Learning Tokenization in Private Federated Learning with Sub-Word Model Sampling

Abstract

Federated learning with differential privacy, i.e. private federated learning (PFL), makes it possible to train models on private data distributed across users’ devices without harming privacy. However, it is only known how to do this for models, such as neural networks, that have a fixed number of parameters, and thus a fixed-dimensional gradient vector. Such models include neural-net language models, but not n-gram language models or, indeed, tokenizers, the topic of this work. Training a tokenizer normally requires access to the training data. An alternative is to train the tokenizer on publicly available data, but this, we show, degrades accuracy for a next-word prediction task by 10–20% across different datasets and models. We propose to take a tokenizer built on public data, use it to train a language model with PFL, and sample from the language model to find a new tokenizer. Retraining with the new tokenizer brings performance to within 2% of the oracle tokenizer, without expending additional privacy budget. Finally, we build a new federated pipeline to update the tokenizer during model training by modifying affected model embeddings.

1 Introduction

Learning a language model (LM) requires a dataset that in many situations is private, resides on people’s devices, and should stay there. In federated learning (McMahan et al., 2017), a central server learns a model by receiving statistics, like parameter updates, from many devices. Though devices send only statistics and not the raw data, federated learning by itself can leak information about the data (Shokri et al., 2017; Song et al., 2017). Private Federated Learning (PFL) (McMahan et al., 2018; Geyer et al., 2017) uses differential privacy (Dwork et al., 2006, 2014) to mitigate the privacy leaks by limiting the user’s impact on the final model.

It is known how to train neural-net language models using PFL (McMahan et al., 2018). However, an important part of language modeling is tokenization: turning text into a sequence of symbols from a fixed-size symbol set. To obtain a tokenizer, published research on private federated learning of language models uses either of two approaches, neither of which are satisfactory. One approach is to train the tokenizer on user data directly. The commonly-used LEAF dataset (Caldas et al., 2018) and works relying on it (Li et al., 2021; Hu et al., 2021; Yu et al., 2020) assume access to the training data to create the tokenizer. This is not relevant to real-world use cases and undermines user privacy. The other approach is to use public unrelated data to obtain the tokenizer (McMahan et al., 2018). This is sensible from a privacy perspective, but as we show the resulting distribution mismatch harms performance, resulting in 10%-20% drop compared to using an “oracle” tokenizer trained directly on users’ private data.

There are two common types of tokenization, which are affected by mismatched distributions in different ways: word and sub-word tokenization. Word-level tokenization assigns an out-of-vocabulary token (OOV) to each unseen word. Text from mismatched distributions will generally contain unseen words, which means the correct word cannot be predicted, and the context becomes less meaningful when predicting the next word. It’s possible to discover new words with the character-level model (Beaufays et al., 2019), however this method requires a separate model, training run, and a dedicated privacy budget. Sub-word tokenization splits some words into multiple smaller tokens. This type of tokenization is generally chosen to minimize the average number of tokens per word. Current centrally trained models use sub-word tokenization such as Byte-Pair Encoding (Sennrich et al., 2016), SentencePiece (Kudo and Richardson, 2018), or WordPieces (Schuster and Nakajima, 2012). Nevertheless, mismatched tokenizations in sub-word methods cause an increase in the number of tokens.
per word, and thus decrease the amount of context
the model can use to predict the distribution of the
next word.

In this work we present a general framework to
approach training language models in private fed-
erated learning by including tokenization as part of
the training pipeline. Our contributions are: (1) we
uncover the performance gaps when the models use
the tokenizer obtained from a different distribution
vs the tokenizer obtained from the underlying dis-
tribution. For word-level tokenization we show that
a tokenizer trained on public data reduces the next-
word prediction accuracy of 10–20 % compared to
a tokenizer estimated on user data. (2) We demon-
strate significant benefits of switching tokenizers
from word to sub-word level, thus eliminating the
out-of-vocabulary problem. (3) We propose a new
method that samples data from an existing model
and uses the data to initialize a new tokenizer and
update the model.

2 Tokenization in language modeling

A language model is a model that assigns proba-
bilities to sequences of tokens. In this paper, it
is always an autoregressive model with parame-
ters \( \theta \): 

\[
P_\theta(s) = P_\theta(t_2|t_1 = \text{BOS}) \cdot P_\theta(t_3|t_1 = \text{BOS}, t_2) \cdots P_\theta(t_n = \text{EOS}|t_1 = \text{BOS}, \ldots, t_{n-1}),
\]

where each term in this equation is normalized
over all possible values of the current token. Lo-
cal normalization is useful when decoding input,
like in speech recognition or a soft keyboard (Hard-
et al., 2018).

For this paper, assume that a corpus is segmen-
ted into sentences. A tokenizer \( \tau \) then converts each
sentence \( s \) into a sequence of \( n \) to-
kins \( \tau(s) = [\text{BOS}, t_2, \ldots, t_{n-1}, \text{EOS}] \). There are
two broad types of tokenizers for language mod-
elling. A word-level tokenizer produces whole
words as tokens. This implies a closed vocabu-
lary, in which new words cannot be represented.
Sub-word tokenizers, on the other hand, split some
words into smaller pieces. Some types of sub-word
tokenizers can allow an open vocabulary, where any
unseen word can still be represented by a sequence
of tokens.

2.1 Word-level tokenization

Papers that study language models in feder-
erated learning commonly use word-level tokeniza-
tion (McMahan et al., 2017). For a vocabulary of
size \( N \) the tokenizer assigns a unique token for top-
\( N \) most popular words in the dataset while other
words receive an out-of-vocabulary token \( \text{OOV} \).

Some papers (e.g. McMahan et al., 2018) build
the tokenizer from a publicly available dataset, oth-
ers including the LEAF benchmark (Caldas et al.,
2018) build the tokenizer from users’ training data.

Word-level schemes can be negatively affected
by the high number of \( \text{OOVs} \) in the test data, reduc-
ing essential context that language models need.

Private federated learning further complicates the
problem as the training data distribution is un-
known to the central server; the model inputs have
only few non-\( \text{OOV} \) tokens during training. For ex-
ample, in Section 5 we observe an increase in \( \text{OOV} \)
share from 5 % to 13 % for the Reddit dataset and
from 2 % to 10 % for the StackOverflow dataset
when initializing the \( N = 10,000 \)-word vocabu-
larv using Wikipedia data. Although Wikipedia
articles look far from the data in both datasets, it is
fair to assume that real-life applications can diverge
even more when participants are using different
dialects, topics, and contexts (we further explore
other datasets in Appendix A).

2.2 Sub-word tokenization

There are two popular schemes for sub-word to-
kenization: byte-pair encoding (BPE) (Sennrich
et al., 2016) and WordPieces (Schuster and Naka-
jima, 2012). We focus on BPE which unlike Word-
Pieces guarantees the absence of \( \text{OOVs} \) as there
exists a token for every byte. However, the number of tokens required to encode each word can change
significantly depending on the dataset that the to-
kenizer was trained on. For example, a tokenizer
trained on data before the COVID-19 pandemic
would not assign the word “covid” a separate token
and instead split the word into multiple parts.

Generating longer token sequences has signifi-
cant implications when training language models.
The greater the number of tokens, the longer the
context a language model needs to keep track of.
Even LSTMs and other RNNs, which in theory can
use arbitrarily long history, have imperfect memory.
Therefore, the model performance can degrade due
to a lack of context as the mismatched tokenizer
splits words into more tokens than optimal.

2.3 Comparing across tokenizations

Comparing language models across tokenizations
is a complex problem. For example, when compar-
ing word-level language models using perplexity,
often \( \text{OOVs} \) are ignored which gives an edge to the
language model with more OOVs, which is the opposite of what is desired. The following sections detail the problems when comparing sub-word language models.

2.3.1 Comparing word-level with sub-word
Since a word-level language model has a closed vocabulary, it outputs probabilities only for the specific set of words. An open-vocabulary language model can output the probability of any sequence. This distinction artificially lowers the perplexity of closed-vocabulary LMs, particularly on data with a large number of OOVs.

One alternative is to compute the perplexity on the sub-word model trained on the dataset with words removed that are OOV for the other model. That allows us to train the model on the same data that the word-level tokenizer would produce. However, it disadvantages the sub-word system, which assigns probability mass to out-of-vocabulary words.

A better alternative, which this paper will use, is to compare model performance the word-level accuracy. The most accurate way would be to find the word with the highest probability by summing over sequences of tokens. However, we choose a simpler, though less accurate method (similar to Likhomanenko et al., 2019): repeatedly generate the best tokens within each word’s bounds and only accept the word as accurate if all generated tokens were correct.

2.3.2 Comparing sub-word with sub-word
It is possible to meaningfully compare perplexities of two language models with different sub-word tokenizations, although with one niggle and one tweak (Mielke, 2019).

First, the niggle. A language model assigns probability mass to all token sequences. However, a single sentence can have multiple corresponding token sequences, one of which will be chosen by the tokenizer. Some of the probability mass will therefore be lost to never-occurring token sequences. However, it is infeasible to sum over all token sequences (Likhomanenko et al., 2019).

Second, the tweak. The danger with comparing perplexities directly is that since models with different tokenizers operate on different sets of tokens the number of tokens needed to encode each sentence is different in general (Mielke, 2019). However, note that all models assign a probability to a sentence (with the niggle above). For the model \( \theta \) trained with the tokenizer \( \tau \), and a sentence \( s \) that contains tokens \( t_1, \ldots, t_n \) where \( t_1 = \text{BOS} \) and \( t_n = \text{EOS} \):

\[
P_{\theta, \tau}(s) \simeq \prod_{i=1}^{n} p(t_i | t_1 \ldots t_{i-1}) =
\]

\[
p(t_2 | \text{BOS}) \cdot \ldots \cdot p(\text{EOS} | \text{BOS}, \ldots, t_{n-1})
\]

(1)

Similarly the model \( \theta' \) trained with the tokenizer \( \tau' \) would output \( P_{\theta', \tau'}(s) = P_{\theta'}(t'_1, \ldots, t'_n) \), where \( n \neq n' \) in general. Both \( P_{\theta, \tau}(s) \) and \( P_{\theta', \tau'}(s) \) represent the probability of outputting the same sentence \( s \) regardless of the tokenization methods or the model and thus are comparable.

The tweak, then, is to use the same denominator in computing the perplexity: the number of words in the sentence instead of number of tokens, which depends on the tokenizer. Therefore we define the perplexity as:

\[
ppl_{\theta, \tau}(s) = \exp \left( -\frac{\log(P_{\theta, \tau}(s))}{\|s\|_w} \right)
\]

(2)

where \( \|s\|_w \) counts the number of words in the sentence \( s \). For a dataset containing many sentences, the probabilities multiply across sentences. To generalize (2), the log-probabilities can be summed and the whole divided by the total word count in the dataset.

3 Private federated learning
In many scenarios, text data is private and contained on people’s devices, and should stay there. Learning a central model is then possible only with federated learning (McMahan et al., 2017). This involves devices sending not the data, but statistics derived from the data (e.g. gradients). However, even from just the statistics, it is often still possible to derive private information including user participation in training (Shokri et al., 2017; Song et al., 2017; Melis et al., 2019).

To mitigate this threat we can combine federated learning with differential privacy (DP) (Dwork et al., 2006, 2014), to give private federate learning (McMahan et al., 2018). Differential privacy gives a strong guarantee: it limits the advantage that a computationally unconstrained adversary has in inferring whether an individual’s data is contained in the data set that the statistics are computed from. \((\epsilon, \delta)\)-differential privacy parametrizes this advantage by \( \epsilon \) (the maximum privacy loss) and \( \delta \) (a slack term).
The common mechanism to provide differential privacy in a federated setting is the Gaussian mechanism that uses moments accountant (Abadi et al., 2016). This can be used in “local differential privacy”, where each device is responsible for adding sufficient noise, or “central differential privacy”, where a trusted third party collects data from individuals and outputs noisy statistics. Central DP is attractive since it does not require adding as much noise. However, for machine learning tasks, a trusted third party is usually unavailable. Luckily, it is known how to replace the trusted third party can be replaced by secure multiparty computation for one particular type of statistic (Goryczka and Xiong, 2015; Bonawitz et al., 2017): a noisy sum over individual contributions that are fixed-length vectors. Phrasing an algorithm for federated learning in terms of such a sum can therefore enable central DP instead of local DP, greatly improving utility in practice.

Examples of algorithms for federated learning that perform gradient-based learning, e.g., for neural networks, are federated SGD and federated averaging (McMahan et al., 2017). In federated SGD, the server holds a model and sends it to a cohort of devices. Each device computes a gradient on its data and sends this gradient back. Critically, each device’s gradient has the same dimensionality, so that the sum of the gradients can be noised to provide differential privacy. The server then updates the model in the direction of the average gradient. Federated averaging (McMahan et al., 2017) is a generalization of federated SGD where each device takes multiple gradient steps and contributes the sum of these gradients. Other techniques for federated optimization (Wang et al., 2021) can further improve performance, however for this paper will use federated averaging, since it works well in practice, and combine it with differential privacy.

3.1 Privately finding vocabulary items

To perform federated learning on data that is not straightforwardly described as summable vectors, custom algorithms are needed. There exist differentially private algorithms to compute a histogram over multisets of elements (e.g., words) distributed over devices. These are called “heavy hitters” algorithms (Bassily et al., 2017; Zhu et al., 2020; Apple, 2017). However, heavy hitters algorithms require a separate privacy budget (since $\epsilon$ essentially add up in DP) and in Section 5 we show that the created vocabulary does not provide a significant performance boost.

Another way of finding vocabulary items privately is to train a neural-net generative model. Beaufays et al. (2019) trains a separate, character-level LSTM model to generate the new words. However, the proposed method only works to discover OOVs in a word-level model and also requires separate training and a privacy budget.

New techniques exist to improve tokenization (Xu et al., 2021; Park et al., 2021; Guo et al., 2021), but they require access to training data.

4 Learning a tokenizer with private federated learning

We focus on a common application of federated learning: training a language model, parameterized by $\theta$, using federated learning with differential privacy. In our setting each user $u_i$ has a dataset $d_i$ of private texts from a private distribution of user data $D$. The trained model will be evaluated against a held-out dataset $D_{test}$, e.g., a mix of all user data, which in practice must be replaced by federated evaluation.

We assume that the central server does not have access to the user data distribution $D$ and can only approximate it with the publicly available dataset $D_{pub}$. We assume the public data is some commonly available dataset, such as Wikipedia (Merity et al., 2017). The tokenizer trained on this public data will be $\tau_{pub}$. For comparison we assume the existence of an oracle tokenizer $\tau_o$ initialized on users’ training data $D$.

Papers that study language models in federated learning commonly use word-level tokenization. While some papers (e.g. McMahan et al., 2018), build the vocabulary using publicly available dataset, others (e.g. Yu et al., 2020; Caldas et al., 2018) explicitly use the federated training data, even though in real-world scenarios the analogous data would be unavailable and it violates privacy guarantees when used in PFL (Li et al., 2021).

**Problem definition.** We aim to obtain a tokenizer that works well on users’ federated data without compromising user privacy. First, we aim to find the appropriate tokenization scheme, and second, given the tokenization scheme obtain the right approximation of user data to train the tokenizer.

**Proposed solution.** We pick a sub-word tokenizer with an open vocabulary that allows the language
model trained with such a tokenizer to represent any word, if inefficiently. It is then possible to query the language model to find new words. This is the core of the Algorithm 1 that this paper introduces.

Figure 1 shows the proposed pipeline. A language model is trained with private federated learning. This results (on the left) in a model matched with an old, stale tokenizer. The next block queries the language model to produce a better tokenizer, with a method that section 4.1 will detail. The block after that updates the language model for the new tokenizer, using reasonable guesses for the new parameters. This results in a new LM-tokenizer combination that can be trained further with PFL.

We assume that the language model obtained with the stale tokenizer is trained with a certain privacy budget. The postprocessing guarantee of differential privacy means that the steps other than private federated learning do not consume any further budget. The function UPDATE in Algorithm 1 performs the on-server steps. The following sections will give more detail.

4.1 New tokenizer from a trained LM

Training a tokenizer requires text data. Since the raw data is not available, we propose to instead sample from the LM matched with the stale tokenizer, as detailed in Algorithm 1. The SAMPLETOKENS function samples from the language model, drawing sequences of tokens according to the probabilities that the model assigns to them. The SAMPLE function then converts these sequences in the old tokenization into word sequences, by decoding with \( \tau_{\text{pub}} \). Once a large enough corpus of word-level sentences has been produced, training a tokenizer proceeds as normally (the TRAINTOKENIZER function is not specified).

4.2 Changing tokenizer on an existing model

After a new tokenizer \( \tau \) has been trained, the language model, trained with \( \tau_{\text{pub}} \), must be updated to work with the new tokenizer. Neural-net language models use an embedding layer to convert the provided tokens into multi-dimensional vectors. It is the embedding vectors that are most important to modify when changing the tokenizer. The rest of the model only consumes the embedding vector. It is not possible to find the optimal parameters without further training of both embeddings and other layers, but we propose an algorithm to find a reasonable starting point, in the function REMAP(\( \tau, \tau_{\text{pub}} \)) in Algorithm 1.

REMAP iterates over the tokens from the new tokenizer \( \tau \) and creates the mapping from the tokens’ embedding in the public tokenizer \( \tau_{\text{pub}} \) to the new token’s embedding. In some cases it is a one-to-one mapping, but when the new token accumulates multiple tokens in \( \tau_{\text{pub}} \) we split the weight equally between each token.

Once we have the mapping \( \text{map} \) we modify the embedding layer of the model by performing matrix multiplication, i.e. \( \theta_{\text{embedding}} = \text{map} \cdot \theta_{\text{embedding}} \). The resulting model can accept the tokens from the new tokenizer \( \tau \), and can participate in future training in federated learning.

5 Experiments

We evaluate effects of tokenizers trained on the distributions matched and mismatched to real data, we test the proposed approach on different datasets for federated learning.

5.1 Experimental setup.

We use two datasets common in the federated learning literature (Kairouz et al., 2019). While both use English, there is nothing about our experiments that is specific to this language, and multilingual
Algorithm 1 Model sampling algorithm

Inputs: model \( \theta \), current sentence \( s \), new tokenizer \( \tau \), public tokenizer \( \tau_{\text{pub}} \), size of the sampled dataset \( \text{corpus.size} \).

function \( \text{SAMPLETOK}\text{ENS}(\theta, s) \)

- \( t_{\text{next}} \sim \theta \cdot t_k | s \)
- if \( t_{\text{next}} = \text{EOS} \) then return \( s \leftrightarrow t_{\text{next}} \)
- else return \( \text{SAMPLETOK}\text{ENS}(\theta, s \leftrightarrow t_{\text{next}}) \)

function \( \text{SAMPLE}(\theta, \tau) \)
return \( \tau.\text{decode}(\text{SAMPLETOK}\text{ENS}(\theta, [\text{BOS}])) \)

function \( \text{REMAP}(\tau_{\text{pub}}, \tau) \)
map = \( \text{zeros}(\tau.\text{size}, \tau_{\text{pub}}.\text{size}) \)
for token, tid \( \leftarrow \tau.\text{vocab} \) do
  tokens = \( \tau_{\text{pub}}.\text{decode}(\text{token}) \)
  for token \( \leftarrow \text{tokens} \) do
    \( \text{tid}_{\text{pub}} = \tau_{\text{pub}}.\text{vocab}[\text{token}] \)
    map[\text{tid}_{\text{pub}}, \text{tid}] = \frac{1}{\text{len}(\text{tokens})}
return map

function \( \text{UPDATE}(\theta, \tau_{\text{pub}}) \)
while \( \text{len}(\text{corpus}) < \text{corpus.size} \) do
  corpus \( \leftarrow \text{SAMPLE}(\theta, \theta, l_{\text{max}}) \)
  \( \tau = \text{TRAIN\text{Tokenizer}}(\text{corpus}) \)
  map = \( \text{REMAP}(\tau_{\text{pub}}, \tau) \)
  \( \theta.\text{embedding} = \text{map} \cdot \theta.\text{embedding} \)
return \( \theta, \tau \)

datasets can further benefit from using Sentence-Piece tokenization (Kudo and Richardson, 2018).

- Reddit data – this dataset is taken from the LEAF benchmark (Caldas et al., 2018) and contains over a million users that have multiple posts on the Reddit platform. As proposed by LEAF, we limit each user to contain at most 1600 tokens and use 10% of users for faster training.

- StackOverflow data – this data is taken from Kaggle (Kaggle, 2021) and processed with the TensorFlow Federated framework. The train split of the dataset contains 342k users and we select at most 1600 tokens per user.

Model parameters. We use an LSTM model with 3 layers, and total parameters of 14M. We also use a transformer language model with 6 layers and the same total number of parameters as the LSTM (see Appendix A). Each model is trained from scratch.

Hyper-parameters. We set the privacy budget to \( \epsilon = 2 \) and \( \delta = 10^{-6} \) and clipping bound to 0.5. The overall population for the moments accountant is assumed to be 10m. We use a cohort size of 20,000 and train all models for 5,000 iterations. We use Adam (Kingma and Ba, 2015) for central optimization with learning rate set to 0.5. For the clients’ local SGD, we train one local epoch with batch size set to 16 and local learning rate set to 0.1.

Vocabulary size. We assume that the tokenizer has a moderate vocabulary size such as 10,000 tokens (we experiment with larger vocabularies in Appendix A). Smaller vocabularies reduce model size and, therefore, might be better for deployment on devices and communication with the global server.

Tokenizer details. To train an initial tokenizer we use a popular and public Wikipedia dataset (Merity et al., 2017). It may seem like the distribution of Wikipedia data is artificially far from the distributions of Reddit and StackOverflow data. However, in real-world scenarios it is unknown how different private data is from public data, and it is important to understand the effects of using disparate distributions on the algorithm we propose. Also, when running the same algorithm with a Reddit tokenizer to find a StackOverflow tokenizer and vice versa, those distributions similarly turn out to be far away from each other. Appendix A shows this.

We use BPE tokenization from the HuggingFace Tokenizer library (Huggingface, 2021) to train the tokenizer. Each user post is surrounded by special tokens BOS and EOS. We also tried WordPieces tokenization which has slightly better performance than BPE but cannot encode all words and is therefore less applicable in a FL setting.

Note on splitting data. Whereas the original LEAF dataset for Reddit proposes to split each user’s data we argue that in real life not every user might have a chance to participate in the training. Therefore, we split users into two distinct training and test sets and evaluate the model on data from the users who have never participated in the training. This results in notably increased test perplexity but provides a clear separation between training and inference modes.
Figure 2: Example of sampling data from the model.

5.2 Comparing tokenization schemes

Table 1 summarizes experiments that use different tokenization schemes. For comparison we use word-level accuracy and perplexity as described in Section 2.3. We also compute statistics on tokenizers: average share of OOV tokens for the word-level scheme and an average number of tokens required to encode one word for the sub-word scheme. The “wiki” tokenizers are trained on the Wikipedia data, and the “oracle” tokenizers directly on the training data.

Word-level tokenization provides very high word-level accuracy when it is trained using “oracle” user training data. However, when the word-level has access to only public “wiki” dataset the performance significantly drops: by 26% for Reddit and 10% for StackOverflow with a significant increase in out-of-vocabulary share. However, BPE tokenizers that use public data perform more consistently and outperform the word-level models trained on public data, but still require large number of tokens per word.

5.3 Learning tokenizer with model sampling

A key part of the proposed algorithm is the sampling from a model that uses a public tokenizer $\tau_{pub}$, but is trained with private federated learning and should represent the words in the actual data. The sampling is implemented as in Algorithm 1.

First, Figure 2 shows samples from the language models on the two data sets. Although clearly the samples are less coherent than the underlying data, it seems plausible that the word occurrences match that data.

Second, Table 1 further investigates the properties of the sampled text. The “BPE sample” rows refer to the method proposed in this paper. A language model with the “wiki” tokenizer is trained with PFL on the first half of the training data. Then samples are drawn from this language model. Then, the language model is trained from scratch on the second half of the training data.

The “BPE HH” rows refer to training with a differentially private “heavy hitters” algorithm (Apple, 2017). Each of the population of the users from the first half of the training set contributes three words from the from the Wikipedia dataset, with a local privacy budget of $\epsilon = 8$. Just like for the sampling approach, the language model is then trained from scratch on the second half of the training data.

First, we examine the difference between the real training data and the data used to train the tokenizers. The column “Data KLD” shows the KL divergence from the training data to the sampled data. The KL divergence is computed from the unigram counts, which are relevant for training a tokenizer, by assuming add-1 smoothing and using the top 10,000 words from the training data. The KL divergence to the training data itself, which the oracle tokenizer is trained on, is 0 by definition. The KL divergence between the actual data and the Wikipedia data, on the other hand, is around 1, for both datasets. Both the heavy hitters algorithm and the algorithm we propose in this paper find a distribution close to the real distribution.

For sub-word tokenizers, the number of tokens per word is relevant. Even though they can represent unseen words by multiple tokens, the language models trained on top of that have a harder task given the longer context on average. The oracle tokenizer has the lowest number of tokens per words.
and the “wiki” tokenizer comes the highest. The “BPE sample” tokenizer comes very close to the oracle tokenizer.

However, heavy hitters experiment shows much smaller gain in performance, i.e. better than “wiki” tokenizer but still worse than our proposed sampling method. Furthermore, it requires a separate privacy budget allocated for the run, while sampling can operate on existing prior model.

### 5.4 Iterative updates

This part implements Algorithm 1 completely. We again initialize the tokenizer on publicly available data. We then train the language model with PFL.

At a point during training, we retrain the tokenizer by sampling. Unlike in the previous section, we update the language model by remapping its embedding layer, and continue training. We sample the same data before and after changing the tokenizer.

Figure 3 shows the results for changing tokenizers at different times. The “Wiki” curve represents the baseline system using public tokenizer $\tau_{pub}$. Each of the other curves takes the system from the “wiki” curve at a different iteration. As expected, the initial remapping of the embedding layer is not perfect and needs finetuning. The graph also shows the tradeoff in when to change tokenizers: too early, e.g. after only 1000 iterations, and the tokenizer is not representative enough yet; too late, e.g. after 4000 iterations, and there is not enough time to converge again.

### 6 Conclusion

This paper has proposed a method that allows a tokenizer to be found together with a language model using private federated learning. First, it has shown that a mismatched tokenizer can cause a significant performance degradation. The key to improving this is to use a sub-word tokenizer which allows new words to be represented as a sequence of tokens. Then, a language model trained with PFL can represent the private data. This paper has presented a method to produce a new tokenizer from that model, and to convert the model to work with the new tokenizer. When this is trained further with private federated learning, it outperforms the language model with the mismatched tokenizer, and gets close to one with the oracle tokenizer.

**Personalization and Fairness.** The problem of out-of-vocabulary words might be more acute for some users that use unique vocabulary, such as dialect, and impact individual performance. Therefore good tokenizers can benefit personalization in federated models (Li et al., 2021; Yu et al., 2020).

### References


Wennan Zhu, Peter Kairouz, Brendan McMahan, Haicheng Sun, and Wei Li. 2020. Federated heavy hitters discovery with differential privacy. In International Conference on Artificial Intelligence and Statistics, pages 3837–3847. PMLR.
This section examines different hyperparameters.

### A.1 Experimental design

First, consider the choice to train the public tokenizer on Wikipedia data. To examine the effect of using a more conversational style corpus. To do this, Table 2 takes a subset of the numbers from Table 1 and adds a scenario where a tokenizer on StackOverflow data is used with Reddit data and vice versa. The cross-dataset numbers are highlighted bold in the table.

First, in terms of the KL divergence the StackOverflow data seems a slightly better model for the Reddit distribution than the Wikipedia data is. However, when using PFL to train on Reddit data, but with a StackOverflow-trained tokenizer, the perplexity deteriorates compared to the Wikipedia-trained tokenizer. Second, the reverse experiment looks a bit better but not hugely better. Though the KL divergence from the StackOverflow data to the Reddit data is significantly better than the KL divergence to the Wikipedia data, some of that advantage disappears in the final trained model.

### A.2 Other hyperparameters

We consider two hyperparameter choices for experiments: first, the privacy budget, and secondly, the cohort size.

Figure 4 shows the effect of different privacy

Then, consider the choice of vocabulary size, here the number of distinct tokens. Table 3 shows the perplexities for the baseline (“Wiki”) and ceiling (“oracle”) experiments. Though the absolute numbers change, the trends do not change.

Similarly for changing model architectures. This paper has presented results on an LSTM model. Table 4 shows results on a Transformer model. Again, though the absolute numbers change, the trends do not change.

### Impact of hyperparameters

This section examines different hyperparameters.

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<th>( \tau )</th>
<th>Data</th>
<th>Data</th>
<th>LM perp.</th>
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</thead>
<tbody>
<tr>
<td>Reddit</td>
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<td>276.5</td>
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<tr>
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<td>BPE StackOverflow</td>
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<td>BPE Reddit</td>
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<td>BPE sample</td>
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<td>1.0629</td>
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<td>BPE Reddit</td>
<td>0.5315</td>
<td>118.8</td>
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<td></td>
<td>BPE StackOverflow</td>
<td>0</td>
<td>108.2</td>
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<td></td>
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Table 3: The effect of varying the vocabulary size.

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<th>Vocab size</th>
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<th>Oracle</th>
<th>StackOverflow Wiki</th>
<th>Oracle</th>
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<td>282.2</td>
<td>136.3</td>
<td>116.8</td>
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<tr>
<td>10,000</td>
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<td>256.9</td>
<td>124.6</td>
<td>108.2</td>
</tr>
<tr>
<td>50,000</td>
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<td>225.4</td>
<td>111.5</td>
<td>101.5</td>
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<tr>
<td>100,000</td>
<td>231.2</td>
<td>217.9</td>
<td>108.9</td>
<td>100.5</td>
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Table 4: The effect of changing model architectures.

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<th>Oracle</th>
<th>StackOverflow Wiki</th>
<th>Oracle</th>
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<td>Transformer</td>
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<td>117.4</td>
<td>107.0</td>
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<td>LSTM</td>
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<td>256.9</td>
<td>124.6</td>
<td>108.2</td>
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</tbody>
</table>
parameters. The effects are not huge, but clearly
differential privacy does impede learning some-
what.

Figure 5 shows the effect of differing cohort
sizes. A larger cohort size implies a better signal-to-
noise ratio when training with differential privacy.
However, for practical reasons it is preferable for
cohorts to be smaller. 10,000 is a happy medium
between good performance and practicality. Also,
again, though the absolute numbers change, the
trends do not change.