Downside Volatility-Managed Portfolios*

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Abstract

Downside volatility and volatility typically comove but are not highly correlated during the most volatile times. We show that portfolios scaled by downside volatility expand the ex post mean-variance frontiers constructed using the original portfolios and volatility-managed portfolios, and improve the Sharpe ratios of the ex post tangency portfolios. Our results follow from the empirical finding that downside volatility-managed portfolios are not spanned by the original portfolios or volatility-managed portfolios. Whereas downside volatility-managed portfolios expand the investment opportunity set, upside volatility-managed portfolios do not.

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Downside volatility and volatility typically comove but are not highly correlated during the most volatile times. We show that portfolios scaled by downside volatility expand the ex post mean-variance frontiers constructed using the original portfolios and volatility-managed portfolios, and improve the Sharpe ratios of the ex post tangency portfolios. Our results follow from the empirical finding that downside volatility-managed portfolios are not spanned by the original portfolios or volatility-managed portfolios. Whereas downside volatility-managed portfolios expand the investment opportunity set, upside volatility-managed portfolios do not.

JEL classification: G11, G12

Keywords: volatility, portfolio management, downside volatility, asymmetry, mean variance

Volatility is of central importance in finance. Since the groundbreaking work of Markowitz (1952) on modern portfolio theory, both academics and practitioners have focused on volatility as a key measure of risk. Academics have documented empirical facts about volatility and proposed models to capture them. Practitioners track the volatility of investment strategies and use market volatility as a gauge for investor sentiment. Volatility has been the topic of countless academic studies and industry reports.

In calculating volatility, positive and negative deviations from the unconditional mean are treated equally. When volatility is high, the investor is equally likely to experience a large upside move as a large downside move. However, investors may not view upside and downside moves as equally risky. Whereas a 10% sharp increase in the market may call for celebration, a 10% steep fall will cause serious agony.

Unlike volatility, which computes standard deviation using all returns, downside volatility is the standard deviation of returns below a threshold. In contrast to the popularity of volatility, downside volatility is a risk measure that has received less attention. Because downside volatility and volatility do not always move in lockstep, tracking both gives the risk manager additional information to make improved decisions. Monitoring volatility and downside volatility allows the investor to differentiate between total risk and downside risk, helping her make more informed investment decisions.

We measure downside volatility as the deviation from the unconditional average returns, rather than from the conditional mean of only downside observations. Our construction provides a simple decomposition: total variance is a weighted average of downside and upside variances. The weights are proportional to the number of downside and upside observations.

Equity return distributions tend to be symmetric, so downside volatility and volatility are generally highly correlated. For eight long-short factors and 49 long-only industry portfolios, the unconditional correlation between downside volatility and volatility is around 90%. Indeed, during economic downturns such as the Great Depression or the Great Financial Crisis, downside volatility and volatility are both extremely high.

At their extremes, volatility and downside volatility can be quite different. When both downside volatility and volatility are elevated, their correlation is considerably lower. The tail correlations between downside volatility and volatility during the historically most volatile 12 months range between -50% and 95%, with most values falling between 20% and 60%. Pooled regressions show that a 1% change in volatility in the most volatile months is associated with only 0.4% to 0.5% increase in downside volatility. Times of extreme volatility are precisely when monitoring both downside volatility and volatility become the most valuable, and when these two measures markedly differ.

Volatility is often used as an input in portfolio management. Moreira and Muir (2017) show that volatility-managed portfolios produce large alphas and have higher Sharpe ratios compared to the original portfolios. They scale well-known factors such as value or momentum by the inverse of their conditional volatilities, and show that these portfolios expand the investor's opportunity set.

We explore the behavior of downside volatility-managed portfolios, which scales return series by their conditional downside volatilities, and find that they expand the investor's opportunity set. Downside volatility-managed factors and industry portfolios compare favorably to their original counterparts. Regressions of downside volatility-managed factors and industry portfolios against the original series result in economically large alphas and low explanatory power. Scaling returns by their conditional downside volatility introduces diversified new return series that are not spanned by the original ones.

We compare downside volatility-managed portfolios to volatility-managed portfolios of Moreira and Muir (2017), and find that our portfolios add value beyond those proposed by Moreira and Muir (2017). In regressions of downside volatility-managed portfolios against volatility-managed portfolios, six of the eight factors have economically large alphas. 39 of the 49 industry portfolios have positive alphas, and 23 of the 49 have alphas exceeding 1% per year.

We also compare downside volatility-managed portfolios and Moreira and Muir (2017) volatility-managed portfolios in a mean-variance framework. While combining volatility-managed portfolios with the original ones leads to higher Sharpe ratios for the ex post tangency portfolio, adding downside volatility-managed further increases the Sharpe ratio. The tangency portfolio formed using the original eight factors has a Sharpe ratio of 1.41. Combining volatility-managed factors and the original eight, the new ex post tangency portfolio has a Sharpe ratio of 1.84. When we add downside volatility-managed portfolios, the Sharpe ratio increases to 2.04. The Shape ratio of the tangency portfolio formed using 49 industry portfolios is 1.28. Adding volatility-managed portfolios improves the Sharpe ratio to 2.28. Adding downside volatility-managed portfolios to the mix raises the Sharpe ratio to 3.22. The Sharpe ratio improvements are statistically significant, with p-values from Ledoit and Wolf (2008) tests near zero. While the investor may not be able to fully realize these Sharpe ratio gains, comparing ex post tangency portfolio characteristics illustrates how investors may expand their opportunity set if they were to include downside volatility-managed portfolios among their choices.

Downside volatility-managed portfolios expand the investment opportunity set, but upside volatility-managed portfolios do not. Upside volatility-managed portfolios do not contain additional information beyond volatility-managed portfolios. In spanning tests, six of the eight upside volatility-managed factors have negative alphas against volatility-managed factors, and 43 of the 49 upside volatility-managed industry portfolios have negative alphas against volatility-managed industry portfolios.

Our results are not driven by our definition of downside volatility. Markowitz (1959) proposed using semi-standard deviation, calculated as the squared deviation from the conditional mean of only negative returns, as a measure of downside risk. We find downside volatility-managed portfolios using the Markowitz (1959) definition also add value to investors by expanding their investment opportunity set: volatility-managed portfolios do not span downside volatility-managed portfolios using semi-standard deviation.

The distinction between upside and downside risk has been made at least since Markowitz (1959), who recommends semi-variance as a replacement for variance as a measure of risk on the grounds it is realistically superior. Among others, Hogan and Warren (1974), Bawa and Lindenberg (1977), and Harlow and Rao (1989) expand this idea into equilibrium asset pricing frameworks. Compared to these papers, our work focuses more on the portfolio management implication of downside volatility rather than a comprehensive asset pricing framework.

The paper closest to our is Moreira and Muir (2017), who document that volatility-managed factors can improve the investor's opportunity set when compared to the original factors. The key difference between Moreira and Muir (2017) and our approach is the variable used to determine portfolio exposure. Moreira and Muir (2017) use volatility, whereas we argue that due to investors' asymmetric preferences for upside versus downside risk, downside volatility could also be useful for portfolio management. The resulting portfolio returns using downside volatility differ from those using volatility and have attractive risk-return properties. We find that downside volatility-managed portfolios can add value beyond volatility-managed portfolios, because downside volatility and volatility are not always highly correlated.

The paper is organized as follows. Section 1 introduces downside volatility and compares it to volatility. In section 2, we scale portfolios by their conditional downside volatilities and measure them against the original portfolios and volatility-managed portfolios. Section 3 shows how downside volatility-managed portfolios can expand the investor's ex post efficient frontier. Section 4 shows that unlike downside volatility, upside volatility is not useful in expanding the investment opportunity set. Section 5 concludes.

1 Downside Volatility

1.1 A Variance Decomposition

Variance of a return series can be calculated by the well-known formula:

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 \tag{1}$$

Where N is the number of observations, x_i are individual observations, and $\bar{x}$ is the unconditional average of all observations. $N-1$ may be used in place of N for a bias correction, but in large samples this distinction is minor.

Downside deviation is often calculated using returns smaller than some target level. We take the average return to be the target level. Define downside and upside variance as the following:

$$\hat{\sigma}_d^2 = \frac{1}{N_d}\sum_{i=1}^{N}(x_i - \bar{x})^2\,\mathbb{1}_{x_i \le \bar{x}}, \quad \hat{\sigma}_u^2 = \frac{1}{N_u}\sum_{i=1}^{N}(x_i - \bar{x})^2\,\mathbb{1}_{x_i > \bar{x}} \tag{2}$$

Where N_d is the number of observations in which $x_i \le \bar{x}$, and N_u is the number of observations in which $x_i > \bar{x}$. $N = N_d + N_u$. $\mathbb{1}_{x_i \le \bar{x}}$ and $\mathbb{1}_{x_i > \bar{x}}$ are indicator function for observations less than (or equal to) and greater than the sample mean.

We can rewrite Equation (1) as the following:

$$\hat{\sigma}^2 = \frac{N_d}{N}\hat{\sigma}_d^2 + \frac{N_u}{N}\hat{\sigma}_u^2$$

Therefore, overall variance is a weighted sum of downside variance and upside variance. Volatility, downside volatility, and upside volatility are obtained by taking square root of variances.

1.2 Data

For our analysis, we use daily and monthly returns for Fama and French. (1992, 2015) factors and characteristic-sorted portfolios from Ken French's website[1]. We obtain daily and monthly returns on the Fama and French. (1992) factors, as well as short-term and long-term reversals, from July 1926 through January 2018, and Fama and French. (2015) factors from July 1963 to January 2018. We also use daily and monthly returns of 49 industry portfolios from French's website.[2]

1.3 Downside Volatility vs. Volatility

We follow Moreira and Muir (2017) and construct monthly volatility and downside volatility using daily returns. For each month from July 1926 to January 2018, we use (1) and (2) to calculate the volatility measures. For comparison, we also compute upside volatility using Equation (2). We calculate volatility measures for long-short factors including market excess returns (RMRF), size (SMB), value (HML), momentum (MOM), profitability (RMW), investments (CMA), short-term reversal (STRev), and long-term reversal (LTRev). We also calculate volatilities for 49 industry portfolios.[3]

Exhibit 1, in the top panel, shows unconditional correlations between volatility and downside volatility. In the bottom panel, we present tail correlations. For each return series, we calculate tail correlations using the 12 months with the highest volatility. The tail correlations illustrate how closely volatility and downside volatility comove during the most

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[2] See Ken French's website for industry definitions.

[3] In robustness checks, we consider alternative horizons to compute volatility measures. Our results are qualitatively unchanged if we use the previous two months, three months, six months, or 12 months to calculate volatility.

volatile times.

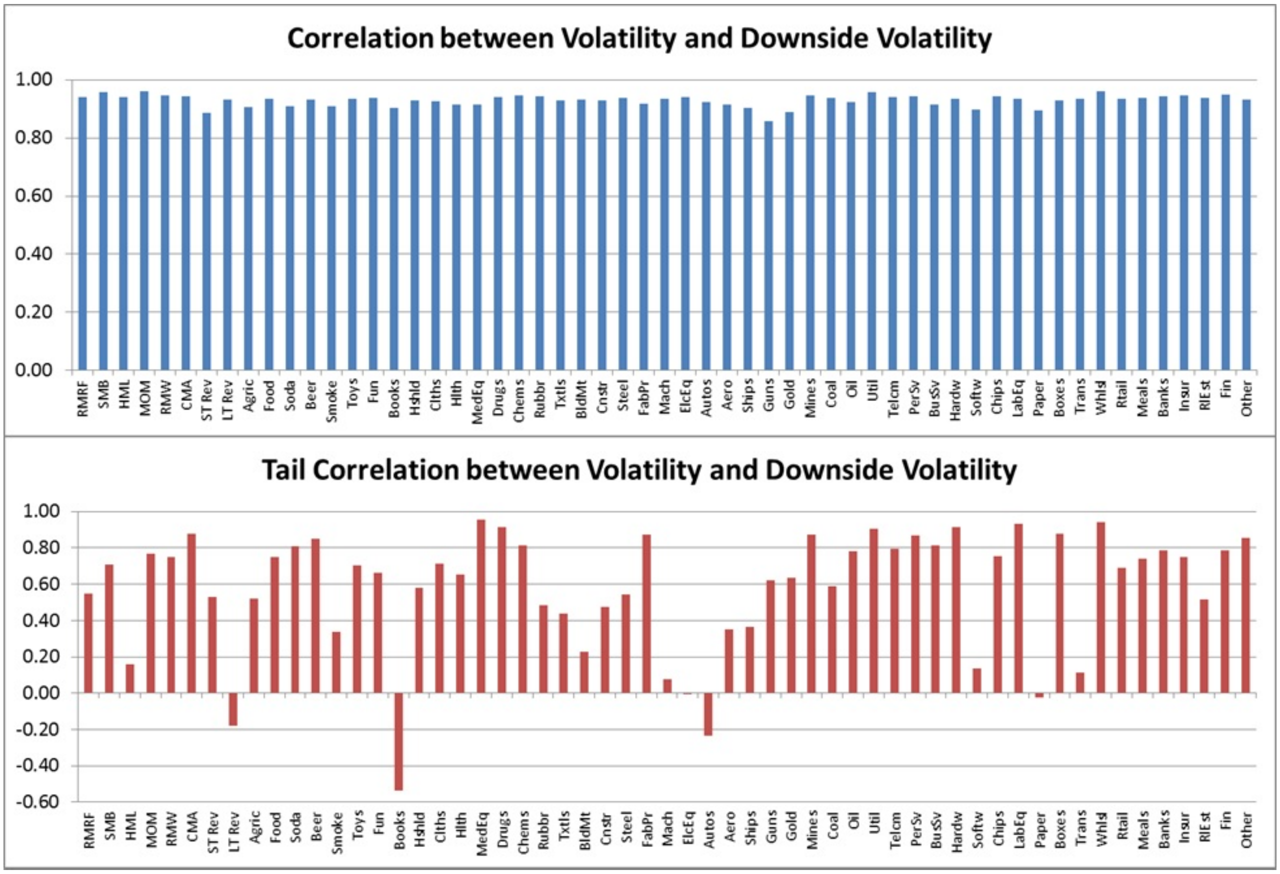

Exhibit 1: **Correlations and Tail Correlations between Volatility and Downside Volatility.**
We compute correlations between volatility and downside volatility for eight factors and 49 industry portfolios. We also report tail correlations, calculated using the 12 most volatile months of each return series.

Across eight factors and 49 industry portfolios, we find that volatility and downside volatility tend to move together. The average unconditional correlation between the two measures is 93%. On average, downside volatility tends to be high when volatility is high.

In the most volatile months, volatility and downside volatility do not necessarily move together. For market excess returns, the unconditional correlation between volatility and downside volatility is 94%. During the most volatile 12 months, the conditional correlation is only 55%. Although volatility and downside volatility tend to comove on average, they may be meaningfully different during the most volatile episodes. The average tail correlation across factors and industry portfolios is only 58%.

Tail correlations are difficult to estimate due to limited data. We compute tail correlations using 12 months in our attempt to capture the tail, at the cost of increased sampling variation. To alleviate this problem, we consider pooling tail measurements across all return series to understand the average relationship between volatility and downside volatility in the tails. For each return series, we collect volatility measures from its most volatile 12 months. We then combine all measures across 8 factors and 49 industry portfolios.

Exhibit 2 presents a scatterplot of downside volatility against volatility in the most volatile months for all portfolios. The line of best fit for the pooled sample shows a regression coefficient of 0.48 and an R-squared of 57%. If volatility and downside volatility were closely related, we would expect a coefficient near 1.0, and a line of best fit to be close to the dashed 45-degree line. Instead, the data show if the volatility in the most volatile months is 1% higher, the downside volatility measured in the same month is likely to be only 0.48% higher; the majority of points on the graph lie below the 45-degree line.

To understand the relationship between volatility and downside volatility in Exhibit 2 more formally, we run pooled regressions of downside volatility on volatility, during the most volatile months:

$$\sigma_{d,t}^i = p\sigma_t^i + \text{Fixed Effects} + \eta_{d,t}^i \tag{3}$$

Where $\sigma_{d,t}^i$ and σ_t^i are the downside volatility and volatility of portfolio i in month t. p is the sensitivity of downside volatility to volatility. $p = 1$ implies volatility and downside volatility perfectly comove in the most volatile months. We include fixed effects to capture any latent differences in downside volatility across time or portfolios not due to differences in volatility. We cluster standard errors by time to allow for arbitrary correlations across portfolios at each point in time.

Exhibit 3 includes the pooled regression results. Column (1) shows a pooled time-series cross-sectional regression of downside volatility on volatility. We get a coefficient of 0.48 with a standard error of 0.05; we can reject $p = 1$ at conventional confidence levels or 5% or 1%. The estimate for p does not change much to the inclusion of portfolio fixed effects (Column (2)), time fixed effects (Column (3)), or both (Column (4)). In each specification, the estimate is between 0.4 and 0.5 and the null hypothesis that $p = 1$ is rejected.

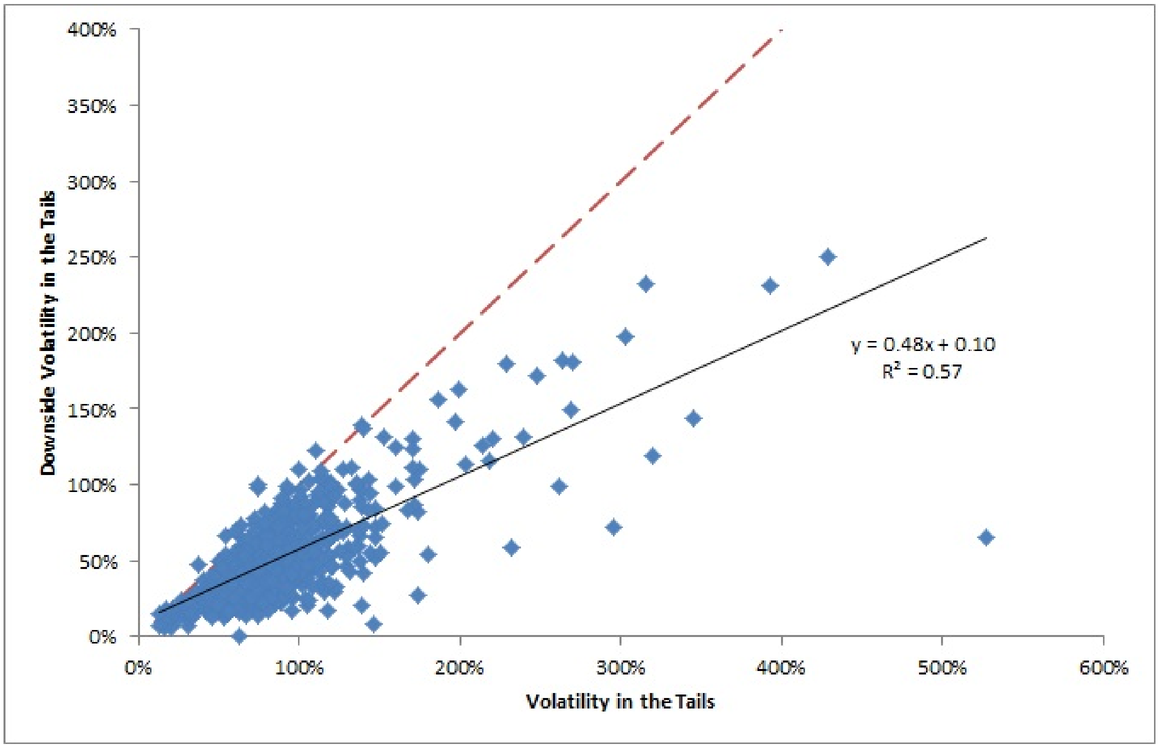

Exhibit 2: **Scatterplot of Volatility and Downside Volatility in the Tails.**
We select volatility and downside volatility measures from the most volatile 12 months of each return series. We then pool these measures across 8 factors and 49 industry portfolios to create the scatterplot. Monthly volatility measures are annualized by multiplying by square root of 12. The solid line is the line of best fit. A 45-degree dashed line is shown.

Above evidence indicates volatility and downside volatility are not the same measure of risk. One criticism of downside volatility is that return distributions tend to be symmetric, so downside volatility does not provide additional information but uses fewer data points in estimation. Our findings show that volatility does not subsume the information in downside volatility. Tracking both quantities may provide a better understanding of the risk profile of a strategy.

Left-Hand Variable = $\sigma^i_{d,t}$				
	(1)	(2)	(3)	(4)
p	0.48	0.50	0.46	0.41
	[0.05]	[0.06]	[0.04]	[0.08]
Nobs	684	684	684	684
Portfolio FE	N	Y	N	Y
Time FE	N	N	Y	Y
R^2	0.57	0.61	0.58	0.62

Exhibit 3: Panel Regressions of Downside Volatility on Volatility.
We select downside volatility and volatility in the most volatile 12 months for each portfolio, and run pooled regressions: $\sigma^i_{d,t} = p\sigma^i_t + \text{Fixed Effects} + \eta^i_{d,t}$. Standard errors clustered by time are shown in square brackets.

2 Downside Volatility-Managed Portfolios

Monitoring volatility and downside volatility gives the investor metrics to measure portfolio risk. Are these quantities useful in portfolio management? To answer this question, we explore changing portfolio exposure based on conditional downside volatility. We construct downside volatility-managed portfolios as follows:

$$f^d_{t+1} = \frac{\gamma}{\hat{\sigma}^2_{d,t}} f_{t+1} \tag{4}$$

f^d_{t+1}, the downside volatility-managed portfolio at time $t+1$, is given as the original portfolio f_{t+1} scaled by last month's downside variance. The constant γ is set such that f^d_{t+1} has the same unconditional standard deviation as f_{t+1}. Our construction follows Moreira and Muir (2017), who propose volatility-managed portfolios:

$$f^{MM}_{t+1} = \frac{c}{\hat{\sigma}^2_t} f_{t+1} \tag{5}$$

Where c is chosen such that f^{MM}_{t+1} and f_{t+1} have the same unconditional standard deviation. The only difference between volatility-managed portfolio f^{MM}_{t+1} and downside volatility-managed portfolio f^d_{t+1} is whether conditional volatility or downside volatility is used to control strategy exposure.

We use spanning tests to study whether downside volatility-managed portfolios improve the investor's opportunity set. We run time-series regressions of downside volatility-managed factor returns on the original factors.

$$f^d_{t+1} = \alpha + \beta f_{t+1} + \epsilon^d_{t+1} \tag{6}$$

In this regression, $\alpha = 0$ indicates downside volatility-managed factors are spanned by the original factors. That is, managing factor exposure using downside volatility cannot improve the investor's risk-return tradeoff. A positive α suggests that the downside volatility-managed factor cannot be spanned by the original factor, and adds additional value for the investor: the investor can obtain superior risk-adjusted returns if she were to invest in some combination of f_{t+1} and f^d_{t+1} rather than 100% in f_{t+1}.

β in the regression measures the comovement between downside volatility-managed factors and the original factors. $\beta = 1$ implies that the downside volatility-managed factor moves one-for-one with the original factor, whereas a positive β less than one means the downside volatility-managed factor moves in the same direction as the original factor, but these moves are smaller.

Exhibit 4 presents the regression results. We observe positive intercepts for six of the eight factors. The exceptions are CMA and $STRev$, which have negative intercepts that are economically and statistically small. Four of the eight intercepts are economically and statistically large, ranging between 3.6% to 8.3% per year. β, the slope coefficient in the regression, is smaller than one for all factors, indicating downside volatility-managed factors do not comove one-for-one with the original factors. Furthermore, R^2 is small for all regressions. These results indicate that downside volatility-managed factors do not share much common comovement with the original factors. Instead, they provide diversifying returns not spanned by the original long-short factors.[4]

Moreira and Muir (2017) show that volatility-managed factors are not spanned by the original factors. They consider some of the same factors we consider, including $RMRF$,

[4]The intercepts are economically and statistically large even if we include Fama and French (1992, 2015) factors. Results are available upon request.

	Left-Hand Variable							
	$RMRF^d$	SMB^d	HML^d	MOM^d	RMW^d	CMA^d	$STRev^d$	$LTRev^d$
α	6.0%	1.6%	4.2%	8.3%	3.6%	-0.8%	-1.5%	0.9%
	(3.5)	(1.5)	(3.8)	(5.0)	(3.8)	(-0.8)	(-1.2)	(0.7)
$RMRF$	0.47							
	(17.5)							
SMB		0.40						
		(14.4)						
HML			0.51					
			(19.4)					
MOM				0.26				
				(8.9)				
RMW					0.45			
					(13.0)			
CMA						0.19		
						(4.9)		
$STRev$							0.07	
							(2.3)	
$LTRev$								0.41
								(14.3)
R^2	0.22	0.16	0.26	0.07	0.21	0.04	0.00	0.16

Exhibit 4: Univariate regressions of downside volatility-managed portfolios on the original factors.
We regress downside volatility-managed factors onto the original factors: $f^d_{t+1} = \alpha + \beta f_{t+1} + \epsilon^d_{t+1}$ Each column is a separate regression. The data are monthly and the sample period is 1926 to 2018 for $RMRF$, SMB, HML, MOM, and $STRev$; 1931 to 2018 for for $LTRev$; 1963 to 2018 for RMW and CMA.

SMB, HML, MOM, RMW, and CMA. One may wonder if our results in Exhibit 4 are a product of the results in Moreira and Muir (2017) and the fact that downside volatility is generally highly correlated with volatility.

To understand the difference between our findings and those in Moreira and Muir (2017), we run time-series regressions of downside volatility-managed portfolios on volatility-managed portfolios:

$$f_{t+1}^{d} = a + b f_{t+1}^{MM} + \eta_{t+1}^{d} \tag{7}$$

Where f_{t+1}^{MM} is the volatility-managed portfolio of Moreira and Muir (2017). Similar to the interpretation of Equation (6), a positive a suggests that the downside volatility-managed portfolio cannot be spanned by the volatility-managed portfolio.

Exhibit 5 shows the results for Equation (7). Four of the eight factors have statistically large intercepts a ranging from 1.9% to 2.7% per year. Six intercepts are positive, and the two negative ones - CMA and $STRev$ - are statistically small. Regression slope, b, is always less than one. R^2 is typically higher than those in Exhibit 4, since volatility and downside volatility are positively correlated so portfolios based on them are also positively correlated.

We also investigate downside volatility-managed industry portfolios using spanning tests in Equation (6) and Equation (7). For both regressions, we subtract off the one-month Treasury bill returns from both the independent and dependent variables, so the regression intercept has the interpretation of risk premium not captured by the right-hand variable. We report the intercepts of Equation (6) and their t-statistics in Exhibit 6.

As the case for factor portfolios, downside volatility-managed industry portfolios are not spanned by the original industry portfolios. 48 of 49 portfolios have positive intercepts and only one has a small negative intercept. 36 industry portfolios have annualized intercepts of over 3%, economically large out-performance relative to the original portfolios after adjust for their exposures. 29 of the 49 intercepts are statistically significant.

Exhibit 7 shows spanning test results for downside volatility-managed industry portfolio on volatility-managed industry portfolios. 19 downside volatility-managed portfolios have annualized intercepts of 2% or larger, marking considerable improvements relative to the volatility-managed portfolios. 10 of the 49 regressions have negative intercepts, indicating

	Left-Hand Variable							
	$RMRF^d$	SMB^d	HML^d	MOM^d	RMW^d	CMA^d	$STRev^d$	$LTRev^d$
a	1.6%	2.0%	2.7%	1.6%	1.9%	-0.8%	-1.2%	1.9%
	(1.4)	(2.2)	(4.1)	(1.1)	(2.7)	(-0.9)	(-0.9)	(2.1)
$RMRF^{MM}$	0.81							
	(45.2)							
SMB^{MM}		0.62						
		(25.9)						
HML^{MM}			0.85					
			(53.9)					
MOM^{MM}				0.55				
				(21.6)				
RMW^{MM}					0.74			
					(28.5)			
CMA^{MM}						0.27		
						(7.1)		
$STRev^{MM}$							0.04	
							(1.4)	
$LTRev^{MM}$								0.73
								(34.9)
R^2	0.65	0.38	0.73	0.30	0.55	0.07	0.00	0.54

Exhibit 5: Univariate regressions of downside volatility-managed portfolios on volatility-managed factors.

We regress downside volatility-managed factors onto volatility-managed factors of Moreira and Muir (2017): $f_{t+1}^d = a + bf_{t+1}^{MM} + \eta_{t+1}^d$ Each column is a separate regression. The data are monthly and the sample period is 1926 to 2018 for $RMRF$, SMB, HML, MOM, and $STRev$; 1931 to 2018 for for $LTRev$; 1963 to 2018 for RMW and CMA.

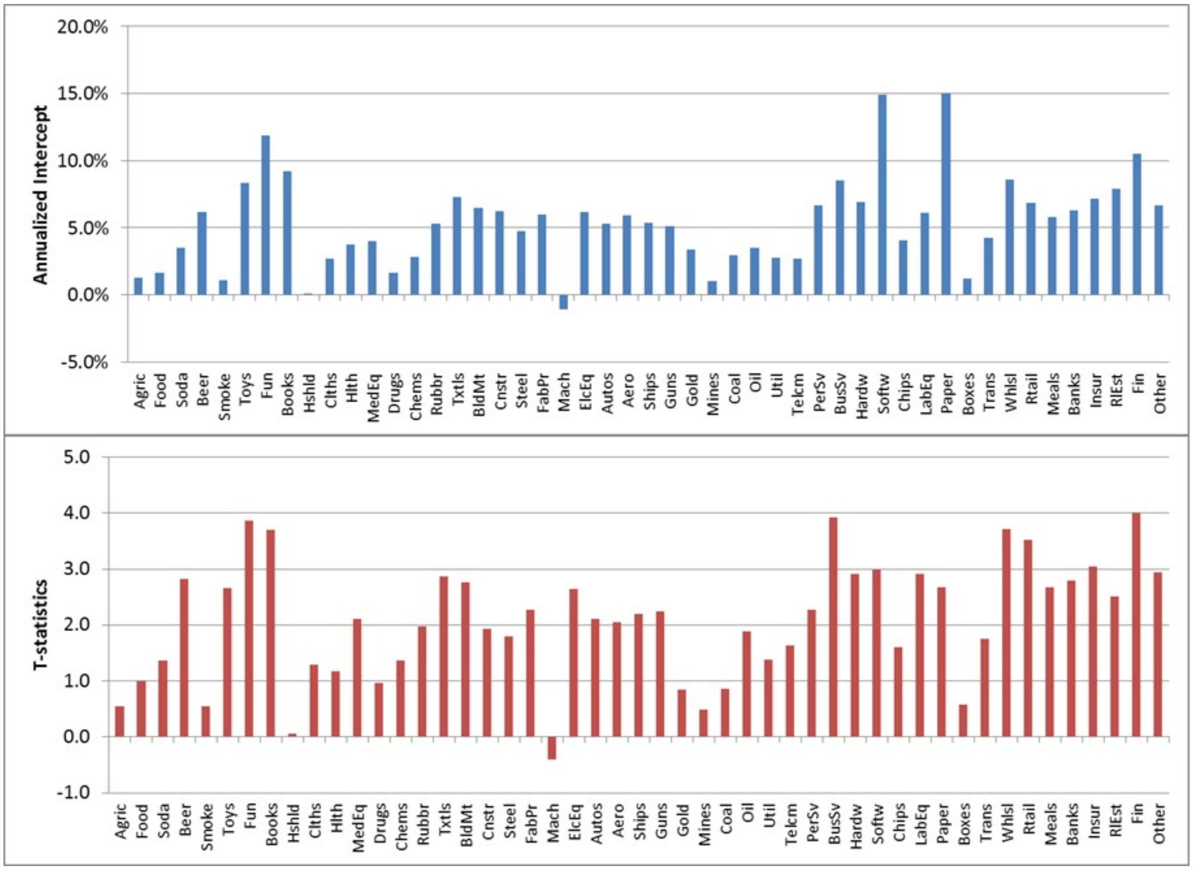

Exhibit 6: **Comparison of Downside Volatility-Managed Industry Portfolios and Original Industry Portfolios.**
We regress excess returns of downside volatility-managed industry portfolios onto the excess returns of the original industry portfolios: $f^d_{t+1} = \alpha + \beta f_{t+1} + \epsilon^d_{t+1}$. Annualized intercepts and t-statistics are shown. The data are monthly and available from 1926 to 2018.

that volatility-managed portfolios span the downside volatility-managed portfolios for those industries. Seven positive intercepts are statistically significant, whereas none of the negative intercepts is statistically significant. Viewed as a whole, downside volatility-managed industry portfolios expand the investment opportunity set compared to volatility-managed portfolios, although there is variation across industries.

The results in this section were produced using the definition of downside volatility from Section 1.1. We examine how the results change using an alternative definition of downside volatility, calculated as the deviation from the conditional mean of only negative returns (Markowitz, 1959). Our results hold under this alternative definition. In regressions of downside volatility-managed portfolios on volatility-managed portfolios, four of the eight

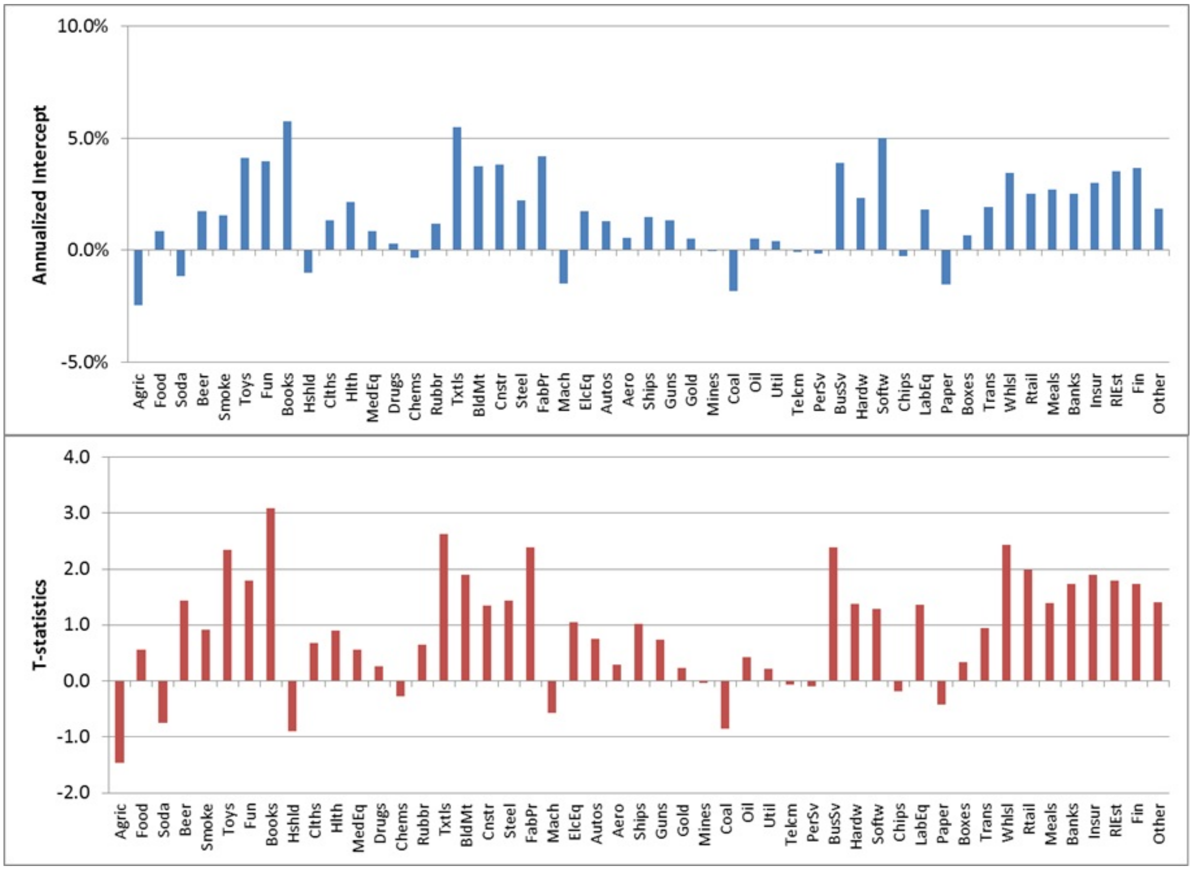

Exhibit 7: **Comparison of Downside Volatility-Managed Industry Portfolios and Volatility Managed Portfolios of Moreira and Muir (2017).**
We regress excess returns of downside volatility-managed industry portfolios onto the excess returns of volatility-managed industry portfolios of Moreira and Muir (2017): $f_{t+1}^{d} = a + bf_{t+1}^{MM} + \eta_{t+1}^{d}$ The data are monthly and available from 1926 to 2018.

factors have large and positive intercepts. None of the intercepts is statistically negative. For industry portfolios, 14 have annualized intercepts greater than 2%, whereas there is no large negative intercepts.

3 Expanding the Ex Post Efficient Frontier

The previous section documents that downside volatility-managed portfolios can expand the investor's opportunity set when compared to the original portfolios or volatility-managed portfolios. The comparisons were all in a univariate setting, between two individual return series. In practice, the investor may invest in more than one factor or more than one industry portfolio, so it is important to understand the overall change to her investment opportunity set. We investigate the extent to which downside volatility-managed portfolios expand the investor's ex post efficient frontier.

Given a set of excess returns μ and variance-covariance matrix Σ, the Sharpe ratio is given as $\frac{w'\mu}{w'\Sigma w}$ where w is a vector of portfolio weights summing up to one. The ex post tangency portfolio, the portfolio with the highest in-sample Sharpe ratio, captures the investment opportunity set for market participants. Its weights are given by the following expression:

$$w^{Tang} = \frac{\Sigma^{-1}\mu}{1\Sigma^{-1}\mu} \tag{8}$$

Where 1 is a conforming vector of ones. Using the ex post tangency portfolio weights, it is easy to show that the ex post tangency portfolio has the following Sharpe ratio (Campbell et al., 1997):

$$SR^{Tang} = \sqrt{\mu'\Sigma^{-1}\mu} \tag{9}$$

We use the above expression to evaluate the expansion of the ex post mean-variance efficient frontier. For each factor or industry portfolio, we estimate Σ and μ using the full sample. We compare the Sharpe ratio of the ex post tangency portfolio formed in three different ways: 1) the original set of portfolios, 2) the original portfolios and volatility-managed portfolios, and 3) the original portfolios, volatility-managed portfolios, and downside volatility-managed portfolios. We present our findings in Exhibit 8.

Consider the Fama and French. (1992) factors $RMRF$, SMB, and HML. The ex post tangency portfolio [1] formed using these three factors has a Sharpe ratio of 0.52. These factors, plus their volatility-managed counterparts, form a tangency portfolio [2] with a Sharpe ratio of 0.71. Including downside volatility-managed factors, the Sharpe ratio of the tangency portfolio [3] rises to 0.87. We formally test for equal Sharpe ratios using the approach of Ledoit and Wolf (2008), which accounts for both heteroscedasticity and heavy tails. We can reject the null of equal Sharpe ratios for the tangency portfolio formed excluding downside volatility-managed portfolios [2] and including downside volatility-managed portfolios

[3], indicating that downside volatility-managed portfolios significantly improves the Sharpe ratio of the tangency portfolio. We can also reject equal Sharpe ratios for excluding and including volatility-managed portfolios ([1] and [2]).

Downside volatility-managed portfolios expand the investment opportunity set formed by both the original and volatility-managed portfolios. The improvements in Sharpe ratio are statistically significant. This pattern holds for different combinations of factors, such as the Carhart (1997) four factors, reversals, the Fama and French. (2015) five factors, and all eight factors.

Portfolio weights in Equation (8) are not constrained to be positive, so the Sharpe ratios in Exhibit 8 could be implied by negative weights on the constituent factor portfolios. This raises the question of implementation costs associated with short position on the underlying factors. Because volatility-managed factors and downside volatility-managed factors are simply scaled versions of the original factors, they have the same underlying stocks, just with potentially different positions. A portfolio which combines the original factors, volatility-managed factors, and downside volatility-managed factors should not be more difficult to trade than the original factors, as the positions on the underlying stocks can be netted. It is possible that after netting the positions, some stocks with long positions in the original factor portfolios receive short positions, and vice versa. Presumably, investors who have enough resources to trade the original long-short factors can construct the new portfolio without much increased slippage.

Sharpe ratio improvements in Exhibit 8 do not necessarily come from short positions in the underlying factor portfolios. The implied short positions are typically modest, smaller than -20% in magnitude, with the exception of SMB and HML's volatility-managed versions in the tangency portfolio of FF3F (-31% and -37%). To understand the effect of short factor positions on Sharpe ratios, we convert the negative positions in Equation (8) to zero and normalize the remaining weights to sum to one. The resulting portfolios including downside volatility-managed factors still show large improvement in Sharpe ratios compared to portfolios excluding these factors. We could also solve a formal portfolio optimization problem maximizing the Sharpe ratio with a non-negative constraint on portfolio weights. The results are similar to simply converting negative weights in Equation (8) to zero because there are only a few small negative positions implied by Equation (8). In fact, downside volatility-managed portfolios provide such strong diversification that equal-weight portfolios

	FF3F	FF3F +Mom	FF5F	FF5F +Mom	FF3F +Revs	FF5F +Revs	FF5F +Mom+Revs
[1] f	0.52	0.98	1.11	1.28	0.90	1.19	1.41
[2] f and f^{MM}	0.71	1.27	1.38	1.71	1.03	1.46	1.84
[3] f, f^{MM}, and f^d	0.87	1.38	1.61	1.91	1.26	1.71	2.08
LW Test for [1] and [2]	0.01	0.00	0.00	0.00	0.02	0.00	0.00
LW Test for [2] and [3]	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Exhibit 8: Sharpe Ratio of Ex Post Tangency Portfolios.
We compare ex post tangency portfolios formed using the original portfolios, volatility-managed portfolios of Moreira and Muir (2017), and downside volatility-managed portfolios. $SR^{Tang} = \sqrt{\mu' \Sigma^{-1} \mu}$ where Σ is the variance-covariance matrix and μ is a vector of mean returns. FF3F consists of $RMRF$, SMB, and HML; FF5F consists of $RMRF$, SMB, HML, CMA, and RMW; Revs contains $STRev$ and $LTRev$. We use the Ledoit and Wolf (2008) approach to test the equality of Sharpe ratios between pairs of portfolios; the null hypothesis is equal Sharpe ratios. P-values are shown in the bottom two rows.

of the original, volatility-managed, and downside volatility-managed factors have higher Sharpe ratios compared to equal-weight portfolios using the original and volatility-managed factors.

Sharpe ratio improvements Exhibit 8 are not driven by increased tail risk. Goetzmann et al. (2007) argue that portfolio managers can manipulate their performance by adjusting tail risk characteristics of the portfolio to optimize the Sharpe ratio. We investigate the tangency portfolios formed using volatility-managed and downside volatility-managed factors, and we find that their tail risks do not appear to be higher compared to the tangency portfolios using the original factors. Tail risk measures including minimum daily return, 1% Value at Risk (VaR), and maximum drawdown for tangency portfolios including downside volatility-managed portfolios [3] in Exhibit 8 are actually smaller than those for the original portfolios, [1]. Therefore, our results are unlikely to be driven by increased tail risk.

We compare the ex post mean-variance frontiers of different factor combinations in Exhibit 9. We solve the portfolio problem of Markowitz (1952): for different levels of portfolio expected returns, minimize the standard deviation for the portfolio. The mean-variance frontier is formed as a combination of many of these individual data points. The solid line traces out the mean-variance frontier associated with the eight factor portfolios. We include the eight volatility-managed factors to construct the frontier shown in a dashed line. We then add the eight downside volatility-managed factors to construct the dash-dotted frontier.

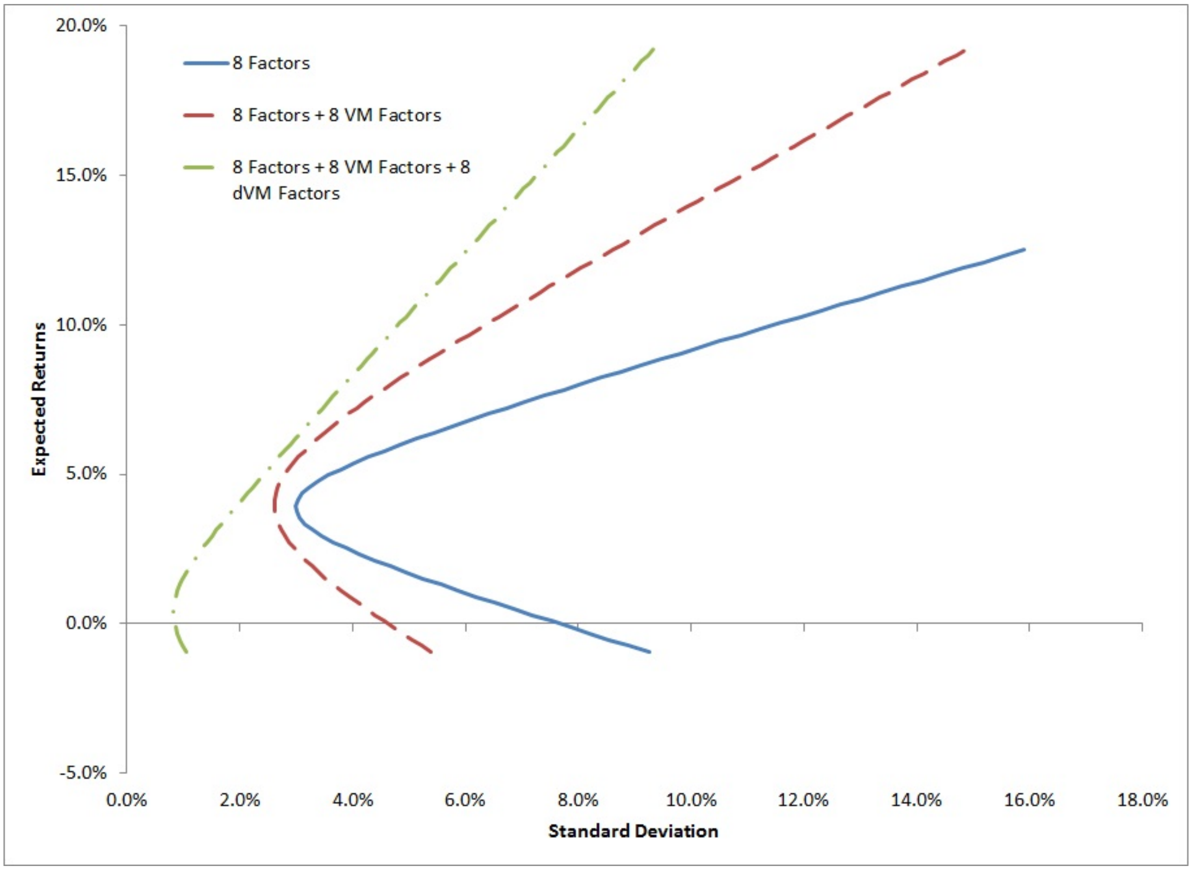

Exhibit 9: **Expansion of the Ex Post Mean-Variance Frontier For Factors.** We construct the ex post mean-variance frontier of eight long-short factor portfolios (solid line): market excess returns, size, value, momentum, profitability, investment, short-term reversal, and long-term reversal. The dashed line traces out the ex post mean-variance frontier of the eight original factors and their volatility-managed counterparts. The dash-dotted line is the ex post mean-variance frontier of the above 16 portfolios plus the eight downside volatility-managed factors.

In a graph of expected returns against standard deviation, each point represents the risk-return characteristics of a specific return stream. For an asset A, the area to the northeast has both lower volatility and higher expected returns. An asset B that lies in this area has a more attractive risk-return tradeoff than asset A. If the investor gains access to asset B, her investment opportunity is improved. Adding volatility-managed factors to the eight original factors improves the investor's opportunity set, as the dashed mean-variance frontier is to the northeast of the solid one. Adding downside volatility-managed factors further expands the investment opportunity set, since the dash-dotted frontier is to the northeast of both the dashed and solid ones.

Exhibit 10 shows the ex post mean-variance efficient frontiers for different industry portfolio combinations. The frontier shown in solid line is associated with the original 49 portfolios; the dashed one is constructed by including volatility-managed industry portfolios, and the dash-dotted frontier by further including downside volatility-managed portfolios. Similar to our findings for factors, while volatility-managed portfolios expand the investment opportunity set compared to the original 49 portfolios, downside volatility-managed industry portfolios provide further expansion to lower-volatility, higher-return regions.

Downside volatility-managed portfolios expand the investor's opportunity set compared to the original portfolios and volatility-managed portfolios of Moreira and Muir (2017). Adding downside volatility-managed portfolios result in tangency portfolios with higher Sharpe ratios. Our findings in this section corroborate our earlier results that downside volatility-managed portfolios cannot be spanned by the original portfolios or volatility-managed portfolios. In both univariate and multivariate settings, downside volatility-managed portfolios provide value to the investor by expanding her investment opportunity set.

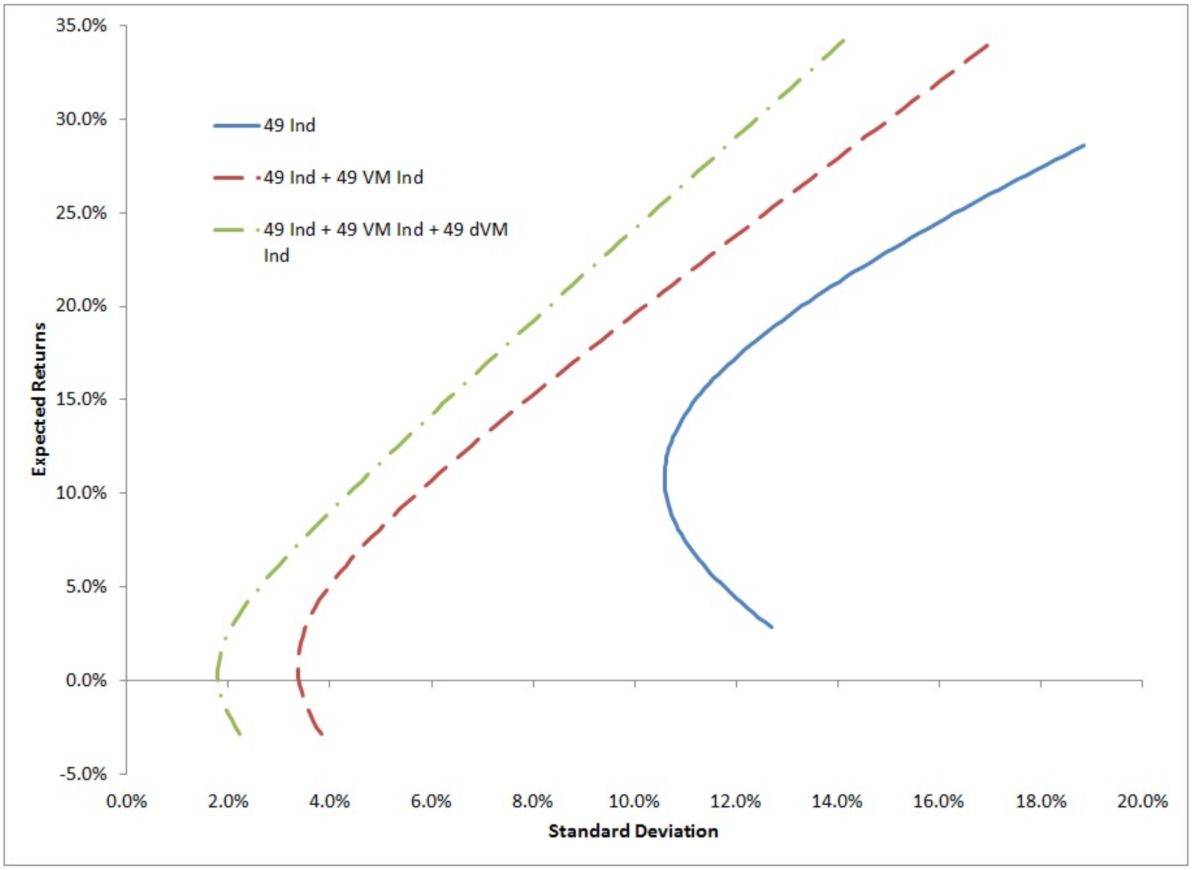

Exhibit 10: **Expansion of the Ex Post Mean-Variance Frontier for 49 Industry Portfolios.**
We compare the ex post mean-variance frontier of 49 industry portfolios (solid line), 49 industry portfolios plus volatility-managed industry portfolios (dashed line), and the above 98 portfolios plus 49 downside volatility-managed portfolios (dash-dotted line).

4 Upside Volatility versus Downside Volatility

One potential criticism of our findings is that they may be mechanical. We have simply found a measure imperfectly correlated to volatility, so naturally managing portfolio exposure using this measure produces returns that are not perfectly correlated with volatility-managed portfolios. Including imperfectly correlated returns can mechanically improve the risk-return tradeoff of a set of portfolios.

To address this concern, we investigate using upside volatility to managed portfolio exposure. Upside volatility is also not perfectly correlated with volatility. If we simply included imperfectly correlated returns to expand the investment opportunity set, upside volatility-managed portfolios should behave similarly to downside volatility-managed portfolios.

Upside volatility-managed portfolios are constructed in the same way as downside volatility-managed portfolios:

$$f^{u}_{t+1} = \frac{\xi}{\hat{\sigma}^{2}_{u,t}} f_{t+1} \tag{10}$$

Where $\hat{\sigma}_u$ is our estimate of upside volatility and ξ is chosen such that the unconditional volatilities of f^{u}_{t+1} and f_{t+1} are the same.

We compare upside volatility-managed portfolios to volatility-managed portfolios f^{MM} using the following regression:

$$f^{u}_{t+1} = a + b f^{MM}_{t+1} + \eta^{d}_{t+1} \tag{11}$$

Exhibit 11 presents the results of Equation (11). Six of the eight regressions have negative intercepts, several of which are economically and statistically large. None of the intercepts is large and positive. These results stand in sharp contracts to those in Exhibit 5, in which most of the intercepts were large and positive. Exhibit 11 shows that upside volatility-managed factors are spanned by volatility-managed factors; the investor is better off investing only in volatility-managed factors rather than diversifying to upside volatility-managed factors.

Exhibit 12 shows regression intercepts and t-statistics of Equation (11) for industry portfolios. Only six of the 49 intercepts are positive. 12 intercepts are reliably negative at the 5% level. Like the case for factors, volatility-managed industry portfolios also span upside volatility-managed portfolios.

Our finding that downside volatility can be used to improve the risk-return tradeoff available to the investor is not simply a mechanical result. Although upside volatility is correlated to volatility in much the same way as downside volatility, upside volatility-managed portfolios do not improve the investor's opportunity set. Volatility-managed portfolios contain all information available in upside volatility-managed portfolios, and subsume the latter

Left-Hand Variable								
	$RMRF^u$	SMB^u	HML^u	MOM^u	RMW^u	CMA^u	$STRev^u$	$LTRev^u$
a_u	0.0%	-1.9%	-2.8%	0.9%	-0.7%	-0.9%	-0.6%	-2.5%
	(0.0)	(-1.8)	(-3.9)	(1.0)	(-1.3)	(-2.0)	(-1.0)	(-3.7)
$RMRF^{MM}$	0.90							
	(70.0)							
SMB^{MM}		0.41						
		(15.0)						
HML^{MM}			0.82					
			(48.3)					
MOM^{MM}				0.86				
				(56.5)				
RMW^{MM}					0.85			
					(41.1)			
CMA^{MM}						0.88		
						(47.6)		
$STRev^{MM}$							0.90	
							(67.1)	
$LTRev^{MM}$								0.85
								(51.6)
R^2	0.82	0.17	0.68	0.75	0.72	0.78	0.80	0.72

Exhibit 11: Univariate regressions of *upside* volatility-managed portfolios on Moreira and Muir (2017) factors.

We regress upside volatility-managed factors onto volatility-managed factors of Moreira and Muir (2017): $f^u_{t+1} = a_u + b_u f^{MM}_{t+1} + \eta^u_{t+1}$ Each column is a separate regression. The data are monthly and the sample period is 1926 to 2018 for $RMRF$, SMB, HML, MOM, and $STRev$; 1931 to 2018 for for $LTRev$; 1963 to 2018 for RMW and CMA.

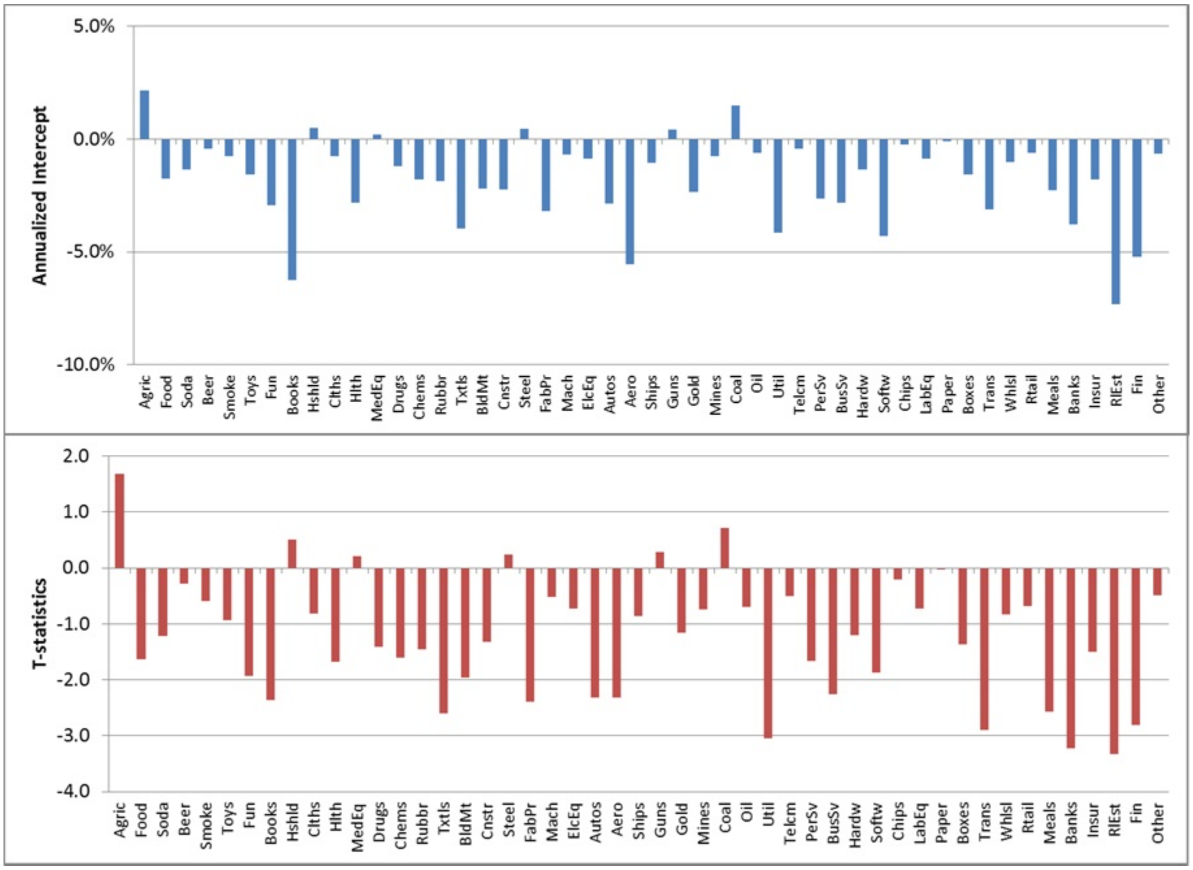

Exhibit 12: **Comparison of *Upside* Volatility-Managed Industry Portfolios and Volatility Managed Portfolios of Moreira and Muir (2017).**
We regress excess returns of upside volatility-managed industry portfolios onto the excess returns of volatility-managed industry portfolios of Moreira and Muir (2017): $f^u_{t+1} = a_u + b_u f^{MM}_{t+1} + \eta^u_{t+1}$ The data are monthly and available from 1926 to 2018.

set in spanning tests.

5 Conclusion

In this paper, we argue that downside volatility is an important risk measure which can provide the investor with additional value not captured by volatility. Although downside volatility and volatility are highly correlated on average, during the most volatile times historically, the two measures behave differently. Whereas the unconditional correlation between downside volatility and volatility is around 90%, conditional correlation during the 12 most volatile months is around 55%. Pooled regression results show that in the tails, a 1% increase in volatility is associated with only a 0.4-0.5% increase in downside volatility. High volatility indicates increased uncertainty in the marketplace, which is precisely the time when tracking both downside volatility and volatility would have been the most beneficial.

We explore the behavior of downside volatility-managed portfolios for long-short factors and industry portfolios. We reduce exposure when downside volatility is high and increase exposure when downside volatility is low. Downside volatility-managed portfolios are not spanned by the original portfolios, or by volatility-managed portfolios of Moreira and Muir (2017). We also investigate the merit of downside volatility-managed portfolios in a mean-variance setting. Adding downside volatility-managed portfolios increase the Sharpe ratio of the tangency portfolio and expands the mean-variance frontier towards more desirable regions. Whereas downside volatility proves to be useful in portfolio management, upside volatility does not help the investor in the same setting.

While we motivate our investigation with asymmetric investor preference for upside and downside volatility, our empirical findings may be consistent with alternative channels. Possible explanations could be risk-based, perhaps rooted in institutional frictions, or behavioral, potentially involving investor sentiment. We acknowledge asymmetric investor preference is a mere motivation and certainly not the only possible explanation consistent with our results.

We have focused only on the U.S. equity market. One interesting research direction would be to expand our analysis to include a broad set of countries. How does downside volatility differ from volatility in international markets? Is downside volatility useful for portfolio management? Another interesting path would be to look across asset classes, including bonds, currencies, and commodities. If the results hold across countries and asset classes, downside volatility may have a common, systematic component. Understanding this common factor brings us closer to building superior investment strategies and risk management systems.

References

Bawa, V. S. and Lindenberg, E. B. (1977). Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics*, 5(2):189–200.

Campbell, J. Y., Lo, A. W., MacKinlay, A. C., et al. (1997). *The econometrics of financial markets*, volume 2. Princeton University Press Princeton, NJ.

Carhart, M. M. (1997). On persistence in mutual fund performance. *Journal of Finance*, 52(1):57–82.

Fama, E. F. and French., K. R. (1992). The cross-section of expected stock returns. *Journal of Finance*, 47(2):427–465.

Fama, E. F. and French., K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116(1):1–22.

Goetzmann, W., Ingersoll, J., Spiegel, M., and Welch, I. (2007). Portfolio performance manipulation and manipulation-proof performance measures. *Review of Financial Studies*, 20(5):1503–1546.

Harlow, W. V. and Rao, R. K. (1989). Asset pricing in a generalized mean-lower partial moment framework: Theory and evidence. *Journal of Financial and Quantitative Analysis*, 24(3):285–311.

Hogan, W. W. and Warren, J. M. (1974). Toward the development of an equilibrium capital-market model based on semivariance. *Journal of Financial and Quantitative Analysis*, 9(1):1–11.

Ledoit, O. and Wolf, M. (2008). Robust performance hypothesis testing with the sharpe ratio. *Journal of Empirical Finance*, 15(5):850–859.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7(1):77–91.

Markowitz, H. (1959). *Portfolio Selection, Efficent Diversification of Investments*. J. Wiley.

Moreira, A. and Muir, T. (2017). Volatility-managed portfolios. *Journal of Finance*, 72(4):1611–1644.