

Wearable Robotics Assisted Bipedal Balance and Locomotion Control with Whole-body Musculoskeletal Simulations

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Abstract:

The design and control of wearable robotics to assist human movement present a significant challenge, largely due to the complexity of human-robot interaction. Traditional development cycles that rely on extensive hardware prototyping and human subject testing are often costly, time-consuming, and pose potential safety risks. We present a novel simulation framework for the rapid, real-time optimization of wearable robotics hardware and control parameters. Our approach leverages a detailed, whole-body musculoskeletal model to simulate dynamic interactions between the human user and an exoskeleton during both locomotion and balance tasks. This collaborative simulation enables the efficient evaluation of various wearable robotics configurations and control strategies, providing deep insights into their effects on human biomechanics, including muscle activation patterns and stability. We demonstrate the framework's efficacy by optimizing hip exoskeletons for assistance, proving that this approach can rapidly provide actionable insights into device design and control, thereby accelerating the development and reducing the risks associated with creating assistive robotic technologies.

Keywords: wearable robotics, balance control, human locomotion, musculoskeletal system

1 Introduction

Bipedal locomotion and balance have been extensively studied in robotics and control, with research demonstrating the inherent challenges of stabilizing underactuated, high-degree-of-freedom systems [1, 2, 3]. In humanoid robotics, stability is often analyzed using criteria such as limit cycles and gait periodicity [4, 5, 6]. However, these challenges are not exclusive to bipedal humanoid robots. Maintaining stable bipedal posture and gait remains a complex task for humans, for these functions rely on precise neuromuscular coordination, multi-sensory integration, and the nonlinear dynamics of the human musculoskeletal system.

Recent advances in musculoskeletal simulation have enabled biomechanically realistic models of human movement, providing unique opportunities to study balance, locomotion, and muscle-level mechanics in simulation [7, 8, 9, 10, 11]. Recent work introduced a full-body human musculoskeletal system that simulates whole-body dynamics [12]. Such models offer controlled, repeatable conditions that are difficult or unsafe to achieve in physical experiments, making them well suited for investigating interactions between humans and assistive devices under various conditions. This capability is particularly relevant for wearable robotics such as exoskeletons, whose influence on human balance, gait, and muscle activation is often difficult to quantify comprehensively in live trials due to hardware limitations, safety concerns, and measurement constraints.

In this work, we extend full-body musculoskeletal simulation to develop a validation platform for exoskeleton–human interaction. Our framework enables rapid evaluation of how wearable robotics morphology and assistance control policies affect human gait, stability, and muscle activation demands. By integrating detailed musculoskeletal dynamics with controllable representations of human-exoskeleton interaction, we can conduct fast design iterations and primary validation prior to hardware prototyping, reducing development cost and risk.

We demonstrate the use of this simulation–validation pipeline for multiple exoskeleton configurations, highlighting its ability to (i) capture the muscle-level effects of different assistance strategies, (ii) quantify their influence on gait and postural balance, and (iii) provide actionable insights for optimizing both mechanical design and control policy.

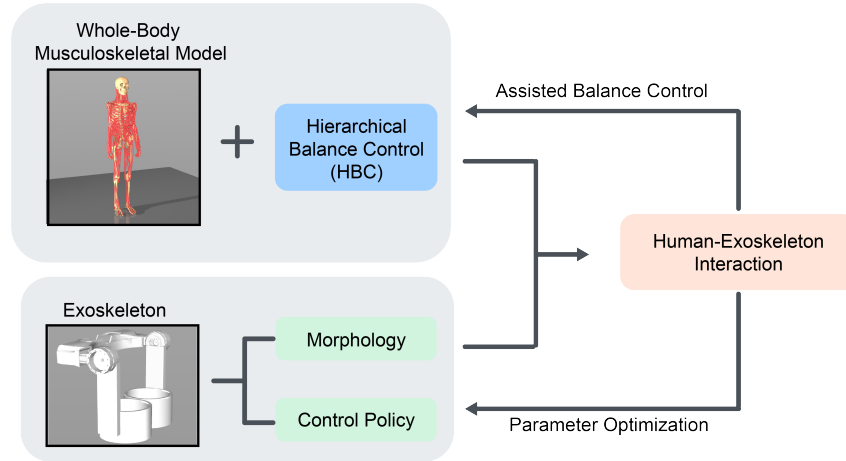


Figure 1: Control and analysis of human musculoskeletal standing and falling. HBC enables training-free balance control and efficient collection of dynamical behavior for balance analysis. Our method supports concurrent control planning for exoskeleton-assisted scenarios, facilitating integrated evaluation and optimization of human-exoskeleton interaction.

2 Related Work

2.1 Wearable robotics for Balance and Locomotion Assistance

Wearable robotics such as lower-limb exoskeletons have been widely used to improve human performance in a variety of locomotion tasks, such as walking, running, and stair climbing, for both able-bodied individuals and those with mobility impairments [13, 14, 15, 16]. They have demonstrated the ability to reduce the metabolic cost of locomotion and restore mobility for individuals with neurological disorders like stroke or spinal cord injury, aiding in gait recovery and daily activities [17, 18]. However, despite their widespread use for locomotion, their application for balance assistance has been less explored, often limited to small-scale studies in controlled lab environments [19, 20].

A major challenge in the development of exoskeletons has been the reliance on traditional control approaches that require extensive, labor-intensive human tests and the creation of handcrafted control laws for each user and activity [21, 17, 22]. This process is not only time-consuming and expensive but also creates a significant bottleneck, making it difficult to personalize controllers and scale them to diverse activities or a larger user base. This has led to a growing need for more advanced, simulation-based methods to streamline the development process [14, 16].

2.2 Musculoskeletal Simulation for Exoskeleton Development

Musculoskeletal simulations are invaluable for studying human movement biomechanics in a non-invasive way [8, 9]. They provide insights into difficult-to-measure metrics like muscle forces and joint loads, making them essential tools for designing [23], controlling [14], and evaluating assistive devices. These models have been used to estimate joint torques [24], serve as control testbeds [25], and optimize design parameters like stiffness and geometry before physical prototyping [26]. They’ve even been customized for specific user populations [15], offering crucial insights for creating safer and more comfortable devices [27].

Despite these advances, a critical need remains for rigorous experimental validation of model-based designs and for improving the fidelity of biomechanical models to better capture the complex dynamics of human-exoskeleton interactions [28, 27]. Many studies use reinforcement learning (RL) algorithms to optimize exoskeletons, which is time-consuming and computationally expensive, often resulting in long training times [14, 29, 16].

Our work aims to address these gaps by presenting a collaborative simulation framework utilizing a full-body human musculoskeletal model and advanced control methods [12, 30, 31] for the rapid, real-time optimization of wearable robotics configurations and control parameters across diverse bipedal balance and locomotion tasks.

3 Method

3.1 Musculoskeletal Model Dynamics

The musculoskeletal model used in this work is the MS-Human-700 model [12]. It comprises of 90 rigid body segments, 206 joints and 700 muscle-tendon units. By actuating its 700 muscle-tendon units, the model can be controlled and perform human-like tasks. The dynamics of the model can be formulated as follow:

$$M(q)\ddot{q} + c(q, \dot{q}) = J_m^T f_m(act) + J_c^T f_c + \tau_{ext}. \quad (1)$$

On the left side of the equation, q stands for generalized coordinates of joints, $M(q)$ stands for the mass distribution matrix, and $c(q, \dot{q})$ stands for Coriolis and the gravitational force. On the right side, J_m and J_c stand for Jacobian matrices that map forces to the generalized coordinates, f_c is the constraint force, $f_m(act)$ stands for actuator forces generated by muscle-tendon units determined by muscle activations (act), and τ_{ext} stands for all external torque when interacting with environments.

MS-Human-700 model is implemented in the MuJoCo physics engine [32]. The actuators of the model in this work are 700 Hill-type [33] muscles. The actuator force generated by each muscle-tendon unit, and the temporal relation between muscle activation act and the input control signal of the musculoskeletal model u can be decided by the following equations:

$$f_m(act) = f_{max} \cdot [F_l(l_m) \cdot F_v(v_m) \cdot act + F_p(l_m)]. \quad (2)$$

$$\frac{\partial act}{\partial t} = \frac{u - act}{\tau(u, act)}, \quad (3)$$

In Eq. (2), F_l and F_v represent force-length and force-velocity functions which are actuator gains, F_p is the passive force that works as actuator bias, and l_m, v_m are normalized length and normalized velocity of the muscle. f_{max} is the maximum isometric muscle force as specified in the model. In the first-order nonlinear system described by Eq. (3), muscle activation act is calculated. The time parameter τ is computed following Millard et al. [34]. τ is the time constant related to the latency in activation and deactivation.

3.2 Postural Control over Muscles

We employed the hierarchical balance control method (HBC) [30] in this work. HBC tackles the very high dimensionality of musculoskeletal model postural control by setting up high-level planning and low-level control. The high-level planning uses Model Predictive Path Integral (MPPI,

105 [35]) to sample low-dimensional target postures, while the low-level control maps the desired target
106 postures back to the 700-dimensional muscle control.

107 3.3 Exoskeleton Control Policy and Optimization

108 As illustrated in Figure 1, the balance and locomotion simulation provides a testbed for strategies
109 and designs of assistive devices, such as a hip exoskeleton, thereby enabling efficient simulation and
110 design of exoskeletal systems. Assisted balance is simulated by applying torque actuation at the hip
111 joints—a widely accepted approach for modeling exoskeleton effects via externally applied forces
112 [36, 37].

113 We implemented a weighted postural PD control over the joint torque actuation placed at the left and
114 right hip flexion joints. The high-level planner is adapted to plan an extra target posture indicating
115 the overall leaning direction of the body, represented by the tilt angle of the pelvis. The control
116 policy of the exoskeleton torque actuation is separated into two parts mixed by a weight: (1) Hip
117 flexion joint angle PD control. (2) Postural PD control. The control is formulated as follow:

$$\tau_e^i = (1 - w) \cdot (k_{p_e} \cdot (q_i^* - q_i) + k_{d_e} \cdot (0 - \dot{q}_i)) + w \cdot (k_{p_t} \cdot (q_t^* - q_t) + k_{d_t} \cdot (0 - \dot{q}_t)). \quad (4)$$

118 $i = 1, 2$ represents the left or right side respectively. τ_e^i is the torque actuation value. k_{p_e} and k_{d_e} are
119 the joint angle PD control constants, while k_{p_t} and k_{d_t} are the postural PD control constants over
120 the tilt angle of the pelvis. q_i^* and q_t^* are the target values of the hip joint angles and the pelvis tilt
121 angle. w is the weight between the two PD control policies.

122 We found that the assistive effect of the exoskeleton was very sensitive to the k_{p_e} , k_{p_e} , k_{p_t} and
123 w values. Therefore, we carried out Bayesian optimization (BO [38, 39]) to determine a set of
124 parameters to ensure performance across trials. We define $\mathbf{x} = (k_{p_e}, k_{p_e}, k_{p_t}, w)$, and formulate the
125 parameter search as a black-box optimization problem:

$$\max_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}) = \mathbb{E}_{\mathbf{x}} \left[- \sum_{t=0}^{T-1} C(s_t, u_t) \right] \quad (5)$$

126 where the objective $f(\mathbf{x})$ is the negative cumulative cost function under parameter \mathbf{x} , averaged over
127 5 independent trials. Given sampled data, We use Gaussian process [40] to model the posterior of the
128 objective function, and utilize the Expected Improvement [41] acquisition function as the criterion
129 to sample the next parameter.

130 4 Experiments

131 In the experiments, we validated the assistance provided by hip exoskeletons on the MS-Human-700
132 model during locomotion and balance tasks, and carried out analysis on the assistive influence of the
133 hip exoskeleton on muscle effort.

134 We used MuJoCo MPC [42] in our experiments to deploy HBC in simulation time. With the very
135 high dimensionality of the MS-Human-700 model, the simulation speed was set to 10%. A 5-second
136 experiment will cost 50 seconds, which is much shorter than the training time of reinforcement
137 learning methods.

138 4.1 Simulation of Assisted Balance

139 We simulated an assisted balance scenario with a hip exoskeleton, as shown in Figure 2a. We opti-
140 mize the parameters of the exoskeleton control policy over 600 iterations. We employed optimized
141 exoskeleton control parameters and validated the effectiveness of this exoskeleton control policy
142 in a perturbation test: Models with and without the exoskeleton assistance are pushed in random
143 directions for 3 times with intervals of 1 second. As shown in Figure 2b, balance with exoskele-
144 ton assistance achieves a higher success rate in maintaining balance under perturbation through the
145 5-second simulation.

146 In Figure 2c, we carried out an ablation study over the assisted balance by recording the muscle ac-
 147 tivations of the gluteus maximus, gluteus medius, and gluteus minimus, which play critical roles in
 148 lower-limb movement and postural stability. We observe a reduction in muscle activation levels with
 149 exoskeleton assistance, highlighting the potential of assistive devices to alleviate muscular effort and
 150 reduce metabolic cost. Such muscle-level data are difficult to obtain in traditional experimental set-
 151 tings due to the limitations of surface EMG and the inaccessibility of deep muscles. Our simulation
 152 pipeline enables validation of exoskeleton effects on balance maintenance. These findings suggest
 153 the utility of musculoskeletal simulations in evaluating and optimizing assistive device performance
 154 prior to costly physical prototyping and human subject testing.

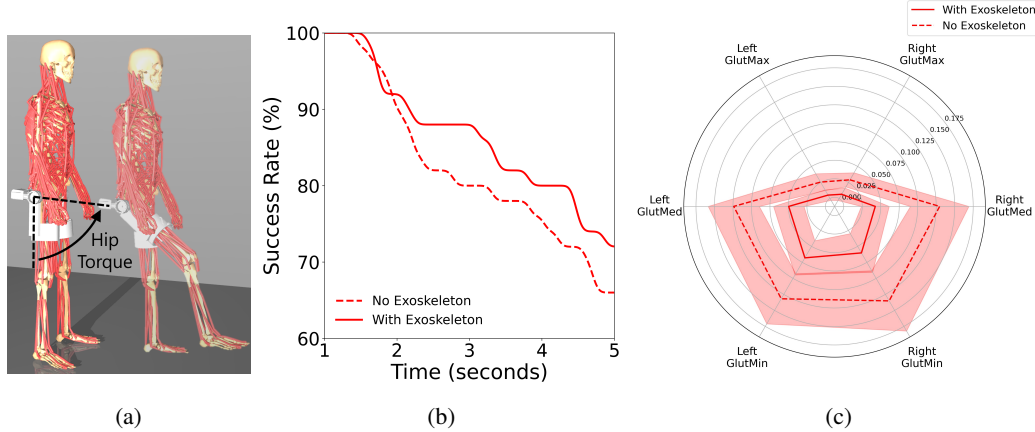


Figure 2: Balancing behavior under exoskeleton assistance. (a) Visualization of the hip exoskeleton device, using joint torques as actuators. (b) Success rate of maintaining standing posture, where the exoskeleton assistance helps better static balance. (c) Activation levels of gluteal muscle activations (GlutMax: gluteus maximus, GlutMed: gluteus medius, GlutMin: gluteus minimus) during the standing simulation, where muscle activation levels are reduced with exoskeleton assistance.

155 4.2 Simulation of Assisted Locomotion

156 We further designed an assisted locomotion scenario to investigate the effect of hip exoskeletons.
 157 The task cost function includes forward velocity, which encourages the model to walk forward. In
 158 the assisted locomotion task, we simulated the interaction between human body and a hip exoskele-
 159 ton by a virtual representation of high-rigidity spring connections located at thighs and waist. The
 160 shape and weight distribution of the hip exoskeleton are considered in simulation, and the actuation
 161 of hip exoskeletons are modeled with actuated joints located at the motor compartment.

162 The control policy of the exoskeleton is crafted by hand based on proportional control and gait
 163 recognition, providing a propulsive torque to the swing leg and a counteracting torque to the stance
 164 leg during the pre-swing phase of a gait cycle. The deployment of various exoskeleton control
 165 policies, as well as testing the effects of different exoskeleton design prototypes are efficient.

166 5 Conclusion

167 In this work, we introduced a simulation-based method for the co-optimization of wearable robotics
 168 design and control with a dynamic, whole-body human musculoskeletal model. We have success-
 169 fully demonstrated its capability as a validation pipeline for evaluating assistive strategies in both
 170 locomotion and balance-critical scenarios. Our experiments confirm that the method can effectively
 171 quantify the influence of different exoskeleton control policies on human muscle effort and postural
 172 stability, enabling rapid, real-time parameter tuning.

173 By providing a high-fidelity testbed for whole-body human-exoskeleton interaction, our work
 174 bridges a critical gap between theoretical modeling and costly physical experiments. While our

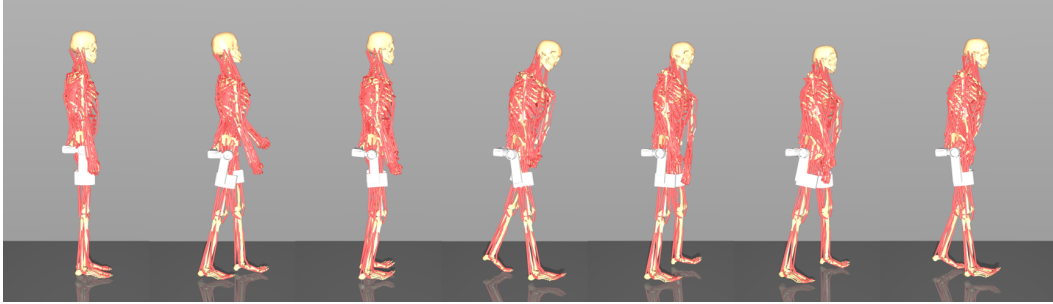


Figure 3: Human musculoskeletal model locomotion task with exoskeleton assistance. HBC enables training-free locomotion assistance demonstration and efficient parameter optimization.

175 current implementation utilizes a simplified representation of the exoskeleton’s mechanical inter-
 176 face, future work will focus on incorporating more complex, realistic device models and leveraging
 177 data-driven optimization techniques to explore the co-adaptation of human and robotic controllers to
 178 accelerate the design cycle for wearable robotics. Ultimately, this simulation-driven approach holds
 179 significant promise for the development of safer, more effective, and highly personalized wearable
 180 robotics tailored to the specific needs of individual users.

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