# Mathematical aspects of physics-informed neural networks architecture for studying hydrology and climate problems

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The global climate change affects the regional climate, resulting in heavy precipitation and river flooding. Therefore, it is important to study hydrological processes in rivers using mathematical modeling in order to construct digital twins of rivers. For the developers of digital twins, it is necessary to perform real-time computations. From a mathematical point of view, the problems of studying flooding processes can be reduced to solving forward and inverse problems in the field of hydrological processes using the equations of Shallow Water Theory. Physics-Informed Neural Networks (PINNs), a key component of digital twins, enable real-time calculations and have seen significant advancements recently. PINNs embed the Partial Differential Equations (PDE) residual into the loss function of the neural network. They have been successfully employed to solve diverse forward and inverse PDE problems. However, one disadvantage of the first generation of PINNs is that they usually have limited accuracy even with many training points, and also place high computational demands on memory resources on GPU cards.

Using Machine Learning in the field of PDEs and physics-based predictions has been seen several exciting developments. Some efforts have focused on learning from data and introducing physics constraints, and others have been developed to learn neural operators. A new area in the development of PINN architectures is related to the direction of the Evolutional Deep Neural Networks.

The famous open source libraries (DeepXDE, Nvidia Modulus, FourCastNet) for PINN and AFNO (Adaptive Fourier Neural Operator) development will be discussed, as well as mathematical aspects in the area of forming the loss function and solving the optimization problem to find hyperparameters for a neural network to solve river hydrological processes and climate problems. Results of solving forward and inverse problems for modeling viscous incompressible fluid motion using synthetic data, historical weather data and PINNs will be presented.