Meso-scale Structures in Signed Networks

Keywords: Signed Networks; Meso-scale; Community; Core-periphery; Balance

Extended Abstract

One of the interests in the study of signed networks is *structural balance theory* [1], which was primarily motivated by social and economic networks [2]. The structural balance theory states that a signed network is balanced if all cycles with an even number of negative edges [1]. In contrast, a signed network is anti-balanced if all cycles have an even number of positive edges [3]. From a mesoscopic perspective, the signed network is balanced if it can be partitioned into communities whose intra-links are all positive and inter-links are all negative.

Traditional balance theory has been widely used as the starting point for community detection methods on signed networks [4–6]. The goal of these methods is to divide the network into separate communities so that nodes within the same community have as many positive connections as possible, while as many negative connections as possible link nodes in different communities. While such "descriptive methods" invariably succeed in finding balanced partitions in signed graphs, they fail to quantify their importance in describing the data and ignore the possibility of the existence of unbalanced meso-scale structures in signed graphs.

In this work, we propose and apply a framework to overcome the limitations of these approaches. Our goal is to characterize and quantify the strength of unbalanced partitions of signed graphs. We achieve this by combining the application of modern community detection techniques based on Stochastic Block Models [7, 8] and a generalization to signed graphs of recently proposed characterizations of mesoscale structures between groups [9]. The work shares overlapping goals in identifying cohesive subgroups and analyzing relational patterns, yet they differ in methodological frameworks and theoretical underpinnings. First, our method is agnostic to structural balance constraints. That is to say, we would expect more unbalanced meso-level structures in empirical signed networks. Second, although the underlying methodology is different, our methods could implicitly recover the traditional structural balance theory if the data shows strong intra-group positivity and inter-group negativity.

One of the case studies we will report is the network of the U.S. Congress. The network illustrates bill co-sponsorship tendencies in the 100th House. Edges indicate significant tendencies to co-sponsor or not, with positive and negative relationships inferred using the Stochastic Degree Sequence Model [10]. Nodes have been labeled as the two main parties in the United States. We are agnostic about the underlying party affiliation when we detect the communities; however, in Fig. 1, our approach finds schematically how communities divide. Furthermore, we find several core-periphery patterns [9], where one sparser community (periphery) is more linked to the other denser group (core), which is more linked to itself. Here, in our case, the mixed communities serve as the core to influence the others.

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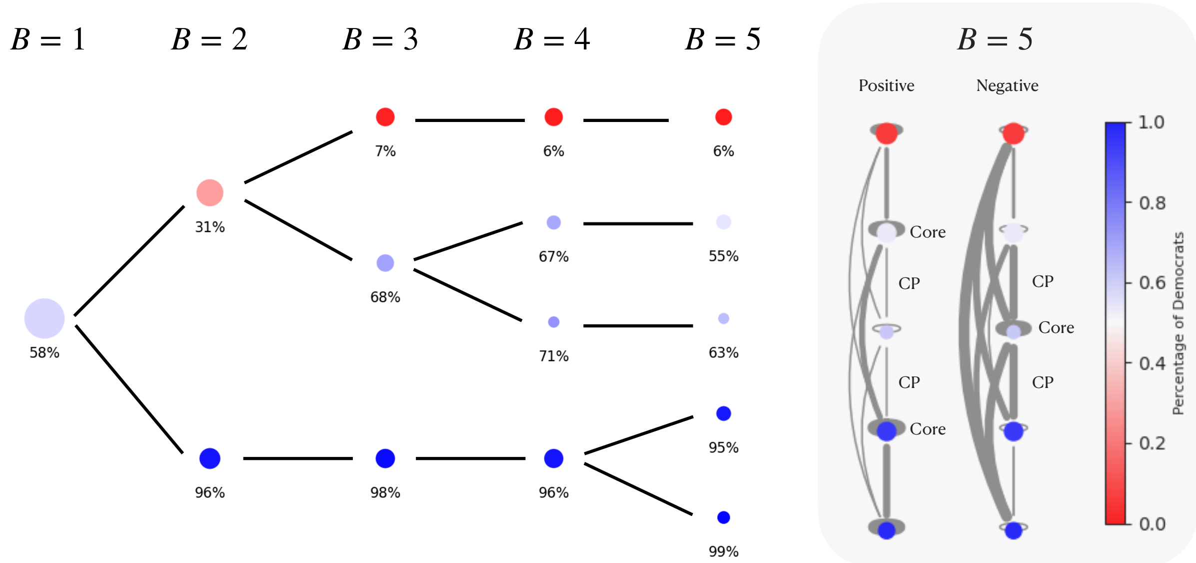

Figure 1: **Affiliation separation in the U.S. Congress network.** The partition into groups was obtained by increasing the number of communities B. The plot shows schematically how communities divide and the type of pairwise interaction they share. The plot on the right shows, for the largest B, the graphical representation of the positive and negative density matrices.