Distinctions Between Causality and Association

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Abstract

In Hume's philosophy, causality represents the logical relationship between a cause and its effect, which is inferred from observation. However, it is crucial to note that the concept of causality differs from association based on observation. In this essay, we will commence by revisiting the concept of causal hierarchy and subsequently delve into a specific case study to explore the nuanced distinctions between causality and association.

1 Introduction

As detailed in Table 1, Judea Pearl, the renowned father of Bayesian networks and a Turing Award recipient, has categorized causality into three distinct hierarchies: association, intervention, and counterfactual [2]. In the context of association, the focus lies on observation and understanding how variables are interconnected. It explores questions such as, 'What insights can be gleaned from a phenomenon about the weather?' and 'What can the presence of a symptom reveal about a disease?' These inquiries can be summarized as 'What if I see...?'

Intervention, on the other hand, centers around taking action and understanding the consequences of doing so. It delves into questions like, 'If I were to take this medication, would it lead to the cure of my ailment?' and 'What outcomes can be anticipated if a particular policy is enacted?' These scenarios can be encapsulated as 'What if I do...? How?'

Lastly, the counterfactual hierarchy revolves around imaginative retrospection. It examines whether a particular event (X) caused another (Y), and contemplates how Y would have transpired had X not occurred or if one had taken different actions. This level of causality is concerned with questions like, 'What if I had not been accepted into Peking University?' and 'Would my research direction have been more successful if I had made alternative choices?' These queries can be summarized as 'What if I had done...? Why?'

Level (Symbol)	**Typical Activity**	**Typical Questions**	**Examples**
1. Association $P(y\|x)$	Seeing, Observing	What if I see ...? How are the variables related? How would seeing X change my belief in Y?	What does a phenomenon tell me about the weather? What does a symptom tell me about a disease?
2.Intervention $P(y\|do(x), z)$	Doing, Intervening	What if I do ...? How? What would Y be if I do X? How can I make Y happen?	If I take this medicine, will my disease be cured? What will happen if we pass the policy?
3.Counterfactual $P(y_x\|x', y')$	Imaging, Retrospection	What if I had done ...? Why? Was it X that caused Y? What if X had not occurred? What if I had acted differently?	what if I had not entered Peking University? would it be better if I had not chosen this research direction?

Table 1: Causality Hierarchy.

2 Causal Effect

We start from a case to investigate the calculation of causal effect and how it differs from the calculation of association.

Suppose that we want to infer whether a drug is useful for survival and there are two possible actions that can be applied to an individual:

- 1("treatment")
- 0("control")

where 1 indicates providing the patient with the drug while 0 indicates not. For each individual in the population, there are two potential outcomes:

- Y(1): outcome if treatment applied
- Y(0): outcome if control applied

where the potential outcome can be survival or not. The causal effect of the action for an individual is the difference between the outcome if they are assigned treatment or control:

$$Y(1) - Y(0) \tag{1}$$

However, when engaging in causal inference, we encounter a common issue wherein, in any given example, we can only observe one of the two potential outcomes for each individual. To clarify, for each patient, we can solely ascertain either the outcome associated with taking drugs or the outcome associated with not taking drugs, thereby presenting a challenge of missing data. As illustrated in Table 2, the 'question mask' signifies the presence of missing data, rendering it impossible to calculate the causal effect for each individual.

i	**T**	**Y**	**Y(1)**	**Y(0)**	**Y(1)-Y(0)**
1	0	0	?	0	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
6	1	1	1	?	?

Table 2: Missing Data of the Experiments.

In Table 2, i denotes a specific individual (i.e., a patient), T denotes observed treatment, Y denotes observed outcome, $Y_i(1)$ denotes the potential outcome under treatment, $Y_i(0)$ denotes the potential outcome under no treatment.

To tackle the issue of missing data, an intuitive solution is to compute the average treatment effect instead of individual treatment effects, leading to the following equation. However, it's important to note that this approach doesn't always yield accurate results:

$$E[Y(1) - Y(0)] = E[Y(1)] - E[Y(0)] = E[Y|T = 1] - E[Y|T = 0] \tag{2}$$

However, $E[Y(1)] - E[Y(0)]$ is the causal difference while $E[Y|T = 1] - E[Y|T = 0]$ is the associational difference. Only when the ignorability is satisfied can we do this:

$$(Y(1), Y(0)) \perp\!\!\!\perp T \tag{3}$$

As depicted in Figure 1, the Bayesian Networks provide a visual representation of the relationships among Drug, Stage, and Survival. When we compute the associational difference conditioned on Drug, it becomes evident that the distribution of Stage varies accordingly. However, this variation does not align with the treatment and control actions, rendering them inherently unequal.

3 Conclusion

In conclusion, it becomes evident that causality differs from association in several key aspects. This distinction serves as a crucial reminder that, although association relationships in the data can be effective in numerous scenarios, they may occasionally lead to inaccuracies, particularly due to their placement within different levels of the causal hierarchy. In such instances, association can obscure the true underlying causal effects, potentially resulting in erroneous decision-making judgments. Such inaccuracies are especially unacceptable in fields like healthcare[3] and autonomous driving[1].

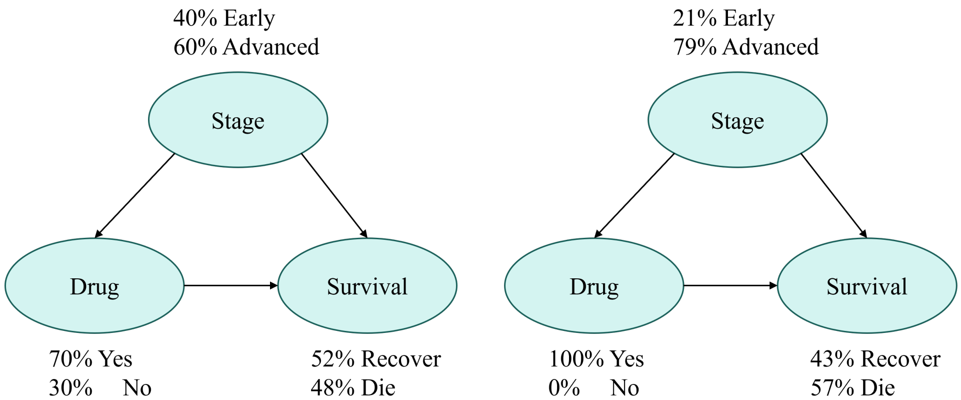

Figure 1: Bayesian Networks for the drug, stage and survival.

References

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[2] Judea Pearl. *Causality*. Cambridge university press, 2009. 1

[3] Athanasios Vlontzos, Daniel Rueckert, Bernhard Kainz, et al. A review of causality for learning algorithms in medical image analysis. *Machine Learning for Biomedical Imaging*, 1(November 2022 issue):1–17, 2022. 2