SLM-MUX: ORCHESTRATING SMALL LANGUAGE MODELS FOR REASONING

Anonymous authors
Paper under double-blind review

ABSTRACT

With the rapid development of language models, the number of small language models (SLMs) has grown significantly. Although they do not achieve state-of-the-art accuracy, they are more efficient and often excel at specific tasks. This raises a natural question: can multiple SLMs be orchestrated into a system where each contributes effectively, achieving higher accuracy than any individual model? Existing orchestration methods have primarily targeted frontier models (e.g., GPT-4) and perform suboptimally when applied to SLMs. To address this gap, we propose a three-stage approach for orchestrating SLMs. First, we introduce SLM-MUX, a multi-model architecture that effectively coordinates multiple SLMs. Building on this, we develop two optimization strategies: (i) a model selection search that identifies the most complementary SLMs from a given pool, and (ii) test-time scaling tailored to SLM-MUX. Our approach delivers strong results: Compared to existing orchestration methods, our approach achieves up to 13.4% improvement on MATH, 8.8% on GPQA, and 7.0% on GSM8K. With just two SLMs, SLM-MUX outperforms Qwen 2.5 72B on GPQA and GSM8K, and matches its performance on MATH. We further provide theoretical analyses to substantiate the advantages of our method. Additional experiments show that the core principle of SLM-MUX extends to open-ended generation tasks (e.g., HumanEval) and benefits other model classes, including frontier LLMs and domain-specific fine-tuned SLMs. In summary, we demonstrate that SLMs can be effectively orchestrated into more accurate and efficient systems through the proposed approach.

1 INTRODUCTION

Recent years have witnessed a surge of small-sized language models (SLMs) containing billions to tens of billions of parameters (Wang et al., 2024a; Javaheripi & Bubeck, 2023; Guo et al., 2025; Allal et al., 2025). While these models may underperform state-of-the-art frontier language models, which usually contain hundreds of billions to trillions of parameters, on any given query, they offer substantially lower inference costs, are more affordable to train and finetune, and allow edge deployment due to their small size (Belcak et al., 2025). Meanwhile, frontier models have reached trillion-parameter scales where further increases in size and training data yield diminishing returns. This mirrors a well-known challenge in computer architecture two decades ago: when enlarging single CPU cores no longer delivered proportional performance gains, computer architects turned to designing multi-core processors, where multiple smaller cores working together enabled sustained improvements. This parallel suggests that combining multiple SLMs could offer a promising alternative to scaling ever-larger frontier models.

Recent works have explored orchestrating multiple LLMs (e.g., GPT-3.5 and GPT-4o), combining them into one system to process an input collaboratively. Representative approaches include Mixture-of-Agents (Wang et al., 2024b), LLM-Debate (Du et al., 2023), and Multi-Agent Verification (Lifshitz et al., 2025). These approaches share a key assumption: that models possess strong reasoning and deliberation abilities, so that interaction through natural language can reliably correct mistakes. However, when applied to SLMs, this assumption no longer holds. Our study finds that *such discussion-based orchestration often fails to improve performance for SLMs*, and in some cases even reduces accuracy by over 5%. Instead of correcting mistakes, SLMs tend to fall into groupthink during interaction, amplifying errors rather than mitigating them. The assumptions that language models can correct each other's answers behind existing orchestration methods do not hold for SLMs (Taubenfeld et al., 2024; Huang et al., 2024; Liu et al., 2023; Fu et al., 2025).

To address this issue, we propose **SLM-MUX**, a multi-model architecture for effectively orchestrating SLMs while avoiding explicit text exchanges between models. Our key insight is that SLM-MUX leverages complementary abilities from different models by selecting outputs based on confidence scores without any model training.

After introducing SLM-MUX, another question arises: which models should be orchestrated together? Not all combinations are effective – if one model is weaker across all dimensions, it provides no benefit when paired with a stronger one. In contrast, combining models with complementary strengths (e.g., one stronger in algebra, another in geometry) allows the system to succeed where a single model would fail.

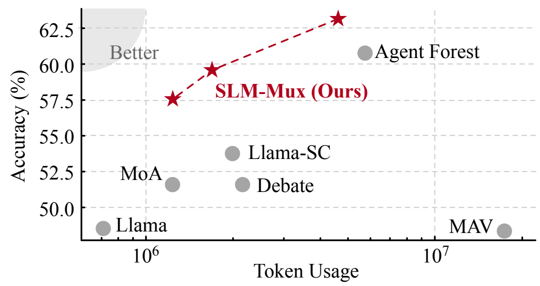


Figure 1: **Head-to-Head Comparison of SLM-MUX with Other Methods.** SLM-MUX outperforms existing methods such as Self-Consistency (SC) (Wang et al., 2023), Mixture-of-Agents (MoA) (Wang et al., 2024b), LLM-Debate (Du et al., 2023), Multi-Agent Verification (MAV) (Lifshitz et al., 2025), and Agent Forest (Li et al., 2024). Results reported on MATH dataset with SLMs.

To address this, we develop a **model selection search strategy** for SLM-MUX, which systematically evaluates and identifies model subsets with complementary strengths. By maximizing union accuracy while penalizing overconfident contradictions, the search procedure finds the most suitable models for a given model budget.

In addition, we explore **compute scaling strategies** for the selected model ensembles to further enhance performance. By adjusting the number of models and samples at inference time, we further boost performance and identify practical sweet spots in the accuracy-compute tradeoff.

Our experiments demonstrate significant improvements across multiple benchmarks. By combining only two SLMs, we achieve accuracy improvements of up to 6.7% on MATH, 5.7% on GPQA, and 4.8% on GSM8K, compared to the best-performing single SLMs in the system. Our method consistently outperforms existing discussion-based approaches for SLMs, with gains of up to 13.4% on MATH, 8.8% on GPQA, and 7.0% on GSM8K. Most importantly, with just two SLMs, SLM-MUX outperforms Qwen2.5-72B on GPQA and GSM8K, and matches its performance on MATH.

Finally, we complement these empirical findings with theoretical and experimental analyses. Our approach shows superiority in multiple scenarios compared with previous methods (Figure 1).

Our main contributions are as follows: **(i) We identify a fundamental limitation of existing orchestration methods:** Through systematic evaluation, we demonstrate that existing discussion-based methods, which show consistent improvements for frontier LLMs, actually harm performance when applied to SLMs. This counterintuitive finding challenges the assumption that orchestration methods transfer across model scales and reveals the need for SLM-specific method. **(ii) We propose SLM-MUX**, a novel multi-model architecture designed specifically for SLMs that avoids the error amplification problems of discussion-based methods. SLM-MUX achieves consistent gains across multiple benchmarks (MATH, GPQA, GSM8K) and significantly outperforms existing discussion-based methods by large margins (up to 11.6% on MATH). **(iii) We develop principled optimization strategies** for the SLM-MUX, including model selection search that identifies complementary model selections and compute scaling strategies, further boosting performance while maintaining efficiency.

2 Related Work

Discussion-based Orchestration Methods. We use discussion-based orchestration to refer to orchestration schemes where multiple LM instances exchange or evaluate natural-language messages (Fu et al., 2025)—such as proposing answers, critiquing or debating, verifying from different aspects, and finally aggregating into one output. Representative approaches include Mixture-of-Agents (Wang et al., 2024b), which uses a dedicated LLM to aggregate outputs from several models; LLM-Debate (Du et al., 2023), where models critique and refine each other's reasoning; and Multi-Agent Verification (Lifshitz et al., 2025), which assigns models to independently evaluate candidate solutions before selecting the final answer. These methods assume that participating models have sufficient reasoning ability to self-correct through interaction. Prior evaluations have been conducted on frontier LLMs, while their effectiveness for SLMs remains unstudied.

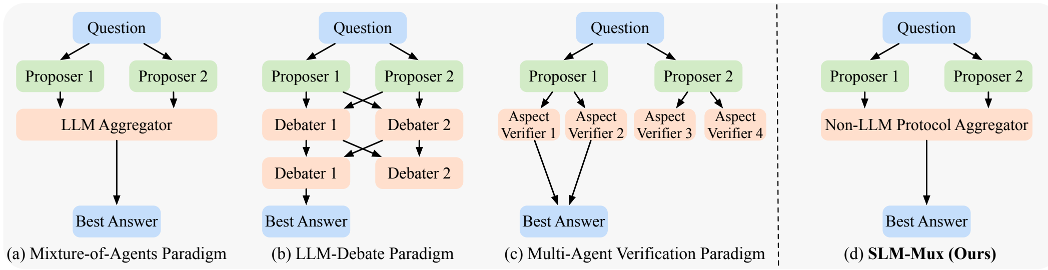


Figure 2: **Comparing SLM-MUX (Ours) with Existing LLM Orchestration Methods.** (a) Mixture-of-Agents, (b) LLM-Debate, (c) Multi-Agent Verification, (d) SLM-MUX (Ours).

Optimization for Multi-LM Orchestration. Given these orchestration methods, some works study how to further improve their performance—e.g., how to select models to include, how to optimize prompts, or how to adapt the architecture for specific tasks (Chen et al., 2023a; Ong et al., 2025; Chen et al., 2024). Prompt and workflow optimization methods (Khattab et al., 2023; Opsahl-Ong et al., 2024; Saad-Falcon et al., 2025; Zhang et al., 2025a) generally assume strong instruction-following ability, which makes them less effective for smaller models with limited such capabilities..

Another line of work is model selection for orchestration (Chen et al., 2025; Poon et al., 2025). These methods often select models based on accuracy, assuming that combining models with higher standalone accuracy will yield stronger orchestrations. However, most selection criteria are not end-to-end: they evaluate models independently without directly assessing the performance of the overall orchestration. This overlooks how models interact with each other—overconfident but incorrect predictions from one model can dominate and suppress correct predictions from others, meaning that the best standalone models may not yield the best orchestration.

Test-time Scaling Strategies. Test-time scaling methods improve performance by using additional computation during inference without retraining (Snell et al., 2024; Muennighoff et al., 2025; Zhang et al., 2025b). A common single-model approach is self-consistency (Trad & Chehab, 2025; Thirukovalluru et al., 2024; Chow et al., 2024), which draws multiple samples from one model and selects the majority answer; accuracy typically improves as the number of samples increases. Agent Forest (Li et al., 2024) extends this idea to multiple models by collecting one output from each model and applying majority voting across all answers.

3 Methods

In this work, we set out to ask two critical questions: given a pool of available SLMs, how can we (i) orchestrate their outputs to achieve the best overall performance, and (ii) select an effective subset of models that maximizes accuracy?

To answer question (i), we present the SLM-MUX (Section 3.1), a simple yet effective orchestration method. To answer question (ii), we propose model selection search (Section 3.2) that identifies complementary subsets from dozens of available SLMs. Finally, we explore compute scaling strategies (Section 3.3) to further enhance the reasoning accuracy during inference.

3.1 SLM-MUX for Orchestrating Multiple Small Language Models

At a high level, our intuition is that we do not need to let SLMs discuss with each other. Instead, we can develop a simple rule-based method that estimates the confidence of each model's answer and then selects the final output from the model with the highest confidence. We term our method **SLM-MUX**, which operates in two phases.

Independent Generation Phase. For a given question, we first let each SLM independently generate multiple candidate responses to the same query prompt with temperature > 0, producing a pool of sampled answers per model.

Confidence Estimation Phase. We evaluate the confidence of each SLM's outputs by measuring their consistency across their own outputs. Intuitively, a model that places higher probability mass on the correct answer will reproduce equivalent answer across samples, whereas an uncertain model will produce varied outputs. For instance, if SLM A produces three equivalent answers while model B produces three different ones, the answers from model A are more consistent and should be selected. This correlation between consistency and correctness is observed by previous papers. (Wang et al.,

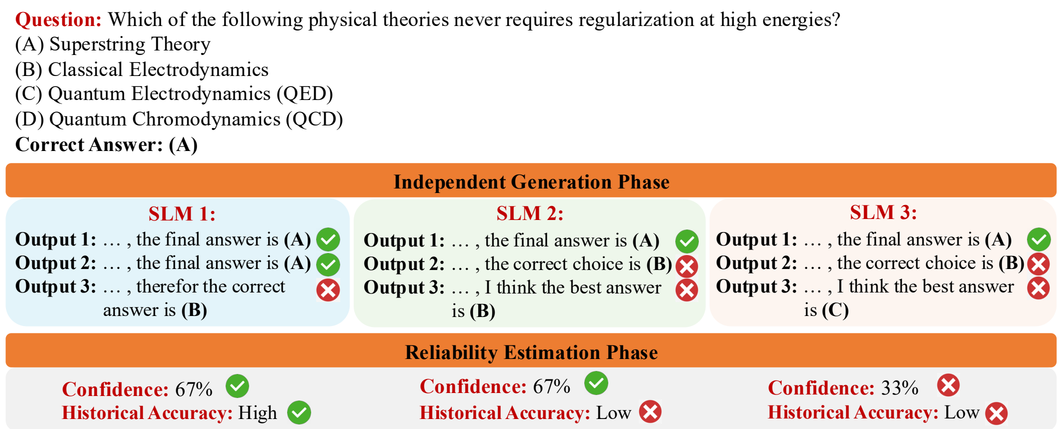


Figure 3: **Illustration of SLM-MUX Workflow.** (1) Each SLM first independently generates multiple outputs for the same question. (2) The most frequent answer from each SLM is selected, and its frequency in the answer pool is used as the confidence score. (3) The answers with the highest confidence score are selected. (4) If multiple answers share the same confidence score, the tie is broken by selecting the answer from the SLM with the highest accuracy on the validation set.

Algorithm 1 SLM-MUX Working Flow

Input: Models $M_1, \dots, M_n$, query x, samples per model k, validation accuracies $a_1, \dots, a_n$
Output: Final answer $\hat{y}$

Independent Generation: each model produces multiple candidate answers independently
for $i = 1, \dots, n$ **do**
  Sample k answers $Y_i = \{y_i^{(1)}, \dots, y_i^{(k)}\}$ from M_i
  Compute $f_i(y) = \frac{1}{k} \sum_{j=1}^{k} \mathbf{1}\left(y_i^{(j)} = y\right)$
  Let $y_i^* = \arg\max_y f_i(y)$ and set $s_i = f_i(y_i^*)$
Confidence Estimation: measure confidence and break ties by validation accuracy
$S_{\max} = \max_i s_i, \quad I^* = \{\, i \mid s_i = S_{\max} \,\}$
if $|I^*| = 1$ **then**
  $i^* \leftarrow$ the unique index in I^*
else
  $i^* \leftarrow \arg\max_{i \in I^*} a_i$
return $\hat{y} = y_{i^*}^*$

2023; Xie et al., 2024; Taubenfeld et al., 2025; Chen et al., 2023b), and we empirically revalidate this observation in Appendix D.1.

In cases where two SLMs are equally consistent but disagree, we use their validation accuracy as a tie-breaker. Prior work has shown that consistency is strongly correlated with correctness, which provides a rationale for this design.

For more details, Algorithm 1 summarizes the workflow step by step. Figure 3 provides a visual example of the workflow. The evaluation of SLM-MUX is presented in Section 4.2.

3.2 Model Selection Search for SLM-MUX Optimization

At a high level, the idea of model selection search is to combine models with complementary skills. The goal is not simply to add more models, but to bring new capabilities as we add models. Figure 4 illustrates this intuition: Qwen2.5-7B consistently outperforms Llama3.2-3B across all subjects, so combining them offers no capability beyond what Qwen2.5-7B already provides. In contrast, Mistral Small 24B and Qwen2.5-7B excel in different subjects, making their combination more effective than either model individually.

We frame model selection as a search problem on the validation set with two competing objectives. Our first objective is **Union Accuracy**, which reflects the overall accuracy of the system. The higher the union accuracy is, the more questions a system can potentially answer. Formally, let $\mathcal{M} = \{m_1, \dots, m_K\}$ denote the set of candidate models and $\mathcal{D}$ the validation set. For each model

$m_i \in \mathcal{M}$, we record the subset of validation instances it solves correctly. Given a candidate subset $S \subseteq \mathcal{M}$, the union accuracy is defined as

$$\text{UnionAcc}(S) = \frac{1}{|\mathcal{D}|} \sum_{x \in \mathcal{D}} \mathbf{1}\{\exists\, m \in S \; : \; m(x) \text{ is correct}\}$$

The second objective is the **Contradiction Penalty**. It captures problematic cases where overconfident wrong answers suppress correct predictions from other models. Consider two SLMs answering the same multiple-choice question three times: the first model consistently outputs "A" (correct), while the second consistently outputs "B" (incorrect but confident). Since SLM-MUX selects based on consistency, both models would appear equally confident, making it impossible to distinguish the correct answer from the confident but wrong one. We define this penalty as the percentage of questions where at least one model consistently gives the wrong answer while another provides the correct answer:

$$\text{Contradiction}(S) = \frac{1}{|\mathcal{D}|} \sum_{x \in \mathcal{D}} \mathbf{1}\left\{ \begin{array}{l} \exists\, m_1 \in S : \; m_1(x) \text{ consistently wrong}, \\ \exists\, m_2 \in S : \; m_2(x) \text{ correct} \end{array} \right\}$$

Here, a model is "consistently wrong" if it produces the same incorrect answer across all sampled generations for that question. The final objective balances these competing factors:

$$\mathcal{O}(S) \;=\; \mathit{UnionAcc}(S) \;-\; \lambda \cdot \mathit{Contradiction}(S),$$

Where λ is a hyperparameter. Since the number of candidate models is not very large, we perform an exhaustive search. We present visualization of the two search objectives and evaluation of the searched model selection in Section 4.3.

The rationale behind this search objective is as follows: UnionAcc represents an optimistic upper bound for SLM-MUX performance. It assumes an ideal selection mechanism capable of identifying the correct answer whenever at least one model provides it, which is unrealistic in practice. Conversely, when $\lambda = 1$, the search objective represents a pessimistic lower bound of SLM-MUX accuracy. This setting assumes that in cases involving confidently wrong answers, the system will invariably select the incorrect one. In practice, due to factors such as tie-breaking rules and the presence of confidently correct answers, such a worst-case scenario will not always happen. Consequently, by employing the objective $\mathcal{O}(S) = \text{UnionAcc}(S) - \lambda \cdot \text{Contradiction}(S)$, we estimate an approximate accuracy between the theoretical upper and lower bounds of the SLM-MUX accuracy.

3.3 Compute Scaling Strategies

Next, we empirically investigate two dimensions of test-time scaling to further enhance the performance of our SLM-MUX with selected models.

Adding More Participating Model Types: As we increase the number of participating model types in the system by adding more SLMs with complementary strengths, we expect the overall accuracy to improve. For each budgeted number of models, we use the search method proposed in Section 3.2 to identify the best selection from the pool.

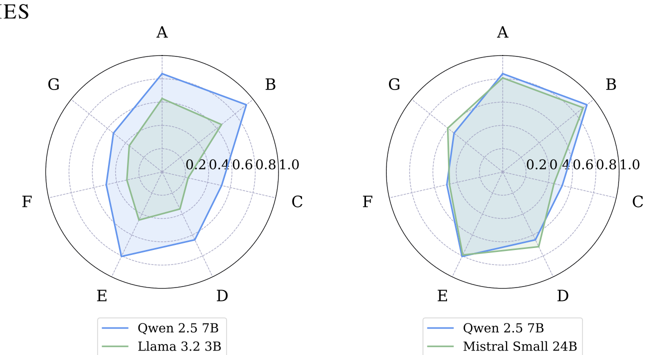


Figure 4: **Comparison of Model Choices**. Accuracy on 7 subjects for two model selection settings on MATH dataset. Subjects are denoted as: A = Prealgebra, B = Algebra, C = Intermediate Algebra, D = Number Theory, E = Counting & Probability, F = Geometry, G = Precalculus.

Drawing More Samples per Model: For a fixed model selection, we can increase the compute budget by scaling the number of samples drawn by each model. Since confidence is evaluated by counting the frequency of majority answers, adding more samples per model is expected to provide a more accurate confidence estimate.

These two compute scaling dimensions are evaluated in Section 4.4.

4 Experiments

In our experiments, we first demonstrate the fundamental limitations of existing discussion-based orchestration methods when applied to SLMs (Section 4.1). We then evaluate the proposed SLM-

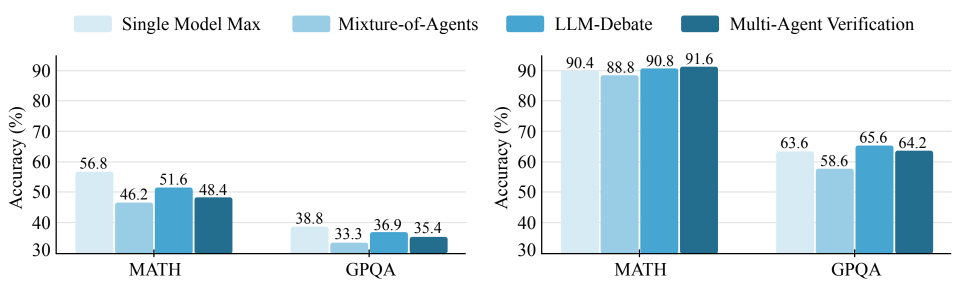
MUX in Section 4.2. In Section 4.3, we access our proposed search strategy. Finally, in Section 4.4, we examine the compute scaling strategies.

4.1 Existing Discussion-Based Orchestration Methods Harm SLM Performance

To understand whether orchestration methods developed for frontier LLMs are suitable for SLMs, we conduct a systematic comparison across model scales. We evaluate three prominent discussion-based methods—LLM-Debate (Du et al., 2023), Mixture-of-Agents (Wang et al., 2024b), and Multi-Agent Verification (Lifshitz et al., 2025) —using identical experimental settings on both SLMs (Llama 3.1 8B (Grattafiori et al., 2024), Mixtral 8×7B (Jiang et al., 2024), Gemma 2 27B) and frontier LLMs (DeepSeek V3 (DeepSeek-AI et al., 2025), Gemini 2.0 Flash (Google Cloud, 2025), GPT-4o (OpenAI et al., 2024)). Evaluation is conducted on MATH and GPQA datasets using original code and prompts.

Results. As shown in Figure 5, discussion-based methods generally outperform the single best-performing models in the frontier LLM group, achieving up to a 2% increase in accuracy. However, when applied to SLMs, these discussion-based methods fail to outperform the best single model in the orchestration, and even incur accuracy drops of up to 5.5%. This performance gap is observed across all three methods and both benchmarks.

To understand this counterintuitive result, we analyze SLM behavior in discussion settings. We find that discussion-based methods amplify rather than correct errors in SLMs due to a key limitation: SLMs tend to exhibit groupthink, reinforcing incorrect reasoning during discussions rather than correcting mistakes. In Appendix C, we provide detailed analysis showing that 59.5% of failures are attributed to groupthink, and that the performance gap persists even after extensive prompt optimization.



(a) SLM Orchestration (b) Large-Scale Model Composition

Figure 5: **Comparison of discussion-based orchestration when invoking SLMs and LLMs.** We compare three orchestration methods (Mixture-of-Agents, LLM-Debate, and Verification) using (a) SLMs (Llama 3.1 8B, Mixtral 8×7B, Gemma 2 27B) and (b) frontier LLMs (DeepSeek V3, Gemini 2.0 Flash, GPT-4o) on the MATH and GPQA datasets. The baseline (*Single-Model Max*) reflects the best performance of individual models. An orchestration is considered successful if it surpasses Single-Model Max. All discussion-based methods are evaluated with temperature=0. The standard deviations of the accuracies are presented in Appendix B.3.

4.2 SLM-MUX Achieves SLM Orchestration Where Existing Methods Fail

To evaluate whether our proposed SLM-MUX can successfully orchestrate SLMs, we test it against the same baselines from Section 4.1. We use Mixtral 8×7B, LLaMA 3.1 8B, and Gemma 2 27B (Team et al., 2024) as base models. We implement the SLM-MUX as follows. First, we generate three rounds of answers with a temperature of 0.3. Next, we compute a confidence score by counting how often the most common answer appears across these rounds. The final answer for each model is chosen as the most frequent one; in the case of a tie, we select the answer from the model with the highest validation accuracy.

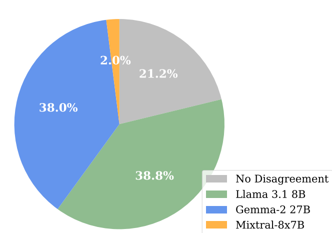


Figure 6: **Final Output Attribution.** We report the percentage of outputs contributed by each model on the MATH dataset for our SLM-MUX. These results are from the same run as in Table 1.

We evaluate three types of baselines. First, we measure the accuracies of individual models and report the best-performing ones. Second, we apply self-consistency to each of the three base models independently, reporting the best-performing result as the *Single-Best-SC* baseline. Next, for comparison with existing discussion-based methods, we include LLM-Debate (Du et al., 2023), Mixture-of-Agents (Wang et al., 2024b), and Multi-Agent Verification (Lifshitz et al., 2025). We follow the original code and prompts described in their papers. Experiments are conducted on three benchmark datasets: MATH (Hendrycks et al., 2021), GPQA (Rein et al., 2023), and GSM8K (Cobbe et al., 2021).

Results. Table 1 summarizes the results. In our experiments, we find that for SLMs, existing orchestration methods do not consistently outperform the strongest individual base models or self-consistency approaches. In contrast, our SLM-MUX yields consistent gains on MATH and GSM8K, and is comparable to Single-Best-SC on GPQA. Compared with other approaches, our method yields up to a 13.4% improvement on MATH, up to 8.8% on GPQA, and up to 7.0% on GSM8K. These results demonstrate that the SLM-MUX itself provides a clear advantage over alternative orchestration approaches at the architectural level.

To better illustrate our proposed SLM-MUX, we plot the output attribution for the MATH experiment (Table 1) in Figure 6. By selecting diverse outputs from the generation, SLM-MUX leverages the complementary strengths of different SLMs.

Method	MATH Acc (%)	GPQA Acc (%)	GSM8K Acc (%)
Mixture-of-Agents	51.4 ± 2.2	33.3 ± 3.4	81.6 ± 1.7
LLM-Debate	51.6 ± 2.2	36.8 ± 3.4	80.8 ± 1.8
Multi-Agent Verification	48.4 ± 2.2	35.3 ± 3.4	86.4 ± 1.5
SLM-MUX (Ours)	$\mathbf{61.8 \pm 1.2}$	42.1 ± 0.3	$\mathbf{87.8 \pm 0.6}$
Single-Best	56.8 ± 2.2	38.9 ± 3.5	84.2 ± 1.6
Single-Best-SC	58.0 ± 2.2	$\mathbf{42.4 \pm 3.5}$	86.8 ± 1.5

Table 1: **Quantitative Results.** Accuracy and standard deviation across MATH, GPQA, and GSM8K. "SC" denotes self-consistency decoding (majority vote over samples from a single model), and "Single-Best-SC" reports the highest accuracy among the three base models when each applies self-consistency individually.

4.3 Model Selection Search Boosts SLM-MUX Performance

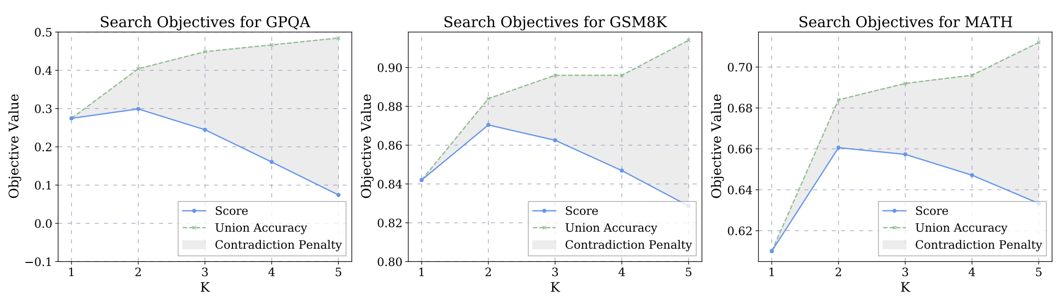


Figure 7: **Union Accuracy and Contradiction Penalty both Increases as more models are added**. We plot the search objectives as the number of models (K) increases from 2 to 5 across three benchmarks. The green line denotes the union accuracy across models, the grey area indicates the contradiction penalty, and the blue line represents the overall search objective score. For each value of K, the plotted quantities are computed for the single model combination that maximizes our model selection objective defined in Section 3.2.

To examine whether model selection search benefits SLM-MUX, we construct a validation set of 500 questions sampled from the training splits of MATH, GPQA, and GSM8K. The candidate pool consists of five SLMs: Gemma 2 27B, Llama 3.1 8B, Mistral Small 24B (Mistral AI, 2025), Mixtral 8×7B, and Qwen2.5-7B (Qwen et al., 2025). For each question, we collect three independent generations per model with temperature 0.5, repeating this process three times to obtain stable accuracy estimates. The search procedure considers orchestrations with $K = 2$ to 5 models and is guided by an objective function mentioned in Section 3, with hyperparameter $\lambda = 1$. The behavior of this objective is illustrated in Figure 7, showing the trade-off as K increases. For simplicity, we select two representative two-model combinations from the search results for evaluation on the test set.

Results. Table 2 summarizes the outcome of the search. The table lists the top-performing two-model combinations identified on the validation set, along with their evaluation on the held-out test set. Across benchmarks, these optimized orchestrations yield consistent improvements over the strongest individual models: accuracy increases by 4.5% on MATH, 4.4% on GPQA, and 4.3% on GSM8K. This contrasts with Section 4.2, where naive three-model combinations provide little to no benefit on GPQA. Figure 7 further illustrates the underlying trade-off: while union accuracy rises with additional models, the contradiction penalty also grows, emphasizing that effective orchestration

Benchmark	Group	Model Selection	Best Single Acc (%)	Composed Acc (%)	Δ (Gain)
MATH	1	Mistral Small 24B Qwen2.5-7B	75.5 ± 1.5	80.0 ± 0.7	+4.5
	2	Qwen2.5-7B Llama 3.1 8B	75.5 ± 1.5	77.7 ± 0.7	+2.2
GPQA	1	Gemma 2 27B Mistral Small 24B	45.1 ± 2.8	49.5 ± 1.8	+4.4
	2	Llama 3.1 8B Mistral Small 24B	45.1 ± 2.8	48.8 ± 0.8	+3.6
GSM8K	1	Mistral Small 24B Qwen2.5-7B	88.5 ± 0.7	92.8 ± 0.6	+4.3
	2	Llama 3.1 8B Mixtral 8×7B	80.8 ± 2.1	85.2 ± 0.7	+4.4

Table 2: **Model Selection Search and Evaluation Results.** We show the top two model groups identified by our search for each benchmark. For each group, we report the accuracy of the best-performing single model within the orchestration, the accuracy achieved by our SLM-MUX, and the resulting performance gain.

requires balancing these competing factors rather than simply enlarging the orchestration size. In Appendix D.3, we show that the SLM-MUX architecture itself yields consistent gains even with randomly selected model combinations; the search procedure provides an effective and data-efficient way to further boost accuracy.

4.4 Compute Scaling Strategies Reveal Optimal Resource Allocation

To evaluate the “Adding More Participating Model Types" dimension of compute scaling, we assess how performance changes as the number of models in the orchestration increases. For each number of models from 2 to 5, we first apply the search method from Section 3.2 to identify the optimal model selection from our pool. We then evaluate SLM-MUX with selected models on the validation set. Figure 9 plots the resulting mean accuracy (blue line, left y-axis) for each value of K. To illustrate the theoretical performance ceiling of each ensemble, we also plot the union accuracy (grey line, right y-axis), defined as the percentage of questions solved by at least one model in the group. For each value of K in Figure 9, we show the single model combination that achieves the highest value of our model selection objective from Section 3.2; the search procedure is used to find the best combination under a fixed K, rather than to choose K itself.

Benchmark	Samples	SLM-MUX	Agent Forest	Δ (Gain)
MATH	2 Best	76.8 ± 0.7 79.5 ± 0.4	72.3 ± 1.5 79.2 ± 0.4	+4.5 +0.3
GPQA	2 Best	46.3 ± 2.3 48.8 ± 1.2	40.4 ± 2.3 47.6 ± 1.4	+5.9 +1.2
GSM8K	2 Best	82.1 ± 0.7 86.5 ± 0.8	77.7 ± 0.2 84.3 ± 0.8	+4.4 +2.2

Table 3: **Comparison of SLM-MUX and Agent Forest.** We compare SLM-MUX and Agent Forest in two settings: **(1)** with 2 samples per model (Samples=2), and **(2)** using the best accuracy found during scaling for each method (Samples=best). In the second setting, the number of samples per model may vary.

For the “Drawing More Samples per Model” dimension, we reuse the two groups of models listed in Table 2. We vary the number of samples per model from 2 to 9 and report the mean accuracy of SLM-MUX over three runs for each sample budget. The results are presented in Figure 8, along with a baseline, Agent Forest (Li et al., 2024), for comparison. To ensure fairness, Agent Forest is reproduced using the same models from Group 2. We report the best accuracy achieved by the SLM-MUX when scaling with Samples per Model and compare it to the accuracy of the single best model in the orchestration, as shown in Table 2.

Results. The effect of “Adding More Participating Model Types” varies substantially across benchmarks. On GPQA, accuracy peaks when combining two models and declines thereafter. On GSM8K, accuracy quickly saturates at two models without further gains. In contrast, on MATH, accuracy continues to improve as additional models are included. Despite these differences, the union accuracy of model orchestration consistently increases with more models, emphasizing the role of output

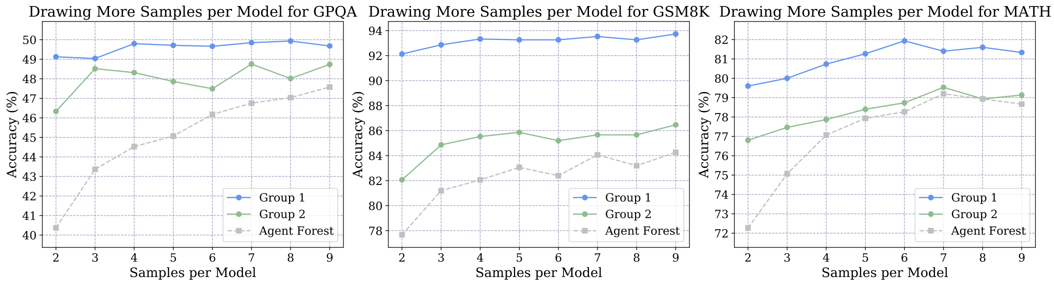


Figure 8: **Drawing More Samples per Model Improves Accuracy**. We report mean accuracy of SLM-MUX as the number of samples per model increases from 2 to 9 across three benchmarks. Group 1 and Group 2 are from Table 2. We also plot the mean accuracy of Agent Forest (Li et al., 2024) in grey line.

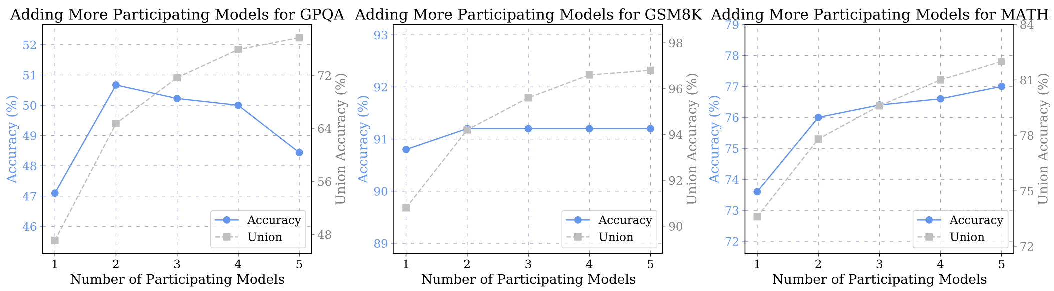


Figure 9: **Adding More Participating Models Affects Accuracy Differently**. We report the mean accuracy (blue line) of the optimal SLM-MUX obtained when using 2 to 5 models across three benchmarks. We also report the union accuracy (grey line), defined in Section 3.2. The blue line (Mean Accuracy) is plotted against the left-hand Y-axis. The grey line (Union Accuracy) is plotted against the right-hand Y-axis. For each K, both curves correspond to the single model combination that maximizes our model selection objective (Section 3.2) under that fixed K.

contradictions among models, as elaborated in Section 3.2. We also validate this scaling behavior on the test set; see Appendix D.4 for details.

"Drawing More Samples per Model" yields more consistent improvements across benchmarks. Moreover, under this setting, our SLM-MUX systematically outperforms Agent Forest, with the largest margin observed on GPQA, where single-model accuracy is lowest.

Benchmark	**Group 1**		**Group 2**		**Qwen2.5-72B Acc (%)**
	Acc (%)	Δ (Gain)	Acc (%)	Δ (Gain)	
MATH	81.9 ± 0.2	+6.4	79.5 ± 0.4	+4.0	82.3 ± 0.5
GPQA	49.9 ± 1.8	+4.8	48.7 ± 1.2	+3.6	44.9 ± 0.5
GSM8K	93.7 ± 0.2	+5.2	86.5 ± 0.8	+5.7	90.4 ± 0.3

Table 4: **Best Accuracy after Sample Scaling beats Larger Model.** Acc indicates the highest accuracy achieved through scaling. Groups 1 & 2 are defined in Table 2. Gain represents the improvement over the best single-model accuracy reported in Table 2. For reference, we also include the performance of the large model Qwen2.5-72B, showing that our composed small models can outperform it on GPQA and GSM8K.

5 Discussion

Mathematical Intuition behind SLM-MUX. Different SLMs have complementary strengths: for any given question, some models are more likely to answer correctly than others. SLM-MUX exploits this by selecting the most self-consistent model's output through a simple rule-based mechanism that requires no inter-model communication.

The key insight is that the confidence score can identify the strongest model. We assume that for each question, there is a unique correct answer, while incorrect answers are scattered rather than clustered. Under this assumption, a model with higher accuracy p_i produces the correct answer more frequently across N samples, leading to a higher confidence score. Therefore, selecting the model with the highest confidence score effectively identifies the model most likely to be correct.

More formally, consider K models where model i has probability p_i of being correct. Let $i^* = \arg\max_i p_i$ denote the strongest model with margin $\gamma = p_{i^*} - \max_{j \neq i^*} p_j > 0$. Under our assumption, the confidence score s_i (the frequency of the most common answer over N samples) concentrates around p_i. Applying Hoeffding's inequality and a union bound, the probability of correctly selecting the strongest model satisfies:

$$\Pr(\hat{i} = i^*) \geq 1 - 2(K-1)\exp\left(-\frac{N\gamma^2}{2}\right).$$

This bound shows that the probability of misidentifying the strongest model decays exponentially with sample size N.

This selection mechanism explains why SLM-MUX outperforms alternatives. Unlike a single fixed model, SLM-MUX performs per-question routing, effectively achieving accuracy $p_{\max}$ by always selecting the strongest available expert. Unlike pooling methods such as Agent Forest that aggregate outputs from all models, SLM-MUX avoids interference from weaker models. For instance, if the strongest model has $p_1 = 0.8$ and a weaker one has $p_2 = 0.3$, pooling their outputs merely dilutes the correct answer's frequency. By isolating the strongest model and selecting its most frequent answer, SLM-MUX preserves the full predictive power of the most reliable source. We provide a more detailed comparative analysis with self-consistency and Agent Forest in Appendix D.2.

Extending SLM-MUX to Open-Ended Generation.. Although the current implementation of SLM-MUX relies on majority voting and is therefore restricted to tasks with discrete answer spaces, the underlying idea of selecting the most self-consistent model is more general. For open-ended generation, one can replace majority voting with alternative consistency estimators, such as LLM-as-a-judge scoring or embedding-based similarity measures. In Appendix E.1, we show a simple extension of SLM-MUX to HumanEval (Chen et al., 2021) using this idea and observe strong empirical gains.

Extending SLM-MUX Beyond Generalist SLMs.. The experiments above focus on general-purpose SLMs. We further evaluate whether the consistency-based selection principle extends to other settings: (1) frontier LLMs such as GPT-4o and Gemini-2.5-Flash, and (2) domain-specific fine-tuned models such as code and math specialists. In both cases, SLM-MUX achieves consistent improvements over the best single model. Full experimental details are provided in Appendix E.

Limitation and Future Work. The SLM-MUX framework has two main limitations. First, its design is static and does not adapt to specific questions. For every query, it uses a fixed group of models that are pre-selected through exhaustive search – a method that is slow and costly when there are many models to choose from. When models are tied, the framework uses their past accuracy on a validation set to decide, which is also a fixed, non-adaptive rule. Second, the way the framework measures model confidence is simple. It relies only on self-consistency – how often a model produces the same answer. This can be a problem because a model can be very consistent while still being incorrect.

Conclusion. This work demonstrates that orchestration methods designed for frontier models paradoxically degrade the performance of SLMs by amplifying errors. To address this, we propose SLM-MUX, a framework that avoids inter-model discussion, instead selecting the most reliable output based on each model's self-consistency. We further introduce a model selection search algorithm to find complementary model combinations. Experiments show our method not only substantially outperforms existing strategies but also enables an ensemble of just two SLMs to surpass the much larger Qwen2.5-72B model on key reasoning benchmarks. In summary, our work validates that intelligently orchestrating multiple efficient models—a "multi-core" approach—is a promising alternative to scaling monolithic models on the path toward more capable AI systems.

References

Loubna Ben Allal, Anton Lozhkov, Elie Bakouch, Gabriel Martín Blázquez, Guilherme Penedo, Lewis Tunstall, Andrés Marafioti, Hynek Kydlíček, Agustín Piqueres Lajarín, Vaibhav Srivastav, et al. Smollm2: When smol goes big–data-centric training of a small language model. *arXiv preprint arXiv:2502.02737*, 2025. 1

Peter Belcak, Greg Heinrich, Shizhe Diao, Yonggan Fu, Xin Dong, Saurav Muralidharan, Yingyan Celine Lin, and Pavlo Molchanov. Small language models are the future of agentic ai. *arXiv preprint arXiv:2506.02153*, 2025. 1

Lingjiao Chen, Matei Zaharia, and James Zou. Frugalgpt: How to use large language models while reducing cost and improving performance, 2023a. URL https://arxiv.org/abs/2305.05176. 3

Lingjiao Chen, Jared Quincy Davis, Boris Hanin, Peter Bailis, Matei Zaharia, James Zou, and Ion Stoica. Optimizing model selection for compound ai systems, 2025. URL https://arxiv.org/abs/2502.14815. 3

Mark Chen, Jerry Tworek, Heewoo Jun, Qiming Yuan, Henrique Ponde de Oliveira Pinto, Jared Kaplan, Harri Edwards, Yuri Burda, Nicholas Joseph, Greg Brockman, Alex Ray, Raul Puri, Gretchen Krueger, Michael Petrov, Heidy Khlaaf, Girish Sastry, Pamela Mishkin, Brooke Chan, Scott Gray, Nick Ryder, Mikhail Pavlov, Alethea Power, Lukasz Kaiser, Mohammad Bavarian, Clemens Winter, Philippe Tillet, Felipe Petroski Such, Dave Cummings, Matthias Plappert, Fotios Chantzis, Elizabeth Barnes, Ariel Herbert-Voss, William Hebgen Guss, Alex Nichol, Alex Paino, Nikolas Tezak, Jie Tang, Igor Babuschkin, Suchir Balaji, Shantanu Jain, William Saunders, Christopher Hesse, Andrew N. Carr, Jan Leike, Josh Achiam, Vedant Misra, Evan Morikawa, Alec Radford, Matthew Knight, Miles Brundage, Mira Murati, Katie Mayer, Peter Welinder, Bob McGrew, Dario Amodei, Sam McCandlish, Ilya Sutskever, and Wojciech Zaremba. Evaluating large language models trained on code, 2021. URL https://arxiv.org/abs/2107.03374. 10

Shuhao Chen, Weisen Jiang, Baijiong Lin, James T. Kwok, and Yu Zhang. Routerdc: Query-based router by dual contrastive learning for assembling large language models, 2024. URL https://arxiv.org/abs/2409.19886. 3

Xinyun Chen, Renat Aksitov, Uri Alon, Jie Ren, Kefan Xiao, Pengcheng Yin, Sushant Prakash, Charles Sutton, Xuezhi Wang, and Denny Zhou. Universal self-consistency for large language model generation, 2023b. URL https://arxiv.org/abs/2311.17311. 4

Yinlam Chow, Guy Tennenholtz, Izzeddin Gur, Vincent Zhuang, Bo Dai, Sridhar Thiagarajan, Craig Boutilier, Rishabh Agarwal, Aviral Kumar, and Aleksandra Faust. Inference-aware fine-tuning for best-of-n sampling in large language models, 2024. URL https://arxiv.org/abs/2412.15287. 3

Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. Training verifiers to solve math word problems, 2021. URL https://arxiv.org/abs/2110.14168. 7

DeepSeek-AI, Aixin Liu, Bei Feng, Bing Xue, Bingxuan Wang, Bochao Wu, Chengda Lu, Chenggang Zhao, Chengqi Deng, Chenyu Zhang, Chong Ruan, Damai Dai, Daya Guo, Dejian Yang, Deli Chen, Dongjie Ji, Erhang Li, Fangyun Lin, Fucong Dai, Fuli Luo, Guangbo Hao, Guanting Chen, Guowei Li, H. Zhang, Han Bao, Hanwei Xu, Haocheng Wang, Haowei Zhang, Honghui Ding, Huajian Xin, Huazuo Gao, Hui Li, Hui Qu, J. L. Cai, Jian Liang, Jianzhong Guo, Jiaqi Ni, Jiashi Li, Jiawei Wang, Jin Chen, Jingchang Chen, Jingyang Yuan, Junjie Qiu, Junlong Li, Junxiao Song, Kai Dong, Kai Hu, Kaige Gao, Kang Guan, Kexin Huang, Kuai Yu, Lean Wang, Lecong Zhang, Lei Xu, Leyi Xia, Liang Zhao, Litong Wang, Liyue Zhang, Meng Li, Miaojun Wang, Mingchuan Zhang, Minghua Zhang, Minghui Tang, Mingming Li, Ning Tian, Panpan Huang, Peiyi Wang, Peng Zhang, Qiancheng Wang, Qihao Zhu, Qinyu Chen, Qiushi Du, R. J. Chen, R. L. Jin, Ruiqi Ge, Ruisong Zhang, Ruizhe Pan, Runji Wang, Runxin Xu, Ruoyu Zhang, Ruyi Chen, S. S. Li, Shanghao Lu, Shangyan Zhou, Shanhuang Chen, Shaoqing Wu, Shengfeng Ye, Shengfeng Ye, Shirong Ma, Shiyu Wang, Shuang Zhou, Shuiping Yu, Shunfeng Zhou, Shuting Pan, T. Wang, Tao Yun, Tian Pei, Tianyu Sun, W. L. Xiao, Wangding Zeng, Wanjia Zhao, Wei An, Wen Liu, Wenfeng Liang, Wenjun Gao, Wenqin Yu, Wentao Zhang, X. Q. Li, Xiangyue Jin, Xianzu Wang, Xiao Bi, Xiaodong Liu, Xiaohan Wang, Xiaojin Shen, Xiaokang Chen, Xiaokang Zhang, Xiaosha Chen, Xiaotao Nie, Xiaowen Sun, Xiaoxiang Wang, Xin Cheng, Xin Liu, Xin Xie, Xingchao Liu, Xingkai Yu, Xinnan Song, Xinxia Shan, Xinyi Zhou, Xinyu Yang, Xinyuan Li, Xuecheng Su, Xuheng Lin, Y. K. Li, Y. Q. Wang, Y. X. Wei, Y. X. Zhu, Yang Zhang, Yanhong Xu, Yanhong Xu, Yanping Huang, Yao Li, Yao Zhao, Yaofeng Sun, Yaohui Li, Yaohui Wang, Yi Yu, Yi Zheng, Yichao Zhang, Yifan Shi, Yiliang Xiong, Ying He, Ying Tang, Yishi Piao, Yisong Wang, Yixuan Tan, Yiyang Ma, Yiyuan Liu, Yongqiang Guo, Yu Wu, Yuan Ou, Yuchen Zhu, Yuduan Wang, Yue Gong, Yuheng Zou, Yujia He, Yukun Zha, Yunfan Xiong, Yunxian Ma, Yuting Yan, Yuxiang Luo, Yuxiang You, Yuxuan Liu, Yuyang Zhou, Z. F. Wu, Z. Z. Ren, Zehui Ren, Zhangli Sha, Zhe Fu, Zhean Xu, Zhen Huang, Zhen Zhang, Zhenda Xie, Zhengyan Zhang, Zhewen Hao, Zhibin Gou, Zhicheng Ma, Zhigang Yan, Zhihong Shao, Zhipeng Xu, Zhiyu Wu, Zhongyu Zhang, Zhuoshu

Li, Zihui Gu, Zijia Zhu, Zijun Liu, Zilin Li, Ziwei Xie, Ziyang Song, Ziyi Gao, and Zizheng Pan. Deepseek-v3 technical report, 2025. URL https://arxiv.org/abs/2412.19437. 6

Yilun Du, Shuang Li, Antonio Torralba, Joshua B. Tenenbaum, and Igor Mordatch. Improving factuality and reasoning in language models through multiagent debate, 2023. URL https://arxiv.org/abs/2305.14325. 1, 2, 6, 7

Tianyu Fu, Zihan Min, Hanling Zhang, Jichao Yan, Guohao Dai, Wanli Ouyang, and Yu Wang. Cache-to-cache: Direct semantic communication between large language models. *arXiv preprint arXiv:2510.03215*, 2025. 1, 2

Google Cloud. Gemini 2.0 flash | generative ai on vertex ai. https://cloud.google.com/vertex-ai/generative-ai/docs/models/gemini/2-0-flash, 2025. Accessed: 2025-09-24. 6

Aaron Grattafiori, Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Alex Vaughan, Amy Yang, Angela Fan, Anirudh Goyal, Anthony Hartshorn, Aobo Yang, Archi Mitra, Archie Sravankumar, Artem Korenev, Arthur Hinsvark, Arun Rao, Aston Zhang, Aurelien Rodriguez, Austen Gregerson, Ava Spataru, Baptiste Roziere, Bethany Biron, Binh Tang, Bobbie Chern, Charlotte Caucheteux, Chaya Nayak, Chloe Bi, Chris Marra, Chris McConnell, Christian Keller, Christophe Touret, Chunyang Wu, Corinne Wong, Cristian Canton Ferrer, Cyrus Nikolaidis, Damien Allonsius, Daniel Song, Danielle Pintz, Danny Livshits, Danny Wyatt, David Esiobu, Dhruv Choudhary, Dhruv Mahajan, Diego Garcia-Olano, Diego Perino, Dieuwke Hupkes, Egor Lakomkin, Ehab AlBadawy, Elina Lobanova, Emily Dinan, Eric Michael Smith, Filip Radenovic, Francisco Guzmán, Frank Zhang, Gabriel Synnaeve, Gabrielle Lee, Georgia Lewis Anderson, Govind Thattai, Graeme Nail, Gregoire Mialon, Guan Pang, Guillem Cucurell, Hailey Nguyen, Hannah Korevaar, Hu Xu, Hugo Touvron, Iliyan Zarov, Imanol Arrieta Ibarra, Isabel Kloumann, Ishan Misra, Ivan Evtimov, Jack Zhang, Jade Copet, Jaewon Lee, Jan Geffert, Jana Vranes, Jason Park, Jay Mahadeokar, Jeet Shah, Jelmer van der Linde, Jennifer Billock, Jenny Hong, Jenya Lee, Jeremy Fu, Jianfeng Chi, Jianyu Huang, Jiawen Liu, Jie Wang, Jiecao Yu, Joanna Bitton, Joe Spisak, Jongsoo Park, Joseph Rocca, Joshua Johnstun, Joshua Saxe, Junteng Jia, Kalyan Vasuden Alwala, Karthik Prasad, Kartikeya Upasani, Kate Plawiak, Ke Li, Kenneth Heafield, Kevin Stone, Khalid El-Arini, Krithika Iyer, Kshitiz Malik, Kuenley Chiu, Kunal Bhalla, Kushal Lakhotia, Lauren Rantala-Yeary, Laurens van der Maaten, Lawrence Chen, Liang Tan, Liz Jenkins, Louis Martin, Lovish Madaan, Lubo Malo, Lukas Blecher, Lukas Landzaat, Luke de Oliveira, Madeline Muzzi, Mahesh Pasupuleti, Mannat Singh, Manohar Paluri, Marcin Kardas, Maria Tsimpoukelli, Mathew Oldham, Mathieu Rita, Maya Pavlova, Melanie Kambadur, Mike Lewis, Min Si, Mitesh Kumar Singh, Mona Hassan, Naman Goyal, Narjes Torabi, Nikolay Bashlykov, Nikolay Bogoychev, Niladri Chatterji, Ning Zhang, Olivier Duchenne, Onur Çelebi, Patrick Alrassy, Pengchuan Zhang, Pengwei Li, Petar Vasic, Peter Weng, Prajjwal Bhargava, Pratik Dubal, Praveen Krishnan, Punit Singh Koura, Puxin Xu, Qing He, Qingxiao Dong, Ragavan Srinivasan, Raj Ganapathy, Ramon Calderer, Ricardo Silveira Cabral, Robert Stojnic, Roberta Raileanu, Rohan Maheswari, Rohit Girdhar, Rohit Patel, Romain Sauvestre, Ronnie Polidoro, Roshan Sumbaly, Ross Taylor, Ruan Silva, Rui Hou, Rui Wang, Saghar Hosseini, Sahana Chennabasappa, Sanjay Singh, Sean Bell, Seohyun Sonia Kim, Sergey Edunov, Shaoliang Nie, Sharan Narang, Sharath Raparthy, Sheng Shen, Shengye Wan, Shruti Bhosale, Shun Zhang, Simon Vandenhende, Soumya Batra, Spencer Whitman, Sten Sootla, Stephane Collot, Suchin Gururangan, Sydney Borodinsky, Tamar Herman, Tara Fowler, Tarek Sheasha, Thomas Georgiou, Thomas Scialom, Tobias Speckbacher, Todor Mihaylov, Tong Xiao, Ujjwal Karn, Vedanuj Goswami, Vibhor Gupta, Vignesh Ramanathan, Viktor Kerkez, Vincent Gonguet, Virginie Do, Vish Vogeti, Vítor Albiero, Vladan Petrovic, Weiwei Chu, Wenhan Xiong, Wenyin Fu, Whitney Meers, Xavier Martinet, Xiaodong Wang, Xiaofang Wang, Xiaoqing Ellen Tan, Xide Xia, Xinfeng Xie, Xuchao Jia, Xuewei Wang, Yaelle Goldschlag, Yashesh Gaur, Yasmine Babaei, Yi Wen, Yiwen Song, Yuchen Zhang, Yue Li, Yuning Mao, Zacharie Delpierre Coudert, Zheng Yan, Zhengxing Chen, Zoe Papakipos, Aaditya Singh, Aayushi Srivastava, Abha Jain, Adam Kelsey, Adam Shajnfeld, Adithya Gangidi, Adolfo Victoria, Ahuva Goldstand, Ajay Menon, Ajay Sharma, Alex Boesenberg, Alexei Baevski, Allie Feinstein, Amanda Kallet, Amit Sangani, Amos Teo, Anam Yunus, Andrei Lupu, Andres Alvarado, Andrew Caples, Andrew Gu, Andrew Ho, Andrew Poulton, Andrew Ryan, Ankit Ramchandani, Annie Dong, Annie Franco, Anuj Goyal, Aparajita Saraf, Arkabandhu Chowdhury, Ashley Gabriel, Ashwin Bharambe, Assaf Eisenman, Azadeh Yazdan, Beau James, Ben Maurer, Benjamin Leonhardi, Bernie Huang, Beth Loyd, Beto De Paola, Bhargavi Paranjape, Bing Liu, Bo Wu, Boyu Ni, Braden Hancock, Bram Wasti, Brandon Spence, Brani Stojkovic, Brian Gamido, Britt Montalvo, Carl Parker, Carly Burton, Catalina Mejia, Ce Liu, Changhan Wang, Changkyu Kim, Chao Zhou, Chester Hu, Ching-Hsiang Chu, Chris Cai, Chris Tindal, Christoph Feichtenhofer, Cynthia Gao, Damon Civin, Dana Beaty, Daniel Kreymer, Daniel Li, David Adkins, David Xu, Davide Testuggine, Delia David, Devi Parikh, Diana Liskovich, Didem Foss, Dingkang Wang, Duc Le, Dustin Holland, Edward Dowling, Eissa Jamil, Elaine Montgomery, Eleonora Presani, Emily Hahn, Emily Wood, Eric-Tuan Le, Erik Brinkman, Esteban Arcaute, Evan Dunbar, Evan Smothers, Fei Sun, Felix Kreuk, Feng Tian, Filippos Kokkinos, Firat Ozgenel, Francesco Caggioni, Frank Kanayet, Frank Seide, Gabriela Medina Florez, Gabriella Schwarz, Gada Badeer, Georgia Swee, Gil Halpern, Grant Herman, Grigory Sizov, Guangyi, Zhang, Guna Lakshminarayanan, Hakan Inan, Hamid Shojanazeri, Han Zou, Hannah Wang, Hanwen Zha, Haroun Habeeb, Harrison Rudolph, Helen Suk, Henry Aspegren, Hunter Goldman, Hongyuan Zhan, Ibrahim Damlaj, Igor Molybog, Igor Tufanov, Ilias Leontiadis, Irina-Elena Veliche, Itai Gat, Jake Weissman, James Geboski, James Kohli, Janice Lam, Japhet Asher, Jean-Baptiste Gaya, Jeff Marcus, Jeff Tang, Jennifer Chan, Jenny Zhen, Jeremy Reizenstein, Jeremy Teboul, Jessica Zhong, Jian Jin, Jingyi Yang, Joe Cummings, Jon Carvill, Jon Shepard, Jonathan McPhie, Jonathan Torres, Josh Ginsburg, Junjie Wang, Kai Wu, Kam Hou U, Karan Saxena, Kartikay Khandelwal, Katayoun Zand, Kathy Matosich, Kaushik Veeraraghavan, Kelly Michelena, Keqian Li, Kiran Jagadeesh, Kun Huang, Kunal Chawla, Kyle Huang, Lailin Chen, Lakshya Garg, Lavender A, Leandro Silva, Lee Bell, Lei Zhang, Liangpeng Guo, Licheng Yu, Liron Moshkovich, Luca Wehrstedt, Madian Khabsa, Manav Avalani, Manish Bhatt, Martynas Mankus, Matan Hasson, Matthew Lennie, Matthias Reso, Maxim Groshev, Maxim Naumov, Maya Lathi, Meghan Keneally, Miao Liu, Michael L. Seltzer, Michal Valko, Michelle Restrepo, Mihir Patel, Mik Vyatskov, Mikayel Samvelyan, Mike Clark, Mike Macey, Mike Wang, Miquel Jubert Hermoso, Mo Metanat, Mohammad Rastegari, Munish Bansal, Nandhini Santhanam, Natascha Parks, Natasha White, Navyata Bawa, Nayan Singhal, Nick Egebo, Nicolas Usunier, Nikhil Mehta, Nikolay Pavlovich Laptev, Ning Dong, Norman Cheng, Oleg Chernoguz, Olivia Hart, Omkar Salpekar, Ozlem Kalinli, Parkin Kent, Parth Parekh, Paul Saab, Pavan Balaji, Pedro Rittner, Philip Bontrager, Pierre Roux, Piotr Dollar, Polina Zvyagina, Prashant Ratanchandani, Pritish Yuvraj, Qian Liang, Rachad Alao, Rachel Rodriguez, Rafi Ayub, Raghotham Murthy, Raghu Nayani, Rahul Mitra, Rangaprabhu Parthasarathy, Raymond Li, Rebekkah Hogan, Robin Battey, Rocky Wang, Russ Howes, Ruty Rinott, Sachin Mehta, Sachin Siby, Sai Jayesh Bondu, Samyak Datta, Sara Chugh, Sara Hunt, Sargun Dhillon, Sasha Sidorov, Satadru Pan, Saurabh Mahajan, Saurabh Verma, Seiji Yamamoto, Sharadh Ramaswamy, Shaun Lindsay, Shaun Lindsay, Sheng Feng, Shenghao Lin, Shengxin Cindy Zha, Shishir Patil, Shiva Shankar, Shuqiang Zhang, Shuqiang Zhang, Sinong Wang, Sneha Agarwal, Soji Sajuyigbe, Soumith Chintala, Stephanie Max, Stephen Chen, Steve Kehoe, Steve Satterfield, Sudarshan Govindaprasad, Sumit Gupta, Summer Deng, Sungmin Cho, Sunny Virk, Suraj Subramanian, Sy Choudhury, Sydney Goldman, Tal Remez, Tamar Glaser, Tamara Best, Thilo Koehler, Thomas Robinson, Tianhe Li, Tianjun Zhang, Tim Matthews, Timothy Chou, Tzook Shaked, Varun Vontimitta, Victoria Ajayi, Victoria Montanez, Vijai Mohan, Vinay Satish Kumar, Vishal Mangla, Vlad Ionescu, Vlad Poenaru, Vlad Tiberiu Mihailescu, Vladimir Ivanov, Wei Li, Wenchen Wang, Wenwen Jiang, Wes Bouaziz, Will Constable, Xiaocheng Tang, Xiaojian Wu, Xiaolan Wang, Xilun Wu, Xinbo Gao, Yaniv Kleinman, Yanjun Chen, Ye Hu, Ye Jia, Ye Qi, Yenda Li, Yilin Zhang, Ying Zhang, Yossi Adi, Youngjin Nam, Yu, Wang, Yu Zhao, Yuchen Hao, Yundi Qian, Yunlu Li, Yuzi He, Zach Rait, Zachary DeVito, Zef Rosnbrick, Zhaoduo Wen, Zhenyu Yang, Zhiwei Zhao, and Zhiyu Ma. The llama 3 herd of models, 2024. URL https://arxiv.org/abs/2407.21783. 6

Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, et al. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning. *arXiv preprint arXiv:2501.12948*, 2025. 1

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset, 2021. URL https://arxiv.org/abs/2103.03874. 7

Jie Huang, Xinyun Chen, Swaroop Mishra, Huaixiu Steven Zheng, Adams Wei Yu, Xinying Song, and Denny Zhou. Large language models cannot self-correct reasoning yet, 2024. URL https://arxiv.org/abs/2310.01798. 1

Kevin Jamieson and Robert Nowak. Best-arm identification algorithms for multi-armed bandits in the fixed confidence setting. In *2014 48th annual conference on information sciences and systems (CISS)*, pp. 1–6. IEEE, 2014.

Mojan Javaheripi and Sébastien Bubeck. Phi-2: The surprising power of small language models. Microsoft Research Blog, 2023. URL https://www.microsoft.com/en-us/research/blog/phi-2-the-surprising-power-of-small-language-models/. 1

Albert Q. Jiang, Alexandre Sablayrolles, Antoine Roux, Arthur Mensch, Blanche Savary, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Emma Bou Hanna, Florian Bressand, Gianna Lengyel, Guillaume Bour, Guillaume Lample, Lélio Renard Lavaud, Lucile Saulnier, Marie-Anne Lachaux, Pierre Stock, Sandeep Subramanian, Sophia Yang, Szymon Antoniak, Teven Le Scao, Théophile Gervet, Thibaut Lavril, Thomas Wang, Timothée Lacroix, and William El Sayed. Mixtral of experts, 2024. URL https://arxiv.org/abs/2401.04088. 6

Omar Khattab, Arnav Singhvi, Paridhi Maheshwari, Zhiyuan Zhang, Keshav Santhanam, Sri Vardhamanan, Saiful Haq, Ashutosh Sharma, Thomas T. Joshi, Hanna Moazam, Heather Miller, Matei Zaharia, and Christopher Potts. Dspy: Compiling declarative language model calls into self-improving pipelines, 2023. URL https://arxiv.org/abs/2310.03714. 3

Junyou Li, Qin Zhang, Yangbin Yu, Qiang Fu, and Deheng Ye. More agents is all you need, 2024. URL https://arxiv.org/abs/2402.05120. 2, 3, 8, 9

Shalev Lifshitz, Sheila A. McIlraith, and Yilun Du. Multi-agent verification: Scaling test-time compute with multiple verifiers, 2025. URL https://arxiv.org/abs/2502.20379. 1, 2, 6, 7

Nelson F. Liu, Kevin Lin, John Hewitt, Ashwin Paranjape, Michele Bevilacqua, Fabio Petroni, and Percy Liang. Lost in the middle: How language models use long contexts, 2023. URL https://arxiv.org/abs/2307.03172. 1

Mistral AI. Mistral small 24b instruct. https://huggingface.co/mistralai/Mistral-Small-24B-Instruct-2501, 2025. Accessed: 2025-09-23. 7

Niklas Muennighoff, Zitong Yang, Weijia Shi, Xiang Lisa Li, Li Fei-Fei, Hannaneh Hajishirzi, Luke Zettlemoyer, Percy Liang, Emmanuel Candès, and Tatsunori Hashimoto. s1: Simple test-time scaling, 2025. URL https://arxiv.org/abs/2501.19393. 3

Isaac Ong, Amjad Almahairi, Vincent Wu, Wei-Lin Chiang, Tianhao Wu, Joseph E. Gonzalez, M Waleed Kadous, and Ion Stoica. Routellm: Learning to route llms with preference data, 2025. URL https://arxiv.org/abs/2406.18665. 3

OpenAI, :, Aaron Hurst, Adam Lerer, Adam P. Goucher, Adam Perelman, Aditya Ramesh, Aidan Clark, AJ Ostrow, Akila Welihinda, Alan Hayes, Alec Radford, Aleksander Mądry, Alex Baker-Whitcomb, Alex Beutel, Alex Borzunov, Alex Carney, Alex Chow, Alex Kirillov, Alex Nichol, Alex Paino, Alex Renzin, Alex Tachard Passos, Alexander Kirillov, Alexi Christakis, Alexis Conneau, Ali Kamali, Allan Jabri, Allison Moyer, Allison Tam, Amadou Crookes, Amin Tootoochian, Amin Tootoonchian, Ananya Kumar, Andrea Vallone, Andrej Karpathy, Andrew Braunstein, Andrew Cann, Andrew Codispoti, Andrew Galu, Andrew Kondrich, Andrew Tulloch, Andrey Mishchenko, Angela Baek, Angela Jiang, Antoine Pelisse, Antonia Woodford, Anuj Gosalia, Arka Dhar, Ashley Pantuliano, Avi Nayak, Avital Oliver, Barret Zoph, Behrooz Ghorbani, Ben Leimberger, Ben Rossen, Ben Sokolowsky, Ben Wang, Benjamin Zweig, Beth Hoover, Blake Samic, Bob McGrew, Bobby Spero, Bogo Giertler, Bowen Cheng, Brad Lightcap, Brandon Walkin, Brendan Quinn, Brian Guarraci, Brian Hsu, Bright Kellogg, Brydon Eastman, Camillo Lugaresi, Carroll Wainwright, Cary Bassin, Cary Hudson, Casey Chu, Chad Nelson, Chak Li, Chan Jun Shern, Channing Conger, Charlotte Barette, Chelsea Voss, Chen Ding, Cheng Lu, Chong Zhang, Chris Beaumont, Chris Hallacy, Chris Koch, Christian Gibson, Christina Kim, Christine Choi, Christine McLeavey, Christopher Hesse, Claudia Fischer, Clemens Winter, Coley Czarnecki, Colin Jarvis, Colin Wei, Constantin Koumouzelis, Dane Sherburn, Daniel Kappler, Daniel Levin, Daniel Levy, David Carr, David Farhi, David Mely, David Robinson, David Sasaki, Denny Jin, Dev Valladares, Dimitris Tsipras, Doug Li, Duc Phong Nguyen, Duncan Findlay, Edede Oiwoh, Edmund Wong, Ehsan Asdar, Elizabeth Proehl, Elizabeth Yang, Eric Antonow, Eric

Kramer, Eric Peterson, Eric Sigler, Eric Wallace, Eugene Brevdo, Evan Mays, Farzad Khorasani, Felipe Petroski Such, Filippo Raso, Francis Zhang, Fred von Lohmann, Freddie Sulit, Gabriel Goh, Gene Oden, Geoff Salmon, Giulio Starace, Greg Brockman, Hadi Salman, Haiming Bao, Haitang Hu, Hannah Wong, Haoyu Wang, Heather Schmidt, Heather Whitney, Heewoo Jun, Hendrik Kirchner, Henrique Ponde de Oliveira Pinto, Hongyu Ren, Huiwen Chang, Hyung Won Chung, Ian Kivlichan, Ian O'Connell, Ian O'Connell, Ian Osband, Ian Silber, Ian Sohl, Ibrahim Okuyucu, Ikai Lan, Ilya Kostrikov, Ilya Sutskever, Ingmar Kanitscheider, Ishaan Gulrajani, Jacob Coxon, Jacob Menick, Jakub Pachocki, James Aung, James Betker, James Crooks, James Lennon, Jamie Kiros, Jan Leike, Jane Park, Jason Kwon, Jason Phang, Jason Teplitz, Jason Wei, Jason Wolfe, Jay Chen, Jeff Harris, Jenia Varavva, Jessica Gan Lee, Jessica Shieh, Ji Lin, Jiahui Yu, Jiayi Weng, Jie Tang, Jieqi Yu, Joanne Jang, Joaquin Quinonero Candela, Joe Beutler, Joe Landers, Joel Parish, Johannes Heidecke, John Schulman, Jonathan Lachman, Jonathan McKay, Jonathan Uesato, Jonathan Ward, Jong Wook Kim, Joost Huizinga, Jordan Sitkin, Jos Kraaijeveld, Josh Gross, Josh Kaplan, Josh Snyder, Joshua Achiam, Joy Jiao, Joyce Lee, Juntang Zhuang, Justyn Harriman, Kai Fricke, Kai Hayashi, Karan Singhal, Katy Shi, Kavin Karthik, Kayla Wood, Kendra Rimbach, Kenny Hsu, Kenny Nguyen, Keren Gu-Lemberg, Kevin Button, Kevin Liu, Kiel Howe, Krithika Muthukumar, Kyle Luther, Lama Ahmad, Larry Kai, Lauren Itow, Lauren Workman, Leher Pathak, Leo Chen, Li Jing, Lia Guy, Liam Fedus, Liang Zhou, Lien Mamitsuka, Lilian Weng, Lindsay McCallum, Lindsey Held, Long Ouyang, Louis Feuvrier, Lu Zhang, Lukas Kondraciuk, Lukasz Kaiser, Luke Hewitt, Luke Metz, Lyric Doshi, Mada Aflak, Maddie Simens, Madelaine Boyd, Madeleine Thompson, Marat Dukhan, Mark Chen, Mark Gray, Mark Hudnall, Marvin Zhang, Marwan Aljubeh, Mateusz Litwin, Matthew Zeng, Max Johnson, Maya Shetty, Mayank Gupta, Meghan Shah, Mehmet Yatbaz, Meng Jia Yang, Mengchao Zhong, Mia Glaese, Mianna Chen, Michael Janner, Michael Lampe, Michael Petrov, Michael Wu, Michele Wang, Michelle Fradin, Michelle Pokrass, Miguel Castro, Miguel Oom Temudo de Castro, Mikhail Pavlov, Miles Brundage, Miles Wang, Minal Khan, Mira Murati, Mo Bavarian, Molly Lin, Murat Yesildal, Nacho Soto, Natalia Gimelshein, Natalie Cone, Natalie Staudacher, Natalie Summers, Natan LaFontaine, Neil Chowdhury, Nick Ryder, Nick Stathas, Nick Turley, Nik Tezak, Niko Felix, Nithanth Kudige, Nitish Keskar, Noah Deutsch, Noel Bundick, Nora Puckett, Ofir Nachum, Ola Okelola, Oleg Boiko, Oleg Murk, Oliver Jaffe, Olivia Watkins, Olivier Godement, Owen Campbell-Moore, Patrick Chao, Paul McMillan, Pavel Belov, Peng Su, Peter Bak, Peter Bakkum, Peter Deng, Peter Dolan, Peter Hoeschele, Peter Welinder, Phil Tillet, Philip Pronin, Philippe Tillet, Prafulla Dhariwal, Qiming Yuan, Rachel Dias, Rachel Lim, Rahul Arora, Rajan Troll, Randall Lin, Rapha Gontijo Lopes, Raul Puri, Reah Miyara, Reimar Leike, Renaud Gaubert, Reza Zamani, Ricky Wang, Rob Donnelly, Rob Honsby, Rocky Smith, Rohan Sahai, Rohit Ramchandani, Romain Huet, Rory Carmichael, Rowan Zellers, Roy Chen, Ruby Chen, Ruslan Nigmatullin, Ryan Cheu, Saachi Jain, Sam Altman, Sam Schoenholz, Sam Toizer, Samuel Miserendino, Sandhini Agarwal, Sara Culver, Scott Ethersmith, Scott Gray, Sean Grove, Sean Metzger, Shamez Hermani, Shantanu Jain, Shengjia Zhao, Sherwin Wu, Shino Jomoto, Shirong Wu, Shuaiqi, Xia, Sonia Phene, Spencer Papay, Srinivas Narayanan, Steve Coffey, Steve Lee, Stewart Hall, Suchir Balaji, Tal Broda, Tal Stramer, Tao Xu, Tarun Gogineni, Taya Christianson, Ted Sanders, Tejal Patwardhan, Thomas Cunninghman, Thomas Degry, Thomas Dimson, Thomas Raoux, Thomas Shadwell, Tianhao Zheng, Todd Underwood, Todor Markov, Toki Sherbakov, Tom Rubin, Tom Stasi, Tomer Kaftan, Tristan Heywood, Troy Peterson, Tyce Walters, Tyna Eloundou, Valerie Qi, Veit Moeller, Vinnie Monaco, Vishal Kuo, Vlad Fomenko, Wayne Chang, Weiyi Zheng, Wenda Zhou, Wesam Manassra, Will Sheu, Wojciech Zaremba, Yash Patil, Yilei Qian, Yongjik Kim, Youlong Cheng, Yu Zhang, Yuchen He, Yuchen Zhang, Yujia Jin, Yunxing Dai, and Yury Malkov. Gpt-4o system card, 2024. URL https://arxiv.org/abs/2410.21276. 6

Krista Opsahl-Ong, Michael J Ryan, Josh Purtell, David Broman, Christopher Potts, Matei Zaharia, and Omar Khattab. Optimizing instructions and demonstrations for multi-stage language model programs, 2024. URL https://arxiv.org/abs/2406.11695. 3

Manhin Poon, XiangXiang Dai, Xutong Liu, Fang Kong, John Lui, and Jinhang Zuo. Online multi-llm selection via contextual bandits under unstructured context evolution. *arXiv preprint arXiv:2506.17670*, 2025. 3

Qwen, :, An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, Fei Huang, Haoran Wei, Huan Lin, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Yang, Jiaxi Yang, Jingren Zhou, Junyang Lin, Kai Dang, Keming Lu, Keqin Bao, Kexin

Yang, Le Yu, Mei Li, Mingfeng Xue, Pei Zhang, Qin Zhu, Rui Men, Runji Lin, Tianhao Li, Tianyi Tang, Tingyu Xia, Xingzhang Ren, Xuancheng Ren, Yang Fan, Yang Su, Yichang Zhang, Yu Wan, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, and Zihan Qiu. Qwen2.5 technical report, 2025. URL https://arxiv.org/abs/2412.15115. 7

David Rein, Betty Li Hou, Asa Cooper Stickland, Jackson Petty, Richard Yuanzhe Pang, Julien Dirani, Julian Michael, and Samuel R. Bowman. Gpqa: A graduate-level google-proof q&a benchmark, 2023. URL https://arxiv.org/abs/2311.12022. 7

Jon Saad-Falcon, Adrian Gamarra Lafuente, Shlok Natarajan, Nahum Maru, Hristo Todorov, Etash Guha, E. Kelly Buchanan, Mayee Chen, Neel Guha, Christopher Ré, and Azalia Mirhoseini. Archon: An architecture search framework for inference-time techniques, 2025. URL https://arxiv.org/abs/2409.15254. 3

Charlie Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. Scaling llm test-time compute optimally can be more effective than scaling model parameters, 2024. URL https://arxiv.org/abs/2408.03314. 3

Amir Taubenfeld, Yaniv Dover, Roi Reichart, and Ariel Goldstein. Systematic biases in llm simulations of debates. In *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing*, pp. 251–267. Association for Computational Linguistics, 2024. doi: 10.18653/v1/2024.emnlp-main.16. URL http://dx.doi.org/10.18653/v1/2024.emnlp-main.16. 1

Amir Taubenfeld, Tom Sheffer, Eran Ofek, Amir Feder, Ariel Goldstein, Zorik Gekhman, and Gal Yona. Confidence improves self-consistency in llms. In *Findings of the Association for Computational Linguistics: ACL 2025*, pp. 20090–20111. Association for Computational Linguistics, 2025. doi: 10.18653/v1/2025.findings-acl.1030. URL http://dx.doi.org/10.18653/v1/2025.findings-acl.1030. 4

Gemma Team, Morgane Riviere, Shreya Pathak, Pier Giuseppe Sessa, Cassidy Hardin, Surya Bhupatiraju, Léonard Hussenot, Thomas Mesnard, Bobak Shahriari, Alexandre Ramé, Johan Ferret, Peter Liu, Pouya Tafti, Abe Friesen, Michelle Casbon, Sabela Ramos, Ravin Kumar, Charline Le Lan, Sammy Jerome, Anton Tsitsulin, Nino Vieillard, Piotr Stanczyk, Sertan Girgin, Nikola Momchev, Matt Hoffman, Shantanu Thakoor, Jean-Bastien Grill, Behnam Neyshabur, Olivier Bachem, Alanna Walton, Aliaksei Severyn, Alicia Parrish, Aliya Ahmad, Allen Hutchison, Alvin Abdagic, Amanda Carl, Amy Shen, Andy Brock, Andy Coenen, Anthony Laforge, Antonia Paterson, Ben Bastian, Bilal Piot, Bo Wu, Brandon Royal, Charlie Chen, Chintu Kumar, Chris Perry, Chris Welty, Christopher A. Choquette-Choo, Danila Sinopalnikov, David Weinberger, Dimple Vijaykumar, Dominika Rogozińska, Dustin Herbison, Elisa Bandy, Emma Wang, Eric Noland, Erica Moreira, Evan Senter, Evgenii Eltyshev, Francesco Visin, Gabriel Rasskin, Gary Wei, Glenn Cameron, Gus Martins, Hadi Hashemi, Hanna Klimczak-Plucińska, Harleen Batra, Harsh Dhand, Ivan Nardini, Jacinda Mein, Jack Zhou, James Svensson, Jeff Stanway, Jetha Chan, Jin Peng Zhou, Joana Carrasqueira, Joana Iljazi, Jocelyn Becker, Joe Fernandez, Joost van Amersfoort, Josh Gordon, Josh Lipschultz, Josh Newlan, Ju yeong Ji, Kareem Mohamed, Kartikeya Badola, Kat Black, Katie Millican, Keelin McDonell, Kelvin Nguyen, Kiranbir Sodhia, Kish Greene, Lars Lowe Sjoesund, Lauren Usui, Laurent Sifre, Lena Heuermann, Leticia Lago, Lilly McNealus, Livio Baldini Soares, Logan Kilpatrick, Lucas Dixon, Luciano Martins, Machel Reid, Manvinder Singh, Mark Iverson, Martin Görner, Mat Velloso, Mateo Wirth, Matt Davidow, Matt Miller, Matthew Rahtz, Matthew Watson, Meg Risdal, Mehran Kazemi, Michael Moynihan, Ming Zhang, Minsuk Kahng, Minwoo Park, Mofi Rahman, Mohit Khatwani, Natalie Dao, Nenshad Bardoliwalla, Nesh Devanathan, Neta Dumai, Nilay Chauhan, Oscar Wahltinez, Pankil Botarda, Parker Barnes, Paul Barham, Paul Michel, Pengchong Jin, Petko Georgiev, Phil Culliton, Pradeep Kuppala, Ramona Comanescu, Ramona Merhej, Reena Jana, Reza Ardeshir Rokni, Rishabh Agarwal, Ryan Mullins, Samaneh Saadat, Sara Mc Carthy, Sarah Cogan, Sarah Perrin, Sébastien M. R. Arnold, Sebastian Krause, Shengyang Dai, Shruti Garg, Shruti Sheth, Sue Ronstrom, Susan Chan, Timothy Jordan, Ting Yu, Tom Eccles, Tom Hennigan, Tomas Kocisky, Tulsee Doshi, Vihan Jain, Vikas Yadav, Vilobh Meshram, Vishal Dharmadhikari, Warren Barkley, Wei Wei, Wenming Ye, Woohyun Han, Woosuk Kwon, Xiang Xu, Zhe Shen, Zhitao Gong, Zichuan Wei, Victor Cotruta, Phoebe Kirk, Anand Rao, Minh Giang, Ludovic Peran, Tris Warkentin, Eli Collins, Joelle Barral, Zoubin Ghahramani, Raia Hadsell, D. Sculley, Jeanine Banks, Anca Dragan, Slav Petrov, Oriol Vinyals, Jeff Dean, Demis Hassabis, Koray Kavukcuoglu, Clement Farabet, Elena

Buchatskaya, Sebastian Borgeaud, Noah Fiedel, Armand Joulin, Kathleen Kenealy, Robert Dadashi, and Alek Andreev. Gemma 2: Improving open language models at a practical size, 2024. URL https://arxiv.org/abs/2408.00118. 6

Raghuveer Thirukovalluru, Yukun Huang, and Bhuwan Dhingra. Atomic self-consistency for better long form generations, 2024. URL https://arxiv.org/abs/2405.13131. 3

Fouad Trad and Ali Chehab. *To Ensemble or Not: Assessing Majority Voting Strategies for Phishing Detection with Large Language Models*, pp. 158–173. Springer Nature Switzerland, 2025. ISBN 9783031821509. doi: 10.1007/978-3-031-82150-9_13. URL http://dx.doi.org/10.1007/978-3-031-82150-9_13. 3

Fali Wang, Zhiwei Zhang, Xianren Zhang, Zongyu Wu, Tzuhao Mo, Qiuhao Lu, Wanjing Wang, Rui Li, Junjie Xu, Xianfeng Tang, Qi He, Yao Ma, Ming Huang, and Suhang Wang. A comprehensive survey of small language models in the era of large language models: Techniques, enhancements, applications, collaboration with llms, and trustworthiness, 2024a. URL https://arxiv.org/abs/2411.03350. 1

Junlin Wang, Jue Wang, Ben Athiwaratkun, Ce Zhang, and James Zou. Mixture-of-agents enhances large language model capabilities, 2024b. URL https://arxiv.org/abs/2406.04692. 1, 2, 6, 7

Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models, 2023. URL https://arxiv.org/abs/2203.11171. 2, 3

Zhihui Xie, Jizhou Guo, Tong Yu, and Shuai Li. Calibrating reasoning in language models with internal consistency, 2024. URL https://arxiv.org/abs/2405.18711. 4

Jiayi Zhang, Jinyu Xiang, Zhaoyang Yu, Fengwei Teng, Xionghui Chen, Jiaqi Chen, Mingchen Zhuge, Xin Cheng, Sirui Hong, Jinlin Wang, Bingnan Zheng, Bang Liu, Yuyu Luo, and Chenglin Wu. Aflow: Automating agentic workflow generation, 2025a. URL https://arxiv.org/abs/2410.10762. 3

Qiyuan Zhang, Fuyuan Lyu, Zexu Sun, Lei Wang, Weixu Zhang, Wenyue Hua, Haolun Wu, Zhihan Guo, Yufei Wang, Niklas Muennighoff, Irwin King, Xue Liu, and Chen Ma. A survey on test-time scaling in large language models: What, how, where, and how well?, 2025b. URL https://arxiv.org/abs/2503.24235. 3

Appendix Overview

The appendix is organized into five main sections. **Section A** states the usage of LLMs in preparing this paper. **Section B** provides experimental details, including visual illustrations, single-model accuracies, and standard deviation calculations. **Section C** analyzes why discussion-based methods fail on SLMs, presenting groupthink analysis and prompt sensitivity studies. **Section D** validates the SLM-MUX design through consistency-accuracy correlation analysis, comparative analysis with voting-based methods, model selection search analysis, and test-set scaling validation. **Section E** demonstrates the generalization of SLM-MUX to open-ended generation, frontier LLMs, and domain-specific models. Finally, Section **F** provides dataset licenses.

A LLM Usage Statement

We used Cursor for coding. Large language models (LLMs) were employed to help polish drafts written by humans, and to assist in searching for related papers. The final choice of related work included in this paper was made entirely by the human authors after careful screening. LLMs were also used for proofreading and for providing suggestions.

B Experimental Details

B.1 Visual Illustrations of SLM-MUX

To more effectively illustrate the workflow of our proposed composition method, we select several representative examples from the logs. We demonstrate them in Figure 10, Figure 11 and Figure 12.

SLM-MUX surpasses majority voting in scenarios with initial disagreement among models.. As illustrated by Figure 10, during the independent generation phase, Gemma-2-27B is the sole model to provide the correct answer. Hence, majority voting applied directly would fail to select the correct author.

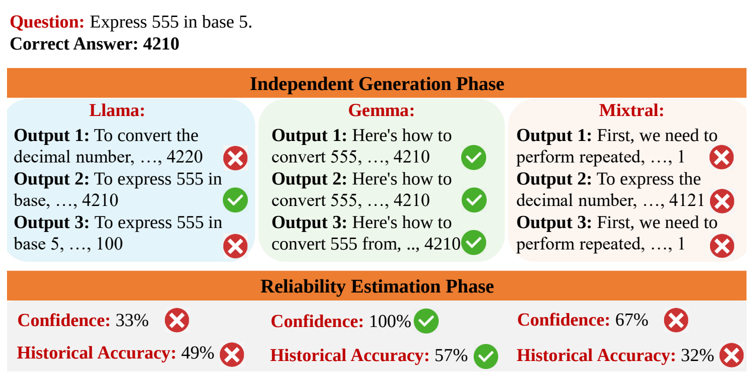


Figure 10: **An illustration of the SLM-MUX method applied to the MATH dataset.** In the independent generation phase, three models are used: LLaMA-3.1-8B (denoted as Llama), Gemma-2-27B (denoted as Gemma), and Mixtral-8×7B (denoted as Mixtral). Because the three models provide different answers at first, so each model is invoked two more times. Gemma obtains the highest confidence score and is therefore selected as the final output.

B.2 Accuracy of Single LLMs

We evaluated the accuracy of single model accuracy under the condition of temperature equal to zero. The results are shown in Table 5 and Table 6.

Model	MATH Acc (%)	GPQA Acc (%)	GSM8K Acc (%)
Llama-3.1-8B	48.6	23.7	84.2
Mixtral-8×7B	31.6	31.9	63.4
Gemma-2-27B	56.8	38.8	81.6

Table 5: **Small Model Base Performance.** Base model accuracy on MATH, GPQA, and GSM8K.

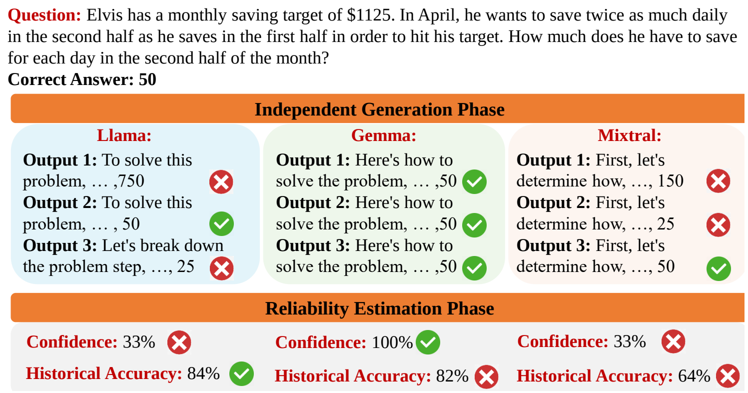


Figure 11: **An illustration of the SLM-MUX method applied to the GSM8K dataset.** In the independent generation phase, different models produce different answers. However, when we invoke each model multiple times, we observe that Llama and Mixtral only yield correct answers approximately one-third of the time. In contrast, Gemma demonstrates stable performance.

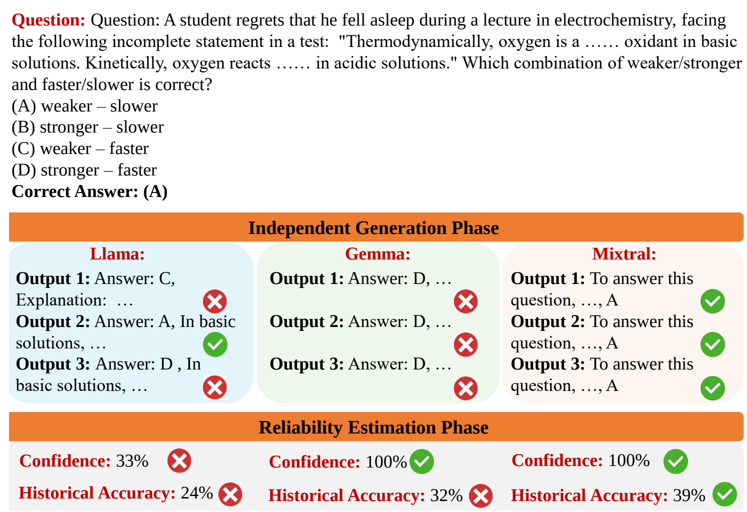


Figure 12: **An illustration of the SLM-MUX method applied to the GPQA dataset.** During the independent generation phase, Gemma and Mixtral obtain the same confidence score. However, considering historical accuracy, Mixtral ranks higher. Therefore, Mixtral's answer is selected as the final output.

B.3 Standard Deviation of the Data Points in Figure 5

Although all experiments are run in a deterministic setting with temperature set to zero, we can still compute the standard deviation of each datapoint by treating the outcome as a Bernoulli variable. Specifically, if there are n_{correct} correct answers and n_{wrong} incorrect answers, the standard deviation

Model	MATH		GPQA	
	Acc (%)	Token Usage	Acc (%)	Token Usage
DeepSeek V3	87.0	419,513	55.1	173,885
Gemini 2.0 Flash	90.4	361,737	63.6	195,576
GPT-4o	79.8	408,410	51.0	212,037

Table 6: **Large Model Base Performance.** Base model performance and token usage on MATH and GPQA datasets. Accuracy is the percentage of correct answers, and token usage reflects total tokens consumed (prompt + response) over the entire dataset for each model.

is

$$\frac{\sqrt{\mathrm{Var}(X)}}{\sqrt{n_{\text{total}}}} = \frac{\sqrt{p(1-p)}}{\sqrt{n_{\text{total}}}} = \frac{\sqrt{\frac{n_{\text{correct}}}{n_{\text{total}}}\left(1 - \frac{n_{\text{correct}}}{n_{\text{total}}}\right)}}{\sqrt{n_{\text{total}}}},$$

where $n_{\text{total}} = n_{\text{correct}} + n_{\text{wrong}}$.

The results are summarized in Table 7.

Table 7: Accuracy and estimated standard deviation on MATH ($n = 500$) and GPQA ($n = 196$) using datapoints from Figure 5.

	MATH ($n = 500$)		GPQA ($n = 196$)	
Method	SLM orchestration	LLM composition	SLM orchestration	LLM composition
Single Model Max	56.8 ± 2.22	90.4 ± 1.32	38.8 ± 3.48	63.6 ± 3.44
Mixture-of-Agents	46.2 ± 2.23	88.8 ± 1.41	33.3 ± 3.37	58.6 ± 3.52
LLM-Debate	51.6 ± 2.23	90.8 ± 1.29	36.9 ± 3.45	65.6 ± 3.39
Multi-Agent Verification	48.4 ± 2.23	91.6 ± 1.24	35.4 ± 3.42	64.2 ± 3.42

C Why Discussion-Based Methods Fail on SLMs

C.1 Groupthink Analysis

We analyze the experiment logs of LLM-Debate using small language models (SLMs) in Section 4.1. Among 500 debate problems, 242 resulted in failure (48.4%). For each of the 242 failed debates, we first used an analyzer LLM to produce a process-focused failure analysis. We then used a separate labeling LLM to classify whether each failed debate was due to groupthink.

The labeling results are shown in Table 8:

These results reinforce our claim that groupthink is a major failure mode in SLM-based LLM-debate.

We provide the exact prompts used by (i) the analyzer LLM to generate the 242 failure analyses (Figure 13) and (ii) the groupthink labeler LLM to classify groupthink (Figure 14). Placeholders such as `{problem}` indicate runtime substitutions by our code.

C.2 Prompt Sensitivity Analysis

A natural concern is whether the performance gap between discussion-based methods and SLM-MUX is due to suboptimal prompt selection rather than inherent limitations. To address this, we conduct a prompt sensitivity analysis on the Mixture-of-Agents (MoA) baseline using the MATH benchmark.

We use Gemini-2.5-Flash to generate 10 diverse aggregator prompts for MoA. Table 9 summarizes the results. Our baseline prompt (46.2%) outperforms the average of the tuned prompts (41.8%) and surpasses 7 out of 10 generated prompts. Even the best tuned prompt (48.4%) remains substantially below the best single model (56.8%) and SLM-MUX (61.8%).

To further stress-test this result, we conducted iterative prompt optimization directly on the test set for 6 rounds, selecting the best-performing prompt at each iteration. Even under this extremely favorable setting for MoA, the accuracy peaked at 55.6%, still below the best single model (56.8%) and far below SLM-MUX (61.8%). This confirms that the performance gap reflects inherent limitations of discussion-based aggregation when applied to SLMs, rather than an artifact of prompt selection.

D Validation of SLM-MUX Design

D.1 Consistency vs Accuracy Correlation

We empirically study how per-question self-consistency correlates with accuracy on four datasets: GSM8K, MATH, GPQA, and HumanEval. For each model–dataset pair, we compute a self-consistency score for every question (as defined in the main text) and group questions into three bins according to this score: *Low* $[0.0, 0.5)$, *Medium* $[0.5, 0.8)$, and *High* $[0.8, 1.0]$. We then measure the empirical accuracy (fraction of correct answers) within each bin.

```
As an expert in analyzing multi-agent AI systems, your task is to
   analyze why an 'LLM Debate' process failed to find the correct
    answer. Your focus should be on the *debate dynamics and
   process*, not just the mathematical details. The goal is to
   understand the failure of the debate methodology itself.

**Ground Truth:**
- **Problem Statement:** {problem}
- **Correct Answer:** {ref_answer}

**Debate Information:**
- **Final Incorrect Answer from System:** {system_answer}

**Analysis of Round 1:**
- **Model `{model_name}` proposed:**
  - Answer: `{extracted_answer}`
  - Reasoning:
```
```
{full_text}
```

```
... (repeats per round and per model)

**Your Analysis Task:**
Based on the debate history, provide a "Debate Failure Analysis".
   Do not focus on simple calculation mistakes. Instead, analyze
   the interaction between the models and the structure of the
   debate. Pinpoint the core reasons the *debate process* failed.
    Consider these questions:

1.  **Error Propagation vs. Correction:** How did initial errors
   influence later rounds? Were there moments where a correct
   idea was introduced but ignored or overruled? Why did the
   debate fail to self-correct?
2.  **Groupthink and Influence Dynamics:** Did the models converge
    on a flawed consensus? Did one or more influential but
   incorrect models lead the group astray? Was there evidence of
   independent reasoning that was shut down?
3.  **Argumentation Quality:** Did the models provide convincing
   but ultimately flawed arguments? Did they effectively
   challenge each other's reasoning, or was the debate
   superficial?
4.  **Critical Failure Point in the Debate:** Identify the single
   most critical turn or moment in the debate that sealed its
   failure. What happened, and why was it so impactful?
5.  **Improving the Debate:** What is the single most important
   change to the debate protocol or dynamics that could have
   prevented this failure? (e.g., different communication rules,
   promoting dissident opinions, etc.)

Provide a concise, expert analysis focusing on the *process*
   failure.
```

Figure 13: Prompt Template for Failure Analysis.

Metric	Count	Rate
Total Debates Analyzed	500	100% of total
Failed Debates (System Error)	242	48.4% of total
Breakdown of Failed Debates:		
Attributed to Groupthink	144	59.5% of failures
Attributed to Other Causes	79	32.6% of failures
Classification Unsuccessful	19	7.9% of failures

Table 8: **Failure Cause Attribution** This table shows the cause attribution for LLM-Debate when involving SLMs.

```
You are an expert analyst of multi-agent LLM debates. Your goal is
    to determine whether the failure primarily involved
   groupthink/conformity dynamics. Groupthink indicators include:
    early flawed consensus, explicit capitulation to a majority,
   social proofing, adopting peers' answers without critique,
   abandoning independent reasoning to match others, or
   reinforcing an incorrect majority despite available dissent.
   Not-groupthink includes failures due to independent arithmetic
   /logic errors, argument complexity/veneer effects without
   convergence, or chaotic divergence with no consensus influence
   . Return STRICT JSON only, with keys: groupthink (bool),
   confidence (float 0-1), reasons (string), cues (array of
   strings).
```

Figure 14: Prompt for Groupthink Classification.

Figure 15 reports the resulting accuracies for two representative SLMs on each dataset. Across GSM8K, MATH, and HumanEval we observe a strong positive relationship between self-consistency and accuracy: questions in the high-consistency bin are substantially more likely to be answered correctly than those in the low-consistency bin. GPQA exhibits a weaker but still positive correlation. Overall, these results provide empirical support for the link between self-consistency and correctness assumed in our method.

D.2 Comparative Analysis with Voting-Based Methods

Since SLM-MUX also involves voting on model outputs, we examine its differences from standard self-consistency and Agent Forest to better explain the source of our improvements.

To explain our stronger performance, we note a limitation of self-consistency methods. Suppose a model has probability p of answering a question correctly. When self-consistency samples N responses, the probability of obtaining the correct answer after majority voting follows a binomial distribution:

$$A(N,p) = \Pr\left(X \geq \lceil \tfrac{N}{2} \rceil\right) = \sum_{k=\lceil N/2 \rceil}^{N} \binom{N}{k} p^k (1-p)^{N-k}, \quad X \sim \text{Binomial}(N,p) \tag{1}$$

Table 9: Prompt sensitivity analysis for MoA on MATH. The baseline prompt used in our experiments is already near-optimal.

Setting	Accuracy
SLM-MUX (Ours)	**61.8%**
Best Single Model	56.8%
Tuned Prompt (Best)	48.4%
Baseline Prompt (Used in Paper)	46.2%
Tuned Prompt (Average)	41.8%
Tuned Prompt (Worst)	25.8%

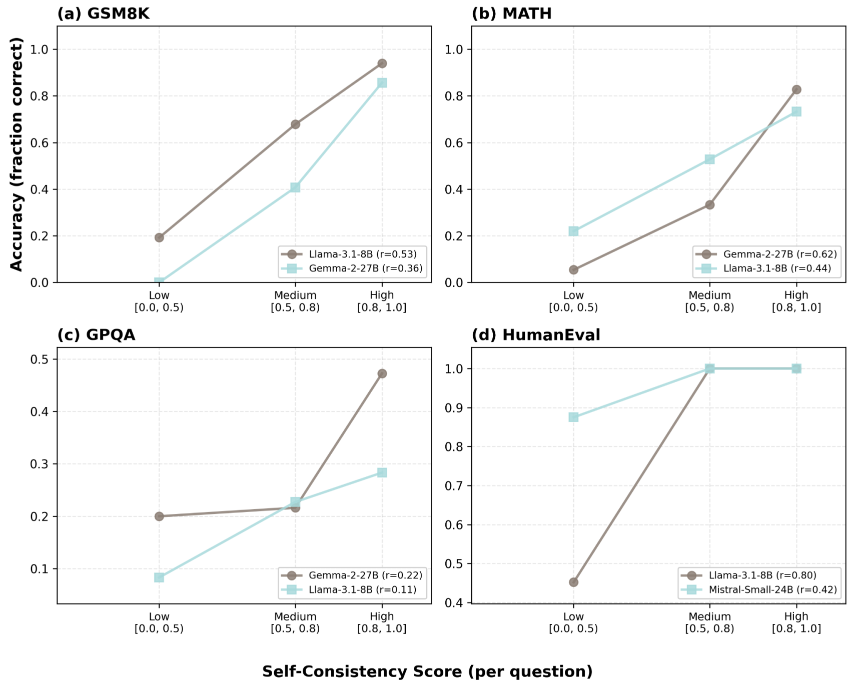


Figure 15: Accuracy as a function of per-question self-consistency score on GSM8K, MATH, GPQA, and HUMANEVAL. Questions are grouped into three bins by self-consistency: *Low* $[0.0, 0.5)$, *Medium* $[0.5, 0.8)$, and *High* $[0.8, 1.0]$. Each line corresponds to a different SLM; legends report the Pearson correlation r between self-consistency and correctness.

We observe that $A(N, p)$ exceeds p only when $p > 0.5$, meaning self-consistency is effective only in this regime. When $p < 0.5$, self-consistency can actually lower overall accuracy.

For any dataset, we can conceptually divide examples into three types of questions. **Type 1**: $p = 100\%$, the model always answers correctly. **Type 2**: $p > 50\%$, the model is more likely than not to be correct. **Type 3**: $p < 50\%$, the model is more likely to be wrong. The overall effect of self-consistency is then the improvement from Type 2, offset by the degradation from Type 3. Improvement occurs only when the dataset contains a sufficiently large proportion of Type 2 questions.

For SLM-MUX, we select the output from the most confident model, so the accuracy can be approximated as $A(N, p_{\max})$, where $p_{\max}$ is the highest probability among the participating models. By routing to the model with the highest $p_{\max}$ on each question, we effectively enlarge the proportion of Type 2 questions, leading to higher overall accuracy.

For the Agent Forest approach, answers are drawn evenly from all models, so its accuracy can be approximated as $A(N, \bar{p})$, where $\bar{p}$ is the average probability across models. This generally results in lower accuracy than SLM-MUX, as weaker models dilute the signal from stronger ones.

D.3 MODEL SELECTION SEARCH ANALYSIS

D.3.1 SEARCH-SET SIZE STABILITY

We analyze how the size of the search set (number of problems used in the search phase) affects the resulting model ranking. For each of the three benchmarks MATH, GSM8K, and GPQA, we treat the full benchmark (approximately 500 problems) as the search pool and first run our search procedure on the full set to obtain a ranking of all candidate models. We then record the models occupying Rank 1, Rank 2, and Rank 3 under this full-set ranking.

Next, we subsample the search set to smaller sizes and re-run the search. Specifically, for each dataset we evaluate search-set sizes of 100, 200, 300, 400, and the full set. At each size, we recompute the ranking over all models and track the ranks of the three models that were Rank 1–3 under the full-set setting.

Figure 16 shows the results. Each curve corresponds to one of the Rank 1/2/3 models under the full-set ranking, and the y-axis reports its rank when the search-set size is changed. Across all three datasets, these models consistently remain within the top three positions even when the search set is reduced to as few as 100 problems, with only minor swaps in their relative order on MATH and GSM8K. This suggests that a search set of a few hundred problems is sufficient to stably identify the top-performing models.

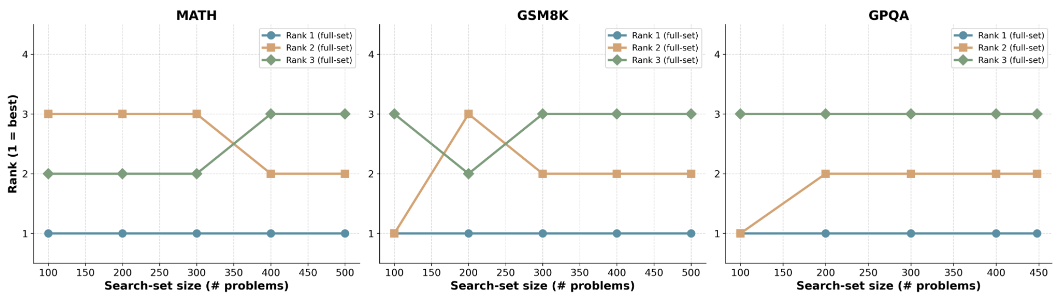


Figure 16: Stability of model rankings with respect to search-set size on MATH, GSM8K, and GPQA. For each dataset, we first determine the top three models using the full search set and then track their ranks as the search-set size is reduced.

D.3.2 Random Model Selection Baseline

A natural question is whether the performance gains of SLM-MUX stem from the model selection search or from the orchestration architecture itself. To isolate the contribution of the architecture, we conduct an experiment where model combinations are selected randomly rather than through our search procedure.

For each dataset, we randomly sample model combinations of size $K = 2, 3, 4$ from our pool of five SLMs and apply SLM-MUX. We compare the resulting accuracy against the best single model within each random pool. Table 10 reports the average performance across all random combinations.

Table 10: Performance of SLM-MUX with randomly selected model combinations. Even without optimized model selection, SLM-MUX consistently outperforms the best single model in the pool.

Dataset	K	SLM-MUX (Random)	Best Single Model	Δ (Gain)
MATH	2	67.2%	66.1%	+1.1
	3	71.5%	69.8%	+1.7
	4	73.7%	71.8%	+1.9
GSM8K	2	84.1%	81.7%	+2.4
	3	87.3%	83.6%	+3.7
	4	88.8%	84.0%	+4.8
GPQA	2	43.0%	41.3%	+1.7
	3	44.5%	44.4%	+0.1
	4	46.3%	46.0%	+0.3

As shown in Table 10, SLM-MUX consistently outperforms the best single model even when the model combination is selected randomly. On MATH and GSM8K, the gains are substantial (up to +4.8 on GSM8K with $K = 4$). On GPQA, where consistency is a weaker signal for correctness, the gains are smaller but still positive.

To further quantify how often SLM-MUX improves over single models, we compute the **Effective Combination Rate**: the percentage of all possible K-model subsets where SLM-MUX outperforms the best single model in that subset. Table 11 reports the results.

On GSM8K, 100% of combinations are effective across all values of K. On MATH and GPQA, the effective rate increases with K, reaching 100% at $K = 4$. Even at $K = 2$, the majority of combinations (60–100%) are effective. These results demonstrate that the space of "workable" model combinations is dense, and one does not need to search extensively to find an effective subset. The

Table 11: Effective Combination Rate: percentage of model combinations where SLM-MUX outperforms the best single model in the subset.

Dataset	$K = 2$	$K = 3$	$K = 4$
MATH	70%	100%	100%
GSM8K	100%	100%	100%
GPQA	60%	80%	100%

model selection search provides additional gains by identifying the optimal combination, but the architecture is robust even without it.

D.4 Test-Set Validation of Scaling Behavior

In Section 3.3, we evaluated the "Adding More Participating Model Types" scaling dimension on the validation set. Here we report the corresponding results on the test set to verify that the observed trends generalize.

Table 12 summarizes the test-set accuracy as the number of participating models K increases from 1 to 5. For each value of K, we use the model combination selected by the search procedure described in Section 3.2. Figure 17 visualizes these results.

Table 12: Test-set accuracy (%) as the number of participating models K increases. For $K = 1$, the best single model is reported.

K	GPQA	GSM8K	MATH
1 (best single)	47.98	90.20	78.20
2	50.21	91.51	80.27
3	51.81	91.29	81.09
4	49.16	91.44	81.00
5	45.18	90.80	80.64

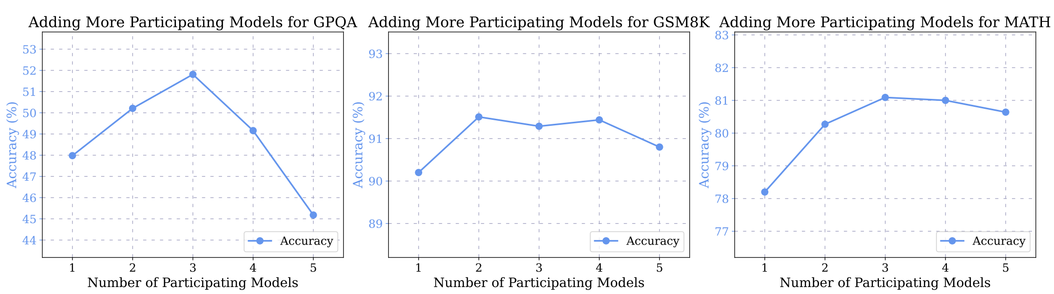


Figure 17: Test-set accuracy as a function of the number of participating models on GPQA, GSM8K, and MATH. The trends mirror those observed on the validation set: GPQA peaks at $K = 3$, GSM8K saturates quickly, and MATH shows continued improvement up to $K = 3$.

E Generalization of SLM-MUX

E.1 Open-Ended Generation (HumanEval)

In the main text, SLM-MUX is instantiated on tasks with discrete answer spaces, where self-consistency can be measured via majority voting over sampled outputs. To check whether the same principle extends to open-ended generation, we apply SLM-MUX to the HumanEval code-generation benchmark.

Consistency estimator for open-ended generation.. On HumanEval, exact-string majority voting is not appropriate, so we replace it with a semantic consistency estimator. For each model and each problem, we sample $N = 5$ code generations with temperature 0.3. We then encode the 5 generations using the pretrained embedding model `Salesforce/codet5p-110m-embedding` and compute pairwise cosine similarities, yielding a 5×5 similarity matrix. From this matrix,

Table 13: Pass@1 on HUMANEVAL for individual SLMs.

Model	Pass@1
Llama-3.1-8B-Instruct	0.178
Qwen2.5-7B-Instruct	0.485
Mistral-Small-24B	0.870
Qwen2.5-Coder-7B	0.893

Table 14: SLM-MUX on HUMANEVAL (Pass@1). Each row corresponds to a pair of SLMs; SLM-MUX selects the output from the model with higher embedding-based consistency.

Setup	Models combined	SLM-MUX (Pass@1)
Exp 1	Llama-3.1-8B-Instruct + Qwen2.5-7B-Instruct	**0.506**
Exp 2	Mistral-Small-24B + Qwen2.5-Coder-7B	**0.939**

we identify the most coherent cluster of generations (of size x) and use $x/5$ as the model's self-consistency score for that problem, analogous to the confidence score derived from majority voting in the discrete-answer setting.

We also experimented with an LLM-as-a-judge–based consistency estimator (using Qwen2.5-7B-Instruct as the judge) and found that the embedding-based estimator exhibits a stronger correlation with ground-truth correctness (Pass@1). All results below therefore use the embedding-based consistency score.

Results.. Table 13 reports the Pass@1 of each individual SLM on HUMANEVAL. Table 14 reports the Pass@1 of SLM-MUX when combining two models at a time; for each problem, SLM-MUX selects the solution from the model with the larger embedding-based consistency score.

E.2 FRONTIER LLMS

We evaluate whether SLM-MUX can exploit complementary strengths between state-of-the-art frontier models. We pair GPT-4o with Gemini-2.5-Flash and apply SLM-MUX on MATH, GPQA, and GSM8K. For each problem, we sample $N = 5$ responses per model at temperature 0.3 and apply self-consistency routing: for each problem, we perform majority voting within each model's samples, then route to the model showing higher agreement.

Table 15 summarizes the results. As a reference, "Perfect Routing" indicates the theoretical upper bound achievable if the system always selects the correct model when at least one succeeds.

Table 15: SLM-MUX performance when applied to frontier LLMs (GPT-4o and Gemini-2.5-Flash).

Benchmark	GPT-4o	Gemini-2.5-Flash	SLM-MUX	Perfect Routing
MATH	73.0%	92.1%	92.8%	94.2%
GPQA	50.7%	51.1%	**60.1%**	73.7%
GSM8K	89.1%	85.7%	89.4%	91.4%

The results reveal two distinct regimes. On GPQA, the two models exhibit complementary error patterns, and SLM-MUX achieves 60.1% accuracy, surpassing the best single model by nearly 10 percentage points. This demonstrates that SLM-MUX effectively exploits complementary strengths even at the frontier scale. On MATH and GSM8K, the Perfect Routing bounds (94.2% and 91.4%) are only marginally higher than the single-model baselines, indicating high overlap in the models' correct predictions. The limited gains on these benchmarks reflect this ceiling rather than a limitation of the routing mechanism.

E.3 DOMAIN-SPECIFIC FINE-TUNED MODELS

Domain-specific fine-tuned models are widely deployed in practice. We evaluate whether SLM-MUX can effectively orchestrate such specialized models by testing on two domains: code generation and mathematical reasoning.

Table 16: SLM-MUX performance when orchestrating domain-specific fine-tuned models.

Domain	Benchmark	Best Single Model	SLM-MUX	Δ (Gain)
Code Generation	HumanEval	89.3%	**93.9%**	+4.6
Math Reasoning	MATH	58.8%	**62.2%**	+3.4

For code generation, we pair Qwen2.5-Coder-7B (a code-specialized model) with Mistral-Small-24B (a general-purpose model) on HumanEval. We use the same embedding-based consistency estimator described in Section E.1. For mathematical reasoning, we pair DeepSeek-Math-7B-RL (a math-specialized model) with Llama-3.1-8B-Instruct on MATH, using standard majority voting for consistency estimation. Both experiments sample $N = 5$ responses per model at temperature 0.3.

As shown in Table 16, SLM-MUX achieves consistent improvements in both domains. On HumanEval, the orchestrated system reaches 93.9% Pass@1, outperforming the code specialist (89.3%) by 4.6 percentage points. On MATH, combining the math specialist with a general-purpose model yields 62.2% accuracy, a 3.4pp improvement. These results demonstrate that SLM-MUX generalizes effectively to domain-specific fine-tuned models, successfully capturing complementary strengths between specialists and generalists.

F Licenses for Datasets

The MATH dataset is licensed under the MIT License.
The GPQA dataset is licensed under the Creative Commons Attribution 4.0 International (CC BY 4.0) License.
The GSM8K dataset is licensed under the MIT License.