

000 001 002 003 004 005 REVISITING NODE AFFINITY PREDICTION 006 IN TEMPORAL GRAPHS 007 008 009

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011 Paper under double-blind review
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ABSTRACT

Node affinity prediction is a common task that is widely used in temporal graph learning with applications in social and financial networks, recommender systems, and more. Recent works have addressed this task by adapting state-of-the-art dynamic link property prediction models to node affinity prediction. However, simple heuristics, such as Persistent Forecast or Moving Average, outperform these models. In this work, we analyze the challenges in training current Temporal Graph Neural Networks for node affinity prediction and suggest appropriate solutions. Combining the solutions, we develop NAVIS - Node Affinity prediction model using Virtual State, by exploiting the equivalence between heuristics and state space models. While promising, training NAVIS is non-trivial. Therefore, we further introduce a dedicated loss function for node affinity prediction. We evaluate NAVIS on TGB and show that it outperforms the state-of-the-art, including heuristics. Our source code is available at <https://anonymous.4open.science/r/NAVIS-0257>

1 INTRODUCTION

Temporal graphs provide a natural way to represent evolving interactions in systems such as trade networks, recommender systems, social platforms, and financial transactions (Kumar et al., 2019; Shetty & Adibi, 2004; Huang et al., 2023). A central challenge in this setting is *future node affinity prediction*: forecasting how strongly a node will interact with other nodes at a future time. This differs from future link prediction, which instead asks whether a particular edge will appear. In contrast, *affinity prediction* requires producing a full ranking over potential neighbors, making it more demanding but also more relevant to many real-world applications (MacDonald et al., 2015; Bertin-Mahieux et al., 2011; Nadiri & Takes, 2022; Shamsi et al., 2022).

In the context of link-level prediction, recent progress in Temporal Graph Neural Networks (TGNNs), including TGN (Rossi et al., 2020), TGAT (Xu et al., 2020), DyGFormer (Yu et al., 2023), and GraphMixer (Cong et al., 2023), has improved state-of-the-art performance. These methods rely on local neighborhood sampling and nonlinear message-passing, which are effective for future link prediction task. However, when applied to node *affinity prediction*, it is evident that they perform worse than simple heuristics such as Persistent Forecast and Moving Average (Huang et al., 2023). This gap suggests that current TGNNs designs do not align well with the inductive biases required for affinity prediction.

Accordingly, this paper aims to answer the following questions: *why do heuristics outperform more sophisticated TGNNs for future node affinity prediction? and can we push TGNNs to do better?* We argue that the advantage arises from a confluence of factors and identify several contributing issues that collectively explain this phenomenon, including: (i) **Expressivity**. Existing TGNNs cannot represent a simple Moving Average of past affinities, because their nonlinear updates and reliance on sampled neighborhoods prevent them from maintaining the required linear memory. (ii) **Loss mismatch**. Cross-entropy, commonly used as a loss function for link prediction, is not well-aligned with the ranking nature of affinity tasks. (iii) **Global temporal dynamics**. Affinities often depend on shared network-wide trends (e.g., regime shifts), which local sampling does not capture. (iv) **Information loss**. TGNNs are broadly categorized into memory-based and non-memory-based architectures. Memory-based models (e.g., TGN (Rossi et al., 2020) and DyRep (Trivedi et al., 2019)) maintain per-node states; however, batch processing of events can cause short-term updates within a batch to be

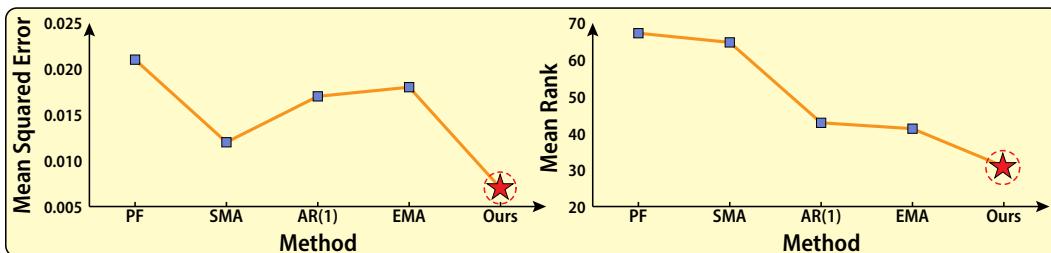


Figure 1: **Synthetic node affinity experiment.** Node affinities depend on a global, regime-switching latent $g(t)$ with nonlinear component $g(t)^2$ and node-specific phases. Baselines relying only on per-node histories (PERSISTENT FORECAST, SMA, EMA) or a local AR(1) SSM cannot recover the shared latent space, leading to higher error. OURS, indicating NAVIS, maintains a virtual global state, achieves the lowest error on both metrics. In Appendix C we provide the full experiment details and baseline descriptions.

missed (Feldman & Baskin, 2024). In contrast, non-memory-based methods recompute embeddings from scratch at prediction time and maintain no states, making them prone to overlooking earlier updates in the evolving graph (Cong et al., 2023; Yu et al., 2023).

These observations guide and motivate our work. Our key idea is that heuristics like persistent Forecast and Moving Average are not arbitrary fixes, but rather special cases of *linear state space models* (SSMs) (Gu et al., 2021; 2022; Eliasof et al., 2025), which naturally provide memory and long-range temporal dependencies. By embedding the structure of SSMs into a learnable TGNN, we can retain the robustness of heuristics while extending their expressivity.

Building on this idea, we introduce NAVIS (Node Affinity prediction with Virtual State). NAVIS maintains both per-node state and a virtual global state that co-evolve with the dynamic graph structure, thereby providing a principled memory mechanism suitable for the requirements of future node affinity prediction. Additionally, to address the loss mismatch, we propose a rank-based objective that is better suited to ordinal affinity outputs. Importantly, we do not claim to have solved the problem entirely: the approach still inherits limitations, for example, in modeling complex multi-hop dependencies. Nonetheless, our results indicate meaningful progress that can shape the future of node affinity prediction.

To illustrate the existing challenges and the effectiveness of NAVIS, we include a synthetic node-affinity experiment in Figure 1. In this controlled setting, node affinities depend on a hidden global process with nonlinear structure. Although simple heuristics outperform state-of-the-art TGNNs, since they capture only per-node history, as TGNNs, they fail to recover the shared latent space. By maintaining a virtual global state, NAVIS achieves the lowest error. Although simplified, this example highlights the importance of global temporal dynamics and motivates one of the design choices in NAVIS. We provide the full experiment details in Appendix C.

Our contributions.

1. We theoretically show that simple heuristics are special cases of linear SSMs, and use this connection to design a TGNN architecture that generalizes them, making it more expressive.
2. We analyze why cross-entropy loss is suboptimal for affinity prediction, and develop a rank-based alternative that improves optimization and aligns with evaluation metrics.
3. We provide extensive experiments on the Temporal Graph Benchmark (TGB) and additional datasets, demonstrating consistent improvements over both heuristics and prior TGNNs. The significance of our experiments is that they validate the importance of aligning model inductive biases and training objectives with the task.

2 BACKGROUND

In this section, we provide essential information related to our work, from basic notations and definitions, to simple future node affinity prediction baselines, which we generalize in NAVIS.

108 **Notations and Definitions.** We consider a *continuous-time dynamic graph* (CTDG) as a stream of
 109 timestamped interactions between ordered node pairs drawn from the node set $\mathcal{V} = \{1, \dots, n\}$. The
 110 CTDG observed up to time t is:

$$\mathcal{G}_t = \{(u_j, v_j, \tau_j, w_j)\}_{j=1}^{J(t)}, \quad (1)$$

111 where $u_j, v_j \in \mathcal{V}$ denote source and target nodes, $\tau_j \in \mathbb{R}^+$ is the interaction time, $w_j \in \mathbb{R}$ is its
 112 weight and $J(t)$ is the number of interactions occurred up to time t . The *future node affinity prediction*
 113 problem seeks, for a query node $u \in \mathcal{V}$ and a future time $t^+ > t$, to estimate the node's affinity to
 114 every other node $v \in \mathcal{V} \setminus \{u\}$ conditioned on \mathcal{G}_t . A parameterized model F_θ produces the predicted
 115 affinity scores vector

$$\mathbf{s} = F_\theta(u, \mathcal{G}_t, t^+) \in \mathbb{R}^{|\mathcal{V}|}. \quad (2)$$

116 Given the ground truth affinities \mathbf{y} realized at t^+ , we learn θ by minimizing a task-specific loss ℓ :

$$\min_{\theta} \sum_{u \in \mathcal{V}} \ell(F_\theta(u, \mathcal{G}_t, t^+), \mathbf{y}). \quad (3)$$

117 **Historical Average.** A simple interaction-level baseline is the historical average, which estimates
 118 each source–destination affinity by the mean weight of all past interactions observed prior to t^+ :

$$\mathbf{s}(v) = \frac{1}{\#u, v} \sum_{(u, v, \tau_j, w_j) \in \mathcal{G}_t} w_j. \quad (4)$$

119 Here $\#_{u, v}$ denotes the number of observed (u, v) interactions up to time t .

120 **Moving Average and State Space Models.** Allowing the tested models to use previous ground truth
 121 affinity vectors at inference, instead of the full fine-grained CTDG, creates additional schemes to
 122 predict the future affinity vectors. The *Persistent Forecast* (PF) heuristic is the most basic one that
 123 utilizes previous affinity vectors. PF outputs the previous affinity vector as future prediction, i.e.:

$$\mathbf{s} = \mathbf{x} \quad (5)$$

124 where \mathbf{x} is the previous (most recent) ground truth affinity vector.

125 Another natural vector-level heuristic for future node affinity prediction is the *Exponential Moving*
 126 *Average* (EMA), which maintains an estimate of a node's affinity vector by exponentially weighting
 127 recent affinity vectors:

$$\mathbf{s} = \mathbf{h}_i = \alpha \mathbf{h}_{i-1} + (1 - \alpha) \mathbf{x} \quad (6)$$

128 here, $\alpha \in [0, 1]$ is the decay parameter and $\mathbf{h}_i, \mathbf{h}_{i-1}$ are hidden states. Note that PF is a specific case
 129 of EMA where $\alpha = 0$.

130 An alternative is the *Simple Moving Average* (SMA) with window size w , which averages over a
 131 finite window. Its recursive form is:

$$\mathbf{s} = \mathbf{h}_i = \frac{w-1}{w} \mathbf{h}_{i-1} + \frac{1}{w} \mathbf{x}. \quad (7)$$

132 EMA and SMA use an infinite geometric decay, retaining long but diminishing memory. Although
 133 these filters can capture long-term dynamics, they are limited to fixed, hand-crafted memory kernels.
 134 To move beyond such ad-hoc designs, we can view EMA through the lens of latent dynamical
 135 systems.

136 Departing from simple averaging approaches, State Space Models (SSMs) provide a principled
 137 framework to model temporal sequences via hidden state evolution and observation processes
 138 (Hamilton, 1994; Aoki, 2013). A discrete linear SSM is defined as:

$$\mathbf{h}_i = \mathbf{A} \mathbf{h}_{i-1} + \mathbf{B} \mathbf{x}, \quad (8)$$

$$\mathbf{s} = \mathbf{C} \mathbf{h}_i + \mathbf{D} \mathbf{x}, \quad (9)$$

139 Where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are learnable matrices.

140 **Beyond Moving Average.** We show that vector-level heuristics are instances of SSMs. This reveals
 141 a clear hierarchy in model expressiveness, where SSMs are more expressive, generalizing Moving
 142 Average.

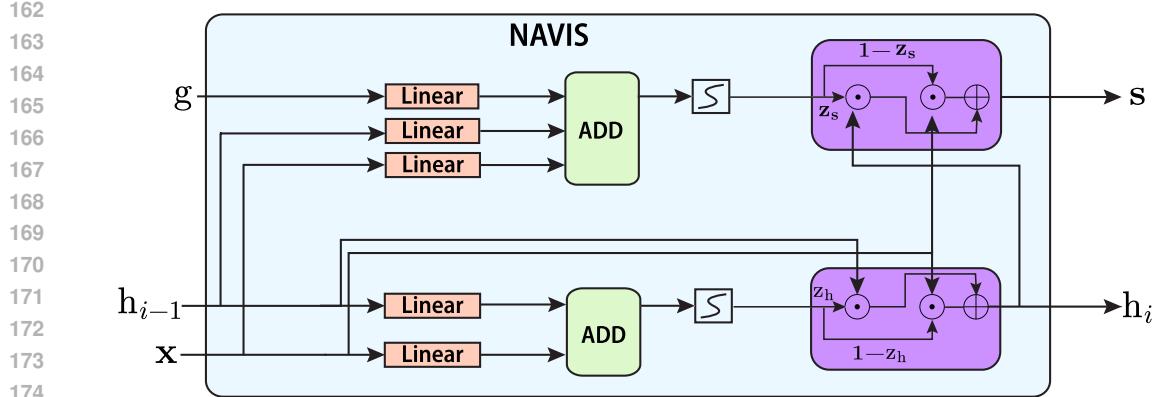


Figure 2: NAVIS architecture for node affinity prediction. The current state and previous affinity vector are projected through linear transformations and aggregated into a new state. A lightweight gated mechanism ensures a persistent, linear input–output. The predicted affinity vector is then produced directly from this state based on the virtual global state.

Theorem 1 (Linear SSMs generalize basic heuristics). *Let \mathcal{H} be the set of basic heuristics (PF, SMA, EMA), $\mathcal{F}_{\text{lin-SSM}}$ be the set of maps realizable by the linear SSM in Equation 8 and Equation 9. Then, the following strict inclusion holds:*

$$\mathcal{H} \subsetneq \mathcal{F}_{\text{lin-SSM}}. \quad (10)$$

Proof. $\mathcal{H} \subsetneq \mathcal{F}_{\text{lin-SSM}}$. First, we show containment (\subseteq). Each heuristic corresponds to a specific choice of $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$:

- EMA(α): $\mathbf{A} = \alpha \mathbf{I}$, $\mathbf{B} = (1 - \alpha) \mathbf{I}$, $\mathbf{C} = \mathbf{I}$, $\mathbf{D} = 0$.
- SMA(w): $\mathbf{A} = \frac{w-1}{w} \mathbf{I}$, $\mathbf{B} = \frac{1}{w} \mathbf{I}$, $\mathbf{C} = \mathbf{I}$, $\mathbf{D} = 0$.
- PF: $\mathbf{A} = 0$, $\mathbf{B} = 0$, $\mathbf{C} = 0$, $\mathbf{D} = I$.

Next, we show the inclusion is strict (\subsetneq). Consider a set of 2×2 SSM matrices with $\mathbf{A} = \text{diag}(\alpha_1, \alpha_2)$, $\mathbf{B} = \text{diag}(1 - \alpha_1, 1 - \alpha_2)$, $\mathbf{C} = \mathbf{I}$, $\mathbf{D} = 0$ where $\alpha_1 \neq \alpha_2$. These models' weights on past inputs cause each entry in the affinity vector to decay at a different rate. This behavior cannot be replicated by the single decay of an EMA. Thus, $\mathcal{F}_{\text{lin-SSM}} \setminus \mathcal{H} \neq \emptyset$. \square

Implications. This hierarchy that stems from Theorem 1 reveals the potential of SSMs: although simple heuristics outperform any state-of-the-art TGNN, a carefully designed SSM-based TGNN can potentially outperform these heuristics because it generalizes them.

3 METHOD

This section contains our key contributions and is organized as follows: in Section 3.1 we detail factors that hinder TGNNs performance for future node affinity prediction. Motivated by these findings, in Section 3.2 we introduce our model. In Section 3.3 we detail how to train it effectively.

3.1 WHAT HINDERS TGNNs IN FUTURE NODE AFFINITY PREDICTION

As established in Section 2, common heuristics like PF and EMA are special cases of SSMs. This advantage of SSMs over heuristics is achieved due to the fact that SSMs can output a linear combination of the previous affinity vector and the previous predicted affinity vector. We now discuss another important theoretical direction: RNN (Elman, 1990), LSTM (Hochreiter & Schmidhuber, 1997) or GRU (Cho et al., 2014) cells — that are commonly used in popular memory-based TGNNs

(Trivedi et al., 2019; Rossi et al., 2020; Tjandra et al., 2024) — cannot express the most basic heuristic of PF, thereby hindering memory-based TGNNs performance.

Theorem 2. *Let $\{h_i\}_{i \geq 0}$ be the hidden states generated by a single standard RNN cell, LSTM cell, or GRU cell driven by inputs $\{x_i\}_{i \geq 1} \subseteq \mathbb{R}^d$. There do not exist parameters of these cells such that, for all t and all input sequences, $h_i = x_i$ (PF).*

Proof. We assume the standard elementwise nonlinearities $\sigma(u) = \frac{1}{1+e^{-u}} \in (0, 1)$ and $\tanh(u) \in (-1, 1)$. Below, we present the equations that define common recurrent models.

$$(RNN) \quad \mathbf{h}_i = \phi(W_h \mathbf{h}_{i-1} + W_x \mathbf{x}_i + \mathbf{b}), \quad \phi = \tanh. \quad (11)$$

$$(LSTM) \quad \begin{aligned} \mathbf{i}_i &= \sigma(W_i[\mathbf{h}_{i-1}; \mathbf{x}_i] + \mathbf{b}_i), & \mathbf{f}_i &= \sigma(W_f[\mathbf{h}_{i-1}; \mathbf{x}_i] + \mathbf{b}_f), \\ \mathbf{o}_i &= \sigma(W_o[\mathbf{h}_{i-1}; \mathbf{x}_i] + \mathbf{b}_o), & \mathbf{g}_i &= \tanh(W_g[\mathbf{h}_{i-1}; \mathbf{x}_i] + \mathbf{b}_g), \\ \mathbf{c}_i &= \mathbf{f}_i \odot \mathbf{c}_{i-1} + \mathbf{i}_i \odot \mathbf{g}_i, & \mathbf{h}_i &= \mathbf{o}_i \odot \tanh(\mathbf{c}_i). \end{aligned} \quad (12)$$

$$(GRU) \quad \begin{aligned} \mathbf{z}_i &= \sigma(W_z[\mathbf{h}_{i-1}; \mathbf{x}_i] + \mathbf{b}_z), & \mathbf{r}_i &= \sigma(W_r[\mathbf{h}_{i-1}; \mathbf{x}_i] + \mathbf{b}_r), \\ \tilde{\mathbf{h}}_i &= \tanh(W[\mathbf{r}_i \odot \mathbf{h}_{i-1}; \mathbf{x}_i] + \mathbf{b}), & & \\ \mathbf{h}_i &= (1 - \mathbf{z}_i) \odot \mathbf{h}_{i-1} + \mathbf{z}_i \odot \tilde{\mathbf{h}}_i. & & \end{aligned} \quad (13)$$

We show that no choice of parameters in RNN, LSTM or GRU yields the map $\mathbf{h}_i = \mathbf{x}_i$.

RNN: $\mathbf{h}_t = \tanh(W_h \mathbf{h}_{t-1} + W_x \mathbf{x}_t + \mathbf{b})$ takes values in $(-1, 1)^d$, while the mapping $\mathbf{x}_t \mapsto \mathbf{h}_t$ is unbounded on \mathbb{R}^d . Hence, equality $\forall \mathbf{x}_i$ is impossible.

LSTM: $\mathbf{h}_i = \mathbf{o}_i \odot \tanh(\mathbf{c}_i)$ with $\mathbf{o}_i \in (0, 1)^d$ and $\tanh(\mathbf{c}_i) \in (-1, 1)^d$ implies $\mathbf{h}_i \in (-1, 1)^d$, again contradicting the unbounded range of \mathbf{h}_i .

GRU: Set $\mathbf{h}_{i-1} = 0$. Then $\mathbf{h}_i = \mathbf{z}_i \odot \tilde{\mathbf{h}}_i$ with $\mathbf{z}_i \in (0, 1)^d$ and $\tilde{\mathbf{h}}_i = \tanh(\cdot) \in (-1, 1)^d$, hence $\mathbf{h}_i \in (-1, 1)^d$ cannot equal \mathbf{x}_i for arbitrary $\mathbf{x}_i \in \mathbb{R}^d$. \square

Theorem 2 has profound implications: any memory-based TGNN that applies standard memory cells cannot represent even the simplest heuristic – Persistent Forecasting (PF), which have been proven empirically to perform exceptionally well on node affinity tasks. This theoretical result is at the underpinnings of our work, and motivates us to generalize heuristics while remaining more expressive. Thus, we design NAVIS as a simple learnable linear SSM. Tjandra et al. (2024) showed that identifying the target node when updating the node state of the source node is a necessary property. Without this property, TGNNs cannot express the persistent forecast heuristic. In the proof of Theorem 2, we explicitly assume that the target node is identified via its corresponding index in the affinity vector of the source node. Hence, Theorem 2 holds even when the target node is identified, revealing another necessary condition for expressing the persistent forecast heuristic.

Current TGNNs Underutilize Available Temporal Information

We argue that a major source of empirical underperformance in TGNNs is the systematic loss of temporal information. Memory-based TGNNs often rely on batching for tractable runtimes; however, batching can obscure multiple interactions that affect the same node within a single batch window, thereby dropping intermediate state transitions (Feldman & Baskin, 2024). In contrast, non-memory architectures, e.g., DyGFormer (Yu et al., 2023), GraphMixer (Cong et al., 2023), and DyGMamba (Ding et al., 2025), avoid within-batch omissions by maintaining a buffer of recent events and recomputing node embeddings on demand. To bound latency, these buffers are fixed in size; once filled, the oldest events are evicted, discarding potentially informative long-term interaction events. This hard truncation differs from EMA, where the influence of older events decays but remains non-zero, allowing previous interaction events to shape future affinity predictions. A further, underexploited source of information is the evolving global graph state. Memory-based TGNNs typically use few message-passing layers per state update, limiting the incorporation of broader context (Rossi et al., 2020), while non-memory methods commonly restrict buffered events to 1-hop

270 neighborhoods. Even new sophisticated state-of-the-art TGNNs do not utilize the full global state (Lu
 271 et al., 2024; Gravina et al., 2024). We leverage these observations to design a TGNN that preserves
 272 fine-grained temporal transitions, retains long-term interaction events, and integrates global graph
 273 context to maximize the use of available information to give an accurate prediction for future node
 274 affinity.

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276

277 3.2 NAVIS: NODE AFFINITY PREDICTION WITH A GLOBAL VIRTUAL STATE

278

279 Motivated by Section 3.1, we propose NAVIS—a node-affinity prediction model that utilizes a linear
 280 state-space mechanism to maintain a state $\mathbf{h} \in \mathbb{R}^d$ for each node, and a virtual global state $\mathbf{g} \in \mathbb{R}^d$,
 281 where $d = |\mathbb{V}|$ denotes the affinity-space dimension. Transitions are computed by a learnable linear
 282 SSM that enforces the output to be a linear combination of the inputs, akin to an EMA, but with
 283 flexibility to allow the coefficient α to be computed at runtime from the current events rather than
 284 being fixed. Concretely, we define NAVIS as the following sequence of update steps:

285

$$\begin{aligned} \mathbf{z}_h &= \sigma(W_{xh}\mathbf{x} + W_{hh}\mathbf{h}_{i-1} + \mathbf{b}_h), \\ \mathbf{h}_i &= \mathbf{z}_h \odot \mathbf{h}_{i-1} + (1 - \mathbf{z}_h) \odot \mathbf{x}, \\ \mathbf{z}_s &= \sigma(W_{xs}\mathbf{x} + W_{hs}\mathbf{h}_i + W_{gs}\mathbf{g} + \mathbf{b}_s), \\ \mathbf{s} &= \mathbf{z}_s \odot \mathbf{h}_i + (1 - \mathbf{z}_s) \odot \mathbf{x}, \end{aligned} \quad (14)$$

290

291

292 where $\mathbf{x}, \mathbf{h}_{i-1}, \mathbf{h}_i, \mathbf{g}, \mathbf{s} \in \mathbb{R}^d$ are the previous affinity vector, previous node state, updated
 293 node state, virtual global vector, and predicted affinity vector, respectively. The parameters
 294 $W_{xh}, W_{hh}, W_{xs}, W_{hs}, W_{gs} \in \mathbb{R}^{1 \times d}$ are learnable weights, $\mathbf{b}_h, \mathbf{b}_s \in \mathbb{R}^d$ are learnable biases, and
 295 σ is the sigmoid function, forcing $\mathbf{z}_h, \mathbf{z}_s$ to be in $[0, 1]$, to maintain conceptual similarity with α in
 296 Equation (6). In Figure 2 we provide a detailed scheme of NAVIS.

297

298 We compute the virtual global vector \mathbf{g} by maintaining a buffer of the most recent previous affinity
 299 vectors, globally. Then, the virtual global vector is computed by performing aggregation over all the
 300 vectors in the buffer. **The goal of the global vector is to detect a global trend (e.g, a new song or a**
 301 **new TV series that is globally streamed) before we are queried about a specific node.** In practice,
 302 aggregating the buffer with the most recent vector selection is efficient and empirically effective, as
 303 we show later in Section 4.

304

305 **Handling a full CTDG.** When a previous affinity vector is unavailable and predictions must rely
 306 solely on interaction weights of the CTDG, we estimate the previous affinity vector of u , \mathbf{x} , via $\hat{\mathbf{x}}$.
 307 We initialize $\hat{\mathbf{x}} = \mathbf{0}$ and, upon each weighted interaction between source node u and destination node
 308 v , add the interaction weight to the v -th entry of $\hat{\mathbf{x}}$. When $\hat{\mathbf{x}}$ is required for prediction, we normalize
 309 it by dividing by the sum of its entries. After computing the future affinity vector of u we again set
 310 $\hat{\mathbf{x}} = \mathbf{0}$.

311

312 **Key Properties of NAVIS.** We note that NAVIS generalizes EMA and other heuristics by allowing
 313 the gates \mathbf{z}_h and \mathbf{z}_s to adapt to new information, in contrast to a fixed α . Large gate values enable the
 314 model to retain long-term information when beneficial. Notably, NAVIS does not rely on neighbors'
 315 hidden states, unlike other memory-based TGNNs, and therefore is compatible with the t-Batch
 316 mechanism (Kumar et al., 2019), enabling efficient batching without missing updates. In addition,
 317 we show in Appendix F that incorporating global information via \mathbf{g} can improve the accuracy of the
 318 predicted affinity vector when global trends affect the nodes' affinities.

319

320 **NAVIS for large-scale graphs.** For graphs with N nodes, the number of learnable parameters in
 321 NAVIS scales as $O(N)$, which can be prohibitive for graphs with millions of nodes. To make NAVIS
 322 practical at this scale, we introduce a sparsified affinity prediction pipeline. Specifically, for each
 323 node we retain only the entries corresponding to candidate target nodes. In real-world settings, such
 324 as streaming services where users and movies are nodes and interactions are edges, we are interested
 325 in each user's affinity to movies rather than to other users. In practice, this substantially reduces the
 326 parameter count of NAVIS. For example, on the tgbn-token dataset (Shamsi et al., 2022), which
 327 records user–token interactions, NAVIS requires about (5,000) parameters, while the graph contains
 328 over (60,000) nodes. We further provide a detailed empirical runtime and memory analysis of NAVIS
 329 in Appendix G.

324 3.3 LEARNING WITH RANK-BASED LOSS: WHY CROSS-ENTROPY FAILS
325

326 With NAV1S specified in Section 3.2, we now turn to the question of *how to train it effectively*.
327 Because most downstream uses of affinity vectors depend on the induced ordering of candidates
328 rather than the actual affinity values (Huang et al., 2023; Tjandra et al., 2024), the choice of the loss
329 function is critical. Most TGNNS use the cross-entropy loss (Luo & Li, 2022; Yu et al., 2023; Tjandra
330 et al., 2024), which treats the output as a categorical distribution and ignores ordinal structure. As we
331 show next, this property is suboptimal, and requires a designated loss to address this limitation.

332 **The Limitation of Cross-Entropy Loss.** Let $\mathbf{y}, \mathbf{s} \in \mathbb{R}^d$ be the ground-truth and predicted affinity
333 vectors. The cross-entropy loss reads:

$$334 \quad \ell_{\text{CE}}(\mathbf{s}, \mathbf{y}) = - \sum_{v \in \mathbb{V}} \mathbf{y}(v) \log[\text{softmax}(\mathbf{s})(v)]. \quad (15)$$

335 By construction, this loss is suboptimal because it penalizes well-ranked predictions with mismatched
336 magnitudes. We formalize this claim in Theorem 3.

337 **Theorem 3** (Cross-Entropy is Suboptimal for Ranking). *There exist infinitely many triplets of \mathbf{y} , a
338 ground-truth affinity vector, and $\mathbf{s}_1, \mathbf{s}_2$, two predicted affinity vectors such that:*

339 $\text{rank}(\mathbf{s}_1) = \text{rank}(\mathbf{y})$ and $\text{rank}(\mathbf{s}_2) \neq \text{rank}(\mathbf{y})$, where $\ell_{\text{CE}}(\mathbf{s}_1, \mathbf{y}) > \ell_{\text{CE}}(\mathbf{s}_2, \mathbf{y})$

340 *Proof.* Set $\mathbf{y} = [0.4, 0.35, 0.25]$, $\mathbf{s}_1 = [0.8, 0.15, 0.05]$ (correct rank), $\mathbf{s}_2 = [0.35, 0.4, 0.25]$ (wrong
341 rank). Then $1.091 = \ell_{\text{CE}}(\mathbf{s}_2, \mathbf{y}) < \ell_{\text{CE}}(\mathbf{s}_1, \mathbf{y}) = 1.105$, where \mathbf{s}_1 ranks the same as \mathbf{y} while \mathbf{s}_2 does
342 not. Since ℓ_{CE} is a continuous function, there are infinitely many such triplets. \square

343 To address this shortcoming, we train NAV1S using *Lambda Loss* (Burges et al., 2006) that is defined
344 as follows:

$$345 \quad \ell_{\text{Lambda}}(\mathbf{s}, \mathbf{y}) = \sum_{y_i > y_j} \log_2 \left(\frac{1}{1 + e^{-\sigma(\mathbf{s}_{\pi_i} - \mathbf{s}_{\pi_j})}} \right) \delta_{ij} |A_{\pi_i} - A_{\pi_j}|, \quad (16)$$

346 where π_i is the index of the node at rank i after sorting the affinity scores and $A_{\pi_i}, \delta_{ij}, D_i$ are defined
347 as:

$$348 \quad A_{\pi_i} = \frac{2^{y_{\pi_i}} - 1}{\text{maxDCG}}, \quad \delta_{ij} = \left| \frac{1}{D_{|i-j|}} - \frac{1}{D_{|i-j|+1}} \right|, \quad D_i = \log_2(1 + i) \quad (17)$$

349 and maxDCG is the maximum Discounted Cumulative Gain (DCG) computed by:
350 $\max_{\pi'} \sum_{i=1}^d \frac{y_{\pi'(i)}}{\log_2(i+1)}$. The loss in Equation (16) was previously shown to be effective for ranking
351 tasks (Burges et al., 2011; Wang et al., 2018). This loss directly optimizes rank-based objectives
352 via pairwise “lambdas” that approximate the gradient of non-differentiable ranking-based metrics,
353 focusing learning on swaps that most impact the final ranking.

354 **Pairwise Margin Regularization.** In our experiments, we discovered that the use of the loss in
355 Equation (16) alone is not sufficient for future node affinity prediction tasks, as we elaborate in
356 Appendix F. Hence, we suggest the following regularization:

$$357 \quad \ell_{\text{Reg}}(\mathbf{s}, \mathbf{y}) = \sum_{y_i > y_j} \max(0, -(s_{\pi_i} - s_{\pi_j}) + \Delta), \quad (18)$$

358 Here, Δ is a hyperparameter that represents the minimum margin required between each pair of
359 affinity scores. The goal of this regularization is to prevent NAV1S from incorrectly learning to shrink
360 the affinity scores to minimize Equation (16).

372 4 EXPERIMENTS
373

374 We evaluate NAV1S across multiple future node affinity prediction benchmarks and compare it with
375 recent state-of-the-art baselines, including both heuristics and TGNNS. Sections 4.1 and 4.2 present
376 our key empirical findings, and Appendix F provides additional ablation studies. Our experiments
377 aim to address the following research questions: **(RQ1)** How does NAV1S perform compared to prior

378 Table 1: NDCG@10 on TGB datasets (\uparrow higher is better). NAVIS is benchmarked against TGNNs
 379 that use all available graph messages. Boldface marks the best method.
 380

381 Method	382 tgbn-trade		383 tgbn-genre		384 tgbn-reddit		385 tgbn-token	
	386 Val.	387 Test	388 Val.	389 Test	390 Val.	391 Test	392 Val.	393 Test
Moving Avg	0.793	0.777	0.496	0.497	0.498	0.480	0.401	0.414
Historical Avg	0.793	0.777	0.478	0.472	0.499	0.481	0.402	0.415
JODIE	0.394 \pm 0.05	0.374 \pm 0.09	0.358 \pm 0.03	0.350 \pm 0.04	0.345 \pm 0.02	0.314 \pm 0.01	–	–
TGAT	0.395 \pm 0.14	0.375 \pm 0.07	0.360 \pm 0.04	0.352 \pm 0.03	0.345 \pm 0.01	0.314 \pm 0.01	–	–
CAWN	0.393 \pm 0.07	0.374 \pm 0.09	–	–	–	–	–	–
TCL	0.394 \pm 0.11	0.375 \pm 0.09	0.362 \pm 0.04	0.354 \pm 0.02	0.347 \pm 0.01	0.314 \pm 0.01	–	–
GraphMixer	0.394 \pm 0.17	0.375 \pm 0.11	0.361 \pm 0.04	0.352 \pm 0.03	0.347 \pm 0.01	0.314 \pm 0.01	–	–
DyGFormer	0.408 \pm 0.58	0.388 \pm 0.64	0.371 \pm 0.06	0.365 \pm 0.20	0.348 \pm 0.02	0.316 \pm 0.01	–	–
DyGMamba	0.393 \pm 0.001	0.374 \pm 0.001	0.359 \pm 0.001	0.351 \pm 0.001	0.347 \pm 0.000	0.314 \pm 0.000	–	–
DyRep	0.394 \pm 0.001	0.374 \pm 0.001	0.357 \pm 0.001	0.351 \pm 0.001	0.344 \pm 0.001	0.312 \pm 0.001	0.151 \pm 0.006	0.141 \pm 0.006
TGN	0.445 \pm 0.009	0.409 \pm 0.005	0.443 \pm 0.002	0.423 \pm 0.007	0.482 \pm 0.007	0.408 \pm 0.006	0.251 \pm 0.000	0.200 \pm 0.005
TGNv2	0.807 \pm 0.006	0.735 \pm 0.006	0.481 \pm 0.001	0.469 \pm 0.002	0.544 \pm 0.000	0.507 \pm 0.002	0.321 \pm 0.001	0.294 \pm 0.001
NAVIS (ours)	0.872\pm0.001	0.863\pm0.001	0.512\pm0.001	0.520\pm0.001	0.564\pm0.001	0.552\pm0.001	0.423\pm0.001	0.444\pm0.001

393 Table 2: NDCG@10 on TGB datasets using only previous ground-truth labels (\uparrow higher is better).
 394 This setting is suited for heuristics. Boldface marks the best method. Baselines have no standard
 395 deviation because they are pre-defined and deterministic.

397 Method	398 tgbn-trade		399 tgbn-genre		400 tgbn-reddit		401 tgbn-token	
	402 Val.	403 Test	404 Val.	405 Test	406 Val.	407 Test	408 Val.	409 Test
Persistent Forecast	0.860	0.855	0.350	0.357	0.380	0.369	0.403	0.430
Moving Avg	0.841	0.823	0.499	0.509	0.574	0.559	0.491	0.508
NAVIS (ours)	0.872\pm0.001	0.863\pm0.001	0.517\pm0.001	0.528\pm0.001	0.584\pm0.001	0.569\pm0.001	0.493\pm0.001	0.513\pm0.001

402 art for future node affinity prediction? **(RQ2)** Does our method generalize across various types of
 403 graphs? **(RQ3)** What is the contribution of each component in NAVIS?

404 **Experimental setup** We compare NAVIS to the following TGNN baselines JODIE(Kumar et al.,
 405 2019), TGAT(Xu et al., 2020), CAWN (Wang et al., 2021b), TCL (Wang et al., 2021a), Graph-
 406 Mixer(Cong et al., 2023), DyGFormer(Yu et al., 2023), DyRep(Trivedi et al., 2019), TGN(Rossi
 407 et al., 2020), TGNv2(Tjandra et al., 2024) and the standard heuristics presented in (Huang et al.,
 408 2023) and in (Tjandra et al., 2024). Following the standard protocols (Huang et al., 2023), we use
 409 a 70%-15%-15% chronological split, train for 50 epochs, use a batch size of 200, and report the
 410 average NDCG@10 (Järvelin & Kekäläinen, 2002) over three runs. We include both future node
 411 affinity prediction settings: (1) using the full fine-grained CTDG up to the prediction time, and (2)
 412 using only previous ground-truth affinity vectors. Previous TGNNs only support the first setting
 413 (Tjandra et al., 2024), and, therefore, only heuristics are included in the second setting comparisons.

415 4.1 NODE AFFINITY PREDICTION ON TGB

416 To answer **(RQ1)**, we use the TGB datasets for node affinity prediction (Huang et al., 2023) :
 417 tgbn-trade, tgbn-genre, tgbn-reddit, and tgbn-token. As shown in Tables 1 and
 418 2, **NAVIS outperforms all baselines in both experimental settings**. It improves over the best-
 419 performing TGNN, TGNv2, by +12.8% on tgbn-trade. Notably, many TGNNs underperform
 420 simple heuristics, which aligns with our theoretical analysis that they are not optimized for ranking
 421 and underutilize available temporal information. In contrast, NAVIS linear design and rank-aware
 422 loss enable superior performance.

424 4.2 GENERALIZATION TO LINK PREDICTION DATASETS

425 To answer **(RQ2)**, we repurpose four temporal link prediction datasets (Wikipedia(Kumar et al.,
 426 2019), Flights(Strohmeier et al., 2021), USLegis(Fowler, 2006), and UNVote(Voeten et al., 2009))
 427 for the future node affinity prediction task. We detail how we adjust these datasets in Appendix B.
 428 TGNNs are known to operate well on these datasets (Yu et al., 2023), and, therefore, should constitute
 429 a strong baseline. As shown in Tables 3 and 4, **NAVIS consistently outperforms both TGNN and**
 430 **heuristic baselines**, with gains ranging from +13.9% to +20% over the second performing TGNN,
 431 suggesting NAVIS generalizes well to many dynamic graph datasets. Similar to the results in Tables 1

432 Table 3: NDCG@10 on converted link prediction datasets (\uparrow higher is better). NAVIS is benchmarked
 433 against TGNNs that use all available graph messages. Boldface marks the best method.
 434

Method	Wikipedia		Flights		USLegis		UNVote	
	Val.	Test	Val.	Test	Val.	Test	Val.	Test
Historical Avg	0.547	0.555	0.487	0.499	0.274	0.287	0.926	0.917
Moving Avg	0.547	0.555	0.029	0.028	0.150	0.154	0.926	0.918
DyRep	0.019 \pm 0.022	0.023 \pm 0.026	0.000 \pm 0.000	0.000 \pm 0.000	0.231 \pm 0.031	0.123 \pm 0.061	0.800 \pm 0.002	0.804 \pm 0.002
DyFormer	0.058 \pm 0.002	0.058 \pm 0.002	–	–	0.271 \pm 0.036	0.220 \pm 0.057	0.817 \pm 0.007	0.809 \pm 0.005
DyGMamba	0.046 \pm 0.003	0.050 \pm 0.002	–	–	0.246 \pm 0.015	0.154 \pm 0.044	0.814 \pm 0.002	0.804 \pm 0.002
TGN	0.056 \pm 0.005	0.065 \pm 0.006	0.249 \pm 0.003	0.227 \pm 0.007	0.219 \pm 0.022	0.190 \pm 0.024	0.807 \pm 0.003	0.792 \pm 0.006
TGNv2	0.478 \pm 0.005	0.433 \pm 0.004	0.326 \pm 0.008	0.299 \pm 0.014	0.323 \pm 0.036	0.253 \pm 0.040	0.824 \pm 0.008	0.813 \pm 0.010
NAVIS (ours)	0.564\pm0.001	0.573\pm0.001	0.489\pm0.001	0.499\pm0.001	0.331\pm0.001	0.347\pm0.001	0.969\pm0.001	0.952\pm0.001

435
 436 Table 4: NDCG@10 on converted link prediction datasets using only previous ground-truth labels (\uparrow
 437 higher is better). Baselines have no standard deviation because they are pre-defined and deterministic.
 438 Boldface marks the best method.

Method	Wikipedia		Flights		USLegis		UNVote	
	Val.	Test	Val.	Test	Val.	Test	Val.	Test
Persistent Forecast	0.499	0.507	0.296	0.307	0.328	0.320	0.963	0.917
Moving Avg	0.538	0.552	0.468	0.482	0.250	0.276	0.963	0.953
NAVIS (ours)	0.559\pm0.001	0.566\pm0.001	0.482\pm0.001	0.494\pm0.001	0.333\pm0.001	0.326\pm0.001	0.971\pm0.001	0.953\pm0.001

451 and 2, other TGNNs fall behind simple heuristics even though they were shown to produce great
 452 results on these datasets for future link prediction tasks, [strengthening that the TGNNs design choices](#)
 453 mentioned earlier in Sections 3.1 and 3.3 are incompatible for future node affinity prediction.

454 4.3 ABLATION STUDY

455 To answer **(RQ3)**, we conduct an ablation study to isolate the contribution of each core component
 456 of NAVIS: (1) the linear state update mechanism (vs. the commonly used GRU), (2) the inclusion
 457 of the global virtual vector g , and (3) the proposed loss (vs. cross-entropy, CE). Full results are
 458 provided in Appendix F. The ablations show that each component contributes substantially to the
 459 overall performance.

460 5 RELATED WORK

461 **Expressivity in Temporal Graphs.** The dominant view of expressivity in graph learning is measured
 462 by the Weisfeiler-Lehman (WL) test’s ability to distinguish non-isomorphic graphs (Gilmer et al.,
 463 2017; Xu et al., 2019). This concept extends to temporal graphs, with temporal-WL (Souza et al.,
 464 2022) for CTDG and supra Laplacian WL (Galron et al., 2025) for DTDG, where models are
 465 evaluated on their capacity to differentiate evolving graph structures (Kazemi et al., 2020; ENNADIR
 466 et al., 2025). We diverge from this perspective by focusing on **functional expressivity**: a model’s
 467 ability to represent specific mathematical operations. While prior models aim to capture complex
 468 graph topology, they often cannot represent a simple Moving Average, a critical function for affinity
 469 prediction.

470 **Heuristics and State Space Models.** According to the recent literature, simple heuristics, like
 471 Moving Average, often outperform complex TGNNs on various relevant benchmarks (Huang et al.,
 472 2023; Cornell et al., 2025), suggesting the problem is fundamentally sequential. Recent work formally
 473 establishes the equivalence between linear SSMs and Moving Average (Eliasof et al., 2025) and
 474 explores their use in language modeling (Gu et al., 2022; Gu & Dao, 2024) and dynamic link
 475 prediction (Li et al., 2024; Ding et al., 2025). While these works connect SSMs to temporal data, our
 476 approach is distinct. We are the first to explicitly leverage the formal equivalence between heuristics
 477 and SSMs to design an architecture, NAVIS, that is purpose-built for node affinity prediction.

478 **Temporal link prediction on weighted dynamic graphs.** Temporal link prediction (TLP) on
 479 weighted dynamic graphs (Qin et al., 2023; Yang et al., 2019; Lei et al., 2019) is the task in which,
 480 given a discrete-time weighted dynamic graph (a sequence of snapshots of the graph at specific

486 points in time), the model is required to predict the next snapshot, i.e., the weighted adjacency
 487 matrix at a future time. Although this task resembles node affinity prediction, there are several
 488 key differences. First, the downstream objectives differ, and accordingly different metrics are used
 489 to measure performance in each setting. In TLP on weighted dynamic graphs, the end goal is to
 490 construct the entire future weighted adjacency matrix. Hence, metrics such as RMSE, MSE, and
 491 MAE are often used. In node affinity prediction, however, the goal is to rank different nodes with
 492 respect to a specific node by their affinity to it, and therefore ranking metrics such as NDCG or MRR
 493 are used. Moreover, TLP on weighted dynamic graphs operates in a discrete-time setting, while node
 494 affinity prediction operates in a continuous-time setting. Consequently, solutions for the former (Qin
 495 et al., 2023; Yang et al., 2019; Lei et al., 2019) operate on the full graph at each change, which may
 496 lead to unreasonable runtime if applied in the latter setting. In addition, upon each query, TLP on
 497 weighted dynamic graphs requires computing the full weighted adjacency matrix, while for node
 498 affinity prediction only the affinity scores between the queried node and the other nodes need to be
 499 computed. These factors are likely to hinder methods for TLP on weighted dynamic graphs from
 500 transferring well to node affinity prediction, and vice versa.

501 6 CONCLUSION

502 In this work, we identified critical gaps in the design of TGNNS for future node affinity prediction.
 503 These gaps, including an inability to express simple heuristics such as moving averages and Persistent
 504 Forecast, often lead these baselines to outperform TGNNS. We further showed that current under-
 505 performance also stems from the use of suboptimal, non-ranking losses such as cross-entropy. To
 506 address this, we introduced NAVIS, a novel architecture grounded in the ability of linear state-space
 507 models to generalize heuristic behavior. By incorporating a virtual state to capture global dynamics
 508 and a carefully designed rank-aware loss, NAVIS preserves the robustness of heuristics while offering
 509 greater expressive power. Extensive experiments demonstrate that NAVIS consistently outperforms
 510 state-of-the-art models and heuristics across multiple benchmarks, underscoring the importance of
 511 aligning a model’s inductive biases and training objectives with the specific demands of the task.

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 514
515 Limitations and future work Although NAVIS has shown strong performance, both theoretically,
 516 by generalizing common heuristics that excel in node affinity prediction, and empirically on TGB
 517 benchmarks, NAVIS remains elementary, and further research in this direction is required. For
 518 example, NAVIS currently utilizes a basic virtual global state based on recency selection from a
 519 global buffer to capture trends in dynamic graphs. Advanced aggregation schemes over the global
 520 buffer (e.g., attention-based mechanisms) and more sophisticated buffer-eviction strategies (e.g.,
 521 non-deterministic eviction) may improve the modeling of global state and further enhance TGNN
 522 performance on node affinity prediction. In addition, NAVIS computes node affinities as a linear
 523 combination of the input and current state, with coefficients in $[0, 1]$ that are adaptive to both.
 524 Consequently, NAVIS cannot, for example, represent certain non-linear functions. This limitation
 525 could be addressed by adding a third non-linear component with an associated coefficient such that
 526 all three coefficients sum to 1. We leave to future work the investigation of how to best combine
 527 linear and non-linear components to enable TGNNs to perform optimally on the node affinity task.

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702 **A DATASETS STATISTICS AND DESCRIPTION**

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704 In our empirical evaluation, we employed the following dynamic graph datasets, each capturing a dis-
 705 tinct dynamic system and providing varied graph structures, edge features, and temporal resolutions:
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- 707 • TGBN-TRADE (MacDonald et al., 2015): models global agricultural commerce among
 708 UN-affiliated countries over 1986–2016 as a time-evolving network: countries are the nodes
 709 and directed links record, for each calendar year, the aggregate value of agricultural goods
 710 moved from one country to another. Because entries are reported yearly, the temporal
 711 resolution is annual. The accompanying learning objective is to anticipate, for a chosen
 712 country, how its overall agricultural trade will be apportioned across partner countries in the
 713 subsequent year—that is, the next-year distribution of trade shares.
- 714 • TGBN-GENRE (Bertin-Mahieux et al., 2011) models listening behavior as a weighted bipartite
 715 graph linking users to musical genres. Nodes consist of users and genre labels; time-stamped
 716 edges indicate that a user listened to a track associated with that genre, with the edge weight
 717 reflecting the fraction of a track’s composition attributed to that genre. The downstream
 718 objective is a ranking problem: for each user, predict the genres they are most likely to
 719 engage with during the upcoming week.
- 720 • TGBN-REDDIT (Nadiri & Takes, 2022) models Reddit as a temporal bipartite graph linking
 721 users and subreddits. Nodes represent both entities, and a timestamped edge records a
 722 user’s post within a subreddit. The dataset covers activity from 2005 through 2019. The
 723 predictive objective is, for each user, to produce a next-week ranking of subreddits by
 724 expected engagement intensity.
- 725 • TGBN-TOKEN (Shamsi et al., 2022) models a bipartite interaction graph linking wallet users
 726 to cryptocurrency tokens. Nodes consist of users and tokens, and directed edges record
 727 transfers from a user to a particular token. Edge weights capture the logarithmic normalized
 728 transaction quantity. The predictive objective is to estimate, for the next week, how often
 729 each user will engage with different classes of tokens.
- 730 • WIKIPEDIA(Kumar et al., 2019): models a bipartite, time-stamped interaction graph derived
 731 from a single month of edit activity. Nodes correspond to editors and articles, and each
 732 interaction edge denotes an edit event. Every edge is annotated with its event time and a
 733 feature vector from the Linguistic Inquiry and Word Count framework (LIWC (Pennebaker
 734 et al., 2001)) summarizing the edit’s linguistic characteristics. The predictive objective for
 735 this converted version of the dataset is to predict the expected engagement intensity of each
 736 editor with existing articles.
- 737 • Flights (Strohmeier et al., 2021): models air traffic patterns during the COVID-19 period
 738 as a temporal network in which airports are nodes and connections represent observed
 739 routes between them. Each connection carries a timestamp and an associated intensity, with
 740 the edge weight recording how many flights operated on that route on a given day. The
 741 downstream objective is to rank future airport destinations intensity, given the source airport.
- 742 • USLegis (Fowler, 2006): models the collaboration dynamics of the U.S. Senate as a temporal,
 743 weighted network: senators are represented as nodes, ties are created whenever a pair co-
 744 sponsors the same bill, and each tie is stamped with the time of occurrence. Edge weights
 745 record the frequency of joint sponsorships within a legislative term, capturing how often
 746 two members work together. The predictive objective is to estimate the joint sponsorship
 747 frequencies for the next term.
- 748 • UNVote (Voeten et al., 2009): models the United Nations General Assembly roll-call record
 749 from 1946 through 2020 as a time-evolving graph: countries are nodes, and an edge appears
 750 between two countries whenever they cast matching affirmative (“yes”) votes on the same
 751 resolution. Each edge carries a timestamp and a weight, where the weight counts how many
 752 times that pair of countries voted “yes” together over the period. The predictive objective is
 753 to estimate the joint positive votes for the next period.
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- 755

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758 Table 5: Statistics of various datasets used in our experiments
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Dataset	Domain	#Nodes	#Edges	Bipartite	Duration
TGBN-TRADE	Economy	255	468,245	False	30 years
TGBN-GENRE	Interaction	1,505	17,858,395	True	4 years
TGBN-REDDIT	Social	11,766	27,174,118	True	15 years
TGBN-TOKEN	Cryptocurrency	61,756	72,936,998	True	2 years
WIKIPEDIA	Social	9,227	157,474	True	1 month
FLIGHTS	Transport	13,169	1,927,145	False	4 months
USLEGIS	Politics	225	60,396	False	12 terms
UNVOTE	Politics	201	1,035,742	False	72 years

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768
769 **B CONVERTING LINK PREDICTION DATASETS TO NODE AFFINITY PREDICTION
770 DATASETS**
771772
773 Each of the future link-prediction datasets we use (Wikipedia, Flights, USLegis, and UNVote)
774 comprises a CTDG with weighted interaction events between node pairs. For each dataset and node,
775 we define the ground-truth future affinity vector as the normalized sum of weighted interaction it
776 received over a specified period (day, week, year, etc.). Immediately before the start of a new period,
777 the task is to predict each node’s affinities for the upcoming period. Table 6 summarizes the periods
778 used for each dataset.
779780 Table 6: Chosen period for link property prediction datasets.
781

Dataset	Period
Wikipedia	Day
Flights	Day
USLegis	Legislative term
UNVote	Year

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783 Our code release includes a step-by-step guide on processing these datasets and integrating them into
784 the TGB framework to support future research on node-affinity prediction.
785786 **C SYNTHETIC EXPERIMENT**
787788
789 We construct a controlled continuous-time dynamic graph (CTDG) where each node $u \in \{1, \dots, N\}$
790 emits events according to a Poisson process (rate λ) over a horizon $[0, T]$. Node affinities at time t
791 are governed by a *shared global latent* $g(t)$ that switches between two damped-oscillatory regimes.
792 Concretely, on a grid with step Δt , $g(t)$ follows a piecewise AR(2) process with coefficients chosen
793 to approximate low- and high-frequency damped cosines; regime switches are exogenous and
794 uncorrelated with any single node’s local history. Each node is assigned a phase ϕ_u , a mixing
795 coefficient β_u , and a small bias γ_u . The instantaneous (pre-softmax) affinity logits are a nonlinear
796 readout of the global state $[g(t), g(t)^2]$,

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$$\ell_u(t) = \beta_u \cos(\phi_u) g(t) + \beta_u \sin(\phi_u) (g(t)^2 - 1) + \gamma_u + \epsilon_u(t),$$

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805 and, for each source node u , we mask self-transitions and apply a softmax over destinations to obtain
806 ground-truth affinity distributions. The forecasting task is one-step prediction of a node’s destination
807 distribution at query times, given only the *previous* ground-truth affinity vectors at inference. We
808 compare per-node, history-only baselines that cannot share information across nodes—*Persistent*
809 *Forecast* (PF; last observation), *Simple Moving Average* (SMA; window $w=5$), *Exponential Moving*
810 *Average* (EMA; $\alpha=0.2$), and a diagonal per-node *AR(1)*.

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Table 7: MRR on TGB datasets (\uparrow higher is better). NAVIS is benchmarked against TGNNs that use all available graph messages. Boldface marks the best method.

Method	tgbn-trade		tgbn-genre		tgbn-reddit		tgbn-token	
	Val.	Test	Val.	Test	Val.	Test	Val.	Test
TGNv2	0.60 \pm 0.01	0.53 \pm 0.01	0.43 \pm 0.01	0.40 \pm 0.01	0.43 \pm 0.01	0.40 \pm 0.01	0.28 \pm 0.01	0.26 \pm 0.01
NAVIS (ours)	0.78 \pm 0.00	0.77 \pm 0.001	0.40 \pm 0.00	0.42 \pm 0.00	0.45 \pm 0.00	0.44 \pm 0.001	0.39 \pm 0.00	0.41 \pm 0.00

Table 8: Recall on TGB datasets (\uparrow higher is better). NAVIS is benchmarked against TGNNs that use all available graph messages. Boldface marks the best method.

Method	tgbn-trade		tgbn-genre		tgbn-reddit		tgbn-token	
	Val.	Test	Val.	Test	Val.	Test	Val.	Test
TGNv2	0.60 \pm 0.01	0.56 \pm 0.01	0.32 \pm 0.01	0.31 \pm 0.01	0.19 \pm 0.01	0.18 \pm 0.01	0.05 \pm 0.00	0.05 \pm 0.00
NAVIS (ours)	0.75 \pm 0.00	0.72 \pm 0.00	0.34 \pm 0.00	0.33 \pm 0.00	0.21 \pm 0.00	0.20 \pm 0.00	0.30 \pm 0.00	0.28 \pm 0.00

D IMPLEMENTATION DETAILS

We initialized each linear layer in NAVIS with values drawn uniformly from the range $[-\sqrt{d}, \sqrt{d}]$, where d is the input dimension of the layer. No normalization of the output vector s were required as the ground true affinity scores in TGB are already normalized. The final loss we used is $\ell_{\text{lambda}} + \mathcal{C}\ell_{\text{Reg}}$, where \mathcal{C} is a regularization coefficient. In the experiments we set $\mathcal{C} = 1$. We adopted standard hyperparameters, consistent with prior work: batch size 200, Adam optimizer, and learning rate 10^{-4} . Following the TGB protocol, all models were trained for 50 epochs, and the checkpoint with the best performance on the validation set was selected. We set the regularization margin to $\Delta = 10^{-3}$. During training, NAVIS computes the loss only over the top-20 affinities, ensuring that loss computation requires constant time and memory. We used a single NVIDIA GeForce RTX 3090 GPU and a single AMD Ryzen 9 7900X 12-Core Processor CPU.

E ADDITIONAL RESULTS

Additional Evaluation Metrics We additionally compare NAVIS to the best performing baseline in each setting and report the results in terms of MRR (the average value of $\frac{1}{\text{RANK}^1}$, where RANK^1 is the predicted rank of the ground-truth top-scored node) and Recall@10 (the average number of top-10 ground-truth nodes ranked within the top-10 predicted scores). We report the results in Tables 7 to 10. From the results, we observe that the performance gap between the baselines increases under the MRR metric but decreases under the Recall@10 metric. This can be explained by our objective, which penalizes mismatches on top-ranked ground-truth nodes (e.g., ranks 1–3) more strongly than on lower-ranked ones (e.g., ranks 8–10).

Noisy data experiment To examine the robustness of NAVIS, we performed an additional experiment on noisy data. We used the tgbn-genre benchmark and, for each node that received its N -th update, we added an extra random update from that node to a random node and weighted the interaction with largest magnitude of affinity. We ran this experiment twice, once with $N = 10$ and a second time with $N = 20$, and did not observe a significant change in the performance of NAVIS on the dataset. This experiment provides empirical evidence that NAVIS is robust to noise such as short-term spikes.

What NAVIS learns? Table 2, the Persistent Forecast baseline performs better than Moving Average on tgbn-trade, while on tgbn-genre the opposite holds. This means that tgbn-genre requires more memory to accurately perform node affinity prediction while in tgbn-trade one needs to rely

864
 865 Table 9: MRR on TGB datasets using only previous ground-truth labels (\uparrow higher is better). This
 866 setting is suited for heuristics. Boldface marks the best method. Baselines have no standard deviation
 867 because they are pre-defined and deterministic.

Method	tgbn-trade		tgbn-genre		tgbn-reddit		tgbn-token	
	Val.	Test	Val.	Test	Val.	Test	Val.	Test
Moving Avg	0.77	0.78	0.31	0.32	0.41	0.40	0.44	0.47
NAVIS (ours)	0.77 ± 0.00	0.78 ± 0.00	0.41 ± 0.00	0.42 ± 0.00	0.47 ± 0.00	0.46 ± 0.00	0.46 ± 0.00	0.48 ± 0.00

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 873 Table 10: Recall@10 on TGB datasets using only previous ground-truth labels (\uparrow higher is better).
 874 This setting is suited for heuristics. Boldface marks the best method. Baselines have no standard deviation
 875 because they are pre-defined and deterministic.

Method	tgbn-trade		tgbn-genre		tgbn-reddit		tgbn-token	
	Val.	Test	Val.	Test	Val.	Test	Val.	Test
Moving Avg	0.74	0.73	0.29	0.30	0.25	0.24	0.30	0.29
NAVIS (ours)	0.74 ± 0.00	0.73 ± 0.00	0.34 ± 0.00	0.34 ± 0.00	0.25 ± 0.00	0.24 ± 0.00	0.30 ± 0.00	0.29 ± 0.00

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 882 more on the newly arrived data. To examine what NAVIS learns in these two regimes, we measured
 883 the average values of z_h and z_s from Equation (14), which control the relative influence of the
 884 memory component. We expect lower values on tgbn-trade to resemble persistent forecast and larger
 885 values on tgbn-genre to resemble moving average. The measured values are $z_h = 0.48$, $z_s = 0.49$
 886 for tgbn-trade and $z_h = 0.90$, $z_s = 0.84$ for tgbn-genre. This alignment between the learned z_h , z_s
 887 and the heuristic that performs better on each dataset provides empirical evidence that NAVIS learns
 888 the intended heuristic behavior in practice.

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 891 **Additional SSM comparison** We further compared NAVIS to an additional general SSM baseline,
 892 the S4 block (Gu et al., 2022), and report the results in Table 11. The comparison was performed
 893 under the same ground-truth label setting as in our main experiments. Both our method and the
 894 S4 block utilized the same learning rate, batch size, and number of training epochs. The results
 895 in Table 11 show that although the S4 block achieves relatively strong performance on the tested
 896 benchmark, NAVIS still outperforms it, indicating that there remains substantial room for improving
 897 general SSM architectures on this benchmark.

F ABLATION OF DESIGN CHOICES

901 Table 12 shows the ablation study results on the four TGB test sets. The results confirm that each
 902 component provides a significant impact on NAVIS performance. The full NAVIS with our suggested
 903 linear state updating-mechanism, global vector, and ranking loss establishes the strongest performance.
 904 Replacing the ranking loss with cross-entropy causes a notable drop in performance, validating our
 905 theoretical motivation. Switching to the GRU mechanism and removing the global signal further
 906 degrades performance, highlighting the importance of each component in the final design.

907 We further ablate on the regularization term of our loss, and performed an additional experiment on
 908 the TGB datasets where the regularization term is not included in the loss computation. We report the
 909 results in Table 13.

910 Applying our proposed regularization not only significantly improves the performance of NAVIS,
 911 but also accelerates convergence, with convergence typically reached in roughly 30 epochs.

912 We also performed an empirical analysis on the effect of the hyperparameters of delta and batch size
 913 on the perfance of NAVIS, and report the results inTable 14. From Table 14 we can see that on the
 914 tested datasets and batch size values has no effect on the performance of NAVIS. In addion, choosing
 915 too small or too large values for delta can slightly hurt the performance of NAVIS.

916 Additionally, we performed an analysis to examine the effect of the global buffer size and its
 917 aggregation scheme on the performance of NAVIS, compared to the recency selection aggregation

918 Table 11: NDCG@10 on TGB datasets using only previous ground-truth labels (\uparrow higher is better).
 919 Boldface marks the best method.
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921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971			
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Method		tgbn-trade	
Val.		Test	
S4		0.819 \pm 0.005	
NAViS (ours)		0.872\pm0.001	
0.796 \pm 0.002		0.451 \pm 0.001	
0.863\pm0.001		0.517\pm0.001	
0.461 \pm 0.001		0.528\pm0.001	

928 Table 12: Ablation study of NAViS components on TGB test sets. (✓) denotes inclusion and (✗) denotes exclusion.
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931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971	931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971	931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971	931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971
State Update	Global vector	Loss	tgbn-trade
Linear	✓	Rank	0.863\pm0.001
GRU	✗	Rank	0.850 \pm 0.001
Linear	✗	Rank	0.857 \pm 0.001
Linear	✓	CE	0.859 \pm 0.001
			0.461 \pm 0.001
			0.530 \pm 0.001
			0.508 \pm 0.001

939 scheme used in previous experiments. We report the results in Table 15. The results in Table 15
 940 show that changing the aggregation scheme or the size of the buffer has little to no effect on the
 941 existing implementation of NAViS. The slight drop in performance when the buffer size is 8 and
 942 the aggregation scheme is mean may be explained by the reliance on relatively old values in the
 943 buffer. We leave the exploration of new and more sophisticated virtual global state mechanisms (e.g.,
 944 attention-based aggregation) for future work.

G MEMORY AND RUNTIME ANALYSIS

949 We conducted a runtime and memory analysis to examine the efficiency of NAViS. In Table 16,
 950 we report the number of parameters of the TGN baselines and NAViS. NAViS not only requires
 951 the fewest parameters among the compared methods, but also scales well to large graphs due to the
 952 sparsification pipeline detailed in Section 3.2.

953 In Table 17, we report a runtime comparison for a single training epoch and inference between
 954 NAViS, heuristics, and TGN baselines. NAViS is more efficient than the TGN baselines and has
 955 a runtime comparable to the heuristics. Since all methods use the same batch size, their throughput
 956 is proportional to their runtime. Since the heuristics do not contain learnable parameters they only
 957 require a single pass over the training data to compute node states before entering the validation and
 958 test phases. Other methods require multiple iterations over the training data (epochs) like standard
 959 deep learning models.

H THEORETICAL REMARKS

963 In Theorem 2 we showed that standard memory cells (RNN, LSTM, and GRU) cannot express the
 964 simple Persistent Forecast heuristic. The proof assumes that the input to the cells may be unbounded,
 965 and since tanh and other nonlinear activation functions have bounded images, this prevents the
 966 memory cells from expressing this heuristic. A natural question is what happens when the input x
 967 is bounded, e.g., in $[0, 1]$ or $[-1, 1]$. Recall that persistent forecast is defined by $h_i = x_i$, while an
 968 RNN cell is defined by $h_i = \tanh(W_h h_{i-1} + W_x x_i + b)$. Denote $f(x_i) = W_h h_{i-1} + W_x x_i + b$. To
 969 obtain $h_i = x_i$, we need $\tanh(f(x_i)) = x_i$, i.e., $\tanh = f^{-1}$ on the relevant range by composition
 970 of functions. However, f^{-1} must be affine, as the inverse of an affine function, which contradicts the
 971 fact that tanh is not affine. The same argument applies to \tilde{h} in the GRU cell from Equation (13). This
 972 also holds for any non-affine activation on $[-1, 1]$, such as ReLU or leaky ReLU. On $[0, 1]$, ReLU

972 Table 13: Results of NAVIS in terms of NDCG@10 on TGB datasets, with and without the suggested
 973 regularization term. (✓) denotes inclusion and (✗) denotes exclusion.

975 976 977 978 979 980 981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020 1021 1022 1023 1024 1025	tgbn-genre		tgbn-reddit		tgbn-token	
	Val.	Test	Val.	Test	Val.	Test
✗	0.510±0.001	0.520±0.001	0.570±0.001	0.553±0.001	0.487±0.001	0.510±0.001
✓	0.517±0.001	0.528±0.001	0.584±0.001	0.569±0.001	0.493±0.001	0.513±0.001

Table 14: Analysis of different batch sized and deltas, results are measured in NDCG@10 on test sets of datasets from TGB, averaged for 3 runs.

983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020 1021 1022 1023 1024 1025	Hyperparameter	tgbn-trade	tgbn-genre
delta=0.1	0.863±0.001	0.526±0.001	
delta=0.01	0.863±0.001	0.528±0.001	
delta=0.001	0.863±0.001	0.527±0.001	
batch size = 100	0.863±0.001	0.528±0.001	
batch size = 200	0.863±0.001	0.528±0.001	
batch size = 400	0.863±0.001	0.528±0.001	

Table 15: Ablation study of NAVIS components on TGB test sets. (✓) denotes inclusion and (✗) denotes exclusion.

994 995 996 997 998 999 1000 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020 1021 1022 1023 1024 1025	Buffer size	Aggregation scheme	tgbn-trade	tgbn-genre
1	Recent Selection		0.863±0.001	0.528±0.001
4	MEAN		0.863±0.001	0.528±0.001
4	Time Decay		0.863±0.001	0.528±0.001
8	MEAN		0.863±0.001	0.527±0.001
8	Time Decay		0.863±0.001	0.528±0.001

is equal to the identity, and hence applying ReLU to inputs normalized to this range allows these memory cells to learn the Persistent Forecast heuristic.

Tjandra et al. (2024) state that *There exists a formulation of TGNv2 that can represent persistent forecasting and moving average of order k*. To prove this claim Tjandra et al. (2024) utilize permutation matrices, block matrix and a dedicated generator vector for the memory module of TGNv2. However, in practice TGNv2 is officially implemented with a GRU cell as the memory module. We showed in Theorem 2 that GRU cell cannot express the simple Persistent Forecast heuristic.

In Theorem 3 we proved that the cross-entropy loss is suboptimal by showing that there exist two rankings such that one ranks the elements identically to the ground truth while the other does not, yet under cross-entropy the latter achieves a smaller loss, contrary to what is desired in the task of node affinity prediction. We then argued that there exist infinitely many such examples since the cross-entropy loss is a continuous function. Consider the cross-entropy loss evaluated on the ground-truth order vector and a correctly ordered score vector as a function of the first entry of the correctly ordered vector (all other entries are fixed). Denote this function by $f(x)$. Let L_{inc} be the cross-entropy loss of a fixed incorrectly ordered vector, and define $\Delta := f(x) - L_{\text{inc}} > 0$. By continuity of f , there exists $\delta > 0$ such that for any x' satisfying $|x' - x| < \delta$ we have $|f(x') - f(x)| < \Delta/2$, which implies $f(x') > L_{\text{inc}}$. Since there are infinitely many such x' , we obtain infinitely many correctly ordered score vectors that incur a larger cross-entropy loss than the incorrectly ordered vector.

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1034 Table 16: Number of parameters of TGNN baselines and NAVIS, compared to the total number of
1035 nodes in each TGB benchmark for node affinity prediction.

Method	tgbn-trade	tgbn-genre	tgbn-reddit	tgbn-token
#Nodes	255	1505	11766	61756
DyGMamba	255125	257963	259998	-
DyGFormer	1027877	1030715	1032750	-
TGN	207655	233713	252398	283401
TGNv2	6433023	6565377	6660282	6817769
NAVIS	1280	2570	3495	5010

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1061 Table 17: Training and inference runtimes of the TGNN baselines, heuristics and NAVIS on the
1062 tgbn-genre dataset.

Method	Training runtime (sec)	Inference runtime (sec)
Persistent Forecast / Moving Avg	34	6
DyGMamba	2080	381
DyGFormer	1440	304
TGN	170	33
TGNv2	130	36
NAVIS	46	8

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