Mixed Material Point Method for nearly incompressible polymeric solids

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Abstract

Exposed to external loadings, polymeric materials, e.g., biological tissues, hydrogels, and elastomers, may undergo extreme, nearly incompressible, self-contacting deformations, which pose significant challenges for numerical modeling employing mesh-based techniques such as the finite element method (FEM) due to i) extreme distortions in the deformed geometry, ii) accuracy issues stemming from volumetric locking effects, and iii) vastly increased computational cost due to complex contact search. As an alternative to FEM, the material point method (MPM) is a continuum-based meshless particle technique attracting considerable interest due to its robustness against extreme distortions and ability to capture contact at no additional cost. An effective technique to overcome locking effects is the two-field mixed formulation with displacement and pressure as independent fields instead of a displacement-based single-field formulation. However, mixed formulations suffer from numerical instabilities in the nearly incompressible limit due to the violation of the inf-sup or Ladyzhenskaya–Babuska–Brezzi (LBB) condition, leading to spurious nodal pressure solutions. The main objective of this paper is to extend the mixed formulations at large strains to the B-spline material point method using the two-scale relation of B-splines; we further develop a subdivision-stabilized mixed MPM and obtain a stable, oscillation-free nodal pressure solution. We assess the stability and accuracy of the mixed MPM through several benchmark problems, including the Cook's membrane and the rigid sphere obstacle on a quasi-compressible hyperelastic substrate (with comparisons to FEM results). Additionally, we test the robustness of the proposed MPM by modeling the compression of a quasi-compressible hyperelastic sphere where the no-slip contact condition is assumed. The proposed methodology provides a robust computational foundation to study extreme deformations observed in practical soft matter applications.

Keywords: Material Point Method, Sub-Division Method, Stabilized u-p formulation, Isogeometric analysis, B-Spline based shape functions

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