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INPUT-ADAPTIVE BAYESIAN MODEL AVERAGING**Anonymous authors**

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## ABSTRACT

This paper studies prediction with multiple candidate models, where the goal is to combine their outputs. This task is especially challenging in heterogeneous settings, where different models may be better suited to different inputs. We propose input adaptive Bayesian Model Averaging (IA-BMA), a Bayesian method that assigns model weights conditional on the input. IA-BMA employs an input adaptive prior, and yields a posterior distribution that adapts to each prediction, which we estimate with amortized variational inference. We derive formal guarantees for its performance, relative to any single predictor selected per input. We evaluate IA-BMA across regression and classification tasks, studying data from personalized cancer treatment, credit-card fraud detection, and UCI datasets. IA-BMA consistently delivers more accurate and better-calibrated predictions than both non-adaptive baselines and existing adaptive methods.

## 1 INTRODUCTION

Many applications require *adaptive predictions*. In personalized medicine, different patients respond differently to the same treatment (Mahajan et al., 2023); in fairness-sensitive domains, predictions need to adapt to subpopulations (Wang et al., 2019; Grother et al., 2019); and in fraud detection, behavioral data is often heteroskedastic and varies substantially across inputs (Varmedja et al., 2019).

When the data is complex, selecting a single model that performs well across all inputs is challenging. This motivates *model averaging* (MA), which produces an *ensemble* of models. This idea dates back at least to the 1960s (see, e.g., (Clemen, 1989) for a historical perspective).

We denote data points by  $x \in \mathcal{X}$ , labels by  $y \in \mathcal{Y}$ , and the space of probability distributions on labels by  $\mathcal{P}(\mathcal{Y})$ . MA combines the predictive distributions of  $m$  models  $\{f_j : \mathcal{X} \rightarrow \mathcal{P}(\mathcal{Y})\}_{j=1}^m$  into a weighted ensemble,  $p_\alpha(y | x) := \sum_{j=1}^m \alpha_j f_j(y | x)$ , with weights  $\alpha_j > 0$  (often constrained to sum to one). MA accounts for the possibility that multiple models can provide plausible explanations of the data.

In classical MA, the same weights  $\alpha_1, \dots, \alpha_m$  are used for all inputs  $x$ . But in practice, different values of the input  $x$  might call for different predictive models. This motivates *adaptive averaging*, where the weights  $\alpha_j$  depend on  $x$ :

$$\alpha : \mathcal{X} \rightarrow \Delta^{m-1}, \quad x \mapsto \alpha(x) = (\alpha_1(x), \dots, \alpha_m(x)). \quad (1)$$

The result is an adaptive weighted prediction,

$$p_\alpha(y | x) := \sum_{j=1}^m \alpha_j(x) f_j(y | x). \quad (2)$$

This model is also known as a *mixture of experts* (Jacobs et al., 1991; Jordan & Jacobs, 1994), where the adaptive weights  $\alpha_j(x)$  are fit to maximize the predictive log likelihood of the data.

In this paper, we take a Bayesian perspective. We assume that the set of predictors  $\mathcal{F} := \{f_1, \dots, f_m\}$  is fixed, and model the selection of a predictor as a random process. Our model constructs a *random selector*  $g : \mathcal{X} \rightarrow \{e_1, \dots, e_m\}$  where  $\{e_j\}_{j=1}^m$  denote  $m$  indicator vectors, i.e.,  $g(x) = e_j$  selects predictor  $f_j$ . Moreover, the prior on  $g$  itself depends on the inputs  $x$ . Therefore, in our model, adaptivity arises not only from the variability of  $g(x)$  across inputs, but also from allowing its prior to vary with  $x$ .

Under this model, MA is a natural consequence of the posterior predictive distribution. Consider a dataset  $\mathcal{D} := \{x_i, y_i\}_{i=1}^n$ . The posterior predictive distribution for a new input  $x$  is

$$p(y | x, \mathcal{D}) = \sum_{j=1}^m f_j(y | x) p(g(x) = e_j | x, \mathcal{D}), \quad (3)$$

where  $p(g(x) = e_j | x, \mathcal{D})$  is a *data dependent posterior* that incorporates both training inputs and labels. Eq. 3 is an ensemble of candidate models, with weights  $\alpha_j(x)$  equal to the posterior over  $g$ :

$$\alpha_j(x) = p(g(x) = e_j | x, \mathcal{D}). \quad (4)$$

Unlike maximum likelihood approaches to MoE, this posterior captures the uncertainty over which predictor is most plausible for each input  $x$ .

Below, we first analyze the theoretical advantages of this adaptive Bayesian model averaging framework and derive finite-sample guarantees that compare its performance to that of any single predictor selected per input (Section 2.1). We then develop input adaptive Bayesian Model Averaging (IA-BMA), by (i) constructing an input adaptive prior, following Slavutsky & Blei (2025), and (ii) employing amortized variational inference to approximate the posterior (Section 3). We evaluate IA-BMA across regression and classification benchmarks (Section 4), and show that IA-BMA achieves substantial gains in both accuracy and calibration compared to existing adaptive, and non-adaptive strategies.

## 1.1 RELATED WORK

MA is regarded as the machine learning analogue of the “Condorcet’s jury” theorem (Mennis, 2006), leveraging the “wisdom of the crowd” to mitigate the inherent uncertainty in model selection. Thus, MA is often used when there are alternative, potentially overlapping hypotheses and no clear justification for selecting a single preferred model. Applications include ecological research (Wintle et al., 2003; Thuiller, 2004; Richards, 2005; Dormann et al., 2008; Lauzeral et al., 2015; Zheng et al., 2024) and medicine (Jiang et al., 2021; Nanglia et al., 2022; Mahajan et al., 2023). More broadly, MA has been adopted in a wide range of machine learning tasks (e.g., Fernández-Delgado et al. (2014), Rokach (2010)).

As a form of model combination, MA is closely related to other ensemble techniques such as bagging (Breiman, 1996) and boosting (Freund, 1995). It is a variant of stacking procedure (Wolpert, 1992), in which outputs of base learners are combined to produce the final prediction.

MA has been shown to reduce prediction errors beyond those of the best individual component model (Dormann et al., 2018; Peng & Yang, 2022) and to mitigate overfitting (Dietterich et al., 2002; Polikar, 2006). In recent years, extensive surveys have reviewed MA (Kulkarni & Sinha, 2013; Woźniak et al., 2014; Gomes et al., 2017; González et al., 2020; Sagi & Rokach, 2018; Wu & Levinson, 2021), with some focusing specifically on decision trees (Rokach, 2016) or neural networks (Ganaie et al., 2022).

A Bayesian method for MA was introduced by Waterhouse et al. (1995), who place a prior directly on the averaging weights. In contrast, we reinterpret MA as a problem of random model selection, leading to *dynamic* model selection in which the choice of model adapts to the specific input. Earlier work on dynamic model selection includes Cao et al. (1995); Giacinto & Roli (1999); Gunes et al. (2003); Didaci et al. (2005); Didaci & Giacinto (2004). However, these approaches focus on selecting a single model for each instance, rather than assigning instance-specific weights to average predictions across multiple models.

**Input adaptive model averaging methods:** Few methods assign input-dependent weights. These date back to Mixture of Experts (MoE) (Jacobs et al., 1991), where a gating network maps the input  $x$  to weights  $\alpha_j(x)$ , estimated by maximizing the induced likelihood. Classical MoE variants jointly train both experts and gates, and an extensive literature explores different expert classes and gating architectures (see (Yuksel et al., 2012) for a review). In our setting, however, we consider the MoE variant in which the gating network is applied on top of pre-trained experts.

Rasmussen & Ghahramani (2001) extended this framework by using Gaussian Processes (GPs) as base models, providing nonparametric flexibility. They adopt a Bayesian perspective with a Dirichlet

108 Process (DP) prior, yielding an infinite mixture. However, here weights and base models are learned  
 109 jointly, and thus only a single family of base predictors is considered.  
 110

111 Although these methods often outperform standard model averaging, maximum-likelihood-based  
 112 assignment tends to concentrate probability mass on the predictor that is most confident about the  
 113 observed outcome  $y$ , frequently resulting in overconfident predictions (Freund & Schapire, 1997;  
 114 Guo et al., 2017). Several approaches proposed alternative strategies for weight assignment.  
 115

116 Woods et al. (1997) proposed a dynamic scheme based on local accuracy estimates. For a test input  
 117  $x$ , its neighborhood is identified (typically via  $k$ -nearest neighbors), and each classifier’s perfor-  
 118 mance in this region is summarized as a local accuracy score. The classifier with the highest score  
 119 is then selected to predict  $x$ .  
 120

121 Similarly, Chan & van der Schaar (2022) proposed an approach that assigns higher weight to mod-  
 122 els whose training domains better cover a test instance. Inputs are mapped into a learned low-  
 123 dimensional space where models with similar predictions are closer together, and weights are set  
 124 via kernel density estimation. Unlike Woods et al. (1997), where similarity is predefined, here it  
 125 is learned from data. Motivated by Tenzer et al. (2022), the method assumes that models making  
 126 random errors on an input are unlikely to agree.  
 127

128 Perhaps most relevant to our work is Bayesian hierarchical stacking (BHS) (Yao et al., 2022), which  
 129 places priors on logit weights, and models them with hierarchical low-rank linear functions. The  
 130 parameters are then estimated by maximizing the expected log predictive density.  
 131

132 Thus, prior work on adaptive model averaging has focused predominantly on methods targeting  
 133 frequentist objectives, with relatively few Bayesian formulations. In contrast to previous approaches,  
 134 our model assumes a fully Bayesian setting in which the selector itself is random and, crucially, is  
 135 defined locally relative to each input  $x$ . This yields an input-dependent prior  $p(g | x)$  rather than  
 136 a global prior  $p(g)$ . In turn, this prior induces an adaptive posterior that corresponds exactly to the  
 137 Bayes-optimal weights, providing a principled approach for adaptive model averaging.  
 138

## 139 2 PROBABILISTIC FORMULATION OF ADAPTIVE MODEL AVERAGING

140 We cast adaptive model averaging as a probabilistic model selection. To reflect that some models  
 141 may be better suited for different inputs, we assume a probabilistic model in which the *selection*  
 142 *function*  $g$  is treated as a random input-dependent variable. For a training set  $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^n$ ,  
 143 and a new input  $x$ , we assume the data generating process  
 144

$$x_i, x \stackrel{\text{iid}}{\sim} p(x), \quad (5)$$

$$g \sim p(g | x, x_{1:n}), \quad (6)$$

$$y_i \sim p(y_i | x_i, g), \quad y \sim p(y | x, g). \quad (7)$$

145 We defer the precise specification of the adaptive prior  $p(g | x, x_{1:n})$  to Section 3.1.  
 146

147 The predictive distribution for  $y$  given a new input  $x$  and the training data is then  
 148

$$p(y | x, \mathcal{D}) = \int p(y | x, \mathcal{D}, g) p(g | x, \mathcal{D}) d\mu(g) = \int p(y | x, g) p(g | x, \mathcal{D}) d\mu(g), \quad (8)$$

149 where  $p(g | x, \mathcal{D})$  is a posterior distribution on the space of functions<sup>1</sup>  $\mathcal{G} := \{g : \mathcal{X} \rightarrow$   
 150  $\{e_1, \dots, e_m\}\}$ .  
 151

152 A draw from the posterior  $g \sim p(g | x, \mathcal{D})$  induces a random index  $J(x)$ , defined by the relation  
 153  $g(x) = e_{j(x)}$ . Using this index, we can rewrite equation 8 as  
 154

$$p(y | x, \mathcal{D}) = \int p(y | x, g) p(g | x, \mathcal{D}) d\mu(g) = \sum_{j=1}^m f_j(y | x) p(J(x) = j | x, \mathcal{D}). \quad (9)$$

155 A formal proof of this equality is outlined in Appendix A.1.  
 156

157 <sup>1</sup>Formally,  $p(g | x, \mathcal{D})$  is a density w.r.t some reference measure  $\mu$  on a space of measurable functions  $\mathcal{G}$ .  
 158

162 Under our model, the predictive distribution is a mixture of the candidate predictions  $f_j(y \mid x)$   
 163 weighted by the posterior probabilities  $p(J(x) = j \mid x, \mathcal{D})$ . In other words, the input adaptive  
 164 weights  $\alpha_j(x)$  *arise directly from the probabilistic formulation* itself, and *they are precisely the*  
 165 *posterior probabilities* of each model being the generator at input  $x$ .

166 A central difficulty, of course, is that the true posterior is unknown. In Section 3, we introduce a  
 167 variational approximation to  $p(J(x) = j \mid \mathcal{D}_i, x)$  that preserves explicit dependence on both  $x$  and  
 168  $\mathcal{D}$ . Before presenting this approximation, we first analyze the performance guarantees that arise  
 169 when the averaging weights are set to the true posterior probabilities  $p(J(x) = j \mid x, \mathcal{D})$ .  
 170  
 171

## 172 2.1 LIKELIHOOD GUARANTEES

173 So far we have seen that the posterior probabilities  $p(J(x) = j \mid x, \mathcal{D})$  arise naturally as input  
 174 adaptive weights under our model. In particular, they are the Bayes-optimal weights, as they recover  
 175 the true predictive distribution.  
 176

177 We now show that this choice also comes with performance guarantees: the posterior-weights pre-  
 178 dictor not only reflects the correct probabilistic formulation, but in expectation achieves likelihood  
 179 performance competitive with any input-specific single-model selector. The next theorem formalizes  
 180 this result (for proof see Appendix A.2).  
 181

182 **Theorem 2.1.** Denote  $\mathcal{D}_i := \{(x_t, y_t)\}_{t=1}^i$ , and consider the posterior weights predictor  $\hat{p}_\alpha^{(i)}$  assign-  
 183 ing  $\alpha_j(x; \mathcal{D}_i) = p(J(x) = j \mid \mathcal{D}_i, x)$  to the  $j$ -th predictor  $f_j$ . Assume that  $\mathbb{E}[\log f_j(Y \mid X)] < \infty$   
 184 for all  $f_j \in \mathcal{F}$ . Then, for any measurable selector  $j^* : \mathcal{X} \rightarrow \{1, \dots, m\}$  and any  $n \geq 1$ ,

$$185 \frac{1}{n} \sum_{i=1}^n \mathbb{E} \left[ \log \hat{p}_\alpha^{(i)}(y_i \mid x_i, \mathcal{D}_{i-1}) \right] \geq \mathbb{E} \left[ \log f_{j^*(x)}(y \mid x) \right] + \frac{1}{n} \sum_{i=1}^n \mathbb{E} \left[ \log \alpha_{j^*(x_i)}^{(i)}(x_i) \right], \quad (10)$$

189 where the expectations are taken w.r.t the population distribution  $(x_i, y_i) \sim p(x, y)$ .  
 190

191 Thus, the posterior weights predictor can match any per-input selector (i.e., a rule that may pick a  
 192 different  $j$  for different  $x$ ), up to a term depending on the gating weights assigned to the chosen  
 193 model at each  $x$ . Put plainly, the posterior mean performs nearly as well as if we could select the  
 194 best expert separately for every  $x$ .  
 195

196 Concretely, for the selector that picks the most probable model,  $j^{(i)}(x) \in \arg \max_{1 \leq j \leq m} \alpha_j^{(i)}(x)$ ,  
 197 the penalty becomes  $\frac{1}{n} \sum_{i=1}^n \mathbb{E} \left[ \log \max_j \alpha_j^{(i)}(x_i) \right]$ , which vanishes as the posterior sharpens, i.e.,  
 198 when  $\max_j \alpha_j^{(i)}(x_i) \rightarrow 1$  in probability.  
 199

200

## 201 3 IA-BMA: INPUT ADAPTIVE BAYESIAN MODEL AVERAGING

202

203 Our goal is to develop a method for estimating this posterior distribution over models. By doing so,  
 204 we obtain an averaging scheme that is consistent with both the training data  $\mathcal{D}$  and the specific input  
 205  $x$ , thereby approximating the true predictive distribution that we ultimately aim to recover.  
 206

207 We begin by formulating the modeling assumptions for an adaptive prior that is conditioned jointly  
 208 on the training covariates and a new input. Building on this prior, we then develop a variational  
 209 inference method to approximate the resulting posterior.  
 210

211

### 212 3.1 ADAPTIVE PRIOR

213

214 Based on the adaptive prior introduced in (Slavutsky & Blei, 2025), we posit a prior that encodes  
 215 the plausibility of each model conditional on both the training covariate  $x_{1:n}$  and a new input  $x$  at  
 which prediction is sought. This prior is defined through an energy-based formulation.  
 216

216 Specifically, for a predictor  $f_j$  we consider the prior induced by the negative energy function  
 217

$$218 \quad 219 \quad 220 \quad E(J = j; x_{1:n}, x) := \int \sum_{i=1}^n \log p(y|x_i, f_j) + \log p(y|x, f_j) dy \quad (11)$$

$$221 \quad 222 \quad p(J = j|x_{1:n}, x) := \frac{1}{Z(f)} \exp(E(J = j; x_{1:n}, x)), \quad (12)$$

223 where the normalizing factor<sup>2</sup> is given by  $Z(f) := \sum_{j=1}^m \exp(E(J = j; x_{1:n}, x))$ .  
 224

225 This prior allows beliefs about model plausibility to adapt to the new input  $x$ . Unlike a prior defined  
 226 solely from the training data, which remains fixed across prediction points, our formulation updates  
 227 the relative weight of each model once  $x$  is observed. This makes the prior *input adaptive*, enabling  
 228 model selection probabilities to shift dynamically with the prediction covariates. To build intuition,  
 229 we next examine a simple analytical example.

230 Thus, IABMA adds a second layer of adaptivity: the prior itself varies with  $x$ , linking each ex-  
 231 pert's prior selection probability to its expected likelihood and propagating this uncertainty into the  
 232 posterior.

233 In section 4 we show that our prior indeed rewards predictors whose likelihood is high locally at  $x$ ,  
 234 and quantify the additional improvement stemming from this prior in Appendix B.6.

235 **A two-model Bernoulli example** Suppose  $y \in \{0, 1\}$ , and consider two candidate logistic models

$$236 \quad 237 \quad 238 \quad p(y = 1 | x, f_j) = \sigma(\beta_j x), \quad \sigma(u) := \frac{1}{1 + e^{-u}}, \quad (13)$$

239 with  $j \in 1, 2$  and slopes  $0 < \beta_2 < \beta_1$ . In this setting, the energy function is given by  
 240

$$241 \quad 242 \quad 243 \quad E(J = j; x_{1:n}, x) = \sum_{i=1}^n \sum_{y \in \{0,1\}} \log p(y | x_i, f_j) + \sum_{y \in \{0,1\}} \log p(y | x, f_j) \quad (14)$$

$$244 \quad 245 \quad 246 \quad = \underbrace{\sum_{i=1}^n \log (\sigma(\beta_j x_i) [1 - \sigma(\beta_j x_i)])}_{=:C_j} + \underbrace{\log (\sigma(\beta_j x) [1 - \sigma(\beta_j x)])}_{=: \ell_j(x)} \quad (15)$$

247 and the adaptive prior is  
 248

$$249 \quad 250 \quad 251 \quad p(J = j | x_{1:n}, x) = \frac{\exp(C_j + \ell_j(x))}{\sum_{k=1}^m \exp(C_k + \ell_k(x))} \quad (16)$$

252 Accordingly, the log-odds between the two models is  
 253

$$254 \quad 255 \quad \log \frac{p(J = 1 | x_{1:n}, x)}{p(J = 2 | x_{1:n}, x)} = (C_1 - C_2) + \ell_1(x) - \ell_2(x). \quad (17)$$

256 Thus, the log-odds depend both on the difference between training baselines  $C_1 - C_2$ , and the change  
 257 induced by conditioning also on the new input  $x$  is  $\delta_x := \ell_1(x) - \ell_2(x)$ .

258 Concretely, suppose the baseline difference is fixed at  $C_1 - C_2 = \log 5 \approx 1.61$ , yielding  $p(J = 1 | \mathcal{D}) = \sigma(\log 5) \approx 0.83$ . Based solely on the training data, the prior thus strongly favors  $f_1$ .  
 259 Now consider a new input  $x = 1$  with  $\beta_2 = 1$ . As  $\beta_1$  increases, the discrepancy  $|\ell_1(1) - \ell_2(1)|$   
 260 grows, and  $\delta_{x=1}$  shifts the likelihood ratio toward  $f_2$ . For example, when  $\beta_1 = 3$ , the prior shifts to  
 261 a mild preference for  $f_1$ , at  $\beta_1 = 5$  it flips to favor  $f_2$ , and by  $\beta_1 = 9$  the preference for  $f_2$  becomes  
 262 very strong. These dynamics, along with additional parameter settings for coefficients and baseline  
 263 differences, are shown in Figure 1.

264 This analysis highlights the interplay between the baseline preference and the input-specific adjust-  
 265 ment introduced by  $x$ . It shows that, in extreme cases, even strong baseline beliefs can be overturned  
 266 by the adaptive correction at the queried input.

267  
 268  
 269 <sup>2</sup>This definition requires integrability of  $\exp(E(J = j; x_{1:n}, x))$ , and thus we assume that  $\exp(E(J = j; x_{1:n}, x))$  is integrable for each  $j$ .

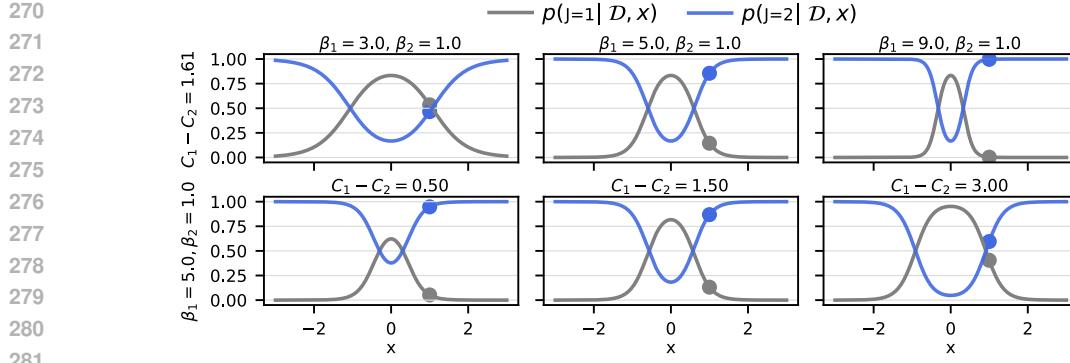


Figure 1: Illustration of the input adaptive prior. Each panel shows the posterior probabilities  $p(J = j | \mathcal{D}, x)$  as functions of  $x$ . *Top*: the baseline log-odds is fixed and  $\beta_1$  varies; larger  $\beta_1$  values increase the influence of  $x$ , producing stronger adaptive corrections. *Bottom*:  $\beta_1, \beta_2$  are fixed while the baseline log-odds  $C_1 - C_2$  varies; stronger baselines yield higher prior preference for  $f_1$ , but input-specific corrections can still substantially reshape the prior at certain  $x$ . The marked point ( $x = 1$ ) highlights how the adaptive prior shifts the relative model probabilities compared to the baseline.

**Evaluation of the prior:** Evaluating the proposed prior requires computing an integral over the outcome space  $\mathcal{Y}$ , and thus depends on whether the outcome space is discrete or continuous. When  $\mathcal{Y}$  is *discrete* (e.g., in classification problems), the integral reduces to a finite sum over all possible outcome values. In this case, the evaluation is straightforward and can be computed exactly without approximation. When  $\mathcal{Y}$  is *continuous*, (e.g., in regression problems), the integral cannot typically be computed in closed form and may even diverge unless we restrict the domain of integration. Thus, to approximate the prior, as in (Slavutsky & Blei, 2025), we employ Monte-Carlo integration where we sample  $K$  possible outcome values uniformly from a predefined integration range  $[y_{\min}, y_{\max}]$  set to large margin upon observed values in training data, and average the Normal log-likelihood (centered at the model’s prediction with unit variance) over the  $K$  samples.

This procedure introduces no meaningful computational overhead: in classification it reduces to a simple summation, and in regression we approximate the expectation using  $K = 64$  Monte-Carlo samples, which we found to be numerically stable in practice. Table 18 confirms that runtime remains comparable to Mixture-of-Experts and DDP models using the same architecture.

### 3.2 AMORTIZED VARIATIONAL POSTERIOR

Equipped with the adaptive prior, we now turn to the estimation of the posterior  $p(J = j | x_{1:n}, y_{1:n}, x) = p(J = j | \mathcal{D}, x)$ , which conditions not only on the covariates  $x$  and  $x_{1:n}$ , but also on the training labels  $y_{1:n}$ . This, in turn, will enable us to assign input adaptive weights for model averaging, bringing them closer to the ideal weights that recover the predictive distribution  $p(y | x)$ .

We do so by fitting variational distributions  $q(f_j; x) \approx p(J = j | \mathcal{D}, x)$  parameterized as functions of the input  $x$ . This yields an *amortized posterior approximation*, which allows us to efficiently evaluate approximate posteriors at multiple inputs  $x$ .

In our case, in the context of a new input  $x$ , the true posterior distribution over predictors is Multinomial  $p(J = j | \mathcal{D}, x) = \rho_j(x)$  for  $j \in \{1, \dots, m\}$ , where each  $\rho_j(x) > 0$  and  $\sum_{j=1}^m \rho_j(x) = 1$ . Thus, we set the variational family to be the set of all multinomial distributions.

$$\mathcal{Q}_x := \{q = (q(J = 1; x), \dots, q(J = m; x) \in \Delta^{m-1}\}. \quad (18)$$

For a given input  $x$ , our goal is to minimize the KL divergence

$$\min_{q \in \mathcal{Q}_x} D_{\text{KL}}(q \| p) := \sum_{j=1}^m q(J = j; x) \log \frac{q(J = j; x)}{p(J = j | \mathcal{D}, x)}. \quad (19)$$

---

324 **Algorithm 1** IA-BMA: Amortized Posterior Learning (IA-BMA)

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325 1: **Inputs:** Training data  $\mathcal{D}$ ; predictors  $\{f_j\}_{j=1}^m$ ; initialization  $\theta_0$ ; learning rate  $\eta$ ; iterations  $K$ .

326 2: **Precompute:** For all  $i = 1, \dots, n$  and predictor  $j = 1, \dots, m$ , store  $\log f_j(y_i | x_i)$ .

327 3: **for**  $k = 1$  **to**  $K$  **do**

328 4:   **for**  $i = 1$  **to**  $n$  **do**

329 5:     **for**  $j = 1$  **to**  $m$  **do**

330 6:       **Prior:** Compute  $p(J = j | x_{-i}, x) \propto \exp(E(J = j; x_{-i}, x_i))$

331 7:       **Posterior:** Compute  $h_{\theta_{k-1}}(x) = (q_{\theta_{k-1}}(J = 1; x_i), \dots, q_{\theta_{k-1}}(J = m; x_i))$

332 8:       **ELBO:** Compute

333 9:       
$$\mathcal{L}(x_i; \theta_{k-1}) = \sum_{j=1}^m q_{\theta_{k-1}}(J = j; x_i) \log f_j(y_i | x_i) - \sum_{j=1}^m q_{\theta_{k-1}}(J = j; x_i) \log \frac{q_{\theta_{k-1}}(J = j; x_i)}{p(J = j | x_{-i}, x_i)}.$$

334 10:     **end for**

335 11:     **Update:**  $\bar{\mathcal{L}}(\theta_{k-1}) \leftarrow \frac{1}{n} \sum_i \mathcal{L}(x_i; \theta_{k-1})$

336 12:   **end for**

337 13:   **end for**

338 14: **Return:**  $\hat{\theta} := \theta_K$

---

343 Note that since the true posterior and the variational family share the same (categorical) form, the  
 344 problem is well-specified: the KL depends only on estimating the probabilities  $P(J = j; x)$ . In  
 345 particular, the variational posterior can recover the true posterior exactly, up to limitations stemming  
 346 from access to finite data.

348 3.3 OPTIMIZATION

350 To minimize the KL divergence in Equation 19, we optimize the evidence lower bound (ELBO)  
 351 on the log-likelihood (Kingma & Welling, 2014; Rezende & Mohamed, 2015; Blei et al., 2017).  
 352 We parameterize the variational distribution with a neural network with weights  $\theta$ , producing  
 353  $h_\theta(x) = (q_\theta(J = 1; x), \dots, q_\theta(J = m; x))$ , and optimize  $\theta$  rather than the output directly. Thus,  
 354 our objective to fit the amortized posterior is

355 
$$\mathcal{L}(\theta; x) = \mathbb{E}_{q_\theta} [\log p(y | x, f_j)] - D_{\text{KL}}(q_\theta \| p(J | x_{1:n}, x)) \quad (20)$$

356 
$$= \sum_{j=1}^m [q_\theta(J = j; x) \log f_j(y | x)] - \sum_{j=1}^m q_\theta(J = j; x) \log \frac{q_\theta(J = j; x)}{p(J = j | x_{1:n}, x)}. \quad (21)$$

359 Note that the expected log-likelihood  $\mathbb{E}_{q_\theta} [\log p(y | x, f_j)]$  reduces to a weighted sum, so no sam-  
 360 pling is required to evaluate our objective. The complete optimization procedure is summarized in  
 361 Algorithm 1.

363 Under the Bayesian formulation, the posterior predictive distribution in Eq. 3 yields Bayes-optimal  
 364 uncertainty. Any deviation of IA-BMA from this ideal arises only from the variational approxima-  
 365 tion—specifically from the expressiveness of the variational family, and quantified by the KL term  
 366 in Eq. 19.

367 **Weight assignment:** After training is complete (see Algorithm 1), with the estimate  $\hat{\theta}$ , for a new  
 368 input  $x$  we compute  $(q_{\hat{\theta}}(J = 1; x), \dots, q_{\hat{\theta}}(J = m; x))$  and assign  $\alpha_j(x) = q_{\hat{\theta}}(J = j; x)$ . This  
 369 yields a predicted value  $\hat{p}_\alpha(y | x) = \sum_{j=1}^m \alpha_j(x) f_j(y | x)$ .

372 4 EXPERIMENTS

374 Our method operates on a fixed pool of pre-trained predictors, rather than learning experts jointly.  
 375 As a result, the absolute scale or capacity of each predictor is inconsequential, and methods that rely  
 376 on ensambles or require joint training of experts are not comparable. For approaches such as MoE  
 377 and DDP, which typically train both experts and gating weights simultaneously, we evaluate variants  
 378 that are applied on top of the same pre-trained experts.

378 Specifically, we compare IABMA against (a) non-adaptive baselines: (i) best single predictor, (ii)  
 379 uniform average over predictors, (iii) accuracy-weighted average, and (iv) classical Bayesian model  
 380 averaging (BMA); and (b) adaptive methods: (i) Mixture of Experts (MoE) (Jacobs et al., 1991), (ii)  
 381 Dynamic Local Accuracy (DLA) (Woods et al., 1997), (iii) Synthetic Model Combination (SMC)  
 382 (Rasmussen & Ghahramani, 2001), (iv) Bayesian Hierarchical Stacking (BHS) (Yao et al., 2022),  
 383 and (vii) dependent Dirichlet process (DDP) with fixed “atoms” to the pre-trained predictors.

384 In each experiment we train the candidate predictors, fit the averaging methods on the training set,  
 385 and evaluate their predictive distributions on the test set.

387 We conduct extensive evaluation across (i) two synthetic benchmarks, including scale and sensi-  
 388 tivity studies, (ii) two large heteroskedastic real-world tasks (personalized medication and credit  
 389 fraud), and (iii) four UCI benchmarks, to verify that adaptivity does not degrade performance in  
 390 low-heteroskedastic settings.

391 Hyperparameters for our method and all baselines were tuned via binary search to maximize av-  
 392 erage performance (accuracy for classification, RMSE for regression) on a held-out repetition ex-  
 393 cluded from the analysis. The selected values and further implementation details are provided in  
 394 Appendix D, with additional data processing and predictor specifications in Section C. Code to  
 395 reproduce all results is included with the submission and will be released publicly upon acceptance.

#### 396 4.1 SIMULATIONS

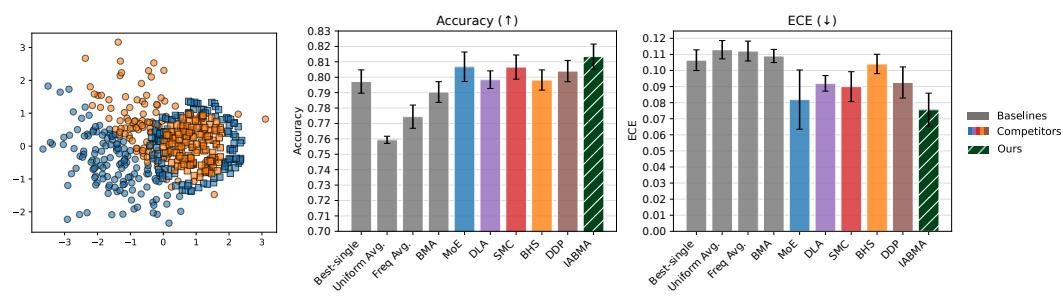
##### 398 4.1.1 LINEAR–CIRCULAR HYBRID CLASSIFICATION

400 We evaluate IA-BMA on a two-dimensional binary task composed of two heterogeneous subpopu-  
 401 lations. Half of the samples follow a linear decision rule and are drawn from a Gaussian cluster near  
 402  $(-1, 0)$ ; the other half lie on a ring around  $(1, 0)$  and follow a circular rule  $y = \mathbb{1}r < 1$ . We use  
 403  $n_{\text{train}} = 1000$ ,  $n_{\text{test}} = 500$ , and train all methods on the raw coordinates  $(x_1, x_2)$ .

404 This construction yields three regions: (i) points linearly separable, (ii) points circularly separable,  
 405 and (iii) an intermediate overlap where the correct predictor switches. Ideal weighting places mass  
 406 on linear models in (i), on circular models in (ii), and mixes softly in (iii).

407 All methods share the same pool of base predictors: polynomial logistic regression (degree 2 and  
 408 3), LDA, and two “soft-circle” classifiers based on radial distance to a learned center. Additional  
 409 details are provided in section C.1

410 **Results:** Figure 2 shows that IA-BMA achieves highest accuracy and lowest ECE compared to all  
 411 non-adaptive baselines, as well as all adaptive methods.



423 Figure 2: Simulation. Left: data (of one repetition). Results for accuracy (middle) and ECE (right)  
 424 are reported for 10 repetitions. IA-BMA achieves highest accuracy and lowest ECE.

##### 426 4.1.2 SCALE AND SENSITIVITY ANALYSIS

428 We evaluate IABMA with respect to: (i) scalability in data dimension, (ii) the number of informative  
 429 (non-noise) features, (iii) the number of predictors, and (iv) the similarity between predictors.

431 As detailed in Appendix C.2, we construct a synthetic setting with two data regimes that share only  
 a subset of the informative features. We vary the data dimension  $d$ , and the number of informative

432 features  $k$ . We construct  $m$  candidate predictors: (i) two per-regime *specialists*,  $m - 3$  *generalists*  
 433 of varying similarity to each other  $\rho$ , and an additional model that exceeds all generalists on both  
 434 regimes but remains inferior to the specialists.

435 For any input  $x$ , the optimal ensemble behavior is to select the specialist corresponding to the sign  
 436 of  $x$ , and never to select one of the suboptimal generalists or the overall-best predictor.

437 **Results:** IABMA consistently outperforms all baselines and selects the correct specialist far more  
 438 often. Table 1 reports specialist selection rates; performance metrics and SDs in Tables 2–6.

440

441

Table 1: Correct specialist proportion across scaling experiments.

Experiment	MoE	DLA	SMC	BHS	DDP	IABMA
Base: $d = 100$ , $k = 30$ , $m = 10$ , $\rho = 0.0$	0.000	0.008	0.051	0.037	0.227	<b>0.948</b>
Dimension increase: $d = 300$	0.000	0.018	0.000	0.020	0.000	<b>0.552</b>
More informative features: $k = 50$	0.000	0.013	0.000	0.022	0.000	<b>0.675</b>
More predictors: $m = 100$	0.493	0.006	0.002	0.114	0.000	<b>0.857</b>
Higher similarity: $\rho = 0.5$	0.000	0.001	0.000	0.022	0.000	<b>0.922</b>

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443

## 4.2 CASE STUDIES

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## 4.2.1 PERSONALIZED CANCER DRUG-RESPONSE

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An important example of heterogeneous data is personalized drug response prediction, where different models may perform better on different subpopulations. We evaluate IA-BMA on this task using the PRISM cancer drug response dataset. The data consists of pairings of molecule-cell line RNA sequence features. For each drug–cell pair we form a continuous response  $y$  so that larger values indicate greater sensitivity. We retain drugs with broad site coverage and construct inputs from the top variance genes. All averaging methods operate over the same four base regressors—Ridge, Histogram-based Gradient Boosting Tree, XGBoost, and a Multilayer perceptron (MLP), each with pre-processing tailored to model class. Additional details are provided in C.3.

446

**Results:** Figure 3 shows that IA-BMA achieves higher  $R^2$  and lower RMSE compared to all other methods. Further analysis is presented in Figures 4–7 which display the weights assigned by each averaging method for randomly selected inputs. The results show that IA-BMA consistently favored the best (or nearly best) model, whereas other methods leaned toward other predictors, with MoE in particular overemphasizing MLP and XGB even when suboptimal.

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## 4.2.2 CREDIT-CARD FRAUD DETECTION

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Another domain characterized by heterogeneous data is fraud detection, where the rarity of fraudulent cases poses an additional challenge. We evaluate IA-BMA on this task using the IEEE-CIS Fraud Detection dataset. The dataset consists from mixed Continuous (such as transaction amount) and high-cardinality categorical features (such as product category), and the target variable  $y \in \{0, 1\}$  indicated where a transaction was fraud. All averaging methods operate over the same base classifiers: Logistic Regression with Lasso penalty, Histogram-based Gradient Boosting Tree, XGBoost, and an MLP. Additional details appear in Appendix C.4.

451

452

**Results:** Figure 3 shows that IA-BMA achieves higher accuracy and lower expected-calibration error compared to all other methods. Since in fraud prediction calibration matters within each bin, we analyzed per-bin confidence  $|p - 0.5|$ , and found that IA-BMA achieves the lowest error in all high-confidence bins ( $> 0.25$ ). The corresponding analysis is shown in Figure B.4.

453

454

## 4.3 EXPERIMENTS ON UCI BENCHMARK DATASETS

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We evaluate IA-BMA on four UCI datasets — two classification (spambase, credit-g) and two regression (bike-sharing, california-housing) — which represent *low-heteroskedasticity scenarios*. In such settings, one should not expect consistent dominance by any method, as the benefits of adaptive model averaging emerge when subpopulations differ substan-

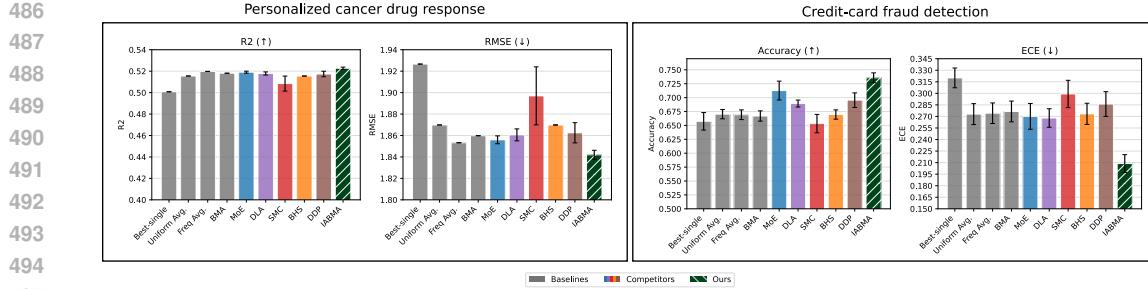


Figure 3: Experimental results for main case studies. Results are reported for 10 repetitions. IA-BMA achieves best results compared to all other averaging method on both case studies.

tially. The value of these experiments lies in confirming that IA-BMA is *non-harmful* in settings where adaptivity is not expected to yield significant gain.

**Results:** IA-BMA yields improvements in RMSE for both regression tasks and in accuracy for both classification tasks, with 9 of 20 pairwise comparisons showing statistically significant gains over five baselines. Experimental details appear in Section C.5. Results are reported in Table 8.

## 5 CONCLUSION

We introduced IA-BMA, a framework that casts model averaging as probabilistic model selection conditioned on the input. Within this formulation, the posterior distribution over models provides the natural, Bayes-optimal choice of input adaptive weights, thereby recovering the true predictive distribution. Our approach is grounded in an input-dependent prior on the selector function and implemented through amortized variational inference of the posterior.

We establish finite-sample bounds showing that the posterior-weights predictor achieves strong likelihood performance compared to any input-specific single-model selector. Empirically, we evaluate IA-BMA across regression and classification tasks, including personalized cancer treatment response, credit-card fraud detection, and standard UCI benchmarks. We show that IA-BMA consistently outperforms both non-adaptive baselines and existing adaptive methods, delivering more accurate and better calibrated predictions.

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702 **A PROOFS**  
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704 **A.1 CHANGE OF MEASURE ARGUMENT**  
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706 Let  $F : X_2 \rightarrow X_1$  be a measurable function between two measure spaces  $(X_1, \mathcal{A}_1, \eta)$  and  
 707  $(X_2, \mathcal{A}_2, \nu)$ . Let  $g : X_1 \rightarrow \mathbb{R}$  measurable function. Recall that the change of variables formula  
 708 is given by

$$709 \quad \int_{X_1} g \, dF_{\#}\eta = \int_{X_1} (g \circ F) \, d\eta, \quad (22)$$

711 where  $F_{\#}\eta$  denotes the push-forward of  $\eta$  through  $F$ .

713 Applying this to our setting, a draw from the posterior  $g \sim p(g | x, \mathcal{D})$  induces a random index  $j(x)$   
 714 defined by the relation  $g(x) = e_{j(x)}$ . Formally, the evaluation map  
 715

$$716 \quad s_x : \mathcal{G} \rightarrow \{1, \dots, m\}, \quad s_x(g) = j(x),$$

717 pushes the posterior measure  $p(g | x, \mathcal{D})$  forward onto a distribution over indices. Using this push-  
 718 forward, we can rewrite equation 8 as

$$719 \quad \int_{\mathcal{G}} p(y | x, g) \, d\mathbb{P}(g | x, \mathcal{D}) = \int_{\mathcal{G}} f_{s_x(g)}(y | x) \, d\mathbb{P}(g | x, \mathcal{D}) \quad (23)$$

$$722 \quad = \int_{\{1, \dots, m\}} f_j(y | x) \, d(E_{x\#}\mathbb{P})(j | x, \mathcal{D}) \quad (24)$$

$$724 \quad = \sum_{j=1}^m f_j(y | x) \, p(j | x, \mathcal{D}). \quad (25)$$

727 **A.2 PROOF OF THEOREM 2.1**  
 728

729 **Theorem.** Denote  $\mathcal{D}_i := \{(x_t, y_t)\}_{t=1}^i$ , and consider the posterior weights predictor  $\hat{p}_{\alpha}^{(i)}$  assigning  
 730  $\alpha_j^{(i)}(x) := p(J(x) = j | \mathcal{D}_i, x)$  to the  $j$ -th predictor  $f_j$ . Assume that  $\mathbb{E}[|\log f_j(Y | X)|] < \infty$  for  
 731 all  $f_j \in \mathcal{F}$ . Then, for any measurable selector  $j^* : \mathcal{X} \rightarrow \{1, \dots, m\}$  and any  $n \geq 1$ ,

$$733 \quad \frac{1}{n} \sum_{i=1}^n \mathbb{E} [\log \hat{p}_{\alpha}^{(i)}(y_i | x_i, \mathcal{D}_{i-1})] \geq \mathbb{E} [\log f_{j^*(x)}(y | x)] + \frac{1}{n} \sum_{i=1}^n \mathbb{E} [\log \alpha_{j^*(x_i)}^{(i)}(x_i)], \quad (26)$$

736 where the expectations are taken w.r.t the population distribution  $(x, y) \sim p(x, y)$ .

738 *Proof.* Define the posterior-weights predictor

$$740 \quad \hat{p}_{\alpha}^{(i)}(y | x, \mathcal{D}_{i-1}) = \sum_{j=1}^m \alpha_j^{(i)}(x) f_j(y | x) \quad (27)$$

742 For a fixed input  $x_i$  and a fixed predictor  $f_k$  we have that

$$744 \quad \log \hat{p}_{\alpha}^{(i)}(y_i | x_i, \mathcal{D}_{i-1}) = \log \left( \sum_{j=1}^m \alpha_j^{(i)}(x_i) f_j(y_i | x_i) \right) \quad (28)$$

$$747 \quad \geq \log \left( \alpha_k^{(i)}(x_i) f_k(y_i | x_i) \right) \quad (29)$$

$$749 \quad = \log f_k(y_i | x_i) + \log \alpha_k^{(i)}(x_i). \quad (30)$$

750 Taking  $\mathbb{E}_{(x, y) \sim p(x, y)} [\cdot | x_i, \mathcal{D}_{i-1}]$ , since  $f_{j^*(x)}(y_i | x_i)$  is independent of  $\mathcal{D}_{i-1}$ ,

$$752 \quad \mathbb{E} [\log \hat{p}_{\alpha}^{(i)}(y_i | x_i, \mathcal{D}_{i-1})] \geq \mathbb{E} [\log f_k(y_i | x_i)] + \mathbb{E} [\log \alpha_k^{(i)}(x_i) | \mathcal{D}_{i-1}]. \quad (31)$$

754 This holds for any  $1 \leq k \leq m$ , hence for  $k = j^*(x_i)$ ,

$$755 \quad \mathbb{E} [\log \hat{p}_{\alpha}^{(i)}(y_i | x_i, \mathcal{D}_{i-1}) | \mathcal{D}_{i-1}] \geq \mathbb{E} [\log f_{j^*(x)}(y_i | x_i)] + \mathbb{E} [\log \alpha_{j^*(x_i)}^{(i)}(x_i) | \mathcal{D}_{i-1}]. \quad (32)$$

756 Taking  $\mathbb{E}[\cdot | \mathcal{D}_{i-1}]$ , by the law of total expectation,

$$758 \mathbb{E} \left[ \log \hat{p}_\alpha^{(i)}(y_i | x_i, \mathcal{D}_{i-1}) \right] \geq \mathbb{E} \left[ \log f_{j^*(x_i)}(y_i | x_i) \right] + \mathbb{E} \left[ \log \alpha_{j^*(x_i)}^{(i)}(x_i) \right]. \quad (33)$$

760 Averaging over  $i$ , we get

$$762 \frac{1}{n} \sum_{i=1}^n \mathbb{E} \left[ \log \hat{p}_\alpha^{(i)}(y_i | x_i, \mathcal{D}_{i-1}) \right] \geq \mathbb{E} \left[ \log f_{j^*(x_i)}(y_i | x_i) \right] + \frac{1}{n} \sum_{i=1}^n \mathbb{E} \left[ \log \alpha_{j^*(x_i)}^{(i)}(x_i) \right].$$

764  $\square$

## 766 B ADDITIONAL EXPERIMENTAL RESULTS

769 In what follows we provide a deeper analysis of the performance of adaptive model averaging methods on the two case-studies.

### 772 B.1 SCALE AND SENSITIVITY ANALYSIS

775 Table 2: Scaling and sensitivity analysis for  $d = 100, k = 30, m = 10, \rho = 0.0$

776 <b>Method</b>	777 <b>Accuracy</b>	778 <b>ECE</b>	779 <b>Correct Specialist</b>	780 <b>Global Predictor</b>	781 <b>Generalists</b>
782 MoE	783 0.904 (0.002)	784 0.076 (0.003)	785 0.000 (0.000)	786 1.000 (0.000)	787 0.000 (0.000)
788 DLA	789 0.825 (0.007)	790 0.100 (0.002)	791 0.008 (0.001)	792 0.966 (0.005)	793 0.015 (0.003)
797 SMC	798 0.680 (0.132)	799 0.302 (0.155)	800 0.051 (0.042)	801 0.481 (0.424)	802 0.414 (0.339)
806 BHS	807 0.821 (0.007)	808 0.097 (0.002)	809 0.037 (0.034)	810 0.164 (0.018)	811 0.724 (0.036)
815 DDP	816 0.910 (0.008)	817 0.061 (0.017)	818 0.227 (0.280)	819 0.773 (0.280)	820 0.000 (0.000)
825 IABMA	826 <b>0.919 (0.004)</b>	827 <b>0.026 (0.007)</b>	828 <b>0.948 (0.017)</b>	829 0.037 (0.006)	830 0.015 (0.019)

785 Table 3: Scaling and sensitivity analysis for  $d = 300, k = 30, m = 10, \rho = 0.0$

786 <b>Method</b>	787 <b>Accuracy</b>	788 <b>ECE</b>	789 <b>Correct Specialist</b>	790 <b>Global Predictor</b>	791 <b>Generalists</b>
796 MoE	797 0.858 (0.003)	798 0.117 (0.003)	799 0.000 (0.000)	800 1.000 (0.000)	801 0.000 (0.000)
805 DLA	806 0.809 (0.005)	807 0.068 (0.004)	808 0.018 (0.005)	809 0.929 (0.004)	810 0.037 (0.001)
814 SMC	815 0.816 (0.005)	816 0.073 (0.004)	817 0.000 (0.000)	818 1.000 (0.000)	819 0.000 (0.000)
823 BHS	824 0.804 (0.005)	825 0.067 (0.004)	826 0.020 (0.002)	827 0.150 (0.005)	828 0.633 (0.011)
832 DDP	833 0.858 (0.003)	834 0.117 (0.003)	835 0.000 (0.000)	836 1.000 (0.000)	837 0.000 (0.000)
841 IABMA	842 <b>0.882 (0.004)</b>	843 <b>0.035 (0.001)</b>	844 <b>0.552 (0.007)</b>	845 0.163 (0.007)	846 0.285 (0.007)

795 Table 4: Scaling and sensitivity analysis for  $d = 100, k = 50, m = 10, \rho = 0.0$

796 <b>Method</b>	797 <b>Accuracy</b>	798 <b>ECE</b>	799 <b>Correct Specialist</b>	800 <b>Global Predictor</b>	801 <b>Generalists</b>
805 MoE	806 0.882 (0.008)	807 0.092 (0.008)	808 0.000 (0.000)	809 1.000 (0.000)	810 0.000 (0.000)
814 DLA	815 0.805 (0.009)	816 0.087 (0.005)	817 0.013 (0.003)	818 0.960 (0.008)	819 0.017 (0.003)
823 SMC	824 0.813 (0.009)	825 0.093 (0.008)	826 0.000 (0.000)	827 1.000 (0.000)	828 0.000 (0.000)
832 BHS	833 0.802 (0.008)	834 0.084 (0.004)	835 0.022 (0.002)	836 0.036 (0.004)	837 0.735 (0.013)
841 DDP	842 0.882 (0.008)	843 0.092 (0.008)	844 0.000 (0.000)	845 1.000 (0.000)	846 0.000 (0.000)
850 IABMA	851 <b>0.902 (0.007)</b>	852 <b>0.029 (0.006)</b>	853 <b>0.675 (0.010)</b>	854 0.325 (0.010)	855 0.000 (0.000)

### 856 B.2 FORMAL STATISTICAL TESTS

864 For the other three heteroskedastic experiments, Table 7 provides p-values for one-sided t-test with  
865 Benjamini–Hochberg correction for multiple comparisons (accuracy for classification, RMSE for  
866 regression). All resulting values are below 0.054.

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Table 5: Scaling and sensitivity analysis for  $d = 100, k = 50, m = 100, \rho = 0.0$ 

Method	Accuracy	ECE	Correct Specialist	Global Predictor	Generalists
MoE	<b>0.926 (0.005)</b>	0.038 (0.002)	0.493 (0.009)	0.507 (0.009)	0.000 (0.000)
DLA	0.758 (0.010)	<b>0.027 (0.005)</b>	0.006 (0.001)	0.955 (0.005)	0.035 (0.005)
SMC	0.725 (0.067)	0.104 (0.151)	0.002 (0.004)	0.755 (0.355)	0.240 (0.347)
BHS	0.757 (0.010)	<b>0.027 (0.005)</b>	0.114 (0.005)	0.019 (0.002)	0.867 (0.005)
DDP	0.900 (0.002)	0.081 (0.004)	0.000 (0.000)	1.000 (0.000)	0.000 (0.000)
IABMA	0.916 (0.004)	0.028 (0.005)	<b>0.857 (0.008)</b>	0.091 (0.007)	0.045 (0.002)

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Table 6: Scaling and sensitivity analysis for  $d = 100, k = 50, m = 100, \rho = 0.5$ 

Method	Accuracy	ECE	Correct Specialist	Global Predictor	Generalists
MoE	0.902 (0.006)	0.073 (0.006)	0.000 (0.000)	1.000 (0.000)	0.000 (0.000)
DLA	0.797 (0.008)	0.137 (0.002)	0.001 (0.001)	0.996 (0.001)	0.002 (0.000)
SMC	0.837 (0.008)	0.163 (0.004)	0.000 (0.000)	1.000 (0.000)	0.000 (0.000)
BHS	0.787 (0.008)	0.126 (0.006)	0.022 (0.003)	0.019 (0.004)	0.952 (0.003)
DDP	0.902 (0.006)	0.073 (0.006)	0.000 (0.000)	1.000 (0.000)	0.000 (0.000)
IABMA	<b>0.919 (0.006)</b>	<b>0.022 (0.005)</b>	<b>0.922 (0.001)</b>	0.077 (0.001)	0.000 (0.000)

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p-values for UCI experiments appear in Table 9.

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Table 7: Paired t-test p-values vs. IABMA on heterogeneous datasets.

Experiment	BMA	MoE	DLA	SMC	BHS	DPP
Cancer	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01
Fraud	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01
Linear-circular	< 0.01	0.054	< 0.01	0.037	< 0.01	< 0.01

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## B.3 CANCER TREATMENT RESPONSE

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To illustrate how different methods allocate weights, we sampled 16 cases as follows: for each classifier  $f_j$ , we randomly selected four examples from those where IA-BMA assigned the highest weight to  $f_j$ . Figures 4–7 display the weights assigned by each averaging method for Ridge, XGB, HGB, and MLP. For each case, we also report the RMSE achieved by the individual classifiers. This analysis shows that in all cases, IA-BMA places the largest weight on the model with either the lowest error or a near-tied second. By contrast, competing methods tend to favor other predictors. In particular, MoE consistently prioritizes MLP or XGB, even in instances where these models are locally suboptimal.

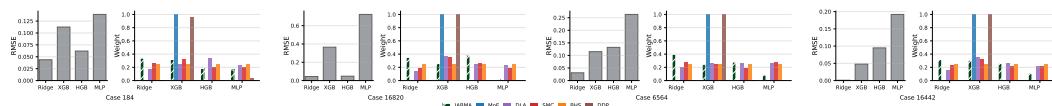


Figure 4: Cases where IA-BMA assigns the highest weight to Ridge.

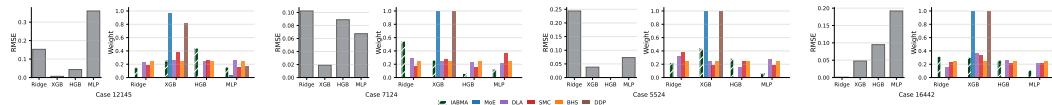


Figure 5: Cases where IA-BMA assigns the highest weight to XGB.

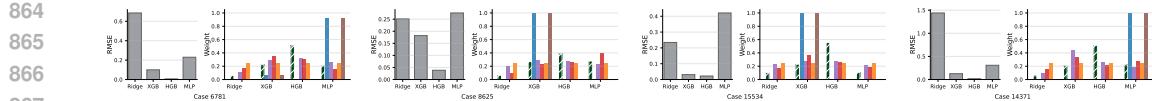


Figure 6: Cases where IA-BMA assigns the highest weight to HGB.

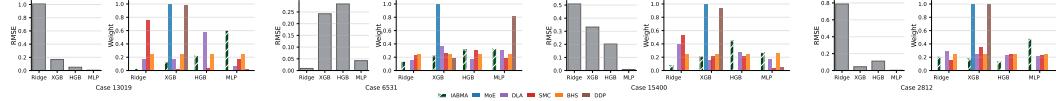


Figure 7: Cases where IA-BMA assigns the highest weight to MLP.

#### B.4 CREDIT CARD FRAUD

Credit card fraud prediction is a highly sensitive area, with risks of false alarms and misreporting, calibration is crucial not only overall but also within each bin. To this end, we analyzed the confidence measure  $|p - 0.5|$  where  $p$  is the estimated probability, which captures certainty for both positive and negative events, and compared the bin-wise errors across averaging methods. Figure B.4 shows that in all bins, IA-BMA attains the lowest error, decreasing with confidence, showing that most wrong predictions occur in low confidence instances.

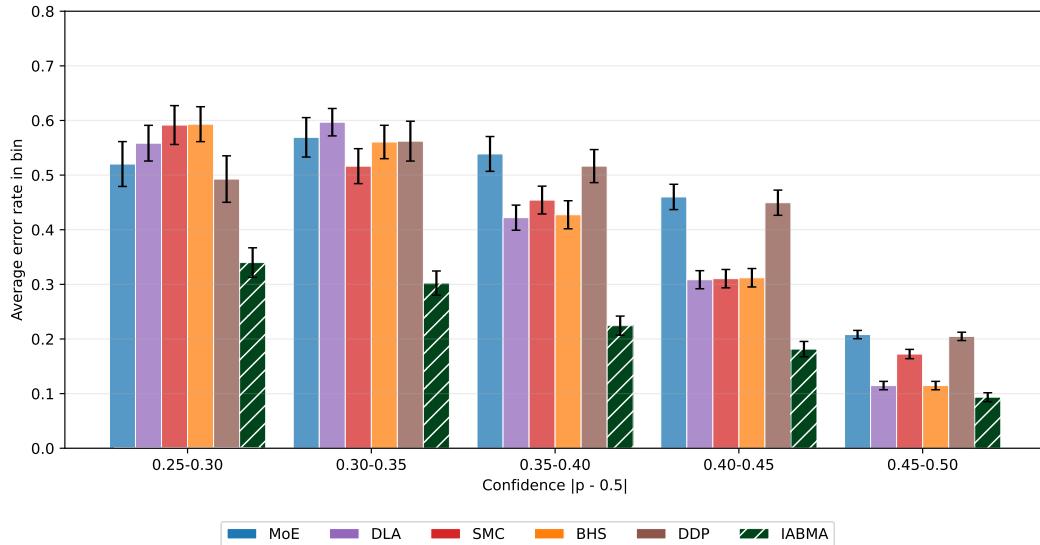


Figure 8: Calibration across confidence bins in credit-card fraud prediction

#### B.5 UCI BENCHMARK DATASETS

Results are reported in Table 8, and p-values for one-sided t-test results with Benjamini–Hochberg correction for multiple comparisons (accuracy for classification, RMSE for regression) in Table 9.

#### B.6 ANALYSIS OF THE EFFECT OF THE ADAPTIVE PRIOR

IA-BMA places an input-dependent prior over selectors  $g$ . Therefore, variability in our model arises not only from variability of  $g(x)$  as in other adaptive methods, but additionally from the adaptivity

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Table 8: UCI benchmarks: mean (sd) across runs.

Dataset	Metric	Best single	Uniform Avg.	Freq Avg.	BMA	MoE	DLA	SMC	BHS	DDP	IA-BMA
Bike-sharing	R2 ( $\uparrow$ )	0.706 (0.022)	0.752 (0.010)	0.773 (0.012)	0.774 (0.014)	0.706 (0.022)	0.781 (0.013)	0.756 (0.010)	0.752 (0.010)	0.753 (0.013)	<b>0.794</b> (0.010)
	RMSE ( $\downarrow$ )	0.582 (0.033)	0.491 (0.021)	0.448 (0.020)	0.447 (0.021)	0.581 (0.033)	0.446 (0.020)	0.483 (0.020)	0.491 (0.021)	0.479 (0.022)	<b>0.433</b> (0.018)
Cal.-housing	R2 ( $\uparrow$ )	0.772 (0.022)	0.840 (0.018)	0.840 (0.017)	0.812 (0.017)	0.778 (0.024)	0.840 (0.018)	0.817 (0.066)	0.840 (0.018)	0.805 (0.031)	<b>0.844</b> (0.014)
	RMSE ( $\downarrow$ )	0.036 (0.004)	0.025 (0.003)	0.025 (0.003)	0.029 (0.003)	0.035 (0.004)	0.025 (0.003)	0.029 (0.010)	0.025 (0.003)	0.031 (0.005)	<b>0.024</b> (0.003)
Credit-g	Accuracy ( $\uparrow$ )	0.634 (0.036)	0.676 (0.029)	0.662 (0.038)	0.648 (0.036)	0.624 (0.036)	0.668 (0.039)	0.626 (0.046)	0.682 (0.039)	0.655 (0.040)	<b>0.684</b> (0.047)
	ECE ( $\downarrow$ )	0.260 (0.034)	0.172 (0.022)	<b>0.169</b> (0.020)	0.174 (0.023)	0.296 (0.035)	0.173 (0.025)	0.222 (0.038)	0.176 (0.25)	0.257 (0.031)	0.175 (0.020)
Spambase	Accuracy ( $\uparrow$ )	0.699 (0.110)	0.702 (0.094)	0.738 (0.044)	0.760 (0.024)	0.760 (0.035)	0.729 (0.052)	0.757 (0.032)	0.646 (0.132)	0.754 (0.064)	<b>0.764</b> (0.032)
	ECE ( $\downarrow$ )	0.114 (0.022)	0.163 (0.049)	0.148 (0.042)	0.169 (0.018)	<b>0.095</b> (0.034)	0.171 (0.051)	0.222 (0.023)	0.180 (0.061)	0.149 (0.042)	0.146 (0.025)

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Table 9: Paired t-test p-values vs. IA-BMA on UCI benchmarks.

Experiment	BMA	MoE	DLA	SMC	BHS
Credit-g	0.104	<b>&lt; 0.01</b>	0.338	<b>&lt; 0.01</b>	0.758
Spambase	0.349	0.380	<b>0.019</b>	0.349	<b>0.016</b>
Bike-sharing	0.316	<b>&lt; 0.01</b>	1.000	<b>&lt; 0.01</b>	<b>&lt; 0.01</b>
California-housing	<b>&lt; 0.01</b>	<b>&lt; 0.01</b>	0.153	0.153	0.153

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943 of the prior itself. To isolate the contribution of this adaptive prior, we evaluate our method under a  
944 uniform prior and compare the resulting behavior.

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The results below (Tables 10 and 11) show that without the adaptive prior, our method still outperforms other methods in most cases, but with a smaller margin.

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Table 10: Impact of the input–adaptive prior on RMSE across regression tasks

Experiment	Best-single	Uniform Avg	Freq Avg	BMA	MoE	DLA	SMC	BHS	DDP	IA-BMA	IA-BMA Uniform
PRISM	1.927	1.870	1.853	1.860	1.856	1.861	1.897	1.870	1.863	1.842	1.853
Bike-Sharing	0.582	0.491	0.448	0.447	0.581	0.433	0.483	0.491	0.479	0.446	1.311
California-Housing	0.036	0.025	0.025	0.029	0.035	0.025	0.029	0.025	0.031	0.024	0.019

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Table 11: Impact of the input–adaptive prior on accuracy across classification tasks

Experiment	Best-single	Uniform Avg	Freq Avg	BMA	MoE	DLA	SMC	BHS	DDP	IA-BMA	IA-BMA Ablation
Fraud small	0.657	0.670	0.669	0.667	0.713	0.689	0.653	0.670	0.695	0.736	0.713
Synthetic Binary	0.797	0.759	0.774	0.790	0.807	0.798	0.807	0.798	0.804	0.813	0.813
Credit-G	0.634	0.684	0.662	0.648	0.624	0.668	0.626	0.682	0.655	0.676	0.575
Spambase	0.699	0.702	0.738	0.760	0.760	0.729	0.757	0.646	0.754	0.764	0.739

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## C EXPERIMENTAL DETAILS

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### C.1 LINEAR–CIRCULAR HYBRID CLASSIFICATION

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#### C.1.1 DATA AND PROCESSING

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We generated a two-dimensional binary dataset with two subpopulations governed by different decision rules. For the *linear* subpopulation, we drew  $n_{\text{lin}} = n_{\text{train}}/2$  training points from a Gaussian

972 cloud centered at  $(-t, 0)$  (with  $t = 1$ ),  
 973

$$974 \quad X^{(\text{lin,train})} \sim \mathcal{N}((-t, 0), 0.1 I_2),$$

975 and assigned labels by a linear rule  $y = \mathbb{1}\{x_1 + x_2 > -t\}$ . For the *circular* subpopulation, we drew  
 976  $n_{\text{circ}} = n_{\text{train}} - n_{\text{lin}}$  points on a ring around  $(t, 0)$  by sampling  $\theta \sim \text{Unif}(0, 2\pi)$  and  $r = \sqrt{U}$  with  
 977  $U \sim \text{Unif}(0, 2)$ , and set

$$978 \quad X^{(\text{circ})} = (t, 0) + (r \cos \theta, r \sin \theta), \quad y = \mathbb{1}\{r < 1\}.$$

980 We used  $n_{\text{train}} = 1,000$  and  $n_{\text{test}} = 500$ ; the train/test splits were generated independently.  
 981

982 Only the two coordinates  $(x_1, x_2)$  were provided as features. A region indicator  $z \in$   
 983  $\{0 \text{ (linear), } 1 \text{ (circular)}\}$  was recorded for analysis but was not used during training.

984 This dataset naturally forms three regimes: (i) linearly separable points, (ii) circularly separable  
 985 points, (iii) an ambiguous overlap region where neither boundary dominates.

### 986 987 C.1.2 CANDIDATE PREDICTORS

988 All averaging methods were evaluated on the same 3 base classifiers:

- 990 1. *Polynomial logistic regression (degrees 2 and 3)*. We fit logistic regression with polynomial  
 991 features of degree  $d \in \{2, 3\}$  (no bias term in the expansion).
- 992 2. *Linear Discriminant Analysis (LDA)*. A linear generative classifier fit on the raw coordinates,  
 993 providing a single linear boundary.
- 994 3. *Soft-circle classifiers (two instances)*. Each instance modeled the positive-class probability  
 995 as a logistic function of radial distance to a fixed center,

$$996 \quad p_{\text{circle}}(y=1 | x) = \sigma(\gamma(R - \|x - c\|)), \quad c = (0.8t, 0), \quad R = 1.0, \quad \gamma = 5.0,$$

998 yielding smooth circular decision regions around  $(t, 0)$ .

999 We include two instances of each predictor to allow the averaging procedure to allocate  
 1000 weight among near-identical experts.

## 1001 1002 C.2 SCALE AND SENSITIVITY ANALYSIS

1003 We evaluate IABMA with respect to four factors: (i) scalability in data dimension, (ii) the number  
 1004 of informative (non-noise) features, (iii) the number of predictors, and (iv) the similarity between  
 1005 predictors.

### 1006 1007 C.2.1 DATA

1008 We generate inputs  $x \in \mathbb{R}^d$  by sampling each coordinate independently from 0, 1 with probability  
 1009 1/2. With probability 1/2 we flip the sign of the entire vector, creating two regimes: positive and  
 1010 negative. Labels  $y$  are assigned by majority vote over  $k$  designated coordinates. The identity of these  
 1011 coordinates differs across regimes, and we control the fraction of shared coordinates  $\rho$  to control  
 1012 heteroskedasticity. Independent Gaussian noise  $\mathcal{N}(0, 0.1)$  is then added to each input, identically  
 1013 for training and test sets.

### 1014 1015 C.2.2 CANDIDATE PREDICTORS

1016 We form a pool of  $m$  logistic predictors. Two *specialists* are trained exclusively on one regime  
 1017 each, performing well on that regime, and poorly on the other. The remaining  $m - 3$  *generalists* are  
 1018 trained on mixtures of the two regimes, yielding weaker per-regime accuracy; varying the mixture  
 1019 proportion  $p$  controls their similarity.

1020 Finally, we include a two-layer MLP (32 and 16 units, ReLU activations) trained on combined  
 1021 balanced data. It is designed to exceed all generalists on both regimes, but remains inferior to the  
 1022 specialists.

1023 For any input  $x$ , the optimal ensemble behavior is to select the specialist corresponding to the sign  
 1024 of  $x$ , and never to select one of the suboptimal generalists or the overall-best predictor.

1026 We set our baseline experiment parameters as  $d = 100, k = 30, \rho = 0.0, m = 11$ . We then vary  
 1027 each.  
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## 1030 C.3 PRISM CANCER EXPERIMENT

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## 1032 C.3.1 DATA AND PROCESSING

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1034 We used the publicly available PRISM cancer drug response dataset. The primary data<sup>3</sup> was com-  
 1035 bined with an RNA-seq expression matrix<sup>4</sup>, cell-line metadata<sup>5</sup>, and tissue labels<sup>6</sup>. All files are  
 1036 available from [https://depmap.org/portal/data\\_page/](https://depmap.org/portal/data_page/).

1037 The PRISM file reports drug–cell line responses with identifiers of the form ACH-#. We normalized  
 1038 all identifiers to the canonical zero-padded format (ACH-XXXXXX). Non-Continuous entries and  
 1039 all observations lacking a primary cancer site were excluded. Responses correspond to log-fold  
 1040 changes (LFC), clipped to the range  $[-6, 6]$ , and the prediction target was defined as  $y = -v$ , where  
 1041  $v$  is the clipped LFC.

1042 We focused on the 40 drugs with the greatest site-level heterogeneity. Specifically, we computed the  
 1043 between-site variance of  $y$  and retained compounds observed in at least 3 distinct sites, with at least  
 1044 5 samples per site and at least 40 samples overall. A minimum per-site coverage threshold of 20  
 1045 samples was enforced. To avoid domination by a few large tissues, we capped each site at  $1.1 \times s$ ,  
 1046 where  $s$  is its sample size. This yielded approximately 18,460 drug–cell line pairs (slight variation  
 1047 across random splits), of which 80% were used for training and 20% for testing.

1048 Gene expression features were restricted to the 100 highest-variance genes. Each gene was stan-  
 1049 dardized to mean 0 and variance 1 based on training statistics. The final feature matrix consisted of  
 1050 standardized gene expression values and a categorical compound indicator.

1051 The full processing code was submitted with this paper and will be released publicly upon accep-  
 1052 tance.  
 1053

1054

## 1055 C.3.2 CANDIDATE PREDICTORS

1056

1057 All averaging methods were evaluated on averaging the same four regression models with repro-  
 1058 cessing pipelines tailored per model:

1059

1. *Ridge regression ( $\ell_2$  regularized linear model)*. Gene features were imputed (median),  
 1060 standardized to zero mean and unit variance, and combined with a dense one-hot encoding  
 1061 of the compound identity.
2. *Histogram-based Gradient Boosting regressor (HGB)*. Tree-based model trained on raw  
 1064 gene values (median imputation only) together with a sparse one-hot encoding of the com-  
 1065 pound identity.
3. *XGBoost regressor (XGB)*. Gradient-boosted decision trees with squared-error objective,  
 1068 trained using the same pre-processing as HGB. We used 400 estimators, learning rate 0.05,  
 1069 maximum depth 8, subsample ratio 0.9, and column subsample ratio 0.8, with  $\ell_1$  and  $\ell_2$   
 1070 regularization.
4. *Multi-layer perceptron (MLP)*. A feed-forward neural network with hidden layers of size  
 1073 (128, 64), ReLU activations, learning rate  $10^{-3}$ , batch size 64, and early stopping based on  
 1074 a 10% validation split. Inputs were preprocessed as for Ridge (dense, imputed, standardized  
 1075 gene features and dense one-hot drug encoding).

1076

<sup>3</sup>Repurposing\_Public\_23Q2\_Extended\_Primary\_Data\_Matrix.csv

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<sup>4</sup>OmicsExpressionProteinCodingGenesTPMLLog1.csv

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<sup>5</sup>Cell\_lines\_annotations\_20181226.txt

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<sup>6</sup>Model.csv

1080 C.4 IEEE-CIS FRAUD EXPERIMENT  
10811082 C.4.1 DATA AND PROCESSING  
10831084 We used the IEEE-CIS credit-card fraud dataset, available at <https://www.kaggle.com/c/ieee-fraud-detection/data>.  
10851086 We removed rows with missing target (`isFraud`) and features with more than 50% missing values.  
1087 To limit explosion in feature dimension, infrequent categories were grouped into a shared rare  
1088 category.1089 In each repetition 80% of the data was used for training and 20% for testing. The training data  
1090 was then reduced to obtained class balance, while in test data class imbalance was maintained. To  
1091 reduced covariate shift in the train-test split we stratified jointly on (`ProductCD`, `card4`) crossed  
1092 with per-row missingness bins and `TransactionAmt` quantile bins, with a fallback “RARE”  
1093 bucket for very small strata. This procedure yielded a stable empirical mix of products, card net-  
1094 works, and spending levels. Specifically, to control the empirical mix of products, card networks,  
1095 and spending levels we stratified jointly on (`ProductCD`, `card4`) crossed with per-row missing-  
1096 ness bins and `TransactionAmt` quantile bins.1097 Continuous features were median-imputed and where appropriate, standardized to zero mean and  
1098 unit variance. Categorical features were imputed to the most frequent level and one-hot encoded,  
1099 with infrequent categories pooled into a rare-level. Class imbalance was addressed within each  
1100 classifier as noted below.1101  
1102 C.4.2 CANDIDATE PREDICTORS  
1103

1104 All averaging methods were evaluated over the same following base classifiers.

1105 1. *Logistic Regression ( $\ell_1$ -penalized)*. We fit a penalized logistic model to the processed fea-  
1106 ture set, using an  $\ell_1$  penalty with strength to encourage sparsity and robustness to correlated  
1107 predictors. We used a saga solver,  $\ell_1$  penalty with regularization strength of 0.05, maximal  
1108 number of iterations as 4000, and tolerance of  $10^{-3}$ .  
1109 2. *XGBoost (XGB)*. We trained a gradient-boosted ensemble of shallow decision trees using  
1110 histogram-based splits and early stopping. Depth, learning rate, and number of estimators  
1111 were selected via a held-out validation set. Hyper parameters were set as maximal bin of  
1112 256, 300 estimators, maximal depth of 5, learning rate 0.1, row subsampling of 0.3, feature  
1113 subsampling of 0.7, and  $\ell_2$  penalty with strength 1.0.  
1114 3. *Histogram-based Gradient Boosting (HGB)*. We train boosted trees with a histogram grow  
1115 policy, subsampling of observations and features, and  $\ell_2$  regularization. Class imbalance  
1116 was addressed via the standard positive-class weight  $\frac{n_{\text{neg}}}{n_{\text{pos}}}$ , estimated from the training ex-  
1117 amples. Hyperparameters (learning rate, depth, estimators, subsampling ratios) were fixed  
1118 based on validation performance and kept constant across comparisons. Hyperparameters  
1119 were set to maximal depth of 4, learning rate 0.07, and  $\ell_2$  regularization with strength 0.5,  
1120 and at most 350 iterations.  
1121 4. *Multi-layer perceptron (MLP)*. We used a feed-forward network with two hidden layers of  
1122 sizes 384, 192 and ReLU activations, trained with weight decay and early stopping on a  
1123 validation split. Weight decay was set to  $\alpha = 3 \cdot 10^{-3}$ , batch size 512, adaptive learning  
1124 rate with initial value of  $10^{-3}$ , early stopping with validation fraction 0.12 and no change  
1125 for 12 iterations, maximal number of iterations as 300, and tolerance  $10^{-4}$ .1126  
1127 C.5 UCI EXPERIMENTS  
11281129 C.5.1 DATA AND PROCESSING  
11301131 We evaluated IA-BMA on standard UCI tasks retrieved from OpenML. We chose datasets with  
1132 relatively large number of observations and features. For *classification*, we used `spambase` (target:  
1133 `class`) and `credit-g` (target: `class`). For *regression*, we used `bike-sharing` (target: `cnt`)  
and `california-housing` (target: `MedHouseVal`).

1134 We replaces common “unknown” tokens (e.g., ?, NA, NaN, unknown) with missing values, strip-  
 1135 ping whitespace on string columns in each dataset, and dropped features whose missing rate ex-  
 1136 ceeded 40%.

1137 We used an 80%/20% train-test split in each repetition. For classification, we performed stratified  
 1138 sampling on the label to preserve class proportions in the test set, and then balanced only the training  
 1139 split by downampling the majority class to the minority size. For regression, we created an approx-  
 1140 imately balanced split by binning the continuous target into 12 quantile bins and stratifying on those  
 1141 bins. All pre-processing statistics (imputation, scaling, and one- hot vocabularies) were computed  
 1142 on the training partition and applied unchanged to the test data.

1143 To encourage diversity among base models, we formed several heterogeneous, partially overlap-  
 1144 ping *feature bundles* and trained each model on a bundle tailored to its strengths. Bundles were  
 1145 constructed from the training data as follows:

- 1147 • **B1:** up to 3 Continuous features with highest absolute Pearson correlation with the target  
 1148 (continuous median-imputed for this computation).
- 1149 • **B2:** up to 3 highest-variance Continuous features.
- 1150 • **B3:** up to 3 categorical features with highest cardinality.
- 1151 • **B4:** up to 5 remaining low-cardinality categorical variables.
- 1152 • **B5:** all categorical features.
- 1153 • **B6:** all Continuous features.
- 1154 • **B7:** the union of **B1** and **B3**.

1155 Continuous features in non-tree models were median-imputed and standardized. Categorical fea-  
 1156 tures were imputed to the most frequent level and one-hot encoded with a minimum frequency  
 1157 threshold of 10 to pool rare levels; unknown categories at test time were ignored.

### 1162 C.5.2 CANDIDATE PREDICTORS

1163 Across all datasets we trained a common set of base learners. For classification: Multinomial Naive  
 1164 Bayes,  $k$ -NN ( $k = 3$ ), Random Forest, Extra Trees, and a linear SVM. For regression: Ridge  
 1165 ( $\alpha=0.05$ ), Lasso ( $\alpha=0.05$ ),  $k$ -NN ( $k=3$ , distance-weighted), Random Forest, and Extra-Trees. To  
 1166 encourage diversity, each model was trained on a subset of features (“feature bundles”).

## 1168 D IMPLEMENTATION DETAILS

1169 In all our experiments the posterior network for IA-BMA and the gating network for MoE were  
 1170 implemented as feed-forward neural networks with hidden layers of size (64, 32, 16) and ReLU  
 1171 activations. We used Adam optimizer for MoE and IA-BMA across all experiments.

1172 Hyperparameters for our method and all baselines were tuned via binary search to maximize average  
 1173 performance (accuracy for classification, RMSE for regression) on a held-out repetition excluded  
 1174 from the analysis. The selected values and running times by experiment and method are reported  
 1175 below.

### 1179 D.1 HYPERPARAMETERS OF ENSEMBLE METHODS

1182 Table 12: Hyperparameters of Mixture-of-Experts

1183 Hyperparameter	1184 Synthetic	1185 PRISM (Cancer)	1186 Fraud (IEEE-CIS)	1187 UCI
1188 Learning rate	$10^{-3}$	$10^{-3}$	$10^{-3}$	$10^{-3}$
1189 Batch size	64	128	64	64
1190 Epochs	10	20	10	10

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Table 13: Hyperparameters of Dynamic Local Accuracy (DLA).

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Table 14: Synthetic Mixture of Experts (SMC).

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Table 15: Bayesian Hierarchical Stacking (BHS).

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Table 16: Input Adaptive Bayesian Model Averaging (IA-BMA)

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Hyperparameter	Synthetic	PRISM (Cancer)	Fraud (IEEE-CIS)	UCI
Neighborhood size $k$	50	50	50	50
Temperature $T$	0.8	1.0	1.0	1.0
Smoothing $\alpha$	1.0	1.0	1.0	1.0

Hyperparameter	Synthetic	PRISM (Cancer)	Fraud (IEEE-CIS)	UCI
Confident-cover threshold	0.6	0.6	0.6	0.6
Cover quantile (reg.)	–	0.30	–	0.30
Min coverage per model	20	20	20	20
Cov. reg. (reg. mix)	0.9 (Gaussian scores)	0.9	0.9	0.7

Hyperparameter	Synthetic	PRISM (Cancer)	Fraud (IEEE-CIS)	UCI
Temperature $T$	1.0	1.0	1.0	1.0
Prior weight	1.0	1.0	1.0	1.0
Slab scale $s_0$	5.0	5.0	5.0	5.0
Learning rate	$5 \times 10^{-3}$	$5 \times 10^{-3}$	$5 \times 10^{-3}$	$10^{-3}$
Batch size	64	128	64	64
Epochs	10	20	10	10

Hyperparameter	Synthetic	PRISM (Cancer)	Fraud (IEEE-CIS)
Learning rate	$10^{-3}$	$10^{-3}$	$10^{-3}$
Batch size	64	128	64
Epochs	10	30	10
KL weight $\lambda_{KL}$	0.05	0.2	0.2

Table 17: IA-BMA (PosteriorNet) hyperparameters per UCI dataset.

Hyperparameter	Spambase (clf)	Credit-g (clf)	Bike-sharing (reg)	Cal housing (reg)
Learning rate	$5 \times 10^{-3}$	$5 \times 10^{-3}$	$1 \times 10^{-3}$	$1 \times 10^{-3}$
Batch size	64	64	64	64
Epochs	10	10	10	10
KL weight $\lambda_{KL}$	0.1	0.1	0.8	3.0

## E RUNTIMES

Overall run-times per method are reported in Table 18. While computational cost scales with number of predictors and data samples, across all experiments run-times of IA-BMA remain consistent with those of MoE and DDP with the same network architectures.

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Table 18: Method runtimes (seconds): mean (sd) across 10 repetitions.

Experiment	MoE	DLA	SMC	BHS	DPP	IA-BMA
Cancer	147.359 (5.282)	22.269 (0.454)	0.072 (0.114)	28.572 (1.167)	271.320 (5.129)	252.985 (5.571)
Fraud	439.502 (129.487)	8.246 (1.719)	688.473 (155.629)	16.622 (3.139)	502.854 (12.831)	461.312 (121.168)
Simulation	5.664 (0.104)	0.218 (0.008)	0.079 (0.004)	1.040 (0.079)	6.381 (0.075)	5.889 (0.038)
Bike-Sharing	25.080 (3.780)	0.868 (0.179)	0.007 (0.001)	21.364 (1.063)	31.889 (3.971)	29.663 (4.094)
Cal. housing	8.510 (0.987)	0.350 (0.041)	0.006 (0.001)	7.281 (0.324)	10.785 (1.122)	9.815 (1.029)
Credit-g	3.178 (0.049)	0.439 (0.017)	0.174 (0.007)	1.184 (0.148)	3.345 (0.071)	3.345 (0.048)
Spambase	16.420 (0.287)	0.642 (0.025)	0.822 (0.159)	1.781 (0.147)	20.831 (5.367)	18.651 (5.122)
Scale ( $m=10, d=100$ )	40.977 (3.83)	1.794 (0.16)	28.174 (9.61)	3.009 (0.26)	48.898 (3.72)	40.333 (4.12)
Scale ( $m=10, d=300$ )	40.517 (3.84)	4.158 (0.17)	85.922 (8.65)	2.501 (0.21)	50.541 (3.68)	40.650 (3.21)
Scale ( $m=100, d=100$ )	41.638 (3.36)	2.169 (0.75)	254.196 (18.17)	2.612 (0.23)	51.914 (3.83)	45.433 (3.55)

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