

# SPEED: Scalable, Precise, and Efficient Concept Erasure for Diffusion Models

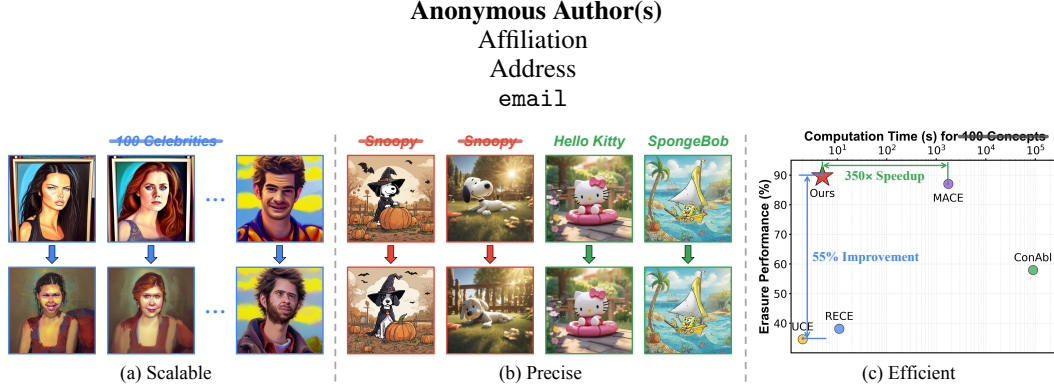


Figure 1: **Three characteristics of our proposed concept erasure method for diffusion models, SPEED.** (a) **Scalable**: SPEED seamlessly scales from single-concept to large-scale multi-concept erasure (e.g., 100 celebrities) without additional design. (b) **Precise**: SPEED precisely removes the target concept (e.g., *Snoopy*) while preserving the semantics for non-target concepts (e.g., *Hello Kitty* and *SpongeBob*). (c) **Efficient**: SPEED immediately erases 100 concepts within 5 seconds, achieving new state-of-the-art (SOTA) performance with a  $350\times$  speedup over competitive methods.

## Abstract

Erasing concepts from large-scale text-to-image (T2I) diffusion models has become increasingly crucial due to the growing concerns over copyright infringement, offensive content, and privacy violations. In scalable applications, fine-tuning-based methods are time-consuming to precisely erase multiple target concepts, while real-time editing-based methods often degrade the generation quality of non-target concepts due to conflicting optimization objectives. To address this dilemma, we introduce SPEED, an efficient concept erasure approach that directly edits model parameters. SPEED searches for a null space, a model editing space where parameter updates do not affect non-target concepts, to achieve scalable and precise erasure. To facilitate accurate null space optimization, we incorporate three complementary strategies: Influence-based Prior Filtering (IPF) to selectively retain the most affected non-target concepts, Directed Prior Augmentation (DPA) to enrich the filtered retain set with semantically consistent variations, and Invariant Equality Constraints (IEC) to preserve key invariants during the T2I generation process. Extensive evaluations across multiple concept erasure tasks demonstrate that SPEED consistently outperforms existing methods in non-target preservation while achieving efficient and high-fidelity concept erasure, successfully erasing 100 concepts within just 5 seconds.

## 1 Introduction

Large-scale text-to-image (T2I) diffusion models [23, 54, 55, 37, 47, 24] have facilitated significant breakthroughs in generating highly realistic and contextually consistent images simply from textual descriptions [11, 44, 16, 7, 48, 42, 12]. Alongside these advancements, concerns have also been raised regarding copyright violations [10, 52], offensive content [49, 64, 66], and privacy concerns [8, 63]. To mitigate ethical and legal risks in generation, it is often necessary to prevent the model

from generating certain concepts, a process termed *concept erasure* [29, 17, 65]. However, removing target concepts without carefully preserving the semantics of non-target concepts can introduce unintended artifacts, distortions, and degraded image quality [17, 40, 49, 65], compromising the model’s reliability and usability. Therefore, beyond ensuring the effective removal of target concepts (*i.e.*, *erasure efficacy*), concept erasure should also maintain the original semantics of non-target concepts (*i.e.*, *prior preservation* [61]).

In this context, recent methods strive to seek a balance between erasure efficacy and prior preservation, broadly categorized into two paradigms: training-based [29, 35, 33] and editing-based [18, 19]. The training-based paradigm fine-tunes T2I diffusion models to achieve concept erasure, incorporating an additional regularization term into the training objective for prior preservation. In contrast, the editing-based paradigm avoids additional fine-tuning by directly modifying model parameters (*e.g.*, projection weights in cross-attention layers [47]), with such modifications derived from a closed-form objective that jointly accounts for erasure and preservation. This efficiency also facilitates editing-based methods to extend to multi-concept erasure without additional designs seamlessly.

However, as the number of target concepts increases, current editing-based methods [18, 19] struggle to balance between erasure efficacy and prior preservation. This can be attributed to the growing conflicts between erasure and preservation objectives, making such trade-offs increasingly difficult. Moreover, these methods rely on weighted least squares optimization, inherently imposing a *non-zero lower bound* on preservation error (see Appx. B.2). In multi-concept settings, this accumulation of preservation errors gradually distorts non-target knowledge, thereby degrading prior preservation. To address the above limitations, we propose Scalable, Precise, and Efficient Concept Erasure for Diffusion Models (SPEED) (see Fig. 1), an editing-based method incorporating null-space constraints. Specifically, we search for the *null space of prior knowledge*, a model editing space where parameter updates do not affect the feature representations of non-target concepts. By projecting the model parameter updates for concept erasure onto such null space, SPEED can minimize the preservation error to zero without compromising erasure efficacy, thereby enabling scalable and precise concept erasure without affecting non-target concepts.

The key contribution of SPEED lies in constructing an effective null space from a set of non-target concepts (*i.e.*, *retain set*). We observe that the existing baseline with null-space constraints [14] confronts a fundamental dilemma during concept erasure: While a small retain set limits the coverage of prior knowledge, enlarging the retain set makes it increasingly difficult to identify an accurate null space. This difficulty arises because a large retain set causes the corresponding feature matrix to approach full rank, necessitating the estimation of its null space to ensure sufficient degrees of freedom for optimization (*i.e.*, for concept erasure). However, this estimation inevitably introduces semantic degradation within the retain set and deteriorating prior preservation (see Fig. 2 and Eq. 4).

In this light, we introduce Prior Knowledge Refinement, a suite of techniques that strategically and selectively refine the retain set to mitigate the semantic degradation in searching for the null space. Particularly, we propose Influence-based Prior Filtering (IPF), which first quantifies the influence of concept erasure on each non-target concept. It then prunes the retain set by removing minimally affected concepts, preventing the correlation matrix from approaching full rank and thus maintaining an accurate null space. Subsequently, to further enhance prior preservation over the resulting retain set, we propose Directed Prior Augmentation (DPA), which expands the retain set with directed, semantically consistent perturbations to improve retain coverage. In addition, we incorporate Invariant Equality Constraints (IEC) to preserve specific representations, such as the [SOT] token, that should remain unchanged during editing. IEC enforces equality constraints on such invariants to regularize the retaining of essential generation properties. We evaluate SPEED on three representative concept erasure tasks, *i.e.*, few-concept, multi-concept, and implicit concept

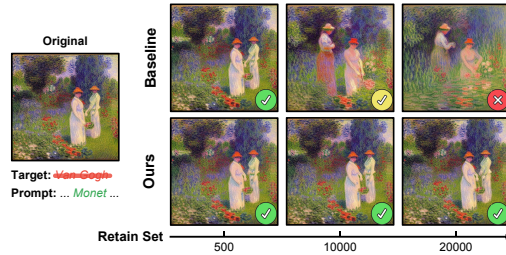


Figure 2: **Semantic degradation with increasing non-target concepts in the retain set.** Baseline null-space constrained method [14] can maintain the non-target semantics given a small retain set (✓). However, as the retain set grows, the rank of corresponding matrix increases, making null space estimation increasingly inaccurate (see Eq. 4) with inevitable approximation errors, thereby degrading *Monet*’s semantics in the retain set (✓ and ✗).

81 erasure, where it consistently exhibits superior prior preservation across all erasure tasks. Overall,  
82 our contributions can be summarized as follows:

- 83 • We propose SPEED, a scalable, precise, and efficient concept erasure method with null-space  
84 constrained model editing, capable of erasing 100 concepts in 5 seconds.
- 85 • We introduce Prior Knowledge Refinement to construct an accurate null space over the retain set  
86 for effective editing. Leveraging three complementary techniques, IPF, DPA, and IEC, our method  
87 balances semantic degradation and retain coverage, enabling precise and scalable concept erasure.
- 88 • Our extensive experiments show that SPEED consistently outperforms existing methods in prior  
89 preservation across various erasure tasks with minimal computational costs.

## 90 2 Related Works

91 **Concept erasure.** Current T2I diffusion models inevitably involve unauthorized and NSFW (Not  
92 Safe For Work) generations due to the noisy training data from web [51, 50]. Apart from applying  
93 additional filters or safety checkers [45, 39, 46], prevailing methods modify diffusion model param-  
94 eters to erase specific target concepts, mainly categorized into two paradigms. The training-based  
95 paradigm fine-tunes model parameters with specific erasure objectives [29, 17, 65] and additional  
96 regularization terms [29, 35, 33]. In contrast, the editing-based paradigm edits model parameters  
97 using a closed-form solution to facilitate efficiency in concept erasure. For example, UCE [18]  
98 modifies model weights by balancing both erasure and preservation error through a weighted least  
99 squares objective and RECE [19] iteratively derives new target concept embeddings. These methods  
100 can erase numerous concepts within seconds, demonstrating superior efficiency in practice.

101 **Null-space constraints.** The null space of a matrix, a fundamental concept in linear algebra, refers to  
102 the set of all vectors that the matrix maps to the zero vector. The null-space constraints are first applied  
103 to continual learning (CL) by projecting gradients onto the null space of uncentered covariances  
104 from previous tasks [58]. Subsequent studies [34, 59, 62, 28, 30] further explore and extend the  
105 application of null space in CL. In model editing, AlphaEdit [14] restricts model weight updates  
106 onto the null space of preserved knowledge, effectively mitigating trade-offs between editing and  
107 preservation. Null-space constraints also apply to various tasks, *e.g.*, machine unlearning [9], MRI  
108 reconstruction [15], and image restoration [60], offering promise for editing-based concept erasure.

## 109 3 Problem Formulation

110 In T2I diffusion models, each concept is encoded by a set of text tokens via CLIP [43], which are  
111 then aggregated into a single concept embedding  $\mathbf{c} \in \mathbb{R}^{d_0}$ . For concept erasure, there are two sets of  
112 concepts: the erasure set  $\mathbf{E}$  and the retain set  $\mathbf{R}$ . The erasure set consists of  $N_E$  target concepts to be  
113 removed, denoted as  $\mathbf{E} = \{\mathbf{c}_1^{(i)}\}_{i=1}^{N_E}$ . The retain set includes  $N_R$  non-target concepts that should be  
114 preserved during editing, denoted as  $\mathbf{R} = \{\mathbf{c}_0^{(j)}\}_{j=1}^{N_R}$ . To enable efficient erasure efficacy for  $\mathbf{E}$  and  
115 prior preservation for  $\mathbf{R}$ , we first formulate a closed-form editing objective in Sec. 3.1, and enhance  
116 it with null-space constrained optimization in Sec. 3.2.

### 117 3.1 Concept Erasure in Closed-Form Solution

118 To effectively erase each target concept  $\mathbf{c}_1^{(i)} \in \mathbf{E}$  (*e.g.*, *Snoopy*), it is specified to be mapped onto an  
119 anchor concept  $\mathbf{c}_*^{(i)}$  that shares general semantics (*e.g.*, *Dog*), termed as an anchor set  $\mathbf{A} = \{\mathbf{c}_*^{(i)}\}_{i=1}^{N_E}$ .  
120 For editing-based methods [40, 18, 19], concept embeddings from the erasure set  $\mathbf{E}$ , anchor set  $\mathbf{A}$ , and  
121 retain set  $\mathbf{R}$  are first organized into three structured matrices:  $\mathbf{C}_1, \mathbf{C}_* \in \mathbb{R}^{d_0 \times N_E}$  and  $\mathbf{C}_0 \in \mathbb{R}^{d_0 \times N_R}$ ,  
122 representing the stacked embeddings of target, anchor, and non-target concepts, respectively. To  
123 derive a closed-form solution for concept erasure, existing methods typically optimize a perturbation  
124  $\Delta$  to model parameters  $\mathbf{W}$ , balancing between erasure efficacy and prior preservation. For example,  
125 UCE [18] formulates concept erasure as a weighted least squares problem:

$$\Delta_{\text{UCE}} = \arg \min_{\Delta} \underbrace{\|(\mathbf{W} + \Delta)\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*\|_{e_1}^2}_{e_1} + \underbrace{\|\Delta\mathbf{C}_0\|_{e_0}^2}_{e_0}, \quad (1)$$

where the erasure error  $e_1$  ensures that each target concept is mapped onto its corresponding anchor concept and the preservation error  $e_0$  minimizes the impact on non-target concepts. This formulation provides a closed-form solution  $\Delta_{\text{UCE}}$  (see Appx. B.1) for parameter updates, achieving computationally efficient optimization. However, as the number of target concepts increases, the accumulated preservation errors  $e_0$ , which prove to share a non-zero bound from Appx. B.2, across multiple target concepts would amplify the distortion on non-target knowledge and degrade prior preservation.

### 3.2 Apply Null-Space Constraints

To mitigate the limitation of weighted optimization in prior preservation, SPEED integrates null-space constraints [58, 14] to achieve prior-preserved model editing by forcing  $e_0 = 0$ . Specifically, the null space of  $\mathbf{C}_0$  is the set of all vectors  $\mathbf{v}$  such that  $\mathbf{v}\mathbf{C}_0 = \mathbf{0}$ . Restricting the parameter update  $\Delta$  to this space ensures that such updates do not interfere with non-target concepts.

To project  $\Delta$  onto null space, we perform singular value decomposition (SVD) on  $\mathbf{C}_0\mathbf{C}_0^\top \in \mathbb{R}^{d_0 \times d_0}$ <sup>1</sup> and have  $\{\mathbf{U}, \mathbf{\Lambda}, \mathbf{U}^\top\} = \text{SVD}(\mathbf{C}_0\mathbf{C}_0^\top)$ , where  $\mathbf{U} \in \mathbb{R}^{d_0 \times d_0}$  contains the singular vectors of  $\mathbf{C}_0\mathbf{C}_0^\top$ , and  $\mathbf{\Lambda}$  is a diagonal matrix of its singular values. The singular vectors in  $\mathbf{U}$  w.r.t. zero singular values form an orthonormal basis for the null space of  $\mathbf{C}_0$ , which we denote as  $\hat{\mathbf{U}}$ . Using this basis, we construct the null-space projection matrix  $\mathbf{P} = \hat{\mathbf{U}}\hat{\mathbf{U}}^\top$ . The process can be formulated as:

$$\{\mathbf{U}, \mathbf{\Lambda}, \mathbf{U}^\top\} = \text{SVD}(\mathbf{C}_0\mathbf{C}_0^\top), \quad \mathbf{U} \in \mathbb{R}^{d_0 \times d_0} \xrightarrow[\text{values}]{\text{zero singular}} \hat{\mathbf{U}} \implies \mathbf{P} = \hat{\mathbf{U}}\hat{\mathbf{U}}^\top. \quad (2)$$

The final update applied to model parameters is  $\Delta\mathbf{P}$ , which projects  $\Delta$  onto the null space of  $\mathbf{C}_0$ . This ensures that updates do not interfere with non-target concepts, satisfying  $\|(\Delta\mathbf{P})\mathbf{C}_0\|^2 = 0$ . To solve for the updates, we minimize the following objective:

$$\Delta_{\text{Null}} = \arg \min_{\Delta} \underbrace{\|(\mathbf{W} + \Delta\mathbf{P})\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*\|^2}_{e_1} + \underbrace{\|(\Delta\mathbf{P})\mathbf{C}_0\|^2}_{e_0=0} + \underbrace{\|\Delta\mathbf{P}\|^2}_{\text{regularization}}, \quad (3)$$

where  $\|\Delta\mathbf{P}\|^2$  is a regularization term to ensure convergence. The preservation term  $\|(\Delta\mathbf{P})\mathbf{C}_0\|^2$  is omitted, as it is guaranteed to be zero by the null-space constraint. This objective enables us to update the model parameters such that target concepts are effectively erased while non-target representations remain unaffected, thereby achieving prior-preserved concept erasure.

## 4 Prior Knowledge Refinement

However, as more diverse non-target concepts are included in the retain set, the rank of the correlation matrix  $\mathbf{C}_0\mathbf{C}_0^\top$  increases<sup>2</sup>. The null space, defined as the orthogonal complement of this span, correspondingly shrinks in dimension:

$$\dim(\text{Null}(\mathbf{C}_0)) = d_0 - \text{rank}(\mathbf{C}_0\mathbf{C}_0^\top). \quad (4)$$

Here, the null space dimension characterizes the degrees of freedom available for editing without affecting the retained concepts. However, as this dimension shrinks, to ensure sufficient degrees of freedom for concept erasure, we are compelled to include singular vectors w.r.t. non-zero singular values in  $\hat{\mathbf{U}}$  following [14], which leads to an approximate null space and induces semantic degradation within the retain set (see Fig. 2). To mitigate this problem, we propose Prior Knowledge Refinement, a structured strategy for refining the retain set to enable accurate null-space construction. It comprises three complementary techniques: Influence-Based Prior Filtering (Sec. 4.1) to discard weakly affected non-target concepts to form a viable null space; Directed Prior Augmentation (Sec. 4.2) to expand the retain set with targeted and semantically consistent variations; and Invariant Equality Constraints (Sec. 4.3) to enforce equality constraints to preserve critical invariants during generation.

<sup>1</sup> $\mathbf{C}_0\mathbf{C}_0^\top$  and  $\mathbf{C}_0$  share the same null space. We operated on  $\mathbf{C}_0\mathbf{C}_0^\top \in \mathbb{R}^{d_0 \times d_0}$  since it has fixed row dimension while  $\mathbf{C}_0 \in \mathbb{R}^{d_0 \times N_R}$  may have high dimensionality depending on concept number  $N_R$ .

<sup>2</sup>We assume that the concepts are not exactly linearly dependent in the representation space, which is generally satisfied in practice due to the semantic diversity and high dimensionality of the embedding space.

#### 4.1 Influence-Based Prior Filtering (IPF)

Given a pre-defined retain set, existing editing-based methods [18, 19] treat all non-target concepts equally when enforcing prior preservation. However, a critical yet overlooked fact is that parameter updates inherently induce output changes over non-target concepts, and these changes vary significantly across different non-target elements. This suggests that not all non-target concepts contribute equally to preserving the model’s prior knowledge, and weakly influenced concepts would provide marginal benefit while introducing additional ranks to narrow the null space.

To this end, we propose an explicit and model-consistent metric, *i.e.*, **prior shift**, to quantify how much a non-target concept is affected by concept erasure. Specifically, we isolate the effect of erasure by solving for a closed-form update  $\Delta_{\text{erase}}$  that minimizes only the erasure error  $e_1$  while discarding the preservation term  $e_0$  from Eq. 1:

$$\Delta_{\text{erase}} = \arg \min_{\Delta} \underbrace{\|(\mathbf{W} + \Delta)\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*\|^2}_{e_1} + \underbrace{\|\Delta\|^2}_{\text{regularization}} = \mathbf{W} (\mathbf{C}_*\mathbf{C}_1^\top - \mathbf{C}_1\mathbf{C}_1^\top) (\mathbf{I} + \mathbf{C}_1\mathbf{C}_1) ^{-1}. \quad (5)$$

where  $\|\Delta\|^2$  is introduced for convergence. Then, for each non-target concept embedding  $\mathbf{c}$ , we define its prior shift as:  $\|\Delta_{\text{erase}}\mathbf{c}\|^2$ . This value offers a faithful reflection of how parameter updates perturb a non-target concept in the feature space with closed-form computation, and can naturally generalize to assessing multi-concept erasure effects. Based on this, we filter the original retain set  $\mathbf{R}$  to focus only on highly influenced concepts:

$$\mathbf{R}_f : \mathbf{R} \mapsto \{\mathbf{c}_0 \in \mathbf{R} \mid \|\Delta_{\text{erase}}\mathbf{c}_0\|^2 > \mu\}, \quad (6)$$

where the mean value  $\mu = \mathbb{E}_{\mathbf{c}_0 \sim \mathbf{R}} [\|\Delta_{\text{erase}}\mathbf{c}_0\|^2]$  serves as a filtering threshold.

#### 4.2 Directed Prior Augmentation (DPA)

To further enhance prior preservation over the resulting retain set with improved retain coverage, an intuitive strategy is to augment the retain set by perturbing non-target embedding  $\mathbf{c}_0$  with random noise [35]. However, this strategy would introduce meaningless embeddings that fail to generate semantically coherent images (*e.g.*, noise image), resulting in excessive preservation with increasing ranks. To search for more semantically consistent concepts, we introduce directed noise by projecting the random noise  $\epsilon$  onto the direction in which the model parameters  $\mathbf{W}$  exhibit minimal variation. This operation ensures the perturbed embeddings express closer semantics to the original concept after being mapped by  $\mathbf{W}$  in Fig. 3. Specifically, we first derive a projection matrix  $\mathbf{P}_{\min}$ :

$$\{\mathbf{U}_{\mathbf{W}}, \Lambda_{\mathbf{W}}, \mathbf{U}_{\mathbf{W}}^\top\} = \text{SVD}(\mathbf{W}), \quad \mathbf{P}_{\min} = \mathbf{U}_{\min} \mathbf{U}_{\min}^\top, \quad (7)$$

where  $\mathbf{U}_{\min} = \mathbf{U}_{\mathbf{W}}[:, -r:]$  denotes the singular vectors w.r.t. the smallest  $r$  singular vectors<sup>3</sup>, which represent the  $r$  least-changing directions of  $\mathbf{W}$  and constrain the rank of the augmented embeddings to a maximum of  $r$ . Then the directed noise  $\epsilon \cdot \mathbf{P}_{\min}$  is used to perturb the original embedding via:

$$\mathbf{c}'_0 = \mathbf{c}_0 + \epsilon \cdot \mathbf{P}_{\min}, \quad \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I}). \quad (8)$$

Given a retain set  $\mathbf{R}$ , the augmentation process can be formulated as follows:

$$\mathbf{R}^{\text{aug}} : \mathbf{R} \mapsto \bigcup_{\mathbf{c}_0 \in \mathbf{R}} \{\mathbf{c}'_{0,k} \mid k = 1, \dots, N_A\}, \quad (9)$$

where  $N_A$  denotes the augmentation times and  $\mathbf{c}'_{0,k}$  represents the  $k$ -th augmented embedding given  $\mathbf{c}_0 \in \mathbf{R}$  using Eq. 8. In implementation, we first filter the original retain set  $\mathbf{R}$  to obtain  $\mathbf{R}_f$  using

<sup>3</sup>Empirically, the model parameter matrix  $\mathbf{W}$  is usually full rank, thus its all singular values are non-zero.

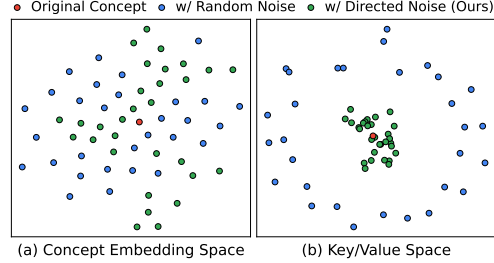


Figure 3: **t-SNE distribution of perturbing the original concept with random noise and our directed noise.** (a) Similar to random noise, our method can span a broad concept embedding space. (b) Our directed noise preserves semantic similarity to the original concept with closer distances in the space mapped by  $\mathbf{W}$ .

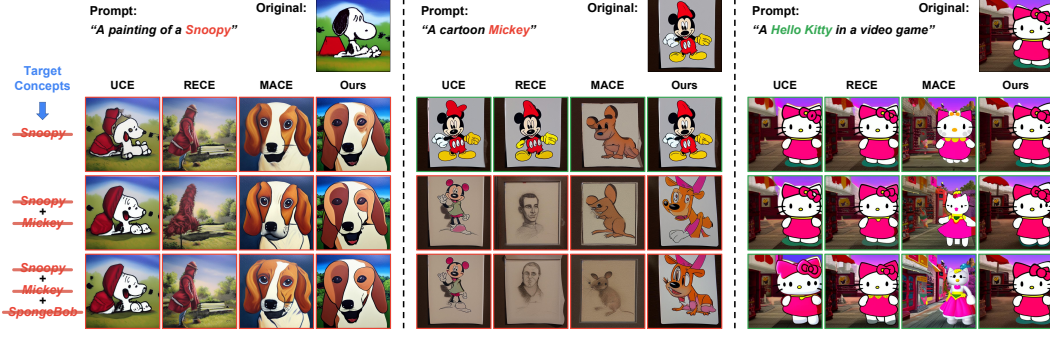


Figure 4: **Qualitative comparison of the few-concept erasure in erasing instances.** The erased and preserved generations are highlighted with **red** and **green** boxes, respectively. Our method exhibits consistent prior preservation with less semantic degradation for non-target concepts. For example, the middle column better retains details such as *Mickey's* hat and button count, and the right column demonstrates more consistent *Hello Kitty* generations along with three concepts erased.

IPF. Subsequently, further augmentation and filtering are applied to  $\mathbf{R}_f$  using DPA and IPF to obtain  $(\mathbf{R}_f)_f^{\text{aug}}$ , and the two filtered retain sets are then combined together to serve as the final refined retain set  $\mathbf{R}_{\text{refine}} = \mathbf{R}_f \cup (\mathbf{R}_f)_f^{\text{aug}}$ .

### 4.3 Invariant Equality Constraints (IEC)

In parallel, we identify certain invariants during the T2I generation process, *i.e.*, intermediate variables that remain unchanged with varying sampling prompts. One such invariant is the CLIP-encoded [SOT] token. Since the encoding process is masked by causal attention and all prompts are prefixed with the fixed [SOT] token during tokenization, its embedding consistently remains unchanged during T2I process. Another invariant is the null-text embedding, as it corresponds to the unconditional generation under the classifier-free guidance [24], which also remains unchanged despite prompt variations. Given the invariance of these embeddings, we consider additional protection measures to ensure their outputs remain unchanged during concept erasure. Specifically, we introduce explicit equality constraints over invariants based on Eq. 3:

$$\min_{\Delta} \underbrace{\|(\mathbf{W} + \Delta\mathbf{P})\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*\|^2}_{e_1} + \underbrace{\|\Delta\mathbf{P}\|^2}_{\text{regularization}}, \quad \text{s.t.} \quad \underbrace{(\Delta\mathbf{P})\mathbf{C}_2 = \mathbf{0}}_{\text{equality constraints}}, \quad (10)$$

where  $\mathbf{C}_2$  denotes the stacked invariant embedding matrix of [SOT] and null-text<sup>4</sup>. Derive the projection matrix  $\mathbf{P}$  from  $\mathbf{R}_{\text{refine}}$ , we can compute the closed-form solution of Eq. 10 using Lagrange Multipliers from Appx. B.3:

$$(\Delta\mathbf{P})_{\text{Ours}} = \mathbf{W} (\mathbf{C}_*\mathbf{C}_1^\top - \mathbf{C}_1\mathbf{C}_1^\top) \mathbf{P}\mathbf{Q}\mathbf{M}, \quad (11)$$

where

$$\mathbf{M} = (\mathbf{C}_1\mathbf{C}_1^\top \mathbf{P} + \mathbf{I})^{-1}, \quad \mathbf{Q} = \mathbf{I} - \mathbf{M}\mathbf{C}_2 (\mathbf{C}_2^\top \mathbf{P}\mathbf{M}\mathbf{C}_2)^{-1} \mathbf{C}_2^\top \mathbf{P}. \quad (12)$$

This closed-form solution enforces the equality constraints by projecting the parameter update onto the subspace orthogonal to the invariant embeddings. Since image generation inevitably depends on these invariant embeddings, such constraints inherently preserve prior knowledge.

## 5 Experiments

In this section, we conduct extensive experiments on three representative erasure tasks, including few-concept erasure, multi-concept erasure, and implicit concept erasure (Appx. D.3), validating our superior prior preservation. The compared baselines include ConAbl [29], MACE [33], RECE [19], and UCE [18], which have achieved SOTA performance across various concept erasure tasks. In implementation, we conduct all experiments on SDv1.4 [1] and generate each image using DPM-solver sampler [32] over 20 sampling steps with classifier-free guidance [24] of 7.5. More implementation details and compared baselines (*e.g.*, SPM [35]) can be found in Appx. C and Appx. D.4.

<sup>4</sup>Since the null-text embeddings are only composed of [EOT] tokens (excluding [SOT]), we use the k-means algorithm [36] to select k centroids to reduce redundancy.

Table 1: **Quantitative comparison of the few-concept erasure in erasing instances (left) and artistic styles (right)** following [35]. Arrows on the headers indicate the preferred direction for each metric, and the best results are highlighted in **bold**. Our method consistently improves prior preservation for non-target and general concepts from MS-COCO (shaded in pink) while achieving effective concept erasure. While our CS is not the lowest for target concept, Appx. D.1 and Fig. 7 show our method is sufficient for erasure, and lower CS may further compromise prior preservation.

| Concept                                                    | Snoopy       | Mickey       | Spongebob    | Pikachu      | Hello Kitty  | MS-COCO      |              |
|------------------------------------------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|                                                            | CS           | CS           | CS           | CS           | CS           | CS           | FID          |
| SD v1.4                                                    | 28.51        | 26.62        | 27.30        | 27.44        | 27.77        | 26.53        | -            |
| Erase <i>Snoopy</i>                                        |              |              |              |              |              |              |              |
|                                                            | CS ↓         | FID ↓        | FID ↓        | FID ↓        | FID ↓        | CS ↑         | FID ↓        |
| ConAbl                                                     | 25.44        | 37.08        | 38.92        | 26.14        | 36.52        | 26.40        | 21.20        |
| MACE                                                       | 20.90        | 105.97       | 102.77       | 65.71        | 75.42        | 26.09        | 42.62        |
| RECE                                                       | <b>18.38</b> | 26.63        | 34.42        | 21.99        | 32.35        | 26.39        | 25.61        |
| UCE                                                        | 23.19        | 24.87        | 29.86        | 19.06        | 27.86        | 26.46        | 22.18        |
| Ours                                                       | 23.50        | <b>23.41</b> | <b>24.64</b> | <b>16.81</b> | <b>21.74</b> | <b>26.48</b> | <b>19.95</b> |
| Erase <i>Snoopy</i> and <i>Mickey</i>                      |              |              |              |              |              |              |              |
|                                                            | CS ↓         | CS ↓         | FID ↓        | FID ↓        | FID ↓        | CS ↑         | FID ↓        |
| ConAbl                                                     | 25.26        | 26.58        | 45.08        | 35.57        | 41.48        | 26.42        | 24.34        |
| MACE                                                       | 20.53        | 20.63        | 112.01       | 91.72        | 106.88       | 25.50        | 55.15        |
| RECE                                                       | <b>18.57</b> | <b>19.14</b> | 35.85        | 26.05        | 40.77        | 26.31        | 30.30        |
| UCE                                                        | 23.60        | 24.79        | 30.58        | 23.51        | 31.76        | 26.38        | 26.06        |
| Ours                                                       | 23.58        | 23.62        | <b>29.67</b> | <b>22.51</b> | <b>28.23</b> | <b>26.47</b> | <b>23.66</b> |
| Erase <i>Snoopy</i> and <i>Mickey</i> and <i>Spongebob</i> |              |              |              |              |              |              |              |
|                                                            | CS ↓         | CS ↓         | FID ↓        | FID ↓        | FID ↓        | CS ↑         | FID ↓        |
| ConAbl                                                     | 24.92        | 26.46        | 25.12        | 46.47        | 48.24        | 26.37        | 26.71        |
| MACE                                                       | 19.86        | 19.35        | 20.12        | 110.12       | 128.56       | 23.39        | 66.39        |
| RECE                                                       | <b>18.17</b> | <b>18.87</b> | <b>16.23</b> | 40.52        | 52.06        | 26.32        | 32.51        |
| UCE                                                        | 23.29        | 24.63        | 19.08        | 29.20        | 38.15        | 26.30        | 28.71        |
| Ours                                                       | 23.69        | 23.93        | 21.39        | <b>21.40</b> | <b>26.22</b> | <b>26.51</b> | <b>24.99</b> |

| Concept               | Van Gogh     | Picasso      | Monet        | P. Gauguin   | Caravaggio   | MS-COCO      |              |
|-----------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|                       | CS           | CS           | CS           | CS           | CS           | CS           | FID          |
| SD v1.4               | 28.75        | 27.98        | 28.91        | 29.80        | 26.27        | 26.53        | -            |
| Erase <i>Van Gogh</i> |              |              |              |              |              |              |              |
|                       | CS ↓         | FID ↓        | FID ↓        | FID ↓        | FID ↓        | CS ↑         | FID ↓        |
| ConAbl                | 28.16        | 77.01        | 63.80        | 63.20        | 79.25        | 26.46        | <b>18.36</b> |
| MACE                  | 26.66        | 69.92        | 60.88        | 56.18        | 69.04        | 26.50        | 23.15        |
| RECE                  | 26.39        | 60.57        | 61.09        | 47.07        | 72.85        | 26.52        | 23.54        |
| UCE                   | 28.10        | 43.02        | 40.49        | 32.62        | 61.72        | 26.54        | 19.63        |
| Ours                  | <b>26.29</b> | <b>35.86</b> | <b>16.85</b> | <b>24.94</b> | <b>39.75</b> | <b>26.55</b> | 20.36        |
| Erase <i>Picasso</i>  |              |              |              |              |              |              |              |
|                       | FID ↓        | CS ↓         | FID ↓        | FID ↓        | FID ↓        | CS ↑         | FID ↓        |
| ConAbl                | 60.44        | 26.97        | 36.23        | 65.23        | 79.12        | 26.43        | 20.02        |
| MACE                  | 59.58        | 26.48        | 37.02        | 46.35        | 66.20        | 26.47        | 22.86        |
| RECE                  | 51.09        | 26.66        | 25.39        | 46.08        | 75.61        | 26.48        | 23.03        |
| UCE                   | 37.58        | 26.99        | <b>16.72</b> | 32.48        | 59.27        | 26.50        | 20.33        |
| Ours                  | <b>19.18</b> | <b>26.22</b> | 19.87        | <b>24.73</b> | <b>43.63</b> | <b>26.51</b> | <b>19.98</b> |
| Erase <i>Monet</i>    |              |              |              |              |              |              |              |
|                       | FID ↓        | FID ↓        | CS ↓         | FID ↓        | FID ↓        | CS ↑         | FID ↓        |
| ConAbl                | 68.77        | 64.25        | 27.05        | 57.33        | 71.88        | 26.45        | 21.03        |
| MACE                  | 61.50        | 48.41        | 25.98        | 49.66        | 65.87        | 26.47        | 22.76        |
| RECE                  | 56.26        | 45.97        | 25.87        | 46.38        | 64.19        | 26.49        | 24.94        |
| UCE                   | 42.25        | <b>38.73</b> | 27.12        | 33.00        | 56.49        | <b>26.51</b> | 21.58        |
| Ours                  | <b>28.78</b> | 41.21        | <b>25.06</b> | <b>27.85</b> | <b>55.20</b> | 26.48        | <b>20.87</b> |

## 5.1 On Few-Concept Erasure

**Evaluation setup.** To compare the few-concept erasure performance with baseline methods, we conduct experiments on instance erasure and artistic style erasure following [35], where all methods are evaluated based on 80 instance templates and 30 artistic style templates, generating 10 images per template per concept. We use two metrics for evaluation: CLIP Score (CS) [43] measuring the text-image similarity and Fréchet Inception Distance (FID) [22] assessing the distributional distance before and after erasure. Following [35], we select non-target concepts with similar semantics to the target concept for comparison and report CS for targets and FID for non-targets in the main paper. Full comparisons are presented in Appx. D.2. We further compare the generations on MS-COCO captions [31], where we generate images with the first 1,000 captions, and report CS and FID to measure general knowledge preservation.

**Analysis and discussion.** Table 1 compares the results of erasing various instance concepts and artistic styles. Our method consistently achieves the lowest FIDs across all non-target concepts, demonstrating superior prior preservation with minimal alteration to the original content. Moreover, we emphasize that our erasure is sufficiently effective, even without achieving the lowest CS, as shown in Fig. 4 and Appx. D.1. In contrast, lower CS values typically indicate over-erasure, which results in excessive degradation of prior knowledge. Notably, with the number of target concepts increasing from 1 to 3, our FID in *Pikachu* rises from 16.81 to 21.40 (4.59 ↑), while UCE increases from 19.06 to 29.20 (10.14 ↑). A similar pattern is observed in *Hello Kitty* (Our 4.48 ↑ v.s. UCE’s 10.29 ↑), showing our robustness in erasing increasing target concepts.

## 5.2 On Multi-Concept Erasure

**Evaluation setup.** Another more realistic erasure scenario is multi-concept erasure, where massive concepts are required to be erased at once. Herein, we follow the experiment setup in [33] for erasing multiple celebrities, where we experiment with erasing 10, 50, and 100 celebrities and collect another 100 celebrities as non-target concepts. We prepare 5 prompt templates for each celebrity concept. For non-target concepts, we generate 1 image per template for each of the 100 concepts, totaling 500 images. For target concepts, we adjust the per-concept quantity to maintain a total of 500 images (e.g., erasing 10 celebrities involves generating 10 images with 5 templates per concept). In evaluation, we adopt GIPHY Celebrity Detector (GCD) [20] and measure the top-1 GCD accuracy, indicated by  $Acc_e$  for erased target concepts and  $Acc_r$  for retained non-target concepts. Meanwhile, the harmonic

Table 2: **Quantitative comparison of the multi-concept erasure in erasing 10, 50, and 100 celebrities.** The best results are highlighted in **bold**. Our method is capable of erasing up to 100 celebrities at once with low  $Acc_e$  (%) and preserving other non-target celebrities with less appearance alteration with high  $Acc_r$  (%), resulting in the best overall erasure performance  $H_o$  (shaded in pink).

|         | Erase 10 Celebrities |                    |                  |              |              | MS-COCO            |                    |                  |              |              | Erase 50 Celebrities |                    |                  |              |              | MS-COCO            |                    |                  |      |       | Erase 100 Celebrities |                    |                  |      |       | MS-COCO |  |  |  |  |
|---------|----------------------|--------------------|------------------|--------------|--------------|--------------------|--------------------|------------------|--------------|--------------|----------------------|--------------------|------------------|--------------|--------------|--------------------|--------------------|------------------|------|-------|-----------------------|--------------------|------------------|------|-------|---------|--|--|--|--|
|         | Acc <sub>e</sub> ↓   | Acc <sub>r</sub> ↑ | H <sub>o</sub> ↑ | CS ↑         | FID ↓        | Acc <sub>e</sub> ↓ | Acc <sub>r</sub> ↑ | H <sub>o</sub> ↑ | CS ↑         | FID ↓        | Acc <sub>e</sub> ↓   | Acc <sub>r</sub> ↑ | H <sub>o</sub> ↑ | CS ↑         | FID ↓        | Acc <sub>e</sub> ↓ | Acc <sub>r</sub> ↑ | H <sub>o</sub> ↑ | CS ↑ | FID ↓ | Acc <sub>e</sub> ↓    | Acc <sub>r</sub> ↑ | H <sub>o</sub> ↑ | CS ↑ | FID ↓ |         |  |  |  |  |
| SD v1.4 | 91.99                | 89.66              | 14.70            | 26.53        | -            | 93.08              | 89.66              | 12.85            | 26.53        | -            | 90.18                | 89.66              | 17.70            | 26.53        | -            |                    |                    |                  |      |       |                       |                    |                  |      |       |         |  |  |  |  |
| ConAbl  | 60.76                | 77.89              | 52.19            | 25.60        | 42.12        | 64.00              | 75.44              | 48.74            | 14.30        | 255.36       | 42.86                | 58.82              | 57.97            | 14.93        | 235.27       |                    |                    |                  |      |       |                       |                    |                  |      |       |         |  |  |  |  |
| UCE     | 0.20                 | 71.19              | 83.10            | 24.07        | 83.81        | 0.00               | 31.94              | 48.41            | 13.45        | 209.93       | 0.00                 | 20.92              | 34.60            | 13.49        | 185.46       |                    |                    |                  |      |       |                       |                    |                  |      |       |         |  |  |  |  |
| RECE    | 0.34                 | 67.43              | 80.44            | 16.75        | 170.65       | 1.03               | 19.77              | 32.95            | 13.49        | 213.39       | 2.43                 | 23.71              | 38.16            | 12.09        | 177.57       |                    |                    |                  |      |       |                       |                    |                  |      |       |         |  |  |  |  |
| MACE    | 1.62                 | 87.73              | 92.75            | 26.36        | 37.25        | 3.41               | 84.31              | 90.03            | 25.45        | 45.31        | 4.80                 | 80.20              | 87.06            | 24.80        | 50.41        |                    |                    |                  |      |       |                       |                    |                  |      |       |         |  |  |  |  |
| Ours    | 1.81                 | 89.09              | <b>93.42</b>     | <b>26.47</b> | <b>30.02</b> | 3.46               | 88.48              | <b>92.34</b>     | <b>26.46</b> | <b>39.23</b> | 5.87                 | 85.54              | <b>89.63</b>     | <b>26.22</b> | <b>44.97</b> |                    |                    |                  |      |       |                       |                    |                  |      |       |         |  |  |  |  |

Table 3: **Duration comparison (s) in erasing multiple celebrities** on one A100 GPU, where  $n$  is the number of target concepts. During data preparation, ConAbl requires pre-sampling 1,000 images ( $t_1$ ) while MACE needs 8 pre-sampled images along with 8 segmentation masks ( $t_2$ ) using SAM [27].  $H_o$  is also included to compare multi-concept erasure performance.

|                         | Training-based  |                    | Editing-based |      |             |
|-------------------------|-----------------|--------------------|---------------|------|-------------|
|                         | ConAbl          | MACE               | UCE           | RECE | Ours        |
| <b>Data Preparation</b> | $n \times 1000$ | $n \times (8 + 8)$ | 0             | 0    | 0           |
| <b>1-concept</b>        | $1 \times 90$   | 55.1               | 1.2           | 1.5  | 3.6         |
| $H_o \uparrow$          | 52.2            | 92.7               | 83.1          | 80.4 | <b>93.4</b> |
| <b>10-concepts</b>      | $10 \times 90$  | 207.0              | 1.5           | 2.5  | 3.8         |
| $H_o \uparrow$          | 48.7            | 90.0               | 48.4          | 33.0 | <b>92.3</b> |
| <b>100-concepts</b>     | $100 \times 90$ | 1735.9             | 2.1           | 11.0 | 5.0         |
| $H_o \uparrow$          | 58.0            | 87.1               | 34.6          | 38.2 | <b>89.6</b> |

Table 4: **Ablation study on proposed components** in erasing *Van Gogh*, with the non-target FID averaged over the other four artistic styles from Table 1. Ablation 1 corresponds to the original objective from [14] in Eq. 3. The ablated components include: IEC (Invariant Equality Constraints), IPF (Influence-based Prior Filtering), RPA (Random Prior Augmentation), and DPA (Directed Prior Augmentation).

| Ablation | Components |     |     |     | Target          | Non-Target       | MS-COCO                        |
|----------|------------|-----|-----|-----|-----------------|------------------|--------------------------------|
|          | IEC        | IPF | RPA | DPA | CS $\downarrow$ | FID $\downarrow$ | CS $\uparrow$ FID $\downarrow$ |
| 1        | ×          | ×   | ×   | ×   | 27.20           | 50.43            | 26.42 26.33                    |
| 2        | ✓          | ×   | ×   | ×   | 27.20           | 48.17            | 26.44 24.95                    |
| 3        | ✓          | ✓   | ×   | ×   | 26.68           | 38.02            | 26.54 20.57                    |
| 4        | ✓          | ✓   | ✓   | ×   | 26.30           | 32.62            | 26.52 20.99                    |
| Ours     | ✓          | ✓   | ×   | ✓   | <b>26.29</b>    | <b>29.35</b>     | <b>26.55 20.36</b>             |
| SD v1.4  | -          | -   | -   | -   | 28.75           | -                | 26.53 -                        |

mean  $H_o = \frac{2}{(1 - Acc_e)^{-1} + (Acc_r)^{-1}}$  is adopted to assess the overall erasure performance. Additionally, we report the results on MS-COCO to demonstrate the prior preservation of general concepts.

**Analysis and discussion.** Table 2 showcases a notable improvement of our method on multi-concept erasure, particularly in prior preservation with the highest  $Acc_r$ . In comparison with the SOTA method, MACE [33], our method achieves superior prior preservation with better  $Acc_r$ , while maintaining comparable erasure efficacy, as reflected in similar  $Acc_e$ , resulting in the best overall erasure performance indicated by the highest  $H_o$ . Meanwhile, our method attains the lowest FID across all methods on MS-COCO. The other methods, UCE [18] and RECE [19], although achieving considerable balance in few-concept erasure, fail to maintain this balance as the number of target concepts increases as shown in Fig. 5, with catastrophic prior damage evidenced by MS-COCO as well. Notably, our method can erase up to 100 celebrities in 5 seconds, whereas MACE requires around 30 minutes ( $\times 350$  time). In real-world scenarios, this efficiency underscores our potential for the instant erasure of massive concepts.

### 5.3 Further Analysis

**Duration comparison.** Table 3 presents the duration comparison in erasing 1, 10, and 100 concepts across different methods. It is obvious that training-based methods necessitate significantly higher computational costs than editing-based ones. In contrast, our method achieves precise multi-concept

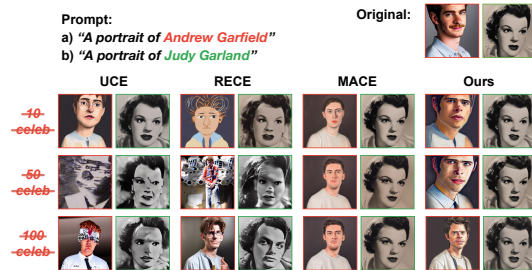


Figure 5: **Quantitative comparison of multi-concept erasure** in erasing celebrities (celeb). The erased and preserved generations are marked with red and green boxes. Our method precisely erases 100 celebrities while preserving generations of other non-target concepts.

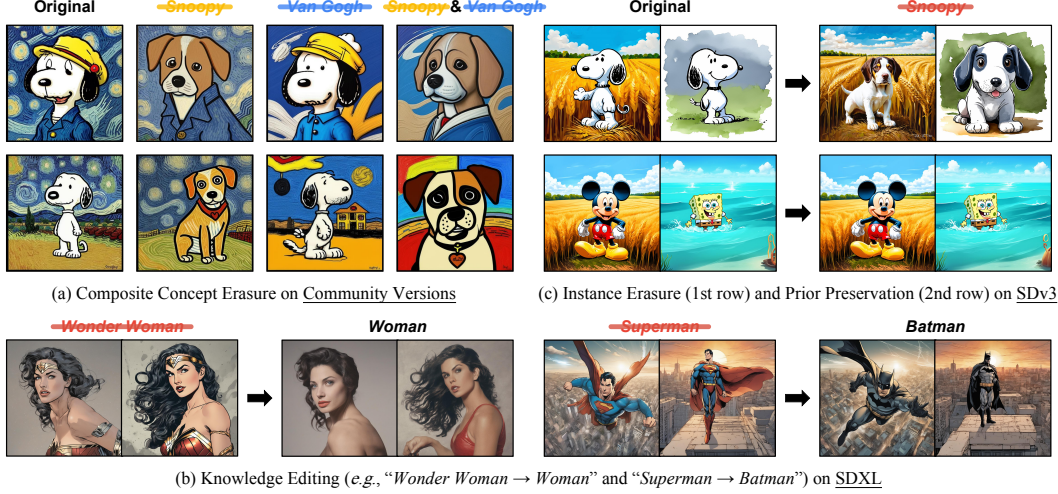


Figure 6: **More applications across various T2I diffusion models.** (a) We conduct composite concept erasure for “*Snoopy + Van Gogh*” on DreamShaper [3] (1st row) and RealisticVision [4] (2nd row). (b) Our method also enables model knowledge editing by specifying the anchor concept on SDXL [42]. (c) Our method can seamlessly transfer to novel DiT-based T2I models, e.g., SDv3 [12].

287 erasure in a remarkably short time, demonstrating superior efficiency while maintaining erasure  
288 performance, as evidenced in Table 2.

289 **Component ablation.** In Table 4, we compare the individual impact of our components on prior  
290 preservation and draw the following conclusions: (1) Impact of IEC (Ablation 1 v.s. 2): IEC reduces  
291 the non-target FID and the MS-COCO FID, demonstrating its effectiveness by preserving invariant  
292 embeddings with equality constraints. (2) Impact of IPF (Ablation 2 v.s. 3): Incorporating IPF results  
293 in a significant improvement in both FIDs, underscoring its critical role in filtering out less-influenced  
294 concepts in the retain set to mitigate semantic degradation. (3) Impact of DPA (Ablation 4 v.s. Ours):  
295 DPA improves RPA with directed noise and leads to a substantial improvement in non-target and  
296 MS-COCO FIDs, highlighting its advantage by introducing semantically similar concepts into the  
297 refined retain set. To conclude, the proposed three components (*i.e.*, IEC, IPF, and DPA) improve  
298 the prior preservation from different perspectives and contribute to our method with the best prior  
299 preservation under null space constraints. More ablations are presented in Appx. D.5.

300 **More applications on other T2I models.** To validate the transferability of our method across versatile  
301 applications, we conduct further experiments on various T2I models with different weights and  
302 architectures, including: (1) Composite concept erasure on DreamShaper [3] and RealisticVision [4]  
303 from Fig 6 (a): Our method can precisely erase the target concept(s) while preserving other non-target  
304 elements within the prompt, such as the *Van Gogh*-style background (2nd column) and the *Snoopy*  
305 character (3rd column). (2) Knowledge editing on SDXL [42] from Fig 6 (b): The arbitrary nature of  
306 anchor concepts allows us to edit the pre-trained model knowledge. Herein, our method effectively  
307 edits the model knowledge while maintaining the overall layout and semantics of the generated  
308 images. (3) Instance erasure on SDv3 [12] from Fig 6 (c): To accommodate the diffusion transformer  
309 (DiT) [41] architecture in T2I models, we adapt our method to a DiT-based model, demonstrating a  
310 well-balanced trade-off between erasure (1st row) and preservation (2nd row) as well.

## 311 6 Conclusion

312 This paper introduced SPEED, a scalable, precise, and efficient concept erasure method for T2I  
313 diffusion models. It formulates concept erasure as a null-space constrained optimization problem,  
314 facilitating effective prior preservation along with precise erasure efficacy. Critically, SPEED  
315 overcomes the inefficacy of editing-based methods in multi-concept erasure while circumventing the  
316 prohibitive computational costs associated with training-based approaches. With our proposed Prior  
317 Knowledge Refinement involving three complementary techniques, SPEED not only ensures superior  
318 prior preservation but also achieves a 350× acceleration in multi-concept erasure, establishing itself  
319 as a scalable and practical solution for real-world applications.

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## A Preliminaries

**T2I diffusion models.** T2I generation has seen significant advancements with diffusion models, particularly Latent Diffusion Models (LDMs) [47]. Unlike pixel-space diffusion, LDMs operate in the latent space of a pretrained autoencoder, reducing computational costs while maintaining high-quality synthesis. LDMs consist of a vector-quantized autoencoder [57, 13] and a diffusion model [11, 23, 53, 26, 55]. The autoencoder encodes an image  $x$  into a latent representation  $z = \mathcal{E}(x)$  and reconstructs it via  $x \approx \mathcal{D}(z)$ . The diffusion model learns to generate latent codes through a denoising process. The training objective is given by [23, 47]:

$$\mathcal{L}_{\text{LDM}} = \mathbb{E}_{z \sim \mathcal{E}(x), c, \epsilon \sim \mathcal{N}(0,1), t} \left[ \|\epsilon - \epsilon_{\theta}(z_t, t, c)\|_2^2 \right], \quad (13)$$

where  $z_t$  is the noisy latent at timestep  $t$ ,  $\epsilon$  is Gaussian noise,  $\epsilon_{\theta}$  is the denoising network, and  $c$  is conditioning information from text, class labels, or segmentation masks [47]. During inference, a latent  $z_T$  is sampled from a Gaussian prior and progressively denoised to obtain  $z_0$ , which is then decoded into an image via  $x_0 \approx \mathcal{D}(z_0)$ .

**Cross-attention mechanisms.** Current T2I diffusion models usually leverage a generative framework to synthesize images conditioned on textual descriptions in the latent space [47]. The conditioning mechanism is implemented through cross-attention (CA) layers. Specifically, textual descriptions are first tokenized into  $n$  tokens and embedded into a sequence of vectors  $e \in \mathbb{R}^{d_o \times n}$  via a pre-trained CLIP model [43]. These text embeddings serve as the key  $\mathbf{K} \in \mathbb{R}^{n \times d_k}$  and value  $\mathbf{V} \in \mathbb{R}^{n \times d_v}$  inputs using parametric projection matrices  $\mathbf{W}_{\mathbf{K}} \in \mathbb{R}^{d_k \times d_o}$  and  $\mathbf{W}_{\mathbf{V}} \in \mathbb{R}^{d_v \times d_o}$ , while the intermediate image representations act as the query  $\mathbf{Q} \in \mathbb{R}^{m \times d_k}$ . The cross-attention mechanism is defined as:

$$\text{Attention}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{softmax} \left( \frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d_k}} \right) \mathbf{V}. \quad (14)$$

This alignment enables the model to capture semantic correlations between the textual input and the visual features, ensuring that the generated images are semantically consistent with the provided text prompts.

## B Proof and Derivation

### B.1 Deriving the Closed-Form Solution for UCE

From Eq. 1, we are tasked with minimizing the following editing objective, where the hyperparameters  $\alpha$  and  $\beta$  correspond to the weights of the erasure error  $e_1$  and the preservation error  $e_0$ , respectively:

$$\min_{\Delta} [\alpha \|(\mathbf{W} + \Delta)\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*\|^2 + \beta \|\Delta\mathbf{C}_0\|^2]. \quad (15)$$

To derive the closed-form solution, we begin by computing the gradient of the objective function with respect to  $\Delta$ . The gradient is given by:

$$\alpha (\mathbf{W}\mathbf{C}_1 - \mathbf{W}\mathbf{C}_* + \Delta\mathbf{C}_1) \mathbf{C}_1^T + \beta \Delta\mathbf{C}_0\mathbf{C}_0^T = 0. \quad (16)$$

Solving the resulting equation yields the closed-form solution for  $\Delta_{\text{UCE}}$ :

$$\Delta_{\text{UCE}} = \alpha \mathbf{W} (\mathbf{C}_*\mathbf{C}_1^T - \mathbf{C}_1\mathbf{C}_1^T) (\alpha\mathbf{C}_1\mathbf{C}_1^T + \beta\mathbf{C}_0\mathbf{C}_0^T)^{-1}. \quad (17)$$

In practice, an additional identity matrix  $\mathbf{I}$  with hyperparameter  $\lambda$  is added to  $(\alpha\mathbf{C}_1\mathbf{C}_1^T + \beta\mathbf{C}_0\mathbf{C}_0^T)^{-1}$  to ensure its invertibility. This modification results in the following closed-form solution for UCE:

$$\Delta_{\text{UCE}} = \alpha \mathbf{W} (\mathbf{C}_*\mathbf{C}_1^T - \mathbf{C}_1\mathbf{C}_1^T) (\alpha\mathbf{C}_1\mathbf{C}_1^T + \beta\mathbf{C}_0\mathbf{C}_0^T + \lambda\mathbf{I})^{-1}. \quad (18)$$

### B.2 Proof of the Lower Bound of $e_0$ for UCE

Herein, we aim to establish the existence of a strictly positive constant  $c > 0$  such that

$$e_0 = \|\Delta_{\text{UCE}}\mathbf{C}_0\|^2 = \|\alpha \mathbf{W} (\mathbf{C}_*\mathbf{C}_1^T - \mathbf{C}_1\mathbf{C}_1^T) (\alpha\mathbf{C}_1\mathbf{C}_1^T + \beta\mathbf{C}_0\mathbf{C}_0^T + \lambda\mathbf{I})^{-1}\mathbf{C}_0\|^2 \geq c > 0. \quad (19)$$

**Assumption B.1.** We assume that  $\alpha, \beta, \lambda \neq 0$ , that  $\mathbf{W}$  is a full-rank matrix, and that  $\mathbf{C}_0 \mathbf{C}_0^\top$  is rank-deficient. Furthermore, we assume that

$$\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top \neq \mathbf{0}.$$

*Proof.* Define the matrix  $\mathbf{M}$  as

$$\mathbf{M} = \alpha \mathbf{C}_1 \mathbf{C}_1^\top + \beta \mathbf{C}_0 \mathbf{C}_0^\top + \lambda \mathbf{I}. \quad (20)$$

Since  $\lambda > 0$  and  $\mathbf{I}$  is positive definite, it follows that  $\mathbf{M}$  is strictly positive definite and therefore invertible.

Rewriting  $e_0$  by defining  $\mathbf{B} = \mathbf{M}^{-1} \mathbf{C}_0$ , we obtain

$$e_0 = \|\alpha \mathbf{W}(\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top) \mathbf{B}\|^2. \quad (21)$$

Applying the singular value bound for matrix products, we have

$$\|\mathbf{X}\mathbf{Y}\| \geq \sigma_{\min}(\mathbf{X})\|\mathbf{Y}\|, \quad (22)$$

where  $\sigma_{\min}(\mathbf{X})$  is the smallest singular value of  $\mathbf{X}$ . Applying this inequality, we obtain

$$\|\mathbf{W}(\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top) \mathbf{B}\| \geq \sigma_{\min}(\mathbf{W})\|(\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top) \mathbf{B}\|. \quad (23)$$

We start with the singular value decomposition (SVD) of the matrix  $\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top$ , given by

$$\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^\top. \quad (24)$$

Here,  $\mathbf{U}$  and  $\mathbf{V}$  are orthogonal matrices, and

$$\mathbf{\Sigma} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r, 0, \dots, 0) \quad (25)$$

is a diagonal matrix containing the singular values  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > 0$ , followed by zeros.

Multiplying both sides by  $\mathbf{B}$ , we obtain

$$(\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top) \mathbf{B} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^\top \mathbf{B}. \quad (26)$$

Define the projection of  $\mathbf{B}$  onto the subspace spanned by the right singular vectors as

$$\mathbf{B}_{\text{proj}} = \mathbf{V}^\top \mathbf{B}. \quad (27)$$

Then, we can rewrite the expression as

$$(\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top) \mathbf{B} = \mathbf{U} \mathbf{\Sigma} \mathbf{B}_{\text{proj}}. \quad (28)$$

Taking norms on both sides and using the fact that orthogonal transformations preserve norms, we get

$$\|(\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top) \mathbf{B}\| = \|\mathbf{\Sigma} \mathbf{B}_{\text{proj}}\|. \quad (29)$$

Since  $\mathbf{\Sigma}$  is a diagonal matrix, its smallest nonzero singular value  $\sigma_r$  provides a lower bound:

$$\|\mathbf{\Sigma} \mathbf{B}_{\text{proj}}\| \geq \sigma_r \|\mathbf{B}_{\text{proj}}\|. \quad (30)$$

Next, we establish a lower bound for  $\|\mathbf{B}_{\text{proj}}\|$ . Given that  $\mathbf{V}$  is composed of right singular vectors, there exists a smallest non-zero singular value  $c_1$  such that:

$$\|\mathbf{B}_{\text{proj}}\| \geq c_1 \|\mathbf{B}\|. \quad (31)$$

Combining these inequalities, we obtain

$$\|(\mathbf{C}_* \mathbf{C}_1^\top - \mathbf{C}_1 \mathbf{C}_1^\top) \mathbf{B}\| \geq \sigma_r \|\mathbf{B}_{\text{proj}}\| \geq \sigma_r c_1 \|\mathbf{B}\|. \quad (32)$$

Since  $\mathbf{M}$  is positive definite, we use the standard norm inequality for an invertible matrix  $\mathbf{M}$ , which states that for any matrix  $\mathbf{X}$ ,

$$\|\mathbf{M}\mathbf{X}\| \leq \|\mathbf{M}\| \|\mathbf{X}\|. \quad (33)$$

Setting  $\mathbf{X} = \mathbf{M}^{-1}\mathbf{C}_0$ , we obtain

$$\|\mathbf{M}\mathbf{M}^{-1}\mathbf{C}_0\| \leq \|\mathbf{M}\| \|\mathbf{M}^{-1}\mathbf{C}_0\|. \quad (34)$$

Since  $\mathbf{M}\mathbf{M}^{-1} = \mathbf{I}$ , the left-hand side simplifies to  $\|\mathbf{C}_0\|$ , yielding

$$\|\mathbf{C}_0\| \leq \|\mathbf{M}\| \|\mathbf{M}^{-1}\mathbf{C}_0\|. \quad (35)$$

Dividing both sides by  $\|\mathbf{M}\|$ , we obtain

$$\|\mathbf{M}^{-1}\mathbf{C}_0\| \geq \frac{1}{\|\mathbf{M}\|} \|\mathbf{C}_0\|. \quad (36)$$

Thus, it follows that

$$\|\mathbf{B}\| = \|\mathbf{M}^{-1}\mathbf{C}_0\| \geq \frac{1}{\|\mathbf{M}\|} \|\mathbf{C}_0\|. \quad (37)$$

Combining the above results, we obtain

$$\|\mathbf{W}(\mathbf{C}_*\mathbf{C}_1^\top - \mathbf{C}_1\mathbf{C}_1^\top)\mathbf{B}\| \geq \sigma_{\min}(\mathbf{W})\sigma_r c_1 \frac{1}{\|\mathbf{M}\|} \|\mathbf{C}_0\|. \quad (38)$$

Squaring both sides, we conclude that

$$e_0 = \|\alpha\mathbf{W}(\mathbf{C}_*\mathbf{C}_1^\top - \mathbf{C}_1\mathbf{C}_1^\top)\mathbf{B}\|^2 \geq \alpha^2 \sigma_{\min}^2(\mathbf{W}) \sigma_r^2 c_1^2 \frac{1}{\|\mathbf{M}\|^2} \|\mathbf{C}_0\|^2. \quad (39)$$

Since all terms on the right-hand side are strictly positive by assumption, we establish the existence of a positive lower bound  $c > 0$  such that

$$e_0 \geq c > 0. \quad (40)$$

This completes the proof.  $\square$

### B.3 Deriving the Closed-Form Solution for SPEED

From Eq. 10, we are tasked with minimizing the following editing objective:

$$\min_{\Delta} \|(\mathbf{W} + \Delta\mathbf{P})\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*\|^2 + \|\Delta\mathbf{P}\|^2, \quad \text{s.t. } (\Delta\mathbf{P})\mathbf{C}_2 = \mathbf{0}. \quad (41)$$

This is a weighted least squares problem subject to an equality constraint. To solve it, we first formulate the Lagrangian function, where  $\Lambda$  is the Lagrange multiplier:

$$\mathcal{L}(\Delta, \Lambda) = \|(\mathbf{W} + \Delta\mathbf{P})\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*\|^2 + \|\Delta\mathbf{P}\|^2 + \Lambda^\top ((\Delta\mathbf{P})\mathbf{C}_2). \quad (42)$$

We compute the gradient of the Lagrangian function in Eq. 42 with respect to  $\Delta$  and set it to zero, yielding the following equation for  $\Delta$ :

$$\frac{\partial \mathcal{L}(\Delta, \Lambda)}{\partial \Delta} = 2((\mathbf{W} + \Delta\mathbf{P})\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*)\mathbf{C}_1^\top \mathbf{P}^\top + 2\Delta\mathbf{P}\mathbf{P}^\top + \Lambda\mathbf{C}_2^\top \mathbf{P}^\top = \mathbf{0}. \quad (43)$$

Given that the projection matrix  $\mathbf{P}$  is derived from  $R_{\text{refine}}$  using Eq. 2,  $\mathbf{P}$  is a symmetric matrix (*i.e.*,  $\mathbf{P} = \mathbf{P}^\top$ ) and an idempotent matrix (*i.e.*,  $\mathbf{P}^2 = \mathbf{P}$ ), the above formulation can be simplified to:

$$\frac{\partial \mathcal{L}(\Delta, \Lambda)}{\partial \Delta} = 2((\mathbf{W} + \Delta\mathbf{P})\mathbf{C}_1 - \mathbf{W}\mathbf{C}_*)\mathbf{C}_1^\top \mathbf{P} + 2\Delta\mathbf{P} + \Lambda\mathbf{C}_2^\top \mathbf{P} = \mathbf{0}. \quad (44)$$

Therefore, we can obtain the closed-form solution for  $\Delta\mathbf{P}$  from this equation:

$$\Delta\mathbf{P} = (\mathbf{W}\mathbf{C}_*\mathbf{C}_1^\top \mathbf{P} - \mathbf{W}\mathbf{C}_1\mathbf{C}_1^\top \mathbf{P} - \frac{1}{2}\Lambda\mathbf{C}_2^\top \mathbf{P})(\mathbf{C}_1\mathbf{C}_1^\top \mathbf{P} + \mathbf{I})^{-1}. \quad (45)$$

Next, we differentiate the Lagrangian function in Eq. 42 with respect to  $\Lambda$  and set it to zero:

$$\frac{\partial \mathcal{L}(\Delta, \Lambda)}{\partial \Lambda} = (\Delta\mathbf{P})\mathbf{C}_2 = \mathbf{0}. \quad (46)$$

For simplicity, we define  $\mathbf{M} = (\mathbf{C}_1\mathbf{C}_1^\top \mathbf{P} + \mathbf{I})^{-1}$ . Then, we substitute the result of Eq. 45 into Eq. 46 and obtain:

$$(\mathbf{W}\mathbf{C}_*\mathbf{C}_1^\top \mathbf{P} - \mathbf{W}\mathbf{C}_1\mathbf{C}_1^\top \mathbf{P} - \frac{1}{2}\Lambda\mathbf{C}_2^\top \mathbf{P})\mathbf{M}\mathbf{C}_2 = \mathbf{0}. \quad (47)$$

Solving this equation leads to:

$$\frac{1}{2}\Lambda = \mathbf{W}(\mathbf{C}_*\mathbf{C}_1^\top - \mathbf{C}_1\mathbf{C}_1^\top)\mathbf{P}\mathbf{M}\mathbf{C}_2(\mathbf{C}_2^\top \mathbf{P}\mathbf{M}\mathbf{C}_2)^{-1}. \quad (48)$$

Substituting Eq. 48 back into Eq. 45, we have the closed-form solution of our objective:

$$(\Delta\mathbf{P})_{\text{SPEED}} = \mathbf{W}(\mathbf{C}_*\mathbf{C}_1^\top - \mathbf{C}_1\mathbf{C}_1^\top)\mathbf{P}\mathbf{Q}\mathbf{M}, \quad (49)$$

where  $\mathbf{Q} = \mathbf{I} - \mathbf{M}\mathbf{C}_2(\mathbf{C}_2^\top \mathbf{P}\mathbf{M}\mathbf{C}_2)^{-1}\mathbf{C}_2^\top \mathbf{P}$  and  $\mathbf{M} = (\mathbf{C}_1\mathbf{C}_1^\top \mathbf{P} + \mathbf{I})^{-1}$ .

Table 5: **The evaluation setup for multi-concept erasure.** This celebrity dataset contains an erasure set with 100 celebrities and a retain set with another 100 celebrities. We experiment with erasing 10, 50, and 100 celebrities with the pre-defined target concepts and the entire retain set is utilized in all cases.

| Group       | Number          | Anchor Concept | Celebrity                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|-------------|-----------------|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Erasure Set | 10              | 'person'       | 'Adam Driver', 'Adriana Lima', 'Amber Heard', 'Amy Adams', 'Andrew Garfield', 'Angelina Jolie', 'Anjelica Huston', 'Anna Faris', 'Anna Kendrick', 'Anne Hathaway'                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|             | 50              | 'person'       | 'Adam Driver', 'Adriana Lima', 'Amber Heard', 'Amy Adams', 'Andrew Garfield', 'Angelina Jolie', 'Anjelica Huston', 'Anna Faris', 'Anna Kendrick', 'Anne Hathaway', 'Arnold Schwarzenegger', 'Barack Obama', 'Beth Behrs', 'Bill Clinton', 'Bob Dylan', 'Bob Marley', 'Bradley Cooper', 'Bruce Willis', 'Bryan Cranston', 'Cameron Diaz', 'Channing Tatum', 'Charlie Sheen', 'Charlize Theron', 'Chris Evans', 'Chris Hemsworth', 'Chris Pine', 'Chuck Norris', 'Courteney Cox', 'Demi Lovato', 'Drake', 'Drew Barrymore', 'Dwayne Johnson', 'Ed Sheeran', 'Elon Musk', 'Elvis Presley', 'Emma Stone', 'Frida Kahlo', 'George Clooney', 'Glenn Close', 'Gwyneth Paltrow', 'Harrison Ford', 'Hillary Clinton', 'Hugh Jackman', 'Idris Elba', 'Jake Gyllenhaal', 'James Franco', 'Jared Leto', 'Jason Momoa', 'Jennifer Aniston', 'Jennifer Lawrence'                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|             | 100             | 'person'       | 'Adam Driver', 'Adriana Lima', 'Amber Heard', 'Amy Adams', 'Andrew Garfield', 'Angelina Jolie', 'Anjelica Huston', 'Anna Faris', 'Anna Kendrick', 'Anne Hathaway', 'Arnold Schwarzenegger', 'Barack Obama', 'Beth Behrs', 'Bill Clinton', 'Bob Dylan', 'Bob Marley', 'Bradley Cooper', 'Bruce Willis', 'Bryan Cranston', 'Cameron Diaz', 'Channing Tatum', 'Charlie Sheen', 'Charlize Theron', 'Chris Evans', 'Chris Hemsworth', 'Chris Pine', 'Chuck Norris', 'Courteney Cox', 'Demi Lovato', 'Drake', 'Drew Barrymore', 'Dwayne Johnson', 'Ed Sheeran', 'Elon Musk', 'Elvis Presley', 'Emma Stone', 'Frida Kahlo', 'George Clooney', 'Glenn Close', 'Gwyneth Paltrow', 'Harrison Ford', 'Hillary Clinton', 'Hugh Jackman', 'Idris Elba', 'Jake Gyllenhaal', 'James Franco', 'Jared Leto', 'Jason Momoa', 'Jennifer Aniston', 'Jennifer Lawrence', 'Jennifer Lopez', 'Jeremy Renner', 'Jessica Biel', 'Jessica Chastain', 'John Oliver', 'John Wayne', 'Johnny Depp', 'Julianne Hough', 'Justin Timberlake', 'Kate Winslet', 'Leonardo DiCaprio', 'Margot Robbie', 'Mariah Carey', 'Melania Trump', 'Meryl Streep', 'Mick Jagger', 'Mila Kunis', 'Milla Jovovich', 'Morgan Freeman', 'Nick Jonas', 'Nicolas Cage', 'Nicole Kidman', 'Octavia Spencer', 'Olivia Wilde', 'Oprah Winfrey', 'Paul McCartney', 'Paul Walker', 'Peter Dinklage', 'Philip Seymour Hoffman', 'Reese Witherspoon', 'Richard Gere', 'Ricky Gervais', 'Rihanna', 'Robin Williams', 'Ronald Reagan', 'Ryan Gosling', 'Ryan Reynolds', 'Shia Labeouf', 'Shirley Temple', 'Spike Lee', 'Stan Lee', 'Theresa May', 'Tom Cruise', 'Tom Hanks', 'Tom Hardy', 'Tom Hiddleston', 'Whoopi Goldberg', 'Zac Efron', 'Zayn Malik'                         |
| Retain Set  | 10, 50, and 100 | -              | 'Aaron Paul', 'Alec Baldwin', 'Amanda Seyfried', 'Amy Poehler', 'Amy Schumer', 'Amy Winehouse', 'Andy Samberg', 'Aretha Franklin', 'Avril Lavigne', 'Aziz Ansari', 'Barry Manilow', 'Ben Affleck', 'Ben Stiller', 'Benicio Del Toro', 'Bette Midler', 'Betty White', 'Bill Murray', 'Bill Nye', 'Britney Spears', 'Brittany Snow', 'Bruce Lee', 'Burt Reynolds', 'Charles Manson', 'Christie Brinkley', 'Christina Hendricks', 'Clint Eastwood', 'Countess Vaughn', 'Dakota Johnson', 'Dane DeHaan', 'David Bowie', 'David Tennant', 'Denise Richards', 'Doris Day', 'Dr Dre', 'Elizabeth Taylor', 'Emma Roberts', 'Fred Rogers', 'Gal Gadot', 'George Bush', 'George Takei', 'Gillian Anderson', 'Gordon Ramsey', 'Halle Berry', 'Harry Dean Stanton', 'Harry Styles', 'Hayley Atwell', 'Heath Ledger', 'Henry Cavill', 'Jackie Chan', 'Jada Pinkett Smith', 'James Garner', 'Jason Statham', 'Jeff Bridges', 'Jennifer Connelly', 'Jensen Ackles', 'Jim Morrison', 'Jimmy Carter', 'Joan Rivers', 'John Lennon', 'Johnny Cash', 'Jon Hamm', 'Judy Garland', 'Julianne Moore', 'Justin Bieber', 'Kaley Cuoco', 'Kate Upton', 'Keanu Reeves', 'Kim Jong Un', 'Kirsten Dunst', 'Kristen Stewart', 'Krysten Ritter', 'Lana Del Rey', 'Leslie Jones', 'Lily Collins', 'Lindsay Lohan', 'Liv Tyler', 'Lizzy Caplan', 'Maggie Gyllenhaal', 'Matt Damon', 'Matt Smith', 'Matthew McConaughey', 'Maya Angelou', 'Megan Fox', 'Mel Gibson', 'Melanie Griffith', 'Michael Cera', 'Michael Ealy', 'Natalie Portman', 'Neil Degrasse Tyson', 'Niall Horan', 'Patrick Stewart', 'Paul Rudd', 'Paul Wesley', 'Pierce Brosnan', 'Prince', 'Queen Elizabeth', 'Rachel Dratch', 'Rachel McAdams', 'Reba McEntire', 'Robert De Niro' |

## 580 C Implementation Details

### 581 C.1 Experimental Setup Details

582 **Few-concept erasure.** We first compare methods on few-concept erasure, a fundamental concept  
583 erasure task, including both instance erasure and artistic style erasure following [35]. For instance  
584 erasure, we prepare 80 instance templates proposed in CLIP [43], such as “a photo of the {Instance}”,  
585 “a drawing of the {Instance}”, and “a painting of the {Instance}”. For artistic style erasure, we use  
586 ChatGPT [38, 5] to generate 30 artistic style templates, including “{Artistic} style painting of the  
587 night sky with bold strokes”, “{Artistic} style landscape of rolling hills with dramatic brushwork”,  
588 and “Sunrise scene in {Artistic} style, capturing the beauty of dawn”. Following [35], we handpick  
589 the representative target and anchor concepts as the erasure set (i.e., *Snoopy*, *Mickey*, *SpongeBob* → ‘  
590 ’ in instance erasure and *Van Gogh*, *Picasso*, *Monet* → ‘art’ in artistic style erasure) and non-target  
591 concepts for evaluation (i.e., *Pikachu* and *Hello Kitty* in instance erasure and *Paul Gauguin* and  
592 *Caravaggio* in artistic style erasure). In terms of the retain set, for instance erasure, we use a scraping  
593 script to crawl Wikipedia category pages to extract fictional character names and their page view  
594 counts with a threshold of 500,000 views from 2020.01.01 to 2023.12.31, resulting in 1,352 instances.  
595 For artistic style erasure, we use the 1,734 artistic styles collected from UCE [18]. In evaluation,  
596 we generate 10 images per template per concept, resulting in 800 and 300 images for each concept  
597 in instance erasure and artistic style erasure, respectively. Moreover, we introduce the MS-COCO

Table 6: **Full quantitative comparison of the few-concept erasure in erasing instances from Table 1 (left).** The best results are highlighted in **bold**, and grey columns are indirect indicators for measuring erasure efficacy on target concepts or prior preservation on non-target concepts.

|                                              | <i>Snoopy</i> |        | <i>Mickey</i> |              | <i>Spongebob</i> |              | <i>Pikachu</i> |              | <i>Hello Kitty</i> |              | <i>MS-COCO</i> |              |
|----------------------------------------------|---------------|--------|---------------|--------------|------------------|--------------|----------------|--------------|--------------------|--------------|----------------|--------------|
|                                              | CS            | FID    | CS            | FID          | CS               | FID          | CS             | FID          | CS                 | FID          | CS             | FID          |
| SD v1.4                                      | 28.51         | -      | 26.62         | -            | 27.30            | -            | 27.44          | -            | 27.77              | -            | 26.53          | -            |
| <i>Erase Snoopy</i>                          |               |        |               |              |                  |              |                |              |                    |              |                |              |
|                                              | CS ↓          | FID ↑  | CS ↑          | FID ↓        | CS ↑             | FID ↓        | CS ↑           | FID ↓        | CS ↑               | FID ↓        | CS ↑           | FID ↓        |
| ConAbl                                       | 25.44         | 98.38  | 26.63         | 37.08        | 26.95            | 38.92        | 27.47          | 26.14        | 27.65              | 36.52        | 26.40          | 21.20        |
| MACE                                         | 20.90         | 165.74 | 23.46         | 105.97       | 23.35            | 102.77       | 26.05          | 65.71        | 26.05              | 75.42        | 26.09          | 42.62        |
| RECE                                         | <b>18.38</b>  | 151.46 | 26.62         | 26.63        | 27.23            | 34.42        | 27.47          | 21.99        | 27.78              | 32.35        | 26.39          | 25.61        |
| UCE                                          | 23.19         | 102.86 | 26.64         | 24.87        | 27.29            | 29.86        | 27.47          | 19.06        | 27.75              | 27.86        | 26.46          | 22.18        |
| Ours                                         | 23.50         | 108.51 | 26.67         | <b>23.41</b> | 27.31            | <b>24.64</b> | 27.48          | <b>16.81</b> | 27.82              | <b>21.74</b> | <b>26.48</b>   | <b>19.95</b> |
| <i>Erase Snoopy and Mickey</i>               |               |        |               |              |                  |              |                |              |                    |              |                |              |
|                                              | CS ↓          | FID ↑  | CS ↓          | FID ↑        | CS ↑             | FID ↓        | CS ↑           | FID ↓        | CS ↑               | FID ↓        | CS ↑           | FID ↓        |
| ConAbl                                       | 25.26         | 106.78 | 26.58         | 57.05        | 26.81            | 45.08        | 27.34          | 35.57        | 27.74              | 41.48        | 26.42          | 24.34        |
| MACE                                         | 20.53         | 170.01 | 20.63         | 142.98       | 22.03            | 112.01       | 24.98          | 91.72        | 23.64              | 106.88       | 25.50          | 55.15        |
| RECE                                         | <b>18.57</b>  | 150.84 | <b>19.14</b>  | 145.59       | 27.29            | 35.85        | 27.37          | 26.05        | 27.71              | 40.77        | 26.31          | 30.30        |
| UCE                                          | 23.60         | 99.30  | 24.79         | 86.32        | 27.32            | 30.58        | 27.38          | 23.51        | 27.74              | 31.76        | 26.38          | 26.06        |
| Ours                                         | 23.58         | 103.62 | 23.62         | 83.70        | 27.34            | <b>29.67</b> | 27.39          | <b>22.51</b> | 27.78              | <b>28.23</b> | <b>26.47</b>   | <b>23.66</b> |
| <i>Erase Snoopy and Mickey and Spongebob</i> |               |        |               |              |                  |              |                |              |                    |              |                |              |
|                                              | CS ↓          | FID ↑  | CS ↓          | FID ↑        | CS ↓             | FID ↑        | CS ↑           | FID ↓        | CS ↑               | FID ↓        | CS ↑           | FID ↓        |
| ConAbl                                       | 24.92         | 112.66 | 26.46         | 63.95        | 25.12            | 102.68       | 27.36          | 46.47        | 27.72              | 48.24        | 26.37          | 26.71        |
| MACE                                         | 19.86         | 175.43 | 19.35         | 140.13       | 20.12            | 143.17       | 19.76          | 110.12       | 21.03              | 128.56       | 23.39          | 66.39        |
| RECE                                         | <b>18.17</b>  | 155.26 | <b>18.87</b>  | 149.77       | <b>16.23</b>     | 178.55       | 27.34          | 40.52        | 27.71              | 52.06        | 26.32          | 32.51        |
| UCE                                          | 23.29         | 101.40 | 24.63         | 88.11        | 19.08            | 140.40       | 27.45          | 29.20        | 27.82              | 38.15        | 26.30          | 28.71        |
| Ours                                         | 23.69         | 103.33 | 23.93         | 86.55        | 21.39            | 109.28       | 27.47          | <b>21.40</b> | 27.76              | <b>26.22</b> | <b>26.51</b>   | <b>24.99</b> |

Table 7: **Full quantitative comparison of the few-concept erasure in erasing artistic styles from Table 1 (right).** The best results are highlighted in **bold**, and the grey columns are indirect indicators for measuring erasure efficacy on target concepts or prior preservation on non-target concepts.

|                       | <i>Van Gogh</i> |              | <i>Picasso</i> |              | <i>Monet</i> |              | <i>Paul Gauguin</i> |              | <i>Caravaggio</i> |              | <i>MS-COCO</i> |              |
|-----------------------|-----------------|--------------|----------------|--------------|--------------|--------------|---------------------|--------------|-------------------|--------------|----------------|--------------|
|                       | CS              | FID          | CS             | FID          | CS           | FID          | CS                  | FID          | CS                | FID          | CS             | FID          |
| SD v1.4               | 28.75           | -            | 27.98          | -            | 28.91        | -            | 29.80               | -            | 26.27             | -            | 26.53          | -            |
| <i>Erase Van Gogh</i> |                 |              |                |              |              |              |                     |              |                   |              |                |              |
|                       | CS ↓            | FID ↑        | CS ↑           | FID ↓        | CS ↑         | FID ↓        | CS ↑                | FID ↓        | CS ↑              | FID ↓        | CS ↑           | FID ↓        |
| ConAbl                | 28.16           | 129.57       | 27.07          | 77.01        | 28.44        | 63.80        | 29.49               | 63.20        | 26.15             | 79.25        | 26.46          | <b>18.36</b> |
| MACE                  | 26.66           | 169.60       | 27.39          | 69.92        | 28.84        | 60.88        | 29.39               | 56.18        | 26.19             | 69.04        | 26.50          | 23.15        |
| RECE                  | 26.39           | 171.70       | 27.58          | 60.57        | 28.83        | 61.09        | 29.58               | 47.07        | 26.21             | 72.85        | 26.52          | 23.54        |
| UCE                   | 28.10           | 133.87       | 27.70          | 43.02        | 28.92        | 40.49        | 29.62               | 32.62        | 26.23             | 61.72        | 26.54          | 19.63        |
| Ours                  | <b>26.29</b>    | 131.02       | 27.96          | <b>35.86</b> | 28.94        | <b>16.85</b> | 29.71               | <b>24.94</b> | 26.24             | <b>39.75</b> | <b>26.55</b>   | 20.36        |
| <i>Erase Picasso</i>  |                 |              |                |              |              |              |                     |              |                   |              |                |              |
|                       | CS ↑            | FID ↓        | CS ↓           | FID ↑        | CS ↑         | FID ↓        | CS ↑                | FID ↓        | CS ↑              | FID ↓        | CS ↑           | FID ↓        |
| ConAbl                | 28.66           | 60.44        | <b>26.97</b>   | 131.45       | 28.72        | 36.23        | 29.68               | 65.23        | 26.20             | 79.12        | 26.43          | 20.02        |
| MACE                  | 28.68           | 59.58        | 26.48          | 137.09       | 28.73        | 37.02        | 29.71               | 46.35        | 26.23             | 66.20        | 26.47          | 22.86        |
| RECE                  | 28.71           | 51.09        | 26.66          | 126.40       | 28.87        | 25.39        | 29.69               | 46.08        | 26.22             | 75.61        | 26.48          | 23.03        |
| UCE                   | 28.72           | 37.58        | 26.99          | 102.21       | 28.92        | <b>16.72</b> | 29.71               | 32.48        | 26.22             | 59.27        | 26.50          | 20.33        |
| Ours                  | 28.76           | <b>19.18</b> | <b>26.22</b>   | 117.71       | 28.88        | 19.87        | 29.75               | <b>24.73</b> | 26.24             | <b>43.63</b> | <b>26.51</b>   | <b>19.98</b> |
| <i>Erase Monet</i>    |                 |              |                |              |              |              |                     |              |                   |              |                |              |
|                       | CS ↑            | FID ↓        | CS ↑           | FID ↓        | CS ↓         | FID ↑        | CS ↑                | FID ↓        | CS ↑              | FID ↓        | CS ↑           | FID ↓        |
| ConAbl                | 28.58           | 68.77        | 27.43          | 64.25        | 27.05        | 96.67        | 29.09               | 57.33        | 26.09             | 71.88        | 26.45          | 21.03        |
| MACE                  | 28.56           | 61.50        | 27.74          | 48.41        | 25.98        | 116.34       | 29.39               | 49.66        | 25.98             | 65.87        | 26.47          | 22.76        |
| RECE                  | 28.63           | 56.26        | 27.88          | 45.97        | 25.87        | 121.28       | 29.43               | 46.38        | 26.20             | 64.19        | 26.49          | 24.94        |
| UCE                   | 28.65           | 42.25        | 27.91          | <b>38.73</b> | 27.12        | 98.37        | 29.58               | 33.00        | 26.16             | 56.49        | <b>26.51</b>   | 21.58        |
| Ours                  | 28.76           | <b>28.78</b> | 27.93          | 41.21        | <b>25.06</b> | 134.11       | 29.66               | <b>27.85</b> | 26.22             | <b>55.20</b> | 26.48          | <b>20.87</b> |

captions [31] to serve as general prior knowledge. In implementation, we use the first 1,000 captions to generate a total of 1000 images to compare CS and FID before and after erasure.

**Multi-concept erasure.** We then compare methods on multi-concept erasure, a more challenging and realistic concept erasure task. Following the experiment setup from [33], we introduce a dataset consisting of 200 celebrities, where their portraits generated by SDv1.4 [1] can be recognizable with exceptional accuracy by the GIPHY Celebrity Detector (GCD) [20]. This dataset is divided into two groups: an erasure set with 10, 50, and 100 celebrities and a retain set with 100 other celebrities. The full list for both sets is presented in Table 5. We experiment with erasing 10, 50, and 100 celebrities with the pre-defined target concepts and the entire retain set is utilized in all cases. In evaluation, we

Table 8: **Evaluation of implicit concept erasure on I2P benchmark.** We report the number of nude body parts (F: Female, M: Male) detected by the NudeNet with threshold = 0.6. The best and second-best results are marked in **bold** and underlined. (Left) Our method effectively removes nude content, even though *nudity* is not explicitly mentioned in prompts from I2P, achieving the second-best total count. (Right) Our method also consistently achieves superior prior preservation for non-target concepts to other methods on MS-COCO.

|         | NudeNet Detection Results on I2P |       |          |      |             |               |             |               |            | MS-COCO      |              |
|---------|----------------------------------|-------|----------|------|-------------|---------------|-------------|---------------|------------|--------------|--------------|
|         | Armpits                          | Belly | Buttocks | Feet | Breasts (F) | Genitalia (F) | Breasts (M) | Genitalia (M) | Total ↓    | CS ↑         | FID ↓        |
| SD v1.4 | 123                              | 134   | 19       | 14   | 258         | 9             | 16          | 3             | 576        | 26.53        | -            |
| ConAbl  | 24                               | 43    | 5        | 6    | 68          | 1             | 6           | 4             | 157        | 26.14        | 39.26        |
| MACE    | 28                               | 19    | 1        | 20   | 37          | 3             | 6           | 5             | 119        | 24.06        | 52.78        |
| RECE    | 17                               | 29    | 3        | 7    | 14          | 1             | 8           | 1             | <b>80</b>  | 25.98        | 40.37        |
| UCE     | 29                               | 42    | 2        | 11   | 36          | 3             | 9           | 7             | 139        | <u>26.24</u> | <u>38.60</u> |
| Ours    | 20                               | 42    | 7        | 3    | 29          | 2             | 5           | 5             | <u>113</u> | <b>26.29</b> | <b>37.82</b> |

prepare five celebrity templates, (i.e., “a portrait of {Celebrity}”, “a sketch of {Celebrity}”, “an oil painting of {Celebrity}”, “{Celebrity} in an official photo”, and “an image capturing {Celebrity} at a public event”) and generate 500 images for both sets. For non-target concepts, we generate 1 image per template for each of the 100 concepts, totaling 500 images. For target concepts, we adjust the per-concept quantity to maintain a total of 500 images (e.g., erasing 10 celebrities involves generating 10 images with 5 templates).

**Implicit concept erasure.** We adopt the same setting in [19] to erase *nudity* → ‘ ’ as the erasure set and ‘ ’ as the retain set. In evaluation, we generate images using all 4,703 prompts in I2P and use NudeNet [6] to identify nude content with the threshold of 0.6.

## C.2 Erasure Configurations

**Implementation of previous works.** In our series of three concept erasure tasks, we mainly compare against four methods: ConAbl<sup>5</sup> [29], MACE<sup>6</sup> [33], RECE<sup>7</sup> [19], and UCE<sup>8</sup> [18], as they achieve SOTA performance across different concept erasure tasks. All the compared methods are implemented using their default configurations from the corresponding official repositories. One exception is that for MACE when erasing 50 celebrities, since it doesn’t provide an official configuration and the *preserve weight* varies with the number of target celebrities, we set it to  $1.2 \times 10^5$  to ensure a consistent balance between erasure and preservation.

**Implementation of SPEED.** In line with previous methods [29, 33, 19, 18], we edit the cross-attention (CA) layers within the diffusion model due to their role in text-image alignment [21]. In contrast, we only edit the value matrices in the CA layers, as suggested by [61]. This choice is grounded in the observation that the keys in CA layers typically govern the layout and compositional structure of the attention map, while the values control the content and visual appearance of the images [56]. In the context of concept erasure, our goal is to effectively remove the semantics of the target concept, and we find that only editing the value matrices is sufficient as shown in Fig. 4 and 5 (further ablation comparison is provided in Appx. D.5). The augmentation times  $N_A$  in Eq. 9 is set to 10 and the augmentation ranks  $r$  in Eq. 7 is set to 1 as ablated in Appx. D.5. Meanwhile, given that eigenvalues are rarely strictly zero in practical applications when determining the null space, we select the singular vectors corresponding to the singular values below  $10^{-1}$  on few-concept and implicit concept erasure and  $10^{-4}$  on multi-concept erasure following [14]. Moreover, since the retain set only includes ‘ ’ in implicit concept erasure, we add an identity matrix  $\mathbf{I}$  with weight  $\lambda = 0.5$  to the term  $(\mathbf{C}_2^T \mathbf{P} \mathbf{M} \mathbf{C}_2)^{-1}$  in Eq. 12 to ensure invertibility following [18].

<sup>5</sup><https://github.com/nupurkmr9/concept-ablation>

<sup>6</sup><https://github.com/Shilin-LU/MACE>

<sup>7</sup><https://github.com/CharlesGong12/RECE>

<sup>8</sup><https://github.com/rohitgandikota/unified-concept-editing>

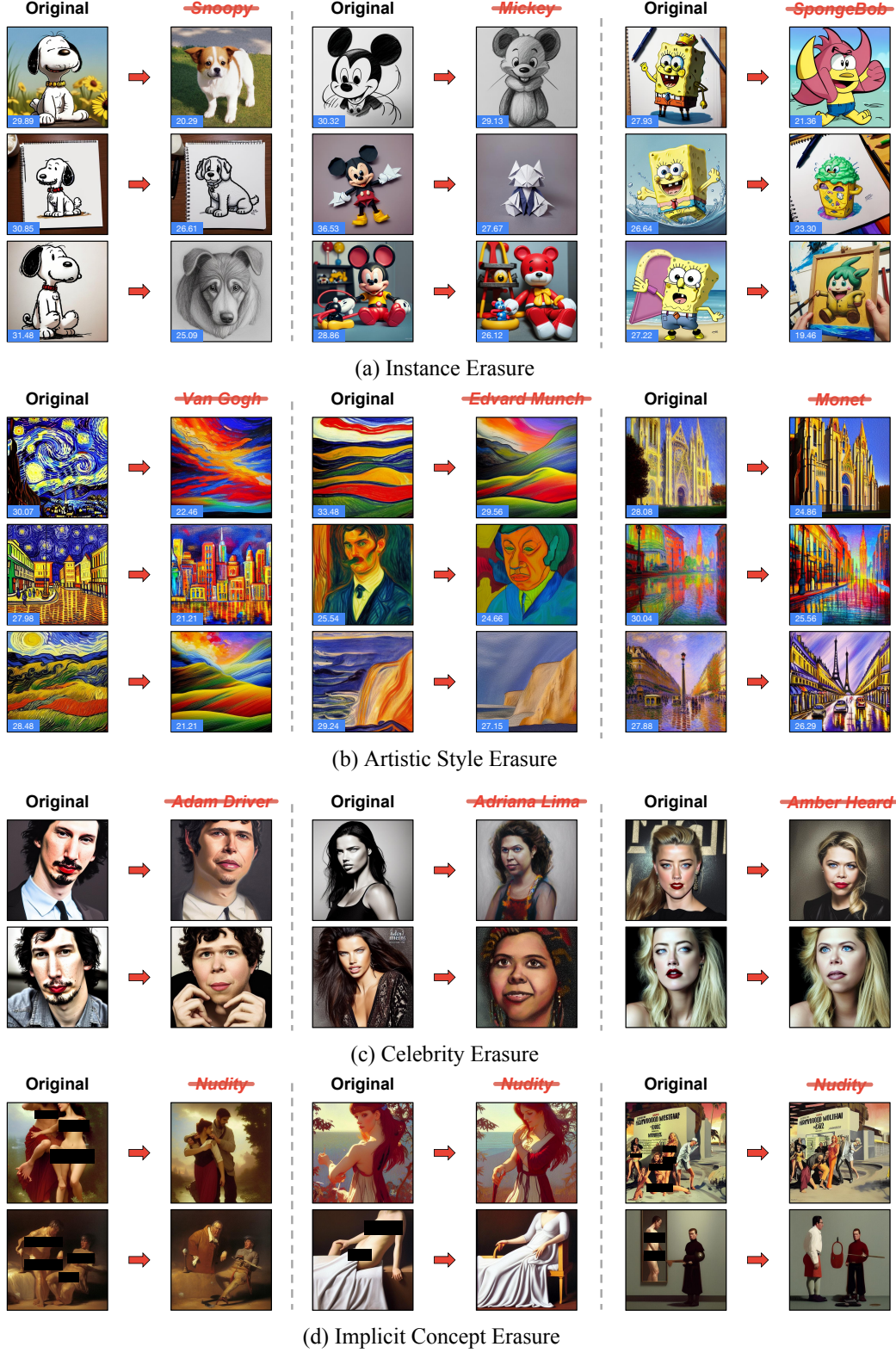


Figure 7: **Qualitative demonstration of our erasure performance** across (a) instance erasure, (b) artistic style erasure, (c) celebrity erasure, and (d) implicit concept erasure. Our method achieves precise erasure efficacy across various scenarios while exhibiting superior prior preservation. The corresponding CS is highlighted in blue, indicating that successful erasure can be achieved without pushing CS much lower, as our results demonstrate sufficient erasure at a moderate level.

Table 9: **Quantitative comparison with SPM and SPM w/o FT (*Facilitated Transport*)**. The best results are highlighted in **bold**, and the grey columns are indirect indicators for measuring erasure efficacy on target concepts or prior preservation on non-target concepts. Our method, which does not achieve the lowest CS but has been proven sufficient in Fig. 9.

| Concept                                                    | <i>Snoopy</i> |        | <i>Mickey</i> |              | <i>Spongebob</i> |              | <i>Pikachu</i> |              | <i>Hello Kitty</i> |              |
|------------------------------------------------------------|---------------|--------|---------------|--------------|------------------|--------------|----------------|--------------|--------------------|--------------|
|                                                            | CS            | FID    | CS            | FID          | CS               | FID          | CS             | FID          | CS                 | FID          |
| SD v1.4                                                    | 28.51         | -      | 26.62         | -            | 27.30            | -            | 27.44          | -            | 27.77              | -            |
| Erase <i>Snoopy</i>                                        |               |        |               |              |                  |              |                |              |                    |              |
|                                                            | CS ↓          | FID ↑  | CS ↑          | FID ↓        | CS ↑             | FID ↓        | CS ↑           | FID ↓        | CS ↑               | FID ↓        |
| SPM                                                        | 23.72         | 116.26 | 26.62         | 31.21        | 27.21            | 31.96        | 27.41          | 19.82        | 27.80              | 30.95        |
| SPM w/o FT                                                 | 23.72         | 116.26 | 26.55         | 43.03        | 26.84            | 42.96        | 27.38          | 25.95        | 27.71              | 42.53        |
| Ours                                                       | <b>23.50</b>  | 108.51 | 26.67         | <b>23.41</b> | 27.31            | <b>24.64</b> | 27.42          | <b>16.81</b> | 27.82              | <b>21.74</b> |
| Erase <i>Snoopy</i> and <i>Mickey</i>                      |               |        |               |              |                  |              |                |              |                    |              |
|                                                            | CS ↓          | FID ↑  | CS ↓          | FID ↑        | CS ↑             | FID ↓        | CS ↑           | FID ↓        | CS ↑               | FID ↓        |
| SPM                                                        | 23.18         | 122.17 | 22.71         | 117.30       | 26.92            | 38.35        | 27.35          | 27.13        | 27.76              | 39.61        |
| SPM w/o FT                                                 | <b>22.45</b>  | 127.95 | <b>21.77</b>  | 127.57       | 25.96            | 61.52        | 27.39          | <b>42.63</b> | 27.14              | <b>68.75</b> |
| Ours                                                       | 23.58         | 103.62 | 23.62         | 83.70        | 27.24            | <b>29.67</b> | 27.39          | <b>22.51</b> | 27.78              | <b>28.23</b> |
| Erase <i>Snoopy</i> and <i>Mickey</i> and <i>Spongebob</i> |               |        |               |              |                  |              |                |              |                    |              |
|                                                            | CS ↓          | FID ↑  | CS ↓          | FID ↑        | CS ↓             | FID ↑        | CS ↑           | FID ↓        | CS ↑               | FID ↓        |
| SPM                                                        | 22.86         | 125.66 | 22.08         | 123.20       | 20.92            | 153.36       | 27.50          | 37.51        | 27.63              | 46.63        |
| SPM w/o FT                                                 | <b>21.80</b>  | 137.98 | <b>20.86</b>  | 139.48       | <b>20.19</b>     | 163.21       | 26.68          | <b>66.15</b> | 26.24              | <b>85.35</b> |
| Ours                                                       | 23.69         | 103.33 | 23.93         | 86.55        | 21.39            | 109.28       | 27.47          | <b>21.40</b> | 27.76              | <b>26.22</b> |

Table 10: **Quantitative comparison with SPM and SPM w/o FT in multi-concept erasure**. The best results are highlighted in **bold**. Our method is capable of erasing up to 100 celebrities at once with low  $Acc_e$  (%) and preserving other non-target celebrities with less appearance alteration with high  $Acc_r$  (%), resulting in the best overall erasure performance  $H_o$  (shaded in pink). **FAIL** indicates that the model collapses with noisy generations ( $Acc_e = Acc_r = 0.00\%$ ).

|            | Erase 10 Celebrities |           |              |              |              | Erase 50 Celebrities |           |              |              |              | Erase 100 Celebrities |           |              |              |              |
|------------|----------------------|-----------|--------------|--------------|--------------|----------------------|-----------|--------------|--------------|--------------|-----------------------|-----------|--------------|--------------|--------------|
|            | $Acc_e$ ↓            | $Acc_r$ ↑ | $H_o$ ↑      | CS ↑         | FID ↓        | $Acc_e$ ↓            | $Acc_r$ ↑ | $H_o$ ↑      | CS ↑         | FID ↓        | $Acc_e$ ↓             | $Acc_r$ ↑ | $H_o$ ↑      | CS ↑         | FID ↓        |
| SD v1.4    | 91.99                | 89.66     | 14.70        | 26.53        | -            | 93.08                | 89.66     | 12.85        | 26.53        | -            | 90.18                 | 89.66     | 17.70        | 26.53        | -            |
| SPM        | 0.00                 | 51.79     | 68.24        | 26.42        | 48.44        | 0.00                 | 0.00      | <b>FAIL</b>  | 26.32        | 52.61        | 0.00                  | 0.00      | <b>FAIL</b>  | 25.15        | 63.20        |
| SPM w/o FT | 0.00                 | 5.08      | 9.68         | 26.38        | 52.23        | 0.00                 | 0.00      | <b>FAIL</b>  | 16.22        | 170.68       | 0.00                  | 0.00      | <b>FAIL</b>  | 14.34        | 245.92       |
| Ours       | 1.81                 | 89.09     | <b>93.42</b> | <b>26.47</b> | <b>30.02</b> | 3.46                 | 88.48     | <b>92.34</b> | <b>26.46</b> | <b>39.23</b> | 5.87                  | 85.54     | <b>89.63</b> | <b>26.22</b> | <b>44.97</b> |

## D Additional Experiments

### D.1 More Demonstrations

We further provide qualitative visualizations of the erasure results in Fig. 7, illustrating the effectiveness of our method in performing precise and targeted concept erasure across diverse scenarios. Specifically, we showcase: (a) instance erasure from Table 1 (left); (b) artistic style erasure from Table 1 (right); (c) celebrity erasure from Table 2; and (d) implicit concept erasure (*e.g.*, nudity) from Table 8. In all cases, our method successfully removes the intended concept while preserving unrelated content, demonstrating its universal erasure applications.

We also evaluate the CLIP score (CS) before and after concept erasure to assess the erasure efficacy. As shown in Figure 8, our method achieves successful erasure of specific concepts such as *Snoopy* and *Mickey* while maintaining moderate CS values (24.18 and 23.44, respectively). This indicates that effective erasure does not require minimizing CS to an extreme. In contrast, RECE obtains the lowest CS (19.79 and 18.75), but this is achieved at the cost of overly aggressive erasure. For example, transforming *Snoopy* into an unrecognizable image and replacing *Mickey* with a generic human figure. While such strategies may enhance erasure efficacy, they also risk compromising prior knowledge unrelated to the target concept. This trade-off is reflected in higher FIDs, as shown in Tables 1 and 2.

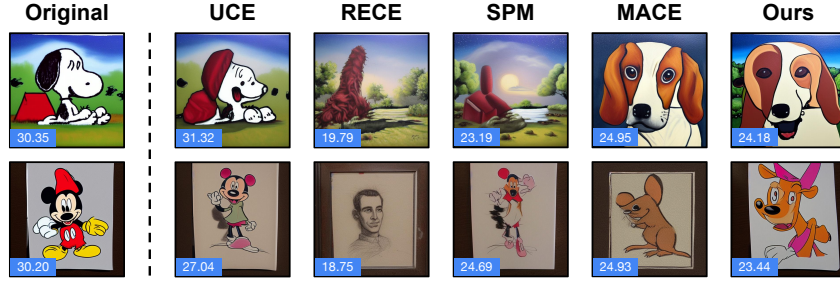


Figure 8: **Comparison of CLIP scores (CS) across different erasure methods.** We compare the results in erasing *Snoopy* and *Mickey*, and highlight the corresponding CS in blue. Our method achieves successful concept erasure with moderate CS values. In contrast, RECE achieves the lowest CS by enabling more aggressive erasure. For example, removing *Snoopy* to the extent of producing a semantically void image, and changing *Mickey* into a generic person. We argue that such over-erasure unnecessarily compromises prior preservation as evidenced by Tables 1 and 2.

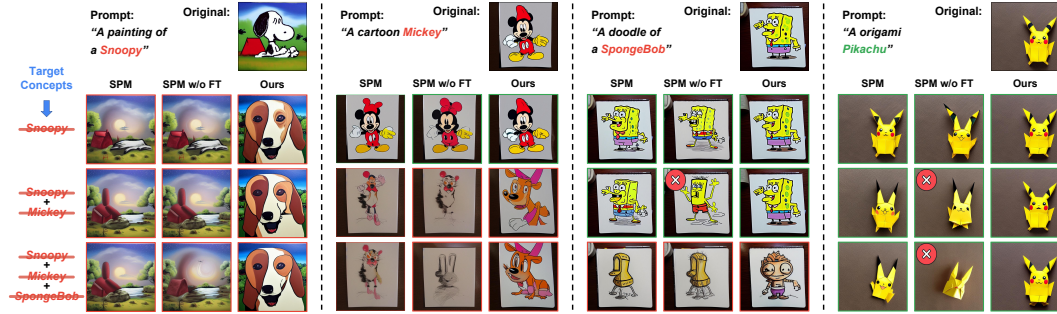


Figure 9: **Qualitative comparison with SPM and SPM w/o FT in erasing single and multiple instances.** The erased and preserved generations are highlighted with red and green boxes, respectively. Our method demonstrates superior prior preservation compared to both SPM and SPM w/o FT. Meanwhile, without the *Facilitated Transport* module, SPM w/o FT shows poorer prior preservation in multi-concept erasure (e.g., marked by  $\otimes$ ) with significant semantic changes compared to original generations.

## 655 D.2 Full Comparison on Few-Concept Erasure

656 We present full quantitative comparisons of few-concept erasure, including both CS and FID, in  
 657 Table 6 and Table 7. Our results demonstrate that our method consistently achieves superior prior  
 658 preservation, as indicated by higher CS and lower FID across the majority of non-target concepts.

## 659 D.3 On Implicit Concept Erasure

660 **Evaluation setup.** We further evaluate the erasure efficacy on implicit concepts, where the target  
 661 concept does not explicitly appear in the text prompt. We conduct experiments on the Inappropriate  
 662 Image Prompt (I2P) benchmark [49], which consists of various implicit inappropriate prompts  
 663 involving violence, sexual content, and nudity. We follow the same setting in [19] to erase *nudity*  
 664  $\rightarrow$  ‘ ’. Specifically, we generate images using all 4,703 text prompts in I2P and use NudeNet [6] to  
 665 identify if the nude content is successfully erased with the threshold of 0.6. Additionally, we report  
 666 the results on MS-COCO to demonstrate the prior preservation of general concepts.

667 **Analysis and discussion.** As shown in Table 8, our method can effectively erase the implicit concept,  
 668 i.e., *nudity*, with the second-best number of detected nude body parts. The SOTA method, RECE [19],  
 669 achieves the best total number by extending the erasure set with more target concepts, but this comes  
 670 at the cost of sacrificing prior preservation on MS-COCO. In contrast, our method achieves the  
 671 best prior preservation, demonstrating effective erasure while maintaining strong prior preservation,  
 672 striking a favorable balance between erasure and preservation.

Table 11: **Ablation study on the edited parameters.** Our scheme on only editing the value matrices achieves a superior balance between erasure efficacy (e.g., target CS of 26.29) and prior preservation (e.g., the lowest FIDs across all non-target concepts).

| Ablation | Parameters |       | Van Gogh     | Picasso      | Monet        | Paul Gauguin | Caravaggio   | MS-COCO      |              |
|----------|------------|-------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|          | Key        | Value | CS ↓         | FID ↓        | FID ↓        | FID ↓        | FID ↓        | CS ↑         | FID ↓        |
| 1        | ✓          | ×     | 27.67        | 42.11        | 26.09        | 28.08        | 52.44        | <b>26.55</b> | <b>18.72</b> |
| 2        | ✓          | ✓     | <b>26.24</b> | 48.41        | 28.65        | 33.79        | 57.23        | 26.53        | 23.20        |
| Ours     | ×          | ✓     | 26.29        | <b>35.86</b> | <b>16.85</b> | <b>24.94</b> | <b>39.75</b> | <b>26.55</b> | 20.36        |

#### D.4 More Baselines

In this section, we compare against more methods because of the page limit in our main paper. Since our method focuses on improving prior preservation and multi-concept erasure performance, we mainly compare it with similar methods, other methods like ESD [17], FMN [65], and SLD [49] are omitted, as they fail to achieve satisfactory prior preservation proved by previous comparisons [35, 33, 61]. The remaining comparable method is SPM<sup>9</sup> [35], which is proposed to improve prior preservation and can scale to multi-concept erasure tasks. Notably, SPM not only fine-tunes the model weights using LoRA [25] but also intervenes in the image generation process through *Facilitated Transport*. Specifically, this module dynamically adjusts the LoRA scale based on the similarity between the sampling prompt and the target concept. In other words, if the prompt contains the target concept or is highly relevant, this scale is set to a large value, whereas if there is little to no relevance, it is set close to 0, functioning similarly to a text filter. We argue that such a comparison with SPM is not fair since we only focus on modifying the model parameters, and therefore, we compare both the original SPM and SPM without *Facilitated Transport* (SPM w/o FT) for a fair comparison. In the latter version, the LoRA scale is set to 1 by default.

The quantitative comparative results are shown in Table 9. It can be seen that our method consistently achieves the best prior preservation compared to both SPM and SPM w/o FT. Even equipped with *Facilitated Transport*, our method achieves the lowest non-target FID (e.g., on *Pikachu* and *Hello Kitty*). This superiority amplifies as the number of target concepts increases as shown in Table 10. For example, with the number of target concepts increasing from 1 to 3, our FID in *Pikachu* rises from 16.81 to 21.40 (4.59 ↑), while SPM increases from 19.82 to 37.51 (17.69 ↑), where a similar pattern is observed in *Hello Kitty* (Our 4.48 ↑ v.s. SPM’s 15.68 ↑).

Once removing the *Facilitated Transport* module, SPM w/o FT shows poorer prior preservation with rapidly increasing FIDs (highlighted by red in Table 9). This indicates that the success of SPM in multi-concept erasure relies on the *Facilitated Transport* module, which dynamically allocates the LoRA scales by calculating the similarity between the sampling prompt and each target concept. For example, when erasing *Snoopy* + *Mickey* + *SpongeBob*, if the sampling prompt is “a photo of *Snoopy*”, SPM will allocate a larger scale to *Snoopy*’s LoRA according to the text similarity. On the contrary, if the sampling prompt is “a photo of *Pikachu*” with the non-target concept, all three LoRA scales will be assigned lower values, thereby preserving the prior knowledge. **We argue that this strategy of dynamically tuning the LoRA scales based on the sampling prompt similarity is vulnerable to attacks and easily bypassed**, especially in white-box attack scenarios, where an attacker can reconstruct the erased concepts by simply modifying the code with extremely low attack costs, e.g., open-source T2I models like Stable Diffusion [1, 2].

#### D.5 Ablation Studies

**Augmentation times.** We ablate the augmentation times  $N_A$  proposed in the Directed Prior Augmentation (DPA) module in Sec. 4.2, which controls the balance between semantic degradation and retain coverage along with the Influence-based Prior Filtering (IPF) module. It can be observed from Fig. 10 (a) that: (1) As  $N_A$  increases, the non-target FID exhibits a trend of first decreasing and then increasing. This suggests that when  $N_A$  is small (i.e.,  $1 \rightarrow 10$ ), augmenting existing non-target concepts with semantically similar concepts facilitates a more comprehensive retain coverage, thereby improving prior preservation. However, when  $N_A$  exceeds a certain threshold (i.e.,  $10 \rightarrow 20$ ), further augmentation of non-target concepts leads to narrowing the null-space derivation with semantic

<sup>9</sup><https://github.com/Con6924/SPM>

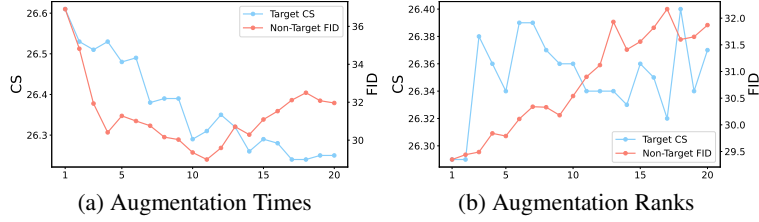


Figure 10: **Ablation study on two parameters, i.e., augmentation times  $N_A$  and augmentation ranks  $r$  of the DPA module.** We report the target CS of erasing *Van Gogh* and the non-target FID averaged over other four styles (i.e., *Picasso*, *Monet*, *Paul Gauguin*, *Caravaggio*).

Table 12: **Ablation study on the importance metrics used in IPF.**

| Metric                | <i>Van Gogh</i> | <i>Picasso</i> | <i>Monet</i> | <i>Paul Gauguin</i> | <i>Caravaggio</i> | MS-COCO      |              |
|-----------------------|-----------------|----------------|--------------|---------------------|-------------------|--------------|--------------|
|                       | CS ↓            | FID ↓          | FID ↓        | FID ↓               | FID ↓             | CS ↑         | FID ↓        |
| w/ Text Similarity    | 26.35           | 36.87          | 19.69        | 25.18               | 41.44             | 26.52        | 20.78        |
| w/ Prior Shift (Ours) | <b>26.29</b>    | <b>35.86</b>   | <b>16.85</b> | <b>24.94</b>        | <b>39.75</b>      | <b>26.55</b> | <b>20.36</b> |

degradation, ultimately degrading prior preservation. (2) Target CS generally shows a declining trend, indicating that the proposed Prior Knowledge Refinement strategy not only improves prior preservation but also exerts a positive impact on erasure efficacy.

**Augmentation ranks.** Another hyperparameter to be ablated is the augmentation ranks  $r$ . From Eq. 7, we introduce the number of the smallest singular values, i.e., augmentation ranks  $r$  in deriving  $\mathbf{P}_{\min} = \mathbf{U}_{\min} \mathbf{U}_{\min}^{\top}$  with  $\mathbf{U}_{\min} = \mathbf{U}_{\mathbf{W}}[:, -r :]$ . Mathematically,  $r$  represents the directions in which the DPA module can augment in the concept embedding space and constrains the rank of the augmented embeddings to a maximum of  $r$ . As shown in Fig. 10 (b), as  $r$  increases, the non-target FID exhibits an overall upward trend, indicating that introducing more ranks does not benefit prior preservation, as it narrows the null space. At the same time, as shown in Table 4, such augmentation by DPA also remains necessary, as it enables more comprehensive coverage of non-target knowledge with semantically similar concepts, leading to improved prior preservation.

**Edited parameters.** We compare the impact on editing different CA parameters in Table 11 and draw the following conclusions: (1) Only editing the key matrices cannot achieve effective erasure, with the target CS being 27.67 (v.s. the original CS of 28.75). This is because they mainly arrange the layout information of the generation and cannot effectively erase the semantics of the target concept. (2) Simultaneously editing both the key and value matrices can achieve effective erasure, but it will also excessively damage prior knowledge. (3) Only editing the value matrices achieves a superior balance between erasure efficacy and prior preservation. Compared to Ablation 2, the editing of key matrices leads to excessive erasure, which is unnecessary in concept erasure.

**Importance metrics in IPF.** In Sec. 4.1, we propose Importance-based Prior Filtering (IPF) in Eq. 6 and evaluate this importance with the metric prior shift  $= \|\Delta_{\text{erase}} \mathbf{c}\|^2$ . Another intuitive and plausible metric is based on text similarity, e.g., the cosine similarity between each non-target embedding  $\mathbf{c}_0$  and each target concept embedding  $\mathbf{c}_1$ , i.e.,  $\cos(\mathbf{c}_0, \mathbf{c}_1)$ . Herein, we conduct an additional ablation study in terms of the metric selection in Table 12. It can be seen that text similarity can also serve as an effective metric for evaluating importance with improved non-target FID while the prior shift provides better prior preservation. This may be because text similarity is implicitly related to importance, while prior shift explicitly reflects the impact of erasure on different concepts from the model updates  $\Delta$ . Moreover, our method can be directly scaled up to multi-concept erasure scenarios, whereas text similarity calculates  $n$  similarities for  $n$  target concepts, requiring additional fusion or selection strategies, introducing accumulated errors during fusion or selection.

## E Limitation

While SPEED demonstrates superior prior preservation, its erasure efficacy may not be as strong as some adversarial training/editing-based methods (e.g., RECE [19]), which explicitly optimize for

robust concept removal. This trade-off arises from SPEED’s emphasis on maintaining non-target knowledge, potentially leading to residual traces of erased concepts in extreme cases. However, due to its efficiency and scalability in multi-concept erasure, an interesting direction for future work is to explore the simultaneous erasure of adversarial examples. Given that null-space constraints inherently minimize the impact on prior knowledge, even with the addition of extra target concepts, SPEED is expected to achieve better prior preservation compared to existing methods while effectively handling adversarial concept erasure.

## **F Ethical Statement**

This work introduces a method for concept erasure in text-to-image diffusion models to address ethical concerns such as copyright infringement, privacy violations, and the generation of offensive content. By precisely removing specific target concepts while preserving the quality and semantics of non-target outputs, the proposed approach enhances the safety, reliability, and controllability of generative models. The method operates through parameter-space editing without requiring access to private data or involving human subjects, ensuring ethical integrity throughout the research process and promoting responsible deployment of generative AI technologies.

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