Data Compressibility Quantifies LLM Memorization

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Abstract

Large Language Models (LLMs) are known to memorize portions of their training data, sometimes even reproduce content verbatim when prompted appropriately. Despite substantial interest, existing LLM memorization research has offered limited insight into how training data influences memorization and largely lacks quantitative characterization. In this work, we build upon the line of research that seeks to quantify memorization through data compressibility. We analyze why prior attempts fail to yield a reliable quantitative measure and show that a surprisingly simple shift from instance-level to set-level metrics uncovers a robust phenomenon, which we term the *Entropy–Memorization (EM) Law*. This law states that a set-level data entropy estimator exhibits a linear correlation with memorization scores.

We validate the EM Law through extensive experiments across a wide range of open-source models and experimental configurations. We further investigate the role of the token space—an implicit yet pivotal factor in our method—and identify an additional variant of the EM Law. Besides, we made a side observation that EM Law enables a simple application to distinguish between LLM train data and test data.

1 Introduction

Large Language Models (LLMs) demonstrate remarkable performance in capturing linguistic patterns and generating coherent text (Vaswani et al., 2017; Radford et al., 2019). It is sweeping across every domain in natural language processing. Alongside dominating performance through various benchmarks, a critical phenomenon has emerged: LLMs are shown to memorize and reproduce verbatim sequences from their training corpora (Carlini et al., 2019; 2020). This memorization behavior has raised growing concerns, particularly regarding privacy leakage and intellectual property protection. For example, studies have shown that LLMs can inadvertently generate personally identifiable information (PII) (Carlini et al., 2020), or proprietary data from books (USAuthorsGuild, 2023; LLMLitigation, 2023) and news articles (Michael, 2023). Most recently, Anthropic reached a USD 1.5 billion settlement with authors over the unauthorized use of copyrighted books, underscoring the growing legal risks surrounding LLM training data (The New York Times, 2025).

As the scaling law (Kaplan et al., 2020) drives LLM developers to expand model capacity and training data for performance improvements, research (Wang et al., 2025; Ippolito et al., 2023) has demonstrated that memorization scales with model size. Broader data exposure in LLM training elevates the risk of leakage for all internet-sourced content. Therefore, advancing the theoretical understanding of the factors that shape memorization has become a crucial and urgent issue in LLM development. Factors can be categorized into three types: prompting strategy (Carlini et al., 2020; Schwarzschild et al., 2024), model training (Chu et al., 2025), and training data.

Existing memorization literature is limited in two aspects. *First*, comparing the first two factors, the role of training data in memorization is under-explored. Existing research limits the scope to data duplication, where researchers find out that data duplication significantly increases memorization (Kandpal et al., 2022; Biderman et al., 2023b). Beyond that, with the belief that highly compressible text is easier to memorize, some attempts have been made to link training data compressibility to memorization (Carlini et al., 2020; Prashanth et al., 2025). However, such attempts fail to identify significant patterns between compressibility and memorization. *Second*, most existing memorization explorations are limited to *qualitative* studies. Most research work typically regards memorization within a binary framework Carlini et al. (2020); Zhang et al.

(2023); Prashanth et al. (2025); Schwarzschild et al. (2024). The only two quantitative studies are by Carlini et al. (2023) and Morris et al. (2025), which investigate model scale, data duplication, and context length.

Concerning the above limitations, this paper tackles an open question: **How to characterize memorization by the compressibility of training data in LLMs quantitatively?** This paper formulates memorization using an integer-valued memorization score (between LLM response and golden answer). In this work, we adopt two metrics related to compressibility: 1) zlib compression (Deutsch & Gailly, 1996; Carlini et al., 2020), which is inspired by prior work, and 2) an entropy (estimator) motivated by their theoretical connection with compressibility in information theory.

This paper identifies a significant defect of previous attempts: metrics are evaluated *instance*-wise. Each instance provides limited and noisy information, since the underlying token space is much smaller than the overall token space. Motivated by this, we instead work on *set*-level. Our set-level approach is also inspired by recent set-level membership inference attack (MIA), "dataset inference" approaches (Maini et al., 2021; 2024), where research has found that instance-level MIA shows limited robustness and works on set-level approaches.

With a simple modification from instance-level approaches to set-level, we demonstrate that the set-level entropy estimator accurately approximates the memorization score. Measuring fitness using linear regression, we achieve the Pearson Correlation $r > 0.9$ across a wide range of popular LLMs. We dub this core finding of the study as **Entropy–Memorization Law**. It suggests that training sequences with higher entropy are strongly correlated with higher memorization scores (i.e., lower proximity between the model's response and ground truth data itself). EM Law is empirically validated on a wide range of pre-trained models, including the OLMo family (Groeneveld et al., 2024), OpenLlama (Geng & Liu, 2023), and Pythia (Biderman et al., 2023b). We also explore EM Law in various experimental setups, including continuation length and inference sampling strategy.

Our entropy estimator enjoys twofold benefits: 1) the metric gives a *quantitative* description of memorization. The quantitative metric advances beyond the traditional binary setting of memorization with qualitative empirical observations. A quantitative metric facilitates the assessment of privacy risks for LLM providers. 2) the metric is *model-agnostic*. A model-agnostic approach is compute-efficient. It does not require backpropagation with a large number of model weights. In contrast, model-aware approaches, such as influence functions (Koh & Liang, 2017; Feldman & Zhang, 2020), typically require Hessian computation or even retraining, which is not affordable on LLMs.

We conduct thorough investigations into EM Law under several dimensions. **First**, we consider an implicit factor that shapes EM Law: the support set over which entropy is defined. We identify that lower memorization-score data comprises *exponentially-linear* fewer unique tokens, and achieves *linearly* higher entropy values given the support size. **Second**, by applying the EM Law to test data, we uncover a simple yet effective method for distinguishing training data from test data. This leads a simple dataset inference attack, enabling privacy auditing.

To summarize, our EM Law advances beyond existing research in the following ways:

- For the first time, we step beyond *instance*-level statistics in LLM memorization and obtain a *set*-level statistics that approximates LLM memorization well (§ 3). We term this as Entropy–Memorization Law. This pattern is preserved under various experimental setups (§ 4 and Appendix C and D).

- Different from previous measures that heavily depend on model or prompting, our set-based entropy estimator is characterized by the training data. To understand this data-centric metric, we explore the token space, an implicit factor within the training data, and identify a variant of EM Law (§ 5).

- Our set-based entropy estimator provides a *quantitative* characterization of memorization. Leveraging this quantitative nature—which most prior work lacks—we further show that the EM Law naturally induces a simple dataset inference attack, enabling practical auditing of privacy risks (§ 6).

2 Preliminaries

To provide background for the study, we first review the efforts to link compressibility to memorization in Section 2.1. Then in Section 2.2, we establish the required notation and explain the experimental setup that we follow through the paper.

2.1 LLM Memorization and its link to compressibility

Prior work on characterizing memorization can be broadly grouped into three categories: (i) model-centric factors, such as training paradigms and model scale (e.g., larger models tend to memorize more (Ippolito et al., 2023)); (ii) prompt-centric factors, such as prompting strategies (e.g., longer prompts tend to elicit more memorization (Carlini et al., 2023)); and (iii) data-centric factors, which focus on properties of the training data itself. For a comprehensive review, we refer readers to Section 7.

Regarding data-centric factors, a line of seminal work (Kandpal et al., 2022; Carlini et al., 2023) proposes that data repetition significantly increases the chances of memorization. Another line of research tries to link data compressibility to memorization based on an implicit suspicion in the community: highly compressible text corresponds to lower memorization difficulty. Such efforts started from the very first LLM data reconstruction work done by Carlini et al. (2020), where researchers use zlib compression to filter out repetitive text. Later, under a binary framework of memorization (i.e., memorization *v.s.* non-memorization), Prashanth et al. (2025) measured the link between text compressibility and memorization, but they did not observe a significant pattern. Note that perplexity, as explored by previous studies (Aerni et al., 2025; Prashanth et al., 2025; Huang et al., 2024), does not fall in the scope of data-centric factors, since it measures the uncertainty of *models*. So far, researchers have yet to establish a satisfactory connection between memorization and compressibility.

Metrics of compressibility   In this paper, we adopt two metrics of compressibility: 1) zlib compression ratio (Deutsch & Gailly, 1996), and 2) an estimated entropy [1] based on empirical point probabilities (Carlton, 1969). The use of zlib compression follows Carlini et al. (2020); while the second entropy-based metric is inspired by the close relationship of compressibility and entropy proved by the source coding theorem (Shannon, 2001). Note that two metrics correspond to two types of lossless coding algorithms in information theory: coding schemes for sources with memory and without memory, i.e., memoryless.

Previous work fails to characterize memorization through compressibility.   Since prior work explores memorization with different experiment setups, we reproduce the result under a unified setup.

Figure 1 presents the result of instance-level zlib compression ratio. The algorithms are detailed in the Appendix B, and the experimental setup will be explained in Section 2.2. In general, we observe a positive linear relationship but with a weak Pearson correlation r around 0.6.

2.2 Experimental Setup

Threat Model   This paper assumes a hypothetical engineer who studies the characterization of training data on an LLM. Therefore, it is necessary for the engineer to have full access to the LLM *and* its training data. This engineer controls for other potential confounders in the memorization score, including prompt strategy and training paradigm.

This paper focuses on *pre-trained*-only LLMs (i.e., the "base" models) and excludes post-trained variants. Post-training procedures can induce substantial distribution shifts in the output space. For example, an LLM that undergoes safety alignment via RLHF (Ouyang et al., 2022) may respond with a generic refusal (e.g., "Sorry, I cannot generate harmful content as a responsible AI.") when prompted with queries that conflict with its safety policies, even if the underlying pre-trained model assigns high likelihood to a direct answer. Moreover, post-training pipelines are rapidly evolving and typically combine multiple training paradigms

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[1]We assume a base-2 logarithm for all entropy calculations throughout the work.

(a) entropy estimator *v.s.* Memorization score

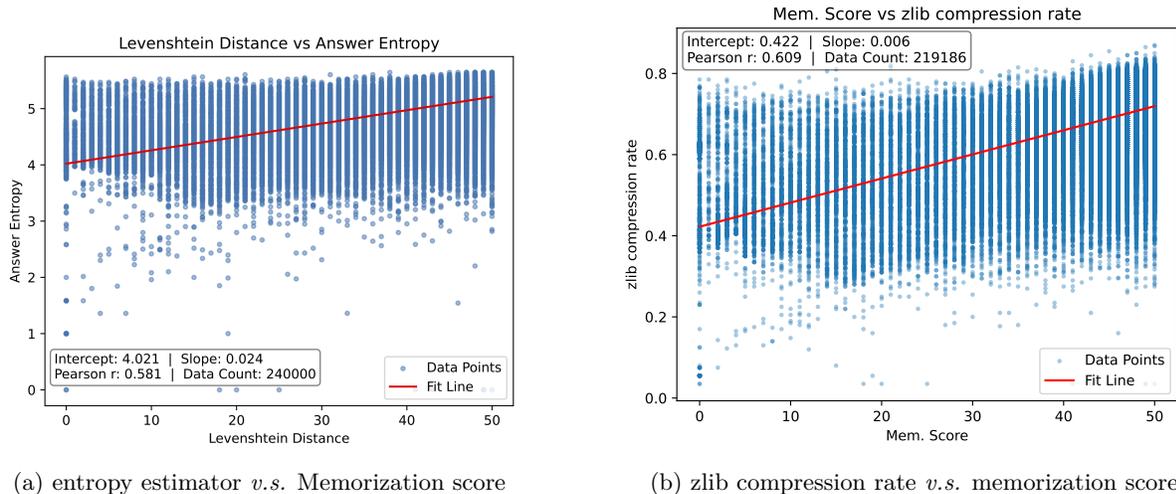(b) zlib compression rate *v.s.* memorization score.

Figure 1: Existing compressibility metrics fail to capture memorization score on OLMo-1B. In this figure, each point represents a sampled text sequence from the LLM training corpus, and sampling is repeated $N = 240,000$ times.

(e.g., instruction fine-tuning, RLHF, and RL on sepcial reasoning domains), making their individual effects difficult to disentangle and analyze.

Choices of LLM and Training Corpus   We selected four LLMs: (1) OLMo (Groeneveld et al., 2024) pre-trained on Dolma (Soldaini et al., 2024) dataset, and (2) OLMo-2 (OLMo et al., 2024) pre-trained on OLMo-2-1124-Mix (OLMo et al., 2024) dataset; (3) OpenLlama (Geng & Liu, 2023) pre-trained on Redpajama (Computer, 2023); (4) Pythia (Biderman et al., 2023b) pre-trained on the Pile (Gao et al., 2020).

Prompting Strategies   In this study, we consider a *Discoverable Memorization* (DM) scenario (Nasr et al., 2025; Carlini et al., 2023; Kandpal et al., 2022; Ippolito et al., 2023). Formally, DM denotes the following: we sample N token sequences from the training dataset. Each sequence is partitioned into (p, s), where p (first $|p|$ tokens) serves as the prompt and s (remaining tokens) serves as the answer for model θ. Afterwards, LLM θ generates a response $r = \theta(p)$. The memorization score measures the difference between two sequences r and s. By default, we set $|p| = 100$ and $|s| = |r| = 50$, following the popular setup in the community (Al-Kaswan et al., 2023). However, variants of continuation length will be discussed in Section 4. We defer the detailed statistics of the sampled datasets to Appendix A. The token sequence sampling strategy is as follows: we repeatedly randomly sample a sequence (with length $= |p + s|$) from the dataset until the number reaches the required number.

Filtering Trivial Memorization   We exclude *trivial* memorization cases where the model's response r exhibits high lexical overlap with the prompt p. For example, the LLM may copy a long URL from the prompt to its response. Such cases are outside the scope of our interest, as memorization is shaped by the prompt rather than the training data. We devise a Longest Common Subsequence (LCS)-based filtering approach. We establish a thresholding strategy based on LCS: samples for which $LCS(p, s) \geq \frac{|s|}{2}$ are excluded from further memorization analysis, while samples below the threshold are retained.

Memorization Score   We adopt the notion of memorization score $d(r, s)$, to measure the differences between response r and answer s at the token level. Following previous memorization work (Dong et al., 2024), we use Levenstein distance, or edit distance (Levenshtein et al., 1966), i.e., $d(r, s) = d_{\text{lev}}(r, s)$. It is defined by the minimal number of single-token edit operations – insertions, deletions, and substitutions – required to transform one sequence into another. A *higher* memorization score indicates *lower* similarity between two sequences.

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Note that we choose not to use a semantics-based memorization score, as semantic-level memorization does not have clear or direct societal implications. For example, authors are suing LLM providers over the use of copyrighted books because model outputs exhibit substantial verbatim or near-verbatim overlap with their texts, not because the models generate passages that are merely semantically similar.

To summarize, the research question in this paper is formulated as follows:

> **Assumption.** A fixed pre-trained LLM θ, a fixed prompting strategy DM to generate p, and a memorization score $d(r, s) = d_{\text{lev}}(\theta(p), s)$.
> **Goal of the study.** Find an approximator function $M(s)$ of memorization score $d(r, s)$.

3 Methodology

3.1 Limitation of prior instance-wise compressibility metrics

As shown in Section 2.1, prior approaches fail to provide a reliable approximation of the memorization score. We attribute this limitation to their *instance*-level computation: each sample spans only a tiny subset of the full token space, making such metrics fundamentally incapable of capturing memorization accurately. More concretely, previous attempts sample instances s_i, which are sequences of tokens, the atomic processing unit in LLMs. In mathematics notations,

$$s_i = (s_i^1, s_i^2, \ldots, s_i^{|s_i|}), \tag{1}$$

where each $s_i^j \in |\mathcal{T}|$ denotes a token from the vocabulary defined by the LLM tokenizer. We argue that each s_i spans a token space that is orders of magnitude smaller than the full token space $|\mathcal{T}|$.

In practice, taking our experimental setup as an example, each LLM generation is limited to at most $|s_i| = 50$ tokens. Thus, each sample contains no more than 50 distinct tokens. In contrast, the full token space–as determined by the tokenizer vocabulary $\mathcal{T}$–is substantially larger; for instance, the OLMo-1B tokenizer has a vocabulary size $|\mathcal{T}| \approx 50{,}000$. This implies that, at the instance level, each sample spans a space that is orders of magnitude smaller than the full token space. Moreover, simply increasing the response length cannot mitigate the issue. The context window size of OLMo-1B is around 4,000 tokens, which is still far beyond what is required for our setting. Therefore, existing compressibility measures are highly noisy, which prevents them from providing a robust estimate in real-world scenarios.

3.2 Set-level compressibility metrics

In this work, we address the limitation of previous work by substantially expanding the size of the token space. More concretely, we consider a simple "level-set" based method. We expand the token space from tokens of *one* instance, to *all* the instances with the same memorization score e. In math notations, for a fixed memorization score e, the new token space is defined as:

$$\mathcal{T}_e = \bigcup \left\{ s_i^j \mid d(r_i, s_i) = e, i \in \{0, \cdots, N-1\}, j \in \{0, \cdots, |s|-1\} \right\}. \tag{2}$$

By adopting the above heuristics to construct set-level estimates, we aim to obtain a token space whose scale is comparable to that of the full vocabulary. Next, we describe how we implement the zlib-based and entropy-based methods at the set level.

zlib method. We adopt zlib compression on the contention of sequences with the same memorization score e:

$$s_e = \bigoplus \left\{ s_i^j \mid d(r_i, s_i) = e, i \in \{0, \cdots, N-1\}, j \in \{0, \cdots, |s|-1\} \right\}. \tag{3}$$

Then, zlib-based compressibility is calculated by the compression rate over the original sequence:

$$M_{\text{zlib}}(s_e) = \frac{|\text{zlib}(s_e)|}{|s_e|}. \tag{4}$$

Entropy method. The empirical probabilities $\hat{p}_e(x)$ are now calculated within the new space $\mathcal{T}_e$:

$$\hat{p}_e(x) = \frac{1}{N|s|} \left| \left\{(i,j) \mid s_i^j = x, d(r_i, s_i) = e\right\} \right|. \tag{5}$$

We then use new empirical probabilities to derive a new level-set-based entropy estimate to approximate the memorization score e.

$$M_{\text{ent}}(s_e) \triangleq -\sum_{x \in \mathcal{T}_e} \hat{p}_e(x) \log \hat{p}_e(x). \tag{6}$$

For clarity, we provide the complete algorithm in Alg. 1.

Algorithm 1: Compute set-level compressibility meatrics.

Input: LLM θ, and its training corpus D.
Output: Plot of $(e, M(s_e))$
1 Sample N prompt-answer pairs$\{(p_i, s_i)\}$ from D; `// Section 2.2`
2 for $i \leftarrow 0$ **to** $N-1$ **do**
3 $\quad$ $r_i \leftarrow \theta(p_i)$ `// Prompt LLM for response`
4 $\quad$ $d(r_i, s_i) \leftarrow d_{\text{lev}}(r_i, s_i)$ `// Memorization score`
5 end
6 for $e \leftarrow 0$ **to** $|s|-1$ **do**
$\quad$ `// zlib method`
7 $\quad$ $s_e \leftarrow \bigoplus \left\{s_i^j \mid d(r_i, s_i) = e, i \in \{0, \cdots, N-1\}, j \in \{0, \cdots, |s|-1\}\right\}$ `// Prepare sequence. Ref: Eq. 3`
8 $\quad$ $M_{\text{zlib}}(s_e) \leftarrow \frac{|\text{zlib}(s_e)|}{|s_e|}$ `// Obtain zlib compression rate. Ref: Eq. 4`
$\quad$ `// Entropy estimator method`
9 $\quad$ $\hat{p}_e \leftarrow \frac{1}{N|s|} \left|\left\{(i,j) \mid s_i^j = x, d(r_i, s_i) = e\right\}\right|$ `// Obtain empirical probabilities. Ref: Eq. 5`
10 $\quad$ $M_{\text{ent}}(s_e) \leftarrow -\sum_{x \in \mathcal{T}_e} \hat{p}_e(x) \log \hat{p}_e(x)$ `// Calculate Entropy. Ref: Eq. 6`
11 $\quad$ Plot $(e, M_{\text{ent}}(s_e))$ and $(e, M_{\text{zlib}}(s_e))$.
12 end

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4 Entropy–Memorization Law
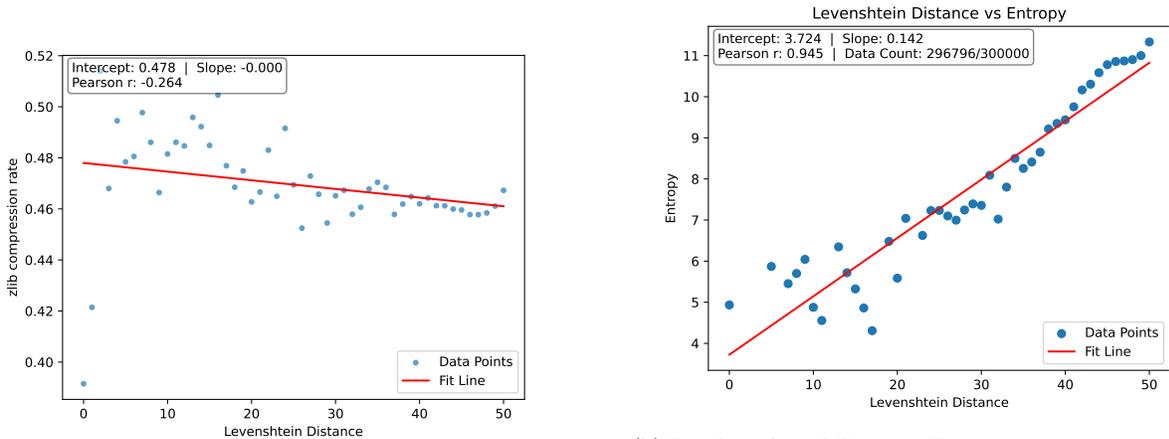
(a) zlib compression rate *v.s.* memorization score.

(b) Level-set-based Entropy Estimator *v.s.* memorization score.

Figure 2: At the set level, compared to zlib compression rate, entropy-based method achieves good approximation of memorization score. The experiments are conducted on OLMo-2-1124-7B.

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We run algorithm 1 on an extensive range of open-dataset LLMs and present the empirical results in Fig. 2. It turns out that the level-set-based entropy estimator is a good approximator of the memorization score, compared with the zlib method.

We observe powerful linear empirical results ($r = 0.972$ and 0.945 respectively) on both plots. It indicates that **the level-set-based entropy estimator is an effective *linear* approximation of memorization score**. We name this discovery as *Entropy-Memorization Law.* We give a formal description of Entropy–Memorization Law, or EM Law:

> **Entropy Memorization Law.** Given a fixed pre-trained LLM θ, a fixed prompting strategy DM to generate p, and a memorization score $d(r, s) = d_{\text{lev}}(\theta(p), s)$. $M_{\text{ent}}(s)$ serves as a linear approximator of the memorization score $d_{\text{lev}}(\theta(p), s)$.

In Figure 3, we provide extensive results on OpenLlama, Pythia-70m-deduped.

(a) OLMo-1B   (b) OpenLlama-13B   (c) Pythia-70m-deduped
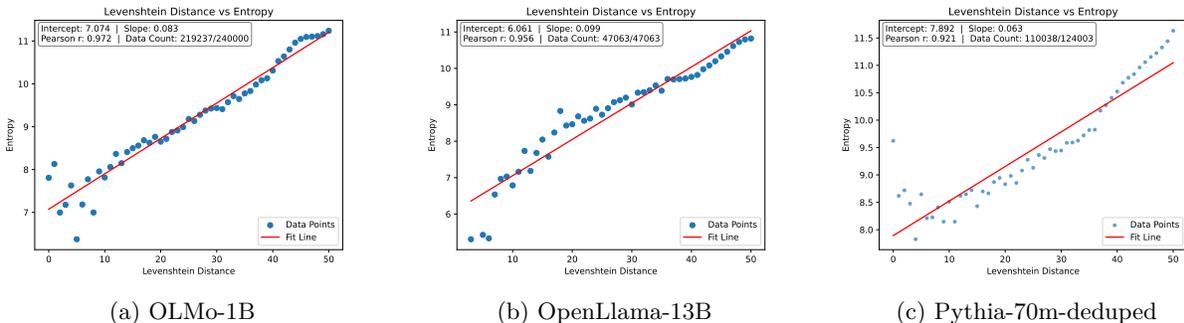
Figure 3: Entropy–Memorization Law on open-dataset LLMs.

We explore various LLM inference sampling strategies, including temperature, top-p and top-k sampling in Appendix C.1. Experiments on a broader range of models are available in Appendix C.2. In appendix D, we also separate the whole datasets into N semantic clusters, and then validate our finding within each cluster. Next, to showcase the validity of EM Law, we present our evaluation under varying continuation lengths.

Entropy–Memorization Law is preserved under varying continuation lengths We explore different continuation token lengths, including $\{10, 20, 30, 40, 50\}$. As a demonstration, we use OLMo-2-1124-7B and its training dataset OLMo-2-1124-Mix in this experiment. Note that we rescaled the memorization score to the range $[0, 1]$ in the plot for better presentation.

Figures 4 present the experimental result when the generation token length varies. The Pearson correlation coefficients (r) remain very high (ranging from 0.92 to 0.98) across all settings, indicating robust fitness under varying token lengths. Although regression lines have different slopes and intercepts, the y-value reaches about 11 when the memorization score $e = 50$, i.e., $M(s_{50}) \approx 11$.
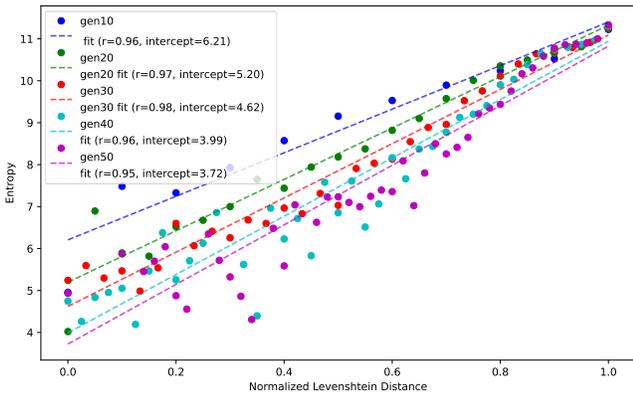

Figure 4: Entropy–Memorization Law under varying generation token lengths.

Another observation is that there is a monotonic increase in the intercept values of the fitted regression lines as the generation token length increases. Information theory may help to justify this: Denote the vocabulary

size as $|\mathcal{T}|$, n-sequence has $|\mathcal{T}|^n$ potential outcomes; hence when n gets larger, the maximum entropy of n-sequence increases.

5 Normalized Entropy–Memorization Law

Our method effectively enlarges the token space and provides a reliable proxy for the memorization score. However, we have not yet conducted a detailed investigation of the underlying token space itself. In fact, token space size plays an implicit yet crucial role in EM Law. In information theory, the maximum entropy is bounded by outcome space cardinality. In fact, the maximum entropy of a discrete random variable is $-\sum_{i=1}^{n} \frac{1}{n} \log \frac{1}{n} = \log n$, where n is the cardinality of the outcome space (i.e., token space size). The upper bound is achieved when the random variable follows a uniform distribution.

In this subsection, for each memorization score e, we report the corresponding token space size $\mathcal{T}_e$. The results are shown in Fig. 5a. The statistic is calculated following the same experimental setup on OLMo-2-1124-7B. The experimental result indicates that the token space size, or unique token count, grows exponentially as the memorization score increases. Low-score memorization occurs within a limited token space. Remarkably, while the full dataset contains tens of millions of tokens, perfect memorization (score=0) occurs within merely $2^1 0$ unique tokens, namely 1/50 of the vocabulary size. While we do not provide a further theoretical justification, we suspect that tokens appearing with higher frequency in the training corpus are more likely to be memorized. Further investigation may require another line of research: measuring memorization at the *token level*. In the context of membership-inference attacks, there are explorations like Tao & Shokri (2025).
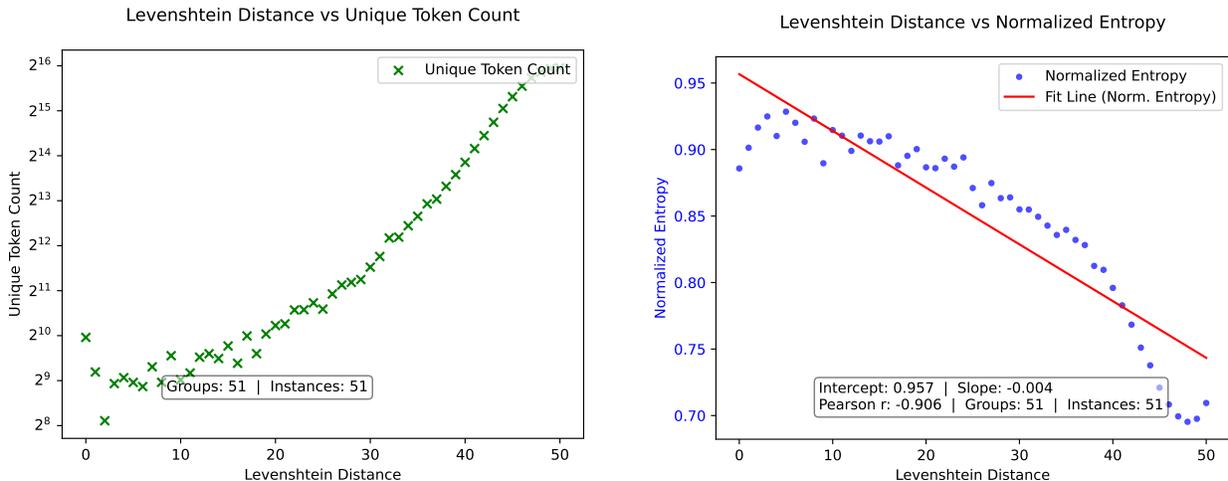
(a) Memorization score *v.s.* token space on OLMo-2-1124-7B. Note: the scale of the y-axis is *exponential*.

(b) Memorization score *v.s.* normalized entropy on OLMo-2-1124-7B.

Figure 5: Memorization score *v.s.* normalized entropy on OLMo-2-1124-7B.

Given a fixed outcome-space cardinality, our entropy estimator characterizes how the empirical probability mass is distributed. To factor out the influence of token-space size, we normalize the observed entropy by the maximum attainable entropy under the same token-space cardinality, and refer to the resulting ratio as the *normalized entropy*:

$$\overline{M}(s_e) \triangleq \frac{M(s_e)}{H_{\max,e}} = \frac{M(s_e)}{\log |\mathcal{T}_e|}, \tag{7}$$

where $\overline{M}(s_e)$ normalizes the entropy estimate $M(s_e)$ by its theoretical maximum $H_{\max,e}$. Values approaching 1 indicate a near-uniform distribution over the token set $\mathcal{T}_e$, while lower values suggest greater non-uniformity.

Following the same setup on OLMo-2-1124-7B, we plot $(e, \overline{M}(s_e))$ using blue dots on Figure 5b to observe how the normalized entropy estimator changes with the memorization score. Interestingly, we observe another linear trend – it indicates that normalized entropy (negatively) linearly decreases as the memorization score increases. In appendix C.3, we discuss the pattern with different sequence lengths.

Although a theoretical justification is not available for the phenomenon, a potential explanation is that high-memorization-score samples tend to follow a more "natural" language distribution, which is known to be long-tailed (Zipf, 2016) and therefore exhibits lower entropy. In contrast, low–memorization-score samples may contain more specialized content—such as code snippets or numerical values—whose distributions deviate from typical natural language regularities.

> **Summary.** By decoupling the entropy estimator to outcome-space cardinality and randomness of distribution, we reveal that 1) lower memorization-score data comprises *exponentially-linear* fewer unique tokens, and 2) achieves *linearly* higher entropy values given the outcome space.

6 Application: Dataset Inference using EM Law

6.1 Entropy Memorization Law on Test Data

Running algorithm 1 on *training* dataset of LLMs, we discovered the Entropy-Memorization Law. This section then explores another question that naturally arises – what happens if we run the same algorithm on *test* dataset? It turns out that the plot behaves very differently from training datasets.

Figure 6 presents the plot of applying the entropy estimator and the normalized entropy estimator to test data. To maintain consistency throughout the paper, we use the terms used for memorization with slight abuse. For example, the memorization score still measures the distance between the ground truth and the model's response, but the model is not actually "memorizing" training data. Following the same setup described in the main body, we select data from LiveBench (White et al., 2025) from 2024-06-25 to 2024-11-25. The time property guarantees that LiveBench is non-member data for OLMo-2-1124-7B. LiveBench is licensed under the CC BY-SA 4.0 International License.
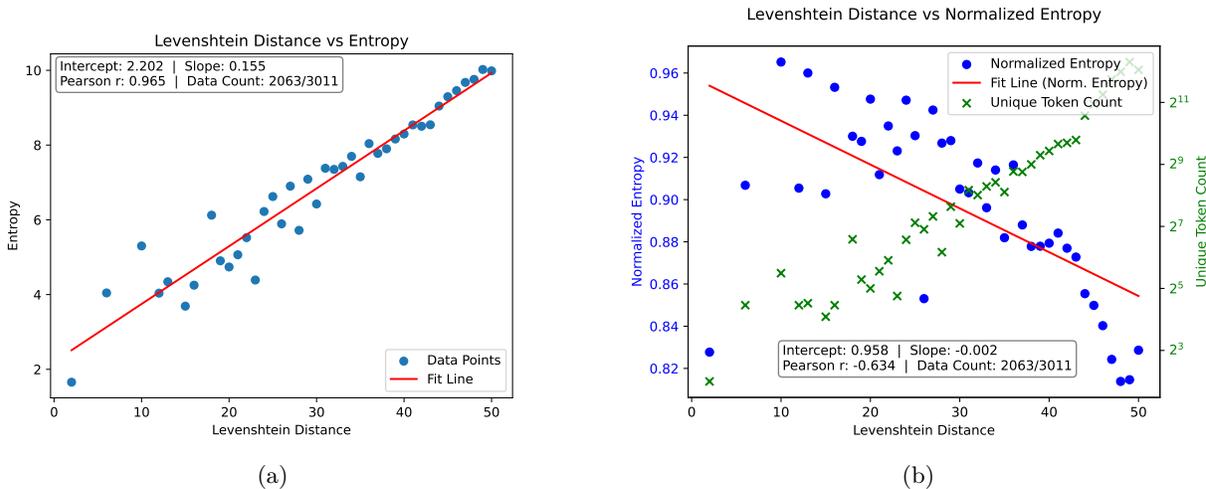

Figure 6: Entropy Memorization Law on OLMo-2 with LiveBench dataset.

With around 3k samples from LiveBench, we observe that:

- Level-set-based entropy is still a good indicator of LLM performance, as demonstrated by the high Pearson's r. The sample space also approximately grows exponentially as the memorization score increases.

- However, compared with training data, the intercept is lower, and the slope is higher. There is less coverage of low memorization scores, especially for memorization scores within the range of 0-10.

- At the low-distance set, it is observed that OLMo-2 is producing low-entropy text. The entropy is much lower than what we observed on train data.

Here is a case study on a 2-distance memorization:

> **Prompt.**
> Please create a valid join mapping between CSV Table A and CSV Table B. Each column in A maps to 0 or 1 columns in B. Return your response as a Python dictionary, formatted as col_name_in_df_a : col_name_in_df_b. Please return only the dictionary.
> CSV Table A: Areas,freq_1,freq_2,freq_3,freq_4,freq_5,freq_6 0.0,0.0,0
> **Response.**
> .0,0.0,0.0,0.0,0.0
> 1.0,0.0,0.0,0.0,0.0,0.0,0.0
> 2.0

Given the prompt in this case study, OLMo-2 generates repeated numbers, exhibiting low entropy. Based on the observations above, we thus derive a simple strategy to distinguish between train data and test data for LLM.

6.2 Data Inference Methodology

Dataset Inferences Dataset Inference (DI) (Maini et al., 2024; 2021) builds on the idea of membership inference attacks (MIA) (Song & Shmatikov, 2019; Galli et al., 2024; Carlini et al., 2022; Mattern et al., 2023; Jagannatha et al., 2021). Both MIA and DI aim to identify whether some suspect data was part of the training data, but they differ on the amount of data required. MIA operates at *instance* (sentence) level; however, DI operates on a *collection* of instances – in reality, the suspect data used for DI could be a book.

In practice, DI can identify potential test set contamination to provide a calibrated performance evaluation of LLMs. DI may also detect unauthorized usage of copyrighted training data, thus promoting the protection of intellectual properties. Moreover, by using more data to determine membership, DI is deemed to be more realistic than MIAs. As Maini et al. (2021) presented, as the size of the training set increases, the success of membership inference degrades to random chance. Our DI method is inspired by several empirical observations. First, over-parametrization of LLMs may lead to overfitting on training data, resulting in a generalization gap between training data and testing data; then, LLMs may perform well on low-entropy testing data, resulting in a low intercept in EM Law. Besides, given a fixed dataset and LLM, empirical evidence suggests that the intercept and slope generated by Algorithm 2 are dependent. This inspires us to develop the following strategy for dataset inference:

> Given an LLM θ and dataset D, run Algorithm 1 and get intercept k. Compare k with a pre-defined threshold τ_k. Assign label 1 (i.e., member) if $k > \tau_k$, assign label 0 otherwise.

Amount of data required for our DI method Effective dataset inference requires reliable entropy estimation and diverse memorization score distributions. In the main body, we have revealed that the frequency of a low memorization score is exponentially smaller than that of a high memorization score. Therefore, we set the minimum sample size to $n = 1,500$, with each sample being a 150-token sequence.

Table 1: Dataset inference result

LLM	Dataset	Intercept	Slope	Prediction	Ground-truth
OLMo-2	LiveBench	2.202	0.155	0	0
Pythia	MIMIR_cc	-2.048	0.251	0	0
Pythia	MIMIR_cc	3.992	0.091	1	1
OLMo-2	OLMo-2-1124-Mix	3.724	0.142	1	1
Pythia	MIMIR_full	6.297	0.092	1	0
Pythia	MIMIR_full	6.166	0.095	1	1
Pythia	MIMIR_tarxiv	1.156	0.174	1	1
Pythia	MIMIR_tarxiv	-0.910	0.0227	0	0
Pythia	MIMIR_wiki	3.006	0.131	1	0
Pythia	MIMIR_wiki	2.894	0.133	1	1

6.3 Experimental Results on Dataset Inference

Selected LLMs. We select OLMo-2-1124-7B ("OLMo-2"), Pythia-6.9B-deduped (Biderman et al., 2023b) ("Pythia").

Selected Datasets. We select LiveBench (White et al., 2025) and MIMIR (Duan et al., 2024). For LiveBench, we use data from 2024-06-25 to 2024-11-25. MIMIR is a public dataset originally for evaluating MIAs on the Pythia suite by re-compiling the Pile (Gao et al., 2020) train/test splits. For MIMIR, we use Pile CC ("cc"), temporal arXiv ("tarxiv"), "wiki" subset, and full dataset ("full") for evaluation. Note that LiveBench and Temporal arXiv are temporal-cutoff-based, while the remaining dataset is i.i.d.-based.

Threshold for each LLM. In our method, we assign a threshold for each LLM. We empirically set τ_k to 0 and 3 for Pythia and OLMo-2, respectively.

Table 1 presents the overall results of our method on the dataset inference task. In general, it achieves desirable accuracy.

6.4 Practical Use of the Dataset Inference Method

In practice, our DI method provides a practical tool for auditing the privacy of training data. An external auditor may apply the dataset inference method as follows:

1. Using a known subset of the training data, the auditor applies the set-level entropy method to obtain reference results. This setup is realistic, as only part of training data is required.

2. The auditor determines an appropriate intercept threshold based on step 1.

3. The auditor collects a candidate dataset that is pending for dataset inference audit, and run set-level entropy method on the dataset.

4. Using the threshold from Step 2, the auditor make a binary decision on the dataset membership.

Advantages of our data inference method It is compute-efficient – it only requires LLM inference on n samples. It does not require any additional shadow or reference models.

7 Related Work

Since the discovery of the memorization phenomenon in the late 2010s (Zhang et al., 2017; Carlini et al., 2019; 2020; Feldman, 2020), the AI Security and Privacy research community has maintained a strong interest in the phenomenon and its implications. The following paragraphs examine how memorization in language models is influenced by key factors, including *training data, model paradigm, and prompting strategy.*

Data shapes memorization. Several studies suggest that (Kandpal et al., 2022; Biderman et al., 2023b) duplicated data significantly increases memorization. Larger models trained on larger datasets show increased memorization (Biderman et al., 2023a;b). Other studies (Tirumala et al., 2022; Wang et al., 2025) investigate how memorization manifests across data with varying semantics and sources.

Model Paradigm shapes memorization. Beyond pre-trained language models, recent work has explored memorization in post-training stages. Chu et al. (2025) demonstrate that supervised fine-tuned (SFT) LLMs exhibit stronger memorization tendencies than those trained with reinforcement learning (RL). Additionally, Nasr et al. (2025) reveals that safety-aligned models still retain memorized data.

Prompting Strategy shapes memorization. Researchers employ three main types of prompting strategies for language models, categorized by threat models. A significant body of work relies on manual efforts or template-based approaches to generate prompts at scale, as seen in Carlini et al. (2019; 2020); Kim et al. (2023). Studies such as Carlini et al. (2020); Kandpal et al. (2022); McCoy et al. (2021) demonstrate that longer prompts substantially increase the likelihood of reproducing memorized training data sequences. Another line of research constructs prompts directly from existing data sources, such as training corpora or web data (Nasr et al., 2025; Carlini et al., 2023; Kandpal et al., 2022; Ippolito et al., 2023; Aerni et al., 2025). Recent advances involve more sophisticated strategies that leverage synergies between LLMs and training data. For instance, Zhang et al. (2023) quantifies how a model's performance on an example x depends on whether x was included in the training data. Additionally, Schwarzschild et al. (2024) adapts GCG (Zou et al., 2023)—a prompt optimization tool designed initially for adversarial attacks—to generate effective extraction prompts.

8 Limitations and Broader Impacts

Limitations *Predictive power of the Entropy-Memorization Law* Due to the limitation of the sample space as discussed in Section 3, our strategy does not enable memorization score prediction at the instance level. However, we see promising results on set-level memorization-related tasks, such as Dataset Inference.

Empirical experiments. Our work adopts a single set of prompting strategy (DM) and memorization score (edit distance) in our memorization experiments. Although the setup is commonly used by other studies, other combinations exist. We want to explore adversarial compression (Schwarzschild et al., 2024), and non-adversarial reproduction (Aerni et al., 2025) in our future work.

Implications and Societal Impact For the research community, we believe that the EM Law provides a useful foundation for theoretically understanding the factors that drive memorization in LLMs. Moreover, it enables a simple and scalable approach for privacy auditing. For practitioners training LLMs, our method allows pre-screening of training datasets to assess potential memorization risks. External auditors may also employ our dataset inference method to detect test-set contamination and identify possible copyright-infringement issues in deployed models.

This paper uses existing open-research LLMs and their corresponding training datasets. To the best of the authors' knowledge, this research does not introduce any additional negative societal impacts.

9 Conclusions

This paper presents the Entropy-Memorization Law: a level-set-based entropy estimator of training data chunks linearly approximates the edit-distance-based memorization score. Further investigation indicates that EM Law is robust under different sequence lengths, sampling strategies, and data clusters with different semantics. By examining vocabulary size, it is revealed that lower memorization-score data comprises *exponentially-linear* fewer unique tokens, and achieves *linearly* higher entropy values given the support size.

For future work, we plan to explore why the proposed level-set-based entropy estimator fits the memorization score so well. Potential theoretical tools include the long-tail theory by Feldman and other researchers (Feldman, 2020; Feldman & Zhang, 2020), and multi-calibration in LLMs (Detommaso et al., 2024). Such efforts may also shed light on interpreting slope and intercept resulting from the EM Law.

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A Details on Experimental Setup

A.1 Composition of Sampled Dataset

We constructed datasets with sizes 240,000 and 300,000, respectively, for Dolma and OLMo-2-1124-Mix. Table 2 and 3 presents the composition of sampled datasets after LCS filtering.

Table 3: Source Counts of OLMo-2-1124-Mix

Dataset	Count
algebraic-stack	797
arxiv	1,464
dclm1	28,130
dclm2	27,984
dclm3	28,014
dclm4	28,163
dclm5	28,049
dclm6	28,065
dclm7	28,060
dclm8	28,091
dclm9	28,040
dclm10	28,032
open-web-math	499
pes2o	4,364
starcoder	5,280
wiki	276
Total	293,308

Table 2: Source Counts of Dolma

Source Dataset	Count
pes2o	39,777
cc	39,743
books	39,598
reddit	39,373
stack	38,865
wiki	23,115
Total	219,186

B   Implementation details of instance-wise compressibility metrics.

Zlib method.   For each instance s, we apply the zlib library to compress the sequence and report its compression rate, computed as the compressed length divided by the original length.

Entropy method.   An instance $s_i = (s_i^1, s_i^2, ..., s_i^{|s_i|})$ is a sequence, where s_i^j is a token. All tokens within s_i form the sample space $\mathcal{T}_i$. Then for each token $x \in \mathcal{T}_i$, the empirical point probabilities $\hat{p}_i(x)$ are calculated as:

$$\hat{p}_i(x) = \frac{1}{|s|} \left| \{ j \mid s_i^j = x \} \right|. \tag{8}$$

$\hat{p}_i(x)$ is the relative frequency of x in the observed sequence. In this attempt, we use entropy estimated by the empirical point probabilities as our approximator $M(s_i)$:

$$M(s_i) \triangleq - \sum_{x \in \mathcal{T}_i} \hat{p}_i(x) \log \hat{p}_i(x) \tag{9}$$

In practice, the distribution for language is unknown. we instead learn from samples. The above estimator approximates entropy by viewing the point probability as samples from the empirical distribution itself.

With the established $M(s_i)$, we are interested in whether $M(s_i)$ is a good approximator of the memorization score $d(r_i, s_i)$. To achieve this, we gather all $(M(s_i), d(r_i, s_i))$ pairs obtained by empirical observations in a scatter plot, and further study their correlation. The detailed algorithm is as follows:

Algorithm 2: Instance-wise entropy estimator.

Input: LLM θ, and its training corpus D
Output: Plot of $(d(r_i, s_i), M(s_i))$
1 Sample N prompt-answer pairs$\{(p_i, s_i)\}$ from D;
2 for $i \leftarrow 0$ **to** $N - 1$ **do**
3 | $r_i \leftarrow \theta(p_i)$;
4 | $\hat{p}_i \leftarrow \text{EmpProb}(r, s)$// Eq. 8
5 | $M(s_i) \leftarrow - \sum_{x \in \mathcal{T}_i} \hat{p}_i(x) \log \hat{p}_i(x)$// Eq. 9
6 | $d(r_i, s_i) \leftarrow d_{\text{lev}}(r_i, s_i)$
7 | Plot $(d(r_i, s_i), M(s_i))$;
8 end

We run algorithm 2 on the OLMo-1B model, and obtain the scatter plot as illustrated in Fig. 1.

C Extended Results on Entropy-Memorization Law

C.1 Additional Results with various sampling strategy of LLMs

In the main body of the paper, we assume a fixed temperature of 0.8. In this subsection, we adopt different sampling strategies of LLMs and discuss how these strategies might shape EM Law. Due to computation constraints, we conduct our experiments on a subset "DCLM1" with OLMo-2-1124-7B. The size of the subset is around 28,000.

We consider combinations of temperature, top-k sampling, and nucleus sampling (top-p). The experimental results are summarized in Tab. 4, and details are shown in Fig. 7 - 12. Under all sampling strategies we have explored, we empirically observe that EM Law holds with $r > 0.92$. Beyond that, we made a few observations here:

- The zero-distance point $(0, M(s_0))$ exhibits a significant deviation from the regression line in both plots.

- Intercept and slope are dependent if we fix the LLM and dataset. The general pattern is that when the intercept increases, the slope decreases. In fact, when the memorization score e= 50. Besides, $M(s_{50}) \approx 11$. Although different regression lines have different slopes and intercepts, the y-value reaches about 11. This might indicate that intercept and slope may have a degree of freedom 1.

- With a fixed temperature, enabling top-k or top-p sampling increases intercept and decreases slope.

- The estimated normalized entropy decreases with the memorization score increasing.

The first two observations are consistent with observation points 2 and 3 in Section 5.

Table 4: Entropy-Memorization Law under different LLM sampling strategy

Strategy	r	Intercept	Slope
Temp=0	0.933	5.490	0.106
Temp=0.5	0.936	5.474	0.106
Temp=0.8	0.926	5.011	0.113
Temp=0.8, top_p=0.5	0.935	5.599	0.103
Temp=1	0.944	4.646	0.118
temp=0.8, top_k=10	0.944	5.138	0.111
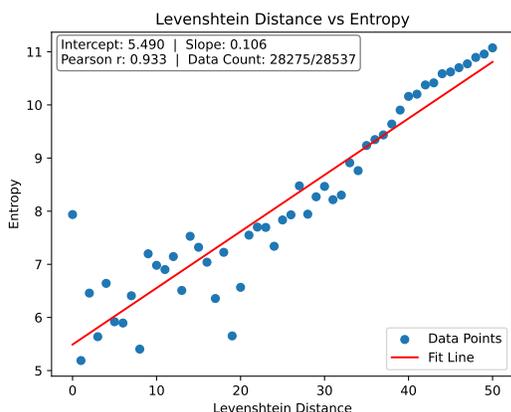
(a)
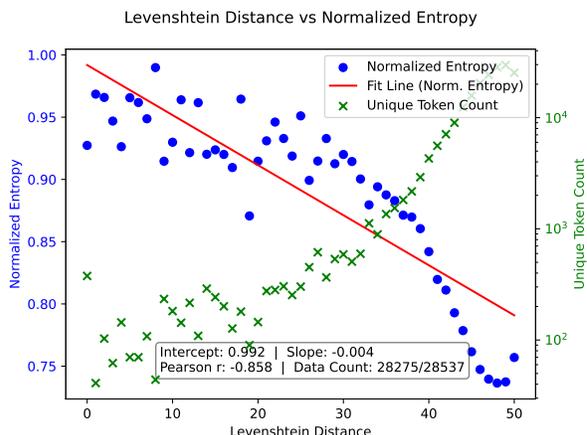
(b)

Figure 7: Temp=0
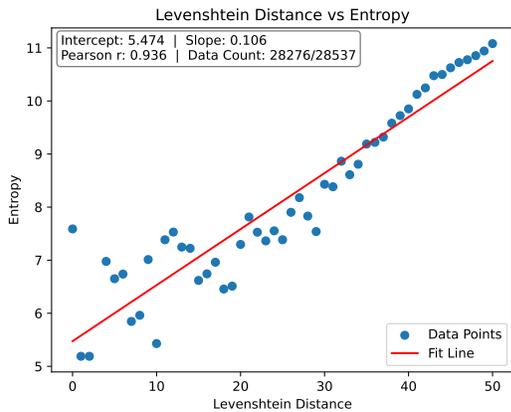
(a)
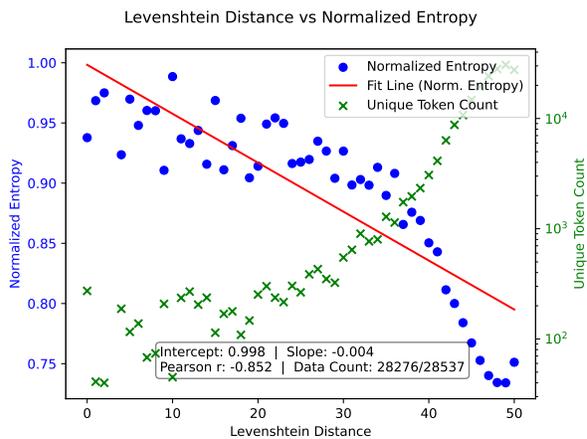
(b)

Figure 8: Temp=0.5

(a)

(b)
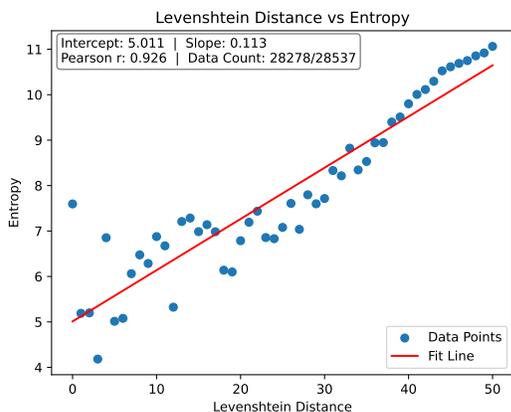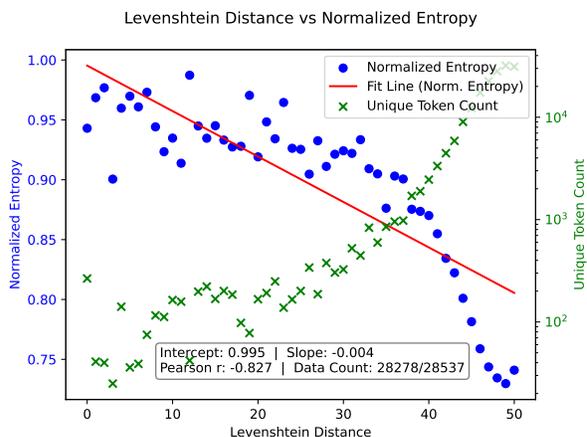
Figure 9: Temp=0.8
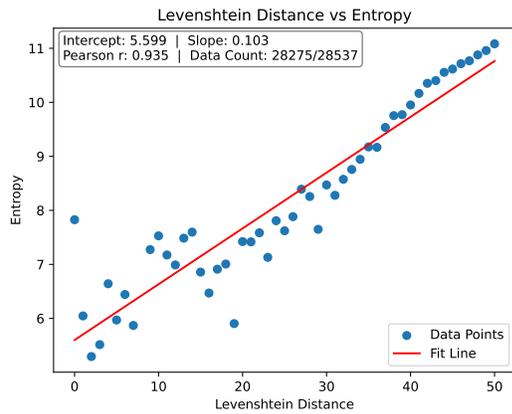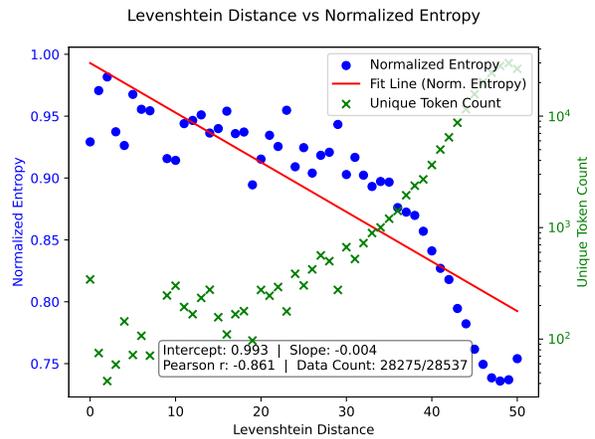
Figure 10: Temp=0.8, Top p=0.5
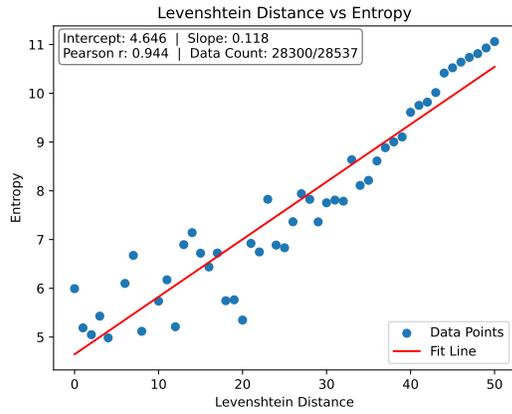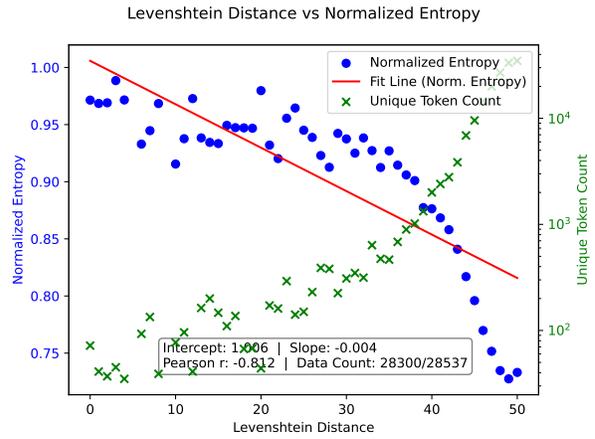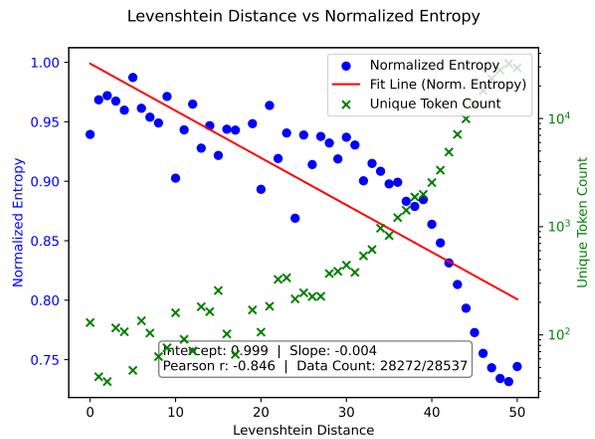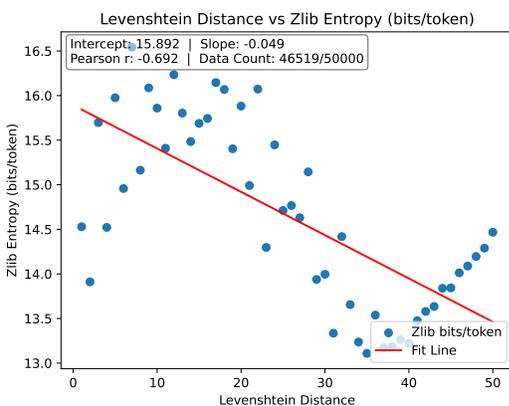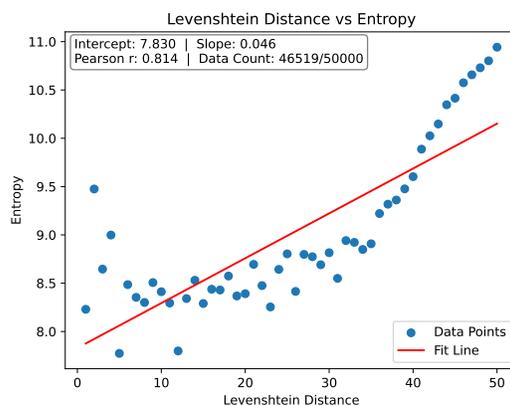

Figure 11: Temp=1
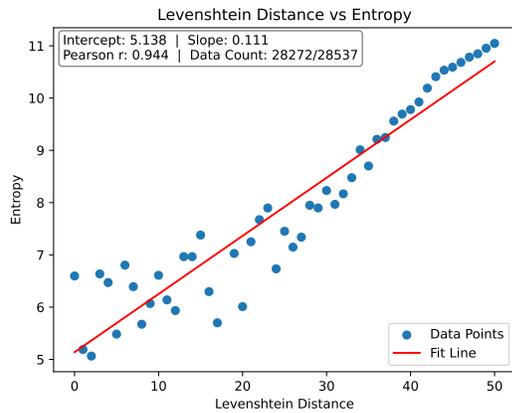
Figure 12: Temp=0.8, top-k=10

C.2 Entropy-Memorization Law

(a) Zlib compression rate *v.s.* memorization score for OpenLlama-7B.

(b) Level-set-based entropy estimate *v.s.* memorization score for OpenLlama-7B.
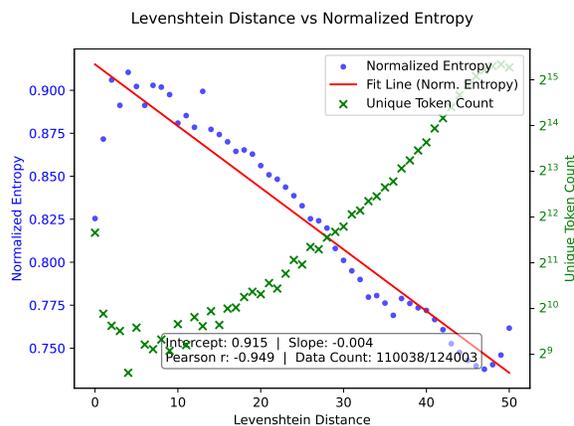
Figure 13: Comparison of compressibility- and entropy-based measurements of memorization for OpenLlama-7B.

The experimental results on OpenLlama-7B are shown in Figure 13a and 13b. The findings are consistent with the pattern observed in the main paper body.

C.3 Normalized Entropy–Memorization Law
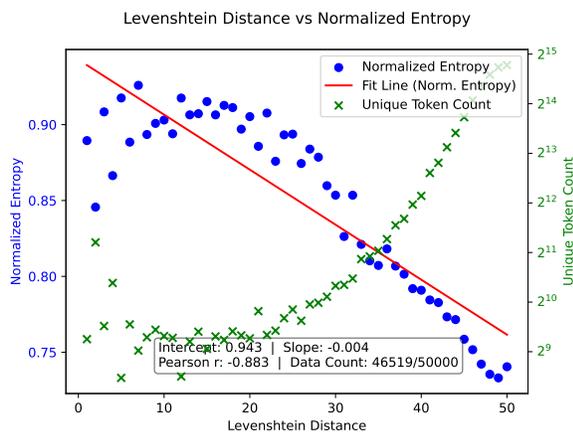
(a) Normalized Entropy–Memorization Law for OpenLlama-7B.

(b) Normalized Entropy–Memorization Law for Pythia-6.9B-dedup.

Figure 14: Normalized Entropy–Memorization Law across different model families.

Fig. 15 presents the (estimated) normalized entropy *v.s.* memorization score. The findings are consistent with the findings that we observed in main paper body. Moreover, $|r|$ increases from 0.82 to 0.97 as generation length decreases.
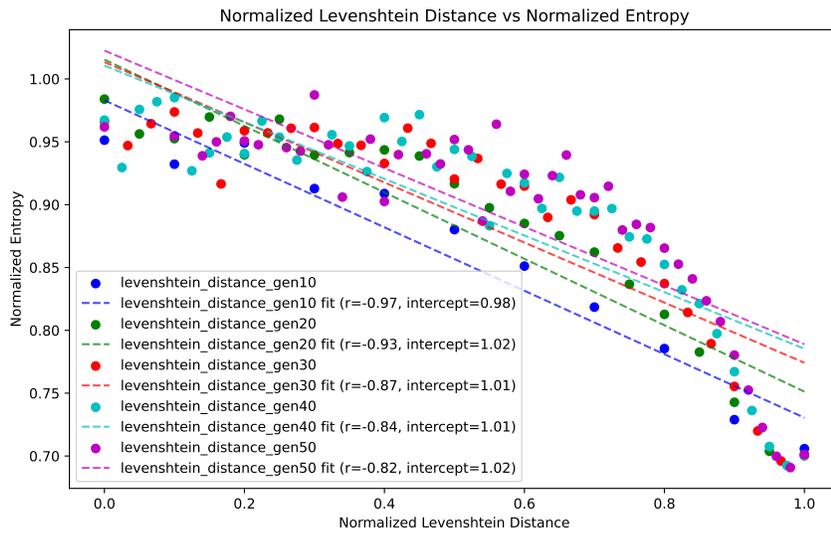
Figure 15: Normalized entropy vs memorization score.

D Entropy-Memorization Law Under Disparate Semantic Data

We employed a semantic-agonistic strategy to sample the dataset in the main body. This section then explores Entropy–Memorization Law under different semantic data. We chunk the sampled dataset into $k = 16$ semantic clusters, develop a strategy to find the semantics of each cluster, and then examine EM Law under these disparate semantic data. This experiment was based on the OLMo-1B model using 240,000 sample pieces.

Semantic Clustering Pipeline   The specific steps are as follows:

1. Extracting semantics of token sequences using sentence embeddings. In this step, sentence embeddings projects a token sequence to a high-dimensional vector space, where semantically similar sequences are mapped to nearby points. Such embedding techniques are implemented by a twisted verison of pre-trained LLM.

2. Clustering. With semantic embeddings, we apply K-Means (Lloyd, 1982) in the latent space and partition the data into $k = 16$ semantic clusters.

3. Identifying semantics of the cluster. Since the clustering methods are performed in a latent space which is not interpretable, we develop an highly-automated pipeline to identify the semantics of each cluster. The core algorithm is differential clustering (Zhang et al., 2025).

4. Run Algorithm 2 on 16 partitions of the dataset. For each cluster, a linear regression is applied. We report the Pearson correlation coefficient, slope, and intercept and visualize the fitted lines.

To implement step 1, we select a popular model *all-mpnet-base-v2* (huggingface, 2025) from Sentence Transformers library in Huggingface as the encoder. In Appendix D.1, we present how we implement step 3, and provide detailed clustering results with labeled semantics.

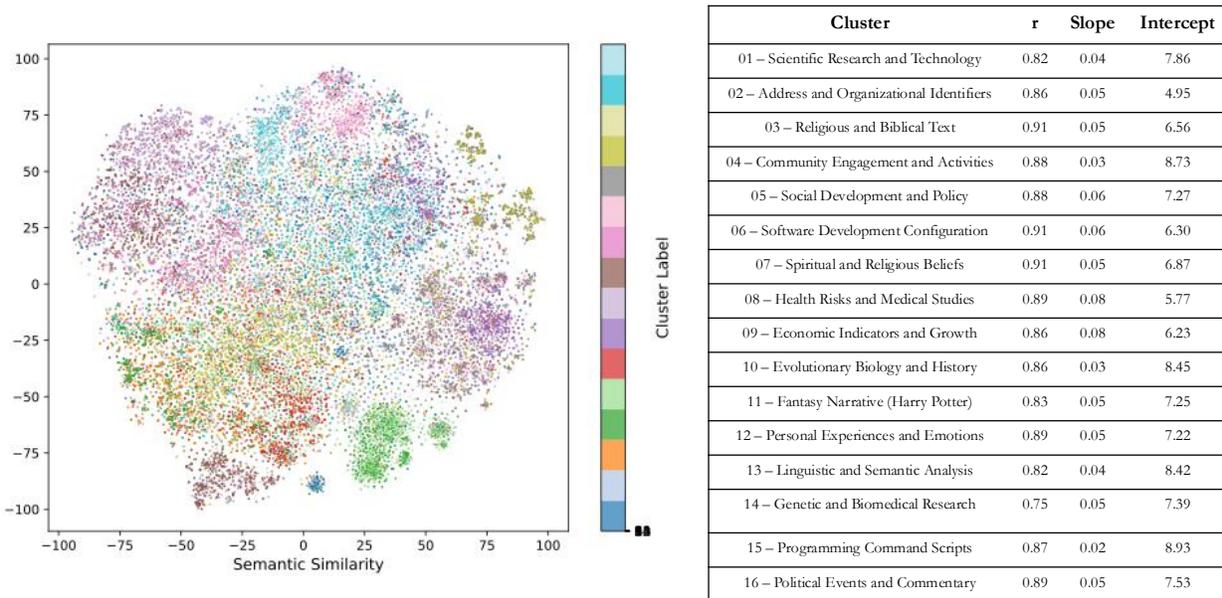| Cluster	r	Slope	Intercept
01 – Scientific Research and Technology	0.82	0.04	7.86
02 – Address and Organizational Identifiers	0.86	0.05	4.95
03 – Religious and Biblical Text	0.91	0.05	6.56
04 – Community Engagement and Activities	0.88	0.03	8.73
05 – Social Development and Policy	0.88	0.06	7.27
06 – Software Development Configuration	0.91	0.06	6.30
07 – Spiritual and Religious Beliefs	0.91	0.05	6.87
08 – Health Risks and Medical Studies	0.89	0.08	5.77
09 – Economic Indicators and Growth	0.86	0.08	6.23
10 – Evolutionary Biology and History	0.86	0.03	8.45
11 – Fantasy Narrative (Harry Potter)	0.83	0.05	7.25
12 – Personal Experiences and Emotions	0.89	0.05	7.22
13 – Linguistic and Semantic Analysis	0.82	0.04	8.42
14 – Genetic and Biomedical Research	0.75	0.05	7.39
15 – Programming Command Scripts	0.87	0.02	8.93
16 – Political Events and Commentary	0.89	0.05	7.53

Figure 16: Clustering sentence embeddings using OLMo-1B pre-training dataset. We apply T-SNE (van der Maaten & Hinton, 2008) for dimension reduction.

Experimental Results   Figure 16 presents overall results. It is verified that Entropy-Memorization Law is observed among all clusters of data. Moreover, another interesting finding is that, in general, different clusters

exhibit distinctive intercept and slope values. For example, cluster 1 (Address and Organizational Identifiers) exhibits low intercept, while cluster 3 (Community Engagement and Activities) and 14 (Programming) exhibit high intercepts.

> We confirm that Entropy-Memorization Law is robust under disparate semantic data clusters. Moreover, intercepts and slopes are different for different semantic data.

D.1 Technical Details on Interpreting Semantics of Each Cluster

To identify the semantics of each cluster, we build a pipeline that significantly reduces human annotation efforts. The pipeline is as follows:

1. Detect distinctive samples within each cluster. Zhang et al. (2025) formulates this task as a *differential clustering* problem and proposes a FINC method. To quantitatively measure semantic distinctions among the 16 clusters obtained via K-means, we conducted 16 FINC comparisons. For each cluster C_i, we set C_i as the novel dataset and the union of the remaining 15 clusters as the reference set. The input to FINC is the sentence embeddings of all instances in the set, and FINC suggests the distinctive samples in the cluster.

2. Keywords summarization. In this stage, we use tri-grams as effective descriptors for naming and interpreting cluster identities. Specifically, we use i) *spaCy* (PyPI, 2025) to perform named entity recognition and dependency parsing to ensure that extracted units are linguistically complete phrases (e.g., *"protective spell harry"*, *"lend broom fly"*), and ii) *YAKE* (Campos et al., 2020) to ranks terms using heuristics such as frequency, context, and positional distribution.

3. Human annotation. Based on summarized keywords, human annotators further summarize the semantics of the cluster.
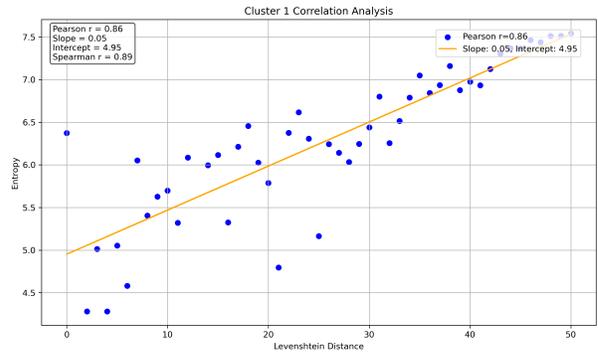
D.2 Entropy–Memorization Plot for Each Cluster

Figure 17 presents the detailed plot for each cluster.

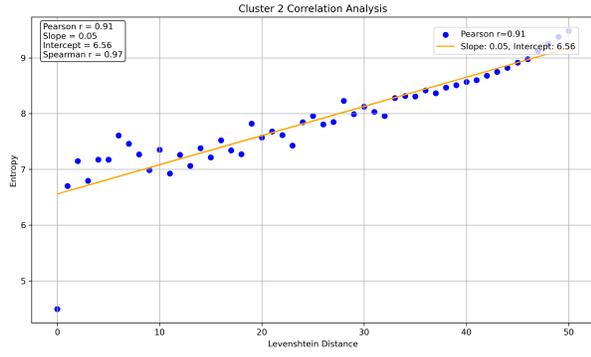## D.3 Interpreting Semantics of Each Cluster
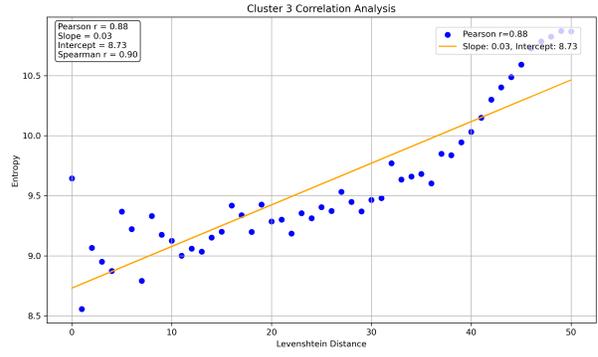
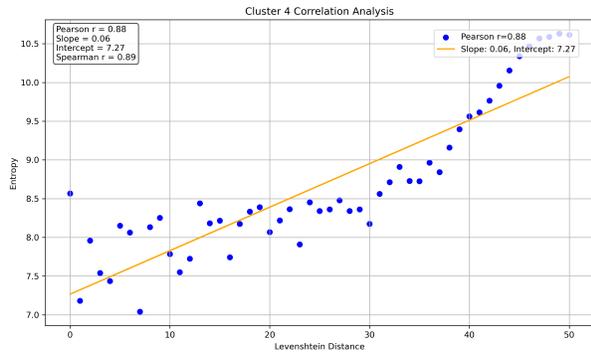Table 5 presents top-5 keywords and human-annotated semantic labels for each cluster.
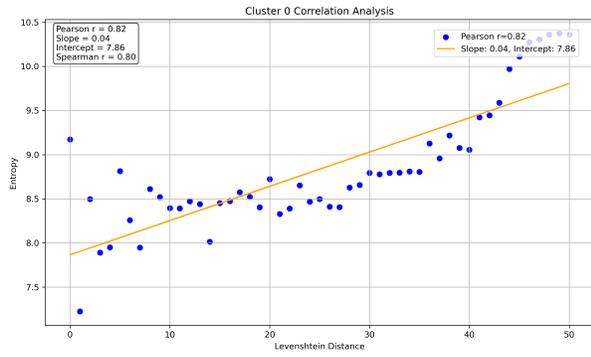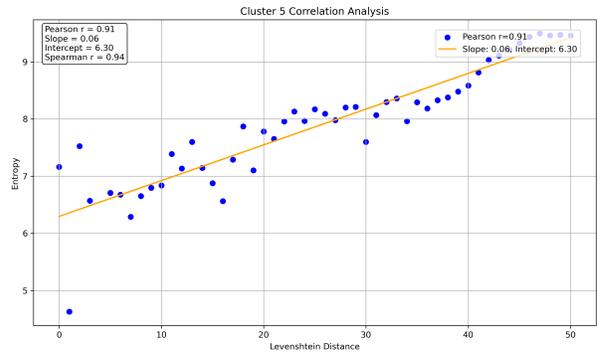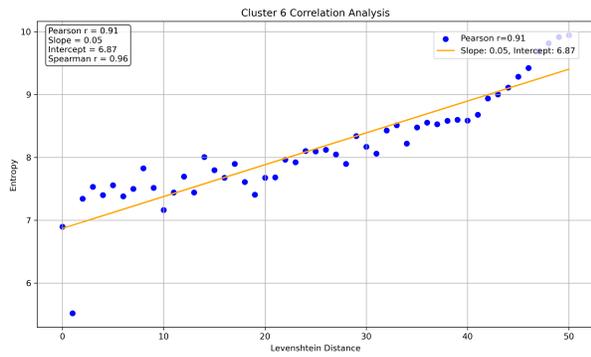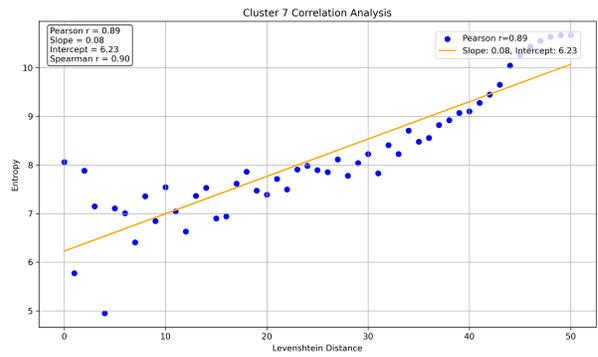
(a) Cluster 0

(b) Cluster 1
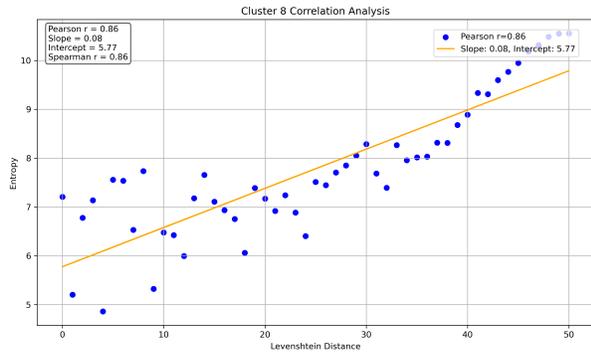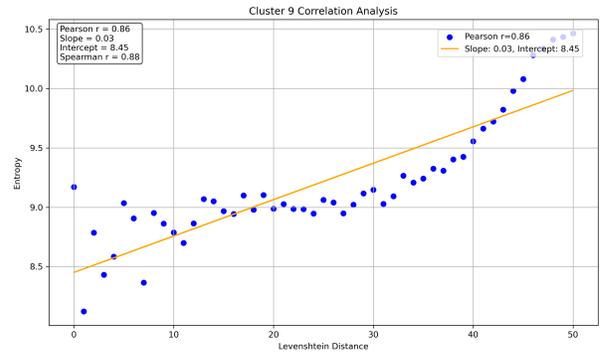
(c) Cluster 2

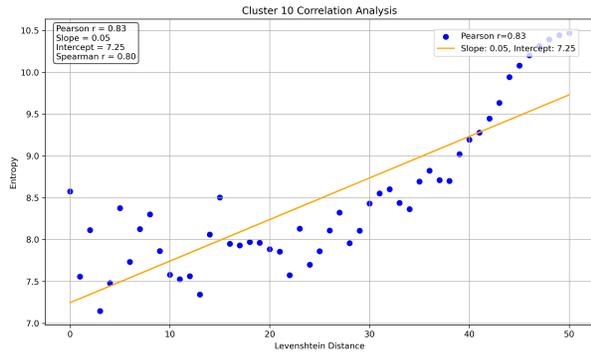(d) Cluster 3

(e) Cluster 4

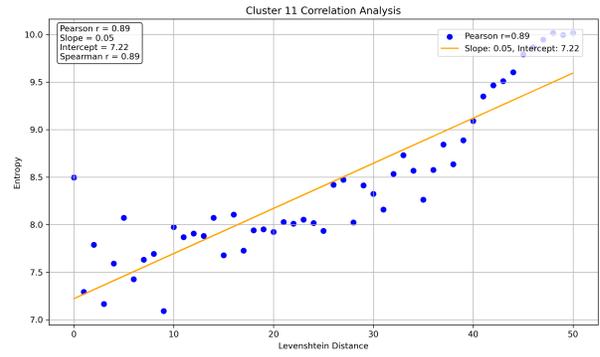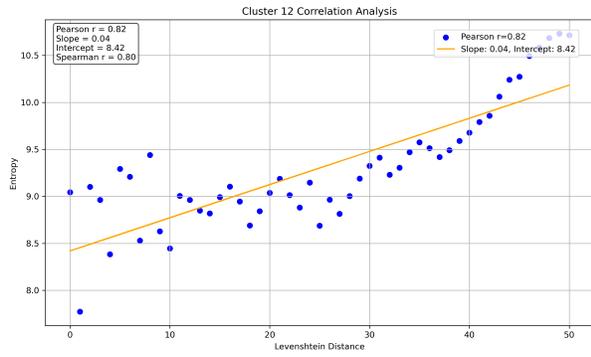(f) Cluster 5

(g) Cluster 6

(h) Cluster 7

Figure 17: Clusters 0–7.
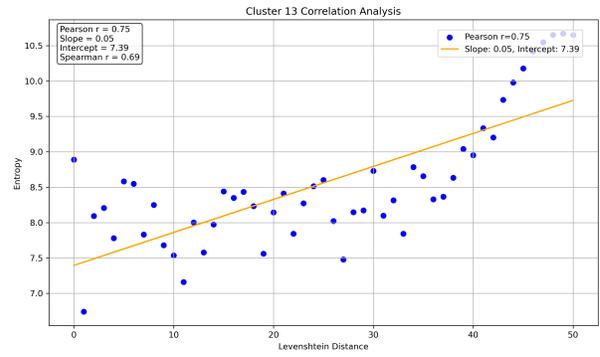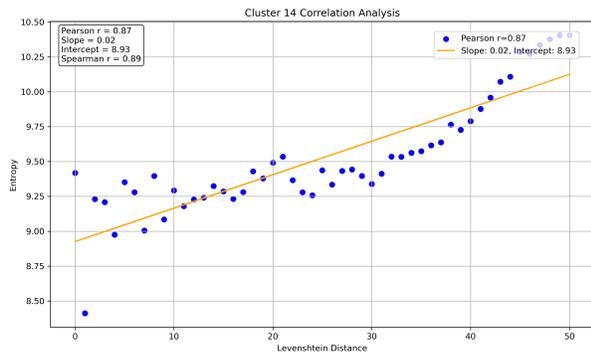
(i) Cluster 8

(j) Cluster 9
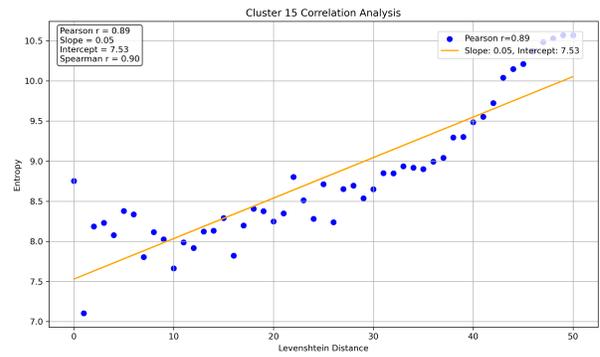
(k) Cluster 10

(l) Cluster 11

(m) Cluster 12

(n) Cluster 13

(o) Cluster 14

(p) Cluster 15

Figure 17: Clusters 8–15.

Table 5: Semantics of each cluster (cluster 0-7)

Cluster 0: Scientific Research and Technology

Score	Keyword Phrase
1.28e-04	translate depth direction
1.28e-04	design fabrication characterization
1.28e-04	pct design fabrication
1.28e-04	manipulation pct design
1.23e-04	sensor control manipulation

Cluster 1: Address and Organizational Identifiers

Score	Keyword Phrase
1.74e-05	street number city
1.54e-05	party committee number
1.54e-05	city number
1.52e-05	type number form
1.46e-05	conduit state number

Cluster 2: Religious and Biblical Texts

Score	Keyword Phrase
7.94e-05	people sword thy
7.94e-05	thy people sword
7.91e-05	jacob son reuban
7.84e-05	son brother house
7.80e-05	son lord hath

Cluster 3: Community Engagement and Activities

Score	Keyword Phrase
1.05e-04	irrigation evaporate leave
1.05e-04	bullet time jump
1.04e-04	community meetup world
1.03e-04	community kid spout
1.02e-04	year electronic music

Cluster 4: Social Development and Policy

Score	Keyword Phrase
1.34e-04	dramatically drive price
1.33e-04	develop skill team
1.33e-04	prefer policy influence
1.33e-04	outwith prefer policy
1.33e-04	sector real passion

Cluster 5: Software Development Configuration

Score	Keyword Phrase
2.39e-05	true plugin proposal
2.29e-05	node optional true
2.27e-05	header content type
2.26e-05	true ellipsis true
2.26e-05	dev true child

Cluster 6: Spiritual and Religious Beliefs

Score	Keyword Phrase
9.67e-05	thy mind thy
9.61e-05	death eternal life
9.54e-05	create sustain universe
9.52e-05	world drive ulterior
9.52e-05	behavior world drive

Cluster 7: Health Risks and Medical Studies

Score	Keyword Phrase
1.06e-04	health increase risk
1.06e-04	evaluate risk factor
1.05e-04	pregnancy increase risk
1.05e-04	disease high risk
1.05e-04	high disease risk

Table 6: Semantics of each cluster (cluster 8-15)

Cluster 8: Economic Indicators and Growth

Score	Keyword Phrase
2.54e-05	billion country oda
2.51e-05	permanent surface runway
2.51e-05	kwh capita growth
2.50e-05	imf intelsat interpol
2.48e-05	price growth rate

Cluster 9: Evolutionary Biology and History

Score	Keyword Phrase
4.49e-04	primatology million year
4.48e-04	mutate gene ancient
4.48e-04	start million million
4.45e-04	paleolithic million year
4.44e-04	account million year

Cluster 10: Fantasy Narrative (Harry Potter)

Score	Keyword Phrase
4.98e-05	dumbledore harry meet
4.95e-05	forward harry ron
4.94e-05	muggle bystander incredulously
4.72e-05	sirius black peril
4.71e-05	harbor end time

Cluster 11: Personal Experiences and Emotions

Score	Keyword Phrase
1.07e-04	hunt season roll
1.06e-04	love good love
1.06e-04	thing work happen
1.05e-04	decision normal result
1.04e-04	understand thing happen

Cluster 12: Linguistic and Semantic Analysis

Score	Keyword Phrase
1.22e-04	interpret apply male
1.20e-04	ingen det finne
1.20e-04	finne ingen det
1.19e-04	title aktivt medlem
1.17e-04	confusion attain agenda

Cluster 13: Genetic and Biomedical Research

Score	Keyword Phrase
8.51e-05	mesenchymal stem cell
8.40e-05	screen gene foster
8.27e-05	interaction show high
8.26e-05	expression level gene
8.25e-05	search perform blast

Cluster 14: Programming Command Scripts

Score	Keyword Phrase
4.64e-06	text text ohm
4.57e-06	delaytimer command
4.03e-06	dark text text
3.94e-06	command runcmd
3.66e-06	cost command type

Cluster 15: Political Events and Commentary

Score	Keyword Phrase
1.41e-04	white house official
1.41e-04	idea african americans
1.38e-04	african americans woman
1.38e-04	end war year
1.37e-04	snc lavalin scandal