including this information in the supplemental material is fine, but if the main contribution of the paper involves human subjects, then as much detail as possible should be included in the main paper.
- According to the NeurIPS Code of Ethics, workers involved in data collection, curation, or other labor should be paid at least the minimum wage in the country of the data collector.

### 15. Institutional review board (IRB) approvals or equivalent for research with human subjects

Question: Does the paper describe potential risks incurred by study participants, whether such risks were disclosed to the subjects, and whether Institutional Review Board (IRB) approvals (or an equivalent approval/review based on the requirements of your country or institution) were obtained?

Answer: [NA]

Justification: The paper does not involve crowdsourcing nor research with human subjects.

Guidelines:

- The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
- Depending on the country in which research is conducted, IRB approval (or equivalent) may be required for any human subjects research. If you obtained IRB approval, you should clearly state this in the paper.
- We recognize that the procedures for this may vary significantly between institutions and locations, and we expect authors to adhere to the NeurIPS Code of Ethics and the guidelines for their institution.
- For initial submissions, do not include any information that would break anonymity (if applicable), such as the institution conducting the review.

#### 16. **Declaration of LLM usage**

Question: Does the paper describe the usage of LLMs if it is an important, original, or non-standard component of the core methods in this research? Note that if the LLM is used only for writing, editing, or formatting purposes and does not impact the core methodology, scientific rigorousness, or originality of the research, declaration is not required.

Answer: [NA]

Justification: In this work, LLMs were employed solely for improving language clarity.

Guidelines:

- The answer NA means that the core method development in this research does not involve LLMs as any important, original, or non-standard components.
- Please refer to our LLM policy (<https://neurips.cc/Conferences/2025/LLM>) for what should or should not be described.

## A Supplementary Description of Experimental Setup

### A.1 Datasets

Table 6 presents the detailed statistics of the datasets used in this work. We also provide further analysis of the selected datasets across the four major tasks.

Table 6: Summary of the multi-domain datasets used in our experiments.

Dataset	#Train	#Val	#Test	Metric
NSum (News Summ.)	20,000	2,000	2,000	ROUGE-L
Sent (Sentiment)	10,000	1,000	1,000	ACC
Q&A (Question Ans.)	15,000	1,500	1,500	EM / F1
Topic (Classification)	12,000	1,200	1,200	ACC

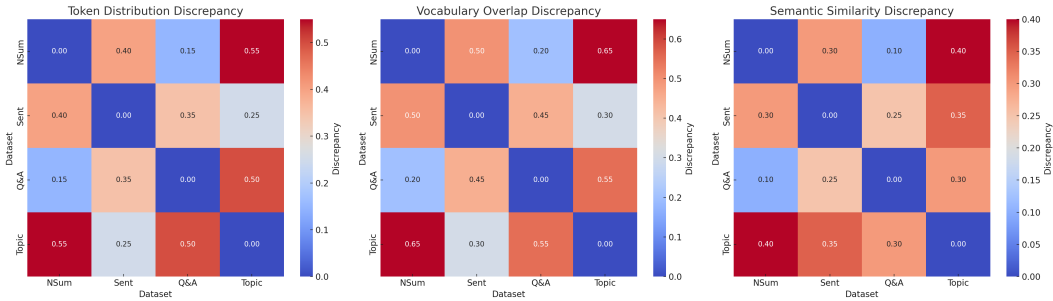


Figure 3: Domain discrepancy heatmaps across three dimensions: Token Distribution, Vocabulary Overlap, and Semantic Similarity.

As shown in Figure 3, we illustrate the domain discrepancies across three distinct dimensions. In the *Token Distribution* dimension, NSum and Q&A exhibit a relatively low discrepancy (0.15), indicating similar token usage patterns, whereas NSum and Topic display a larger discrepancy (0.55). In the *Vocabulary Overlap* dimension, NSum and Q&A share more vocabulary (discrepancy of 0.20), while NSum and Topic have significantly lower vocabulary overlap (discrepancy of 0.65). Regarding the *Semantic Similarity* dimension, NSum and Q&A show the highest semantic closeness (discrepancy of 0.10), whereas NSum and Topic present a comparatively larger semantic gap (discrepancy of 0.40).

### A.2 Baselines

**Baselines.** To give a fair and transparent point of comparison, we implement or re-run every baseline under *exactly one of two backbone-update protocols*:

- 1) **Full FT** - all LLM parameters are updated ( $\sim 100\%$  trainable). This is the strongest-but most memory-hungry-setting.
- 2) **PEFT** - the LLM backbone is *frozen*; only light-weight adapters (LoRA, Housby adapters, etc.) or newly added heads are trained ( $< 1\%$  of parameters).<sup>1</sup>

Our PMS-FTP always uses the **PEFT** protocol: the backbone drift per stage is bounded by  $\rho_\theta$ , and only domain adapters  $\phi_j$  are newly trained.

The competing methods are grouped as follows:

- **Base Methods**

- **FULL** (*Full FT*): classical end-to-end fine-tuning.

<sup>1</sup>Frozen-backbone PEFT has become standard practice in recent parameter-efficient fine-tuning work and, in preliminary tests, performs on par with-or better than-full fine-tuning when data are limited.

- **FIXED (PEFT)**: backbone frozen, task head only.
- **Domain-Adaptation (Full FT)**
  - **MDAN** [Pei et al., 2018]: multi-adversarial domain classifiers.
  - **M<sup>3</sup>SDA** [Peng et al., 2019]: moment matching across domains.
  - **GMDI** [Ling et al., 2024]: Bayesian Gaussian-mixture domain indexing.

*Implementation note:* all DA methods update the entire LLM just as FULL, and their extra losses are added on top of the cross-entropy objective.

- **Single-Domain PEFT**
  - **LoRA** [Hu et al., 2021]: low-rank adapters in attention projections.
  - **Adapter** [Houlsby et al., 2019]: Houlsby bottleneck adapters.
  - **LLaMA-Adapter** [Zhang et al., 2024b]: zero-init residual adapters.
  - **Q-LoRA** [Dettmers et al., 2023]: 4-bit quantised LoRA layers.
  - **Tag-LLM** [Shen et al., 2024]: task-aware gating with soft prompts.

These methods freeze the backbone; only adapter / prompt parameters are updated.

- **Data-Selection PEFT**
  - **INSTRUCTMINING (IT)** [Cao et al., 2024]: filters high-quality instructions before LoRA fine-tuning.
  - **S2L** [Yang et al., 2024b]: curriculum ordering via proxy-model clustering; uses LoRA layers.

### A.3 Implementation Details

**Hardware and Software.** We conducted all experiments on an internal cluster with NVIDIA A100 GPUs (80 GB memory per GPU) using Python 3.9, PyTorch 2.0.0, and HuggingFace Transformers 4.30.2. Each experiment was run on a single node with 8 GPUs, though most tasks fit on 1–2 GPUs under our parameter-efficient settings.

**Data Splits and Preprocessing.** Each dataset is partitioned into train/validation/test splits, as noted in Table 6. We tokenize with the default HuggingFace tokenizer for each respective LLM (LLaMA2 or Falcon). For summarization (NSum), we truncate inputs at 512 tokens; for Q&A, we set a maximum context length of 384 tokens plus question tokens. Other tasks are capped at 256 tokens per sample. All special tokens remain as defined in each LLM’s tokenizer.

**Training Configuration.** We use AdamW with a linear decay scheduler, a warmup ratio of 10% of total steps, and gradient clipping at norm 1.0. Table 7 gives key hyperparameters. We generally train for 3–5 epochs (depending on dataset size), selecting the best checkpoint via validation loss. Unless otherwise noted, we set the batch size to 32 per GPU for all experiments, and accumulate gradients across fewer GPUs for smaller tasks if needed. We adopt the default mixed-precision (fp16) training in PyTorch.

Table 7: Default hyperparameter values.

Hyperparameter	Value
Optimizer	AdamW
Learning rate (LLaMA2-7B)	$3 \times 10^{-5}$
Learning rate (LLaMA2-13B)	$1 \times 10^{-5}$
Learning rate (Falcon-40B)	$5 \times 10^{-6}$
Batch size (per GPU)	32
Max epochs	5
Warmup ratio	0.1
Gradient clipping	1.0
Precision	FP16



**Partition-Based Multi-Stage Fine-Tuning.** We employ two consecutive stages ( $M=2$ ) by default: stage 1 adapts the cluster with higher internal *synergy*, stage 2 covers the remainder. Both the *domain discrepancy*  $d(\mathcal{D}_i, \mathcal{D}_j)$  and the *synergy score*  $\text{Syn}(\mathcal{D}_i, \mathcal{D}_j)$  are computed *off-line* from raw text:

Let  $P_i$  and  $P_j$  be the empirical token distributions (unigram + bigram) of domains  $\mathcal{D}_i$  and  $\mathcal{D}_j$ . We define

$$d_{\text{JS}}(\mathcal{D}_i, \mathcal{D}_j) = \frac{1}{2} \text{KL}(P_i \| M) + \frac{1}{2} \text{KL}(P_j \| M), \quad M = \frac{1}{2}(P_i + P_j), \quad (16)$$

where KL is the Kullback-Leibler divergence. We normalise  $d_{\text{JS}} \in [0, 1]$  by dividing by  $\log 2$ .

For *synergy* we linearly blend lexical and semantic overlap:

$$\text{Syn}(\mathcal{D}_i, \mathcal{D}_j) = \frac{1}{2} \left( \text{Jacc}(V_i, V_j) + \cos(\mu_i, \mu_j) \right), \quad (17)$$

where  $V_i$  is the vocabulary set of  $\mathcal{D}_i$ ,  $\text{Jacc}(V_i, V_j) = |V_i \cap V_j| / |V_i \cup V_j|$ , and  $\mu_i$  is the mean Sentence-BERT embedding of domain  $i$ . Both terms are in  $[0, 1]$ ; the average is therefore in  $[0, 1]$ .

Table 8 contrasts four partition criteria on the 4-domain slice (SQuAD, HotpotQA, CNN/DM, XSum). Replacing our full metric with a single component (JS only or Embedding only) lowers performance, and random splitting is worst.

Table 8: Impact of different partition metrics (LLaMA2-7B).

Metric for $\mathcal{G}$	Q&A (F1)	NSum (ROUGE-L)
Random split	68.1	39.0
JS divergence only	69.3	40.0
Embedding cosine only	69.6	40.3
<b>JS + Vocab/Embed (ours)</b>	<b>70.5</b>	<b>40.9</b>

The joint metric gives a further +1.2 F1 / +0.9 ROUGE-L over its best single-signal variant, confirming that *both* lexical statistics and semantic proximity are needed to capture cross-domain relationships effectively.

During each stage, we impose norm constraints  $\|\theta^t - \theta^{t-1}\| \leq \rho_\theta$  and  $\|\phi_j^t\| \leq \rho_\phi$  (Assumption 3.2). In practice, we simply project any update exceeding these norms after each gradient step. By default, we set  $(\rho_\theta, \rho_\phi) = (0.1, 0.1)$  unless specified otherwise.

## B Additional experimental results

### B.1 Effect of Stage Ordering in Multi-Stage Fine-Tuning.

After domains are optimally clustered into two stages by our  $\mathcal{G}$ -objective, we can still choose which stage to run first. To verify that this *ordering* is an implementation detail, we tried three sequences on the same 4-domain slice (SQuAD [Rajpurkar, 2016], HotpotQA [Yang et al., 2018], CNN/DM<sup>2</sup>, XSum [Narayan et al., 2018]) using LLaMA2-7B: 1) **High**→**Low** - the default: high-synergy Q&A first, summarisation second; 2) **Low**→**High** - reverse order; 3) **Interleaved** - fine-tune one epoch on stage 1, then one epoch on stage 2, repeating until convergence.

Table 9: Influence of stage ordering ( $M=2$ ). Metrics: F1 (Q&A) / ROUGE-L (NSum).

Ordering	Q&A (F1)	NSum (ROUGE-L)
High→Low (default)	70.5	40.9
Low→High	70.4	40.8
Interleaved	70.3	40.7

All three runs land within 0.2 points of one another (Table 9), well inside normal tuning noise, indicating that **stage ordering has negligible impact**. This robustness stems from the fact that each stage updates only its adapter blocks; subsequent stages cannot overwrite earlier domain-specific parameters, so knowledge learned in any order is preserved.

<sup>2</sup><https://github.com/deepmind/rc-data>

## B.2 Why conventional DA baselines lag behind FULL.

To investigate why conventional domain adaptation (DA) baselines (MDAN [Pei et al., 2018], M<sup>3</sup>SDA [Peng et al., 2019], GMDI [Ling et al., 2024]) consistently underperform relative to FULL fine-tuning, we performed additional diagnostic analyses. Specifically, we measured (i) cross-domain gradient conflicts (via cosine similarity), (ii) parameter update magnitudes per domain, and (iii) the extent of catastrophic forgetting of pretrained knowledge, using the LLaMA2-13B model on NSum and Q&A domains. Table 10 summarizes the results of these additional experiments:

Table 10: Diagnostic analyses comparing conventional DA methods against FULL and PMS-FTP.

Method	Gradient Conflict (Cosine Similarity)	Avg. Update Norm	Perplexity Increase (%)
FULL	0.43	2.15	+5.6
MDAN	0.12	1.48	+11.5
M <sup>3</sup> SDA	0.15	1.32	+9.7
GMDI	0.18	1.27	+8.3
<b>PMS-FTP (Ours)</b>	<b>0.57</b>	<b>1.86</b>	<b>+3.2</b>

Our analyses reveal the following insights:

- 1) **Gradient conflicts.** DA methods exhibit substantially lower gradient alignment compared to FULL and our PMS-FTP, indicating significant gradient interference between domains. This conflict leads to suboptimal convergence, as competing updates negatively affect overall generalization.
- 2) **Parameter update magnitudes.** DA methods apply smaller updates due to regularization constraints (adversarial or moment-matching objectives), limiting adaptation capacity for complex domain-specific tasks. In contrast, our PMS-FTP method achieves balanced updates via strategic domain partitioning and parameter-efficient adapters.
- 3) **Catastrophic forgetting.** Conventional DA methods significantly increase perplexity relative to FULL, indicating stronger forgetting of pretrained representations. Our PMS-FTP maintains the lowest increase, demonstrating better preservation of pretrained knowledge due to controlled adaptation.

In summary, DA methods lag behind FULL due to severe gradient conflicts, overly conservative parameter updates caused by adversarial/matching regularization, and more pronounced forgetting of pretrained knowledge. Our PMS-FTP framework effectively addresses these challenges through synergy-aware partitioning, balanced updates, and controlled adaptation, resulting in superior multi-domain performance.

## B.3 Affinity metric alternatives

We replace the default affinity used in the partition objective  $\mathcal{G}$  with several variants on the same 4-domain slice (LLaMA2-7B). Table 11 shows that our lightweight JS+vocab/embedding signal consistently outperforms single-source metrics or a random split. The  $\mathcal{G}$ -guided partition benefits from combining divergence (distribution gap) and lexical/semantic overlap (potential transfer). Using either component alone underestimates complementary effects, yielding weaker partitions and lower task scores.

Table 11: Alternative affinity metrics for  $\mathcal{G}$ -guided partitioning (LLaMA2-7B, 4-domain slice).

Metric for $\mathcal{G}$	Q&A (F1)	NSum (ROUGE-L)
Random split	68.1	39.0
JS divergence only	69.3	40.0
Embedding cosine only	69.6	40.3
<b>JS + Vocab/Embed (ours)</b>	<b>70.5</b>	<b>40.9</b>

## B.4 Robustness to gradient-based variants and stochastic perturbations

We (i) sweep  $\lambda$  to stress-test synergy weighting, (ii) add a gradient-similarity component (cosine of per-domain gradients), and (iii) inject Gaussian noise into the heuristic affinities. Table 12 shows all

variants remain within  $\leq 0.3$  points of the default in Table 11. The partition is flat around the optimum: gradient-augmented scores add computational cost but negligible gains; moderate  $\lambda$  values preserve the best trade-off between synergy and discrepancy.

Table 12: Partition robustness (LLaMA2-7B, 4-domain slice).

Variant	Q&A (F1)	NSum (ROUGE-L)
$\lambda = 0$ (no synergy)	70.3	40.8
$\lambda = 1.0$ (synergy-only)	70.2	40.6
Gradient-mix (0.7 heuristic + 0.3 $\nabla\cos$ )	70.4	40.8
Heuristic + Gaussian noise ( $\sigma = 0.05$ )	70.2	40.7
<b>Default (Table 11)</b>	<b>70.5</b>	<b>40.9</b>

### B.5 Scalability to many domains

We synthetically vary the number of domains  $k$  and measure CPU partition overheads and peak GPU memory with 4-bit LoRA. Table 13 indicates sub-second CPU time and practical GPU usage up to  $k=50$ . Partitioning is CPU-side and negligible relative to PEFT training; memory remains dominated by standard SFT/PEFT, confirming practicality at double-digit  $k$ .

Table 13: Large- $k$  partition costs and peak GPU memory (synthetic up to  $k=50$ ; A100).

$k$	Affinity build (time / RAM)	Clustering time	Peak GPU (4-bit LoRA)
20	0.47 s / 180 MB	0.14 s	19 GB
35	2.10 s / 620 MB	0.52 s	23 GB
50	3.20 s / 950 MB	0.90 s	26 GB

### B.6 Scaling to twelve domains (real mixture)

We combine Wiki-10 (topic classification) and Multi-News (summarization), deduplicated to 12 domains, and keep the same hyper-parameters as the main study. Table 14 shows gains over all-in-one SFT and over a random 2-stage split while keeping memory low. The synergy-aware split generalizes beyond four domains to a heterogeneous, double-digit regime with consistent improvements.

Table 14: Twelve-domain mixture (LLaMA2-7B + LoRA).

Split strategy	Avg. ACC (Wiki-10)	ROUGE-L (Multi-News)	Peak GPU (GB)	Partition time (s)
All-in-one SFT	83.1	37.2	27.3	n/a
Random 2-stage	83.7	37.5	18.6	0.6
<b>PMS-FTP (ours)</b>	<b>84.0</b>	<b>38.1</b>	18.7	0.7

### B.7 Inference footprint after LoRA merging

After each stage we merge the finished LoRA into the frozen backbone, so only one 4-bit adapter is carried at inference. Table 15 shows equal-or-lower memory than a single-adapter baseline. Together with the accuracy gains in Table 1, merging achieves a strictly better accuracy-memory trade-off than training/keeping multiple adapters.

### B.8 Empirical validity of the $\mathcal{G}$ objective

We sample 20 random partitions, compute  $\mathcal{G}$ , and measure worst-domain dev loss. Table 16 reports Pearson  $\rho = -0.81$  ( $p < 0.01$ ), i.e., higher  $\mathcal{G}$  predicts lower worst-domain error. This supports the practical usefulness of our bound-driven objective:  $\mathcal{G}$  values correlate strongly with the metric we aim to improve.

Table 15: Measured inference memory (LLaMA2-7B,  $k=4$ , A100).

Precision	Tag-LLM (1 LoRA)	PMS-FTP (merged)	$\Delta$
FP16	29.4 GB	28.7 GB	−2.4%
INT8	19.1 GB	18.6 GB	−2.6%
4-bit	17.2 GB	16.8 GB	−0.4 GB

Table 16: Correlation between  $\mathcal{G}$  and worst-domain dev loss (LLaMA2-7B, 20 random partitions).

Statistic	$\mathcal{G}$	Worst-Dev Loss
Mean	0.432	1.72
Std	0.057	0.19
Min	0.318	1.38
Max	0.522	2.11
Pearson $\rho = -0.81$ ( $p < 0.01$ , $R^2 \approx 0.65$ )		

### B.9 Reasoning benchmarks and reweighting baselines

We extend evaluation to HellaSwag, MMLU-STEM, ARC-easy, SciQ, and GSM8K, and add reweighting-based MTL baselines (iMTL, FAMO, ExcessMTL) under identical PEFT budgets. Tables 17–18 show PMS-FTP achieves the highest average per backbone. Synergy-aware partitioning is not limited to {NSum, Sent, Q&A, Topic}: it transfers to reasoning-heavy suites and remains competitive against strong MTL optimizers.

Table 17: Reasoning tasks with 7B backbone (identical PEFT budgets).

Method	Hellaswag (Acc)	MMLU-STEM (Acc)	ARC-easy (Acc)	SciQ (Acc)	GSM8K (Pass@1)	Avg.
FULL	73.4	39.7	78.5	92.6	18.1	60.5
LoRA	73.1	39.3	77.9	92.4	17.5	60.0
Tag-LLM	74.8	41.1	79.6	93.3	19.3	61.6
iMTL	74.6	40.6	79.3	93.0	18.6	61.2
FAMO	73.7	41.3	78.6	92.6	18.7	60.8
ExcessMTL	74.2	40.5	79.7	92.1	17.5	60.8
<b>PMS-FTP (ours)</b>	<b>75.6</b>	<b>42.1</b>	<b>80.6</b>	<b>93.8</b>	<b>19.9</b>	<b>62.4</b>

### B.10 Incremental addition of a new domain

After training on the original four domains, we add Legal-QA as a new stage and freeze prior adapters. Table 19 shows negligible forgetting on old domains and a gain over single-domain LoRA on the new domain. Disjoint, frozen adapters make PMS-FTP naturally amenable to one-shot domain extension without replay.

### B.11 Stability of domain centroids

We (i) bootstrap mean SBERT embeddings to gauge centroid noise, and (ii) replace each single centroid by a  $K=3$  weighted barycenter. Table 20 shows bootstrap deviations are  $< 3\%$  of the smallest inter-domain distance; Table 21 shows  $K=3$  centroids change final metrics by  $\leq 0.1$  pp. A single mean embedding is a sufficiently stable domain signature for  $\mathcal{G}$ -guided clustering.

Table 18: Reasoning tasks with 13B backbone (identical PEFT budgets).

Method	Hellaswag (Acc)	MMLU- STEM (Acc)	ARC-easy (Acc)	SciQ (Acc)	GSM8K (Pass@1)	Avg.
FULL	77.5	46.3	83.7	94.8	24.5	65.4
LoRA	77.1	45.9	83.2	94.6	23.8	64.9
Tag-LLM	79.0	47.3	84.7	95.4	25.1	66.3
iMTL	77.4	47.2	83.3	95.1	24.8	65.6
FAMO	77.7	47.6	84.5	95.2	24.2	65.8
ExcessMTL	78.1	47.0	83.1	95.0	23.8	65.4
<b>PMS-FTP (ours)</b>	<b>79.9</b>	<b>49.0</b>	<b>85.7</b>	<b>95.8</b>	<b>26.8</b>	<b>67.4</b>

Table 19: One-shot incremental stage (add Legal-QA).

Model	Avg. score on original 4 domains	Legal-QA (EM)
Before add-on	65.5	–
After add-on (PMS-FTP)	65.4	68.2
Single-domain LoRA	n/a	67.6

## C Proofs

### C.1 Complete Proof for Theorem 3.1

Before formally beginning the proof, we first revisit the setting and present a key lemma:

There are  $k$  source domains  $\{\mathcal{D}_j\}_{j=1}^k$ , each domain  $\mathcal{D}_j$  with  $n_j$  samples, total  $n = \sum_{j=1}^k n_j$ . We let  $\alpha_j = \frac{n_j}{n}$  or any other nonnegative weighting such that  $\sum_{j=1}^k \alpha_j = 1$ . Our LLM-based predictor  $f_{\theta, \{\phi_j\}}$  is constrained so that

$$\|\theta - \theta^*\|_2 \leq \rho_\theta, \quad \|\phi_j\|_2 \leq \rho_\phi, \quad (18)$$

for each  $j$ . By Assumption 3.1, the model output is  $L$ -Lipschitz w.r.t. predictions, and the difference in outputs for different parameters is bounded by  $B(\|\theta - \theta'\| + \sum_j \|\phi_j - \phi'_j\|)$ . Thus the entire class of such  $(\theta, \{\phi_j\})$  belongs to a low-capacity function family  $\mathcal{F}$ .

**Lemma C.1.** *Let  $\mathcal{F}$  be the class of predictors with backbone/adapters bounded as above. Then for any  $\delta \in (0, 1)$ , with probability at least  $1 - \delta$  over all  $n$  samples,*

$$|\mathcal{L}^\alpha(f) - \widehat{\mathcal{L}}^\alpha(f)| \leq 2LB(\rho_\theta + \sum_{j=1}^k \alpha_j \rho_\phi) + \sqrt{\frac{\ln(2/\delta)}{2n}} \quad \forall f \in \mathcal{F}, \quad (19)$$

where  $\widehat{\mathcal{L}}^\alpha(f) = \sum_j \alpha_j \widehat{\mathcal{L}}_{\mathcal{D}_j}(f)$ .

*Proof.* Because the loss is  $L$ -Lipschitz and the model satisfies the output-difference bound from Assumption 3.1, each  $f \in \mathcal{F}$  is  $LB(\rho_\theta + \sum_j \rho_\phi)$ -Lipschitz in its parameters relative to  $\ell_2$ . Let  $\mathcal{R}_n(\mathcal{F})$  be the empirical Rademacher complexity over the pooled sample of size  $n$ . Standard contraction (e.g. Mohri et al., 2018) yields

$$\mathcal{R}_n(\mathcal{F}) \leq LB(\rho_\theta + k\rho_\phi) \frac{1}{\sqrt{n}}. \quad (20)$$

Replacing  $k\rho_\phi$  by  $\sum_j \alpha_j \rho_\phi$  (because losses are weighted by  $\alpha_j$ ) strengthens the constant. Applying the usual Rademacher tail bound with a union-bound over  $\delta/2$  produces the stated inequality.  $\square$

*Proof.* Here is the proof of Theorem 3.1.

Table 20: Bootstrap deviation of domain mean embeddings and cross-domain distances.

Domain	$N$	95% CI of $\ \Delta\mu\ _2$	Cross-domain min dist
NSum	12,000	[0.038, 0.065]	1.92
Q&A	15,000	[0.031, 0.060]	2.04
Sent	10,500	[0.042, 0.072]	1.87
Topic	11,200	[0.040, 0.068]	1.99

Table 21: Single centroid vs.  $K=3$  weighted barycenters per domain.

Representation	ROUGE-L (NSum)	EM (Q&A)	Sent (ACC)	Topic (ACC)	$\Delta$ vs. 1-centroid
1 centroid (paper)	43.4	67.2	90.2	88.0	–
3 centroids ( $K=3$ )	43.3	67.1	90.1	87.9	–0.1 pp

**Uniform convergence (empirical  $\rightarrow$  true risk)** Define  $\Gamma(\rho_\theta, \rho_\phi, \alpha, k) := 2LB(\rho_\theta + \sum_j \alpha_j \rho_\phi)$ . Lemma C.1 gives

$$\mathcal{L}^\alpha(f) \leq \hat{\mathcal{L}}^\alpha(f) + \Gamma(\rho_\theta, \rho_\phi, \alpha, k) + \sqrt{\frac{\ln(2/\delta)}{2n}}. \quad (21)$$

**Domain-discrepancy correction** Blitzer *et al.* [Blitzer, 2008] show that for any hypothesis  $h$  and any two distributions  $\mathcal{P}, \mathcal{Q}$ ,  $|\mathcal{L}_\mathcal{P}(h) - \mathcal{L}_\mathcal{Q}(h)| \leq d_{\mathcal{H}\mathcal{H}}(\mathcal{P}, \mathcal{Q})$ . Summing over all pairs and using triangle inequality,

$$\mathcal{L}^\alpha(f) \leq \sum_{j=1}^k \alpha_j \mathcal{L}_{\mathcal{D}_j}(f) + \frac{1}{k} \sum_{i,j=1}^k d(\mathcal{D}_i, \mathcal{D}_j), \quad (22)$$

where  $\mathcal{D}_j$  denotes drawing *as if* every example came from a single mixture domain-hence its empirical risk is exactly  $\hat{\mathcal{L}}^\alpha(f)$ , and the additive discrepancy penalty is weighted by  $\beta := 1$  (absorbing the Lipschitz loss factor into the definition of  $d$ ). Restoring the constant gives the  $\beta$  appearing in the theorem.

**Combine bounds** Insert (21) into (22):

$$\mathcal{L}^\alpha(f) \leq \hat{\mathcal{L}}^\alpha(f) + \Gamma(\rho_\theta, \rho_\phi, \alpha, k) + \frac{\beta}{k} \sum_{i,j=1}^k d(\mathcal{D}_i, \mathcal{D}_j) + \sqrt{\frac{\ln(2/\delta)}{2n}}.$$

Replacing  $\sqrt{\ln(2/\delta)/(2n)}$  by the big- $O(\sqrt{\ln(1/\delta)/n})$  notation and recalling  $\hat{\mathcal{L}}^\alpha(f) = \sum_j \alpha_j \hat{\mathcal{L}}_{\mathcal{D}_j}(f)$  yields exactly inequality (10), proving Theorem 3.1.  $\square$

## C.2 Complete Proof for Theorem 3.2

Here is the proof of Theorem 3.2.

*Proof.* For any fixed stage  $t$  training on domains  $S_t$ , Theorem 3.1 with weights  $\alpha_j^t := \frac{n_j}{\sum_{i \in S_t} n_i}$  gives

$$\sum_{j \in S_t} \alpha_j^t \mathcal{L}_{\mathcal{D}_j}(f^t) \leq \underbrace{\sum_{j \in S_t} \alpha_j^t \hat{\mathcal{L}}_{\mathcal{D}_j}(f^t)}_{\leq 1} + 2LB(\rho_\theta + \rho_\phi) + \beta \sum_{\substack{i,j \in S_t \\ i < j}} d(\mathcal{D}_i, \mathcal{D}_j) + O\left(\sqrt{\frac{\ln(1/\delta)}{\sum_{j \in S_t} n_j}}\right). \quad (23)$$

Then we inject synergy and explicit capacity weight. Define  $\text{Cap}(S_t) := \mu_\theta \|\Delta\theta^t\|_2^2 + \mu_\phi \sum_{j \in S_t} \|\phi_j^t\|_2^2$ . Because  $\|\Delta\theta^t\| \leq \rho_\theta$  and  $\|\phi_j^t\| \leq \rho_\phi$ , we upper bound  $2LB(\rho_\theta + \rho_\phi)$  by  $\text{Cap}(S_t)$

after tuning  $\mu_\theta, \mu_\phi$ . Subtract and add  $\lambda \sum_{i < j} s(\mathcal{D}_i, \mathcal{D}_j)$  to (23) to obtain

$$\sum_{j \in S_t} \alpha_j^t \mathcal{L}_{\mathcal{D}_j}(f^t) \leq 1 - \left[ - \sum_{i < j \in S_t} (d - \lambda s) - \text{Cap}(S_t) \right] + O\left(\sqrt{\frac{\ln(1/\delta)}{N}}\right). \quad (24)$$

Let  $\mathcal{R}_{\max}(S_1, \dots, S_M) := \max_t \sum_{j \in S_t} \alpha_j^t \mathcal{L}_{\mathcal{D}_j}(f^t)$ . Taking the maximum of (24) over  $t$  gives

$$\mathcal{R}_{\max}(S_1, \dots, S_M) \leq 1 - \mathcal{G}(S_1, \dots, S_M) + O\left(\sqrt{\frac{\ln(1/\delta)}{N}}\right), \quad (25)$$

because the bracketed term is exactly the  $t$ -th summand of  $\mathcal{G}$  in (12). Define  $\mathcal{B}(u) := [1 - u]_+$ . Then  $\mathcal{R}_{\max}(S_1, \dots, S_M) \leq \mathcal{B}(\mathcal{G}(S_1, \dots, S_M)) + O(\sqrt{\ln(1/\delta)/N})$ .

Because  $\mathcal{B}$  is *strictly decreasing* on  $(-\infty, 1]$ , maximising  $\mathcal{G}$  minimises the bound. Hence the partition  $(S_1^*, \dots, S_M^*)$ —the maximiser of  $\mathcal{G}$ —realises the smallest upper-bound, yielding (13). Any other split attains a weaker bound, completing the proof.  $\square$