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Direct Quantification for Stability from Linear Dynamical Systems on Networks

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Stability, the ability of a system to return to a steady state after perturbation, is a fundamental property of networks, critical in contexts ranging from ecological networks to power grids. Despite its importance, existing approaches to stability assessment, such as linear stability analysis based on the dominant eigenvalue of the Jacobian matrix, offer only heuristic insights since they ignore the remainder of the eigenspectrum and do not directly quantify deviation from stability. Moreover, approaches to quantify the contribution of individual nodes to the stability (or lack thereof) of the system, such as eigenvector centrality, also fail to capture the complete dynamic response of individual nodes under perturbation. These limitations drive the need for methods that can directly quantify how network systems may be expected to deviate from stability, especially for directed networks.
Here, we introduce a novel technique for directly quantifying the expected deviation from stability as a function of the directed network structure, assuming linear dynamics around a fixed point. Our measure for the deviation from stability, D_{st}, is computed analytically via a power series expansion of network's weighted, directed connectivity matrix (building on formulations of the network covariance matrix in this fashion [1, 2]). The expansion can be interpreted as a weighted sum over process motifs, which serve as a dynamic building blocks of network behavior. Our results is that the deviation from stability directly corresponds to a weighted sum of convergent paired walks on the network [1, 3] (Figure 1).

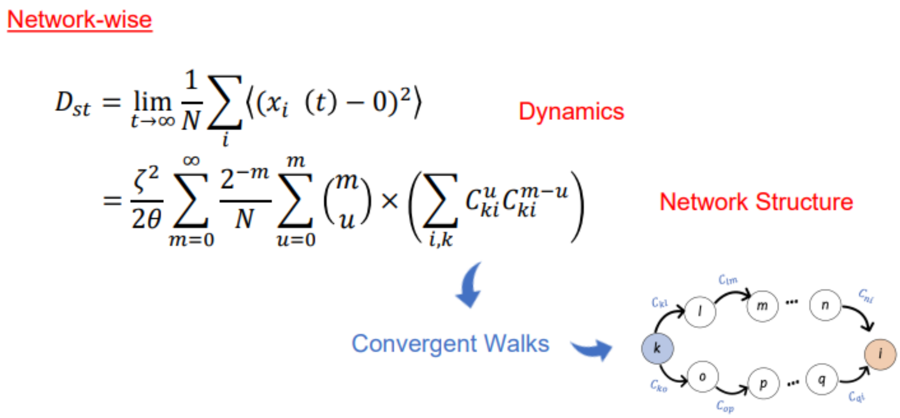
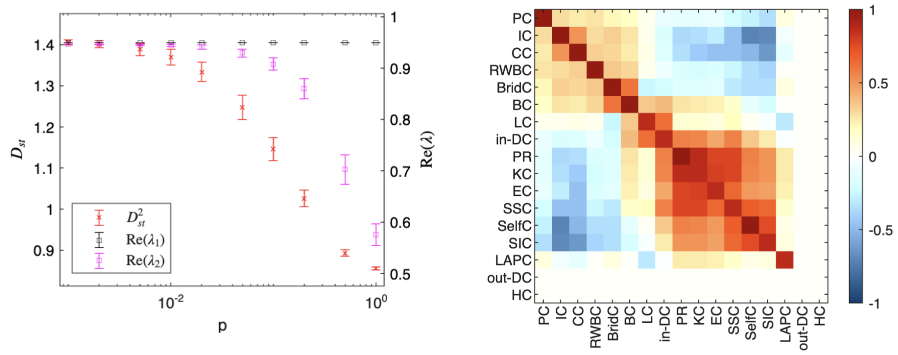


Figure 1: **The dual convergent walks in the networks.** Its impact on stability D_{st} is represented by the (weighted) count of convergent walks that either originate (driving) or terminate (driven) at the specific node.

Compared to heuristic approaches that rely solely on the dominant eigenvalue, our method provides a more comprehensive, accurate, and interpretable characterization of network stability. This is clearly demonstrated in our experiments on 100-node Watt-Strogatz networks, where the largest eigenvalue remains unchanged across randomization of the network in the small-world transition, failing to reflect the underlying changes where our measure D_{st} shows that the dynamics actually become substantially more stable (Figure 2 (a)). While the second-largest eigenvalue shows partial sensitivity, our proposed measure D_{st} captures the full spectrum of structural variation. This highlights the method's ability to reveal dynamic properties that traditional eigenvalue-based metrics overlook.



(*a*) (*b*)

Figure 2: **(a) Stability measurement throughout a small-world transition on an N-100 Watts-Strogatz ring networks. (b) Mean pairwise correlation heatmap of 17 centralities in direceted networks.** The networks are generated as Watts-Strogatz rings with $N = 100$ nodes, varying the rewiring probability p to induce a source node randomization. Parameters are fixed at initial in-degree $d = 4$, self-weights $C_{ii} = 0.25$, and cross weights on connected nodes of $C_{ij} = 0.175$. The largest eigenvalues are constrained to remain unchanged.

At the node level, our framework introduces two centrality measures: stability influence (SIC) and stability susceptibility centrality (SSC), which measure how individual nodes contribute to the network's deviation from stability as sources and targets, respectively, of the convergent walks in (Figure 1). In our comparative analysis involving 17 centrality metrics, SIC and SSC demonstrate unique informational value (Figure 2 (b)), as evidenced by their low Spearman correlation with most existing measures. Notably, their similarity to self-communicability suggests a meaningful connection with all three considering directed weighted walk counts. These results confirm that our method not only captures network-level stability but also provides unique insights at the node-level, surpassing heuristic centrality measures in both depth and clarity.

As a mathematical work without obvious potential for unethical exploitation, there are no clear ethical considerations at the moment.

References

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